Optimal attitude control of satellites

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Section 1

Introduction

Aims and objectives

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- To propose algorithms for the attitude control of satellites.

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- To propose algorithms for the attitude control of satellites.
- Using optimal control techniques.

What is attitude control?

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  - Output: Angular velocities and orientation (Euler angles, quaterion, etc...).

What is optimal control?

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Optimal control problem

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Optimal control problem

Minimize in $\boldsymbol{u} : [0, T] \longrightarrow \mathbb{R}^m$

$$\boldsymbol{u} \longmapsto \int_0^T F(t, \boldsymbol{x}(t), \boldsymbol{u}(t))\ dt$$

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Minimize in $\boldsymbol{u} : [0, T] \longrightarrow \mathbb{R}^m$

$$\boldsymbol{u} \longmapsto \int_0^T F(t, \boldsymbol{x}(t), \boldsymbol{u}(t))\ dt$$

subject to

$$\begin{cases} \boldsymbol{x}'(t) = \boldsymbol{f}(t, \boldsymbol{x}(t), \boldsymbol{u}(t)) \quad \forall t \in [0, T] \\ \boldsymbol{x}(0) = \boldsymbol{x}_0 \\ \boldsymbol{x}(T) = \boldsymbol{x}_T \\ \qquad \textit{Pointwise constraints} \end{cases}$$

Current state of the art

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  - PID, PI, PD, etc: [Wiśniewski, 1996, Tudor, 2011, Sekhavat et al., 2011, Yang, 2012, Bråthen, 2013].

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  - Neural networks [Battipede et al., 2003].

Current state of the art

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- The vast majority rely on a previous linearisation of the equations of motion.

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- The present work deals with the nonlinear system.

Section 2

Problem statement

Nonlinear equations of motion

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Rigid body rotating in space.

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$$\boldsymbol{J}\frac{d\omega}{dt} + \boldsymbol{\omega} \times J\omega = \mathbf{u}$$

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- **Angular velocity** of the body-fixed frame with respect to the inertial frame: $\boldsymbol{\omega}$
- **Tensor of inertia**: J

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where:

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- **Tensor of inertia**: J
- **Torques** exerted on the satellite: $\boldsymbol{u}$ (This will be the control variable)

Nonlinear equations of motion

Nonlinear equations of motion

Kinematic equations of motion

$$\begin{cases} \dfrac{d\boldsymbol{\omega}}{dt} = -\dfrac{1}{2}\boldsymbol{\omega} \times \boldsymbol{\epsilon} + \dfrac{1}{2}\eta\boldsymbol{\omega} \\ \dfrac{d\eta}{dt} = -\dfrac{1}{2}\boldsymbol{\omega}^t\boldsymbol{\epsilon} \end{cases}$$

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where:

- **Unit quaternion** representing orientation of the body-fixed frame with respect to the inertial frame: $\boldsymbol{q} = (\epsilon, \eta)$

Nonlinear optimal control problem

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Find $\boldsymbol{u} : [0, T] \longrightarrow \mathbb{R}^3$

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In addition to be a minimizer of a functional

$$\boldsymbol{u} \longmapsto \int_0^T F(t, \boldsymbol{\omega}, \boldsymbol{q}, \boldsymbol{u})$$

Linearised equation of motion

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Reformulation of the nonlinear equations

$$\begin{cases} J\boldsymbol{\omega}' = -\boldsymbol{\omega} \times J\boldsymbol{\omega} + \boldsymbol{u} \\ \boldsymbol{\epsilon}' = \dfrac{1}{2} G(\boldsymbol{\epsilon})\boldsymbol{\omega} \end{cases}$$

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where

$$G(\boldsymbol{\epsilon}) = \begin{pmatrix} \sqrt{1-||\boldsymbol{\epsilon}||^2} & -\epsilon_3 & \epsilon_2 \\ \epsilon_3 & \sqrt{1-||\boldsymbol{\epsilon}||^2} & -\epsilon_1 \\ -\epsilon_2 & \epsilon_1 & \sqrt{1-||\boldsymbol{\epsilon}||^2} \end{pmatrix}$$

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[Yang, 2010]

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Once $\boldsymbol{\epsilon}$ has been calculated, we use $|\boldsymbol{q}| = 1$ to compute $\boldsymbol{\eta}$.

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$$\boldsymbol{x} = (\boldsymbol{\omega}, \boldsymbol{\epsilon})$$

Section 3

Numerical approximation

- Nonlinear problem

- Nonlinear problem $\rightarrow$ variational reformulation.

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- Linearised problem

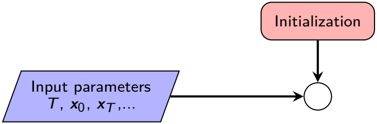
- Nonlinear problem $\rightarrow$ variational reformulation.
- Linearised problem $\rightarrow$ LQR techniques.

