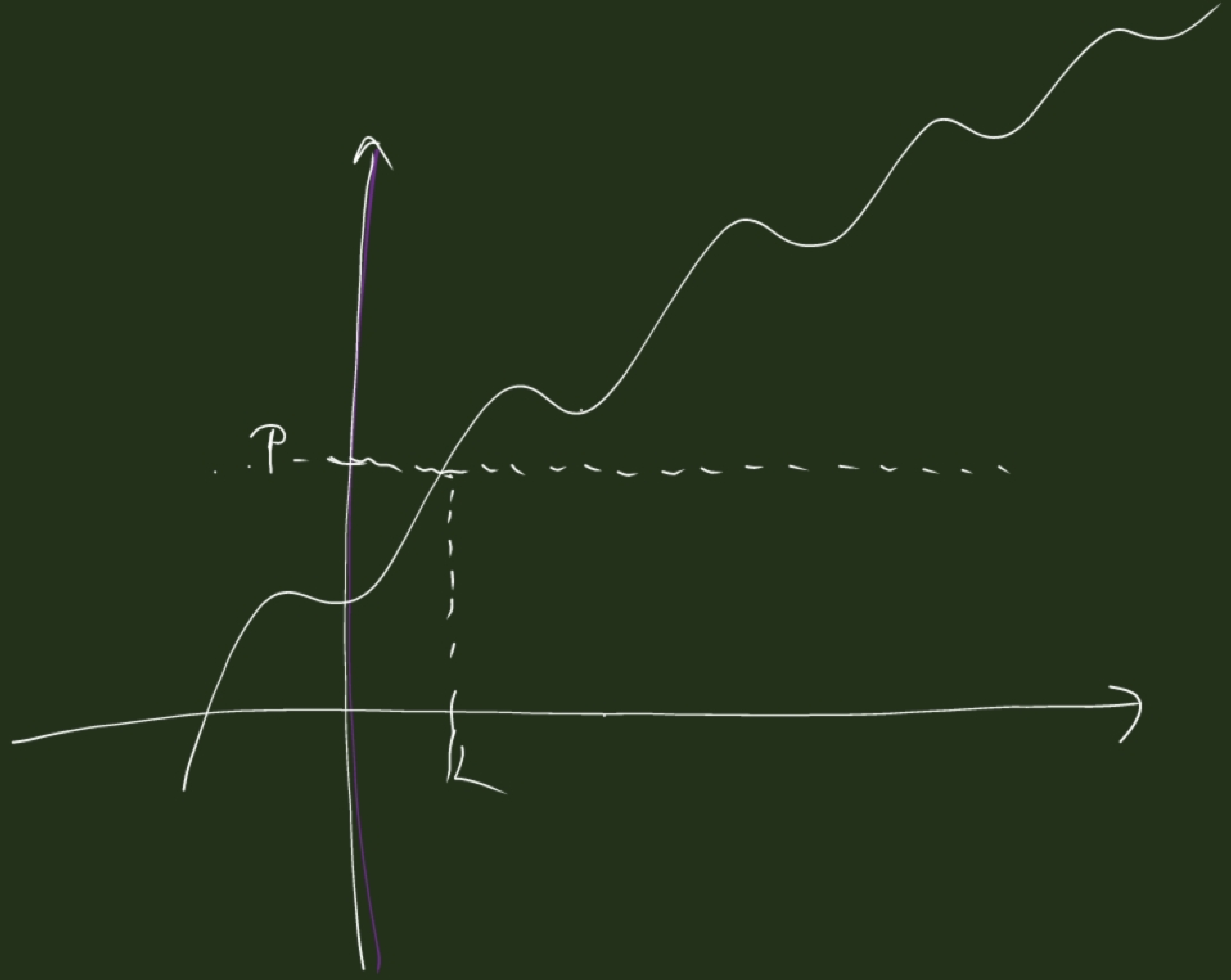
