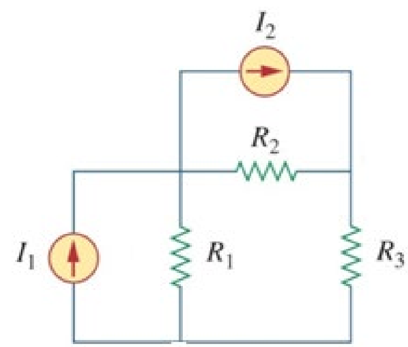
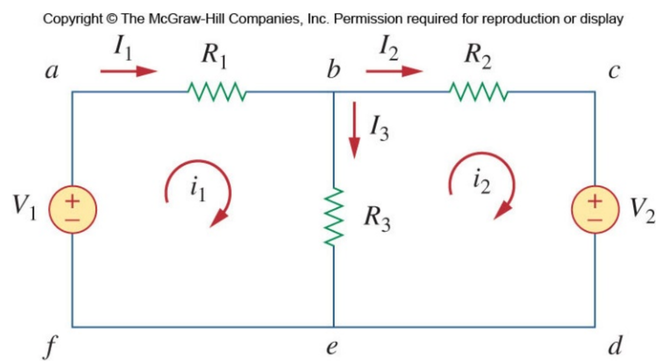
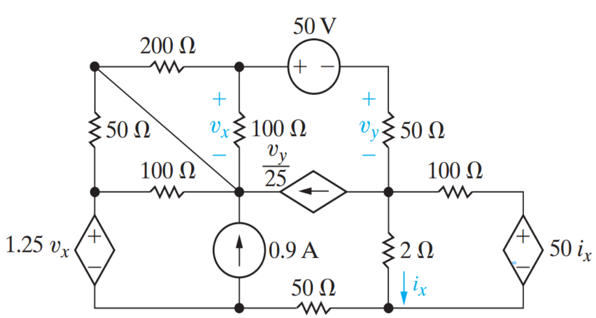
Lecture 6
- RC/RL First-Order Circuits

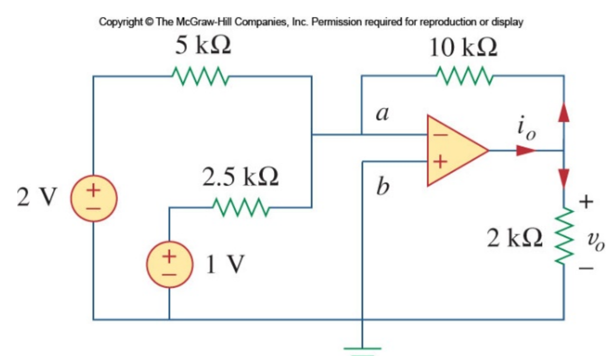
Temporal Behavior of Circuit Responses

- Till now we discussed <u>static</u> analysis of a circuit
  - Responses at a given time depend only on inputs at that time.
  - Circuit responds to input changes infinitely fast.



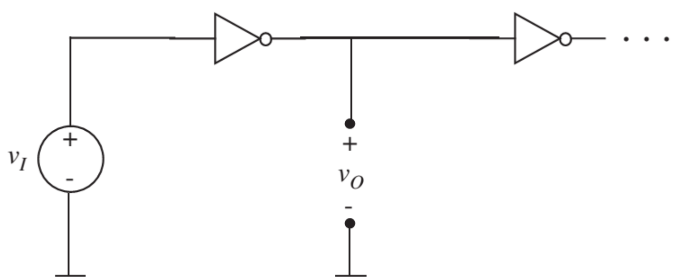


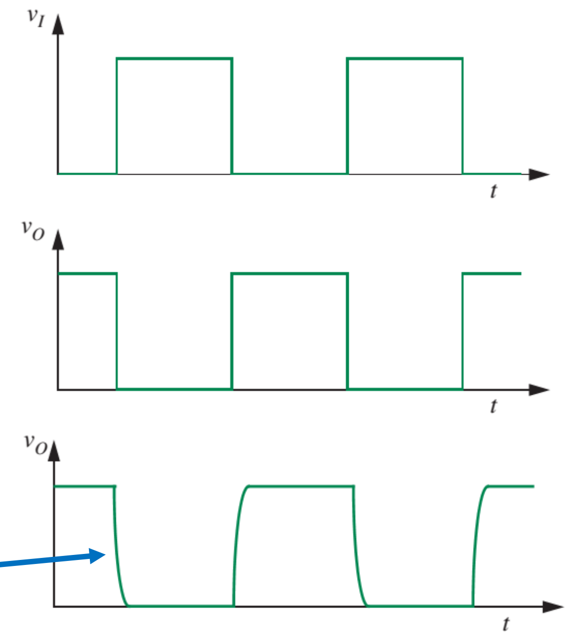




Temporal Behavior of Circuit Responses

- From now on we start to discuss <u>dynamic</u> circuit
  - **Time-varying** sources and responses





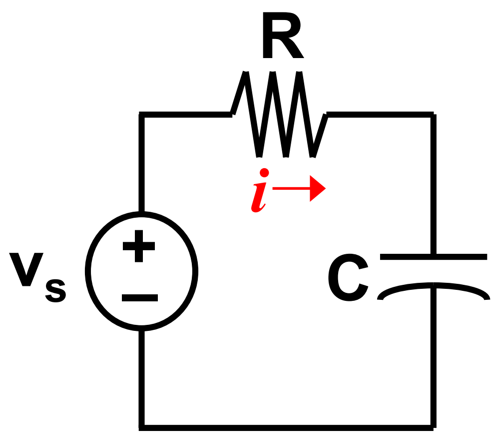
We have to introduce capacitors and inductors to explain such temporal behavior.

Outline

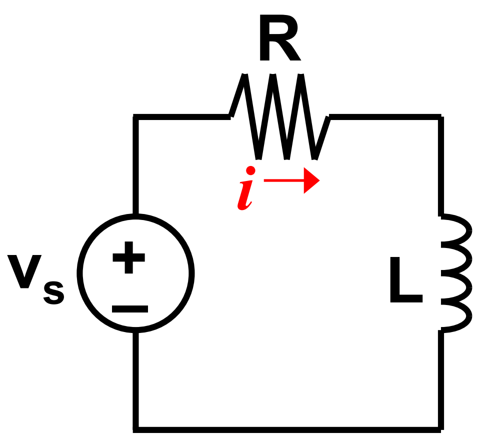
- Natural response of RC/RL circuits
- Step response of RC/RL circuits

RC and RL Circuits

- A circuit that contains only sources, resistors and a capacitor is called an ***RC circuit***.

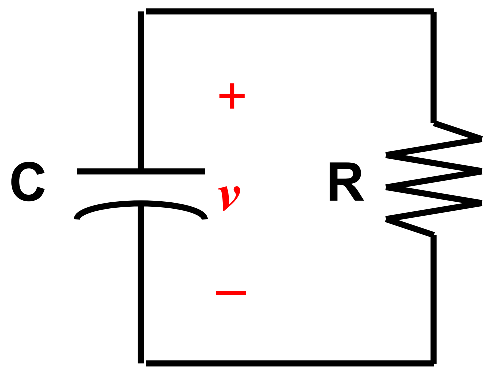


- A circuit that contains only sources, resistors and an inductor is called an ***RL circuit***.



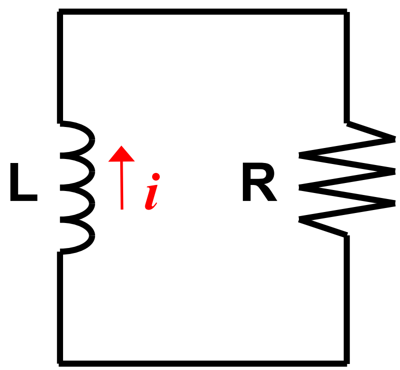
RC and RL Circuits

RC Circuit



- Capacitor voltage cannot change instantaneously
- In steady state, a capacitor behaves like an open circuit

RL Circuit

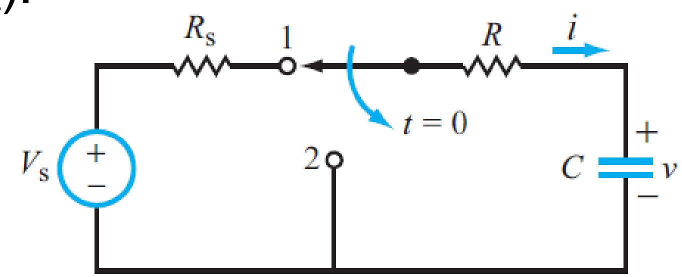


- Inductor current cannot change instantaneously
- In steady state, an inductor behaves like a short circuit.

