Part 2: Clustering (45 points)

Unsupervised data mining methods are fairly diverse. At one extreme, there are the pure statistical summarization techniques that are the mainstay of exploratory data analysis, somewhere in between are data transformations like principal components analysis, which have a deeper mathematical underpinning but a fairly mechanical formulation, and at the other extreme you have unsupervised models that make strong assumptions about the data distribution. Clustering methods fall into that latter category: they make an assumption about instance similarity given attributes and use this modeling assumption to provide an interpretable division of a dataset.

The strong modeling assumptions made by clustering algorithms have varying benefits and drawbacks. In some cases, the modeling assumption of a clustering algorithm adheres to our intuitions about the data. For example, we may use a Gaussian mixture model for normally distributed data, and get extremely useful results. In other cases, the modeling assumptions are clearly incorrect, but these assumptions may provide a simpler formulation that pragmatically works quite well for a dataset. As an example, in mixture modeling for text corpora (like the news example in class), we make the assumption that documents are built by randomly drawing words independently of each other (but dependent on the cluster) -- this is not how journalists write news articles, but it turns out this simplifying assumption allows fairly powerful models. In the worst case, the modeling assumptions conflict with the underlying data generating process and impede our ability to understand the data. For example, in class we discussed how distance functions do not always work as we expect, and two people who both like food and music may have the same distance as two people who both like food, music, hiking, biking, camping, and rock climbing, even though we expect that the first pair will have far less to talk about than the second pair.

In this part of the assignment, we'll use the scikit-learn implementations of clustering (http://scikit-learn.org/stable/modules/clustering.html) in the clustering and mixture modules. We'll explore four different clustering methods (k-Means (http://scikit-learn.org/stable/modules/generated/sklearn.cluster.KMeans.html), Gaussian mixture models (http://scikit-learn.org/stable/modules/mixture.html#gmm), and Agglomerative clustering (http://scikit-learn.org/stable/modules/generated/sklearn.cluster.AgglomerativeClustering.html)) and try to better understand the modeling assumptions each makes.

In [6]:
```
## Preliminaries

#Show plots in the notebook
%matplotlib inline

from sklearn import datasets, preprocessing, cross_validation, feature_
from sklearn import linear_model, svm, metrics, ensemble, neighbors, mi
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
import urllib2
import random
import math

# Helper functions
def folds_to_split(data,targets,train,test):
    data_tr = pd.DataFrame(data).iloc[train]
    data_te = pd.DataFrame(data).iloc[test]
    labels_tr = pd.DataFrame(targets).iloc[train]
    labels_te = pd.DataFrame(targets).iloc[test]
    return [data_tr, data_te, labels_tr, labels_te]

def computeLabelPercentage(data, axis=0):
    return data.groupby(by=axis).size().apply(lambda x: x/data.count())
```

Question 1: Implementing the basic k-Means algorithm (15 points)

Let's work through the writing the code for a basic k-means implementation. Assume you have data this is either a numpy ndarray or pandas DataFrame.

1. Write a function `sample_centroids` that takes as parameters k and your data, then picks k (distinct) random data points as centroids. (The function `random.sample` may be helpful)
2. Write a function `find_closest_centroid` that takes as parameters a list of centroids and your data, then computes the distance of each instance to each of k centroids, and returns the minimum centroid for each instance. Use euclidean distance in your implementation.
3. Write a function `update_centroid` that takes as a parameter a dataset containing the instances mapped to a particular centroid and returns the new centroid
4. Put these together in a function `kmeans` that samples centroids and loops over the steps of finding the closest centroid for each instance and updating the centroids until no centroid assignments change.
5. Put it all together: generate some sample data using `make_blobs` and confirm that your k-Means implementation works. At each iteration, plot the data points and cluster assignments. (See the scikit-learn clustering comparison (http://scikit-learn.org/stable/auto_examples/cluster/plot_cluster_comparison.html) example if you need some help with figuring out the plotting)

In [7]:
```
# 1. sample_centroids

def sample_centroids(k, data):
    data_clean = data
    indexes = random.sample(data.index, k)
    centroids = data.iloc[indexes]
    return [centroids, indexes]
```

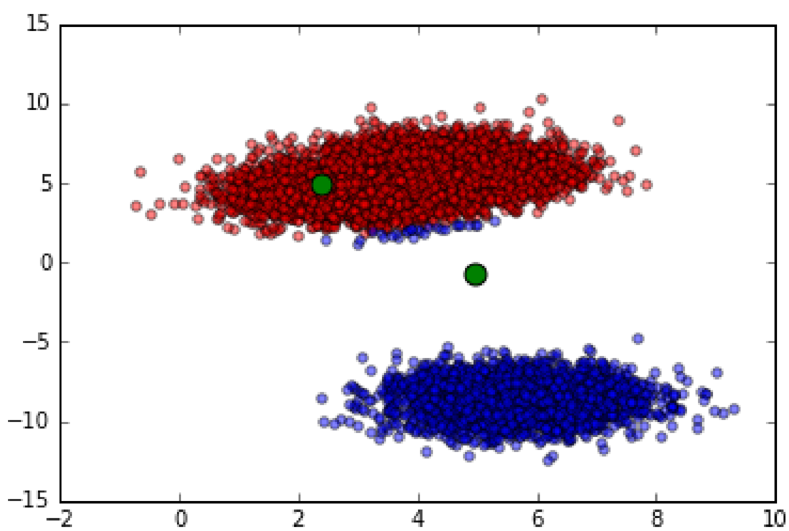
In [8]:
```
# 2. find_closest_centroid

def find_closest_centroid(centroids, data):
    data_values = data.values
    inst_cent_map = []

    for row in range(0, len(data_values)):
        centroid_distances = []

        for centroid in range(0, len(centroids)):
            distance = 0
            cent = centroids[centroid]

            for col in range(0, len(data_values[0])):
                x = cent[col]
                y = data_values[row][col]
                distance += (x - y)**2

            distance = math.sqrt(distance)
            centroid_distances.append(distance)
        closest_centroid = centroid_distances.index(min(centroid_distan
        inst_cent_map.append([data_values[row], centroids[closest_centr
    return inst_cent_map
```

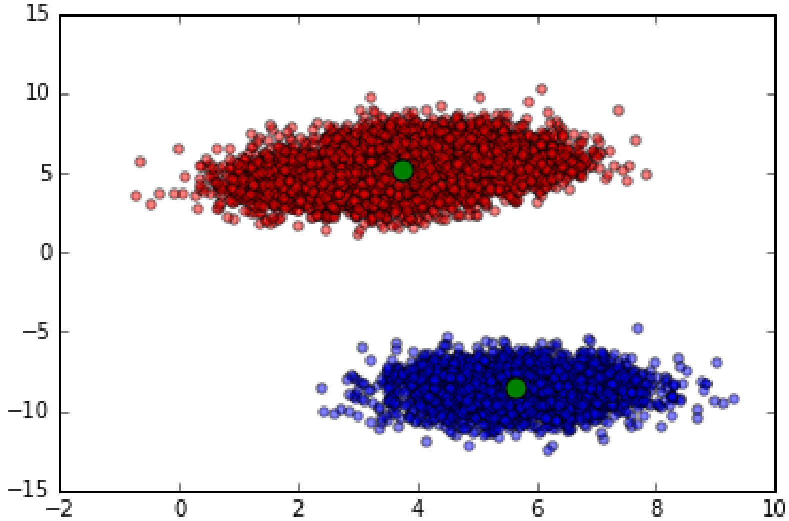
In [9]:
```
# 3. update_centroid

def update_centroid(instances_centroid_map, k_means):
    new_centroids = []
    for i in range(0, k_means):
        same_centroid_instances = []

        added_initial_centroid = 0
        for instance in range(0, len(instances_centroid_map)):
            instance_features = instances_centroid_map[instance][0]
            centroid_points = instances_centroid_map[instance][1]
            centroid_group = instances_centroid_map[instance][2]

            if (centroid_group == i):
                if (added_initial_centroid != 1):
                    same_centroid_instances.append(centroid_points)
                    added_initial_centroid = 1
                same_centroid_instances.append(instance_features)

        new_centroid = []
        if (len(same_centroid_instances) > 0):
            for feature in range(0, len(same_centroid_instances[0])):
                feature_total = 0
                total_instances = len(same_centroid_instances)

                for inst in range(0, total_instances):
                    instance = same_centroid_instances[inst]
                    feature_total += instance[feature]

                new_centroid.append(round(feature_total / total_instanc

            new_centroids.append([new_centroid, total_instances])
    return new_centroids
```

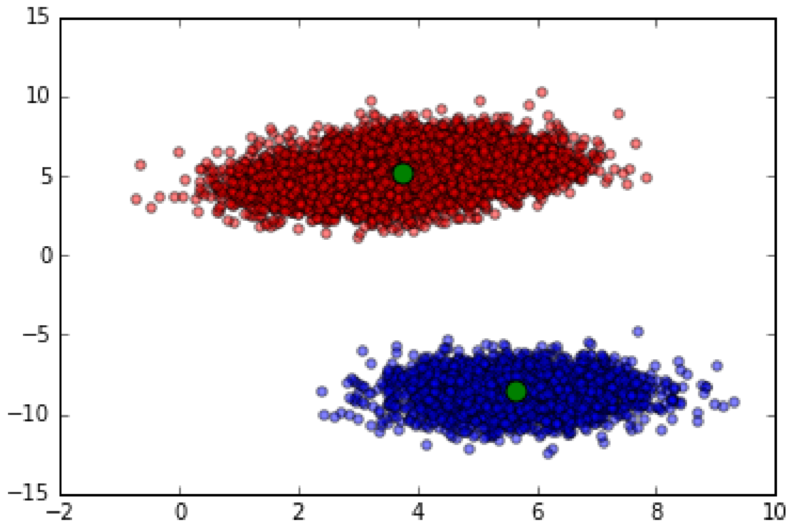
In [10]:

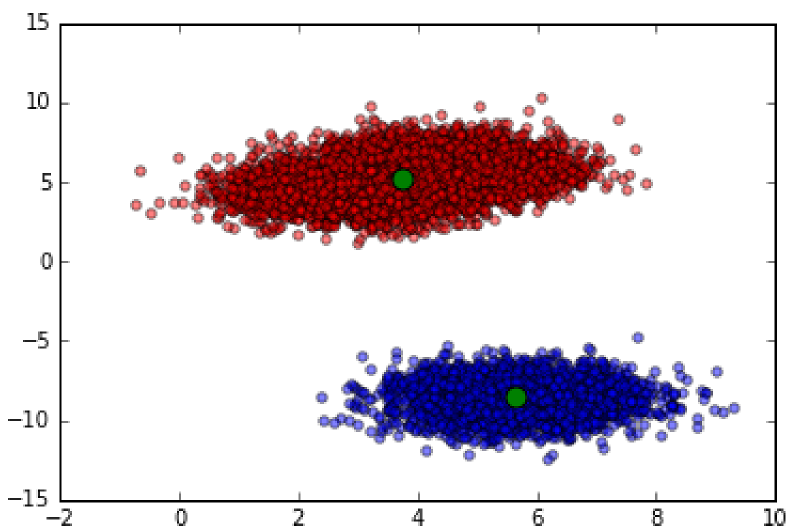
```
# 4. Generating data and putting it all together
import matplotlib.pyplot as plt
import matplotlib.patches as mpatches


[data, targets] = datasets.make_blobs(n_samples=10000, n_features=2,
                    random_state=20160217)

data = pd.DataFrame(data)
targets = pd.DataFrame(targets)


def k_means():
    # Sample centroids
    data_clean = data
    k_means = 2
    [centroids, indexes] = sample_centroids(k_means, data)

    # init total_instances
    total_instances = []
    for i in range(0, k_means):
        total_instances.append(1)

    data_clean.drop(data_clean.index[indexes], inplace=True)
    centroids = centroids.values

    centroid_updated = 1;
    while (centroid_updated == 1):

        centroid_updated = 0

        instances_centroids_map = find_closest_centroid(centroids, data

        old_centroids = centroids
        centroids_info = update_centroid(instances_centroids_map, k_mea
        centroids = []

        new_total_instances = []
        for i in range(0, len(centroids_info)):
            centroids.append(centroids_info[i][0])
            new_total_instances.append(centroids_info[i][1])

        if total_instances == new_total_instances:
            centroid_updated = 0
            centroids = old_centroids
        else:
            centroid_updated = 1
            total_instances = new_total_instances

        x_points_red = []
        y_points_red = []

        x_points_blue = []
        y_points_blue = []
```

```
        x_centroids = []
        y_centroids = []

        instances_centroids_map = find_closest_centroid(centroids, data
        for inst in range(0, len(instances_centroids_map)):
            instance = instances_centroids_map[inst][0]

            if (centroids[0] == instances_centroids_map[inst][1]):
                x_points_red.append(instance[0])
                y_points_red.append(instance[1])
            else:
                x_points_blue.append(instance[0])
                y_points_blue.append(instance[1])

        for cent in range(0, len(centroids)):
            x_centroids.append(centroids[cent][0])
            y_centroids.append(centroids[cent][1])

        plt.scatter(x_points_red, y_points_red, c='r', alpha=0.5)
        plt.scatter(x_points_blue, y_points_blue, c='b', alpha=0.5)

        plt.scatter(x_centroids, y_centroids, c='g', s=100)
        plt.show()


    return [centroids, data]


[centroids, data] = k_means()
```









Flower Arrangements with Python

In this assignment, we'll look at three different clustering algorithms. The first is k-Means -- you're pretty familiar with that algorithm since you just implemented it. Not that your code isn't beautiful, but we'll be using the scikit-learn K-Means (http://scikit-learn.org/stable/modules/generated/sklearn.cluster.KMeans.html) to make sure we have consistent results

Next, is hierarchical clustering, also known as agglomerative clustering. Hierarchical clustering starts out with each instance as its own cluster, then merges the two closest clusters, thus reducing the number of clusters by 1. This merging continues until only the desired number of clusters remain. The main implementation question in hierarchical clustering is how to compute the distance between two clusters of points. The sklearn implementation (http://scikit-learn.org/stable/modules/generated/sklearn.cluster.AgglomerativeClustering.html) offers a few options.

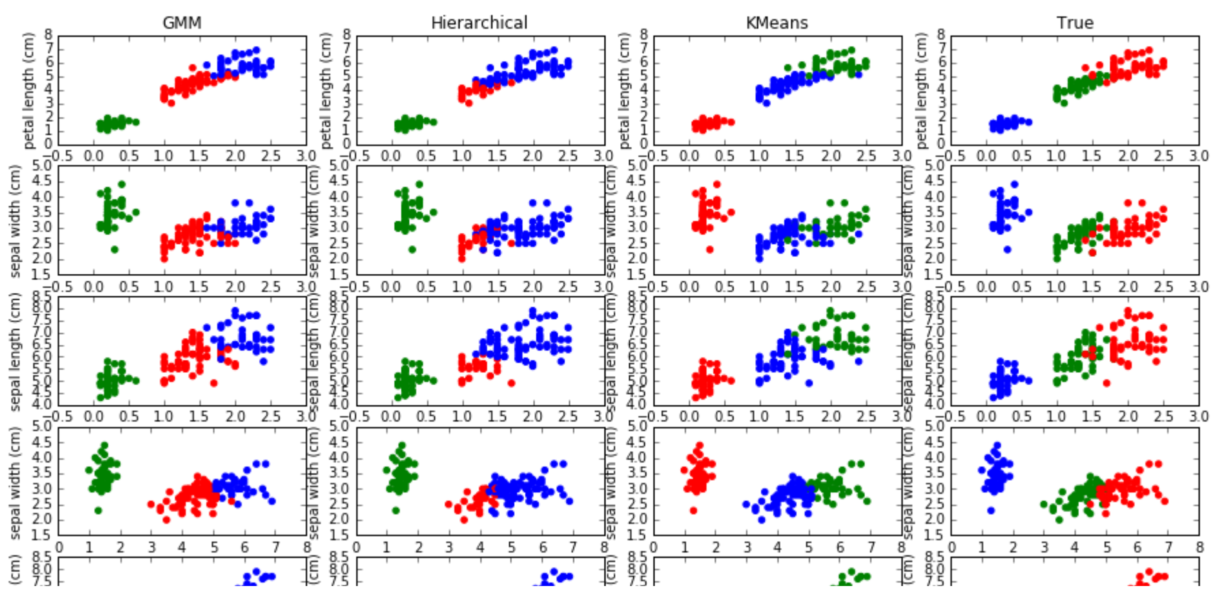
- average: compute all pairwise distances between the two clusters, and then take the average of these distances
- complete (also known as maximum) linkage: compute all pairwise distances and use the largest distance
- Ward: compute the change in cluster variance after all possible cluster merges and choose the merge that results in the smallest variance increase

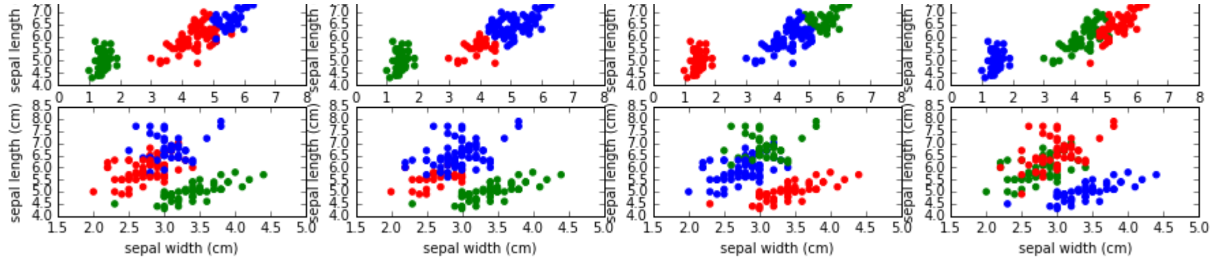
Finally, there are Gaussian Mixture Models (http://scikit-learn.org/stable/modules/mixture.html#gmm). These models assume that the attribute values in each dimension follow a Gaussian (or normal) distribution. One nice feature of this assumption is that the Gaussian models can account for differing attribute variances. For example, if an "income" attribute has a very large range, and a "years of education" has a small range, the fit variance for the former attribute will be large, and for the latter attribute will be small. Of course, you already know how to achieve a somewhat similar result in other algorithms, right?

