

# Assignment 3

In this assignment, you'll explore linear regression and logistic regression, the perceptron, and support vector machines: popular approaches which share a similar form of producing output in the form of a weighted combination of attributes, but with very different approaches to learning the underlying model. The assignment explores two of the general data mining tasks: regression and classification, and takes a closer look at multiclass classification with these techniques.

Similar to previous assignments, you're expected to respond to each question with your answer in a Markdown cell and clearly labeled code supporting your answer in a code cell. When you submit the assignment, you should upload the two notebooks (.ipynb files) corresponding to your solutions and also generate a PDF of each notebook that includes the answers, code, and all intermediate output. In total, you will submit four files: two notebooks and two PDFs generated from those notebooks. This assignment is due on 2/16/16 at 11:59pm.

## Part 1: Linear and Logistic Regression, Perceptrons (40 points)

We'll start by looking at a simple example of linear regression. As with many of the other data mining algorithms we've encountered, we'll be using the implementation available in the `linear_model` module of scikit-learn. Now would be a good time to go read the [documentation \(http://scikit-learn.org/stable/modules/linear\\_model.html\)](http://scikit-learn.org/stable/modules/linear_model.html) on linear models and refresh your memory on the high-level ideas. Once you've done that, we'll go through a simple example.

```
In [15]: ## Preliminaries

#Show plots in the notebook
%matplotlib inline

from sklearn import datasets, preprocessing, cross_validation, feature_
from sklearn import linear_model, svm, metrics, ensemble
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import urllib2

# Helper functions
def folds_to_split(data,targets,train,test):
    data_tr = pd.DataFrame(data).iloc[train]
    data_te = pd.DataFrame(data).iloc[test]
    labels_tr = pd.DataFrame(targets).iloc[train]
    labels_te = pd.DataFrame(targets).iloc[test]
    return [data_tr, data_te, labels_tr, labels_te]
```

## Linear Regression on the Diabetes Dataset

The first dataset we'll explore is the [Diabetes dataset \(http://scikit-learn.org/stable/modules/generated/sklearn.datasets.load\\_diabetes.html#sklearn.datasets.load\\_diabetes\)](http://scikit-learn.org/stable/modules/generated/sklearn.datasets.load_diabetes.html#sklearn.datasets.load_diabetes) one of the pre-packaged datasets available in scikit-learn. There aren't too many details about the dataset out there, but the common description is that it contains 10 physiological variables (age, weight, blood pressure) for 442 patients, and the target is an indication of disease progression in years. Like any good data miners, let's poke around and check out the data before we get started.

```
In [16]: # Load the data
diabetes = datasets.load_diabetes();
# Put it into pandas DataFrames
diabetes_data_df = pd.DataFrame(diabetes.data);
diabetes_target_df = pd.DataFrame(diabetes.target)

# How many attributes and records are there?
print diabetes_data_df.shape

# What are the descriptive statistics?
print diabetes_data_df.describe()
# How are the labels distributed?
diabetes_target_df.hist()

# How are the attributes distributed?
plt.figure()
axes = diabetes_data_df.boxplot()
```

(442, 10)

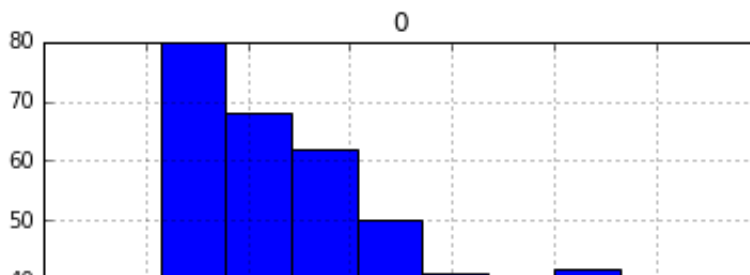
	0	1	2	3
count	4.420000e+02	4.420000e+02	4.420000e+02	4.420000e+02
mean	-3.634285e-16	1.308343e-16	-8.045349e-16	1.281655e-16
std	4.761905e-02	4.761905e-02	4.761905e-02	4.761905e-02
min	-1.072256e-01	-4.464164e-02	-9.027530e-02	-1.123996e-01
25%	-3.729927e-02	-4.464164e-02	-3.422907e-02	-3.665645e-02
50%	5.383060e-03	-4.464164e-02	-7.283766e-03	-5.670611e-03
75%	3.807591e-02	5.068012e-02	3.124802e-02	3.564384e-02
max	1.107267e-01	5.068012e-02	1.705552e-01	1.320442e-01