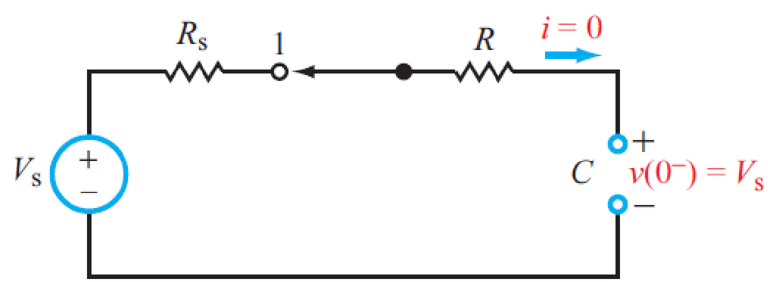
[Source: Berkeley]

Natural Response of a Charged Capacitor

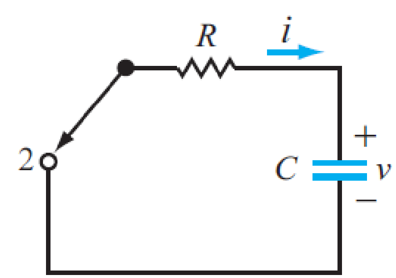
Behavior (*i.e.*, current and voltage) when stored energy in the inductor or capacitor is released to the resistive part of the network (containing no independent sources).



(a) $t = 0^-$ is the instant just before the switch is moved from terminal 1 to terminal 2;

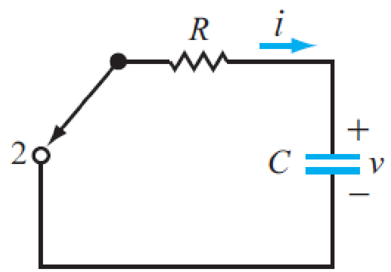


(b) $t = 0$ is the instant just after it was moved, $t = 0$ is synonymous with $t = 0^+$.



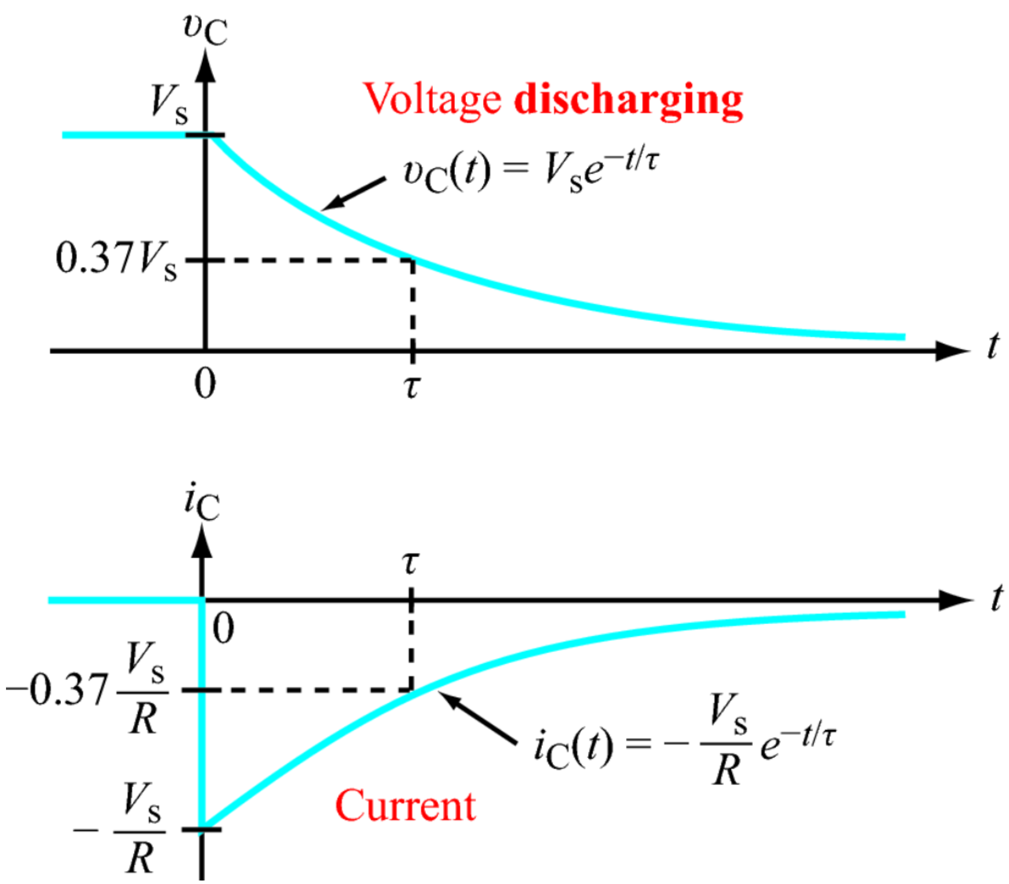
[Source: Berkeley]

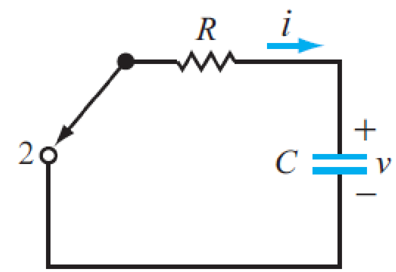
Natural Response of a Charged Capacitor



[Source: Berkeley]

Natural Response of RC



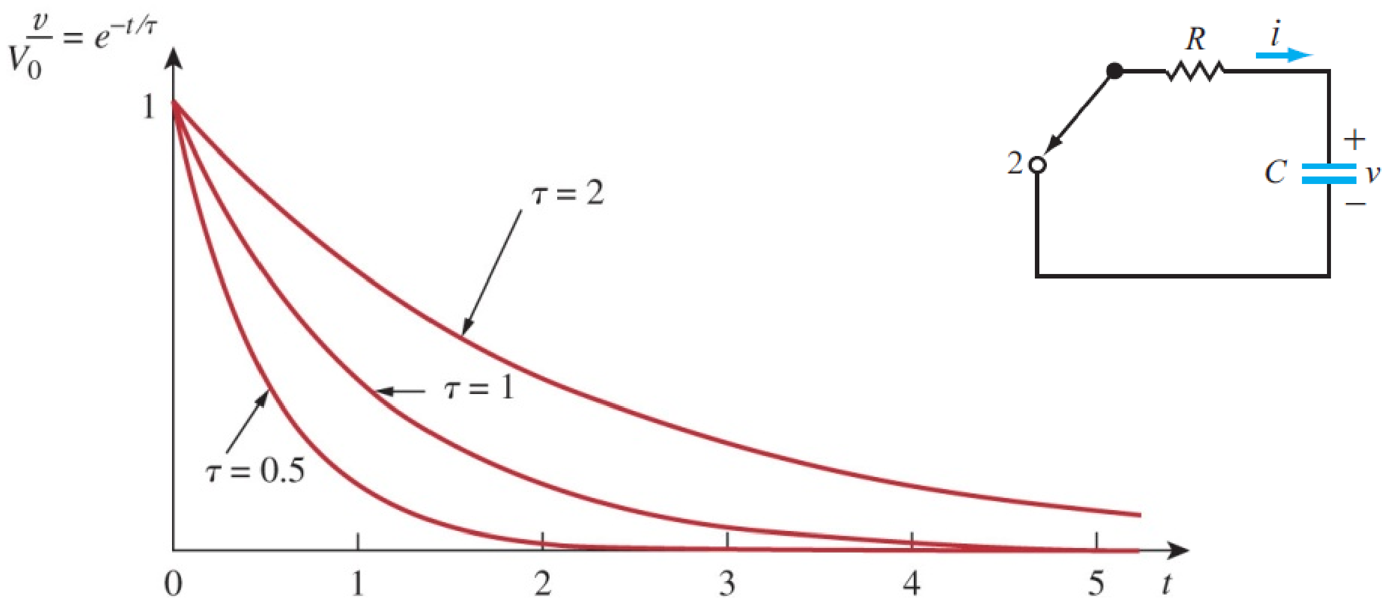


Time constant: $\tau = RC$

[Source: Berkeley]

Time Constant τ $(= RC)$

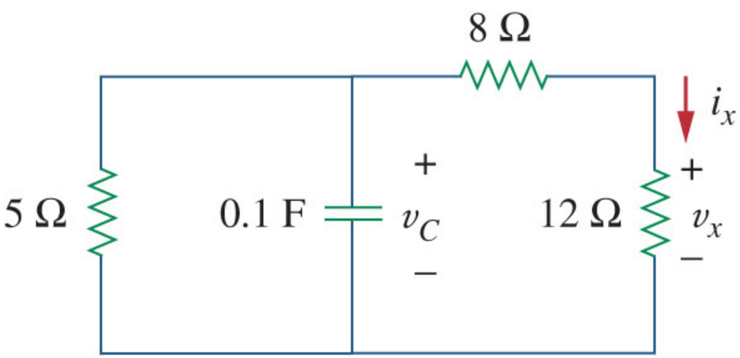
- A circuit with a small time constant has a fast response and vice versa.