Let's look at three different clustering algorithms on the Iris dataset. In the plot below, the columns correspond to the three clustering algorithms and ground truth - the flower species. The rows correspond to a combination of two of the four attributes (petal length, petal width, sepal length, sepal width), of which there are six (4 choose 2). Given the four attributes, you'll have to think high-dimensionally to really get a sense of the clusterings. You can see that all of the clustering algorithms roughly correspond to the true "clusters" that are based on the the flower species. However, each behaves slightly differently. Hierarchical clustering with complete linkage will prefer small, compact clusters. You can see in the last row of the figure how the red cluster for hierarchical clustering remains small, while the other algorithms have more mixing. In that last row, you can also notice how the Gaussian mixture model estimates a higher variance for the attributes in the blue cluster (which have outliers in the top right), resulting in more cluster mixing. By comparison, the same cluster in k-Means has its center pulled towards the extreme which results in a smaller cluster focused on the outliers.

In [11]:
```
iris = datasets.load_iris()
print pd.DataFrame(iris.data).describe()
clusterers = {
    'KMeans':cluster.KMeans(n_clusters=3, init='random', random_state=2
    'Hierarchical':cluster.AgglomerativeClustering(n_clusters=3, linkag
    'GMM':mixture.GMM(n_components=3, random_state=20160217),
    }

clusterings = {'True':iris.target}

for clusterer in clusterers.iterkeys():
    clusterings[clusterer]=clusterers[clusterer].fit_predict(iris.data)

colors = np.array([x for x in 'bgrcmykbgrcmykbgrcmykbgrcmyk'])
colors = np.hstack([colors] * 20)
plt.figure(figsize=(15,10))
pnum=1
for x in range(3,0,-1):
    for y in range(x-1, -1, -1):
        for clusterer in sorted(clusterings.keys()):
            plt.subplot(6,4,pnum)
            if pnum < 5: plt.title(clusterer)
            plt.xlabel(iris.feature_names[x])
            plt.ylabel(iris.feature_names[y])
            plt.scatter(iris.data[:,x], iris.data[:,y], color=colors[cl
            pnum+=1
```

```
                0           1           2           3
count  150.000000  150.000000  150.000000  150.000000
mean     5.843333    3.054000    3.758667    1.198667
std      0.828066    0.433594    1.764420    0.763161
min      4.300000    2.000000    1.000000    0.100000
25%      5.100000    2.800000    1.600000    0.300000
50%      5.800000    3.000000    4.350000    1.300000
75%      6.400000    3.300000    5.100000    1.800000
max      7.900000    4.400000    6.900000    2.500000
```





In [12]:
```
# Load Heart Data
heart_data = urllib2.urlopen("http://archive.ics.uci.edu/ml/machine-lea
heart = pd.read_csv(heart_data, quotechar='"', skipinitialspace=True, n
heart=heart.dropna()
heart_attrs = heart.ix[:,:-1]
heart_labels = heart.ix[:,-1]
heart_processed = pd.DataFrame(preprocessing.StandardScaler().fit_trans
heart_labels_values = heart_labels.values
```

Question 2: Clustering Heart Disease (15 points)

In this exercise, you'll be clustering the heart disease data we've seen in a couple of assignments already. I've loaded and processed the data for you. You'll be using the three clusterers from the example above (KMeans, Hierarchical, GMM), using the same `random_state` settings and initialization as in the example. Cluster the processed heart data using five clusters and the three clustering algorithms.

- Compute the cluster sizes. What do you notice about the cluster sizes for each algorithm?
- Compute the mean value of the HeartDisease column (heart_labels) for each cluster. How well do the clusters correspond to differing heart disease conditions?
- Write a function that `wss` computes the within-cluster sum of square errors (MSE of a cluster) and a function `bss` to compute the between-cluster sum of square errors (see the lecture slides!), and report both metrics for each of the clustering methods.
- For each of the clusterings (15 total), choose one attribute that has a different distribution than the overall data, and discuss how the distribution differs for that attribute.
- Make a figure showing boxplots of each attribute in each clustering, as well as a figure with a boxplot of all the data. (The figure should have three rows [one for each clustering algorithm] and six columns [one for all the data, five for each of the clusterings], and each boxplot will have twelve attributes.

Answers 2.

1. Cluster sizes
Clustering Method: Hierarchical
Cluster: 0 size: 131
Cluster: 1 size: 129
Cluster: 2 size: 13
Cluster: 3 size: 1
Cluster: 4 size: 23

Clustering Method: KMeans
Cluster: 0 size: 39
Cluster: 1 size: 65
Cluster: 2 size: 62
Cluster: 3 size: 46
Cluster: 4 size: 85

Clustering Method: GMM
Cluster: 0 size: 60
Cluster: 1 size: 89
Cluster: 2 size: 78
Cluster: 3 size: 43
Cluster: 4 size: 27

2. Cluster means

Clustering Method: Hierarchical
Cluster: 0 mean: 0.4275
Cluster: 1 mean: 1.1705
Cluster: 2 mean: 2.5385
Cluster: 3 mean: 0.0
Cluster: 4 mean: 1.7826

Clustering Method: KMeans
Cluster: 0 mean: 1.0
Cluster: 1 mean: 1.9846
Cluster: 2 mean: 0.2097
Cluster: 3 mean: 1.6522
Cluster: 4 mean: 0.2824

Clustering Method: GMM
Cluster: 0 mean: 2.2667
Cluster: 1 mean: 0.1124
Cluster: 2 mean: 1.0385
Cluster: 3 mean: 1.093
Cluster: 4 mean: 0.2593

3. WSS
Hierarchical: 3151.18560967
KMeans: 2605.73353663
GMM: 2770.36699076

BSS
Hierarchical: 709.81439033
KMeans: 1255.26646337
GMM: 1090.63300924

4. One different attribute for each mode for each cluster.
See bellow for boxplots for each.
Hierarchical
Cluster 0 - Attr 5 - all same values (-0.411450), except one point (2.430427)
Cluster 1- Attr 5 - all same values (-0.411450), except one point (2.430427)
Cluster 2 - Attr 5 - same values, no distribution -0.411450
Cluster 3- Attr 5 - there's only one point here
Cluster 4 - Attr 5 - same values, no distribution 2.430427

KMeans
Cluster 0 - Attr 5 - same values, no distribution - 2.430427
Cluster 1- Attr 5 - all same values (-0.411450), except one point (2.430427)
Cluster 2 - Attr 5 - same values, no distribution - -0.41145
Cluster 3 - Attr 5 - same values, no distribution - -0.41145
Cluster 4 - Attr 5 - same values, no distribution - -0.41145

GMM
Cluster 0 - Attr 2- same values, no distribution - 0.87388
Cluster 1 - Attr 8 - all same values, no distribution - -0.696419
Cluster 2 - Attr 5 - same values, no distribution - -0.41145
Cluster 3 - Attr 5 - same values, no distribution - 2.430427
Cluster 4 - Attr 5 - same values, no distribution - -0.41145

In [13]:

```
# Part 1 Cluster Count
from sklearn import mixture, cluster

clusterers = {
    'KMeans':cluster.KMeans(n_clusters=5, init='random', random_state=2
    'Hierarchical':cluster.AgglomerativeClustering(n_clusters=5, linkag
    'GMM':mixture.GMM(n_components=5, random_state=20160217),
    }

clusterings = {'True':heart_labels}

for clusterer in clusterers.iterkeys():
    clusterings[clusterer]=clusterers[clusterer].fit_predict(heart_proc


hieararchical_count = {}
kmeans_count = {}
gmm_count = {}

# Cluster sizes
for clusterer in clusterers.iterkeys():
    print "Clustering Method: " + clusterer
    for cluster in range(0, 5):
        bincount = np.bincount(clusterings[clusterer])
    for cluster in range(0, len(bincount)):
        print "Cluster: " + str(cluster) + " size: " + str(bincount[clu
        if (clusterer == 'KMeans'):
            kmeans_count[str(cluster)] = bincount[cluster]
        elif clusterer == 'Hierarchical':
            hieararchical_count[str(cluster)] = bincount[cluster]
        else:
            gmm_count[str(cluster)] = bincount[cluster]
```

```
Clustering Method: Hierarchical
Cluster: 0 size: 131
Cluster: 1 size: 129
Cluster: 2 size: 13
Cluster: 3 size: 1
Cluster: 4 size: 23
Clustering Method: KMeans
Cluster: 0 size: 39
Cluster: 1 size: 65
Cluster: 2 size: 62
Cluster: 3 size: 46
Cluster: 4 size: 85
Clustering Method: GMM
Cluster: 0 size: 60
Cluster: 1 size: 89
Cluster: 2 size: 78
Cluster: 3 size: 43
Cluster: 4 size: 27
```

In [14]:
```
# Part 2 Cluster means
from sklearn import mixture, cluster

clusterers = {
    'KMeans':cluster.KMeans(n_clusters=5, init='random', random_state=2
    'Hierarchical':cluster.AgglomerativeClustering(n_clusters=5, linkag
    'GMM':mixture.GMM(n_components=5, random_state=20160217),
    }

clusterings = {'True':heart_labels}

for clusterer in clusterers.iterkeys():
    clusterings[clusterer]=clusterers[clusterer].fit_predict(heart_proc

hieararchical_means = {}
kmeans_means = {}
gmm_means = {}

# Cluster sizes
for clusterer in clusterers.iterkeys():
    print "Clustering Method: " + clusterer
    for cluster in range(0, 5):
        cluster_mean = 0
        cluster_count = 0
        for i in range(0, len(clusterings[clusterer])):
            if (clusterings[clusterer][i] == cluster):
                cluster_mean += heart_labels_values[i]
                cluster_count +=1

        cluster_mean = round(float(cluster_mean / float(cluster_count))
        if (clusterer == 'KMeans'):
            kmeans_means[str(cluster)] = cluster_mean
        elif clusterer == 'Hierarchical':
            hieararchical_means[str(cluster)] = cluster_mean
        else:
            gmm_means[str(cluster)] = cluster_mean

        print "Cluster: " + str(cluster) + " mean: " + str(cluster_mean
```

```
Clustering Method: Hierarchical
Cluster: 0 mean: 0.4275
Cluster: 1 mean: 1.1705
Cluster: 2 mean: 2.5385
Cluster: 3 mean: 0.0
Cluster: 4 mean: 1.7826
Clustering Method: KMeans
Cluster: 0 mean: 1.0
Cluster: 1 mean: 1.9846
Cluster: 2 mean: 0.2097
Cluster: 3 mean: 1.6522
Cluster: 4 mean: 0.2824
Clustering Method: GMM
Cluster: 0 mean: 2.2667
Cluster: 1 mean: 0.1124
Cluster: 2 mean: 1.0385
Cluster: 3 mean: 1.093
Cluster: 4 mean: 0.2593
```

In [15]:

```
# Part 3 WSS
from sklearn import mixture, cluster
pd.set_option('display.height', 1500)
pd.set_option('display.max_rows', 1500)

clusterers = {
    'KMeans':cluster.KMeans(n_clusters=5, init='random', random_state=2
    'Hierarchical':cluster.AgglomerativeClustering(n_clusters=5, linkag
    'GMM':mixture.GMM(n_components=5, random_state=20160217),
    }

clusterings = {'True':heart_labels}

for clusterer in clusterers.iterkeys():
    clusterings[clusterer]=clusterers[clusterer].fit_predict(heart_proc


def wss(kmeans):
    wss = 0
    # Iterate over each cluster
    for cluster in range(0, 5):
        cluster_instances = []
        # Iterate over each instance of the dataset
        for i in range(0, len(kmeans)):
            # If that dataset instance is in the right cluster
            if (kmeans[i] == cluster):
                cluster_instances.append(i)

        heart_cluster = heart_processed.iloc[cluster_instances]
        cols = heart_cluster.shape[1]

        for col in range(0, cols):
            feature_vals = heart_cluster[col].values
            attr_mean = 0
            for i in range(0, len(feature_vals)):
                attr_mean += feature_vals[i]

            attr_mean = float(attr_mean / float(len(feature_vals)))

            for i in range(0, len(feature_vals)):
                point = (feature_vals[i] - attr_mean)**2
                wss+= point
    return wss

for clusterer in clusterers.iterkeys():
    clusterings[clusterer]=clusterers[clusterer].fit_predict(heart_proc


for clusterer in clusterers.iterkeys():
    print str(clusterer) + " WSS:"
    if clusterer == 'KMeans':
        print wss(clusterings[clusterer])
    elif clusterer == 'Hierarchical':
        print wss(clusterings[clusterer])
```

```
        print wss(clusterings[clusterer])
    else:
        print wss(clusterings[clusterer])
```

```
height has been deprecated.

Hierarchical WSS:
3151.18560967
KMeans WSS:
2605.73353663
GMM WSS:
2770.36699076
```

In [16]:

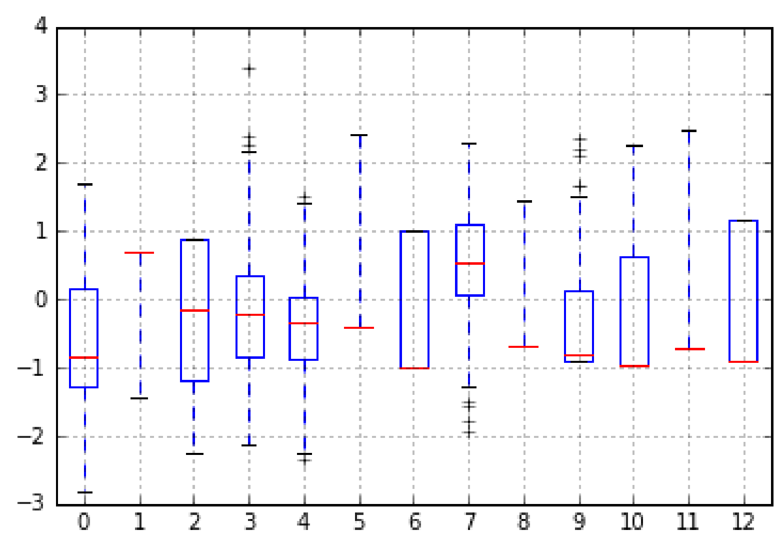
```
# Part 3 BSS
from sklearn import mixture, cluster

def bss(kmeans):
    bss = 0
    mean_of_means = 0

    mean_of_means = heart_processed.mean()
    for cluster in range(0, 5):
        cluster_instances = []
        # Iterate over each instance of the dataset
        for i in range(0, len(kmeans)):
            # If that dataset instance is in the right cluster
            if (kmeans[i] == cluster):
                cluster_instances.append(i)

        heart_cluster = heart_processed.iloc[cluster_instances]
        attrs_means = heart_cluster.mean()

        for i in range(0, len(attrs_means)):
            bss += len(cluster_instances) * (attrs_means[i] - mean_of_m

    return bss

# print bss(hieararchical_means, hieararchical_count)
for clusterer in clusterers.iterkeys():
    print str(clusterer) + " BSS:"
    if clusterer == 'KMeans':
        print bss(clusterings[clusterer])
    elif clusterer == 'Hierarchical':
        print bss(clusterings[clusterer])
    else:
        print bss(clusterings[clusterer])
```

```
Hierarchical BSS:
709.81439033
KMeans BSS:
1255.26646337
GMM BSS:
1090.63300924
```