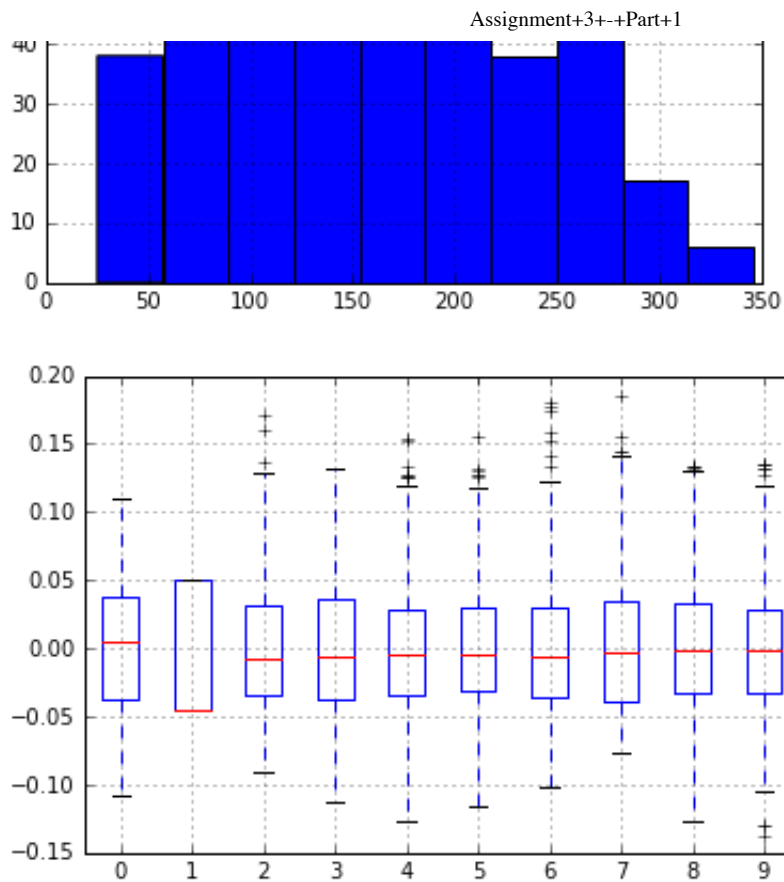
	5	6	7	8
count	4.420000e+02	4.420000e+02	4.420000e+02	4.420000e+02
mean	1.327024e-16	-4.574646e-16	3.777301e-16	-3.830854e-16
std	4.761905e-02	4.761905e-02	4.761905e-02	4.761905e-02
min	-1.156131e-01	-1.023071e-01	-7.639450e-02	-1.260974e-01
25%	-3.035840e-02	-3.511716e-02	-3.949338e-02	-3.324879e-02
50%	-3.819065e-03	-6.584468e-03	-2.592262e-03	-1.947634e-03
75%	2.984439e-02	2.931150e-02	3.430886e-02	3.243323e-02
max	1.987880e-01	1.811791e-01	1.852344e-01	1.335990e-01

/Users/viktorjankov/anaconda/lib/python2.7/site-packages/ipykernel/\_main\_.py:17: FutureWarning:  
The default value for 'return\_type' will change to 'axes' in a future release.

To use the future behavior now, set return\_type='axes'.

To keep the previous behavior and silence this warning, set return\_type='dict'.





Notice anything interesting about the data? The attributes have been normalized - each has a similar mean and standard deviation. Now let's train our first regression model.

```
In [17]: # First we'll do a simple train-test split:
[dbt_tr_data, dbt_te_data,
 dbt_tr_target, dbt_te_target] = cross_validation.train_test_split(diab

# Create the LinearRegression classifier
lr = linear_model.LinearRegression()

# Learn the linear regression model
lr.fit(dbt_tr_data, dbt_tr_target)

# Print out the coefficient of determination (R^2)
print "R^2:\t",lr.score(dbt_te_data,dbt_te_target)

# Peek at the predictions
dbt_te_predict = lr.predict(dbt_te_data)
print dbt_te_predict

# And also the mean squared error:
print "MSE:\t", metrics.mean_squared_error(dbt_te_target, dbt_te_predic

# Which attributes were important to this prediction? We can find out k
print lr.coef_
```

```

R^2:    0.533716155537
[ 124.87699265  241.04779116  239.55002222  262.61331893  245.760759
35
  215.38130957   98.25024738  167.10341193   86.15467868  149.231306
48
  139.44688429  121.07843815  145.93392542  113.89242688  137.000019
59
  106.07596308  188.8720697   151.64642212  147.17548161  193.976209
2
  193.24395545  123.38347892  244.93128416  221.35017651   73.231771
37
  175.0689274   270.81318019   76.43093304  147.64128689  106.932261
73
   72.18329933  246.22062792  155.49372733  132.93631466  122.141629
7
   70.14373998  182.1432308   259.16039335  119.69523429  217.322246
65
  156.87184212  209.45289148  152.68147644   97.91484124   99.479911
66
   60.35999351  163.47427462  175.63241326  165.73475646  120.171374
96
  143.1380446   65.50402188  161.25927147  165.59617116  127.563187
62
  145.47983532  119.2772516   140.89228502  138.44957973  234.270316
41
   70.42410076  219.75216763   96.49453974  169.97467564   75.491765
72
  141.43569069  146.08650475   97.22882088  235.72934302  115.606263
81
  140.86586191   83.8398695   226.79822353  232.14151232  153.569348
61
   85.00662969  185.94162205  100.59978839   80.74525585  160.501532
47
  136.07622533  111.57342257  150.66490274  181.52512021  203.544638
09
  205.67406873  142.71143086  127.42691032  139.56003045  151.354371
13
  242.54261548  206.12199133  145.09312121  120.10821335  248.910441
86
  177.46641126  145.28356072   79.89910472  155.73208421  113.432393
24
  162.18337986   99.99524699  191.84896583  154.24943403  166.819089
55
  104.90171597  220.98892005  134.51002344   38.0086503   203.293999
74
  181.12997599]
MSE:    2919.42264017
[ 11.18121228 -219.22917848  492.74550531  360.62854712 -880.756963
56
  560.47207888  138.68964051  208.07342467  723.26859805   47.007418
34]

```

Hopefully you can see the similarities between how to build a LinearRegression model ([http://scikit-learn.org/stable/modules/generated/sklearn.linear\\_model.LinearRegression.html#sklearn.linear\\_model.LinearRegression](http://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LinearRegression.html#sklearn.linear_model.LinearRegression)) and the previous assignment on DecisionTrees. The big difference is that the predictions are no longer values (much like the training targets). One consequence is that we can't use the same metrics as for classification. One metric that we've discussed (and that linear regression attempts to minimize) is the squared error ([https://en.wikipedia.org/wiki/Mean\\_squared\\_error](https://en.wikipedia.org/wiki/Mean_squared_error)) or MSE. If you understand how to use the `.score()` function, you can leap to the default metric that regression reports (via the `.score()` function) -- the coefficient of determination ([https://en.wikipedia.org/wiki/Coefficient\\_of\\_determination](https://en.wikipedia.org/wiki/Coefficient_of_determination)) ( $R^2$ ) -- shouldn't be too hard to understand. Make sure you understand what it measures.