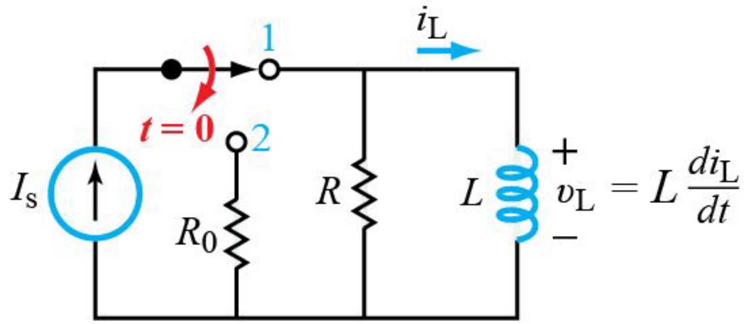
[Source: Berkeley]

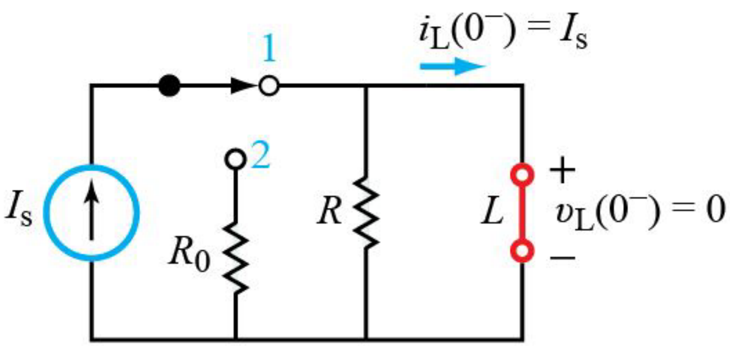
Example

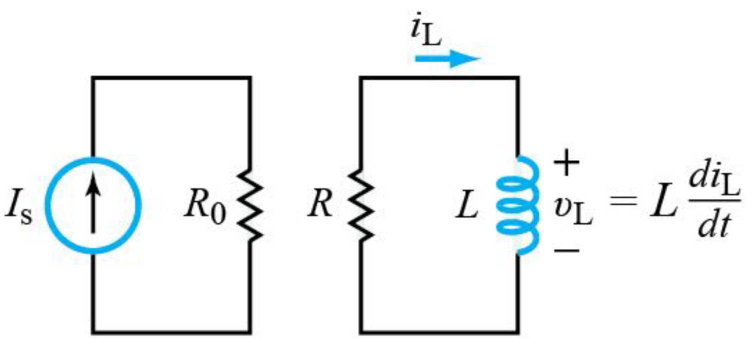
- In the circuit below, let $v_C(0) = 15\text{V}$. Find v_C, v_x, and i_x for $t > 0$.



Natural Response of the RL Circuit

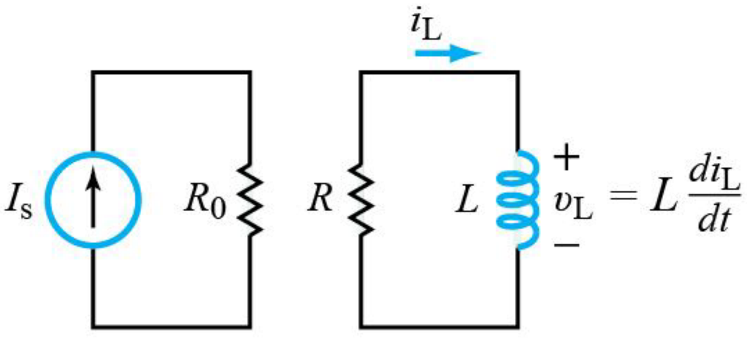




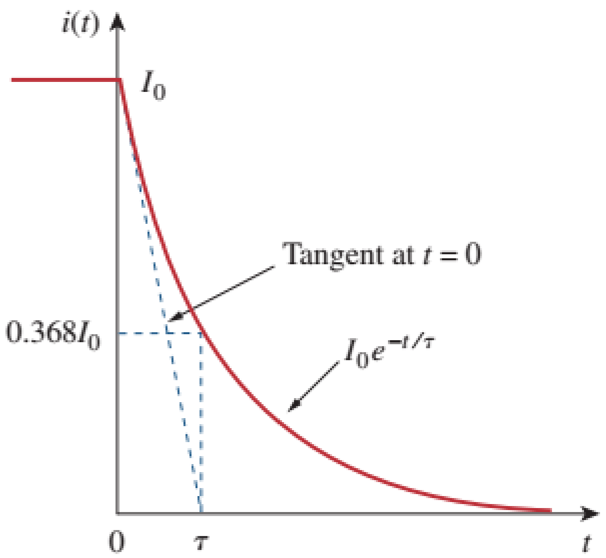


[Source: Berkeley]

Natural Response of the RL Circuit



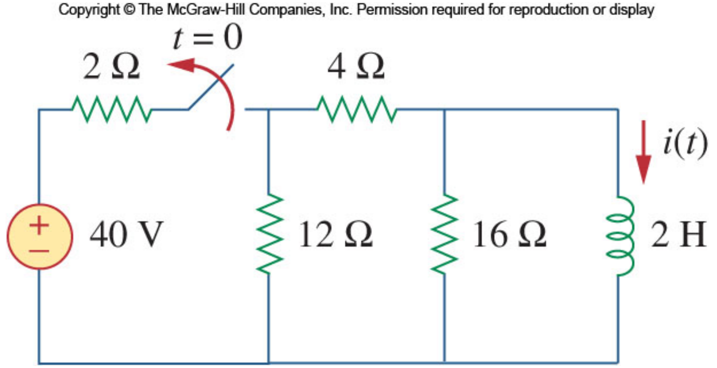
Natural Response of the RL Circuit



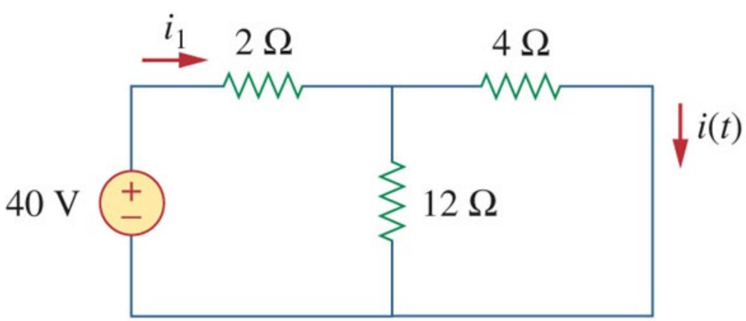
[Source: Berkeley]

Example

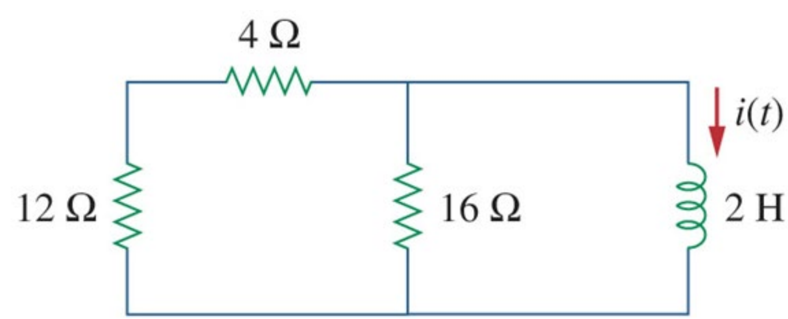
- The switch in the circuit below has been closed for a long time. At $t = 0$, the switch is opened. Calculate $i(t)$ for $t > 0$.