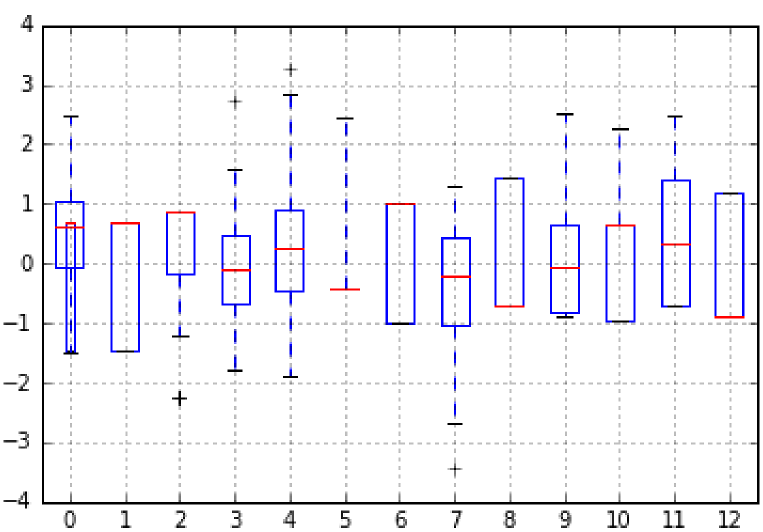
In [17]:

```
# # Part #4 Attribute difference
from sklearn import mixture, cluster

clusterers = {
    'KMeans':cluster.KMeans(n_clusters=5, init='random', random_state=2
    'Hierarchical':cluster.AgglomerativeClustering(n_clusters=5, linkag
    'GMM':mixture.GMM(n_components=5, random_state=20160217),
    }

clusterings = {'True':heart_labels}

for clusterer in clusterers.iterkeys():
    clusterings[clusterer]=clusterers[clusterer].fit_predict(heart_proc

pnum = 1
for clusterer in clusterers.iterkeys():
    for i in range(0, 5):
        cluster_index = []
        for instance in range(0, len(clusterings[clusterer])):
            if i == clusterings[clusterer][instance]:
                cluster_index.append(instance)

        print str(clusterer) + " Cluster: " + str(i)

        df = pd.DataFrame(heart_processed.iloc[cluster_index])
        print df.head(10)


        fig = plt.figure()
        ax = plt.subplot(111)
        ax.boxplot(df.ix[:, i].values, positions = [i])

        df.boxplot(return_type='axes')

    #     ax.set_xlim(-0.5, 9.5)
        plt.show()
```

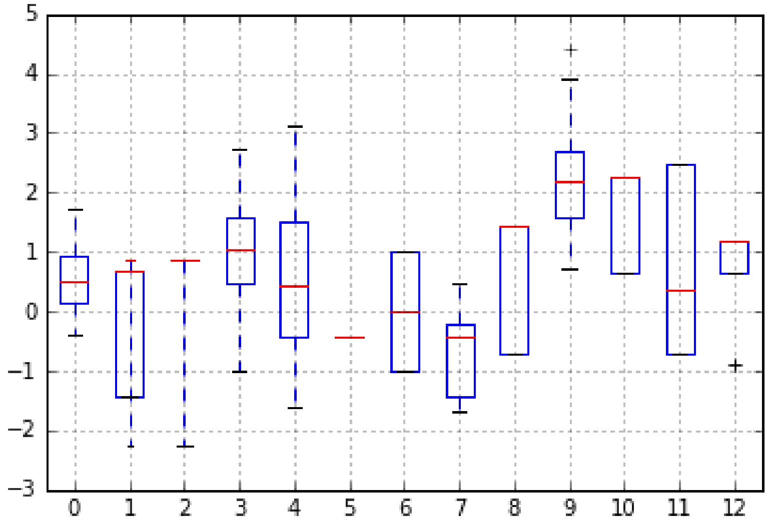
```
Hierarchical Cluster: 0
           0         1         2         3         4         5
6  \
3  -1.941680  0.691095 -0.164289 -0.095506  0.051047 -0.411450 -1.00
3419
4  -1.498933 -1.446980 -1.202459 -0.095506 -0.835103 -0.411450  1.01
0199
5   0.161372  0.691095 -1.202459 -0.659431 -0.218651 -0.411450 -1.00
3419
10  0.272059  0.691095  0.873880  0.468418 -1.066272 -0.411450 -1.00
3419
13 -1.166872  0.691095 -1.202459 -0.659431  0.301480 -0.411450 -1.00
3419
14 -0.281376  0.691095 -0.164289  2.272976 -0.931424  2.430427 -1.00
3419
15  0.272059  0.691095 -0.164289  1.032342 -1.528611 -0.411450 -1.00
3419
16 -0.724124  0.691095 -1.202459 -1.223355 -0.353500 -0.411450 -1.00
3419
17 -0.060002  0.691095  0.873880  0.468418 -0.160859 -0.411450 -1.00
3419
19 -0.613437  0.691095 -1.202459 -0.095506  0.359273 -0.411450 -1.00
3419

           7         8         9        10        11        12
3   1.633010 -0.696419  2.099753  2.264145 -0.721976 -0.894220
4   0.978071 -0.696419  0.295874 -0.976583 -0.721976 -0.894220
5   1.240047 -0.696419 -0.219520 -0.976583 -0.721976 -0.894220
10 -0.069831 -0.696419 -0.563116  0.643781 -0.721976  0.655877
13  1.021734 -0.696419 -0.906712 -0.976583 -0.721976  1.172577
14  0.541445 -0.696419 -0.477217 -0.976583 -0.721976  1.172577
15  1.065396 -0.696419  0.467672 -0.976583 -0.721976 -0.894220
16  0.803421 -0.696419 -0.047722  2.264145 -0.721976  1.172577
17  0.454120 -0.696419  0.124076 -0.976583 -0.721976 -0.894220
19  0.934409 -0.696419 -0.391318 -0.976583 -0.721976 -0.894220
```



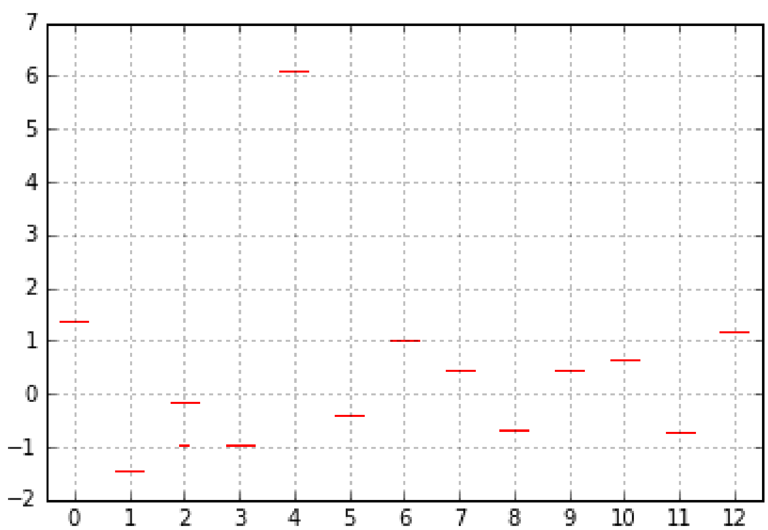
```
Hierarchical Cluster: 1
           0         1         2         3         4         5  
6   \
1   1.378929  0.691095  0.873880  1.596266  0.744555 -0.411450  1.01
0199
2   1.378929  0.691095  0.873880 -0.659431 -0.353500 -0.411450  1.01
0199
7   0.272059 -1.446980  0.873880 -0.659431  2.054515 -0.411450 -1.00
3419
8   0.936181  0.691095  0.873880 -0.095506  0.128103 -0.411450  1.01
0199
11  0.161372 -1.446980 -1.202459  0.468418  0.898668 -0.411450  1.01
0199
18 -0.724124 -1.446980 -0.164289 -0.095506  0.532650 -0.411450 -1.00
3419
20  1.046868  0.691095 -2.240629 -1.223355 -0.700254 -0.411450  1.01
0199
21  0.382746 -1.446980 -2.240629  1.032342  0.686763  2.430427  1.01
0199
22  0.382746  0.691095 -1.202459 -0.659431  0.706027 -0.411450  1.01
0199
23  0.382746  0.691095 -0.164289  0.017278 -0.449820 -0.411450  1.01
0199

           7         8         9        10        11        12  
1  -1.816334  1.435916  0.381773  0.643781  2.478425 -0.894220  
2  -0.899420  1.435916  1.326662  0.643781  1.411625  1.172577  
7   0.585108  1.435916 -0.391318 -0.976583 -0.721976 -0.894220  
8  -0.113493 -0.696419  0.295874  0.643781  0.344824  1.172577  
11  0.148482 -0.696419  0.209975  0.643781 -0.721976 -0.894220  
18 -0.462794 -0.696419 -0.734914 -0.976583 -0.721976 -0.894220  
20 -0.244481  1.435916  0.639470  0.643781 -0.721976 -0.894220  
21  0.541445 -0.696419 -0.047722 -0.976583 -0.721976 -0.894220  
22  0.454120 -0.696419  0.639470  0.643781 -0.721976 -0.894220  
23  1.021734 -0.696419  1.842056 -0.976583  1.411625  1.172577  
```



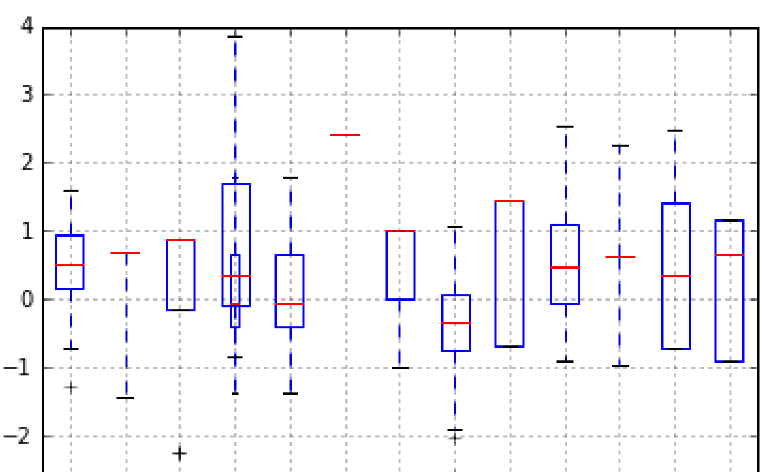
```
Hierarchical Cluster: 2
            0         1         2         3         4        5        
6   \
6    0.825494 -1.446980  0.873880  0.468418  0.397801 -0.41145  1.01
0199
27   1.268242 -1.446980 -2.240629  1.032342 -0.411292 -0.41145 -1.00
3419
68   0.493433  0.691095  0.873880  2.160191  1.515120 -0.41145  1.01
0199
90   0.825494 -1.446980  0.873880  1.596266 -1.605668 -0.41145  1.01
0199
120  0.936181 -1.446980  0.873880  1.032342  3.075514 -0.41145  1.01
0199
122  0.050685  0.691095  0.873880  0.468418 -0.584669 -0.41145 -1.00
3419
135  1.710989  0.691095  0.873880  0.750380 -1.413027 -0.41145 -1.00
3419
168  1.710989  0.691095 -0.164289  1.596266  0.417065 -0.41145 -1.00
3419
179  0.161372 -1.446980  0.873880  0.130063  3.114042 -0.41145  1.01
0199
181  0.493433  0.691095 -2.240629  2.611330  0.436329 -0.41145  1.01
0199

            7         8         9        10        11        12  
6    0.454120 -0.696419  2.185652  2.264145  1.411625 -0.894220  
27  -1.554358 -0.696419  1.326662  2.264145 -0.721976 -0.894220  
68  -0.419131  1.435916  2.013854  2.264145 -0.721976  1.172577  
90  -0.200818 -0.696419  4.419026  2.264145  2.478425  1.172577  
120  0.192145 -0.696419  2.529248  0.643781  2.478425  1.172577  
122 -1.685346  1.435916  3.903632  2.264145 -0.721976  1.172577  
135 -1.074070  1.435916  1.326662  2.264145 -0.721976  1.172577  
168 -1.641684  1.435916  1.584359  0.643781  0.344824  1.172577  
179  0.017494  1.435916  0.725369  0.643781  1.411625  1.172577  
181 -0.200818 -0.696419  2.701046  2.264145 -0.721976  1.172577  
```



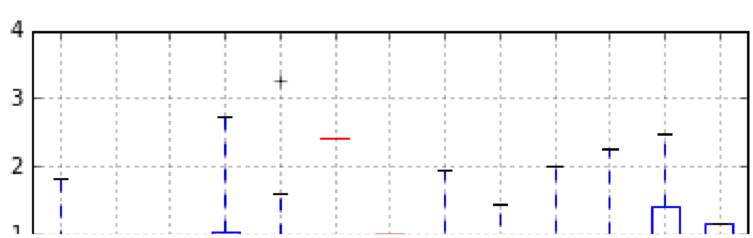

```
Hierarchical Cluster: 3
            0         1          2          3          4         5
6   \
151  1.378929 -1.44698 -0.164289 -0.941393  6.099981 -0.41145  1.010
199

           7         8          9         10         11         12
151  0.45412 -0.696419  0.467672  0.643781 -0.721976  1.172577
```

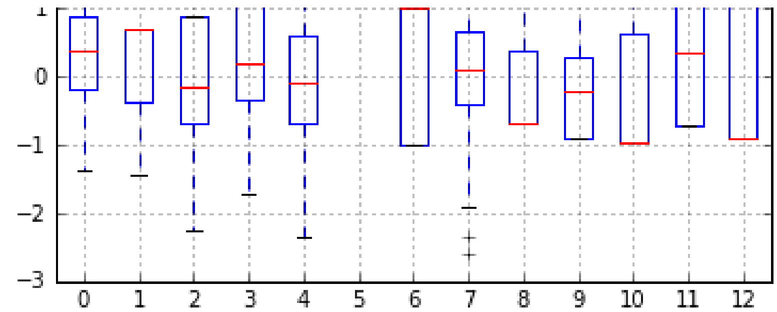


```
Hierarchical Cluster: 4
            0         1         2         3         4         5         
6  \
0    0.936181  0.691095 -2.240629  0.750380 -0.276443  2.430427  1.0
10199
9   -0.170689  0.691095  0.873880  0.468418 -0.854367  2.430427  1.0
10199
12   0.161372  0.691095 -0.164289 -0.095506  0.166631  2.430427  1.0
10199
31   0.604120  0.691095  0.873880 -0.828608 -0.334236  2.430427 -1.0
03419
39   0.714807  0.691095 -0.164289  1.032342 -0.083802  2.430427 -1.0
03419
49  -0.170689  0.691095 -0.164289 -0.095506 -0.969952  2.430427  1.0
10199
83   1.489615  0.691095 -0.164289  2.724115  0.513386  2.430427  1.0
10199
110  0.161372  0.691095  0.873880 -0.377469  0.031783  2.430427  1.0
10199
112 -1.277559 -1.446980  0.873880  0.017278  1.804082  2.430427  1.0
10199
117  0.936181  0.691095  0.873880 -0.095506  1.592176  2.430427  1.0
10199

            7         8         9        10        11        12
0    0.017494 -0.696419  1.068965  2.264145 -0.721976  0.655877
9    0.235807  1.435916  1.756157  2.264145 -0.721976  1.172577
12  -0.331806  1.435916 -0.391318  0.643781  0.344824  0.655877
31   0.454120  1.435916  0.295874 -0.976583  1.411625  1.172577
39  -0.550119  1.435916 -0.047722  0.643781 -0.721976 -0.894220
49   0.104820 -0.696419  0.124076  2.264145 -0.721976 -0.894220
83   0.017494  1.435916  0.467672  0.643781 -0.721976  1.172577
110 -0.244481  1.435916  0.124076  0.643781  0.344824 -0.894220
112 -0.593782  1.435916  1.670258  0.643781 -0.721976  1.172577
117 -0.768432  1.435916  0.639470 -0.976583  2.478425  1.172577
```