Constrained minimization

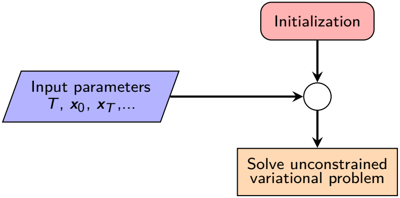
Constrained minimization

Initialization

Constrained minimization



Constrained minimization



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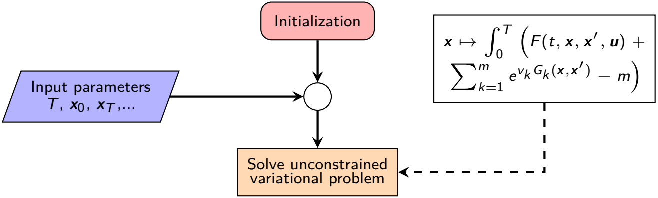

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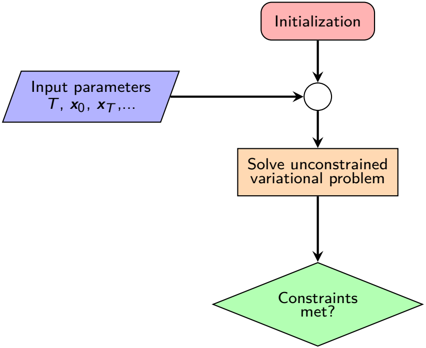

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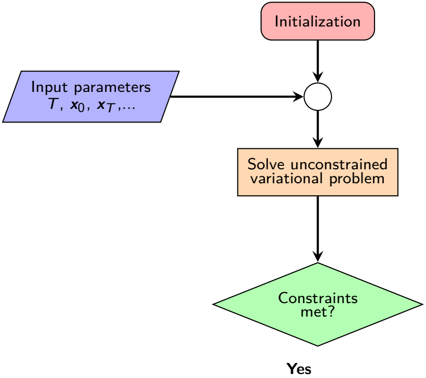

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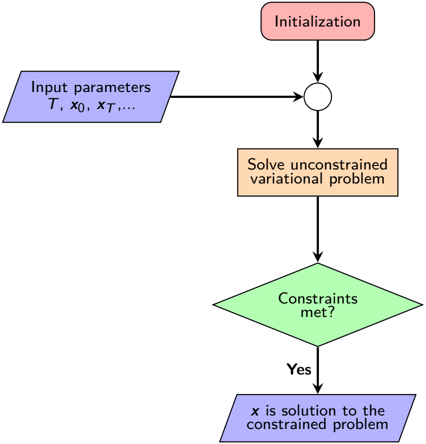

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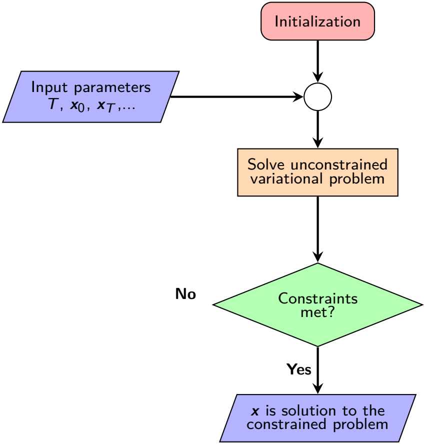

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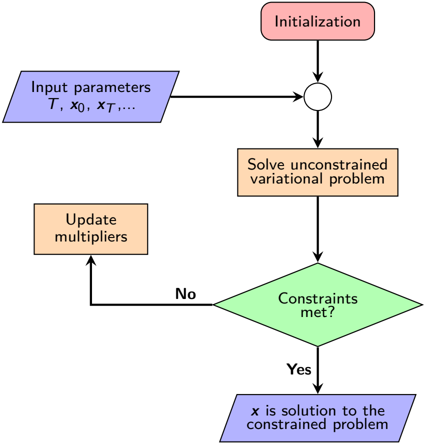

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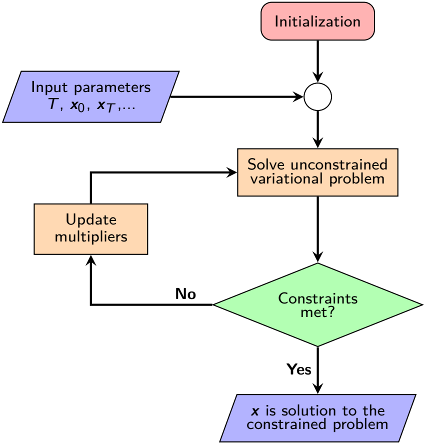

