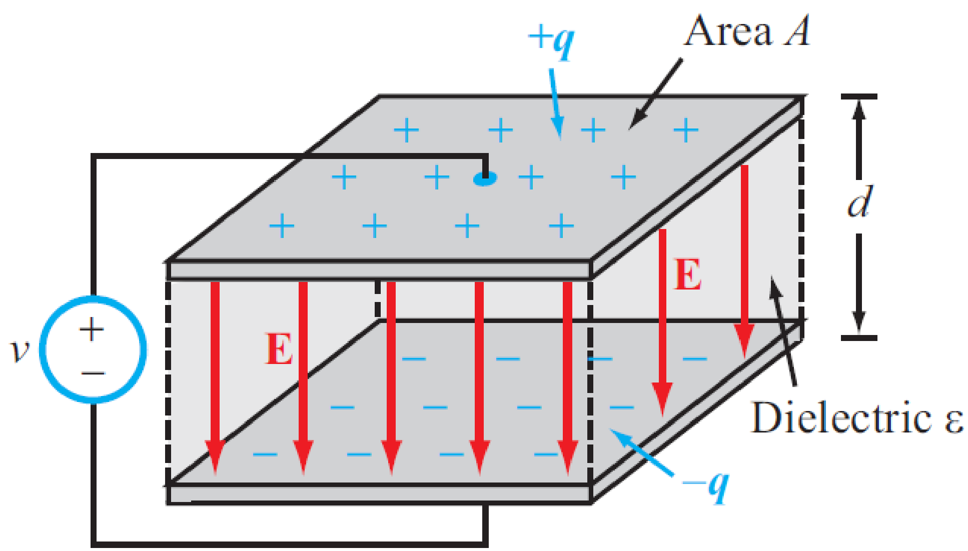
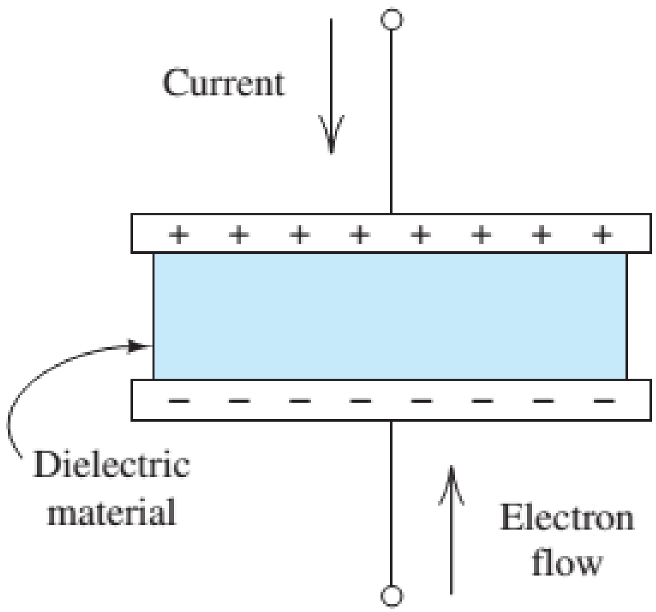
Lecture 5
- Capacitors and Inductors

Capacitors

Storage element that stores energy in electric field



Parallel plate capacitor

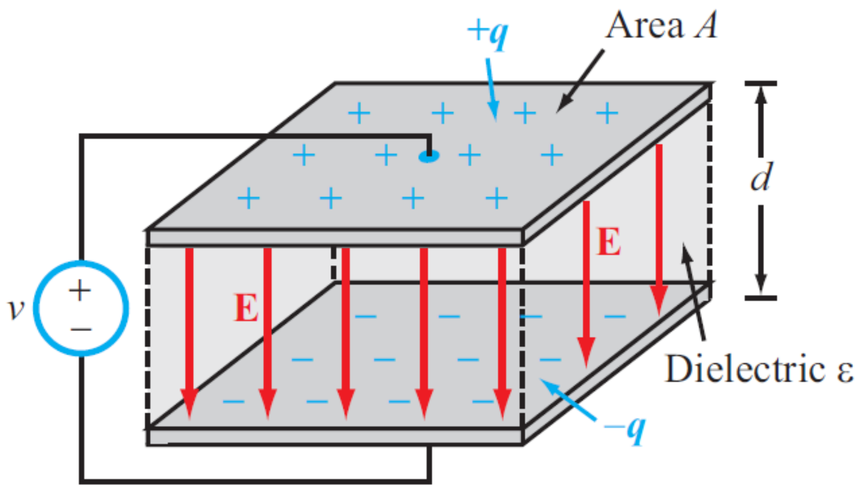


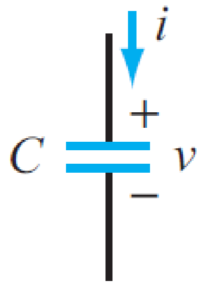
(a) As current flows through a capacitor, charges of opposite signs collect on the respective plates

Does DC voltage generate current flow through a capacitor?

Does AC voltage generate current flow through a capacitor?