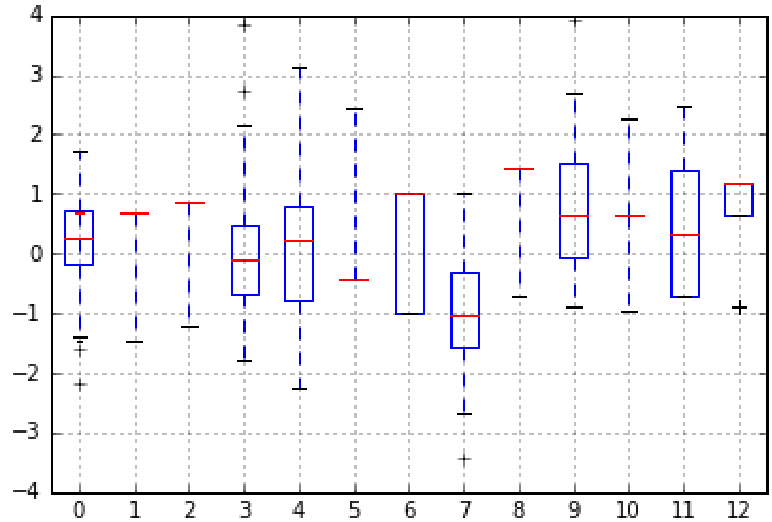



```
KMeans Cluster: 0
           0          1          2          3          4          5  
6  \
0   0.936181  0.691095 -2.240629  0.750380 -0.276443  2.430427  1.01
0199
12  0.161372  0.691095 -0.164289 -0.095506  0.166631  2.430427  1.01
0199
14 -0.281376  0.691095 -0.164289  2.272976 -0.931424  2.430427 -1.00
3419
21  0.382746 -1.446980 -2.240629  1.032342  0.686763  2.430427  1.01
0199
31  0.604120  0.691095  0.873880 -0.828608 -0.334236  2.430427 -1.00
3419
39  0.714807  0.691095 -0.164289  1.032342 -0.083802  2.430427 -1.00
3419
43  0.493433  0.691095 -0.164289  1.032342 -0.680990  2.430427 -1.00
3419
48  1.157555 -1.446980 -0.164289  0.468418  3.268155  2.430427  1.01
0199
49 -0.170689  0.691095 -0.164289 -0.095506 -0.969952  2.430427  1.01
0199
63 -0.060002 -1.446980 -0.164289  0.186456  1.091309  2.430427 -1.00
3419

           7          8          9         10         11         12
0   0.017494 -0.696419  1.068965  2.264145 -0.721976  0.655877
12 -0.331806  1.435916 -0.391318  0.643781  0.344824  0.655877
14  0.541445 -0.696419 -0.477217 -0.976583 -0.721976  1.172577
21  0.541445 -0.696419 -0.047722 -0.976583 -0.721976 -0.894220
31  0.454120  1.435916  0.295874 -0.976583  1.411625  1.172577
39 -0.550119  1.435916 -0.047722  0.643781 -0.721976 -0.894220
43  0.323132 -0.696419  0.467672 -0.976583 -0.721976 -0.894220
48  0.323132 -0.696419 -0.219520 -0.976583  0.344824 -0.894220
49  0.104820 -0.696419  0.124076  2.264145 -0.721976 -0.894220
63  0.890746 -0.696419 -0.906712 -0.976583 -0.721976 -0.894220
```

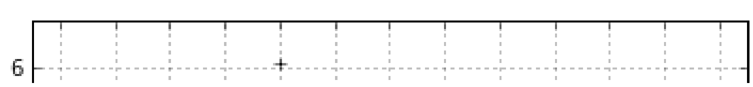


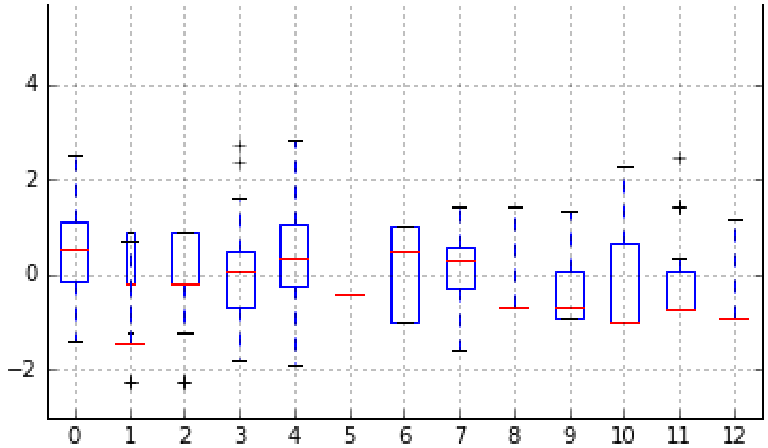



```
KMeans Cluster: 1
          0         1        2         3         4         5         
6  \
1   1.378929  0.691095  0.87388  1.596266  0.744555 -0.411450  1.010
199
2   1.378929  0.691095  0.87388 -0.659431 -0.353500 -0.411450  1.010
199
9  -0.170689  0.691095  0.87388  0.468418 -0.854367  2.430427  1.010
199
24  0.604120  0.691095  0.87388 -0.095506 -0.796575 -0.411450  1.010
199
29 -1.609620  0.691095  0.87388 -1.223355 -1.547875 -0.411450  1.010
199
36 -1.277559  0.691095  0.87388 -0.659431 -1.355234 -0.411450  1.010
199
37  0.272059  0.691095  0.87388  1.032342  0.551914 -0.411450  1.010
199
38  0.050685  0.691095  0.87388  0.017278  2.035251 -0.411450 -1.003
419
54  0.604120  0.691095  0.87388 -0.095506  0.108839 -0.411450 -1.003
419
55 -0.060002  0.691095  0.87388 -0.433861  0.359273 -0.411450  1.010
199

          7         8         9        10        11        12
1  -1.816334  1.435916  0.381773  0.643781  2.478425 -0.894220
2  -0.899420  1.435916  1.326662  0.643781  1.411625  1.172577
9   0.235807  1.435916  1.756157  2.264145 -0.721976  1.172577
24 -0.768432  1.435916  1.154864  0.643781  1.411625  1.172577
29 -1.554358  1.435916  0.811268  0.643781 -0.721976  1.172577
36 -1.292383  1.435916  1.240763  0.643781 -0.721976  1.172577
37 -1.641684  1.435916 -0.391318  0.643781  0.344824  0.655877
38 -0.768432  1.435916  0.124076  0.643781  0.344824  1.172577
54 -0.244481  1.435916  0.295874 -0.976583  0.344824  1.172577
55 -1.772671  1.435916  0.983066  0.643781  0.344824  1.172577
```

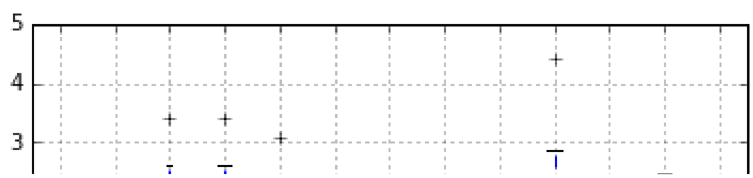


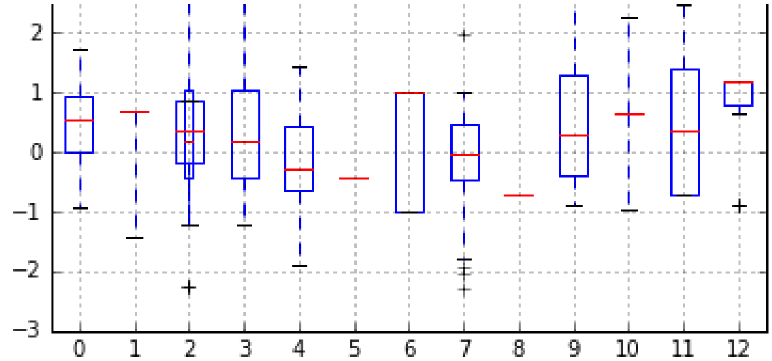
```
KMeans Cluster: 2
           0         1         2         3         4        5      
6  \
7   0.272059 -1.446980  0.873880 -0.659431  2.054515 -0.41145 -1.003
419
11  0.161372 -1.446980 -1.202459  0.468418  0.898668 -0.41145  1.010
199
18 -0.724124 -1.446980 -0.164289 -0.095506  0.532650 -0.41145 -1.003
419
25 -0.502750 -1.446980 -0.164289 -0.659431 -0.546141 -0.41145 -1.003
419
26  0.382746 -1.446980 -0.164289 -0.659431  1.784818 -0.41145 -1.003
419
27  1.268242 -1.446980 -2.240629  1.032342 -0.411292 -0.41145 -1.003
419
30  1.600302 -1.446980 -2.240629  0.468418 -0.160859 -0.41145 -1.003
419
32  1.046868  0.691095 -0.164289  0.468418  1.688497 -0.41145 -1.003
419
42  1.821676 -1.446980 -1.202459  1.596266  1.052781 -0.41145 -1.003
419
44  0.714807 -1.446980  0.873880 -0.095506  1.592176 -0.41145  1.010
199

           7         8         9        10        11       12
7   0.585108  1.435916 -0.391318 -0.976583 -0.721976 -0.89422
11  0.148482 -0.696419  0.209975  0.643781 -0.721976 -0.89422
18 -0.462794 -0.696419 -0.734914 -0.976583 -0.721976 -0.89422
25  0.366795 -0.696419  0.467672  0.643781 -0.721976 -0.89422
26  0.978071 -0.696419 -0.906712 -0.976583 -0.721976 -0.89422
27 -1.554358 -0.696419  1.326662  2.264145 -0.721976 -0.89422
30  0.061157 -0.696419  0.639470 -0.976583  1.411625 -0.89422
32  0.366795 -0.696419 -0.906712 -0.976583 -0.721976 -0.89422
42  0.541445 -0.696419 -0.563116 -0.976583  1.411625 -0.89422
44  0.847083 -0.696419 -0.906712 -0.976583 -0.721976 -0.89422
```



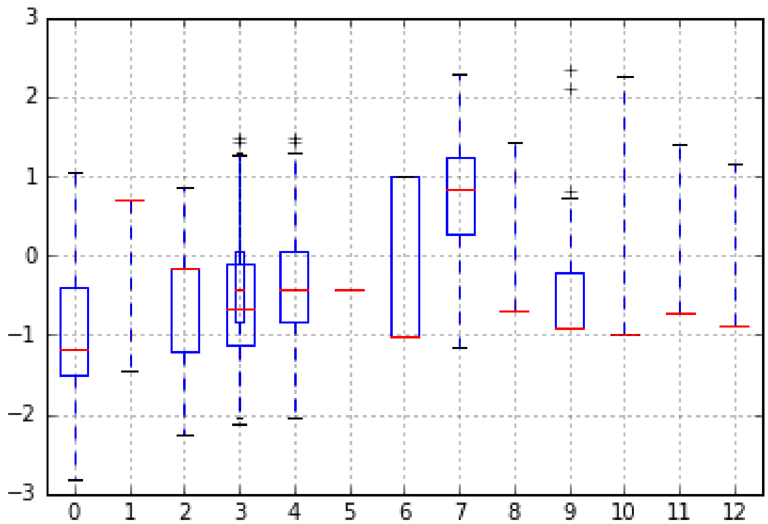



```
KMeans Cluster: 3
          0         1         2         3         4        5         
6   \
6   0.825494 -1.446980  0.873880  0.468418  0.397801 -0.41145  1.010
199
8   0.936181  0.691095  0.873880 -0.095506  0.128103 -0.41145  1.010
199
10  0.272059  0.691095  0.873880  0.468418 -1.066272 -0.41145 -1.003
419
23  0.382746  0.691095 -0.164289  0.017278 -0.449820 -0.41145  1.010
199
33  0.493433  0.691095  0.873880  0.186456 -0.257179 -0.41145 -1.003
419
40  1.157555 -1.446980  0.873880  1.032342 -0.430556 -0.41145  1.010
199
45  0.382746  0.691095 -0.164289 -1.110570 -0.334236 -0.41145  1.010
199
47 -0.502750  0.691095  0.873880  1.032342 -0.083802 -0.41145  1.010
199
51  1.157555  0.691095  0.873880 -0.659431 -1.355234 -0.41145 -1.003
419
56 -0.502750  0.691095 -0.164289  0.468418 -0.276443 -0.41145 -1.003
419

          7         8         9        10        11        12
6   0.454120 -0.696419  2.185652  2.264145  1.411625 -0.894220
8  -0.113493 -0.696419  0.295874  0.643781  0.344824  1.172577
10 -0.069831 -0.696419 -0.563116  0.643781 -0.721976  0.655877
23  1.021734 -0.696419  1.842056 -0.976583  1.411625  1.172577
33  0.497783 -0.696419 -0.477217  0.643781 -0.721976  1.172577
40 -1.554358 -0.696419 -0.047722  0.643781  2.478425  1.172577
45  0.672433 -0.696419  1.240763  0.643781  0.344824  1.172577
47 -0.943082 -0.696419  1.326662  0.643781 -0.721976  1.172577
51 -0.419131 -0.696419 -0.563116 -0.976583 -0.721976  1.172577
56  0.585108 -0.696419 -0.391318  0.643781  0.344824  1.172577
```

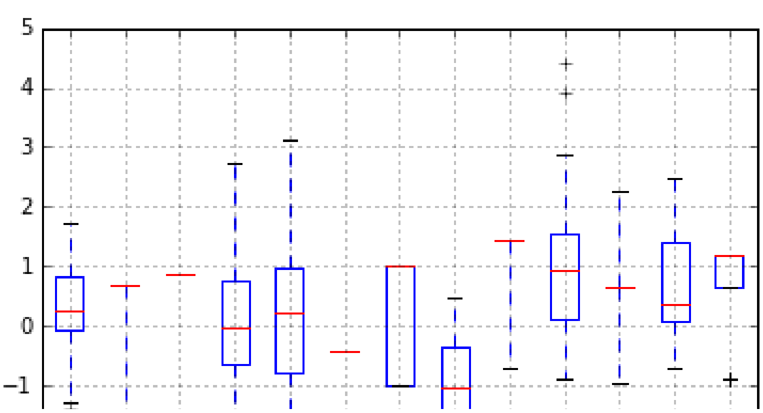


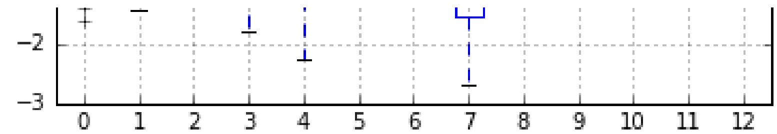



```
KMeans Cluster: 4
           0         1         2         3         4        5          
6  \
3  -1.941680  0.691095 -0.164289 -0.095506  0.051047 -0.41145 -1.003
419
4  -1.498933 -1.446980 -1.202459 -0.095506 -0.835103 -0.41145  1.010
199
5   0.161372  0.691095 -1.202459 -0.659431 -0.218651 -0.41145 -1.003
419
13 -1.166872  0.691095 -1.202459 -0.659431  0.301480 -0.41145 -1.003
419
15  0.272059  0.691095 -0.164289  1.032342 -1.528611 -0.41145 -1.003
419
16 -0.724124  0.691095 -1.202459 -1.223355 -0.353500 -0.41145 -1.003
419
17 -0.060002  0.691095  0.873880  0.468418 -0.160859 -0.41145 -1.003
419
19 -0.613437  0.691095 -1.202459 -0.095506  0.359273 -0.41145 -1.003
419
20  1.046868  0.691095 -2.240629 -1.223355 -0.700254 -0.41145  1.010
199
22  0.382746  0.691095 -1.202459 -0.659431  0.706027 -0.41145  1.010
199

           7         8         9        10        11        12
3   1.633010 -0.696419  2.099753  2.264145 -0.721976 -0.894220
4   0.978071 -0.696419  0.295874 -0.976583 -0.721976 -0.894220
5   1.240047 -0.696419 -0.219520 -0.976583 -0.721976 -0.894220
13  1.021734 -0.696419 -0.906712 -0.976583 -0.721976  1.172577
15  1.065396 -0.696419  0.467672 -0.976583 -0.721976 -0.894220
16  0.803421 -0.696419 -0.047722  2.264145 -0.721976  1.172577
17  0.454120 -0.696419  0.124076 -0.976583 -0.721976 -0.894220
19  0.934409 -0.696419 -0.391318 -0.976583 -0.721976 -0.894220
20 -0.244481  1.435916  0.639470  0.643781 -0.721976 -0.894220
22  0.454120 -0.696419  0.639470  0.643781 -0.721976 -0.894220
```