The above example used a simple train-test split, but we can be more confident of our conclusions with cross-validation. Let's use cross-validation to experiment with the same dataset and take a look at the results. The example provides you with some tricks that you might have missed in the last assignment, so use them below and learning to use it should be helpful.



```

In [18]: foldnum = 0
fold_results = pd.DataFrame()
for train, test in cross_validation.KFold(len(diabetes.data), n_folds=1
    foldnum+=1
    [dbt_tr_data, dbt_te_data,
     dbt_tr_target, dbt_te_target] = folds_to_split(diabetes.data,diabe

    lr = linear_model.LinearRegression()
    lr.fit(dbt_tr_data, dbt_tr_target)
    # We could print out our results
    print "Fold %d\t\t R^2 metric = %03.3f \t\t MSE = %03.1f " % (foldn
                                                lr.sc
                                                metri

                                                )

    # But a nicer way to store them is in a DataFrame
    fold_results.loc[foldnum, 'R^2'] = lr.score(dbt_te_data, dbt_te_tar
    fold_results.loc[foldnum, 'MSE'] = metrics.mean_squared_error(dbt_t

    # By the way, if you were searching over parameters in an inner for
    # you could store those in your results DataFrame just as easily, f
    # for param in params.keys():
    #     for paramVal in params[param]:
    #         paramDict={};
    #         paramDict[param]=paramVal
    #         dtree = tree.DecisionTreeClassifier(random_state=20160121,
    #         dtree.fit(mushroom_train,mushroom_train_labels)
    #         fold_results.loc[foldnum,'%s=%s' % (param, paramVal)]=dtre

#Now let's look at the results:
print fold_results
#And compute the mean error across folds:
print fold_results.mean()

```

Fold 1	R <sup>2</sup> metric = 0.556	MSE = 2533.8
Fold 2	R <sup>2</sup> metric = 0.231	MSE = 2870.8
Fold 3	R <sup>2</sup> metric = 0.354	MSE = 3512.7
Fold 4	R <sup>2</sup> metric = 0.622	MSE = 2759.2
Fold 5	R <sup>2</sup> metric = 0.266	MSE = 3555.7
Fold 6	R <sup>2</sup> metric = 0.618	MSE = 2900.4
Fold 7	R <sup>2</sup> metric = 0.418	MSE = 3696.3
Fold 8	R <sup>2</sup> metric = 0.435	MSE = 2282.3
Fold 9	R <sup>2</sup> metric = 0.434	MSE = 4122.9
Fold 10	R <sup>2</sup> metric = 0.686	MSE = 1769.7

	R <sup>2</sup>	MSE
1	0.556144	2533.848109
2	0.230561	2870.767711
3	0.353578	3512.723509
4	0.621905	2759.227129
5	0.265876	3555.677943
6	0.618193	2900.380412
7	0.418159	3696.281878
8	0.435152	2282.279598
9	0.434370	4122.939981
10	0.685685	1769.684057
R <sup>2</sup>	0.461962	
MSE	3000.381033	

dtype: float64

## Question 1: Linear Regression, Ridge Regression Lasso (20 points)

Now it's your turn to perform a linear regression experiment! Use the [Boston Housing Prices Dataset](http://scikit-learn.org/stable/modules/generated/sklearn.datasets.load_boston.html#sklearn.datasets.load_boston) ([http://scikit-learn.org/stable/modules/generated/sklearn.datasets.load\\_boston.html#sklearn.datasets.load\\_boston](http://scikit-learn.org/stable/modules/generated/sklearn.datasets.load_boston.html#sklearn.datasets.load_boston)) built into scikit-learn to better understand regression. You can load it with `datasets.load_boston` and use the `.DESCR` field to learn more about the dataset.

1. Perform exploratory data analysis on the Boston Housing data.
  - From your exploratory data analysis, what do you notice about the attribute values?
  - Which parameter to the `LinearRegression` model could you use to deal with this issue?
  - Perform 10-fold cross-validation (with `shuffle=True` and `random_state=2016020`) with a `LinearRegression` model that uses the parameter identified above. Report the average coefficient of determination and mean squared error metric on the test set (averaged across folds).
2. Two very popular options in Linear Regression are the [Lasso method](https://en.wikipedia.org/wiki/Lasso_(statistics)) ([https://en.wikipedia.org/wiki/Lasso\\_\(statistics\)](https://en.wikipedia.org/wiki/Lasso_(statistics))) and [Ridge Regression](https://en.wikipedia.org/wiki/Tikhonov_regularization) ([https://en.wikipedia.org/wiki/Tikhonov\\_regularization](https://en.wikipedia.org/wiki/Tikhonov_regularization)). These are implemented in the [Lasso](http://scikit-learn.org/stable/modules/generated/sklearn.linear_model.Lasso.html#sklearn.linear_model.Lasso) ([http://scikit-learn.org/stable/modules/generated/sklearn.linear\\_model.Lasso.html#sklearn.linear\\_model.Lasso](http://scikit-learn.org/stable/modules/generated/sklearn.linear_model.Lasso.html#sklearn.linear_model.Lasso)).

and Ridge ([http://scikit-learn.org/stable/modules/generated/sklearn.linear\\_model.Ridge.html#sklearn.linear\\_model.Ridge](http://scikit-learn.org/stable/modules/generated/sklearn.linear_model.Ridge.html#sklearn.linear_model.Ridge)) models in sklearn.