V-I Relationship of Capacitors

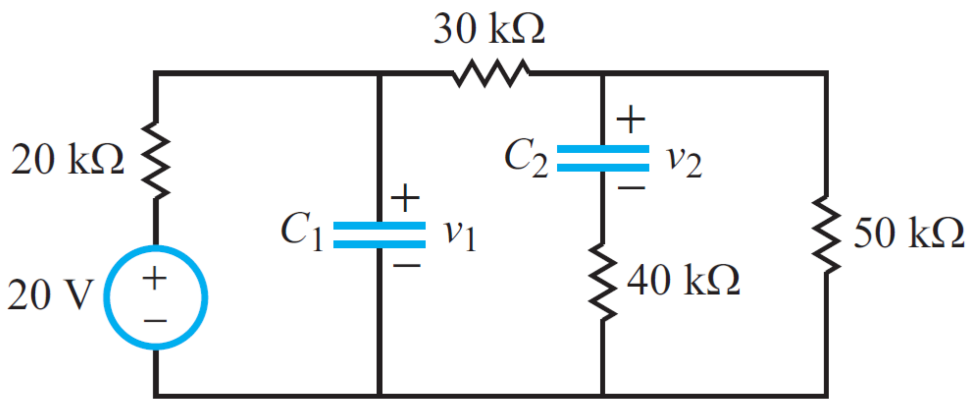




$$i = C\frac{dv}{dt}$$

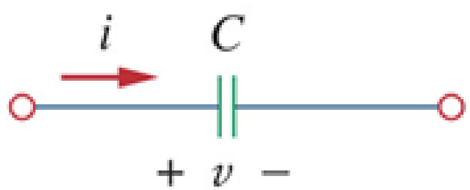
[Source: Berkeley]

Example



[Source: Berkeley]

Stored Energy

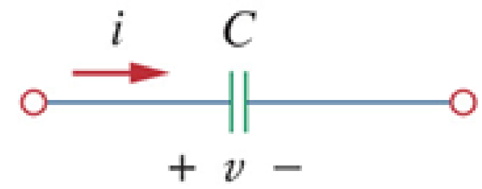


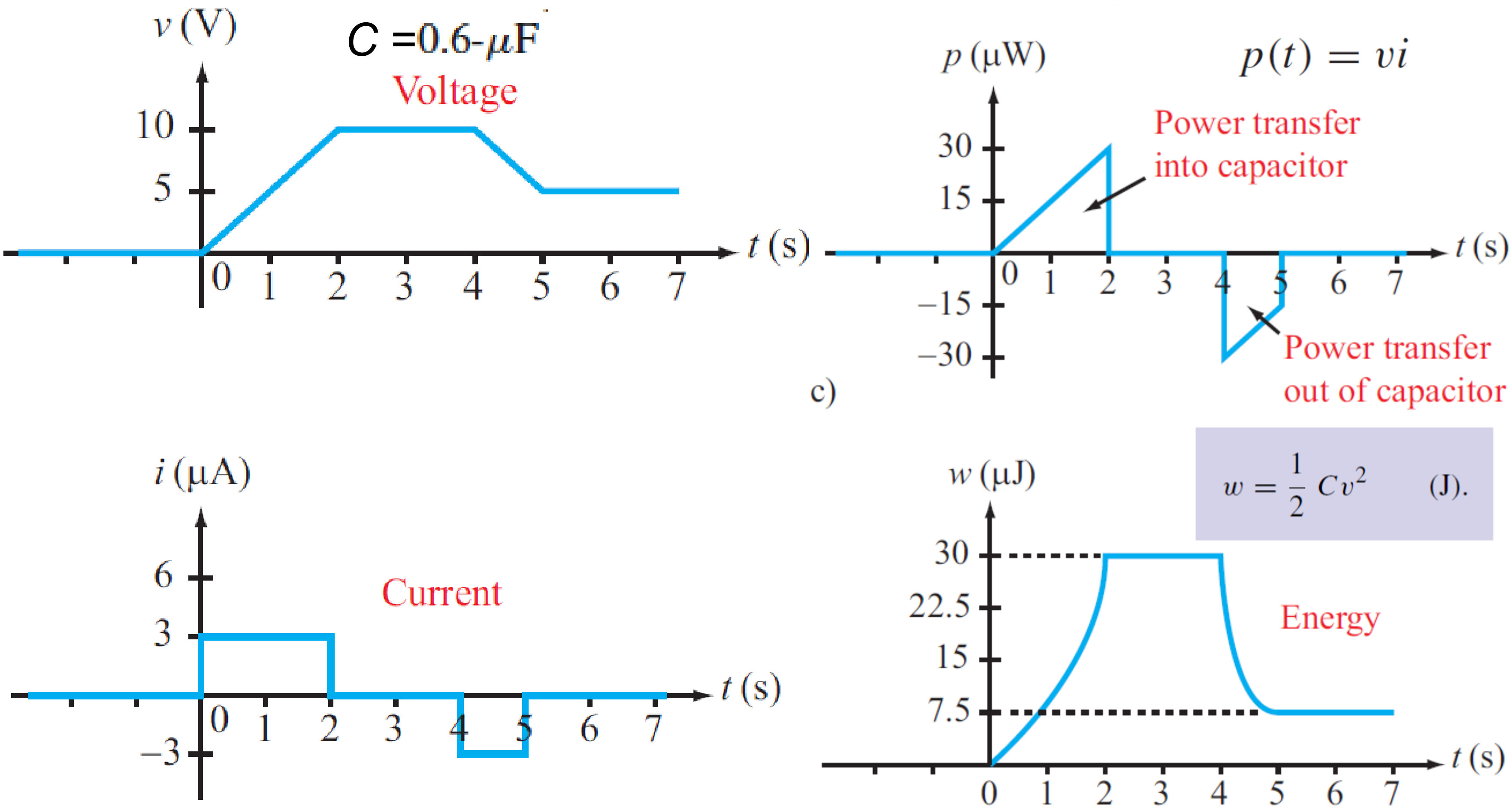
$$i = C\frac{dv}{dt}$$

- The instantaneous power delivered to the capacitor is

- The energy stored in a capacitor is:

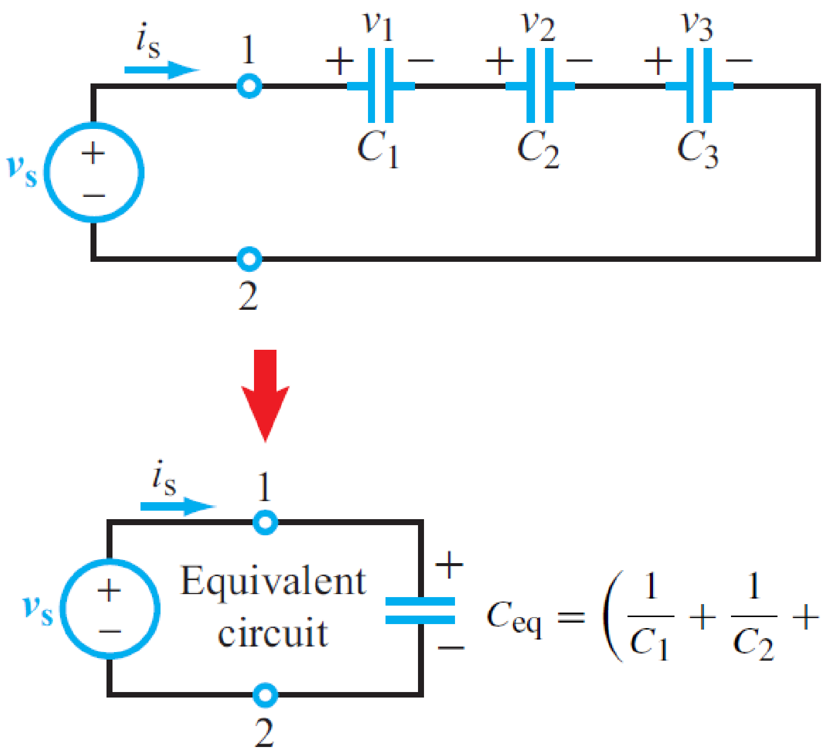
Capacitor Response





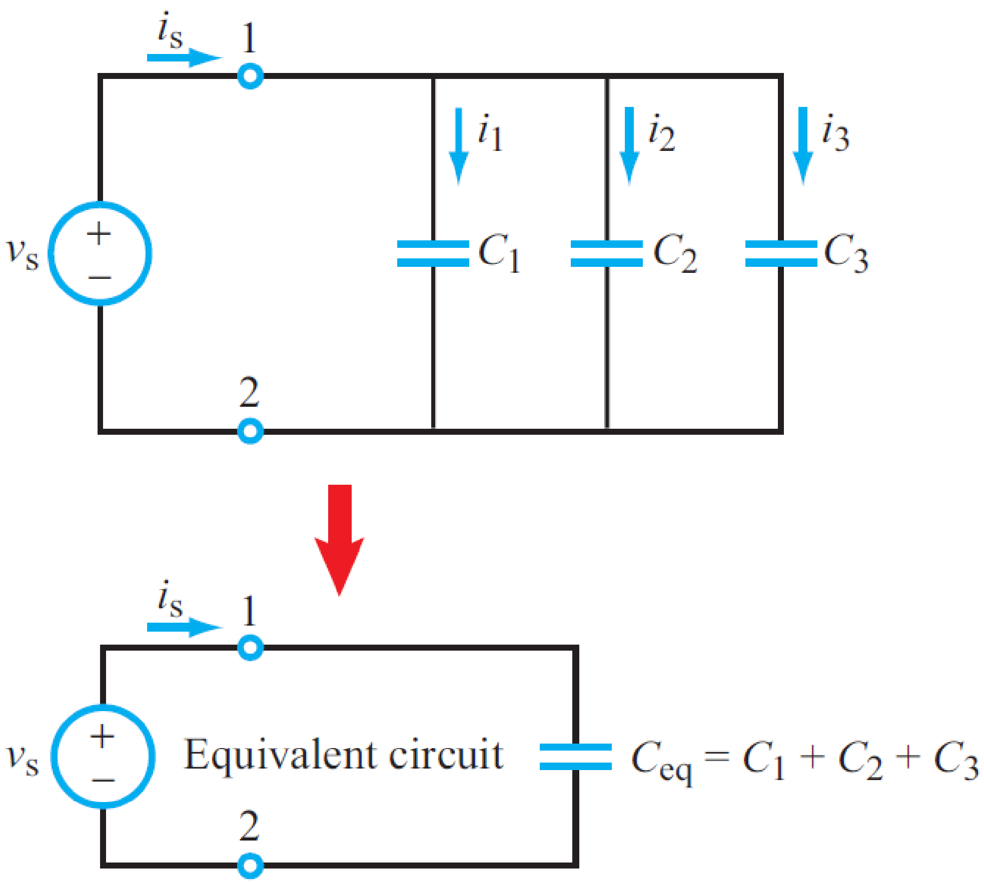
Capacitors in Series

Combining In-Series Capacitors

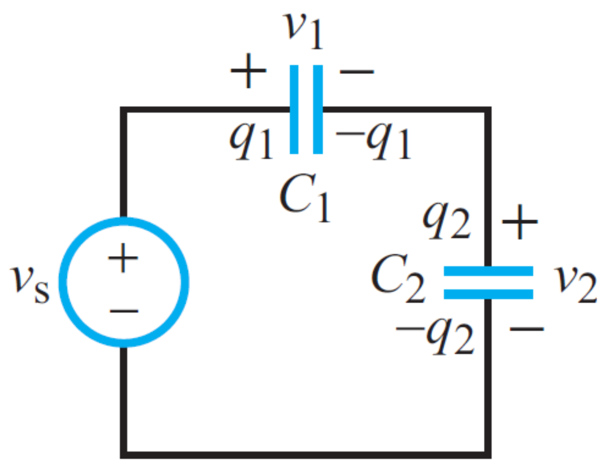


$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$

Capacitors in Parallel



Voltage Division

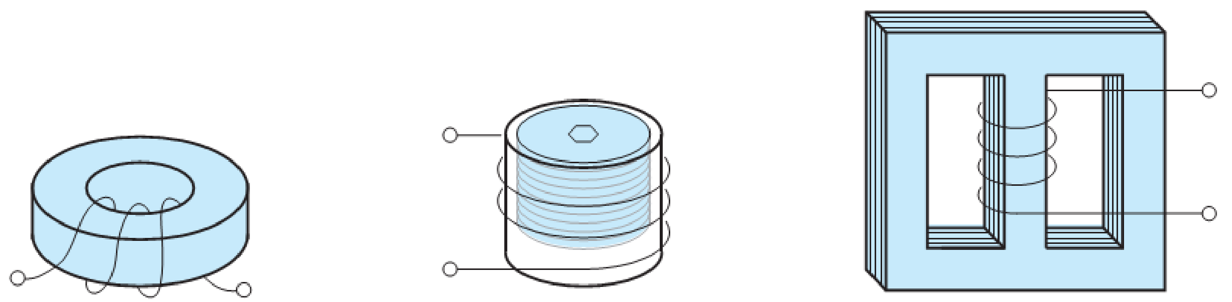


$$v_1 = \left(\frac{C_2}{C_1 + C_2}\right) v_s$$

$$v_2 = \left(\frac{C_1}{C_1 + C_2}\right) v_s$$

Inductors

- A storage element that stores energy in magnetic field.
  - They have applications in power supplies, transformers, radios, TVs, radars, and electric motors.