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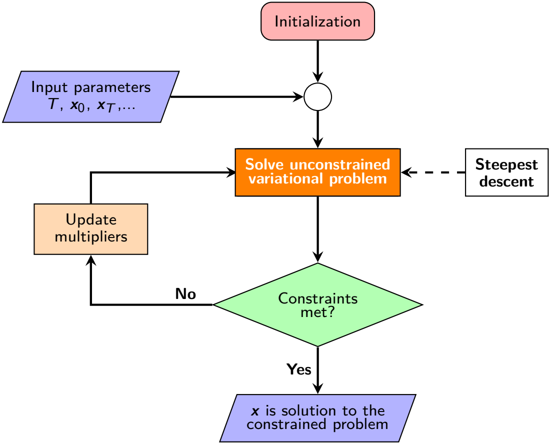

Algorithm for the solution of the LQR

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Initialization

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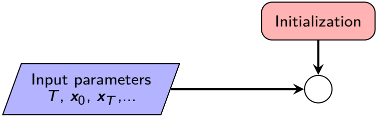

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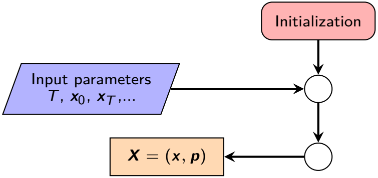

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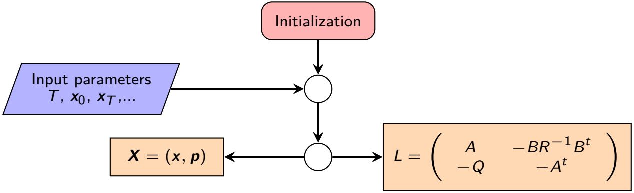

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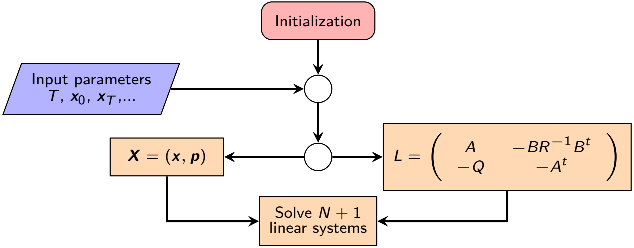

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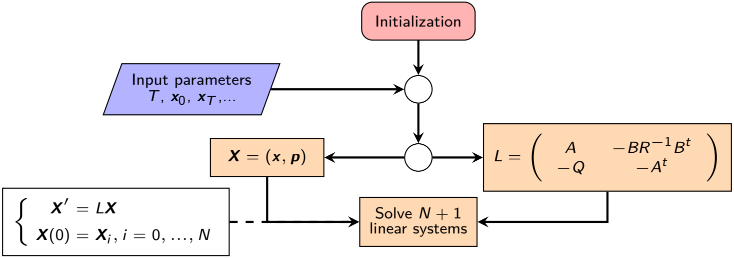

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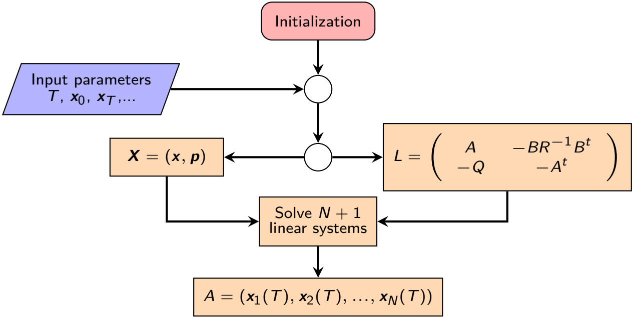

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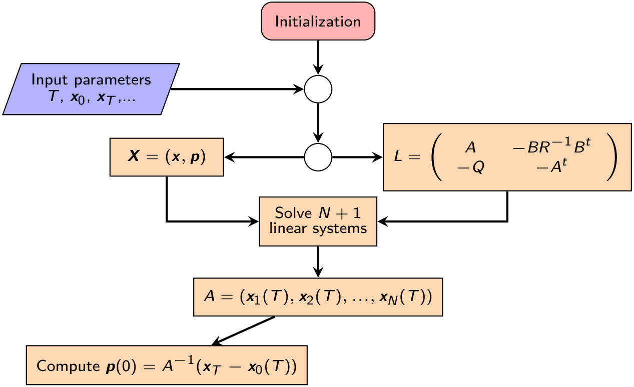