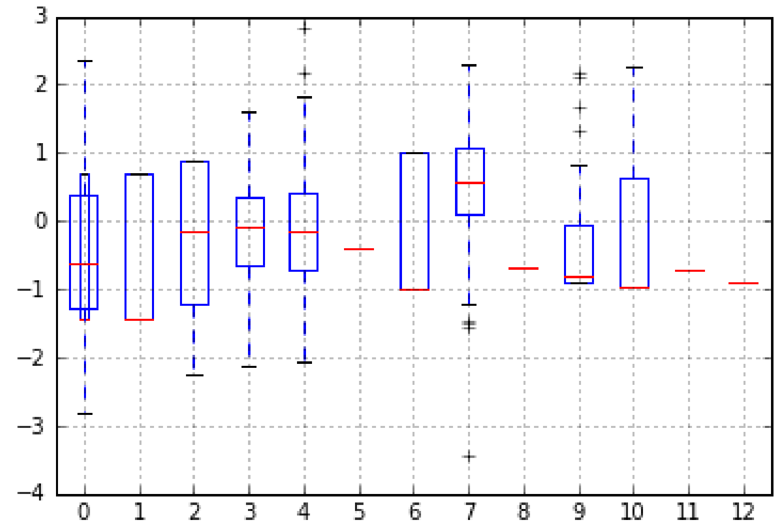

3
2
1
0
−1
−2
−3
0
1
2
3
4
5
6
7
8
9
10
11
12

```
GMM Cluster: 0
          0         1        2         3         4        5         
6   \
1   1.378929  0.691095  0.87388  1.596266  0.744555 -0.41145  1.0101
99
2   1.378929  0.691095  0.87388 -0.659431 -0.353500 -0.41145  1.0101
99
6   0.825494 -1.446980  0.87388  0.468418  0.397801 -0.41145  1.0101
99
24  0.604120  0.691095  0.87388 -0.095506 -0.796575 -0.41145  1.0101
99
29 -1.609620  0.691095  0.87388 -1.223355 -1.547875 -0.41145  1.0101
99
36 -1.277559  0.691095  0.87388 -0.659431 -1.355234 -0.41145  1.0101
99
37  0.272059  0.691095  0.87388  1.032342  0.551914 -0.41145  1.0101
99
38  0.050685  0.691095  0.87388  0.017278  2.035251 -0.41145 -1.0034
19
40  1.157555 -1.446980  0.87388  1.032342 -0.430556 -0.41145  1.0101
99
47 -0.502750  0.691095  0.87388  1.032342 -0.083802 -0.41145  1.0101
99

          7         8         9        10        11        12
1  -1.816334  1.435916  0.381773  0.643781  2.478425 -0.894220
2  -0.899420  1.435916  1.326662  0.643781  1.411625  1.172577
6   0.454120 -0.696419  2.185652  2.264145  1.411625 -0.894220
24 -0.768432  1.435916  1.154864  0.643781  1.411625  1.172577
29 -1.554358  1.435916  0.811268  0.643781 -0.721976  1.172577
36 -1.292383  1.435916  1.240763  0.643781 -0.721976  1.172577
37 -1.641684  1.435916 -0.391318  0.643781  0.344824  0.655877
38 -0.768432  1.435916  0.124076  0.643781  0.344824  1.172577
40 -1.554358 -0.696419 -0.047722  0.643781  2.478425  1.172577
47 -0.943082 -0.696419  1.326662  0.643781 -0.721976  1.172577
```

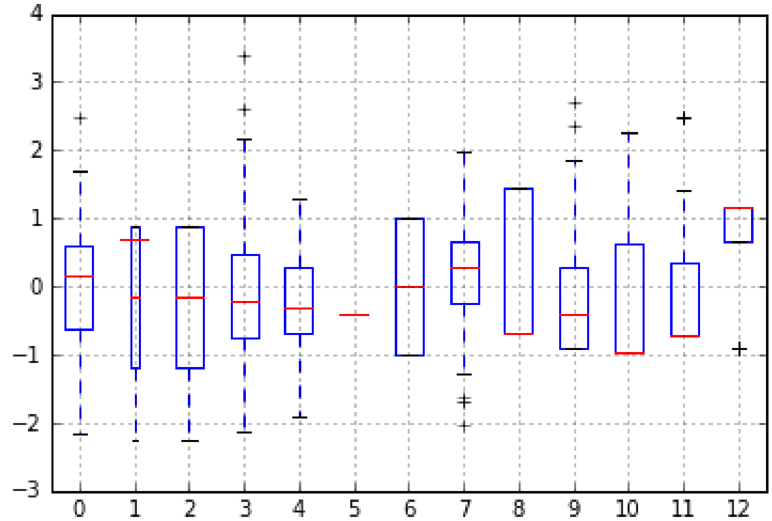




```
GMM Cluster: 1
          0         1         2         3         4        5
6   \
3  -1.941680  0.691095 -0.164289 -0.095506  0.051047 -0.41145 -1.003
419
4  -1.498933 -1.446980 -1.202459 -0.095506 -0.835103 -0.41145  1.010
199
5   0.161372  0.691095 -1.202459 -0.659431 -0.218651 -0.41145 -1.003
419
11  0.161372 -1.446980 -1.202459  0.468418  0.898668 -0.41145  1.010
199
15  0.272059  0.691095 -0.164289  1.032342 -1.528611 -0.41145 -1.003
419
17 -0.060002  0.691095  0.873880  0.468418 -0.160859 -0.41145 -1.003
419
18 -0.724124 -1.446980 -0.164289 -0.095506  0.532650 -0.41145 -1.003
419
19 -0.613437  0.691095 -1.202459 -0.095506  0.359273 -0.41145 -1.003
419
22  0.382746  0.691095 -1.202459 -0.659431  0.706027 -0.41145  1.010
199
25 -0.502750 -1.446980 -0.164289 -0.659431 -0.546141 -0.41145 -1.003
419

          7         8         9        10        11        12
3   1.633010 -0.696419  2.099753  2.264145 -0.721976 -0.89422
4   0.978071 -0.696419  0.295874 -0.976583 -0.721976 -0.89422
5   1.240047 -0.696419 -0.219520 -0.976583 -0.721976 -0.89422
11  0.148482 -0.696419  0.209975  0.643781 -0.721976 -0.89422
15  1.065396 -0.696419  0.467672 -0.976583 -0.721976 -0.89422
17  0.454120 -0.696419  0.124076 -0.976583 -0.721976 -0.89422
18 -0.462794 -0.696419 -0.734914 -0.976583 -0.721976 -0.89422
19  0.934409 -0.696419 -0.391318 -0.976583 -0.721976 -0.89422
22  0.454120 -0.696419  0.639470  0.643781 -0.721976 -0.89422
25  0.366795 -0.696419  0.467672  0.643781 -0.721976 -0.89422
```

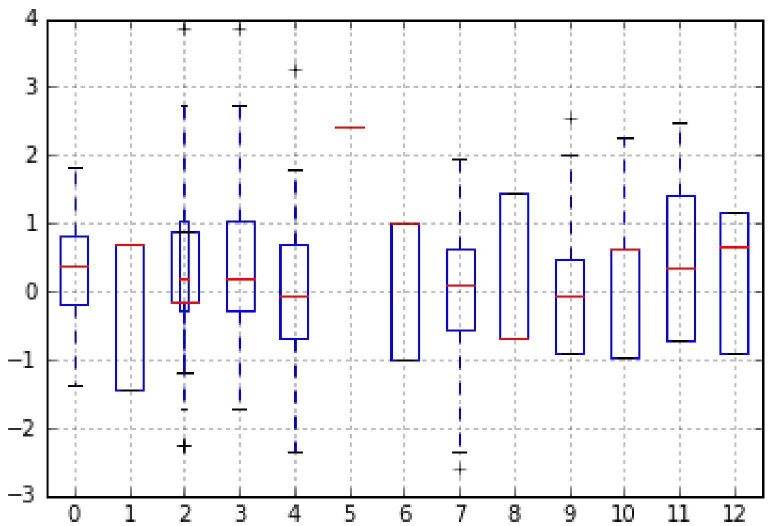



```
GMM Cluster: 2
           0         1         2         3         4        5         
6  \
8   0.936181  0.691095  0.873880 -0.095506  0.128103 -0.41145  1.010
199
10  0.272059  0.691095  0.873880  0.468418 -1.066272 -0.41145 -1.003
419
13 -1.166872  0.691095 -1.202459 -0.659431  0.301480 -0.41145 -1.003
419
16 -0.724124  0.691095 -1.202459 -1.223355 -0.353500 -0.41145 -1.003
419
20  1.046868  0.691095 -2.240629 -1.223355 -0.700254 -0.41145  1.010
199
23  0.382746  0.691095 -0.164289  0.017278 -0.449820 -0.41145  1.010
199
33  0.493433  0.691095  0.873880  0.186456 -0.257179 -0.41145 -1.003
419
34 -1.166872  0.691095 -0.164289 -0.095506 -0.276443 -0.41145 -1.003
419
41 -1.609620  0.691095 -2.240629  0.468418 -0.931424 -0.41145 -1.003
419
45  0.382746  0.691095 -0.164289 -1.110570 -0.334236 -0.41145  1.010
199

           7         8         9        10        11        12
8  -0.113493 -0.696419  0.295874  0.643781  0.344824  1.172577
10 -0.069831 -0.696419 -0.563116  0.643781 -0.721976  0.655877
13  1.021734 -0.696419 -0.906712 -0.976583 -0.721976  1.172577
16  0.803421 -0.696419 -0.047722  2.264145 -0.721976  1.172577
20 -0.244481  1.435916  0.639470  0.643781 -0.721976 -0.894220
23  1.021734 -0.696419  1.842056 -0.976583  1.411625  1.172577
33  0.497783 -0.696419 -0.477217  0.643781 -0.721976  1.172577
34  1.283709  1.435916 -0.563116 -0.976583 -0.721976 -0.894220
41  1.240047  1.435916  0.295874 -0.976583 -0.721976  1.172577
45  0.672433 -0.696419  1.240763  0.643781  0.344824  1.172577
```




```
GMM Cluster: 3
            0         1         2         3         4         5  
6   \
0   0.936181  0.691095 -2.240629  0.750380 -0.276443  2.430427  1.01
0199
9  -0.170689  0.691095  0.873880  0.468418 -0.854367  2.430427  1.01
0199
12  0.161372  0.691095 -0.164289 -0.095506  0.166631  2.430427  1.01
0199
14 -0.281376  0.691095 -0.164289  2.272976 -0.931424  2.430427 -1.00
3419
21  0.382746 -1.446980 -2.240629  1.032342  0.686763  2.430427  1.01
0199
31  0.604120  0.691095  0.873880 -0.828608 -0.334236  2.430427 -1.00
3419
39  0.714807  0.691095 -0.164289  1.032342 -0.083802  2.430427 -1.00
3419
43  0.493433  0.691095 -0.164289  1.032342 -0.680990  2.430427 -1.00
3419
48  1.157555 -1.446980 -0.164289  0.468418  3.268155  2.430427  1.01
0199
```

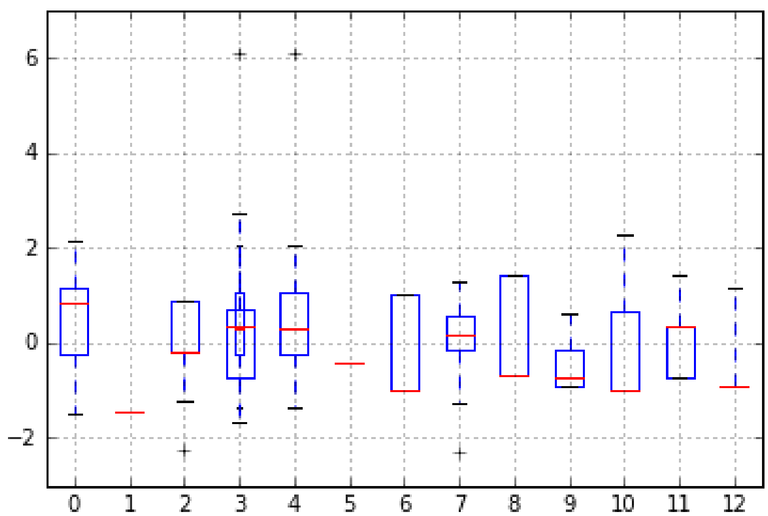
```
49 -0.170689  0.691095 -0.164289 -0.095506 -0.969952  2.430427  1.01
0199

           7         8         9        10        11        12
0   0.017494 -0.696419  1.068965  2.264145 -0.721976  0.655877
9   0.235807  1.435916  1.756157  2.264145 -0.721976  1.172577
12 -0.331806  1.435916 -0.391318  0.643781  0.344824  0.655877
14  0.541445 -0.696419 -0.477217 -0.976583 -0.721976  1.172577
21  0.541445 -0.696419 -0.047722 -0.976583 -0.721976 -0.894220
31  0.454120  1.435916  0.295874 -0.976583  1.411625  1.172577
39 -0.550119  1.435916 -0.047722  0.643781 -0.721976 -0.894220
43  0.323132 -0.696419  0.467672 -0.976583 -0.721976 -0.894220
48  0.323132 -0.696419 -0.219520 -0.976583  0.344824 -0.894220
49  0.104820 -0.696419  0.124076  2.264145 -0.721976 -0.894220
```




```
GMM Cluster: 4
            0        1         2         3         4        5
6   \
7     0.272059 -1.44698  0.873880 -0.659431  2.054515 -0.41145 -1.003
419
30    1.600302 -1.44698 -2.240629  0.468418 -0.160859 -0.41145 -1.003
419
42    1.821676 -1.44698 -1.202459  1.596266  1.052781 -0.41145 -1.003
419
50   -1.498933 -1.44698 -1.202459 -1.505317 -0.950688 -0.41145 -1.003
419
61   -0.945498 -1.44698 -0.164289  0.581203 -1.355234 -0.41145  1.010
199
77   -0.392063 -1.44698 -0.164289  0.468418  1.168366 -0.41145  1.010
199
101   0.272059 -1.44698  0.873880 -0.208291  1.072045 -0.41145  1.010
199
113   0.825494 -1.44698 -0.164289 -0.095506  0.301480 -0.41145 -1.003
419
```

```
148  0.604120 -1.44698 -0.164289 -1.674494  1.361007 -0.41145 -1.003
419
151  1.378929 -1.44698 -0.164289 -0.941393  6.099981 -0.41145  1.010
199
```

	7	8	9	10	11	12
7	0.585108	1.435916	-0.391318	-0.976583	-0.721976	-0.894220
30	0.061157	-0.696419	0.639470	-0.976583	1.411625	-0.894220
42	0.541445	-0.696419	-0.563116	-0.976583	1.411625	-0.894220
50	0.803421	-0.696419	-0.906712	-0.976583	0.344824	-0.894220
61	0.454120	1.435916	0.295874	2.264145	-0.721976	-0.894220
77	-0.331806	-0.696419	0.381773	-0.976583	0.344824	-0.894220
101	0.410458	-0.696419	-0.906712	-0.976583	0.344824	-0.894220
113	-2.296622	-0.696419	0.124076	0.643781	0.344824	1.172577
148	0.454120	-0.696419	-0.906712	-0.976583	0.344824	-0.894220
151	0.454120	-0.696419	0.467672	0.643781	-0.721976	1.172577