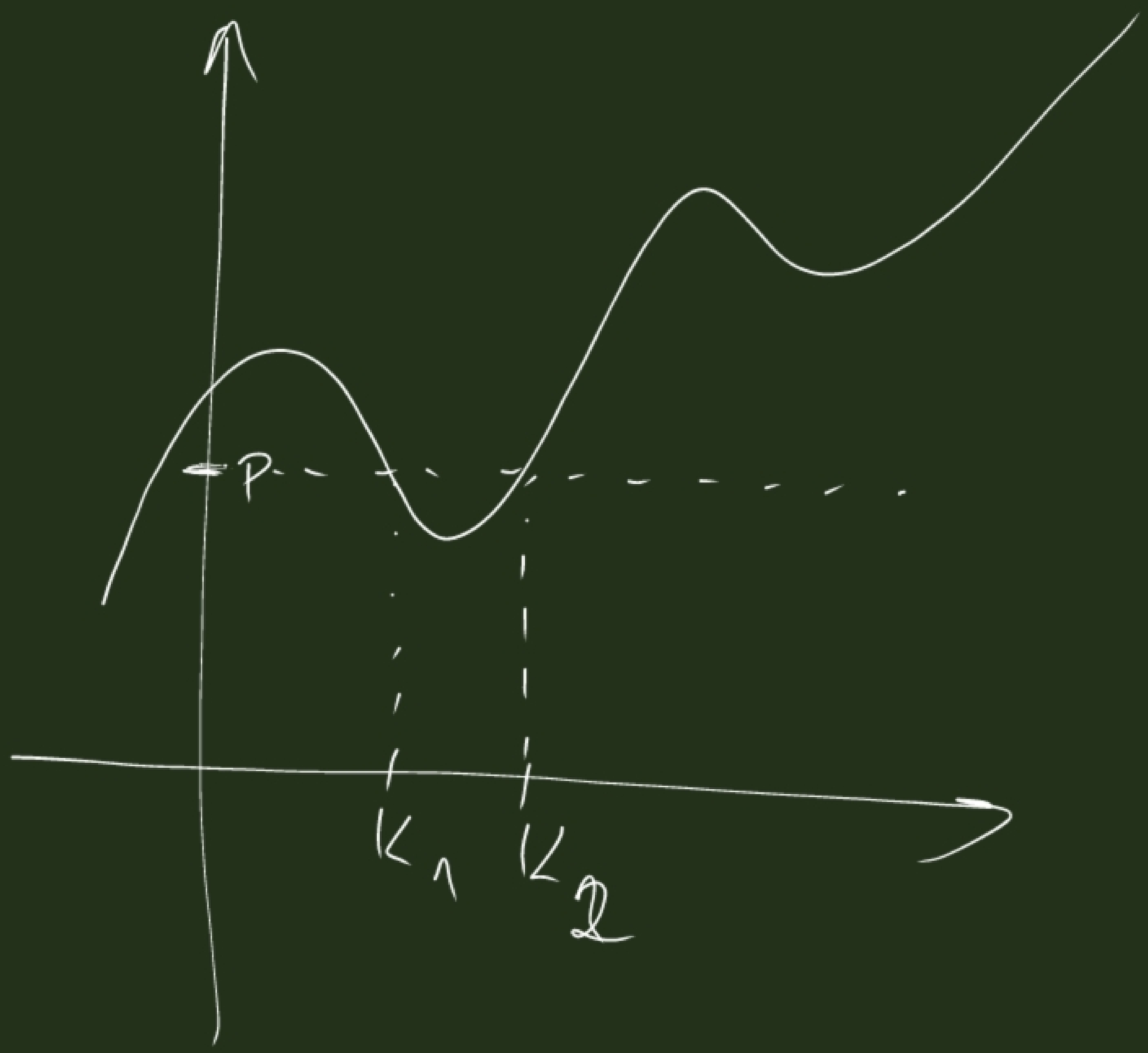