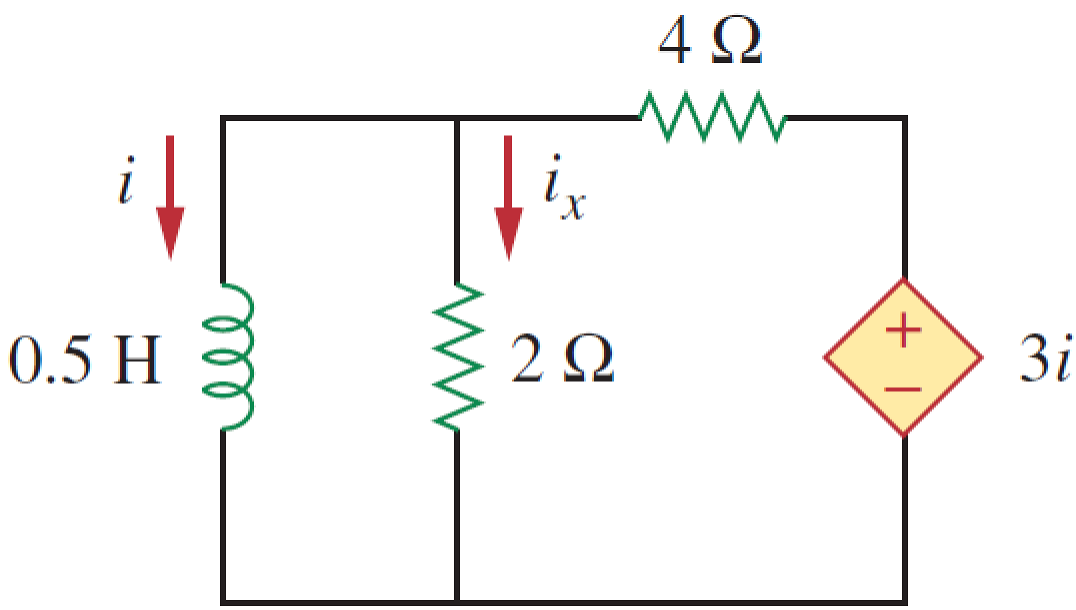
When $t < 0$



When $t > 0$

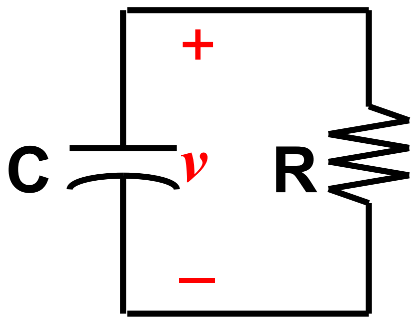


Assuming that $i(0) = 10$ A, calculate $i(t)$ and $i_x(t)$



Natural Response Summary

RC Circuit



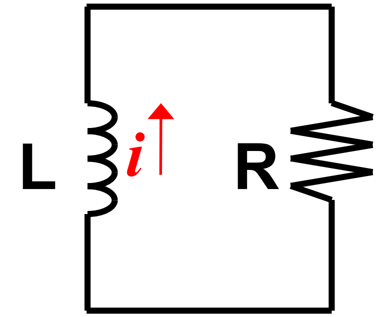
- Capacitor voltage cannot change instantaneously

$$v(0^-) = v(0^+)$$

$$v(t) = v(0)e^{-t/\tau}$$

- time constant $\tau = RC$

RL Circuit



- Inductor current cannot change instantaneously

$$i(0^-) = i(0^+)$$

$$i(t) = i(0)e^{-t/\tau}$$

- time constant $\tau = \dfrac{L}{R}$

[Source: Berkeley]

7.16 Determine the time constant for each of the circuits in Fig. 7.96.

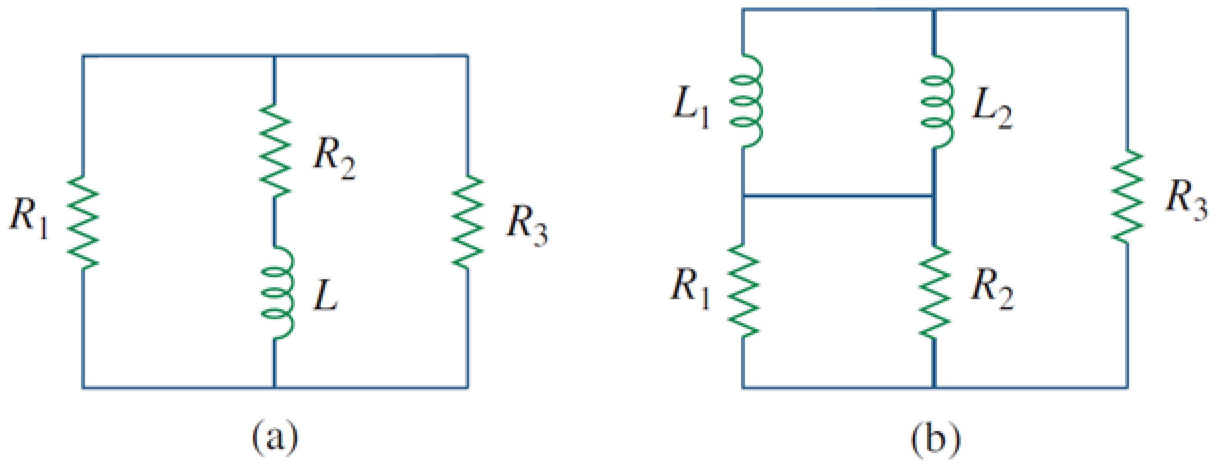


Figure 7.96
For Prob. 7.16.

7.19 In the circuit of Fig. 7.99, find $i(t)$ for $t > 0$ if $i(0) = 6$ A.

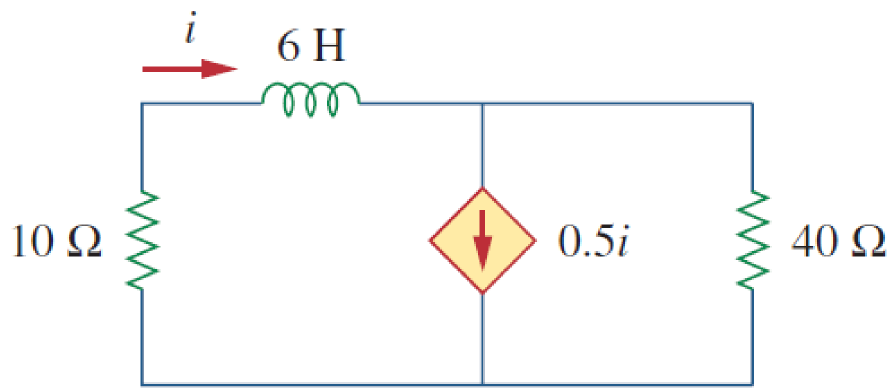


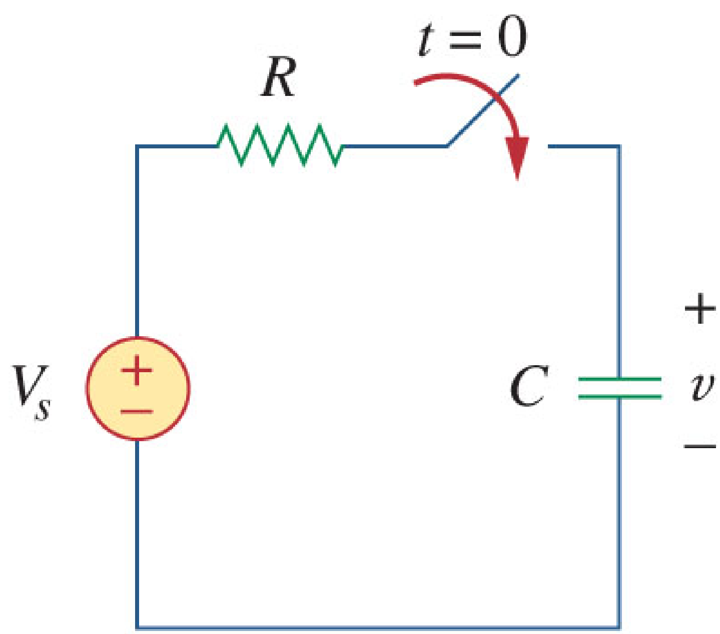
Figure 7.99
For Prob. 7.19.

Outline

- Natural response of RC/RL circuits
- Step response of RC/RL circuits

Step Response of RC Circuit

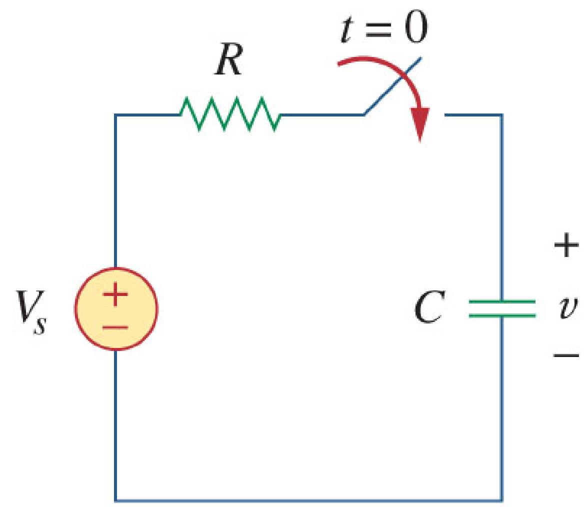
- When a DC source is suddenly applied to a RC circuit, the circuit response is known as the step response.



