

Number Representations

$123_{10} \rightarrow$ One Hundred Twenty Three Base 10

$$= 100_{10} + 20_{10} + 3_{10}$$

$$= 1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$$

Each place represents a power of 10. Hence, a Base 10 number

$$\begin{array}{ccc} 1 & 2 & 3_{10} \\ \uparrow & \uparrow & \uparrow \\ 10^2 & 10^1 & 10^0 \end{array}$$

A Base 10 number has 10 digits

$$\text{Base } 10 = \{0, 1, 2, 3, \dots, 9\}$$

\uparrow
10 digits

The digits in Base 10 are
are 0 to 9!

Computers do not work in Base 10. They work in Base 2. We prefer using Base 8 and Base 16 since it saves space and can easily be converted.

Base 2 \rightarrow 2 digits = $\{0, 1\}$

Base 8 \rightarrow 8 digits = $\{0, 1, 2, \dots, 7\}$

Base 16 \rightarrow $\{0, 1, 2, \dots, 9, A, B, C, D, E, F\}$

Equivalence				Equivalence			
BASE		Number		Representation		BASE	
2	8	10	16	2	8	10	16
0	0	0	0	1000	10	8	8
1	1	1	1	1001	11	9	9
10	2	2	2	1010	12	10	A
11	3	3	3	1011	13	11	B
100	4	4	4	1100	14	12	C
101	5	5	5	1101	15	13	D
110	6	6	6	1110	16	14	E
111	7	7	7	1111	17	15	F

The ease of conversion

$$8 = 2^3 \quad 16 = 2^4$$

Base 2 can easily be converted into Base 8 and Base 16.

Base 8 is 3 places of Base 2

Base 16 is 4 places of Base 2

$$10101011101_2$$

Look at previous table to help

$$\underline{\underline{10101011101}}_2$$

$$2 \quad 5 \quad 3 \quad 5_8$$

Every 3 Places

$$\underline{\underline{10101011101}}_2$$

$$5 \quad 5 \quad D_{16}$$

Every 4 Places

Which is equivalent in Base 10

$$\begin{aligned} 2 \times 8^3 + 5 \times 8^2 + 3 \times 8^1 + 5 \times 8^0 &= 5 \times 16^2 + 5 \times 16^1 + D \times 16^0 \\ 1024 + 320 + 24 + 5 &= 1280 + 80 + 13 \\ \underline{\underline{1373_{10}}} &= \underline{\underline{1373_{10}}} \end{aligned}$$

The conversion should be equivalent!

It Checks

Additional Examples

$$\begin{array}{ccc} 1 & 2 & 3 \\ 001 & 010 & 011 \end{array}_2 = ? \text{ in Base 2, 10, 16}$$

Every 3 places
from Base 2

$$\underline{\underline{001010011}}_2$$

$$\begin{array}{ccc} 0 & 5 & 3 \\ & & 16 \end{array}$$

Every 4 places
from Base 2

$$123_8 = 1 \times 8^2 + 2 \times 8^1 + 3 \times 8^0$$
$$64 + 16 + 3 = \underline{\underline{83_{10}}}$$

$$1010011_2 = 1 \times 2^6 + 1 \times 2^4 + 1 \times 2^1 + 1 \times 2^0$$
$$64 + 16 + 2 + 1 = \underline{\underline{83_{10}}}$$

$$53_{16} = 5 \times 16^1 + 3 \times 16^0$$
$$80 + 3 = \underline{\underline{83_{10}}}$$

The conversion back to Base 10 shows the equivalence of all 3 representations.

Another Example

$$123_{16} = ? \text{ in Base } 2, 8, 10$$

$$\begin{array}{ccc} 1 & 2 & 3_{16} \\ 0001 & 0010 & 0011_2 \end{array} \quad \text{Use 4 places for Base 2}$$

$$\begin{array}{ccc} \underline{1} & \underline{0010} & \underline{0011}_2 \\ 4 & 4 & 3_8 \end{array} \quad \text{From Base 2 use every 3 places}$$

$$443_8 = 100100011_2$$

$$4 \times 8^2 + 4 \times 8^1 + 3 \times 8^0 = 1 \times 2^8 + 1 \times 2^5 + 1 \times 2^1 + 1 \times 2^0$$

$$256 + 32 + 3 = 256 + 32 + 2 + 1$$

$$291_{10} = 291_{10}$$

The conversion to Base 10 and its equivalence shows how correct the results are by agreement!

Test your knowledge. Complete the table by calculating and filling in the unknowns. Show the conversions!

Base
10

Base
2

Base
8

Base
16

127

=

=

=

=

10101

=

=

=

=

71

=

=

=

=

AB