P
L


P
K_1
K_2

26. fejezet 12/a

$D_f = \{x \in \mathbb{R} \mid x^3 + x + 1 \neq 0\}$

3 gyök, legnagyobb: $\tilde{x}$

$D_f := (\tilde{x}, +\infty)$

Ig., hogy $\lim\limits_{x \to +\infty} \frac{x^4 - 2x^3 + x^2 + 7}{x^3 + x + 1} = +\infty$.

Def.: $\forall P > 0 : \exists K > 0 : \forall x \in D_f, x > K : \boxed{f(x) > P}$

Legyen $P > 0$ tetszőleges rögz., ehhez keresünk K-t.

$$f(x) = \frac{x^4 - 2x^3 + \overbrace{x^2 + 7}^{>0}}{x^3 + x + 1} > \frac{\frac{1}{2}x^4}{3x^3} = \frac{x}{6} \quad \boxed{(x > 4)}$$

$$x^4 - 2x^3 + \underbrace{x^2 + 7}_{\geq 0} \geq x^4 - 2x^3 = \frac{1}{2}x^4 + \frac{1}{2}x^4 - 2x^3 = \frac{1}{2}x^4 + \underbrace{x^3}_{\substack{\geq 0 \\ x \geq 0}} \underbrace{\left(\frac{1}{2}x - 2\right)}_{\substack{\geq 0 \\ x > 4}} \geq \frac{1}{2}x^4$$

$$\underbrace{x}_{\substack{\leq x^3 \\ x \geq 1}} + \underbrace{1}_{\substack{\leq x^3 \\ x \geq 1}}$$

$$x^3 + x + 1 \leq 3x^3 \quad (x > 1)$$

elég: $\frac{x}{6} > P \longrightarrow \boxed{x > 6P}$

$\Longrightarrow K := \max\{4, 6P\}$ jó választás, igaz a def.

12/f Ig., hogy $\lim\limits_{x\to+\infty} \frac{2x^3-x^2+3}{x^3+2x-5} = 2$.

Def.

$\forall \varepsilon>0 : \exists K>0 : \quad \forall x\in D_f,\ x>K \quad : \quad |f(x)-2|<\varepsilon$.

Legyen $\varepsilon>0$ tetsz., rögz., ehhez keresünk K-t.

$$|f(x)-2| = \left|\frac{2x^3-x^2+3}{x^3+2x-5} - 2\cdot\overbrace{\frac{x^3+2x-5}{x^3+2x-5}}^{=1}\right| =$$

$$= \left|\frac{2x^3-x^2+3-2(x^3+2x-5)}{x^3+2x-5}\right| = \left|\frac{-x^2-4x+13}{x^3+2x-5}\right| =$$

$$= \left|\frac{-x^2-4x+13}{x^3+2x-5}\right| = \left|\frac{x^2+4x-13}{x^3+2x-5}\right| = \frac{x^2+4x-13}{x^3+2x-5} \quad (x>3) \quad < \varepsilon$$

$$x^2+4x-13=0$$

$$x_{1,2} = \frac{-4 \pm \sqrt{16+4\cdot 13}}{2} = \ldots$$

(legyen x_2 a nagyobbik gyök)

$$\frac{x^2+4x\overbrace{-13}^{\le 0}}{\underbrace{x^3+2x}_{>0}-5} < \frac{5x^2}{\frac{1}{2}x^3} = 10\cdot\frac{1}{x} \quad (x>10)$$

$$x^3-5 \underset{}{\overset{x>1}{>}} x^3-5x^2 = \frac{1}{2}x^3 + \overbrace{\frac{1}{2}x^3-5x^2}^{>0} > \frac{1}{2}x^3 \quad (x>10)$$

$$x^3-3x^2-x = x^3-\underbrace{(3x^2+x)}_{<4x^2} > x^3-4x^2 = \frac{1}{2}x$$

elég: $\frac{10}{x} < \varepsilon \longrightarrow x > \frac{10}{\varepsilon}$

$$K := \max\left\{(3), 10, \frac{10}{\varepsilon}\right\}.$$

12/c, Igy., hogy: $\lim\limits_{x\to+\infty} \frac{x^3+x^2-2x-3}{9-4x^2} = -\infty$

azt mutatjuk meg, hogy $\lim\limits_{x\to+\infty} -f(x) = +\infty$.