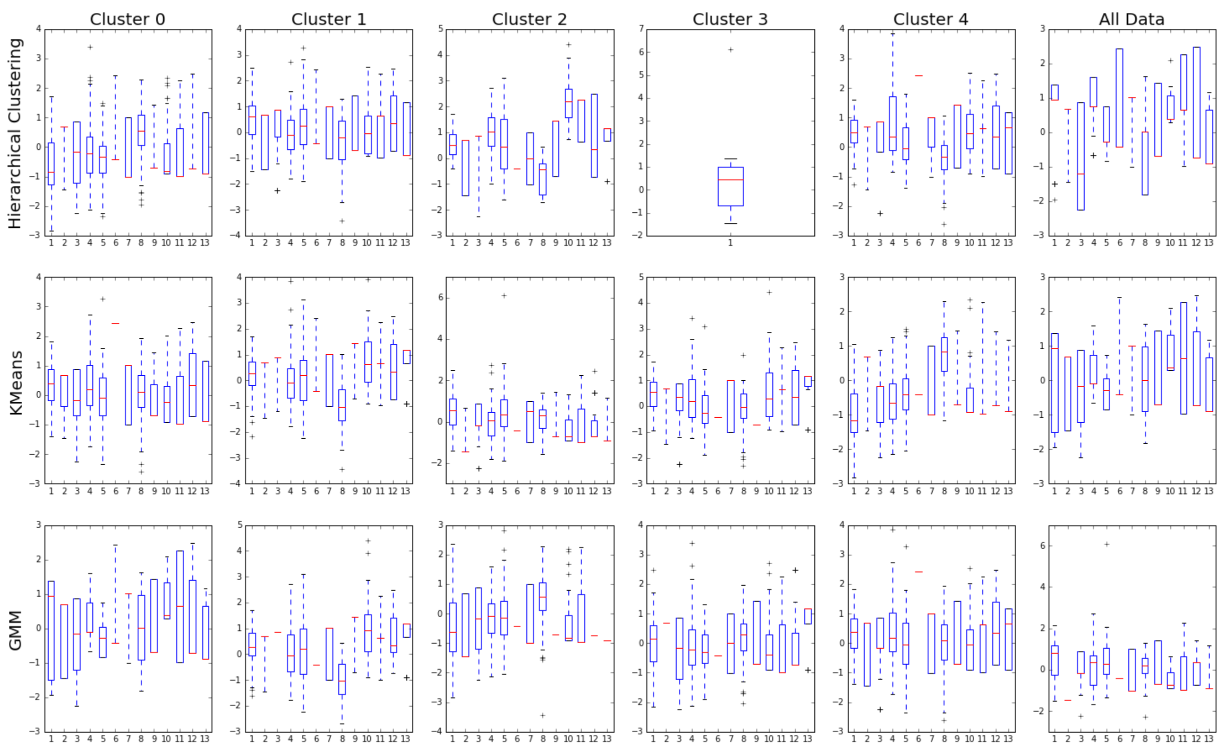


In [18]:

```
# # Part #5 Plotting boxplots of attributes
from sklearn import mixture, cluster

clusterers = {
    'KMeans':cluster.KMeans(n_clusters=5, init='random', random_state=2
    'Hierarchical':cluster.AgglomerativeClustering(n_clusters=5, linkag
    'GMM':mixture.GMM(n_components=5, random_state=20160217),
    }

clusterings = {'True':heart_labels}

for clusterer in clusterers.iterkeys():
    clusterings[clusterer]=clusterers[clusterer].fit_predict(heart_proc

all_values = []

for clusterer in clusterers.iterkeys():
    print "Model: " + clusterer
    for i in range(0, 5):
        cluster_index = []
        for instance in range(0, len(clusterings[clusterer])):
            if i == clusterings[clusterer][instance]:
                cluster_index.append(instance)

        df = pd.DataFrame(heart_processed.iloc[cluster_index])
#         print "Cluster: " + str(i)
#         print df.head(10)
        all_values.append(df)

    all_values.append(pd.DataFrame(heart_processed.iloc[clusterings[clu

font = 20
small_font = 20
f, ax = plt.subplots(3, 6, figsize=(25,15))
ax[0][0].boxplot(all_values[0].values)
ax[0][0].set_title("Cluster 0", fontsize=small_font)
ax[0][0].set_ylabel("Hierarchical Clustering", fontsize=font)

ax[0][1].boxplot(all_values[1].values)
ax[0][1].set_title("Cluster 1", fontsize=small_font)

ax[0][2].boxplot(all_values[2].values)
ax[0][2].set_title("Cluster 2", fontsize=small_font)

ax[0][3].boxplot(all_values[3].values)
ax[0][3].set_title("Cluster 3", fontsize=small_font)

ax[0][4].boxplot(all_values[4].values)
ax[0][4].set_title("Cluster 4", fontsize=small_font)

ax[0][5].boxplot(all_values[5].values)
ax[0][5].set_title("All Data", fontsize=small_font)

ax[1][0].boxplot(all_values[6].values)
```

```
ax[1][0].boxplot(all_values[6].values)
ax[1][0].set_ylabel("KMeans", fontsize=font)
ax[1][1].boxplot(all_values[7].values)
ax[1][2].boxplot(all_values[8].values)
ax[1][3].boxplot(all_values[9].values)
ax[1][4].boxplot(all_values[10].values)
ax[1][5].boxplot(all_values[11].values)

ax[2][0].boxplot(all_values[11].values)
ax[2][0].set_ylabel("GMM", fontsize=font)
ax[2][1].boxplot(all_values[12].values)
ax[2][2].boxplot(all_values[13].values)
ax[2][3].boxplot(all_values[14].values)
ax[2][4].boxplot(all_values[15].values)
ax[2][5].boxplot(all_values[16].values)

plt.show()
```

```
Model: Hierarchical
Model: KMeans
Model: GMM
```



Question 3: Clustering Census Data (15 points)

This question is a bit trickier. You'll be clustering data with a mix of numeric and nominal attributes. I've encoded the one ordinal attribute for you. You'll be looking at a few different techniques to work with this sort of data.

1. Using census_data_scaled, run KMeans and GMM clustering, while increasing the number of clusters to 100 taking steps of 4 (e.g. range(1,100,4)). Plot the WSS with

respect to the number of clusters. It may also be helpful to plot the change in WSS as the number of clusters increase. Where does the WSS error seem to plateau?

2. One of the issues we've noted is that distance functions often behave nonintuitively in very high dimensions. Let's try to address this problem in a couple of ways. One option is to project into a lower dimensional space. We used PCA for this in Assignment 1.
   - Project the census data into a 3-dimensional space and a 7-dimensional space using PCA.
   - Repeat the clustering experiment, plotting the WSS for both settings. Do you notice a difference in how the WSS decreases?
   - Choose a value for the number of clusters based on the plot. Pick one of the clusters from the 7D decomposition in PCA space and examine that cluster in the original attribute space. What are the salient attribute values for that cluster?
3. In some cases, the data is naturally high-dimensional. However, in this situation the high dimensionality of the data is the consequence of our preprocessing. Many of the attributes in the data arise from nominal attributes that have been translated into a OneHot encoding. If these nominal attributes could be encoded more densely, our clusterings might be more useful. We can try to encode the nominal attributes with many values (WorkClass, MaritalStatus, Occupation, NativeCountry) in the original data differently. You could, for example, encode Marital Status as Married or Single, re-encode countries into continents, re-encode WorkClass into Government/non-Government, and Occupation into white-collar or blue-collar. These are just examples, and you can choose your own encoding strategy.
   - Re-encode the nominal attributes using a scheme of your choice
   - Repeat the experiment, plotting the WSS again. What do you notice?
   - Choose a number of clusters based on the WSS plot. Pick one cluster and report the salient attributes for that cluster.

Answers 3

1. The WSS error seems to plateau around 70 clusters for GMM and around 80 for KMeans

2. The WSS values decrease very differently than the high dimensinality data.
Furhtermoer, between the 3 and 7 D data, the error decreases at the same speed, but at different values. The shape is the same while the actual error values are different. Higher for the 7D data.
Also when performing PCA the error starts to plateau much earlier, at around 10 clusters.
Furhtermore, the lower the dimensinality of the data, the lower the WSS error.
For PCA 3 dimensions, the error starts out at 300,000 and goes down to 9000 for each
For PCA 7 dimesions, the error starts out at around 600,000 and goes down to about 50,000, so not as low as the lower dimensional PCA 3.

Plotting one of the clusters of the 7D space and the equivalent cluster with 88 features, we see that only few features have a good feature value distribution. Most features in the original dataset are have a very small range of values. There are 9 features which are relevant: 0, 3, 4, 5, 6, 54, 74, 77 and 82. We see that the choice of a 7D PCA is good, though we could have probably increased it to 9 capture the 2 additional features that might have contributed to the clustering.

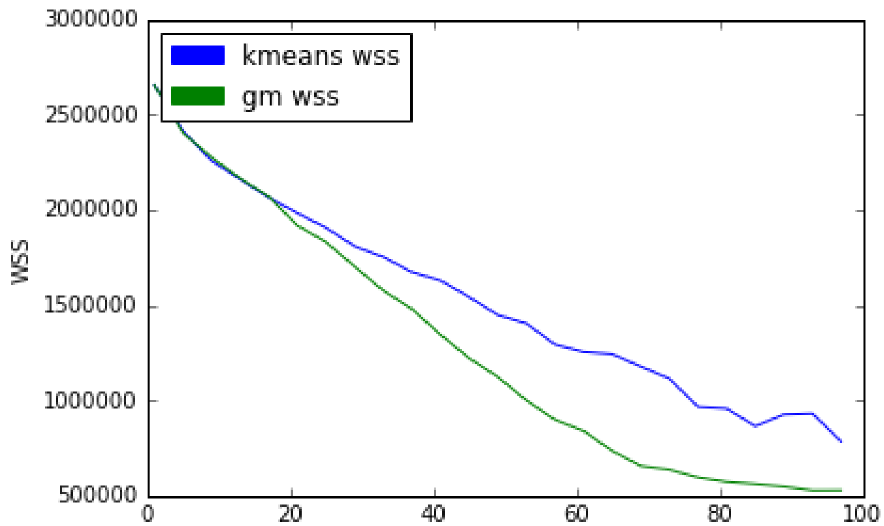
3. Reencoding the high dimensional nominal values and plotting the WSS we notice that the plot starts to look very similar to the plot of the PCA WSS error. This makes sense, because with the reenconding we are doing the same as what PCA does, reduces the dimensionality of the attributes. Thus the simiar shape makes sense.
It's important to note that the WSS error is much higher than the PCA, yet lower than performing one-hot encoding.
Salient attributes: 0, 3, 4, 8, 10, 57, 70, 72, 73, 79, 80, 83

In [19]:
```
census_data = urllib2.urlopen("http://archive.ics.uci.edu/ml/machine-le
census_orig = pd.read_csv(census_data, quotechar='"', skipinitialspace=
                              names=['Age','WorkClass', 'FnlWgt', 'Edu
                                     'Occupation', 'Relationship', 'Ra
                                     'CapitalGain', 'CapitalLoss', 'Ho
                                     'NativeCountry', 'Label'],
                              na_values="?", index_col=False)

census_orig = census_orig.dropna()
census_orig = census_orig.drop('FnlWgt',1)
education_translation = {'Preschool':0,
                        '1st-4th':1,
                         '5th-6th':2,
                         '7th-8th':3,
                         '9th':4,
                         '10th':5,
                         '11th':6,
                         '12th':7,
                         'HS-grad':8,
                         'Some-college':9,
                         'Assoc-voc':10,
                         'Assoc-acdm':11,
                         'Bachelors':12,
                         'Masters':13,
                         'Prof-school':14,
                         'Doctorate':15
                        }
census_orig['Education'] = census_orig['Education'].apply(lambda x: edu

# Convert labels from strings to boolean
label_encoder = preprocessing.LabelEncoder()
census_labels = pd.DataFrame(label_encoder.fit_transform(census_orig.il

# Convert nominal attributes to encoded versions
attr_encoder = feature_extraction.DictVectorizer(sparse=False)
census_data = pd.DataFrame(attr_encoder.fit_transform(census_orig.iloc[
census_data.columns = attr_encoder.get_feature_names()

census_data_scaled = pd.DataFrame(preprocessing.StandardScaler().fit_tr
#census_data_scaled.columns = attr_encoder.get_feature_names()
```

In [20]:
```
# Part 1
from sklearn import mixture, cluster

def wss(kmeans, cluster_size):
    wss = 0
    # Iterate over each cluster
    for cluster in range(0, cluster_size):
        cluster_instances = []
        # Iterate over each instance of the dataset
        for i in range(0, len(kmeans)):
            # If that dataset instance is in the right cluster
            if (kmeans[i] == cluster):
                cluster_instances.append(i)


        census_cluster = census_data_scaled.iloc[cluster_instances]
        cols = census_cluster.shape[1]

        for col in range(0, cols):
            feature_vals = census_cluster[col].values
            attr_mean = 0
            for i in range(0, len(feature_vals)):
                attr_mean += feature_vals[i]

            attr_mean = float(attr_mean / float(len(feature_vals)))

            for i in range(0, len(feature_vals)):
                point = (feature_vals[i] - attr_mean)**2
                wss+= point
    return wss

cluster_size = []
kmeans_wss_vals = []
gmm_wss_vals = []
for cluster_num in range(1, 100, 4):
    km = cluster.KMeans(n_clusters=cluster_num, init='random', random_s
    gmm = mixture.GMM(n_components=cluster_num, random_state=20160217)

    kmeans_wss = wss(km.fit_predict(census_data_scaled), cluster_num)
    gmm_wss = wss(gmm.fit_predict(census_data_scaled), cluster_num)

    cluster_size.append(cluster_num)
    kmeans_wss_vals.append(kmeans_wss)
    gmm_wss_vals.append(gmm_wss)

    print "Cluster: " + str(cluster_num)
    print "KM wss : " + str(kmeans_wss) + " GMM wss: " + str(gmm_wss)
```

```
Cluster: 1
KM wss : 2654256.00002 GMM wss: 2654256.00002
Cluster: 5
KM wss : 2413587.05166 GMM wss: 2403845.90346
Cluster: 9
KM wss : 2257288.92756 GMM wss: 2275604.63554
Cluster: 13
KM wss : 2159732.92708 GMM wss: 2164168.37649
Cluster: 17
KM wss : 2065185.73405 GMM wss: 2069432.46524
Cluster: 21
KM wss : 1982692.4696 GMM wss: 1917221.74872
Cluster: 25
KM wss : 1905639.43897 GMM wss: 1830700.96148
Cluster: 29
KM wss : 1808786.1577 GMM wss: 1702252.06306
Cluster: 33
KM wss : 1752780.28775 GMM wss: 1576220.37798
Cluster: 37
KM wss : 1672867.74956 GMM wss: 1479942.97078
Cluster: 41
KM wss : 1629500.98015 GMM wss: 1345238.04516
Cluster: 45
KM wss : 1542574.00487 GMM wss: 1221620.49957
Cluster: 49
KM wss : 1448685.99897 GMM wss: 1123931.39958
Cluster: 53
KM wss : 1404466.77561 GMM wss: 1000544.07076
Cluster: 57
KM wss : 1293064.34984 GMM wss: 898638.329022
Cluster: 61
KM wss : 1254866.25114 GMM wss: 840297.592565
Cluster: 65
KM wss : 1243397.80166 GMM wss: 735228.388188
Cluster: 69
KM wss : 1177783.52734 GMM wss: 655924.623116
Cluster: 73
KM wss : 1114758.41641 GMM wss: 637116.979107
Cluster: 77
KM wss : 966211.433307 GMM wss: 595922.428696
Cluster: 81
KM wss : 958060.178007 GMM wss: 573118.084469
Cluster: 85
KM wss : 864980.122724 GMM wss: 561486.245808
Cluster: 89
KM wss : 926380.048568 GMM wss: 548736.064057
Cluster: 93
KM wss : 931768.420137 GMM wss: 528466.64443
Cluster: 97
KM wss : 784998.311987 GMM wss: 528959.304985
```

In [21]:
```
plt.plot(cluster_size, kmeans_wss_vals, 'b', cluster_size, gmm_wss_vals
blue_patch = mpatches.Patch(color='blue', label='kmeans wss')
green_patch = mpatches.Patch(color='green', label='gm wss')

plt.legend(handles=[blue_patch, green_patch], loc='upper left')

plt.ylabel("WSS")
plt.show()
```