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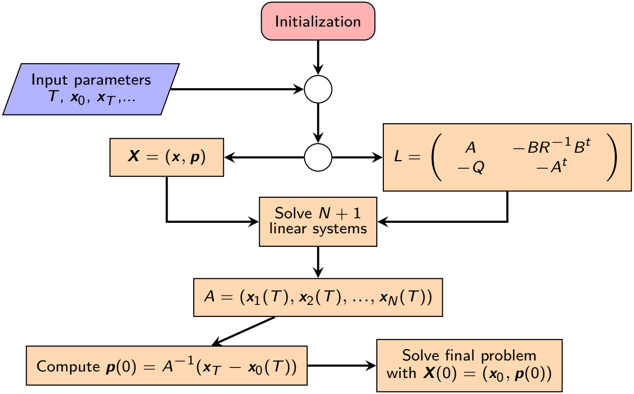

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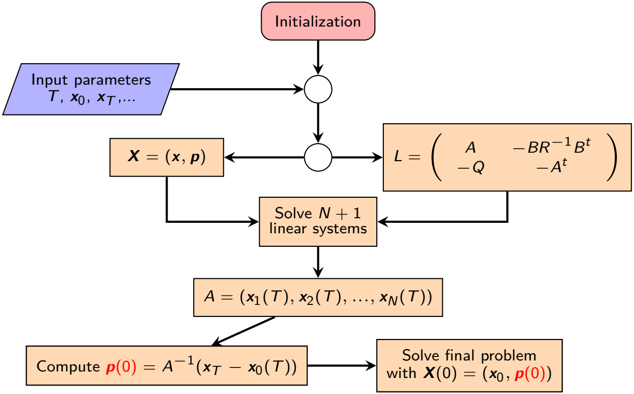

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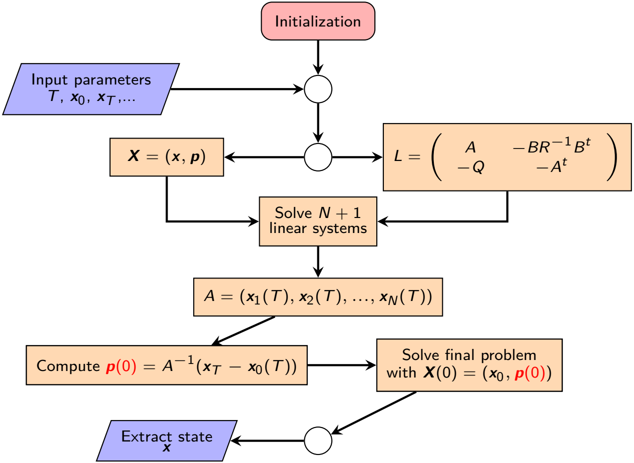

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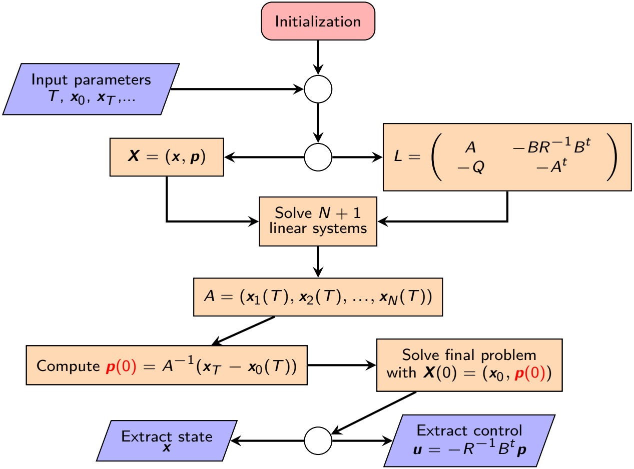

Section 4

Results

All simulations were performed with a functional of the type

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Functional

$$\boldsymbol{u} \longmapsto \frac{1}{2}\int_0^T \boldsymbol{x}^t Q \boldsymbol{x} + \boldsymbol{u}^t R \boldsymbol{u}$$

Linearised model

Linearised model – Detumbling

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- Objective:
  - Dissipate rotational energy.
- Initial state:
  - Tumbling ($\boldsymbol{\omega} \neq \mathbf{0}$).
  - (pitch,roll,yaw) $= (0, 0, 0)$*rad*.
- Final state:
  - At rest ($\boldsymbol{\omega} = \mathbf{0}$).
  - (pitch,roll,yaw) $= (0, 0, 0)$*rad*.