- Any conductor has inductance, but the effect is typically enhanced by coiling the wire up.



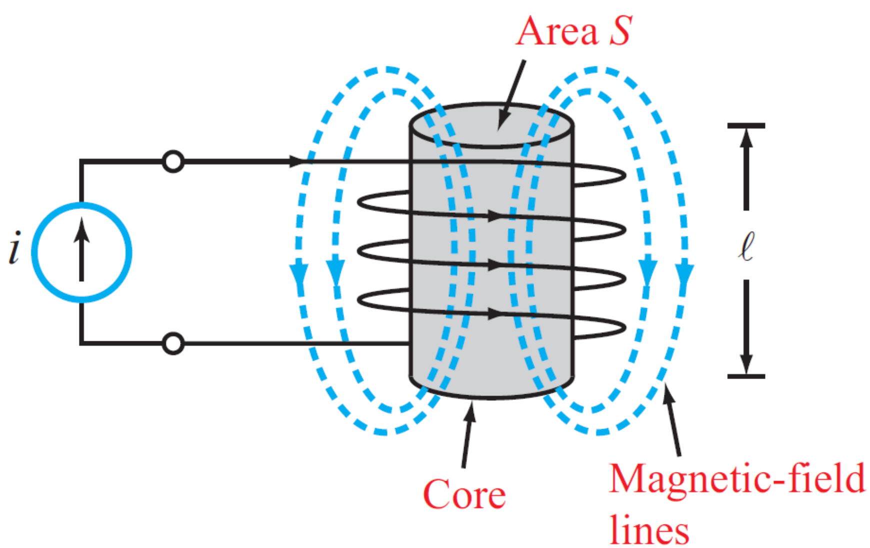
(a) Toroidal inductor

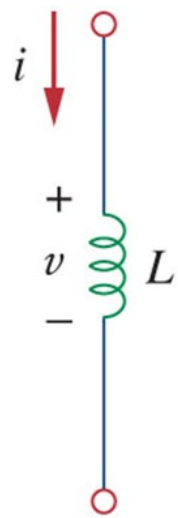
(b) Coil with an iron-oxide slug that can be screwed in or out to adjust the inductance

(c) Inductor with a laminated iron core

$$L = \frac{N^2 \mu S}{l}$$

V-I Relationship of Inductors

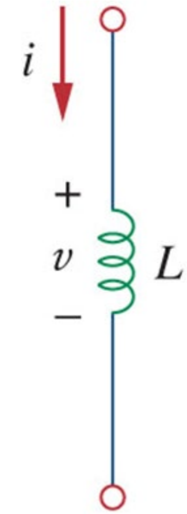




$$v = L\frac{di}{dt}$$

Energy Stored in an Inductor

- The power delivered to the inductor is:

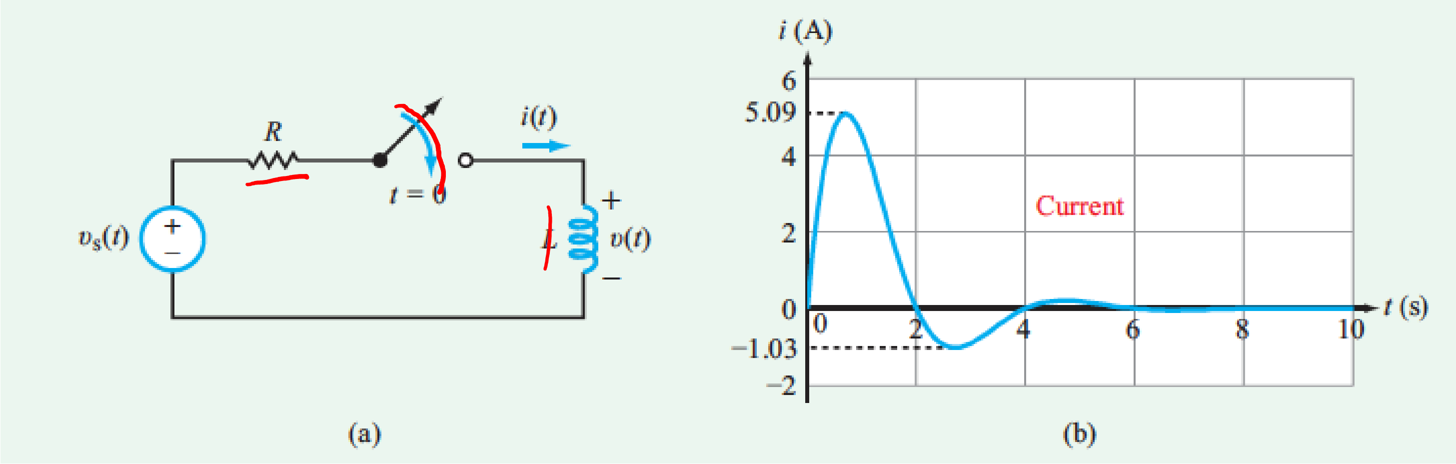


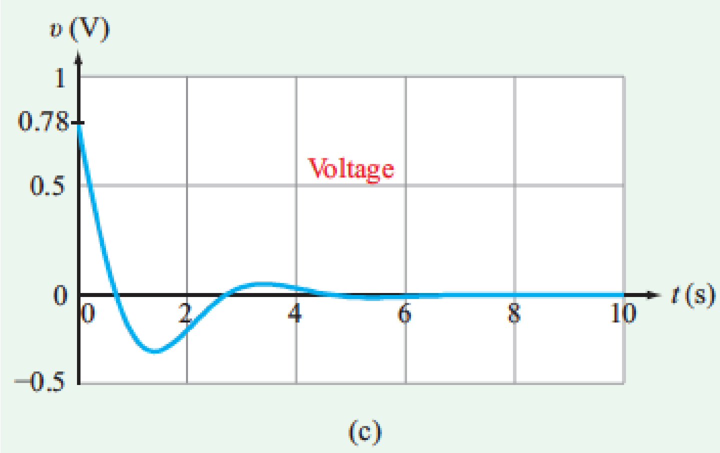
$$v = L\frac{di}{dt}$$

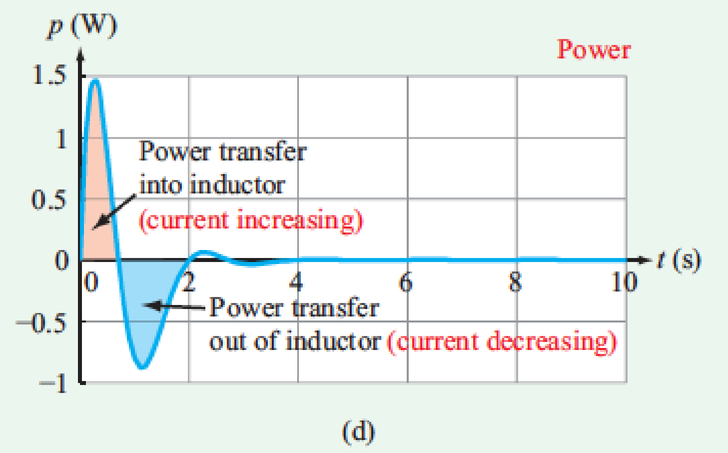
- The energy stored is:

Inductor Response

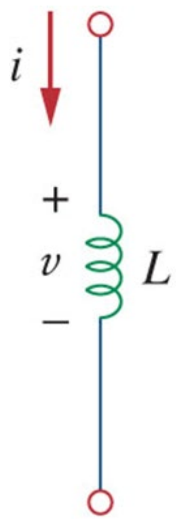
$$i(t) = 10e^{-0.8t}\sin(\pi t/2)\text{ A}, \qquad (\text{for } t \geq 0)$$



