① a) $v_1 = (1, 2, 2, -1)$, $v_2 = (4, 3, 9, -4)$, $v_3 = (5, 8, 9, -5)$

v_1, v_2, v_3 Ö, F?

$$\lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 v_3 = 0$$

$$\lambda_1 \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \\ -1 \end{pmatrix} + \lambda_2 \cdot \begin{pmatrix} 4 \\ 3 \\ 9 \\ -4 \end{pmatrix} + \lambda_3 \cdot \begin{pmatrix} 5 \\ 8 \\ 9 \\ -5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \Leftrightarrow \begin{cases} \lambda_1 + 4\lambda_2 + 5\lambda_3 = 0 \\ 2\lambda_1 + 3\lambda_2 + 8\lambda_3 = 0 \\ 2\lambda_1 + 9\lambda_2 + 9\lambda_3 = 0 \\ -\lambda_1 - 4\lambda_2 - 5\lambda_3 = 0 \end{cases}$$

$$\begin{cases} \lambda_1 + 4\lambda_2 + 5\lambda_3 = 0 & (1) \\ 2\lambda_1 + 3\lambda_2 + 8\lambda_3 = 0 & (2) \\ 2\lambda_1 + 9\lambda_2 + 9\lambda_3 = 0 & (3) \\ -\lambda_1 - 4\lambda_2 - 5\lambda_3 = 0 & (4) \end{cases}
\quad \xrightarrow[\substack{(3)-2\cdot(-1) \\ (4)+(1)}]{(2)-2\cdot(-1)} \quad
\begin{cases} \lambda_1 + 4\lambda_2 + 5\lambda_3 = 0 \\ 0 - 5\lambda_2 - 2\lambda_3 = 0 \\ 0 + \lambda_2 - \lambda_3 = 0 \\ \cancel{0 + 0 + 0 = 0} \end{cases}$$

$$\begin{matrix} (1) \\ (2) \\ (3) \end{matrix}
\begin{cases} \lambda_1 + 4\lambda_2 + 5\lambda_3 = 0 \\ -5\lambda_2 - 2\lambda_3 = 0 \\ \lambda_2 - \lambda_3 = 0 \end{cases}
\Rightarrow \begin{matrix} \lambda_1 = 0 \\ \\ \lambda_2 = \lambda_3 = 0 \end{matrix}
\Longrightarrow v_1, v_2, v_3 \ \text{F}$$

$(3) \Rightarrow \lambda_2 = \lambda_3$

$(2) \Rightarrow \lambda_2 = \frac{2}{5}\lambda_3$ $\Longrightarrow \lambda_3 = 0 \Rightarrow \lambda_2 = 0$

b) $v_1=(1,2,3,1)$, $v_2=(2,2,1,3)$, $v_3=(-1,2,7,-3)$

$$\lambda_1 v_1+\lambda_2 v_2+\lambda_3 v_3=0$$

$$\begin{cases}\lambda_1+2\lambda_2-\lambda_3=0\\ 2\lambda_1+2\lambda_2+2\lambda_3=0\\ 3\lambda_1+\lambda_2+7\lambda_3=0\\ \lambda_1+3\lambda_2-3\lambda_3=0\end{cases}
\xrightarrow[(4)-(1)]{\substack{(2)-2\cdot(1)\\(3)-3\cdot(1)}}
\begin{cases}\lambda_1+2\lambda_2-\lambda_3=0\\ -2\lambda_2+4\lambda_3=0\\ -5\lambda_2+10\lambda_3=0\\ \lambda_2-2\lambda_3=0\end{cases}
\xrightarrow[\substack{(2)+2\cdot(4)\\(3)+5(4)}]{(1)-2\cdot(4)}
\begin{cases}\lambda_1+3\lambda_3=0\\ \cancel{0=0}\\ \cancel{0=0}\\ \lambda_2-2\lambda_3=0\end{cases}$$

$$\begin{cases}\lambda_1+3\lambda_3=0\\ \lambda_2-2\lambda_3=0\end{cases}
\longrightarrow
\begin{cases}\lambda_3=-\frac{1}{3}\lambda_1\\ \lambda_3=\frac{1}{2}\lambda_2\end{cases}
\Rightarrow
\boxed{\begin{matrix}2\lambda_1=3\lambda_2\\ \lambda_1\in\mathbb{R}\end{matrix}}
\longrightarrow \ddot{O}$$

$\lambda_1=1$, $\lambda_2=\frac{2}{3}$, $\lambda_3=-\frac{1}{3}$

$$\begin{cases} \lambda_1 + 3\lambda_3 = 0 \\ \lambda_2 - 2\lambda_3 = 0 \end{cases} \longrightarrow \boxed{\lambda_1 = -3\lambda_3,\ \lambda_2 = 2\lambda_3,\ \lambda_3 \in \mathbb{R}}$$

2, $v_1 = (1, 2, 3, 1)$, $v_2 = (2, 2, 1, 3)$, $v_3 = (-1, 2, 7, -3)$

v_1, v_2, v_3 Ö

Tétel $\Rightarrow$ elhagyható az egyik vektor úgy, hogy $\mathrm{Span}(v_1, v_2, v_3)$ nem változik.

$\rightarrow \exists\, \lambda_1, \lambda_2, \lambda_3$ nemtriv. : $\lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 v_3 = 0$

$$\boxed{-3v_1 + 2v_2 + v_3 = 0}$$

$$-3v_1 + 2v_2 + v_3 = 0$$

$$\boxed{v_3 = 3v_1 - 2v_2} \leftarrow$$

$$\mathrm{Span}(v_1, v_2, v_3) = \mathrm{Span}(v_1, v_2)$$

elemei:

$$\lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 v_3 = \lambda_1 v_1 + \lambda_2 v_2 + \lambda_3 (3v_1 - 2v_2) = (\lambda_1 + 3\lambda_3)v_1 + (\lambda_2 - 2\lambda_3)v_2$$

$$= \mu_1 v_1 + \mu_2 v_2 \quad (\mu_1, \mu_2 \in \mathbb{R})$$