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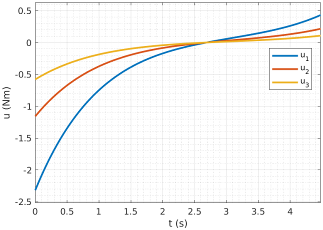

Figure: Control

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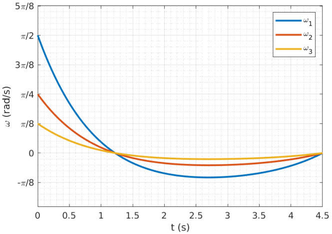

Figure: Angular velocities

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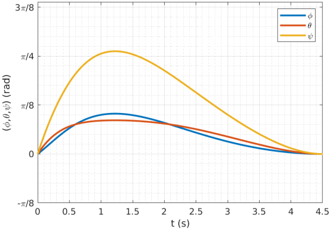

Figure: Euler angles

Simulation parameters

Name	Symbol	Value	Unit
Final time	T	4.5	s
Tensor of inertia	J	I_3	$kg\ m^2$
Integrand $F = \frac{1}{2}(\boldsymbol{x}^t Q \boldsymbol{x} + \boldsymbol{u}^t R \boldsymbol{u})$	Q	I_6	
	R	I_3	
Initial angular velocity	$\boldsymbol{\omega}_0$	$(\pi/2, \pi/4, \pi/8)$	rad/s
Final angular velocity	$\boldsymbol{\omega}_T$	$(0, 0, 0)$	rad/s
Initial orientation (Euler angles)	Φ_0	$(0, 0, 0)$	rad
Final orientation (Euler angles)	Φ_T	$(0, 0, 0)$	rad

Table: Simulation parameters

Simulation results

Name	Symbol	Value	Unit
Achieved final angular velocity	$\boldsymbol{\omega}(T)$	$(4.22, 2.11, 2.14)10^{-3}$	rad/s
Error in final angular velocity	$\lvert\boldsymbol{\omega}(T) - \boldsymbol{\omega}_T\rvert$	$8.39 \cdot 10^{-3}$	rad/s
Achieved final orientation	$\Phi(T)$	$(8.48, 1.63, 3.29)10^{-3}$	rad
Error in final orientation	$\lvert\Phi(T) - \Phi_T\rvert$	$3.77 \cdot 10^{-2}$	rad
Cost	$I(\boldsymbol{u})$	4.95	
CPU time		$1.68 \cdot 10^{-3}$	s

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Table: Simulation results

Simulation parameters

Name	Symbol	Value	Unit
Final time	T	4.5	s
Tensor of inertia	J	I_3	$kg\ m^2$
Integrand $F = \frac{1}{2}(\boldsymbol{x}^t Q \boldsymbol{x} + \boldsymbol{u}^t R \boldsymbol{u})$	Q	I_6	
	R	I_3	
Initial angular velocity	$\boldsymbol{\omega}_0$	$(\pi/2, \pi/4, \pi/8)$	rad/s
Final angular velocity	$\boldsymbol{\omega}_T$	$(0, 0, 0)$	rad/s
Initial orientation (Euler angles)	Φ_0	$(0, 0, 0)$	rad
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Table: Simulation parameters.

Tensor of inertia J

Tensor of inertia J proportional to the identity

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$$\begin{cases} J\boldsymbol{\omega}' = -\cancel{\boldsymbol{\omega} \times J\boldsymbol{\omega}} + \boldsymbol{u} \\ \boldsymbol{\epsilon}' = -\dfrac{1}{2}\boldsymbol{\omega} \times \boldsymbol{\epsilon} + \dfrac{1}{2}\eta\boldsymbol{\omega} \\ \eta' = -\dfrac{1}{2}\boldsymbol{\omega}^t\boldsymbol{\epsilon} \end{cases}$$

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Nonlinear effects are less noticeable.

If J is not proportional to the identity

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Identical configuration, except:

$$J = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0.75 & 0 \\ 0 & 0 & 0.5 \end{pmatrix} kg\ m^2$$

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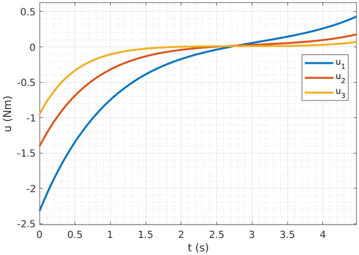

Figure: Control

If J is not proportional to the identity

Identical configuration, except:

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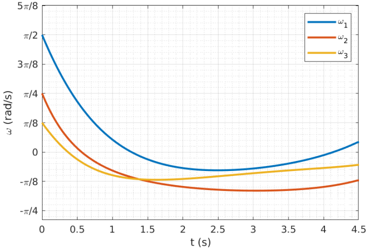

Figure: Angular velocities

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Identical configuration, except:

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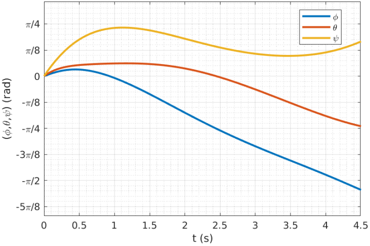