$$v(0^-) = v(0^+) = v_0$$

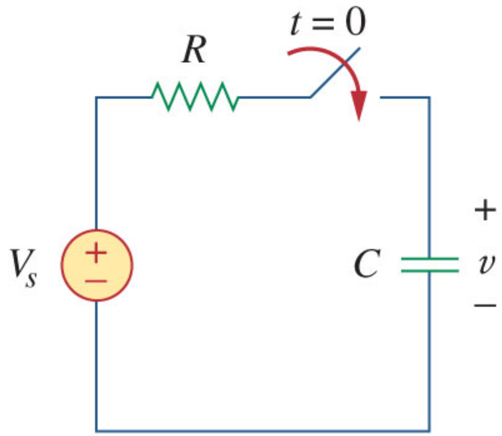
(a)

Step Response of the RC Circuit

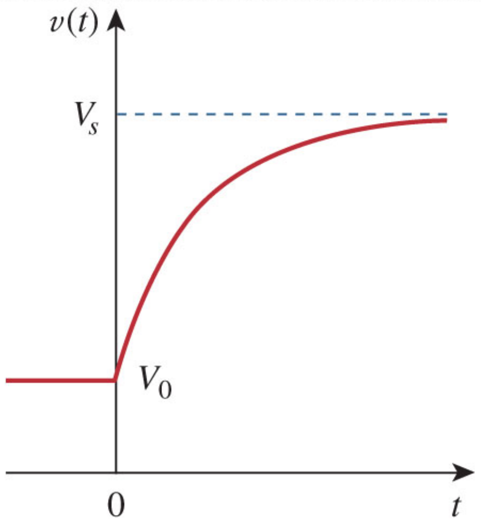


$$v(0^-) = v(0^+) = v_0$$

Step Response of the RC Circuit



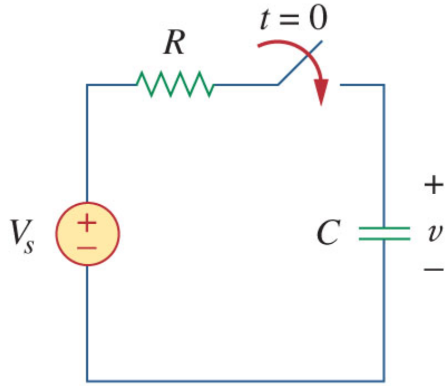
$$v(0^-) = v(0^+) = v_0$$



$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & t > 0 \end{cases}$$

- This is known as the <u>complete response</u>, or total response.

Forced Response



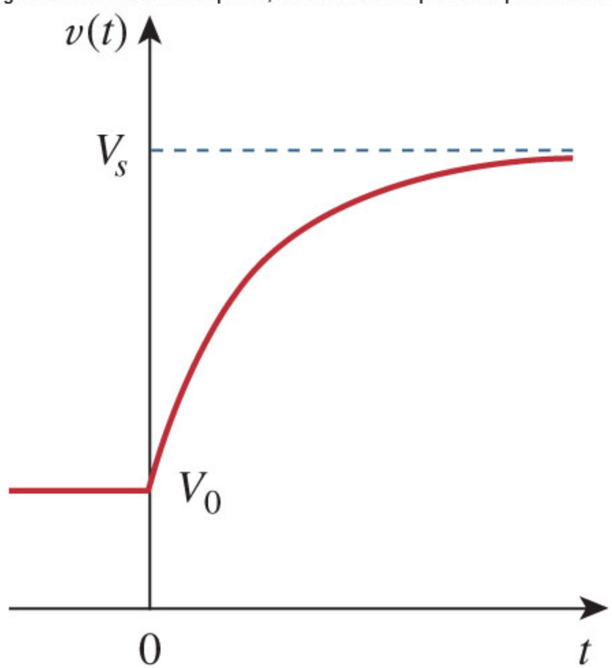
$$v(0^-) = v(0^+) = v_0$$

- The complete response

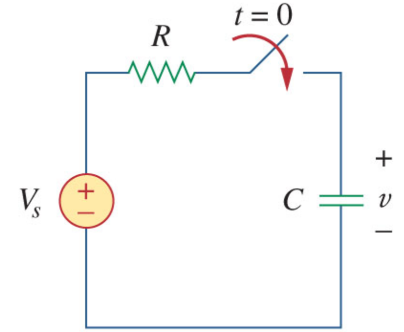
$$v(t) = \begin{cases} V_0, & t < 0 \\ V_s + (V_0 - V_s)e^{-t/\tau} & t > 0 \end{cases}$$

can be written as:

$$\boxed{v = v_n + v_f}$$



Another Perspective



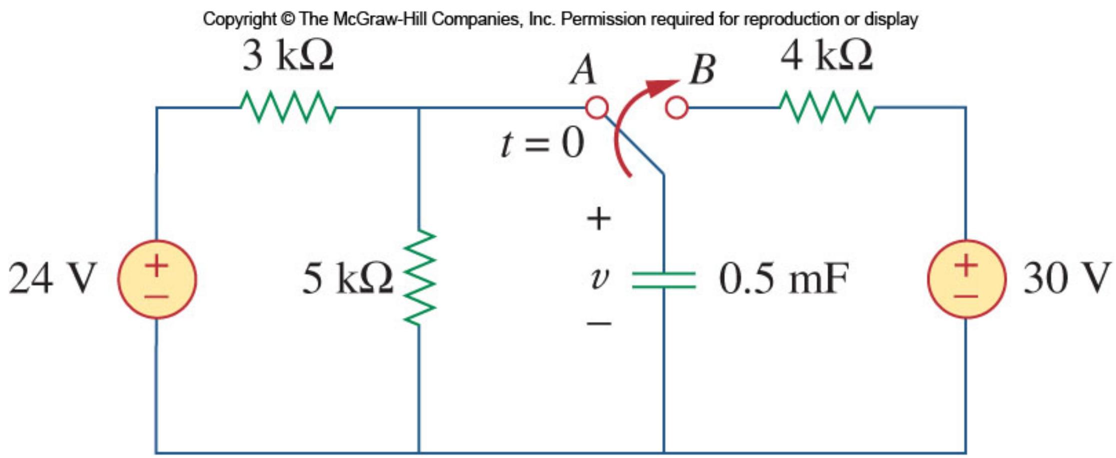
$$v(t)=\begin{cases} V_0, & t<0 \\ V_s+(V_0-V_s)e^{-t/\tau} & t>0 \end{cases}$$

- Another way to look at the response is to break it up into the <u>transient response</u> and the <u>steady state response</u>:

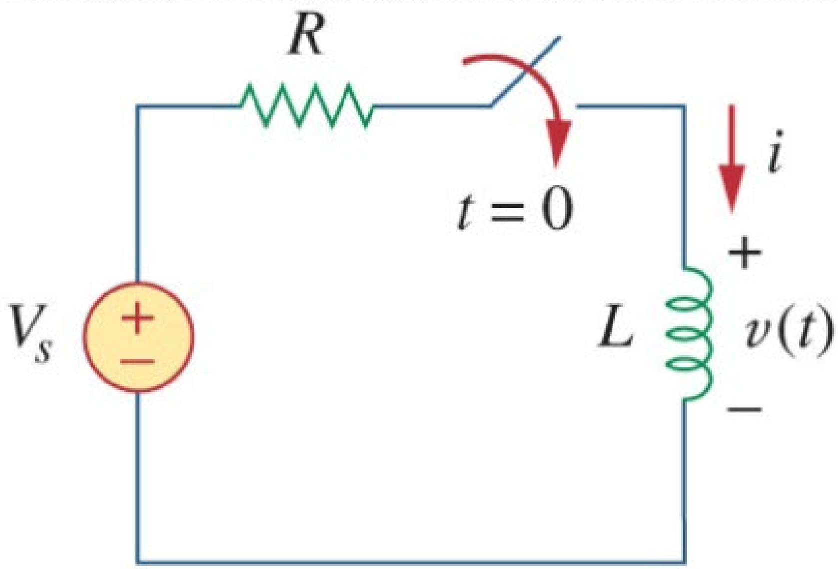
$$v(t) = \underbrace{v(\infty)}_{\text{steady } v_{ss}} + \underbrace{[v(0)-v(\infty)]e^{-t/\tau}}_{\text{transient } v_t}$$

Example

- The switch has been in position A for a long time. At $t = 0$, the switch moves to B. Find $v(t)$.