- What *one word* captures what Lasso and Ridge do? (It's in the first sentence of both Wikipedia articles and sklearn documentation pages)
- Perform 10-fold CV with normal LinearRegression, Lasso, and Ridge. Compare the attribute weights (`coef_`) of each of these methods. What do you observe?
- Report the average  $R^2$  and MSE for each method on the test set across folds. How do these variants of linear regression perform?

Hint: If you'd like to understand Ridge Regression better, you might want to look at this [example](http://scikit-learn.org/stable/auto_examples/linear_model/plot_ols_ridge_variance.html) ([http://scikit-learn.org/stable/auto\\_examples/linear\\_model/plot\\_ols\\_ridge\\_variance.html](http://scikit-learn.org/stable/auto_examples/linear_model/plot_ols_ridge_variance.html)).

## Answers 1:

### 1.

**a)** From the exploratory data analysis, we noticed that the range of the values is huge, which is not good. For example, most of the attributes have a value below 100, while the 9th and 11th attribute have values in the hundreds. Between 300 and 700 for the 9th attribute [TAX] and around 400 for the 11th attribute [B]

**b)** To deal with this issue, we'll use the `normalize=True` parameter to the LinearRegression model

**c)** Averages:

$R^2$  0.713643

MSE 23.657502

### 2.

**a)** Regularization. Both Lasso and Ridge regression are used for regularization to prevent overfitting

**b)** Looking at the attribute weights, we see several differences between the different models. First, Lasso gets completely rid of most features, by setting the weights of 0. It only keeps 2 or 3 features in most cases as the only relevant ones to predict the median house price. The Ridge regression model on the other hand, keeps most features, but we can notice that the weights are slightly different to the weights of the original linear regression (i.e. there are only small changes to the weights).

**c)**

Averages:

Linear Regression:

$R^2$  0.713643

MSE 23.657502

Lasso Regularization:

$R^2$  0.580869

MSE 35.198977

Ridge Regression:

$R^2$  0.708486

MSE 24.133678

TTest for LinearRegression vs RidgeRegression

Ttest\_relResult(statistic=1.2154248843325626, pvalue=0.25512413165702003)

TTest for Lasso vs RidgeRegression

Ttest\_relResult(statistic=-9.4005571317290304, pvalue=5.9711873467613727e-06)

TTest for LinearRegression vs Lasso

Ttest\_relResult(statistic=8.7799598190605064, pvalue=1.0449626130453215e-05)

The Lasso regression model performs significantly worse than the linear regression and the ridge regression model. This means that the model is not overfitted, and most features are relevant to predicting the median house prices. So when Lasso gets rid of some features, the model performs worse.

The Ridge regression performs slightly worse than the regular linear regression, but according to the ttest, this difference is not significant.

```
In [19]: # Load the data  
boston = datasets.load_boston();  
  
# Brief description of the data  
print boston.DESCR
```

## Boston House Prices dataset

## Notes

-----

## Data Set Characteristics:

:Number of Instances: 506

:Number of Attributes: 13 numeric/categorical predictive

:Median Value (attribute 14) is usually the target

## :Attribute Information (in order):

- CRIM	per capita crime rate by town
- ZN	proportion of residential land zoned for lots over 25,000 sq.ft.
- INDUS	proportion of non-retail business acres per town
- CHAS	Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)
- NOX	nitric oxides concentration (parts per 10 million)
- RM	average number of rooms per dwelling
- AGE	proportion of owner-occupied units built prior to 1940
- DIS	weighted distances to five Boston employment centres
- RAD	index of accessibility to radial highways
- TAX	full-value property-tax rate per \$10,000
- PTRATIO	pupil-teacher ratio by town
- B	$1000(B_k - 0.63)^2$ where $B_k$ is the proportion of blacks by town
- LSTAT	% lower status of the population
- MEDV	Median value of owner-occupied homes in \$1000's

:Missing Attribute Values: None

:Creator: Harrison, D. and Rubinfeld, D.L.

This is a copy of UCI ML housing dataset.

<http://archive.ics.uci.edu/ml/datasets/Housing> (<http://archive.ics.uci.edu/ml/datasets/Housing>)

This dataset was taken from the StatLib library which is maintained at Carnegie Mellon University.

The Boston house-price data of Harrison, D. and Rubinfeld, D.L. 'Hedonic prices and the demand for clean air', J. Environ. Economics & Management, vol.5, 81-102, 1978. Used in Belsley, Kuh & Welsch, 'Regression diagnostics...', Wiley, 1980. N.B. Various transformations are used in the table on

pages 244-261 of the latter.

The Boston house-price data has been used in many machine learning papers that address regression problems.

**\*\*References\*\***

- Belsley, Kuh & Welsch, 'Regression diagnostics: Identifying Influential Data and Sources of Collinearity', Wiley, 1980. 244-261.
- Quinlan, R. (1993). Combining Instance-Based and Model-Based Learning. In Proceedings on the Tenth International Conference of Machine Learning, 236-243, University of Massachusetts, Amherst. Morgan Kaufmann.
- many more! (see <http://archive.ics.uci.edu/ml/datasets/Housing>) (<http://archive.ics.uci.edu/ml/datasets/Housing>)

```
In [20]: # Part 1a) Explore the dataset
# Put it into pandas DataFrames
boston_data_df = pd.DataFrame(boston.data);
boston_target_df = pd.DataFrame(boston.target)

# How many attributes and records are there?
print boston_data_df.shape

# What are the descriptive statistics?
print boston_data_df.describe()

# How are the labels distributed?
boston_target_df.hist()

# How are the attributes distributed?
plt.figure()
axes = boston_data_df.boxplot()
```