Figure: Euler angles

If J is not proportional to the identity – Simulation results

Name	Symbol	Value	Unit
Achieved final angular velocity	$\boldsymbol{\omega}(T)$	$(1.33, -3.81, -1.71)10^{-1}$	*rad/s*
Error in final angular velocity	$\lvert\boldsymbol{\omega}(T) - \boldsymbol{\omega}_T\rvert$	$4.38 \cdot 10^{-1}$	*rad/s*
Achieved final orientation	$\Phi(T)$	$(-1.71, -0.75, 0.52)$	*rad*
Error in final orientation	$\lvert\Phi(T) - \Phi_T\rvert$	1.94	*rad*
Cost	$I(\boldsymbol{u})$	4.52	
CPU time		$2.56 \cdot 10^{-3}$	s

Table: Simulation results.

If J is not proportional to the identity – Simulation results

Name	Symbol	Value	Unit
Achieved final angular velocity	$\boldsymbol{\omega}(T)$	$(1.33, -3.81, -1.71)10^{-1}$	rad/s
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Cost	$I(\boldsymbol{u})$	4.52	
CPU time		$2.56 \cdot 10^{-3}$	s

Table: Simulation results.

If J is not proportional to the identity – Simulation results

- Error in angular velocity:

$$|\boldsymbol{\omega}(T) - \boldsymbol{\omega}_T| = 0.44\ rad/s \approx 4\ rpm$$

- Error in orientation (Euler angles):

$$|\phi(T) - \phi_T| = 1.94\ rad \approx 111^\circ$$

If J is not proportional to the identity – Simulation results

The linearised model is not able to steer the system to the desired state.

Nonlinear model

Nonlinear model – Detumbling

Nonlinear model – Detumbling

- Objective:
  - Dissipate rotational energy.
- Initial state:
  - Tumbling ($\boldsymbol{\omega} \neq \mathbf{0}$).
  - (pitch,roll,yaw) $= (0, 0, 0) rad$.
- Final state:
  - At rest ($\boldsymbol{\omega} = \mathbf{0}$).
  - (pitch,roll,yaw) $= (0, 0, 0) rad$.

Simulation parameters

Simulation parameters

Parameters are set as in the previous case:

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Name	Symbol	Value	Unit
Final time	T	4.5	s
Integrand $F = \frac{1}{2}(\boldsymbol{x}^t Q \boldsymbol{x} + \boldsymbol{u}^t R \boldsymbol{u})$	Q	I_7	
	R	I_3	
Initial angular velocity	$\boldsymbol{\omega}_0$	$(\pi/2, \pi/4, \pi/8)$	rad/s
Final angular velocity	$\boldsymbol{\omega}_T$	$(0, 0, 0)$	rad/s
Initial orientation (Euler angles)	Φ_0	$(0, 0, 0)$	rad
Final orientation (Euler angles)	Φ_T	$(0, 0, 0)$	rad

Table: Simulation parameters

Simulation results

Simulation results

The nonlinear model behaves well for any choice of J.

Simulation results

$$J = I_{3\times 3}$$

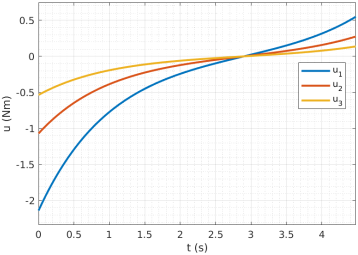

Figure: Control.

$$J = diag(1, 0.75, 0.5)$$

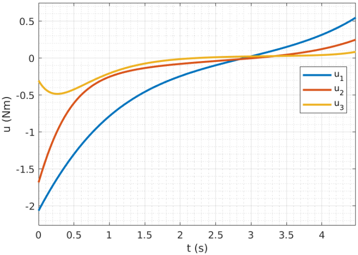

Figure: Control.

Simulation results

$J = I_{3\times3}$

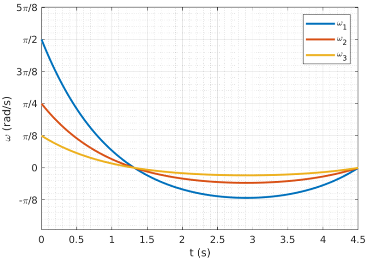

Figure: Angular velocities.

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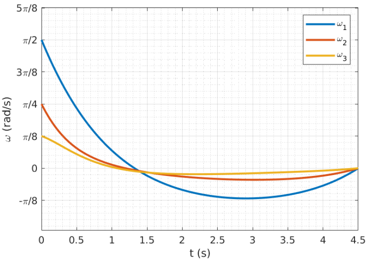

Figure: Angular velocities.