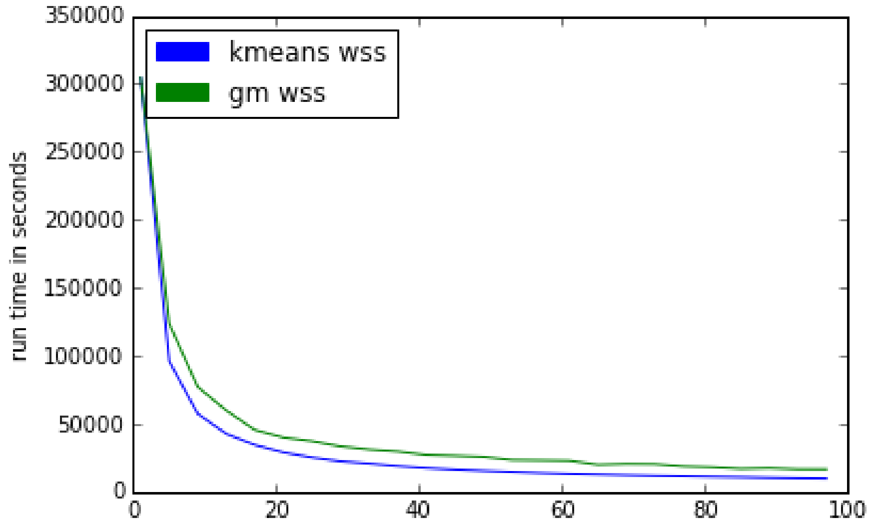
In [22]:

```
# Part 2
from sklearn.decomposition import PCA

def wss(kmeans, cluster_size, census_data_scaled):
    wss = 0
    # Iterate over each cluster
    for cluster in range(0, cluster_size):
        cluster_instances = []
        # Iterate over each instance of the dataset
        for i in range(0, len(kmeans)):
            # If that dataset instance is in the right cluster
            if (kmeans[i] == cluster):
                cluster_instances.append(i)


        census_cluster = census_data_scaled.iloc[cluster_instances]
        cols = census_cluster.shape[1]

        for col in range(0, cols):
            feature_vals = census_cluster[col].values
            attr_mean = 0
            for i in range(0, len(feature_vals)):
                attr_mean += feature_vals[i]

            if len(feature_vals) == 0:
                attr_mean = 0
            else:
                attr_mean = float(attr_mean / float(len(feature_vals)))

            for i in range(0, len(feature_vals)):
                point = (feature_vals[i] - attr_mean)**2
                wss+= point
    return wss

pca_3 = pd.DataFrame(PCA(n_components=3).fit_transform(census_data_scal
pca_7 = pd.DataFrame(PCA(n_components=7).fit_transform(census_data_scal


pca_list = [pca_3, pca_7]

for pca_data in pca_list:

    pca_cluster_size = []
    pca_kmeans_wss_vals = []
    pca_gmm_wss_vals = []
    print pca_data.head()
    print "-----------PCA VALUES------------"
    for cluster_num in range(1, 100, 4):
        km = cluster.KMeans(n_clusters=cluster_num, init='random', rand
        gmm = mixture.GMM(n_components=cluster_num, random_state=201602

        kmeans_wss = wss(km.fit_predict(pca_data), cluster_num, pca_dat
        gmm_wss = wss(gmm.fit_predict(pca_data), cluster_num, pca_data)
```

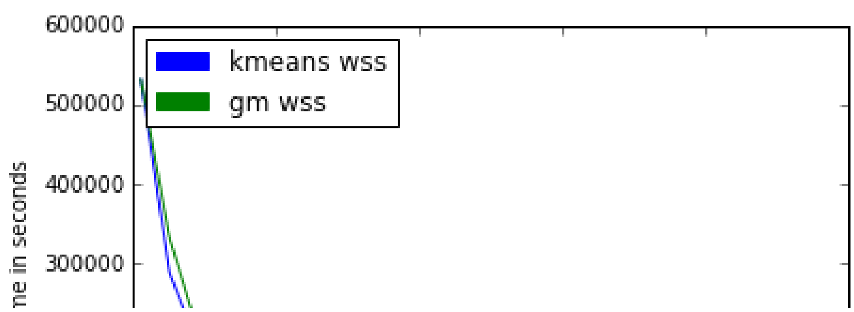
```
        pca_cluster_size.append(cluster_num)
        pca_kmeans_wss_vals.append(kmeans_wss)
        pca_gmm_wss_vals.append(gmm_wss)

        print "Cluster: " + str(cluster_num)
        print "KM wss : " + str(kmeans_wss) + " GMM wss: " + str(gmm_ws

    plt.plot(pca_cluster_size, pca_kmeans_wss_vals, 'b', pca_cluster_si
    blue_patch = mpatches.Patch(color='blue', label='kmeans wss')
    green_patch = mpatches.Patch(color='green', label='gm wss')

    plt.legend(handles=[blue_patch, green_patch], loc='upper left')

    plt.ylabel("run time in seconds")
    plt.show()
```

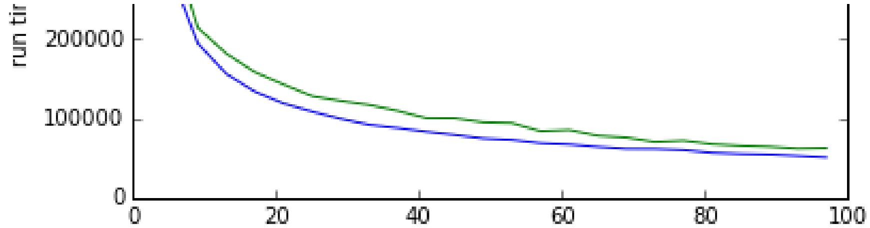
```
          0         1         2
0  0.607939 -2.269147 -0.763535
1 -2.788245 -1.317830 -0.023245
2  0.803568  0.708483 -1.199120
3 -0.901320  2.529662  1.173713
4  1.204012 -1.421205  4.418078
-----------PCA VALUES------------
Cluster: 1
KM wss : 304659.136888 GMM wss: 304659.136888
Cluster: 5
KM wss : 96032.5125022 GMM wss: 123704.17403
Cluster: 9
KM wss : 57337.0908372 GMM wss: 77060.8308305
Cluster: 13
KM wss : 42569.6730684 GMM wss: 59749.8019737
Cluster: 17
KM wss : 34404.8244546 GMM wss: 45424.5730271
Cluster: 21
KM wss : 28910.7311625 GMM wss: 39878.0591016
Cluster: 25
KM wss : 25178.0482025 GMM wss: 37192.1741238
Cluster: 29
KM wss : 22388.6911411 GMM wss: 33357.8453997
Cluster: 33
KM wss : 20720.9293346 GMM wss: 31064.6806697
Cluster: 37
KM wss : 18869.0939049 GMM wss: 29592.1449597
Cluster: 41
KM wss : 17442.4623923 GMM wss: 27061.9355364
Cluster: 45
KM wss : 16289.070252 GMM wss: 26403.8788113
Cluster: 49
KM wss : 15342.8269028 GMM wss: 25622.0442064
Cluster: 53
KM wss : 14523.5707016 GMM wss: 23377.8640907
Cluster: 57
KM wss : 13770.5697844 GMM wss: 23164.7934616
Cluster: 61
KM wss : 13126.232313 GMM wss: 22890.0196648
Cluster: 65
KM wss : 12652.3087242 GMM wss: 19910.7798498
Cluster: 69
KM wss : 12169.8127832 GMM wss: 20461.5181309
Cluster: 73
KM wss : 11735.8321891 GMM wss: 20320.4479115
Cluster: 77
KM wss : 11417.8883367 GMM wss: 18680.9543263
Cluster: 81
KM wss : 11030.1787208 GMM wss: 18109.7412403
Cluster: 85
KM wss : 10717.4728204 GMM wss: 16978.6376745
Cluster: 89
KM wss : 10312.2236635 GMM wss: 17390.6841564
Cluster: 93
```

```
KM wss : 10000.1615502 GMM wss: 16621.9947978
Cluster: 97
KM wss : 9737.05356624 GMM wss: 16575.505536
```



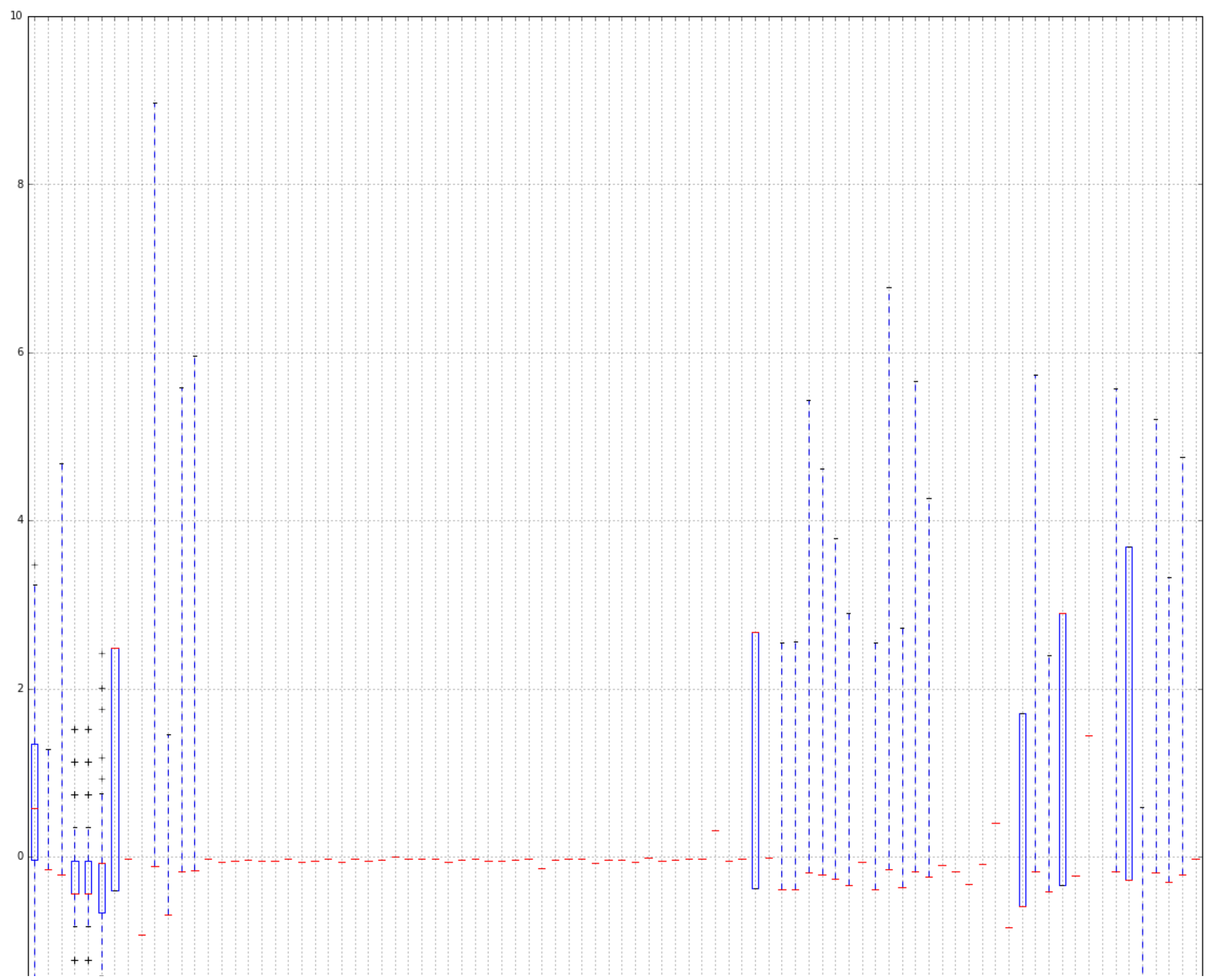
```
          0         1         2         3         4         5         
6
0  0.607939 -2.269147 -0.763535  1.360590 -0.782690 -1.889694 -0.572
049
1 -2.788245 -1.317830 -0.023245 -0.209085 -0.328010 -0.509259 -1.239
226
2  0.803568  0.708483 -1.199120 -0.276066  0.336155 -1.341190  2.139
172
3 -0.901320  2.529662  1.173713 -0.648880 -3.038289  1.467002  1.158
316
4  1.204012 -1.421205  4.418078  0.123548  1.472908  2.488223 -2.084
141
-----------PCA VALUES------------
Cluster: 1
KM wss : 531780.675644 GMM wss: 531780.675644
Cluster: 5
KM wss : 288863.707382 GMM wss: 332772.115569
Cluster: 9
KM wss : 194777.021722 GMM wss: 214440.735736
Cluster: 13
KM wss : 156218.276029 GMM wss: 181784.370583
Cluster: 17
KM wss : 133779.584715 GMM wss: 158381.643779
Cluster: 21
KM wss : 118918.354128 GMM wss: 142873.48907
Cluster: 25
KM wss : 108698.145359 GMM wss: 128234.688623
```

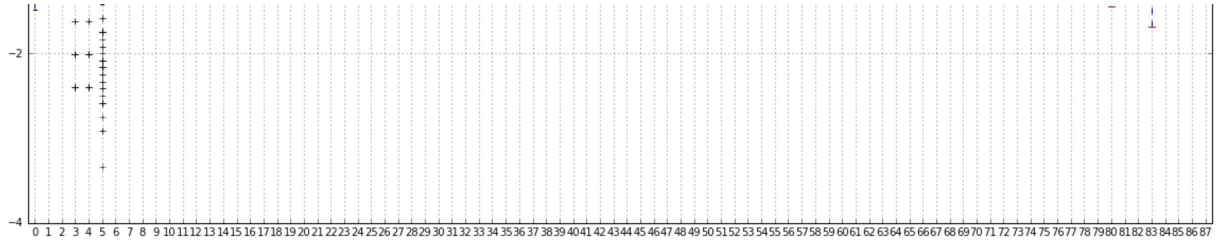



run tir
200000
100000
0
0
20
40
60
80
100

In [23]:
```
# Part 2 salient attribute
from sklearn.decomposition import PCA
import matplotlib.pyplot as plt

pca_data = pd.DataFrame(PCA(n_components=7).fit_transform(census_data_s
cluster_size = 40

gmm = mixture.GMM(n_components=cluster_size, random_state=20160217)

gmm_predict_pca = gmm.fit_predict(pca_data)

cluster_instances = []
# Iterate over each instance of the dataset
for i in range(0, len(gmm_predict_pca)):
    # If that dataset instance is in the right cluster
    if (gmm_predict_pca[i] == 5):
        cluster_instances.append(i)

pca_cluster = pca_data.iloc[cluster_instances]
normal_cluster = census_data_scaled.iloc[cluster_instances]

plt.figure(figsize=(20,20))

normal_cluster.boxplot(return_type='axes')
```

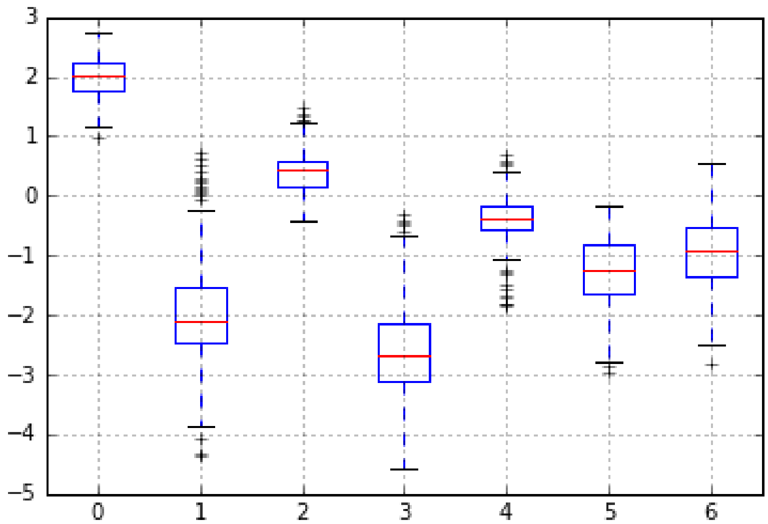
Out[23]: <matplotlib.axes._subplots.AxesSubplot at 0x112891e10>





