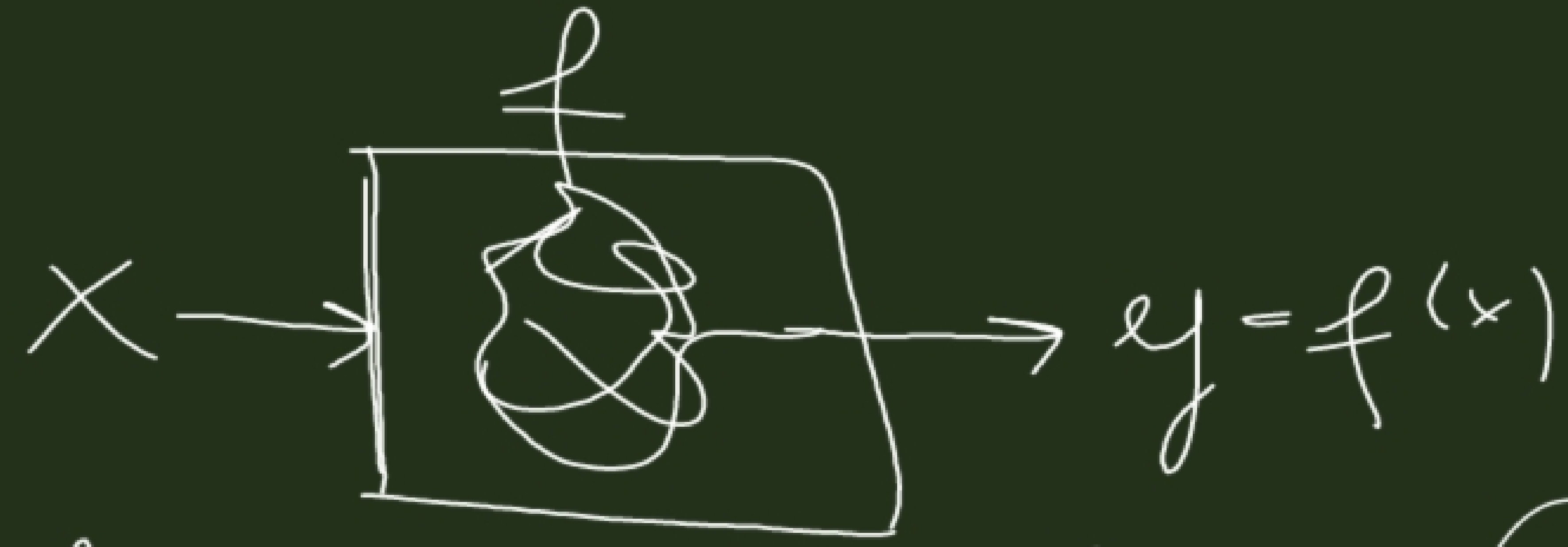
24, 25, 26 fej. : Analízis 0.