(506, 13)

	0	1	2	3	4	
5 \						
count	506.000000	506.000000	506.000000	506.000000	506.000000	5
06.000000						
mean	3.593761	11.363636	11.136779	0.069170	0.554695	
6.284634						
std	8.596783	23.322453	6.860353	0.253994	0.115878	
0.702617						
min	0.006320	0.000000	0.460000	0.000000	0.385000	
3.561000						
25%	0.082045	0.000000	5.190000	0.000000	0.449000	
5.885500						
50%	0.256510	0.000000	9.690000	0.000000	0.538000	
6.208500						
75%	3.647423	12.500000	18.100000	0.000000	0.624000	
6.623500						
max	88.976200	100.000000	27.740000	1.000000	0.871000	
8.780000						

	6	7	8	9	10	
11 \						
count	506.000000	506.000000	506.000000	506.000000	506.000000	5
06.000000						
mean	68.574901	3.795043	9.549407	408.237154	18.455534	3
56.674032						
std	28.148861	2.105710	8.707259	168.537116	2.164946	
91.294864						
min	2.900000	1.129600	1.000000	187.000000	12.600000	
0.320000						
25%	45.025000	2.100175	4.000000	279.000000	17.400000	3
75.377500						
50%	77.500000	3.207450	5.000000	330.000000	19.050000	3
91.440000						
75%	94.075000	5.188425	24.000000	666.000000	20.200000	3
96.225000						
max	100.000000	12.126500	24.000000	711.000000	22.000000	3
96.900000						

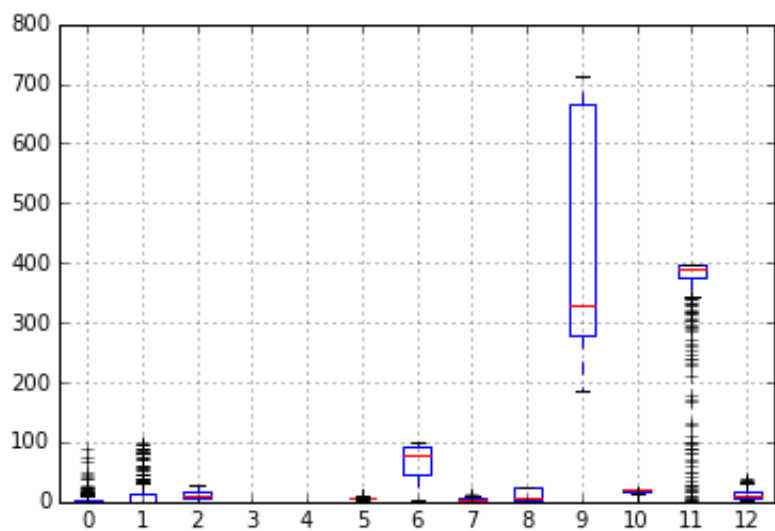
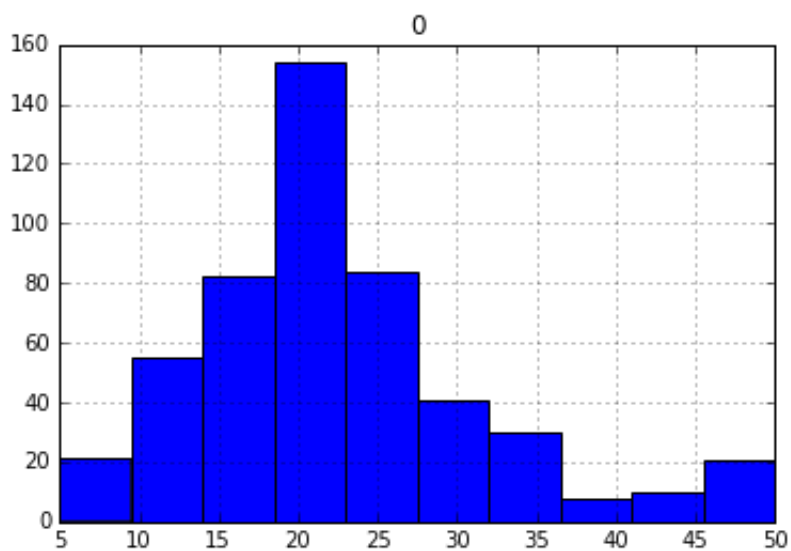
	12
count	506.000000
mean	12.653063
std	7.141062
min	1.730000
25%	6.950000
50%	11.360000
75%	16.955000
max	37.970000

/Users/viktorjankov/anaconda/lib/python2.7/site-packages/ipykernel/\_  
 \_main\_\_.py:17: FutureWarning:  
 The default value for 'return\_type' will change to 'axes' in a futur  
 e release.

To use the future behavior now, set return\_type='axes'.

To keep the previous behavior and silence this warning, set return

To keep the previous behavior and silence this warning, set `return_type='dict'`.



```

In [21]: # Part 1b) Cross Validation
foldnum = 0
fold_results = pd.DataFrame()
for train, test in cross_validation.KFold(len(boston.data), n_folds=10,
    foldnum+=1
    [boston_tr_data, boston_te_data,
    boston_tr_target, boston_te_target] = folds_to_split(boston.data,b

lr = linear_model.LinearRegression(normalize=True)
lr.fit(boston_tr_data, boston_tr_target)

# But a nicer way to store them is in a DataFrame
fold_results.loc[foldnum, 'R^2'] = lr.score(boston_te_data, boston_
fold_results.loc[foldnum, 'MSE'] = metrics.mean_squared_error(bosto

#Now let's look at the results:
print fold_results
#And compute the mean error across folds:
print fold_results.mean()