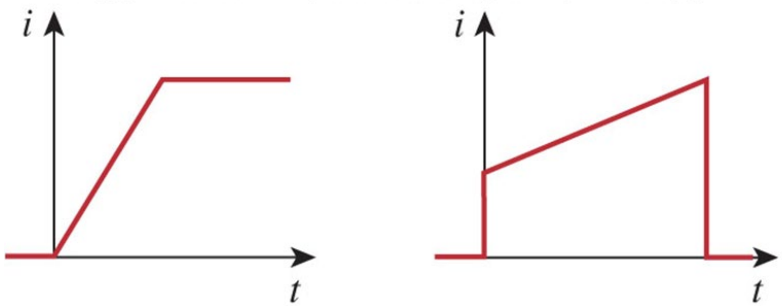




Important Property of Inductors

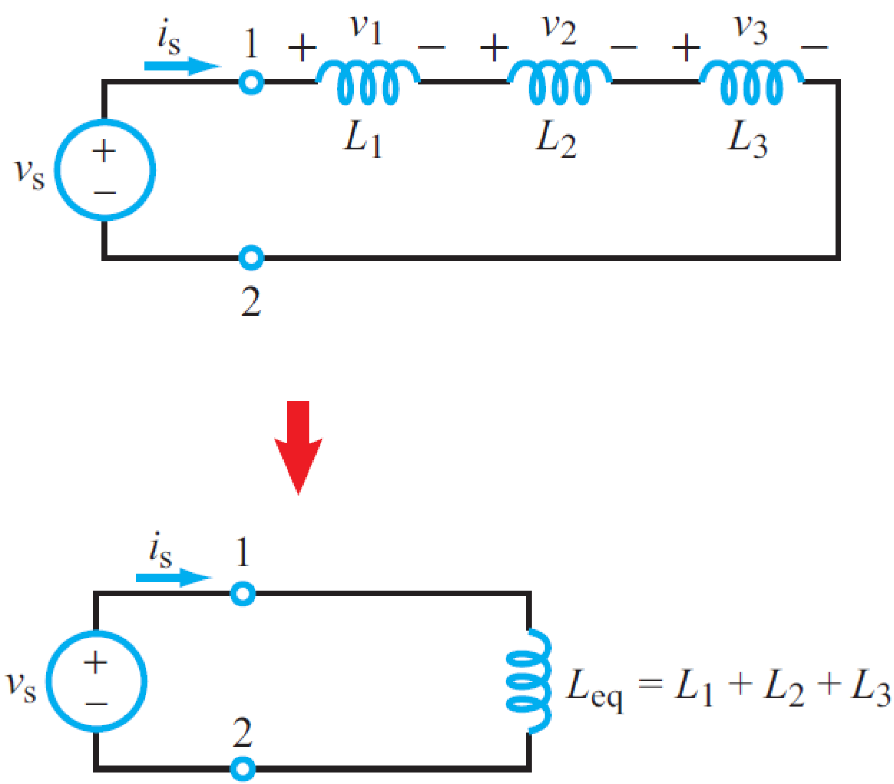


$$v = L\frac{di}{dt}$$



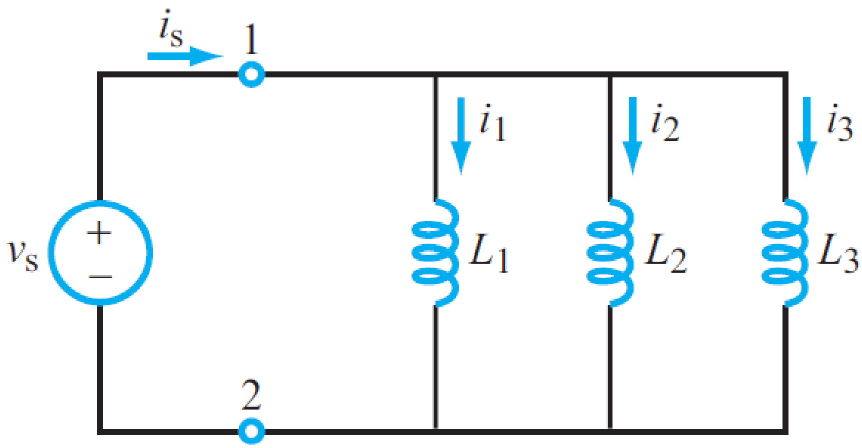
[Source: Berkeley]

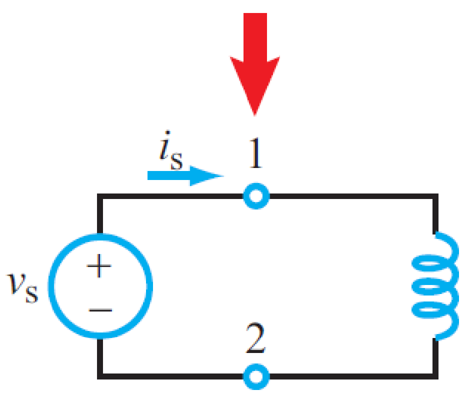
Inductors in Series



Inductors in Parallel

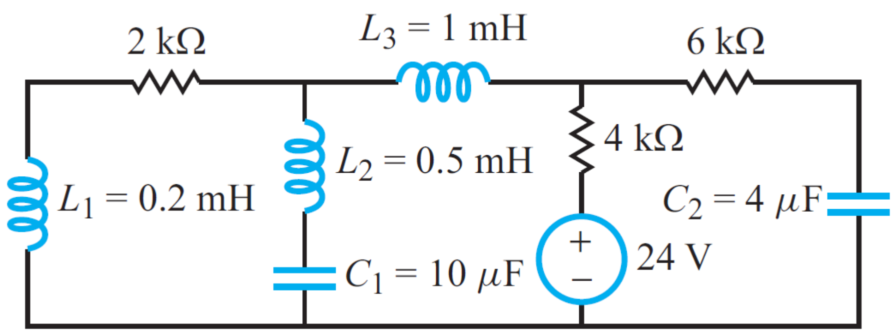
Combining In-Parallel Inductors

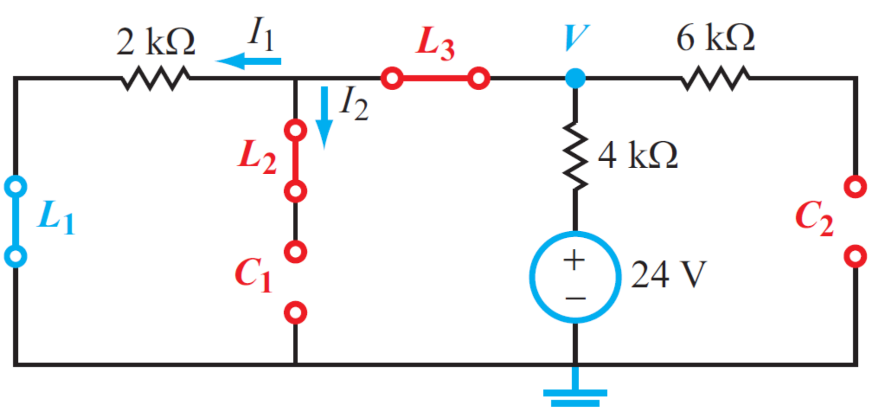




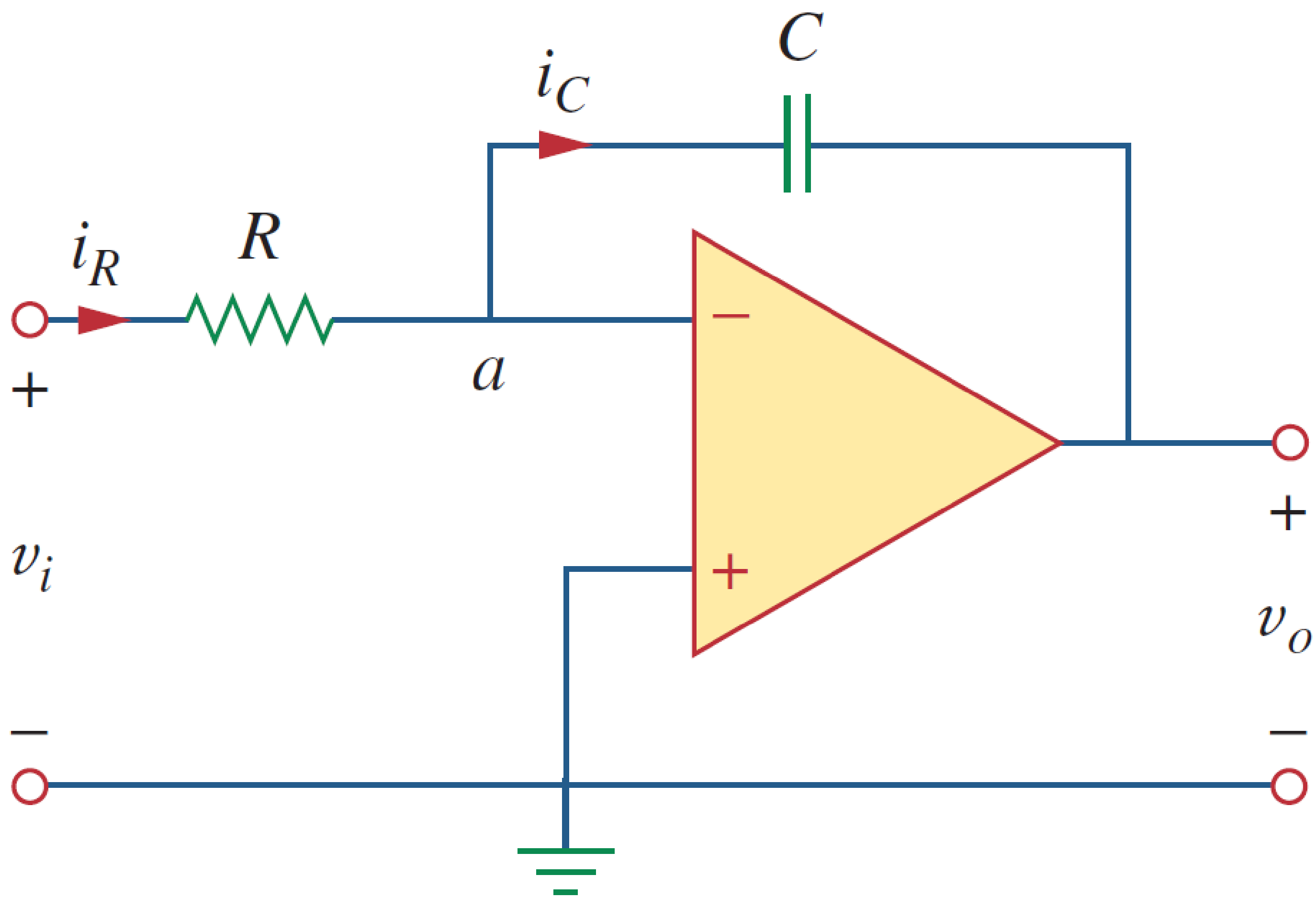
$$L_{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)^{-1}$$

Example


2 kΩ
$L_3 = 1$ mH
6 kΩ
$L_2 = 0.5$ mH
4 kΩ
$L_1 = 0.2$ mH
$C_2 = 4\ \mu$F
+
–
24 V
$C_1 = 10\ \mu$F