$\forall P>0 : \exists K>0 : \forall x\in D_f, x>K : -f(x) > P$.

Legyen $P>0$ tetsz. rögz., ehhez keressünk K-t.

$$-f(x) = \frac{x^3+x^2-2x-3}{4x^2-9} \underset{x>\sqrt{10}}{>} \frac{\frac{1}{2}x^3}{4x^2} = \frac{1}{8}x \qquad (x>\sqrt{10})$$

$$x^3 + \underbrace{x^2}_{>0} - 2x - 3 > x^3 - 2x - 3 = x^3 - (\underbrace{2x+3}_{<5x}) > x^3 - 5x =$$

$$= \frac{1}{2}x^3 + \frac{1}{2}x^3 - 5x = \frac{1}{2}x^3 + x(\underbrace{\tfrac{1}{2}x^2 - 5}_{\substack{>0 \\ x>\sqrt{10}}}) > \frac{1}{2}x^3 \quad x>1$$

elég: $\frac{x}{8} > P \longrightarrow x > 8P$

$\rightarrow K := \max\{\sqrt{10}, 8P\}$

(tanalgi javító 2h)

$f:\mathbb{R}\to\mathbb{R},\quad f(x):=x^2-10x+9 \quad (x\in[6,+\infty))$ $\boxed{f^{-1}=?}$

① f injektív: $x, t\in D_f=[6,+\infty)$

$$f(x)-f(t)=x^2-10x+\cancel{9}-(t^2-10t+\cancel{9})=$$

$$=x^2-t^2-10x+10t=$$

$$=(x-t)(x+t)-10(x-t)=$$

$$=\underbrace{(x-t)}_{\neq 0}\underbrace{(x+t-10)}_{\geq 2>0}\neq 0 \quad \text{ha } x\neq t \Rightarrow f \text{ inj.}$$

$\Rightarrow f$ inv.

② f^{-1} megadása:

- $\mathcal{R}_{f^{-1}} = \mathcal{D}_f = [6, +\infty)$
- $\mathcal{D}_{f^{-1}} = \mathcal{R}_f = \{ y \in \mathbb{R} \mid \exists x \in \mathcal{D}_f : \boxed{f(x) = y} \} =$

$$= \{ y \in \mathbb{R} \mid \exists x \in [6, +\infty) : x^2 - 10x + 9 = y \}$$

$$x^2 - 10x + 9 - y = 0$$

$$\sqrt{4(16+y)} = 2 \cdot \sqrt{16+y}$$

$$x_{1,2} = \frac{10 \pm \sqrt{100 - 4(9-y)}}{2} = \frac{10 \pm \sqrt{64 + 4y}}{2} = 5 \pm \sqrt{16+y}$$

$$x_{1,2} = 5 \pm \sqrt{16+y}$$

1. $x_{1,2} \in \mathbb{R} \iff \boxed{y \geq -16}$

2. $x_1 \geq 6$ vagy $x_2 \geq 6$ $\quad \left(5 - \sqrt{16+y} \geq 6 \text{ vagy } 5 + \sqrt{16+y} \geq 6\right)$