```

	R^2	MSE
1	0.717454	23.926098
2	0.620171	34.779133
3	0.783767	18.841212
4	0.556936	30.379589
5	0.769709	16.603228
6	0.693492	22.287915
7	0.828865	19.256283
8	0.631266	33.237697
9	0.796774	16.961117
10	0.737993	20.302751
R^2	0.713643	
MSE	23.657502	
dtype:	float64	

In [22]:

*# Part 2 Using Lasso and Ridge Regression*

```

from scipy import stats

foldnum = 0
lr_fold_results = pd.DataFrame()
lasso_fold_results = pd.DataFrame()
ridge_fold_results = pd.DataFrame()

for train, test in cross_validation.KFold(len(boston.data), n_folds=10,
    foldnum+=1
    [boston_tr_data, boston_te_data,
      boston_tr_target, boston_te_target] = folds_to_split(boston.data,b

    lr = linear_model.LinearRegression(normalize=True)
    lr.fit(boston_tr_data, boston_tr_target)

    lasso = linear_model.Lasso(alpha=0.1, normalize=True, random_state=
    lasso.fit(boston_tr_data, boston_tr_target)

    ridge = linear_model.Ridge(alpha=0.1, normalize=True, random_state=
    ridge.fit(boston_tr_data, boston_tr_target)

    # But a nicer way to store them is in a DataFrame
    lr_fold_results.loc[foldnum, 'R^2'] = lr.score(boston_te_data, bost
    lr_fold_results.loc[foldnum, 'MSE'] = metrics.mean_squared_error(b

    lasso_fold_results.loc[foldnum, 'R^2'] = lasso.score(boston_te_data
    lasso_fold_results.loc[foldnum, 'MSE'] = metrics.mean_squared_error

    ridge_fold_results.loc[foldnum, 'R^2'] = ridge.score(boston_te_data
    ridge_fold_results.loc[foldnum, 'MSE'] = metrics.mean_squared_error

#Now let's look at the results:
print "TTest for LinearRegression vs RidgeRegression"
print stats.ttest_rel(lr_fold_results['R^2'].values, ridge_fold_results
print "\n"

print "TTest for Lasso vs RidgeRegression"
print stats.ttest_rel(lasso_fold_results['R^2'].values, ridge_fold_resu
print "\n"

print "TTest for LinearRegression vs Lasso"
print stats.ttest_rel(lr_fold_results['R^2'].values, lasso_fold_results
print "\n"

#And compute the mean error across folds:
print "Linear Regression:"
print lr_fold_results.mean()
print "\n"

print "Lasso Regularization:"
print lasso_fold_results.mean()

```

```
print "\n"

print "Ridge Regression: "
print ridge_fold_results.mean()
```

TTest for LinearRegression vs RidgeRegression

Ttest\_relResult(statistic=1.2154248843325628, pvalue=0.25512413165702003)

TTest for Lasso vs RidgeRegression

Ttest\_relResult(statistic=-9.4005571317290268, pvalue=5.971187346761393e-06)

TTest for LinearRegression vs Lasso

Ttest\_relResult(statistic=8.7799598190605046, pvalue=1.0449626130453218e-05)

Linear Regression:

R<sup>2</sup> 0.713643  
MSE 23.657502  
dtype: float64

Lasso Regularization:

R<sup>2</sup> 0.580869  
MSE 35.198977  
dtype: float64

Ridge Regression:

R<sup>2</sup> 0.708486  
MSE 24.133678  
dtype: float64

## Logistic Regression

Now let's look at a different application for regression algorithms: classification. As we learned in class, regression algorithm can be adapted to a binary classification fairly simply. Instead of predicting some continuous value, we predict the class. So, for example, instead of predicting the diabetes condition of a patient we would predict whether or not a patient has diabetes by having the label be 1 when the patient has diabetes and 0 when the patient does not have diabetes.

In this setting the regression algorithm tries to learn some combination of features to get 0 for the negative instance and 1 for the positive instances. Of course, linear regression algorithms produce continuous values, not just 1s and 0s, so we have to convert the

continuous values into 1s and 0s. Usually this is done by rounding values above 0.5 to 1 and the remaining values to 0 (using, for example, the numpy [round](http://docs.scipy.org/doc/numpy-1.10.0/reference/generated/numpy.round.html) (<http://docs.scipy.org/doc/numpy-1.10.0/reference/generated/numpy.round.html>) function).

