4. Exponenciális feladatok

2, $2^{x+3} + 4^{1-\frac{x}{2}} = 33$

$2^3 \cdot 2^x + 4 \cdot 4^{-\frac{x}{2}} = 33$

$8 \cdot 2^x + 4 \cdot \frac{1}{2^x} = 33$

$y := 2^x \quad (y > 0)$

$8y + \frac{4}{y} = 33 \quad / \cdot y$

$8y^2 - 33y + 4 = 0$

(új változó: $y = 2^x$)

$4^{-\frac{x}{2}} = \frac{1}{4^{\frac{x}{2}}} = \frac{1}{(4^x)^{\frac{1}{2}}} = \frac{1}{(4^{\frac{1}{2}})^x} = \frac{1}{(\sqrt{4})^x} = \frac{1}{2^x}$

$y_{1,2} = \frac{33 \pm \sqrt{961}}{16}$

$y_1 = 4 > 0$

$y_2 = \frac{1}{8} > 0$

$y_1 = 4 \Rightarrow 2^x = 4 \Rightarrow x_1 = 2$

$y_2 = \frac{1}{8} \Rightarrow \underline{2^x} = \frac{1}{8} = 8^{-1} = (2^3)^{-1} = \underline{2^{-3}} \Rightarrow x_2 = -3$

$M = \{-3, 2\}$

$\boxed{3c}$ $3^{x+2}\cdot 2^{x} - 2\cdot \underbrace{36^{x}}_{(6^{x})^{2}} + 18 = 0$ (új változó: $y = 6^{x}$)

$$9\cdot 6^{x} - 2\cdot (6^{x})^{2} + 18 = 0 \qquad y := 6^{x} \quad (y > 0)$$

$$-2y^{2} + 9y + 18 = 0 \qquad /\cdot(-1)$$

$$2y^{2} - 9y - 18 = 0 \longrightarrow y_{1,2} = \frac{9 \pm \sqrt{225}}{4}$$

$y_1 = -\frac{3}{2} < 0$

$y_2 = 6 > 0$, $6 = 6^{x}$

$\downarrow$

$x = 1$

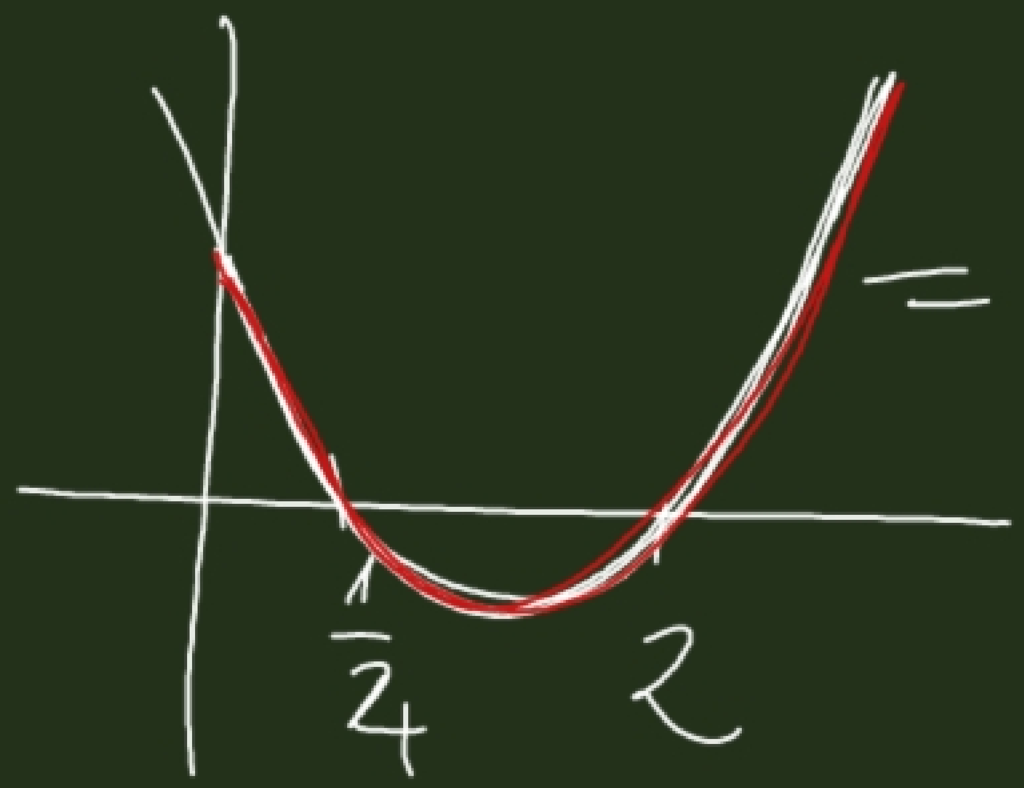
$M = \{1\}$

$\boxed{3/f}$ $4^{x+1} - 9\cdot 2^x + 2 > 0 \qquad (y = 2^x)$

$4\cdot(2^x)^2 - 9\cdot 2^x + 2 > 0 \qquad y := 2^x > 0 \; (y > 0)$

$\boxed{4y^2 - 9y + 2 > 0} \qquad y_{1,2} = \dfrac{9 \pm \sqrt{81-32}}{8} = \dfrac{9 \pm \sqrt{49}}{8} = \dfrac{9 \pm 7}{8} \begin{cases} 2 \\ \frac{1}{4} \end{cases}$