Simulation results

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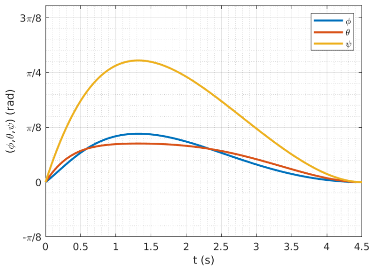

Figure: Euler angles.

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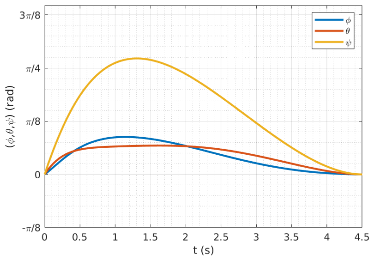

Figure: Euler angles

J proportional to the identity – Simulation results

- Error in angular velocity:
$$|\boldsymbol{\omega}(T) - \boldsymbol{\omega}_T| = 7 \cdot 10^{-3} \ rad/s \approx 0.07 \ rpm$$
- Error in orientation (Euler angles):
$$|\phi(T) - \phi_T| = 1.59 \cdot 10^{-4} \ rad \approx 0.009^\circ$$

J not proportional to the identity – Simulation results

- Error in angular velocity:

$$|\boldsymbol{\omega}(T) - \boldsymbol{\omega}_T| = 6.1 \cdot 10^{-3}\ rad/s \approx 0.06\ rpm$$

- Error in orientation (Euler angles):

$$|\phi(T) - \phi_T| = 1.55 \cdot 10^{-4}\ rad \approx 0.009^\circ$$

J not proportional to the identity – Simulation results

The nonlinear model is able to steer the system to the desired state.

Nonlinear model

Nonlinear model – Single axis, rest to rest

Nonlinear model – Single axis, rest to rest

- Objective:
  - Rotate $\pi/2$ *rad* around the z axis.
- Initial state:
  - At rest ($\boldsymbol{\omega} = \mathbf{0}$).
  - (pitch,roll,yaw) $= (0, 0, 0)$*rad*.
- Final state:
  - At rest ($\boldsymbol{\omega} = \mathbf{0}$).
  - (pitch,roll,yaw) $= (\pi/2, 0, 0)$*rad*.
- Inequality constraints on control: $|u_i| \leq 1Nm\ i = 1, 2, 3$.

Nonlinear model – Single axis, rest to rest

- Objective:
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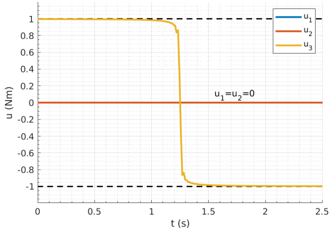

Figure: Control

Nonlinear model – Single axis, rest to rest

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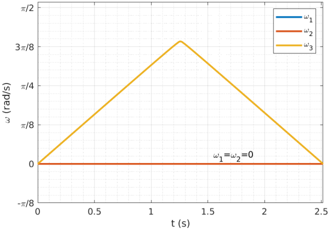

Figure: Angular velocities

Nonlinear model – Single axis, rest to rest

- Objective:
  - Rotate $\pi/2$ *rad* around the *z* axis.
- Initial state:
  - At rest ($\boldsymbol{\omega} = \mathbf{0}$).
  - (pitch,roll,yaw) $= (0, 0, 0)$*rad*.
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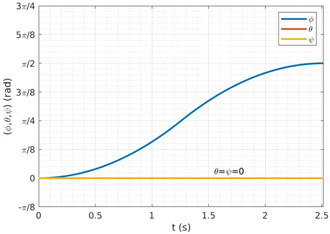

Figure: Euler angles

Simulation parameters

Simulation parameters

Name	Symbol	Value	Unit
Final time	T	$2.55, 2.525, 2.52$	s
Tensor of inertia	J	$diag(0.5, 0.75, 1)$	$Kg\ m^2$
Integrand $F = \frac{1}{2}(\boldsymbol{x}^t Q\boldsymbol{x} + \boldsymbol{u}^t R\boldsymbol{u})$	Q R	0_6 0_3	
Initial angular velocity	$\boldsymbol{\omega}_0$	$(0, 0, 0)$	rad/s
Final angular velocity	$\boldsymbol{\omega}_T$	$(0, 0, 0)$	rad/s
Initial orientation (Euler angles)	Φ_0	$(0, 0, 0)$	rad
Final orientation (Euler angles)	Φ_T	$(\pi/2, 0, 0)$	rad

Table: Simulation parameters

Bang-bang effect

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When inequality constraints on control are imposed

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For example: $|u_i| \leq 1\ Nm \quad i = 1, 2, 3$

Bang-bang effect

When inequality constraints on control are imposed

For example: $|u_i| \leq 1\ Nm \quad i = 1, 2, 3$

The control approaches a *bang-bang* strategy as the time interval is reduced.