However, there's still something messy about this setting. Linear regression wants to predict continuous values and we're conscripting it to produce 1s and 0s. What if we had a regression algorithm that, by design, predicted 1s and 0s? This is exactly what logistic regression does. Let's look at an example of the difference, taken from the [scikit-learn](http://scikit-learn.org/stable/auto_examples/linear_model/plot_logistic.html) documentation ([http://scikit-learn.org/stable/auto\\_examples/linear\\_model/plot\\_logistic.html](http://scikit-learn.org/stable/auto_examples/linear_model/plot_logistic.html))

```
In [23]: # Code source: Gael Varoquaux
# License: BSD 3 clause

# this is our test set, it's just a straight line with some
# Gaussian noise
xmin, xmax = -5, 5
n_samples = 100
np.random.seed(0)
X = np.random.normal(size=n_samples)
y = (X > 0).astype(np.float)
X[X > 0] *= 4
X += .3 * np.random.normal(size=n_samples)

X = X[:, np.newaxis]

#Learn a Linear Regression model
lr = linear_model.LinearRegression()
lr.fit(X, y)

# Learn a Logistic Regression model
lgr = linear_model.LogisticRegression(C=1e5)
lgr.fit(X, y)

# Plot the training data (as black dots)
plt.scatter(X.ravel(), y, color='black', zorder=20)

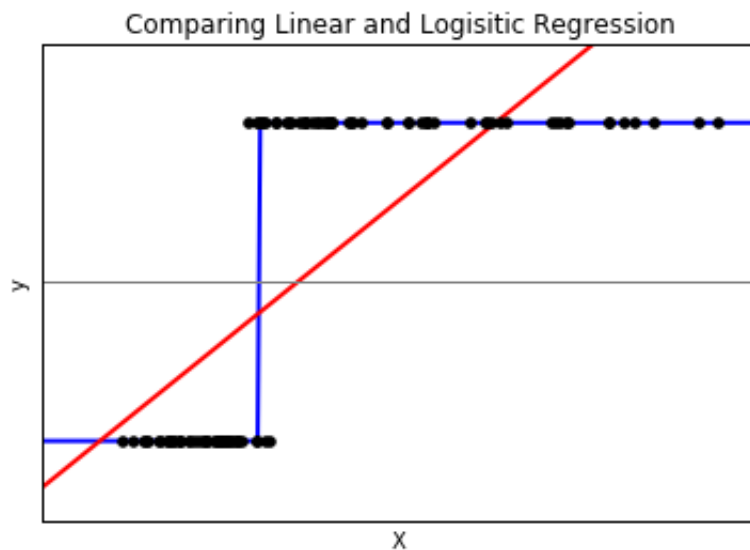
#Create some test data
X_test = np.linspace(-5, 10, 300).reshape(300,1)

# Plot the predictions of the logistic regression model (blue line)
plt.plot(X_test, lgr.predict(X_test), color='blue', linewidth=2)

# Plot the prediction of the linear regression model (red line)
plt.plot(X_test, lr.predict(X_test), color='red', linewidth=2)

# Make the plot prettier
plt.axhline(.5, color='.5')
plt.ylabel('y')
plt.xlabel('X')
plt.xticks(())
plt.yticks(())
plt.ylim(-.25, 1.25)
plt.xlim(-4, 10)
plt.title('Comparing Linear and Logisitic Regression')
plt.show()
```





The black dots in the plot above are the training data. The linear regression model fits that training data with the line shown in red. The logistic regression model fits the same training data with the line shown in blue. As you can see, the logistic regression model is a much closer fit to the binary training data than the linear regression model.

Now let's explore logistic regression with a different data set. The Adult dataset (<http://archive.ics.uci.edu/ml/datasets/Adult>) consists of 14 attributes from census data and tries to predict whether an individual has an income exceeding \$50K. I've covered some of the basic steps to get the data loaded, but you can explore the original data `census_orig` (which has readable, nominal attributes) and the transformed data `census_data` and `census_labels` to get a better understanding of the dataset.

```
In [24]: census_data = urllib2.urlopen("http://archive.ics.uci.edu/ml/machine-le
census_orig = pd.read_csv(census_data, quotechar='"', skipinitialspace=
                        names=['Age', 'WorkClass', 'FnlWgt', 'Edu
                        'Occupation', 'Relationship', 'Ra
                        'CapitalGain', 'CapitalLoss', 'Ho
                        'NativeCountry', 'Label'],
                        na_values="?", index_col=False)

census_orig = census_orig.dropna()

# Convert labels from strings to boolean
label_encoder = preprocessing.LabelEncoder()
census_labels = pd.DataFrame(label_encoder.fit_transform(census_orig.il

# Convert nominal attributes to encoded versions
attr_encoder = feature_extraction.DictVectorizer(sparse=False)
census_data = pd.DataFrame(attr_encoder.fit_transform(census_orig.iloc[
census_data.columns = attr_encoder.get_feature_names()
```

## Question 2: Logistic Regression (20 points)

1. Learn linear and logistic regression classifiers for the census data. Perform 10-fold cross-validation (with `shuffle=True` and `random_state=20160202`) and report the mean accuracy of each classifier. Remember, you have to transform the linear regression result to get a classification.
2. One of the important aspects data mining is data normalization. Use the `StandardScaler` to normalize your data. Now re-run the same 10-fold experiment for both classifiers. What changed?
3. The Lasso and Ridge methods above are also part of logistic regression, although you must specify them slightly differently. You can specify the parameter `penalty='l1'` or `penalty='l2'` to specify Lasso or Ridge, respectively. The importance of regularization can be specified using the parameter `C`. Perform 10-fold CV with the L1 and L2 penalties and `C=[0.1, 10]`. Report the results of each combination (for a total of 4) averaged across folds.

Note: if this were a real experiment, you would want to do your parameter search with a validation set.

## Answers 2:

**1.**

Linear Regression Mean Accuracy 0.833996

Logistic Regression Accuracy 0.790465

**2.**

The accuracies changed between the two models. Now with the data normalized, the Logistic Regression performs better than the Linear Regression, whereas in the original, non-standardized data, the reverse was true. It's worthy to note that the accuracy of the Logistic Regression improved significantly ~5%

Linear Regression Mean Accuracy 0.833864

Logistic Regression Accuracy 0.848551

**3.**

Lasso Log Regression

L1 0.1: 0.848087

L1 10: 0.848551

Ridge Log Regression:

L2 0.1: 0.848485

L2 10: 0.848551

```
In [25]: # Part 1 no normalization
from sklearn.metrics import accuracy_score

#maybe do your EDA here?
foldnum = 0
lr_fold_results = pd.DataFrame()
lgr_fold_results = pd.DataFrame()

for train, test in cross_validation.KFold(len(census_data), n_folds=10,
    foldnum+=1

    [adult_tr_data, adult_te_data,
      adult_tr_target, adult_te_target] = folds_to_split(census_data,cen

    lr = linear_model.LinearRegression()
    lr.fit(adult_tr_data, adult_tr_target)

    prediction = lr.predict(adult_te_data)
    classified_prediction = []
    for value in range(len(prediction)):
        classified_prediction.append(round(prediction[value]))
    adult_te_classified_data = pd.DataFrame(classified_prediction)

    lgr = linear_model.LogisticRegression(C=1e5)
    lgr.fit(adult_tr_data, np.reshape(adult_tr_target.values,[len(adult

    # But a nicer way to store them is in a DataFrame
    lr_fold_results.loc[foldnum, 'LinearReg Mean Accuracy'] = accuracy_
    lgr_fold_results.loc[foldnum, 'LogRegr Accuracy'] = lgr.score(adult

#Now let's look at the results:
print lr_fold_results
#And compute the mean error across folds:
print lr_fold_results.mean()

print lgr_fold_results
print lgr_fold_results.mean()
```

```
LinearReg Mean Accuracy
1          0.840570
2          0.833941
3          0.834881
4          0.839523
5          0.830902
6          0.831565
7          0.834218
8          0.827586
9          0.832560
10         0.834218
LinearReg Mean Accuracy    0.833996
dtype: float64

LogRegr Accuracy
1          0.783560
2          0.790520
3          0.797082
4          0.802056
5          0.781167
6          0.790119
7          0.793103
8          0.783820
9          0.784483
10         0.798740
LogRegr Accuracy    0.790465
dtype: float64
```

```

In [26]: # Part 2 normalizing the data
from sklearn import preprocessing

sscaler = preprocessing.StandardScaler()
census_data_norm = pd.DataFrame(sscaler.fit_transform(census_data.value

#maybe do your EDA here?
foldnum = 0
lr_fold_results = pd.DataFrame()
lgr_fold_results = pd.DataFrame()

for train, test in cross_validation.KFold(len(census_data_norm), n_fold
    foldnum+=1

    [adult_tr_data, adult_te_data,
     adult_tr_target, adult_te_target] = folds_to_split(census_data_nor

    lr = linear_model.LinearRegression()
    lr.fit(adult_tr_data, adult_tr_target)

    prediction = lr.predict(adult_te_data)
    classified_prediction = []
    for value in range(len(prediction)):
        classified_prediction.append(round(prediction[value]))
    adult_te_classified_data = pd.DataFrame(classified_prediction)

    lgr = linear_model.LogisticRegression(C=1e5)
    lgr.fit(adult_tr_data, np.reshape(adult_tr_target.values,[len(adult

    # But a nicer way to store them is in a DataFrame
    lr_fold_results.loc[foldnum, 'LinearReg Mean Accuracy'] = accuracy_
    lgr_fold_results.loc[foldnum, 'LogRegr Accuracy'] = lgr.score(adult

#Now let's look at the results:
print lr_fold_results
#And compute the mean error across folds:
print lr_fold_results.mean()

print lgr_fold_results
print lgr_fold_results.mean()

```

```
LinearReg Mean Accuracy
1          0.842559
2          0.832615
3          0.835212
4          0.839191
5          0.830570
6          0.831233
7          0.832560
8          0.826260
9          0.832891
10         0.835544
LinearReg Mean Accuracy    0.833864
dtype: float64

LogRegr Accuracy
1          0.857474
2          0.846536
3          0.855106
4          0.852122
5          0.842838
6          0.845159
7          0.852122
8          0.838196
9          0.848806
10         0.847149
LogRegr Accuracy    0.848551
dtype: float64
```

```

In [27]: # Part 3 Lasso and Ridge using Logistic Regression
foldnum = 0
fold_l1_01_results = pd.DataFrame()
fold_l1_10_results = pd.DataFrame()

fold_l2_01_results = pd.DataFrame()
fold_l2_10_results = pd.DataFrame()

for train, test in cross_validation.KFold(len(census_data_norm), n_fold
    foldnum+=1

    [adult_tr_data, adult_te_data,
     adult_tr_target, adult_te_target] = folds_to_split(census_data_nor

    lgr_l1_01 = linear_model.LogisticRegression(penalty = 'l1', C=0.1)
    lgr_l1_10 = linear_model.LogisticRegression(penalty = 'l1', C=10)
    lgr_l2_01 = linear_model.LogisticRegression(penalty = 'l2', C=0.1)
    lgr_l2_10 = linear_model.LogisticRegression(penalty = 'l2', C=10)

    lgr_l1_01.fit(adult_tr_data, np.reshape(adult_tr_target.values,[len
    lgr_l1_10.fit(adult_tr_data, np.reshape(adult_tr_target.values,[len

    lgr_l2_01.fit(adult_tr_data, np.reshape(adult_tr_target.values,[len
    lgr_l2_10.fit(adult_tr_data, np.reshape(adult_tr_target.values,[len

    # But a nicer way to store them is in a DataFrame
    fold_l1_01_results.loc[foldnum, 'L1 0.1: '] = lgr_l1_01.score(adult
    fold_l1_10_results.loc[foldnum, 'L1 10: '] = lgr_l1_10.score(adult_

    fold_l2_01_results.loc[foldnum, 'L2 0.1: '] = lgr_l2_01.score(adult
    fold_l2_10_results.loc[foldnum, 'L2 10: '] = lgr_l2_10.score(adult_

#And compute the mean error across folds:
print fold_l1_01_results.mean()
print fold_l1_10_results.mean()
print fold_l2_01_results.mean()
print fold_l2_10_results.mean()

L1 0.1:      0.848087
dtype: float64
L1 10:       0.848551
dtype: float64
L2 0.1:      0.848485
dtype: float64
L2 10:       0.848551
dtype: float64

```