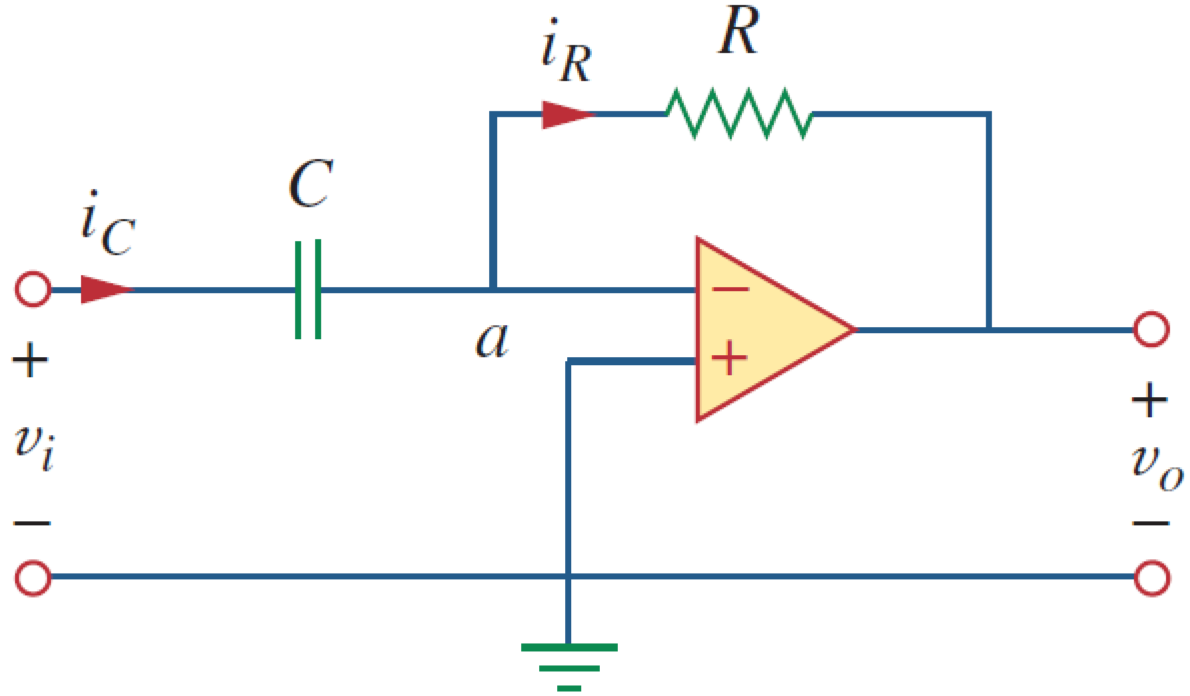

2 kΩ
I_1
L_3
V
6 kΩ
I_2
L_2
4 kΩ
L_1
C_2
C_1
+
–
24 V

Integrator



[Source: Berkeley]

Differentiator

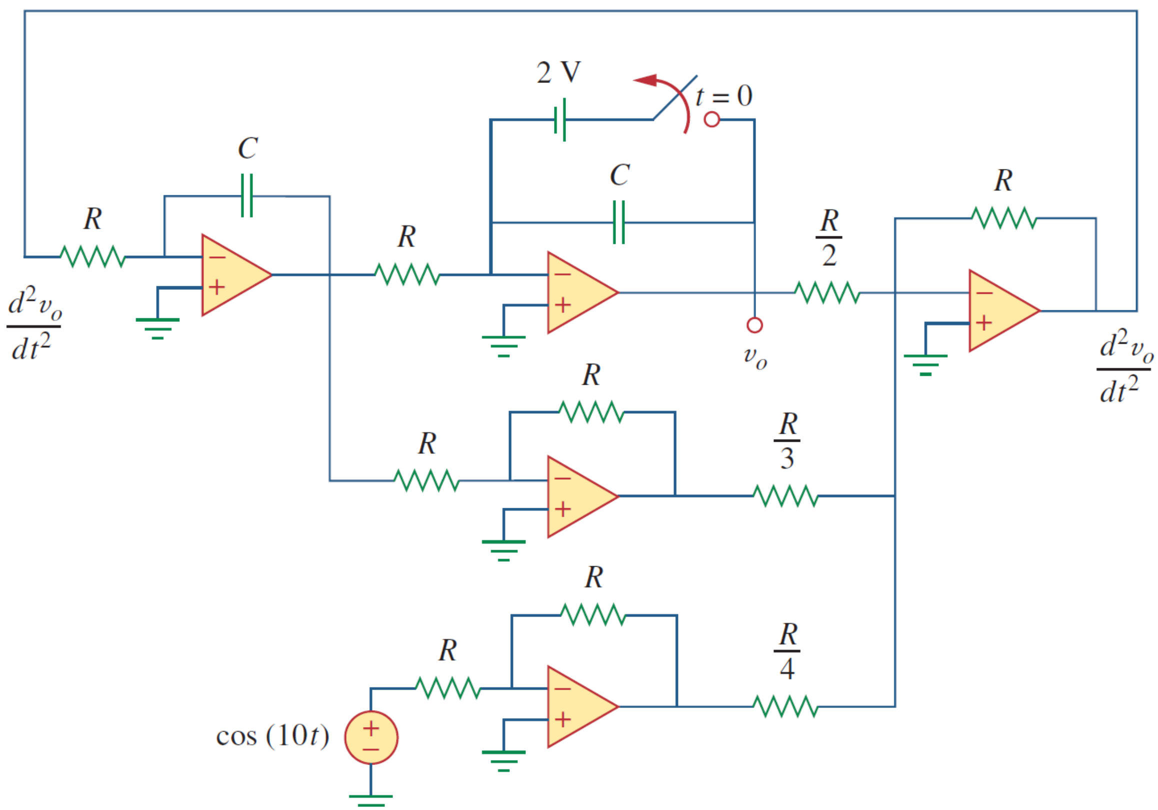


[Source: Berkeley]

Analog Computer

$$\frac{d^2v_o}{dt^2} + 3\frac{dv_o}{dt} + 2v_o = 4\cos 10t, \qquad t > 0$$

subject to $v_o(0) = 2,\ v'_o(0) = 0.$



Summary of Capacitors and Inductors

Table 5-4: Basic properties of R, L, and C.

Property	R	L	C
i–v relation	$i = \frac{v}{R}$	$i = \frac{1}{L}\int_{t_0}^{t} v\, dt' + i(t_0)$	$i = C\frac{dv}{dt}$
v-i relation	$v = iR$	$v = L\frac{di}{dt}$	$v = \frac{1}{C}\int_{t_0}^{t} i\, dt' + v(t_0)$
p (power transfer in)	$p = i^2R$	$p = Li\frac{di}{dt}$	$p = Cv\frac{dv}{dt}$
w (stored energy)	0	$w = \frac{1}{2}Li^2$	$w = \frac{1}{2}Cv^2$
Series combination	$R_{eq} = R_1 + R_2$	$L_{eq} = L_1 + L_2$	$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$
Parallel combination	$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$	$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2}$	$C_{eq} = C_1 + C_2$
dc behavior	no change	short circuit	open circuit
Can v change instantaneously?	yes	yes	no
Can i change instantaneously?	yes	no	yes

[Source: Berkeley]