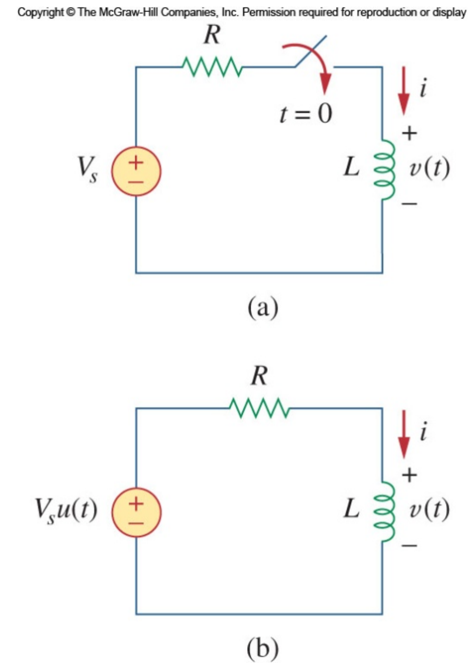


Step Response of the RL Circuit



$$i(0^+) = i(0^-) = I_0$$

Step Response of the RL Circuit



(a)

(b)

- We will use the transient and steady state response approach.
- We know that the transient response will be an exponential:

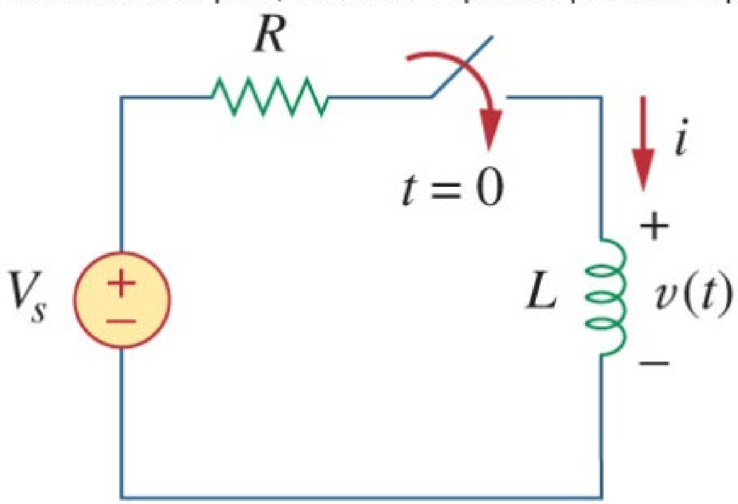
$$i_t = Ae^{-t/\tau}$$

- After a sufficiently long time, the current will reach the steady state:

$$i_{ss} = \frac{V_s}{R}$$

Step Response of RL Circuit

- This yields an overall response of:



$$i = Ae^{-t/\tau} + \frac{V_s}{R}$$

$$i\left(0^+\right) = i\left(0^-\right) = I_0 \qquad A = I_0 - \frac{V_s}{R}$$

$$i\left(t\right) = \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right)e^{-t/\tau}$$

Response of a Circuit

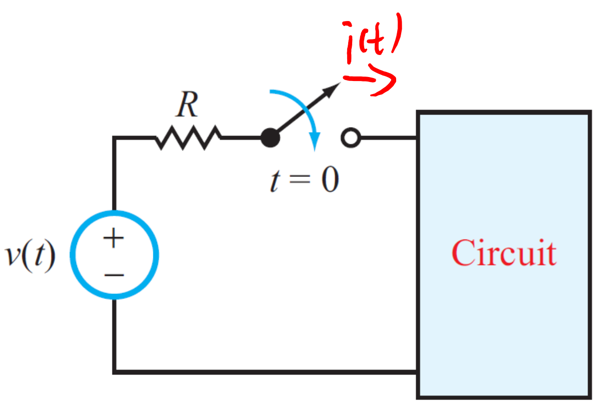
- ***Circuit (dynamic) response***
  - the reaction of a certain voltage or current in the circuit to change, such as the adding of a new source, the elimination of a source, in the circuit configuration.
- ***Transient response***
  - Behavior when voltage or current source are **suddenly** applied to or removed from the circuit due to switching.
  - Temporary behavior

The solution of a linear circuit, called dynamic response, can be decomposed into

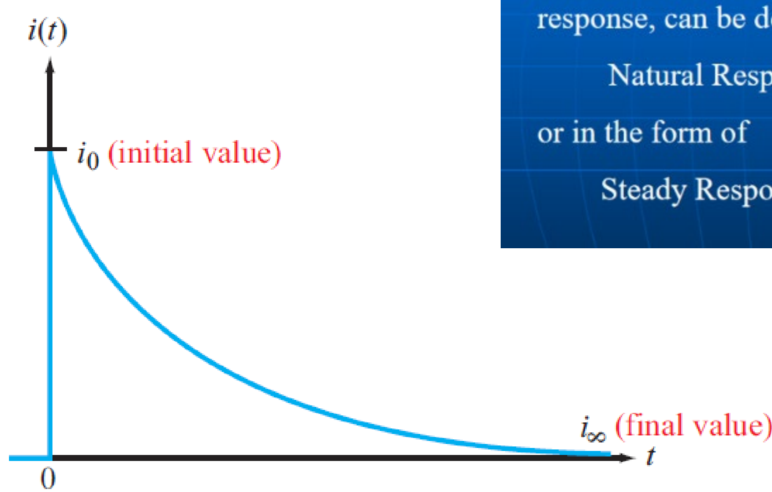
Natural Response + Forced Response

or in the form of

Steady Response + Transient Response

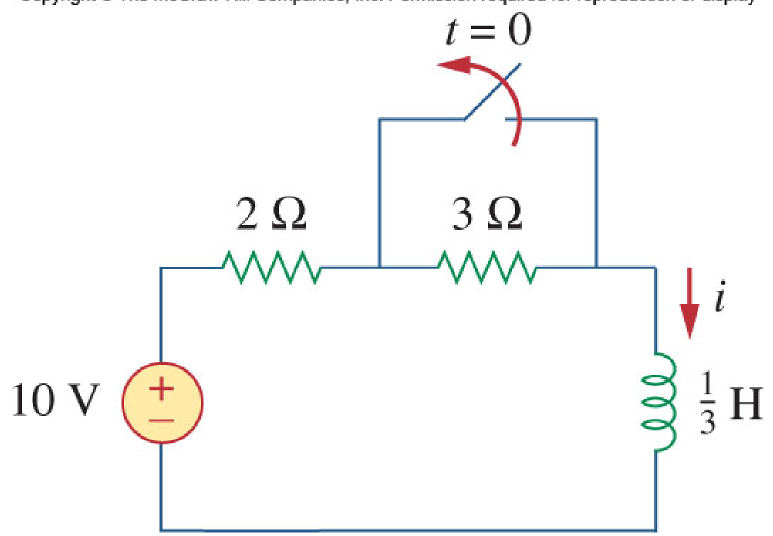


[Source: Berkeley]



Example

- Find $i(t)$ in the circuit for $t > 0$. Assume that the switch has been closed for a long time.



General Procedure for Finding RC/RL Response

1. **Identify the variable of interest**
   - For RL circuits, it is usually the inductor current $i_L(t)$.
   - For RC circuits, it is usually the capacitor voltage $v_c(t)$.

2. **Determine the initial value (at $t = t_0^-$ and t_0^+) of the variable**
   - Recall that $i_L(t)$ and $v_c(t)$ are continuous variables:

     $i_L(t_0^+) = i_L(t_0^-)$ and $v_c(t_0^+) = v_c(t_0^-)$

   - Assuming that the circuit reached steady state before t_0: use the fact that an inductor behaves like a short circuit in steady state or that a capacitor behaves like an open circuit in steady state.

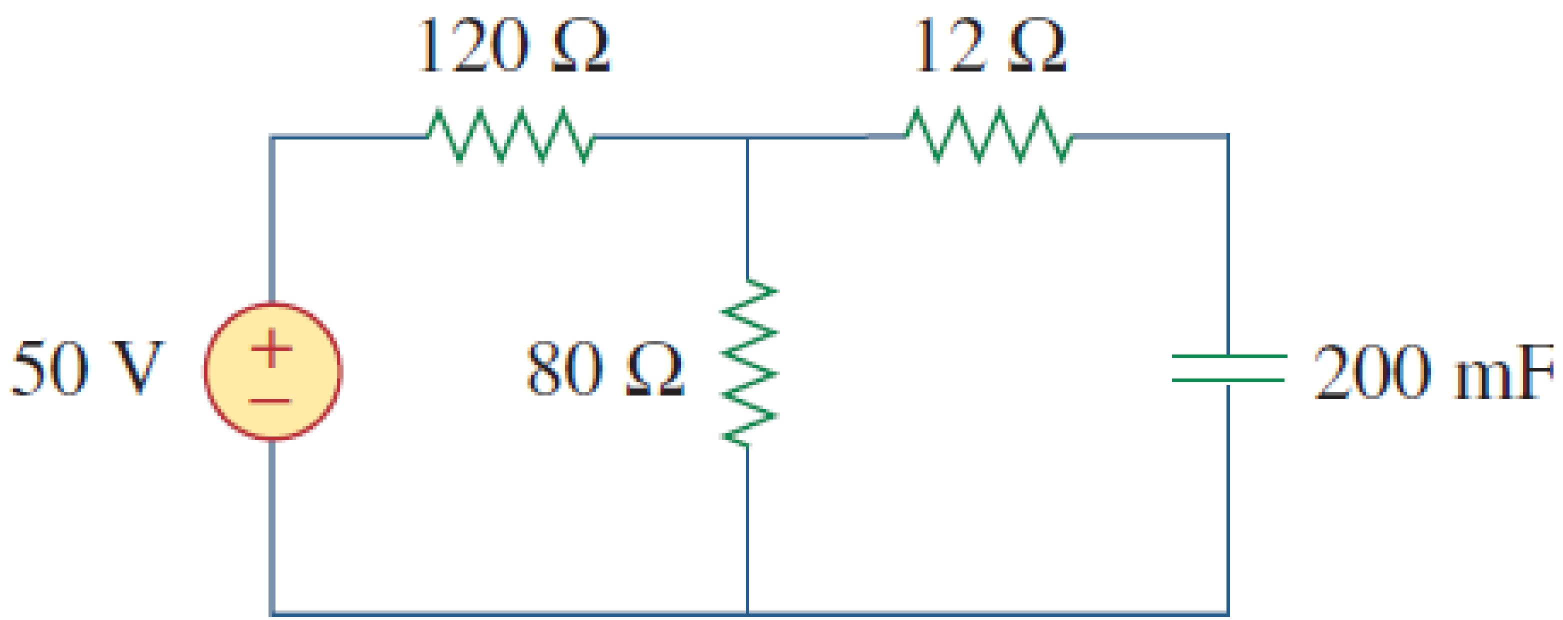
[Source: Berkeley]