elég: $5 + \sqrt{16+y} \geq 6$

$$\sqrt{16+y} \geq 1$$

$$16 + y \geq 1$$

$$\boxed{y \geq -15}$$

$\longrightarrow \mathcal{R}_f = [-15, +\infty)$

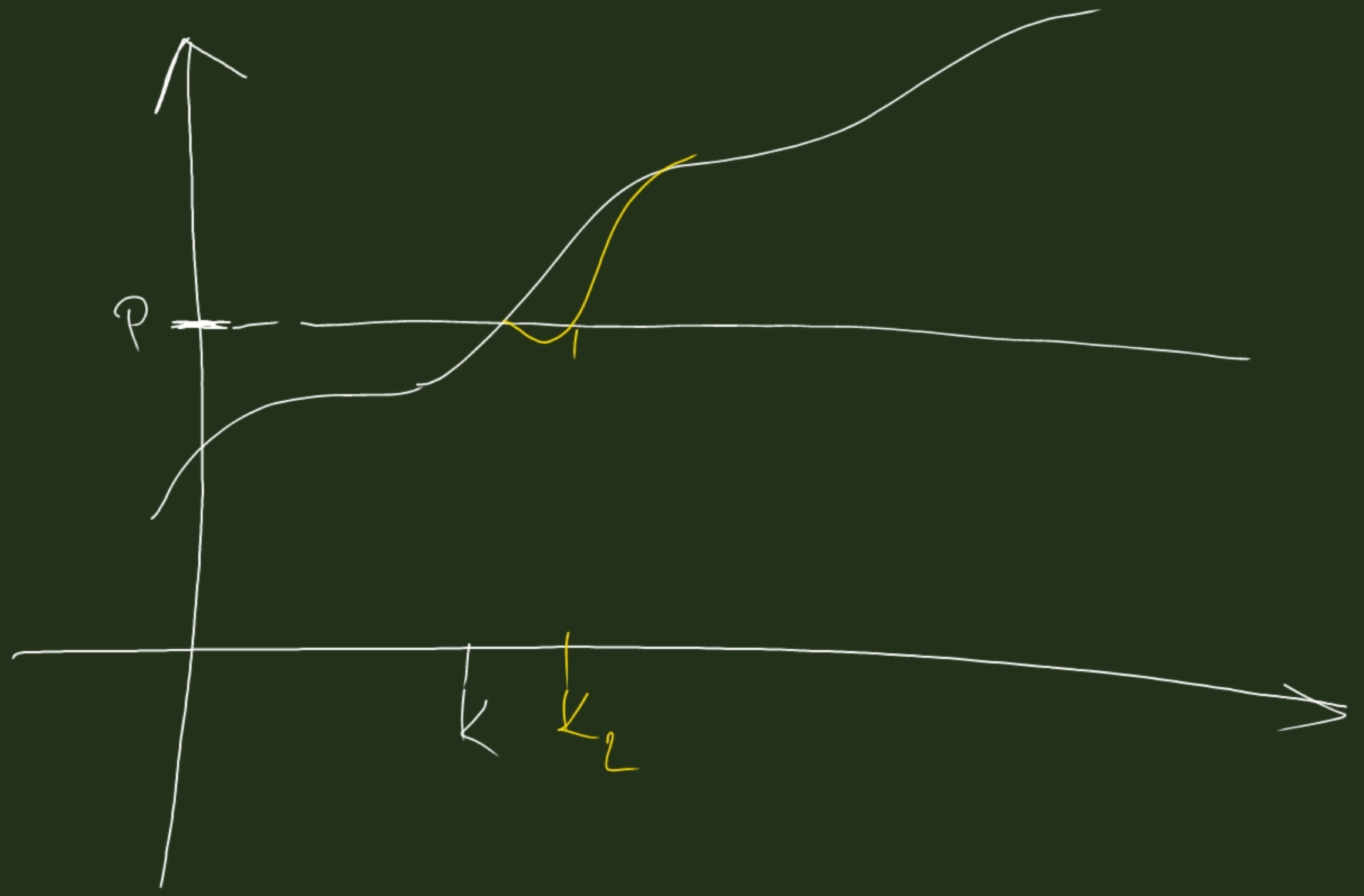
$y \in D_{f^{-1}} = [-15, +\infty)$

$f^{-1}(y) = \ldots$

$y \in D_{f^{-1}}$, $f^{-1}(y) = x \iff \boxed{f(x) = y}$ $(x \geqslant 6,\ y \geqslant -15)$

$$\boxed{x^2 - 10x + 9 = y}$$

$$x = 5 + \sqrt{16 + y}$$


P
K
K
2