Függvények

függvény grafikonja

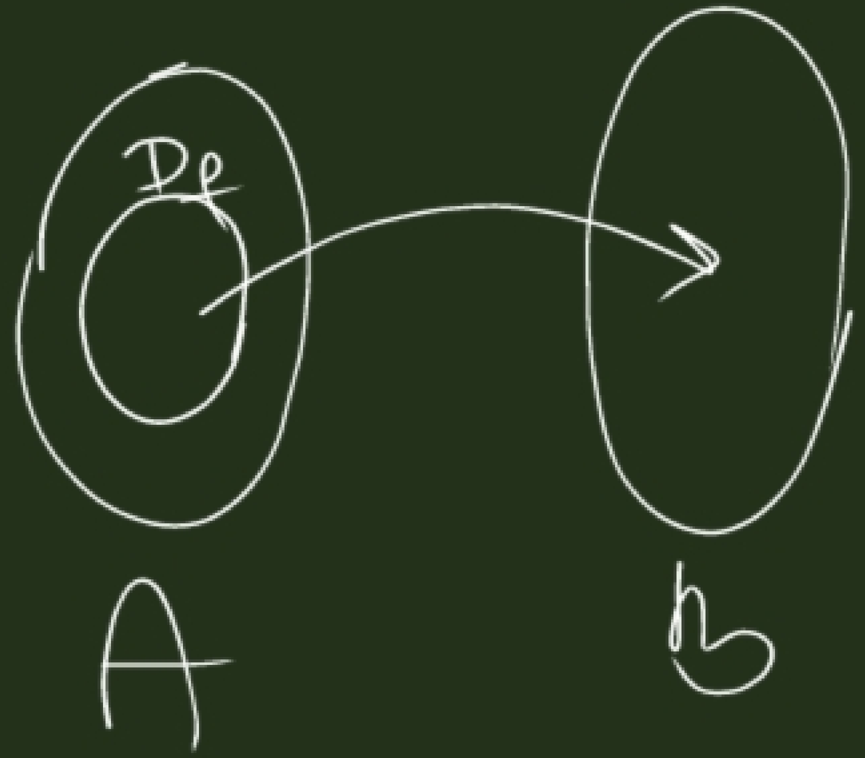
$x \longmapsto y = f(x)$



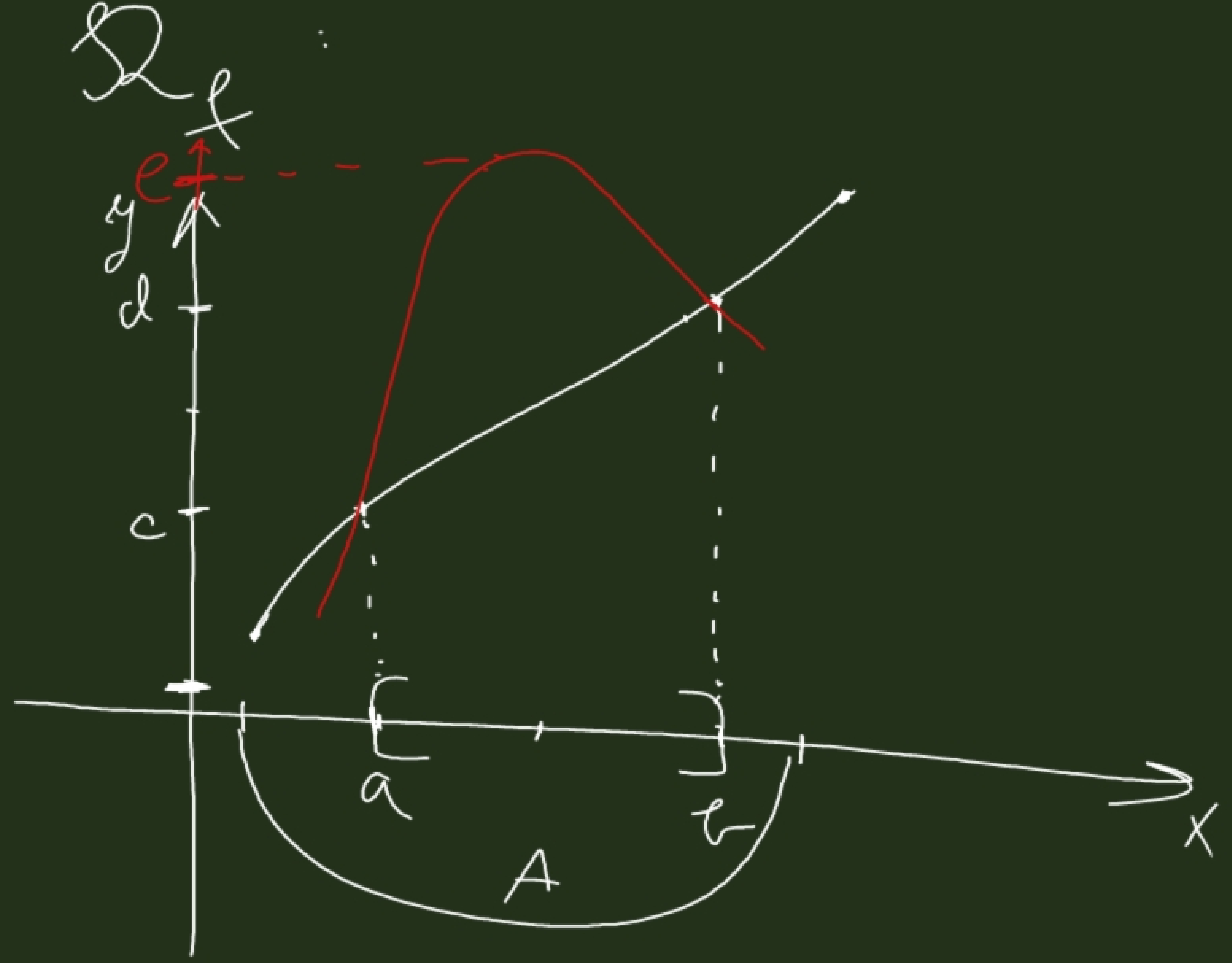
$f: A \longrightarrow B$

értelmezési tart.:

$D_f := \{x \in A \mid f(x) \text{ értelmes}\}$ $\quad f: D_f \longrightarrow R_f$



értékkészlet: $\longrightarrow$ köv. oldal



$$\boxed{f : A \to B}$$

$$f([a,b]) = [c,d]$$

$$f([a,b]) = [c,e]$$

(szorg. $f([a,b]) = [f(a), f(b)]$
mikor igaz)

$$f([a,b]) = \{ y \in B \mid \exists x \in [a,b] : y = f(x) \}$$