Procedure (cont'd)

3. **Calculate the final value of the variable** (as $t \rightarrow \infty$)
   - Again, make use of the fact that an inductor behaves like a short circuit in steady state ($t \rightarrow \infty$) or that a capacitor behaves like an open circuit in steady state ($t \rightarrow \infty$).

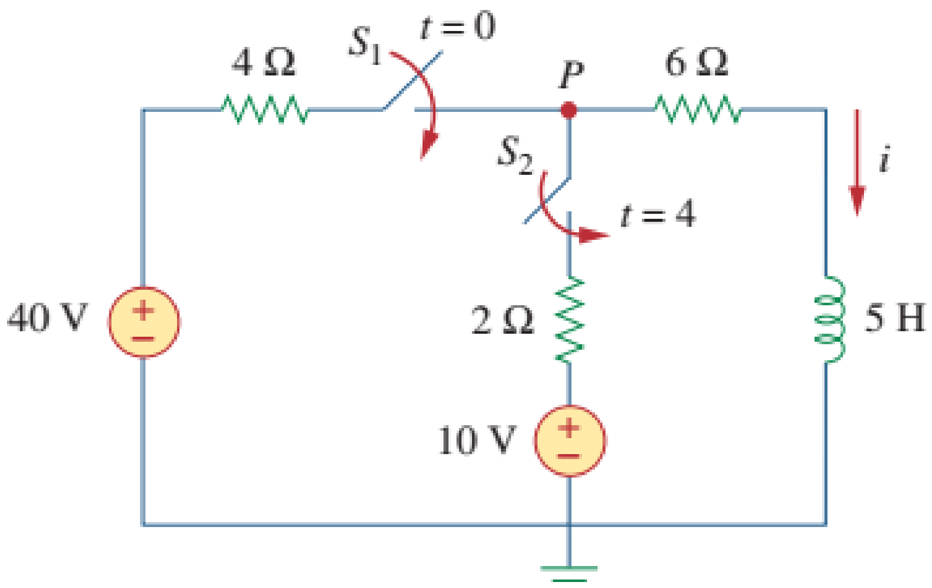
4. **Calculate the time constant for the circuit**
   - $\tau = CR$ for an RC circuit where R is the Thévenin equivalent resistance "seen" by the capacitor.
   - $\tau = L/R$ for an RL circuit, where R is the Thévenin equivalent resistance "seen" by the inductor.

Find the time constant for the *RC* circuit



Sequential switch

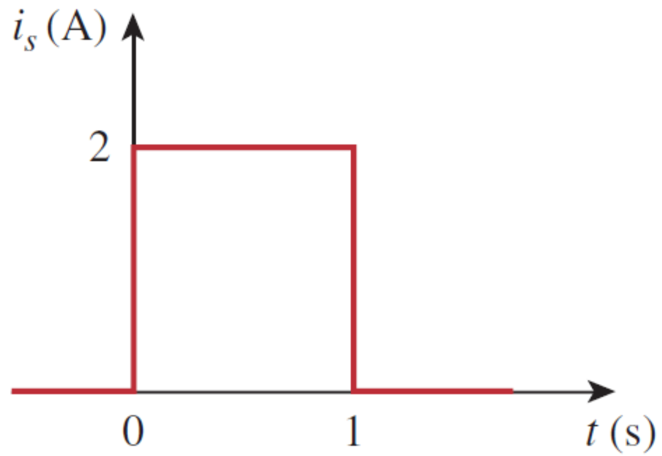
At $t = 0$, switch 1 in Fig. 7.53 is closed, and switch 2 is closed 4 s later. Find $i(t)$ for $t > 0$. Calculate i for $t = 2$ s and $t = 5$ s.



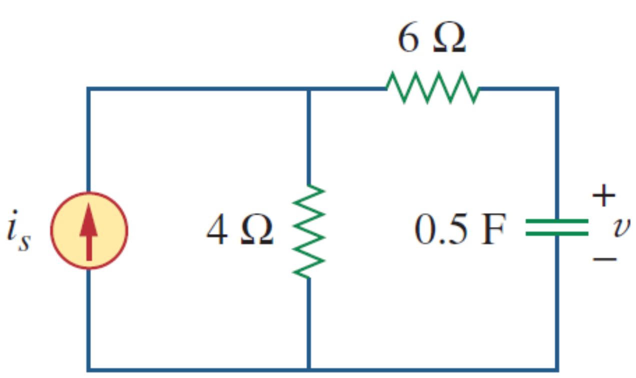
We need to consider the three time intervals $t \le 0$, $0 \le t \le 4$, and $t \ge 4$ separately. For $t < 0$, switches S_1 and S_2 are open so that $i = 0$. Since the inductor current cannot change instantly,

$$i(0^-) = i(0) = i(0^+) = 0$$

7.49 If the waveform in Fig. 7.116(a) is applied to the circuit of Fig. 7.116(b), find $v(t)$. Assume $v(0) = 0$.



(a)



(b)

1^{st} Order Circuit with OPA

7.73 For the op amp circuit of Fig. 7.138, let $R_1 = 10\ \text{k}\Omega$, $R_f = 20\ \text{k}\Omega$, $C = 20\ \mu\text{F}$, and $v(0) = 1V$. Find v_0.