$4(y-2)(y-\frac{1}{4}) > 0$



$2^x = y \in (0, \frac{1}{4}) \cup (2, +\infty)$

$x \in$

$$y \in \left(0, \frac{1}{4}\right) \cup (2, +\infty)$$

$0 < y < \frac{1}{4}$ vagy $y > 2$

$\underbrace{0 < 2^x}_{\text{igaz } \forall x \in \mathbb{R} \text{ esetén}} < \frac{1}{4} = 2^{-2}$ vagy $2^x > 2 = 2^1 \longrightarrow$ exp. fgv. szig. mon. növ. $(2 > 1)$

$x < -2$ vagy $x > 1$

$$x \in (-\infty, -2) \cup (1, +\infty) = M$$

8) $3^{2+\log_9 25} + 25^{1-\log_5 2} + 10^{-\lg 4} =$

$$= 3^2 \cdot 3^{\log_9 25} + 25 \cdot \frac{1}{25^{\log_5 2}} + \frac{1}{10^{\lg 4}} =$$

$$= 9 \cdot \left(9^{\frac{1}{2}}\right)^{\log_9 25} + 25 \cdot \frac{1}{\left(5^2\right)^{\log_5 2}} + \frac{1}{4} =$$

$\lg := \log_{10}$

$\ln := \log_e$

$$\boxed{\left(a = b^{\log_b a}\right)} = 9 \cdot 9^{\frac{1}{2}\log_9 25} + 25 \cdot \frac{1}{4} + \frac{1}{4} = \underbrace{9 \cdot 5}_{\frac{90}{2}} + \underbrace{\frac{25}{4} + \frac{1}{4}}_{\frac{13}{2}} =$$

$$= \frac{103}{2}$$

Logaritmikus feladatok

$\boxed{15/b}$ $\log_{25}\left[\underbrace{\frac{1}{5}\cdot\log_3\underbrace{\left(2-\log_{\frac{1}{2}}\underbrace{x}_{>0}\right)}_{>0}}_{>0}\right]=-\frac{1}{2}$

$\log_a b=\log_a c \iff b=c$
sz.m.

$\log_a b<\log_a c$
$\iff \begin{cases} b<c & (a>1)\\ b>c & (a<1)\end{cases}$
sz.m.

kikötések:

- $x>0$
- $2-\log_{\frac{1}{2}}x>0 \iff \log_{\frac{1}{2}}x<2=\log_{\frac{1}{2}}\left(\frac{1}{2}\right)^2$

$\frac{1}{2}<1$, sz.m.-a

$x>\frac{1}{4}$

$\bullet \ \frac{1}{5} \cdot \log_3\left(2 - \log_{\frac{1}{2}} x\right) > 0$

$\log_3\left(2 - \log_{\frac{1}{2}} x\right) > \log_3 3^0 = \log_3 1$

S2. m. n.

$2 - \log_{\frac{1}{2}} x > 1$

$\log_{\frac{1}{2}} x < 1 = \log_{\frac{1}{2}} \frac{1}{2}$

s2. m. cs.

$x > \frac{1}{2}$

$\boxed{x \in M := \left(\frac{1}{2}, +\infty\right)}$

$$\log_{25}\left[\frac{1}{5}\cdot\log_3\left(2-\log_{\frac{1}{2}}(x)\right)\right] = -\frac{1}{2} = \log_{25} 25^{-\frac{1}{2}} = \log_{25}\frac{1}{5}$$

sz. m.

$$\cancel{\frac{1}{5}}\cdot\log_3\left(2-\log_{\frac{1}{2}}(x)\right) = \cancel{\frac{1}{5}}\,1 = \log_3 3$$

sz. m.

$$2-\log_{\frac{1}{2}}(x) = 3$$

$$\left(x=\frac{1}{2}^{-1}\right.$$

$$\log_{\frac{1}{2}}(x) = -1 = \log_{\frac{1}{2}}\frac{1}{2}^{-1}$$

sz. m.

$$x = 2 \in H \checkmark \Rightarrow M = \{2\}$$

$\boxed{15/c}$ $\log_3(x+1) - \log_3(x+10) = 2 \cdot \log_3(4,5) - 4$

kikötések:

- $x+1>0 \iff x>-1 \rightarrow H := (-1, +\infty)$
- $x+10>0 \iff x>-10$

$$\log_3\left(\frac{x+1}{x+10}\right) = \log_3\left(\left(\frac{9}{2}\right)^2\right) - \log_3 81 =$$

$$= \log_3\left(\frac{\frac{\cancel{81}}{4}}{\cancel{81}}\right) = \log_3\left(\frac{1}{4}\right)$$

$\log_3\left(\frac{9}{2}\right) = \log_3(9) - \log_3(2)$

$$\log_3\left(\frac{x+1}{x+10}\right) = \log_3\left(\frac{1}{4}\right)$$

$$x \in M = \{2\} \quad (HF)$$

$\boxed{15/d}$ $$\log_2(x-2) + \log_2(x+3) = \underbrace{\overbrace{1 + \underbrace{2\cdot\log_4(3)}_{\log_4 9}}^{=\log_4 36}}_{}$$

$1 = \log_4 4$

$$= \log_4 36 = \frac{\log_2 36}{\log_2 4} = \frac{2\cdot\log_2 6}{2} = \log_2 6$$

kikötések:

$x > 2$ és $x > -3 \longrightarrow H := (2, +\infty)$

egyenlet mo-a:

$$\log_2(x-2)(x+3) = \log_2 6 \longrightarrow x \in M = \{3\}$$

$x_1 = -4 \notin H \quad x_2 = 3$

15/e $\log_{32}(2x) - \log_8(4x) + \log_2(x) = 3$

kikötés: $x > 0$

$H := (0, +\infty)$

$$\log_{32}(2x) = \frac{\log_2 2x}{\log_2 32} = \frac{1}{5}\cdot \log_2(2x) = \frac{1}{5}\left(\log_2(2) + \log_2(x)\right) = \frac{1}{5} + \frac{1}{5}\cdot \log_2(x)$$

$$\log_8(4x) = \frac{1}{3}\cdot \log_2(4x) = \frac{1}{3}\left(\log_2(4) + \log_2(x)\right) = \frac{2}{3} + \frac{1}{3}\log_2(x)$$

$$\underbrace{\frac{1}{5}\log_2(2x)}_{\frac{1}{5}+\frac{1}{5}\cdot\log_2(x)} - \frac{1}{3}\cdot \log_2(4x) + \log_2(x) = 3 \quad \left(\text{cél: } y = \log_2(x)\right)$$

$$\frac{1}{5} + \frac{1}{5}\cdot\log_2(x) - \frac{2}{3} - \frac{1}{3}\log_2(x) + \log_2(x) = 3 \to y := \log_2(x)$$

$$13y = 52$$

$$y = 4 \quad \log_2(x) = 4 \Rightarrow x \in M = \{16\}$$

$$\log_a\left(a^b + a^c\right)$$

$$\log_a\left(\underbrace{a^{b+c}}_{a^b \cdot a^c}\right) = b + c$$

$\boxed{15/g}$ $x^{\overbrace{2\cdot\lg^2(x)-\frac{3}{2}\cdot\lg(x)}^{>0}}=\overbrace{\sqrt{10}}^{>0}$ $\boxed{\text{kik}: x>0}$

$\left(\lg^2(x)=\left[\lg(x)\right]^2\right)$

$\lg\left(x^{2\cdot\lg^2(x)-\frac{3}{2}\cdot\lg(x)}\right)=\lg(\sqrt{10})=\frac{1}{2}$

$(\text{cél}: y:=\lg(x))$

$\left(2\cdot\lg^2(x)-\frac{3}{2}\cdot\lg(x)\right)\cdot\lg(x)=\frac{1}{2}$

$y:=\lg(x)$

$2y^3-\frac{3}{2}y^2-\frac{1}{2}=0 \quad /\cdot 2$

$4y^3-3y^2-1=0 \qquad y=1, \rightarrow \lg(x)=1$

$(y-1)(\underbrace{4y^2+y+1}_{D<0})=0 \qquad x=10\in H$

$M=\{10\}$

15/j $\log_{\frac{1}{2}}\left(\frac{3-x}{3x-1}\right) \geq \underset{\log_{\frac{1}{2}} 1}{\overset{\shortparallel}{0}}$

$\log_{\frac{1}{2}}(\cdot)$ sz. m. cs.

kik.: $\frac{3-x}{3x-1} > 0$, $3x-1 \neq 0$

$x \in H = \left(\frac{1}{3}, 3\right)$

$$\frac{3-x}{3x-1} \leq 1 \qquad / \cdot (3x-1), \text{ mert } x \in H \text{ esetén } 3x-1>0$$

$$3-x \leq 3x-1$$

$$4x \geq 4$$

$$x \geq 1$$

$x \in M = [1, +\infty) \cap H = [1, 3)$