Bang-bang effect

Bang-bang effect

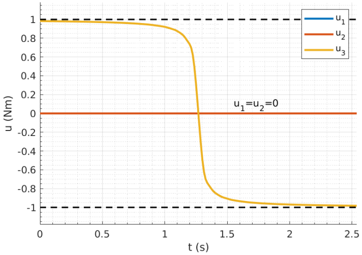


Figure: Control for $T = 2.55s$.

Bang-bang effect

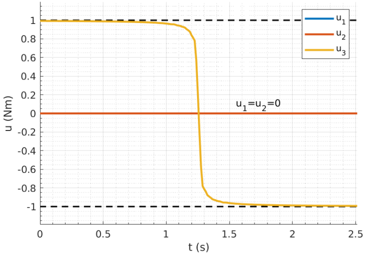


Figure: Control for $T = 2.525s$.

Bang-bang effect

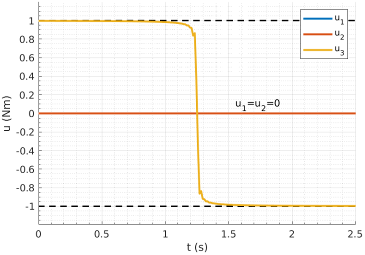
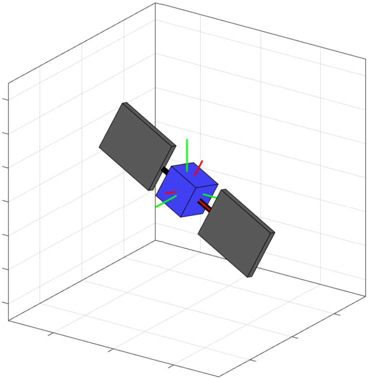


Figure: Control for $T = 2.52s$.

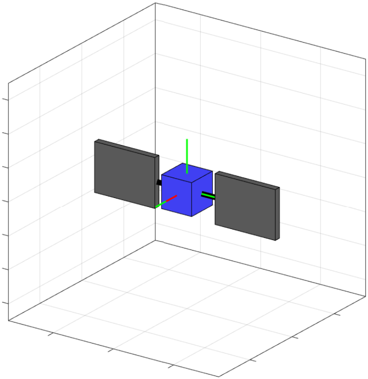
Simulation results

- Error in angular velocity:
$$|\boldsymbol{\omega}(T) - \boldsymbol{\omega}_T| = 2.84 \cdot 10^{-3} \ rad/s \approx 0.03 \ rpm$$
- Error in orientation (Euler angles):
$$|\phi(T) - \phi_T| = 3.8 \cdot 10^{-3} \ rad \approx 0.2^\circ$$

Animation – Detumbling



Animation – Single axis, rest to rest



Section 5

Conclusions and future work

Conclusions

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- Two different approaches have been employed:

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  - Linearised model.

Conclusions

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    - Nonlinear model (Main aim of the work).

Conclusions

- Two different approaches have been employed:
  - Linearised model.
  - Nonlinear model (Main aim of the work).
- Both have proven successful in certain situations.

Comparison

Advantages of the nonlinear approach:

Advantages of the linear approach:

Comparison

Advantages of the nonlinear approach:

- Accuracy.

Advantages of the linear approach:

Comparison

Advantages of the nonlinear approach:

- Accuracy.
- Constraints.

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- Accuracy.
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- Larger class of integrands.

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Advantages of the linear approach:

- Computation time.

Comparison

Advantages of the nonlinear approach:

- Accuracy.
- Constraints.
- Larger class of integrands.
- Complex manoeuvres.

Advantages of the linear approach:

- Computation time.
  - Linearised model: $CPU\ time \sim 10^{-3} s$.
  - Nonlinear model: $CPU\ time \sim 10 s$.

Simulations

Simulations

- Fortran 95.

Simulations

- Fortran 95.
- Compiled with gfortran.

Simulations

- Fortran 95.
- Compiled with gfortran.
- Linux machine.

Simulations

- Fortran 95.
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- Linux machine.
- Intel Core i7 4510U.

Future work

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- Controllability.

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- Influence of:

Future work

- Controllability.
- Influence of:
  - Magnetic actuation.

Future work

- Controllability.
- Influence of:
  - Magnetic actuation.
  - Disturbances

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Future work

- Controllability.
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  - Magnetic actuation.
  - Disturbances, such as atmospheric drag.
- Underactuated satellites.
- Uncertainty.
- Incomplete measurements.
- Improvement of numerical procedures.
- Implementation and testing.

Thank you.