$$R_f = f(D_f) = \{ y \in B \mid \exists x \in D_f : y = f(x) \}$$

24. fejezet, feladatok

1. $D \subset \mathbb{R}$, $D = ?$

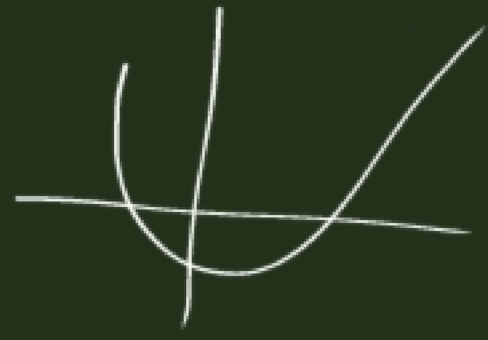
a, $f(x) := \sqrt{\dfrac{2x^3-1}{x}} \qquad (x \in D)$

$$D_f = \{ x \in \mathbb{R} \mid \underbrace{f(x) \text{ értelmes}}_{x \neq 0 \quad \frac{2x^3-1}{x} \geq 0} \}$$

HF.

b, $f(x) = \sqrt{\lg(x^2-5x+7)}$

$$D_f = \{ x \in \mathbb{R} \mid x^2-5x+7 > 0,\ \lg(x^2-5x+7) \geq 0 \}$$



lg szig. mon. növ.

$x^2 - 5x + 7 \geq 1$

24/4,5 HF

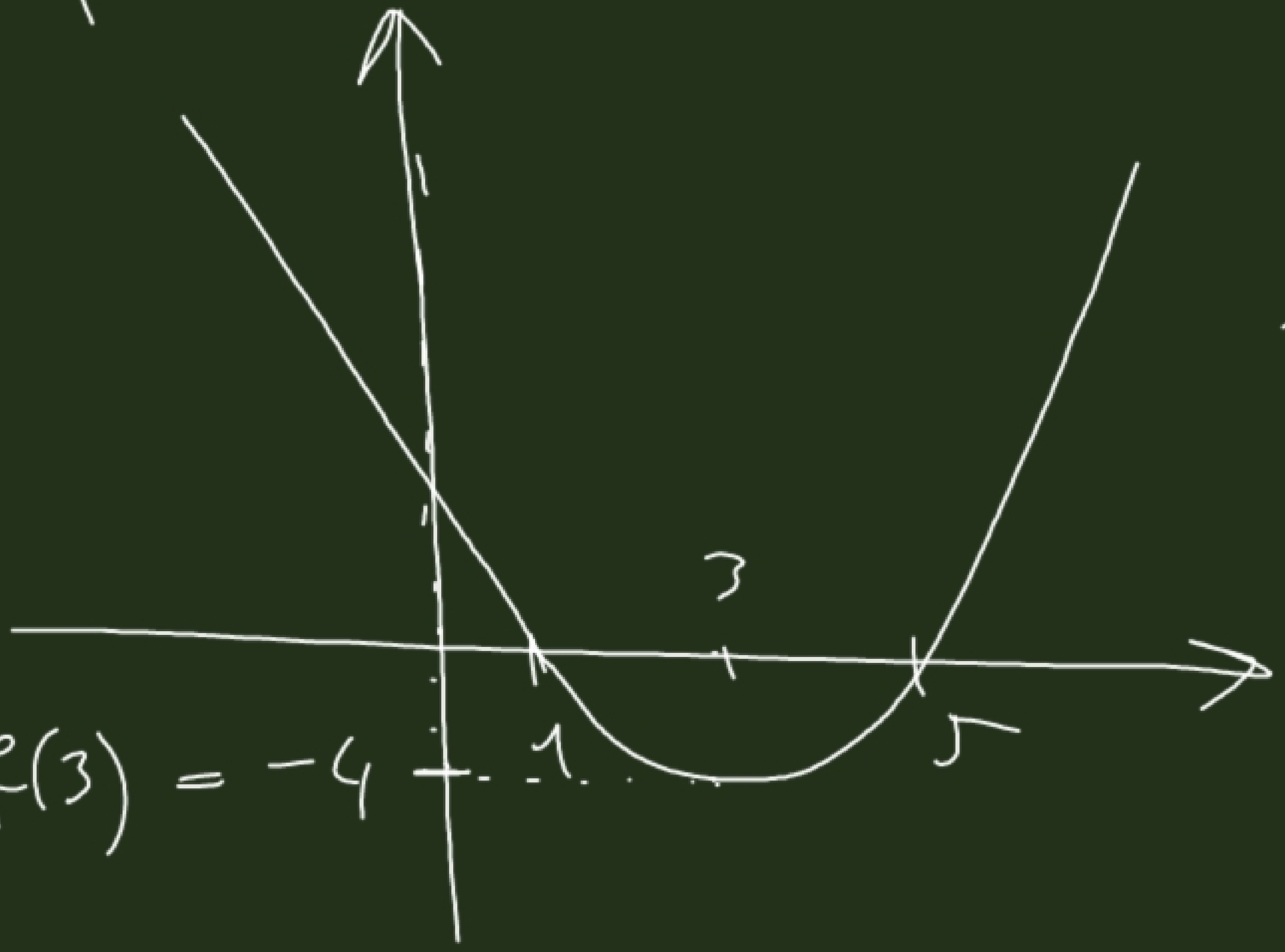
25. fejezet, feladat

$(x-5)(x-1)$

3) $f: \mathbb{R} \to \mathbb{R}$, $f(x) := x^2 - 6x + 5$ $(x \in D_f)$

$(D_f = \mathbb{R})$

$R_f = ?$



$f(3) = -4$

$R_f = [-4, +\infty)$

$$R_f = \{ y \in \mathbb{R} \mid \exists x \in D_f : y = f(x) \} =$$

$$= \{ y \in \mathbb{R} \mid \exists x \in \mathbb{R} : \underbrace{x^2 - 6x + 5 = y}_{\text{milyen } y \text{ esetén van megoldás?}} \}$$

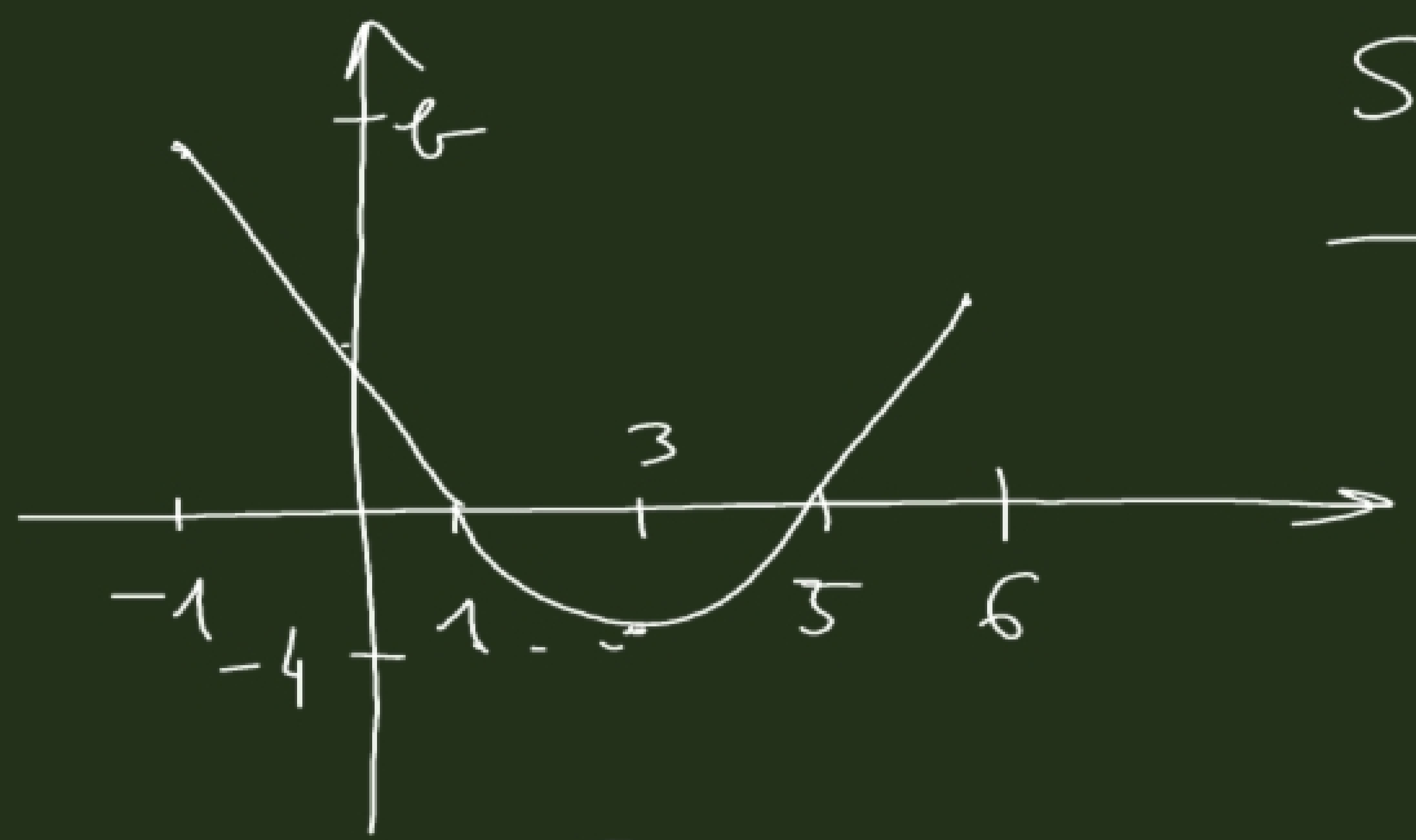
$$x^2 - 6x + 5 - y = 0$$

$$\sqrt{36 - 4(5-y)} = \sqrt{4(9-(5-y))} = 2 \cdot \sqrt{4+y}$$

$$x_{1,2} = \frac{6 \pm \sqrt{36 - 4(5-y)}}{2} = \frac{6 \pm 2 \cdot \sqrt{4+y}}{2} = 3 \pm \underbrace{\sqrt{4+y}}_{\geq 0}$$

$$R_f = [-4, +\infty) \Leftarrow \boxed{y \geq -4}$$

b, $f(x) := x^2 - 6x + 5$, $\quad D_f := [-1, 6]$



segítség: $R_f = [-4, \underset{f(-1)}{\underset{\shortparallel}{12}}]$

$$R_f = \{ y \in \mathbb{R} \mid \exists x \in D_f : f(x) = y \} =$$

$$= \{ y \in \mathbb{R} \mid \exists x \in [-1, 6] : \underbrace{x^2 - 6x + 5 = y}_{\text{milyen } y \text{ esetén van } [-1,6]\text{-beli mo.}} \}$$

$x^2 - 6x + 5 - y = 0 \quad \exists$ valós mo. $\Leftrightarrow \boxed{y \geq -4}$

$x_{1,2} = 3 \pm \sqrt{y+4}$

$x_1, x_2 \overset{?}{\in} [-1, 6]$

1. $-1 \leq x_1 \leq 6 \Leftrightarrow -1 \leq 3 + \sqrt{y+4} \leq 6 \Leftrightarrow -4 \leq \sqrt{y+4} \leq 3$

$\Leftrightarrow 0 \leq \sqrt{y+4} \leq 3 \Leftrightarrow 0 \leq y+4 \leq 9 \Leftrightarrow -4 \leq y \leq 5$

$\boxed{y \in [-4, 5]}$

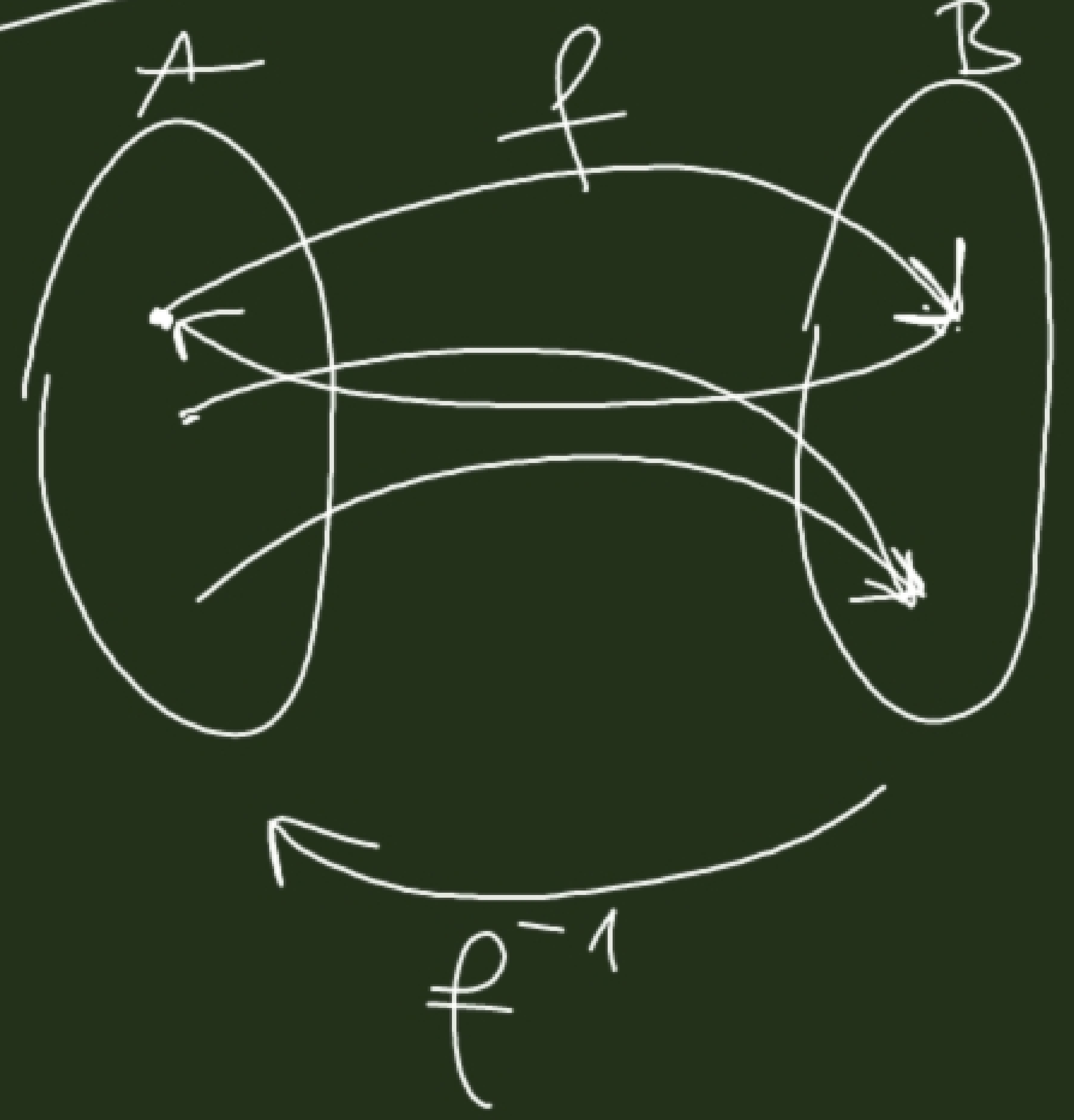
2. $-1 \leq x_2 \leq 6 \Leftrightarrow \dots \Leftrightarrow \boxed{y \in [-4, 12]}$

Kell: $y \in [-4, 5] \cup [-4, 12] = [-4, 12]$

$\Rightarrow R_f = [-4, +\infty) \cap [-4, 12] = \underline{\underline{[-4, 12]}}$

$\boxed{\text{HF}}$ 3/c

Invertálhatóság



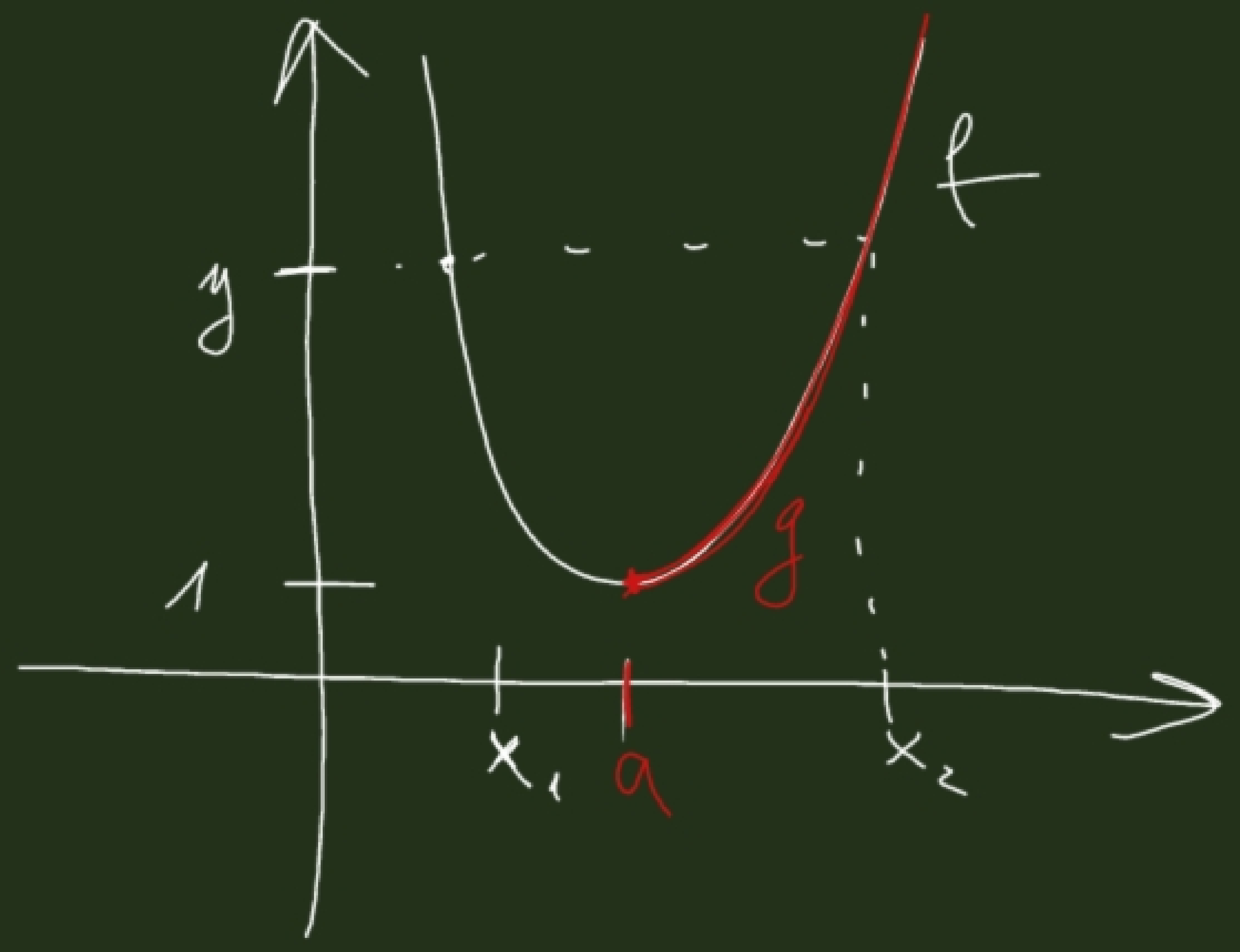
$$f^{-1}: B \to A$$

$$f(x) = y \iff f^{-1}(y) = x$$

"Def"

- $D_{f^{-1}} := R_f$
- $R_{f^{-1}} := D_f$
- $y \in D_{f^{-1}}: \quad f^{-1}(y) = x \iff f(x) = y$

Mikor invertálható egy függvény?



$R_f = [1, +\infty)$

$D_g = [a, +\infty)$, $g(x) = f(x)$

f nem invertálható

g invertálható

f invertálható $\Longleftrightarrow$ f injektív

$\boxed{\text{Def}}$ f injektív, ha $\underbrace{\forall x, t \in D_f : \quad x \neq t \Rightarrow f(x) \neq f(t).}$

$$\Updownarrow$$

$\forall x, t \in D_f : \quad f(x) = f(t) \Longrightarrow x = t$

$\boxed{8}$

d, $f(x) := \dfrac{3x+2}{x-1}$ $\quad (x \in (1, +\infty))$ $\quad \exists f^{-1}$?, ha igen, akkor $f^{-1} = ?$

① belátjuk, hogy f injektív:

$$f(x) = f(t) \iff f(x) - f(t) = 0$$

$$f(x) - f(t) = \ldots = \frac{5 \cdot (t-x)}{(x-1)(t-1)} \qquad (x, t \in (1, +\infty))$$

$$\Rightarrow \quad f(x) = f(t) \iff \frac{5 \cdot (t-x)}{(x-1)(t-1)} = 0 \iff x = t$$

$\Rightarrow$ f injektív $\Rightarrow$ f invertálható.

$$\left(f(x) - f(t) = \frac{3x+2}{x-1} - \frac{3t+2}{t-1} = \frac{(3x+2)(t-1) - (3t+2)(x-1)}{(x-1)(t-1)} = \right.$$

$$= \frac{\cancel{3xt} - 3x + 2t \cancel{-2} - (\cancel{3xt} - 3t + 2x \cancel{-2})}{(x-1)(t-1)} =$$

$$\left. = \frac{-5x+5t}{(x-1)(t-1)} = \frac{5(t-x)}{(x-1)(t-1)} \right)$$

② f^{-1} megadása:

- $\mathcal{R}_{f^{-1}} = D_f = (1, +\infty)$
- $D_{f^{-1}} = \mathcal{R}_f = \left\{ y \in \mathbb{R} \;\middle|\; \exists x \in (1, +\infty) : \boxed{\frac{3x+2}{x-1} = y} \right\}$

$$\frac{3x+2}{x-1} = y \Rightarrow x = \ldots \; ?$$

$$3x + 2 = y(x-1)$$

$$3x - y \cdot x = -2 - y \quad (y \neq 3)$$

$$x = \frac{-2-y}{3-y} > 1 \iff \overset{HF}{\ldots\ldots} \iff \boxed{y > 3} \Rightarrow \mathcal{R}_f = (3, +\infty)$$

$$\Rightarrow D_{f^{-1}} = (3, +\infty)$$

- $f^{-1}(y) = \dfrac{-2-y}{3-y}$

HF: 8/b