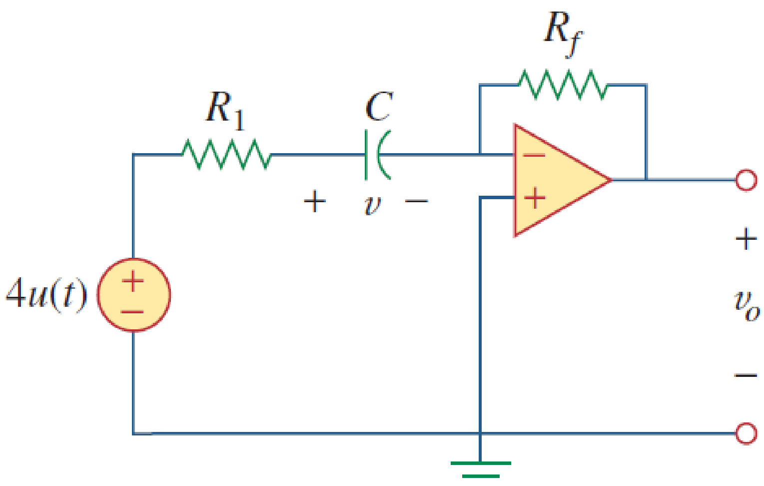
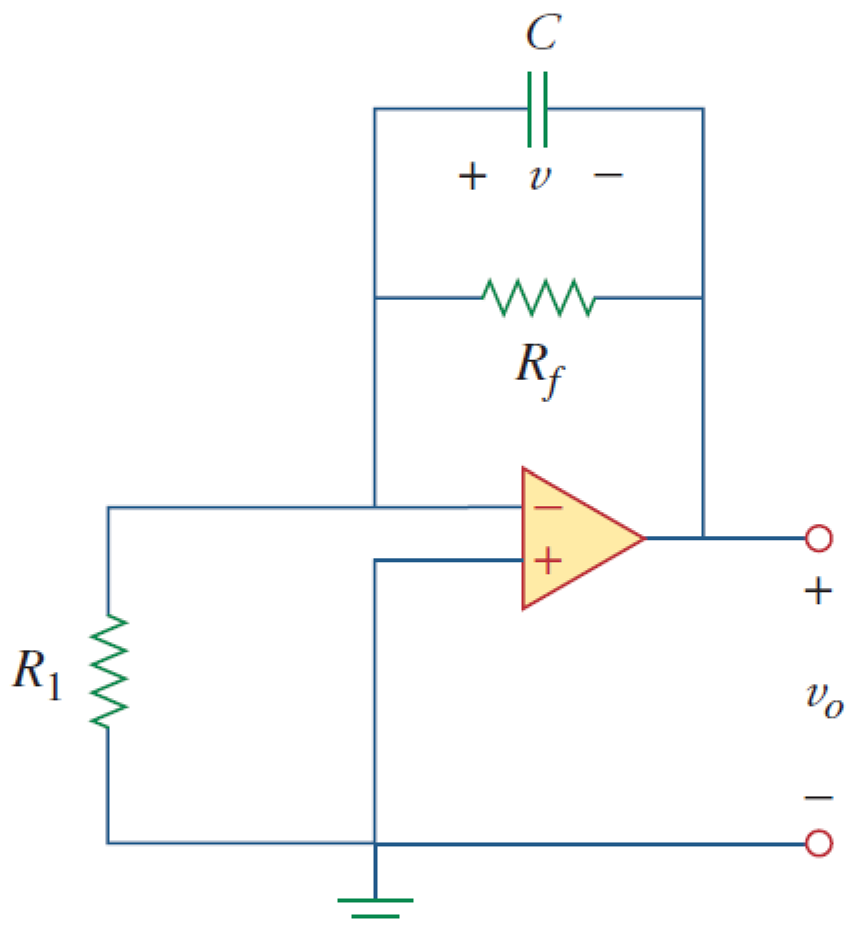
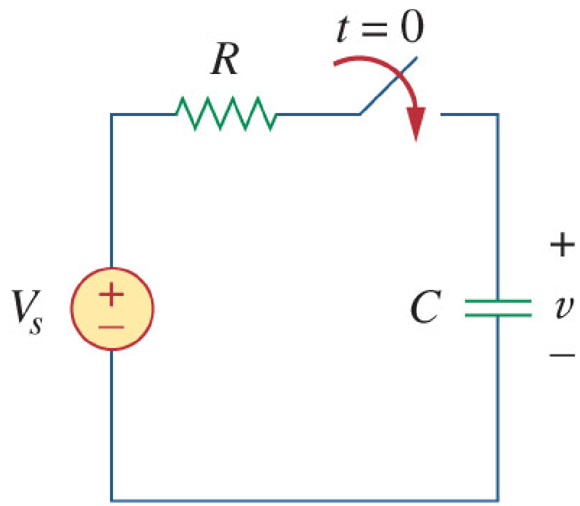


Figure 7.138
For Prob. 7.73.

For the op amp circuit in Fig. 7.56, find v_o for $t > 0$ if $v(0) = 4$ V. Assume that $R_f = 50$ kΩ, $R_1 = 10$ kΩ, and $C = 10$ μF.



Other kinds of excitations

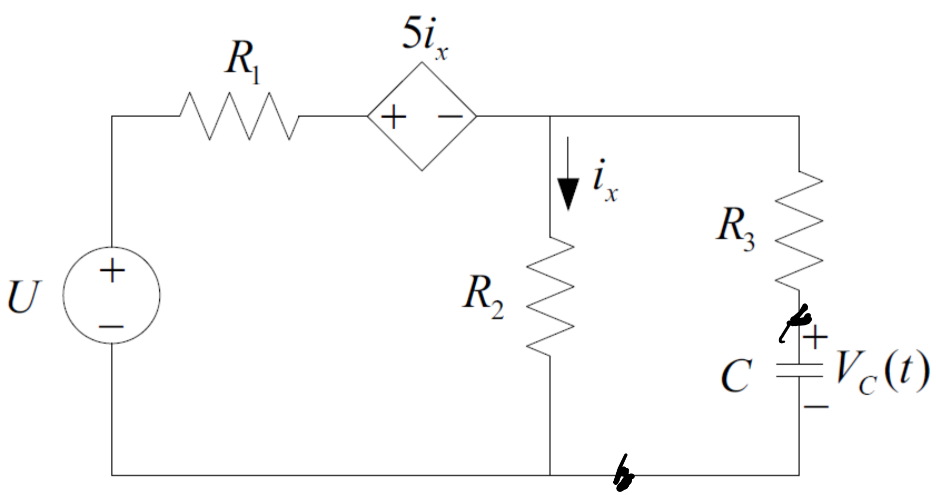


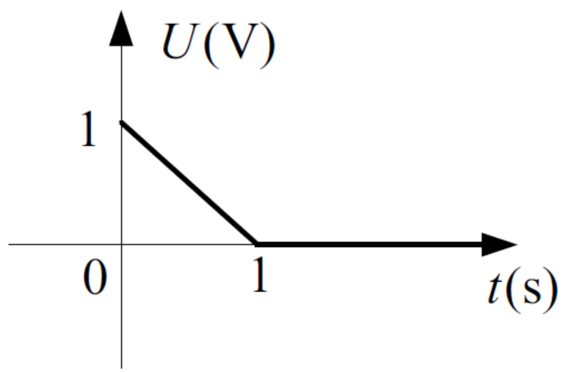
$$V_s = e^{\omega t},\quad wt, \qquad\qquad V_s = \cos(\omega t)$$

$$v(0^-) = v(0^+) = v_0$$

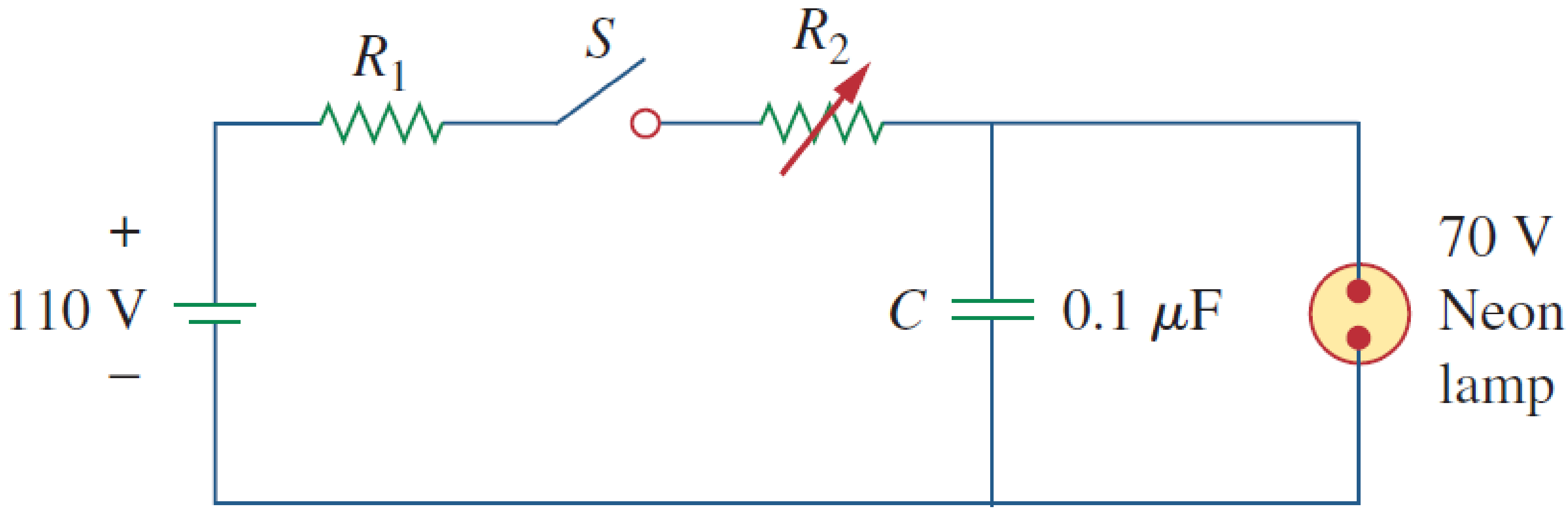
Other kinds of excitations

In the circuit below, $R_1 = 10\ \Omega$, $R_2 = 5\ \Omega$, $R_3 = 10\ \Omega$, $C = 10\ \text{mF}$. When $t < 0$, the input voltage (U) is $1\ \text{V}$. When $t = 0$, the input voltage begins to change as shown in the plot below. When $t > 1\,\text{s}$, $U = 0$. Assume that the circuit reaches steady state before $t = 0$. Determine the expression for $V_C(t)$ when $t \geq 0$.





Applications



Activation of a switch at the time $t = 0$ in a certain circuit caused the voltage across a $L = 20\,\mathrm{mH}$ inductor to exhibit the voltage response:

$$v(t) = 4e^{-0.2t}\,\mathrm{mV} \quad t > 0$$

Determine $i(t)$ for $t > 0$, given the energy stored in the inductor at $t = \infty$ is $0.64\,\mathrm{mJ}$.