In [24]:
```
# Part 2 PCA boxplot
pca_cluster.boxplot(return_type='axes')
```

Out[24]: <matplotlib.axes._subplots.AxesSubplot at 0x111eafbd0>



In [25]:
```
# Part 3
census_data = urllib2.urlopen("http://archive.ics.uci.edu/ml/machine-le
census_orig = pd.read_csv(census_data, quotechar='"', skipinitialspace=
                              names=['Age','WorkClass', 'FnlWgt', 'Edu
                                     'Occupation', 'Relationship', 'Ra
                                     'CapitalGain', 'CapitalLoss', 'Ho
                                     'NativeCountry', 'Label'],
                              na_values="?", index_col=False)

census_orig = census_orig.dropna()
census_orig = census_orig.drop('FnlWgt',1)
education_translation = {'Preschool':0,
                        '1st-4th':1,
                         '5th-6th':2,
                         '7th-8th':3,
                         '9th':4,
                         '10th':5,
                         '11th':6,
                         '12th':7,
                         'HS-grad':8,
                         'Some-college':9,
                         'Assoc-voc':10,
                         'Assoc-acdm':11,
                         'Bachelors':12,
                         'Masters':13,
                         'Prof-school':14,
                         'Doctorate':15
                        }
census_orig['Education'] = census_orig['Education'].apply(lambda x: edu
```

In [26]:

```
martial_status_translation = {'Separated':0,
                         'Widowed':0,
                         'Divorced':0,
                         'Married-spouse-absent':1,
                         'Never-married':0,
                         'Married-AF-spouse':1,
                         'Married-civ-spouse':1
                        }

relationship_translation = {'Own-child':1,
                         'Wife':1,
                         'Unmarried':0,
                         'Other-relative':1,
                         'Husband':1,
                         'Not-in-family': 0
                        }

work_class_translation = {'Self-emp-inc':'Non-Government',
                         'State-gov':'Government',
                         'Without-pay':'Non-Government',
                         'Private':'Non-Government',
                         'Local-gov':'Government',
                         'Self-emp-not-inc':'Non-Government',
                         'Federal-gov':'Government'
                        }
occupation_translation = {'Farming-fishing':'blue-collar',
                             'Armed-Forces':'blue-collar',
                              'Craft-repair':'blue-collar',
                              'Other-service':'white-collar',
                              'Transport-moving':'blue-collar',
                              'Prof-specialty':'white-collar',
                              'Sales':'white-collar',
                              'Exec-managerial':'white-collar',
                              'Handlers-cleaners':'blue-collar',
                              'Adm-clerical':'white-collar',
                              'Protective-serv':'white-collar',
                              'Tech-support':'white-collar',
                              'Priv-house-serv':'blue-collar',
                              'Machine-op-inspct':'white-collar'
                              }
native_country_transformation = {'Canada': 'NA',
                                 'Hong': 'ASIA',
                                 'Dominican-Republic': 'CA',
                                 'Italy': 'EUROPE',
                                 'Ireland': 'EUROPE',
                                 'Outlying-US(Guam-USVI-etc)': 'ASIA',
                                 'Scotland': 'EUROPE',
                                 'Cambodia': 'ASIA',
                                 'France': 'EUROPE',
                                 'Peru': 'SA',
                                 'Laos': 'ASIA',
                                 'Ecuador': 'CA',
                                 'Iran': 'ASIA',
```

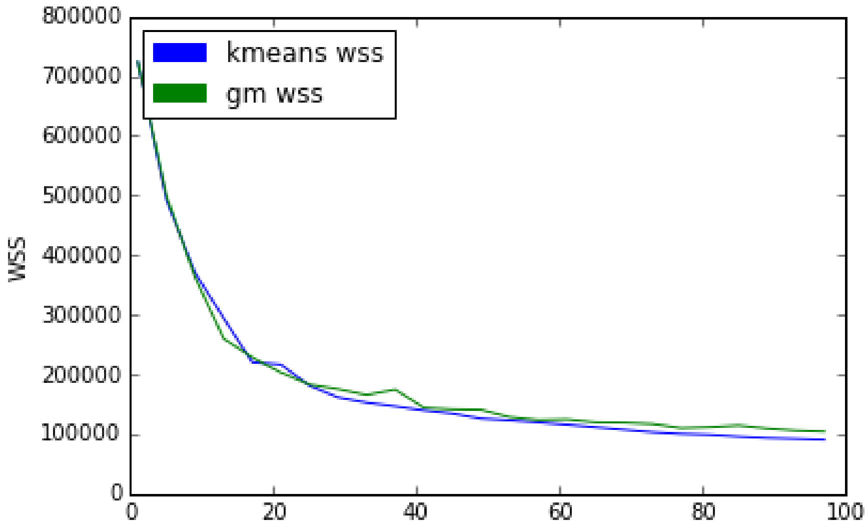
```
                                 'Cuba': 'CA',
                                 'Guatemala': 'SA',
                                 'Germany': 'EUROPE',
                                 'Thailand': 'ASIA',
                                 'Haiti': 'CA',
                                 'Poland': 'EUROPE',
                                 'Holand-Netherlands': 'EUROPE',
                                 'Philippines': 'ASIA',
                                 'Vietnam': 'ASIA',
                                 'Hungary': 'EUROPE',
                                 'England': 'EUROPE',
                                 'South': 'CA',
                                 'Jamaica': 'CA',
                                 'Honduras': 'CA',
                                 'Portugal': 'EUROPE',
                                 'Mexico': 'CA',
                                 'El-Salvador': 'SA',
                                 'India': 'ASIA',
                                 'Puerto-Rico': 'CA',
                                 'China': 'ASIA',
                                 'Yugoslavia': 'EUROPE',
                                 'United-States': 'NA',
                                 'Trinadad&Tobago': 'CA',
                                 'Greece': 'EUROPE',
                                 'Japan': 'ASIA',
                                 'Taiwan': 'ASIA',
                                 'Nicaragua': 'CA',
                                 'Columbia': 'SA'}

census_orig['MaritalStatus'] = census_orig['MaritalStatus'].apply(lambd
census_orig['Relationship'] = census_orig['Relationship'].apply(lambda
census_orig['WorkClass'] = census_orig['WorkClass'].apply(lambda x: wor
census_orig['Occupation'] = census_orig['Occupation'].apply(lambda x: o
census_orig['NativeCountry'] = census_orig['NativeCountry'].apply(lambd
```

In [27]:
```
# Convert labels from strings to boolean
label_encoder = preprocessing.LabelEncoder()
census_labels = pd.DataFrame(label_encoder.fit_transform(census_orig.il

# Convert nominal attributes to encoded versions
attr_encoder = feature_extraction.DictVectorizer(sparse=False)
census_data = pd.DataFrame(attr_encoder.fit_transform(census_orig.iloc[
census_data.columns = attr_encoder.get_feature_names()

census_data_scaled_transformed = pd.DataFrame(preprocessing.StandardSca
#census_data_scaled.columns = attr_encoder.get_feature_names()
```

In [29]:
```
# Part 3 Plotting wss
from sklearn import mixture, cluster

def wss(kmeans, cluster_size):
    wss = 0
    # Iterate over each cluster
    for cluster in range(0, cluster_size):
        cluster_instances = []
        # Iterate over each instance of the dataset
        for i in range(0, len(kmeans)):
            # If that dataset instance is in the right cluster
            if (kmeans[i] == cluster):
                cluster_instances.append(i)


        census_cluster = census_data_scaled_transformed.iloc[cluster_in
        cols = census_cluster.shape[1]

        for col in range(0, cols):
            feature_vals = census_cluster[col].values
            attr_mean = 0
            for i in range(0, len(feature_vals)):
                attr_mean += feature_vals[i]

            attr_mean = float(attr_mean / float(len(feature_vals)))

            for i in range(0, len(feature_vals)):
                point = (feature_vals[i] - attr_mean)**2
                wss+= point
    return wss

cluster_size = []
kmeans_wss_vals = []
gmm_wss_vals = []
for cluster_num in range(1, 100, 4):
    km = cluster.KMeans(n_clusters=cluster_num, init='random', random_s
    gmm = mixture.GMM(n_components=cluster_num, random_state=20160217)

    kmeans_wss = wss(km.fit_predict(census_data_scaled_transformed), cl
    gmm_wss = wss(gmm.fit_predict(census_data_scaled_transformed), clus

    cluster_size.append(cluster_num)
    kmeans_wss_vals.append(kmeans_wss)
    gmm_wss_vals.append(gmm_wss)

    print "Cluster: " + str(cluster_num)
    print "KM wss : " + str(kmeans_wss) + " GMM wss: " + str(gmm_wss)
```

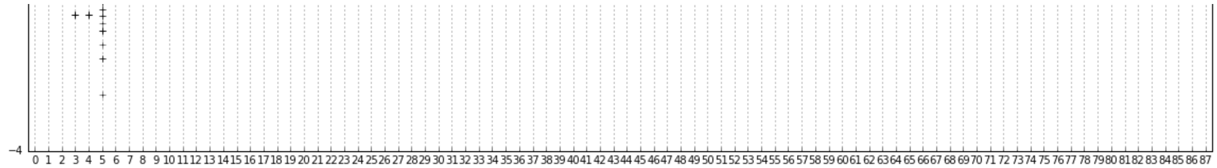
```
Cluster: 1
KM wss : 723888.000004 GMM wss: 723888.000004
Cluster: 5
KM wss : 492429.705842 GMM wss: 499703.121708
Cluster: 9
KM wss : 370431.097226 GMM wss: 362805.551685
Cluster: 13
KM wss : 294261.962079 GMM wss: 259872.786937
Cluster: 17
KM wss : 220227.067138 GMM wss: 228357.23344
Cluster: 21
KM wss : 216517.091194 GMM wss: 202982.547842
Cluster: 25
KM wss : 181168.577799 GMM wss: 183007.140588
Cluster: 29
KM wss : 161262.112427 GMM wss: 175355.574745
Cluster: 33
KM wss : 152499.370529 GMM wss: 165425.2983
Cluster: 37
KM wss : 146394.944824 GMM wss: 174520.754463
Cluster: 41
KM wss : 139603.624081 GMM wss: 144054.481554
Cluster: 45
KM wss : 134380.445659 GMM wss: 141416.546179
Cluster: 49
KM wss : 125829.492452 GMM wss: 140611.220053
Cluster: 53
KM wss : 122980.486992 GMM wss: 128431.744274
Cluster: 57
KM wss : 119579.75442 GMM wss: 123476.215474
Cluster: 61
KM wss : 115743.333467 GMM wss: 124506.593976
Cluster: 65
KM wss : 111449.211269 GMM wss: 119714.445448
Cluster: 69
KM wss : 107267.157003 GMM wss: 119096.610484
Cluster: 73
KM wss : 103044.82988 GMM wss: 116943.157121
Cluster: 77
KM wss : 100190.681263 GMM wss: 110052.241624
Cluster: 81
KM wss : 98714.9060005 GMM wss: 111586.280982
Cluster: 85
KM wss : 95692.6497576 GMM wss: 114402.242063
Cluster: 89
KM wss : 93245.4641332 GMM wss: 109390.000737
Cluster: 93
KM wss : 91932.3811497 GMM wss: 105908.241896
Cluster: 97
KM wss : 90327.4640868 GMM wss: 104254.14054
```

In [30]:
```
plt.plot(cluster_size, kmeans_wss_vals, 'b', cluster_size, gmm_wss_vals
blue_patch = mpatches.Patch(color='blue', label='kmeans wss')
green_patch = mpatches.Patch(color='green', label='gm wss')

plt.legend(handles=[blue_patch, green_patch], loc='upper left')

plt.ylabel("WSS")
plt.show()
```



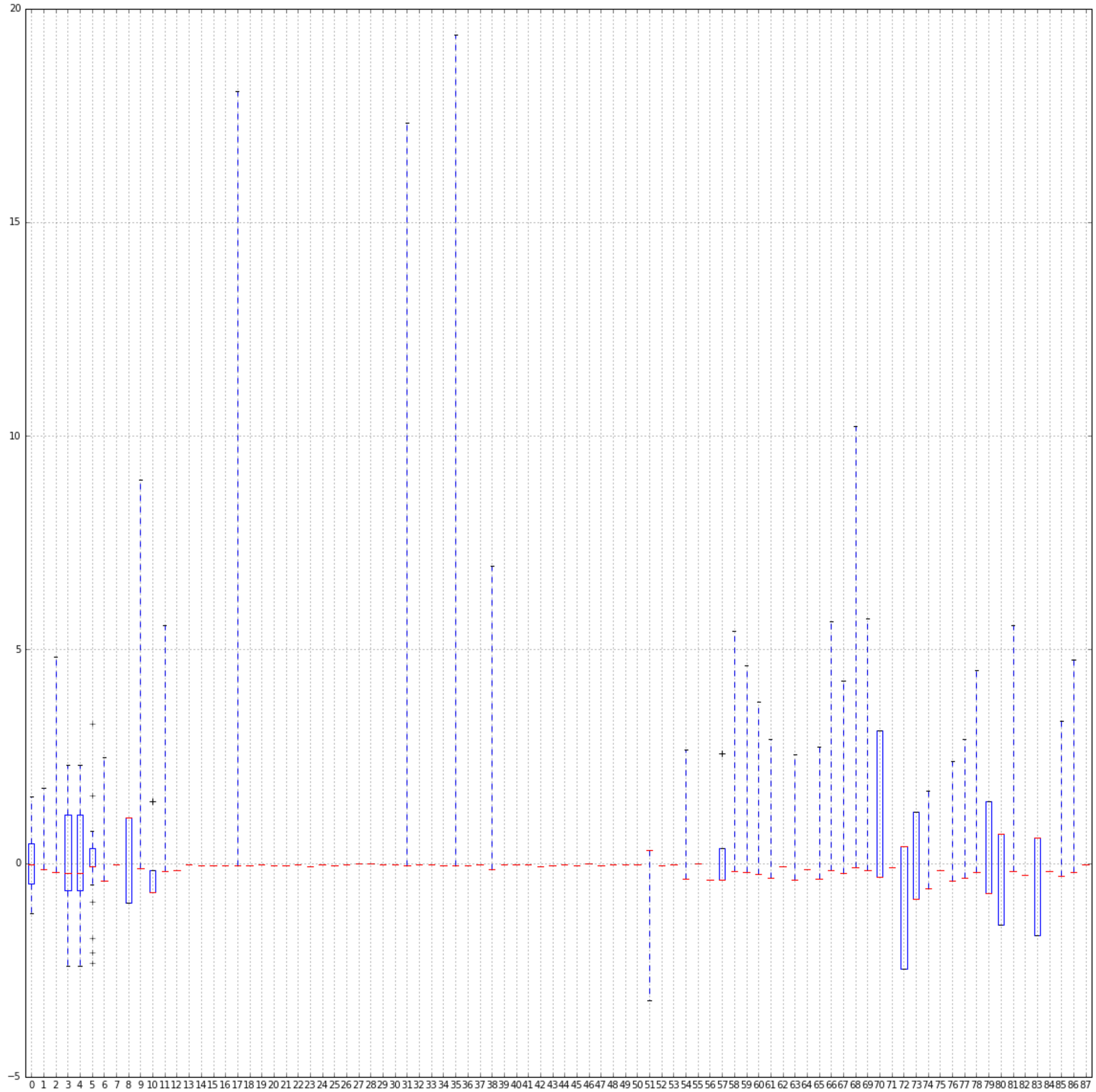
In [31]:
```
# Part 2 salient attribute
from sklearn.decomposition import PCA
import matplotlib.pyplot as plt

cluster_size = 40
gmm = mixture.GMM(n_components=cluster_size, random_state=20160217)

gmm_predict = gmm.fit_predict(census_data_scaled_transformed)

cluster_instances = []
# Iterate over each instance of the dataset
for i in range(0, len(gmm_predict)):
    # If that dataset instance is in the right cluster
    if (gmm_predict_pca[i] == 5):
        cluster_instances.append(i)

transformed_cluster = census_data_scaled.iloc[census_data_scaled_transf
normal_cluster = census_data_scaled.iloc[cluster_instances]

plt.figure(figsize=(20,20))

normal_cluster.boxplot(return_type='axes')
```

Out[31]: <matplotlib.axes._subplots.AxesSubplot at 0x1129634d0>


−4
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87

In [32]:
```
plt.figure(figsize=(20,20))
transformed_cluster.boxplot(return_type='axes')
```

Out[32]: <matplotlib.axes._subplots.AxesSubplot at 0x109493f90>



In [ ]: