20/a) $k \in \mathbb{R}$, $k = ?$

$P(x) := (2x^2 + x + k)$-ból $(x+3)$-at ki lehessen emelni?

$$P(x) = \underbrace{(x+3)}_{(x-(-3))} \cdot Q(x) + \text{konstans pol.} \quad (Q \text{ polinom})$$

$$\Updownarrow$$

$x_0 := -3$ gyöke P-nek (Horner : HF)

$$P(-3) = 18 - 3 + k = 0 \Leftrightarrow k = -15$$

Kiemelés: HF (k-val)

"Horner-plusz" (nem a feladatok között)

$x_0 = 3$, $P(x) := x^5 - 8x^4 + 16x^3 + 18x^2 - 81x + 54 \overset{?}{=} (x-3)\cdot\underbrace{Q(x)}_{\text{polinom}}$

e.h.	1	−8	16	18	−81	54
3	1	−5	1	21	−18	0
3	1	−2	−5	6	0	
3	1	1	−2	0		
3	1	4	$10 \neq 0$			

$\to P(x) = (x-3)\cdot(x^4 - 5x^3 + x^2 + 21x - 18)$

$\to P(x) = (x-3)\cdot(x-3)(x^3 - 2x^2 - 5x + 6)$

$\to P(x) = (x-3)(x-3)\cdot(x-3)(x^2 + x - 2)$

$\to P(x) = (x-3)^3(x^2 + x - 2)$

$\to$ az $x_0 = 3$ háromszoros gyök

2. fejezet

1. a, $P(x) := \underbrace{x^2 - 6x}_{(x-3)^2} + 3 = (x - \Delta)^2 + \square \qquad (\Delta, \square \in \mathbb{R})$

$$= \underbrace{x^2 - 6x + 9}\underbrace{- 9 + 3} =$$

$$= (x-3)^2 - 6 = 0$$

$$(x-3)^2 = 6 \quad / \sqrt{}$$

$$x - 3 = \pm\sqrt{6}$$

$$x = 3 \pm \sqrt{6}$$

b, $P(x) := 2x^2 + 7x - 1 = 2\left(x^2 + \frac{7}{2}x\right) - 1 =$

főegyüttható

$$= 2\left(\underbrace{\left(x + \frac{7}{4}\right)^2}_{x^2 + \frac{7}{2}x + \frac{49}{16}} - \frac{49}{16}\right) - 1 = 2\left(x + \frac{7}{4}\right)^2 - \frac{49}{8} - 1^{=\frac{8}{8}} =$$

$$= 2\left(x + \frac{7}{4}\right)^2 - \frac{57}{8} = 0$$

$$2\left(x + \frac{7}{4}\right)^2 = \frac{57}{8}$$

$$\left(x + \frac{7}{4}\right)^2 = \frac{57}{16}$$

$$x + \frac{7}{4} = \pm \frac{\sqrt{57}}{4}$$

$$x = -\frac{7}{4} \pm \frac{\sqrt{57}}{4}$$

② Viète-képletek: ax^2+bx+c, gyökei: x_1, x_2

$$\boxed{x_1+x_2=-\frac{b}{a}, \quad x_1\cdot x_2=\frac{c}{a}}$$

(szorgalmi: biz.)

$$ax^2+bx+c=a\cdot(x-x_1)(x-x_2)$$

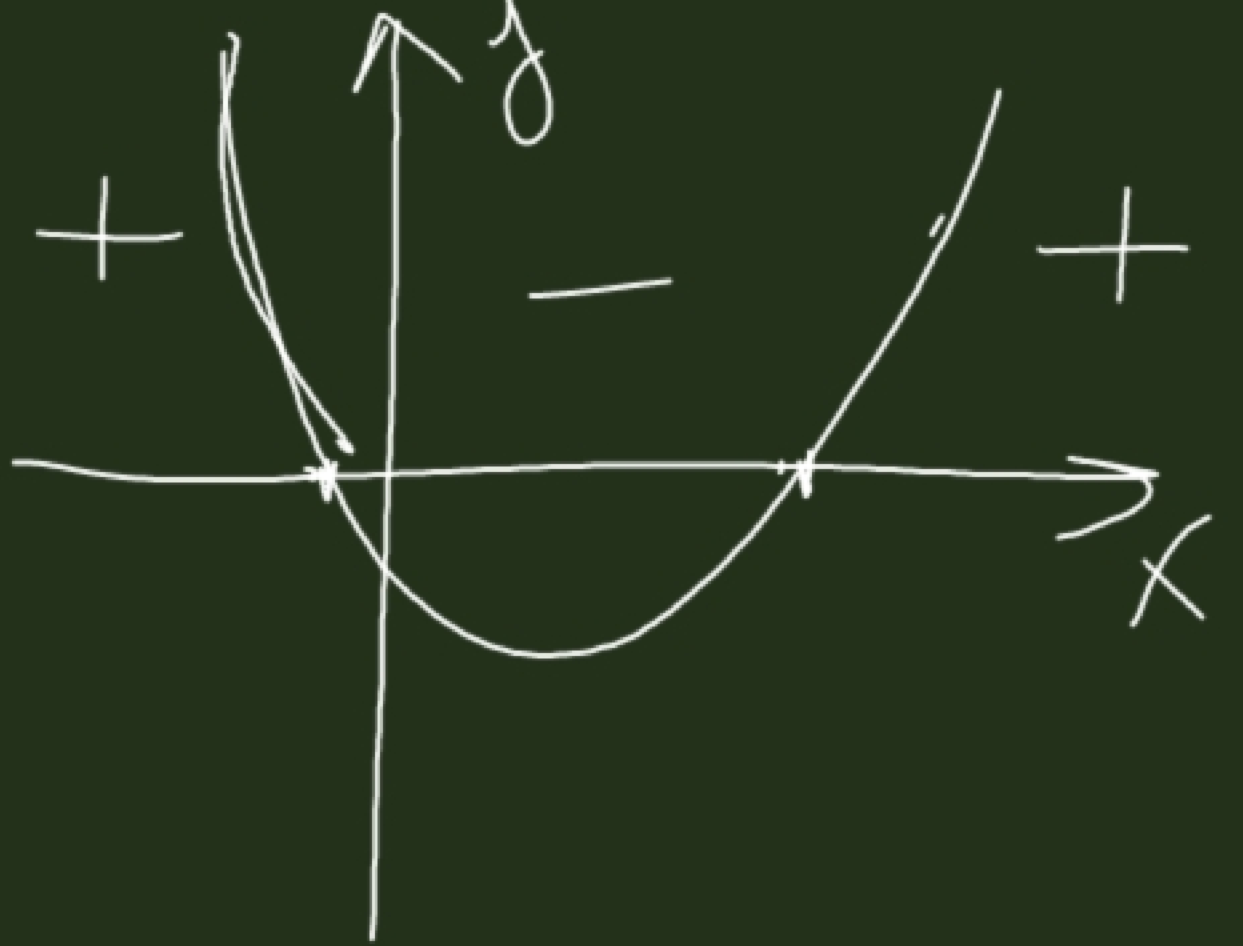
$P(x)=x^2-6x+3$, $a:=1$, $b:=-6$, $c:=3$

- $x_1+x_2=-\frac{-6}{1}=6$
- $x_1\cdot x_2=3$
- $x_1^2+x_2^2=(x_1+x_2)^2-2x_1\cdot x_2=6^2-2\cdot 3=30$
- $|x_1-x_2|=\sqrt{(x_1-x_2)^2}=\sqrt{\overbrace{(x_1^2+x_2^2)}^{30}-2x_1x_2}=\sqrt{30-2\cdot 3}=\sqrt{24}$
- $\frac{1}{x_1}+\frac{1}{x_2}=\frac{x_2+x_1}{x_1\cdot x_2}=\frac{6}{3}=2$,

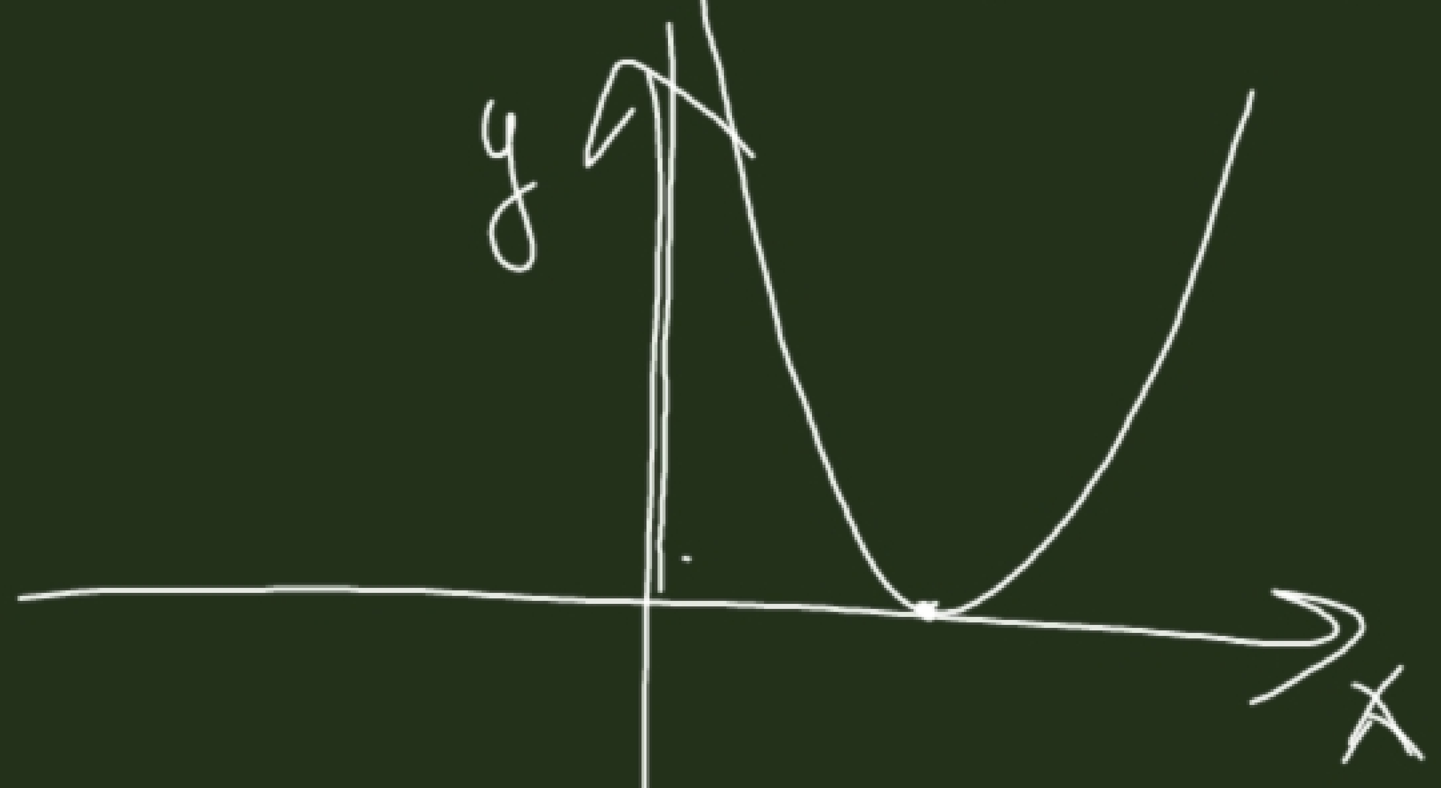
$\boxed{\text{HF: } P(x)=2x^2+7x-1}$

Egyenlőtlenségek

$P(x) := ax^2 + bx + c$, $\quad D := b^2 - 4ac \quad (a \neq 0)$

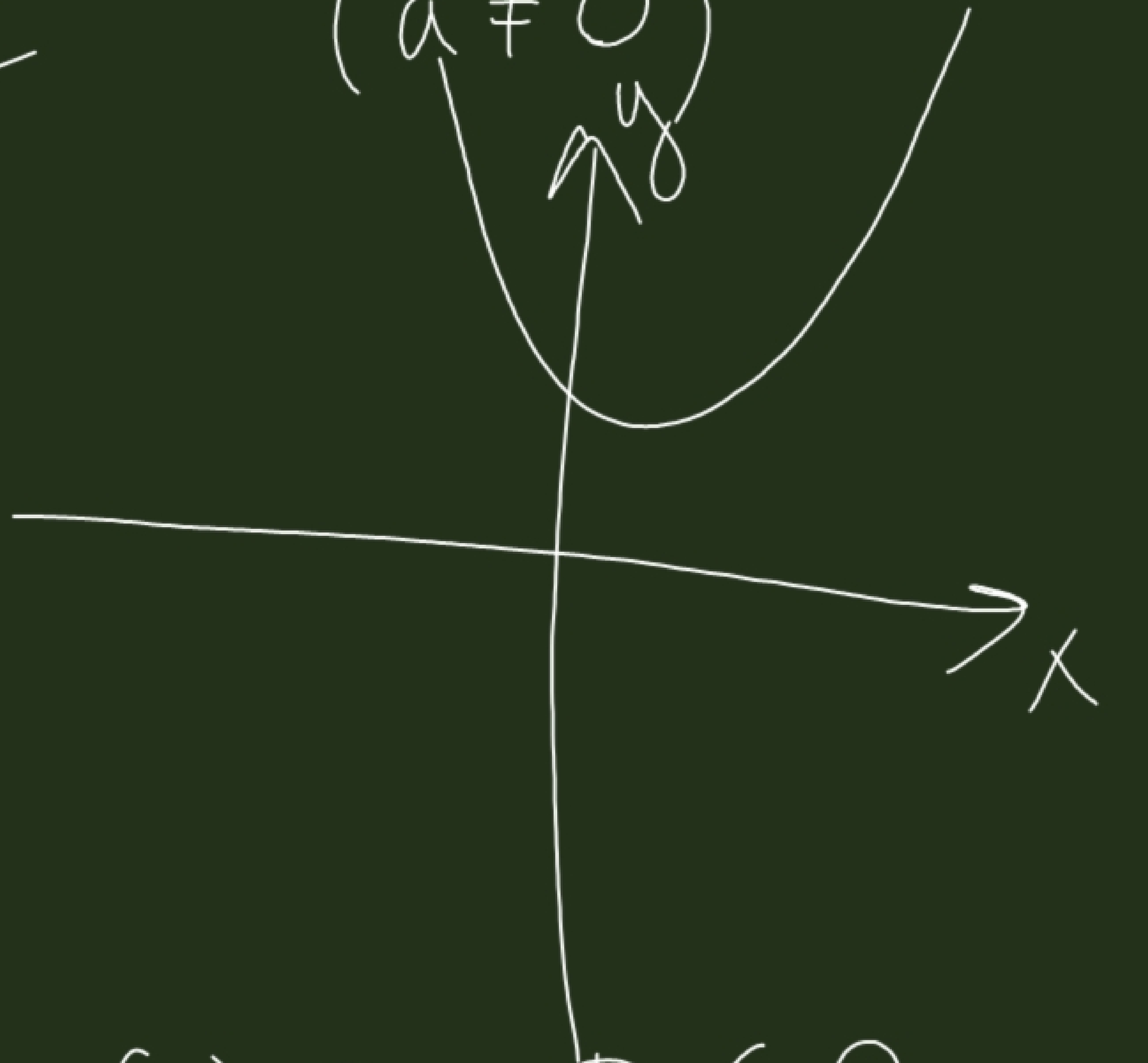


$a > 0, \ D > 0$



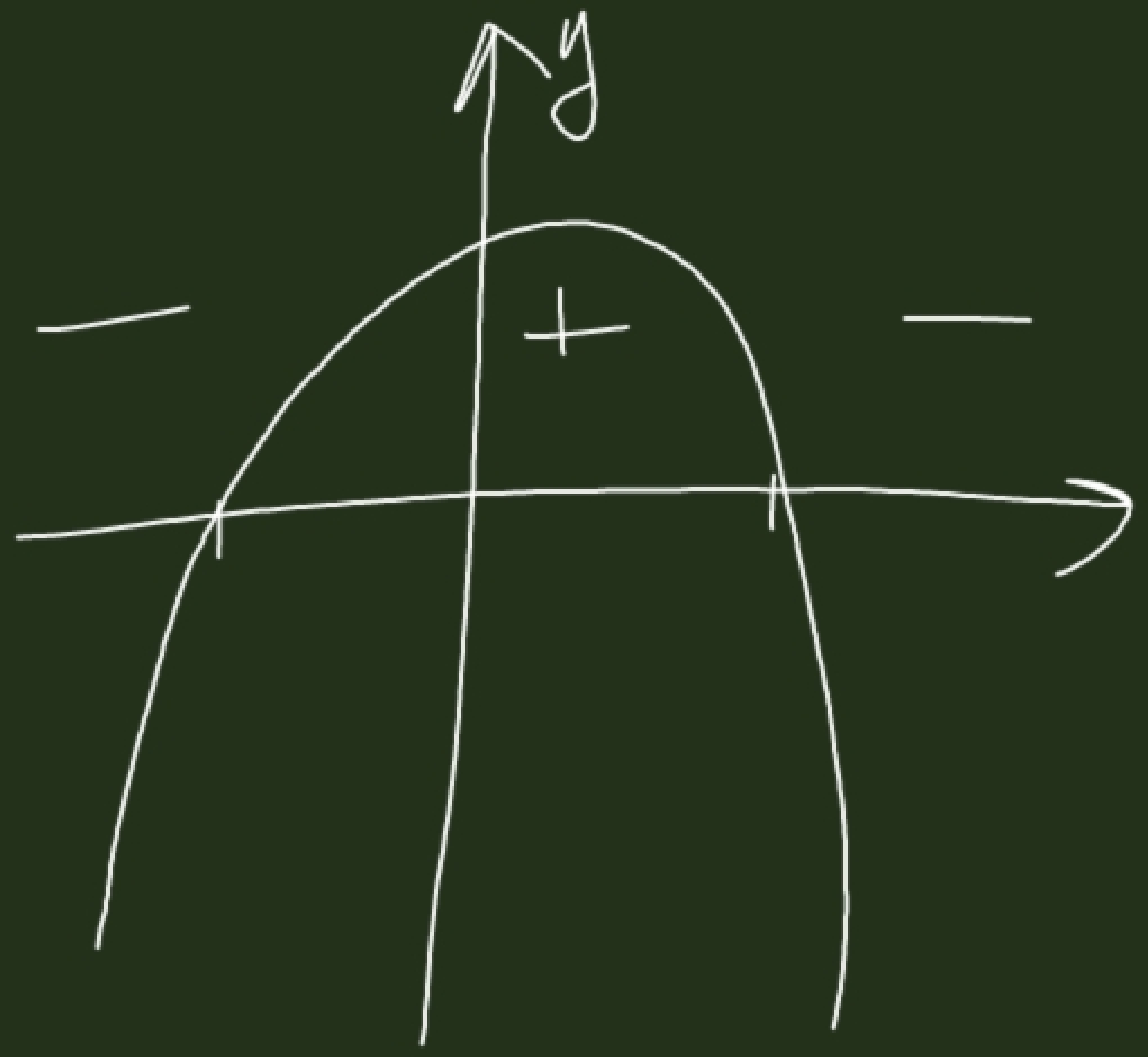
$a > 0 \quad D = 0$

$P(x) \geq 0$

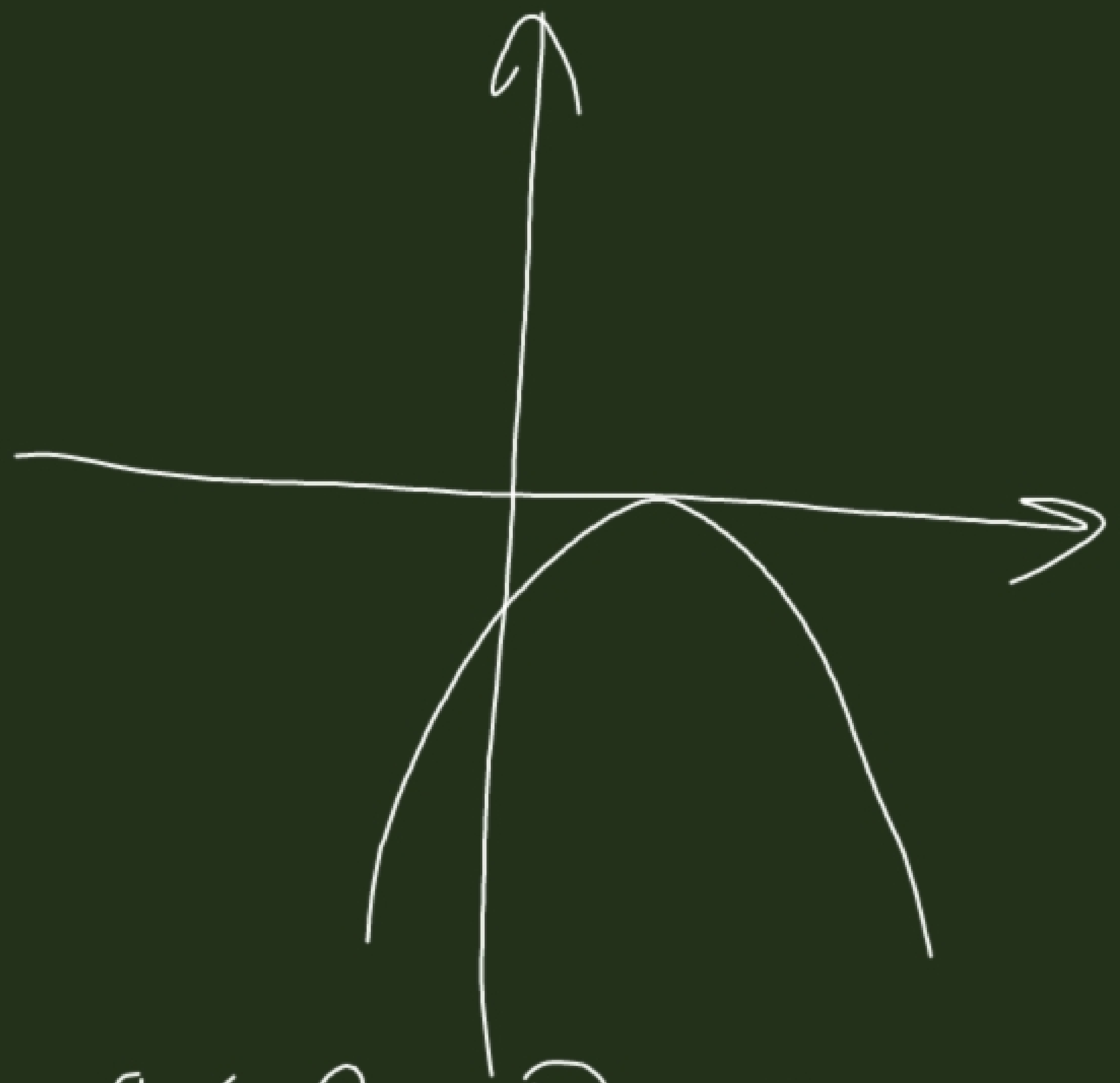


$a > 0, \ D < 0$ ↳ nincs valós mo.

$\boxed{P(x) > 0}$

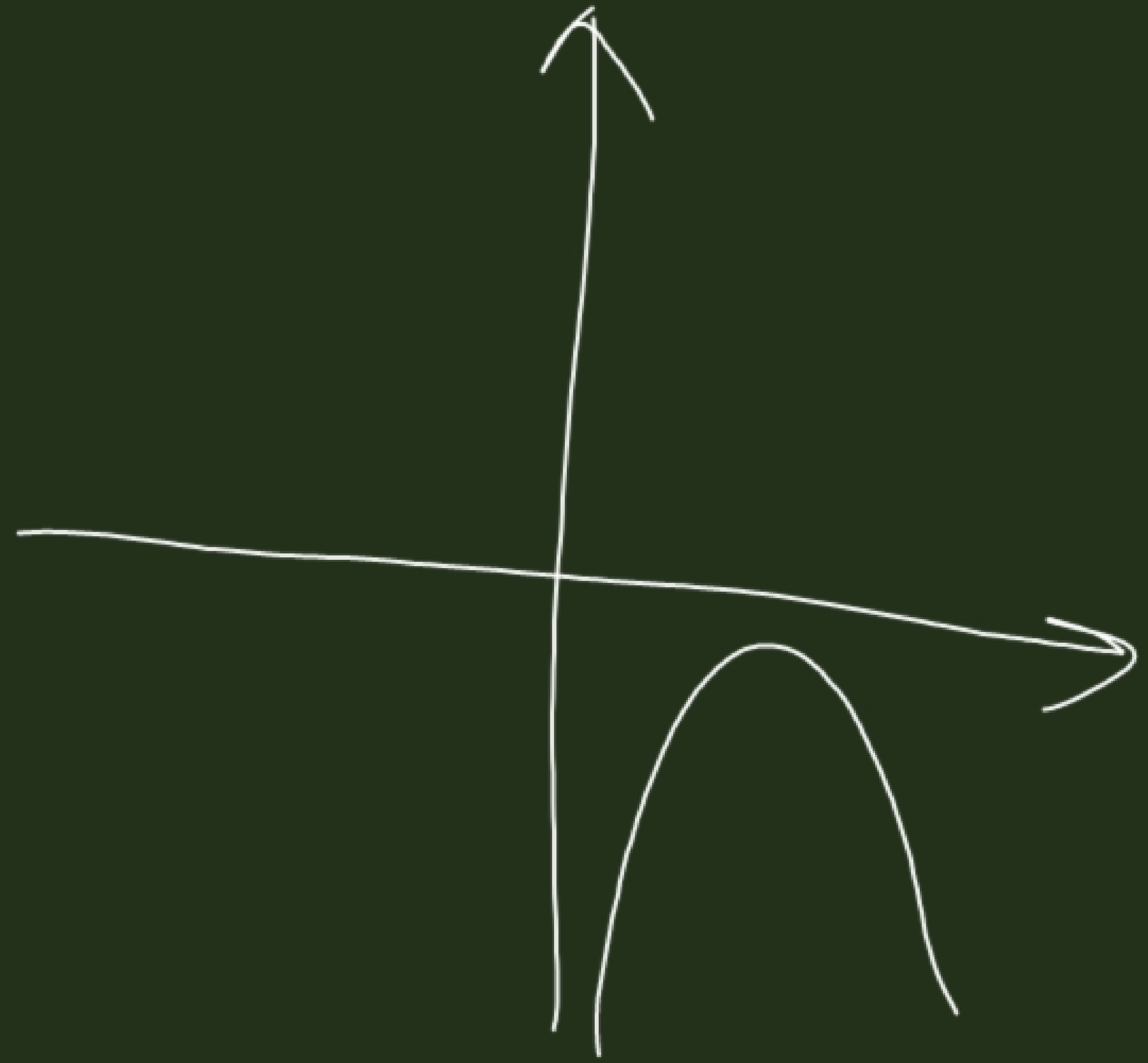


$a<0, D>0$



$a<0, D=0$

$P(x) \leq 0$



$a<0, D<0$

$P(x) < 0$

3/b $\dfrac{3x^2+7x-4}{x^2+2x-3} < 2 \quad /-2$

$\boxed{\text{0-ra rendezés}}$

$$\frac{3x^2+7x-4}{x^2+2x-3} - 2 < 0$$

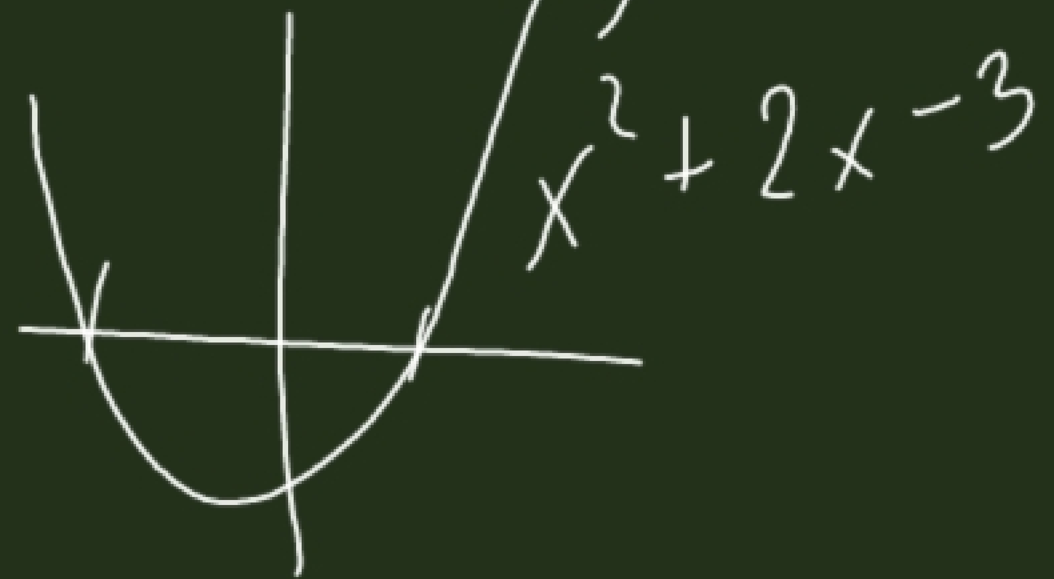
$$\frac{3x^2+7x-4-2(x^2+2x-3)}{x^2+2x-3} = \frac{x^2+3x+2}{x^2+2x-3} < 0$$

Kikötés:

$$x^2+2x-3=0$$

$$x_{1,2} = \frac{-2\pm\sqrt{4+12}}{2} = \frac{-2\pm 4}{2} \begin{cases} 1 \\ -3 \end{cases}$$

$$x \in \mathbb{R} \setminus \{1, -3\}$$



$$\frac{x^2+3x+2}{x^2+2x-3} < 0$$

Sz.: $x^2+3x+2=0$

$$x_{1,2} = \frac{-3 \pm \sqrt{\overset{=1}{9-8}}}{2} = \begin{cases} -1 \\ -2 \end{cases}$$

n: $x_{1,2} = -3,\ 1$

	$x<-3$	-3	$-3<x<-2$	-2	$-2<x<-1$	-1	$-1<x<1$	1	$x>1$
sz.	+	+	+	0	−	0	−	+	+
n.	+	0	−	−	−	−	−	0	+
t	+	h	−	0	+	0	−	ł	+

$M := (-3,-2) \cup (-1,1)$

a megoldáshalmaz

nyílt intervallum: $(a; b)$

zárt — " — $[a; b]$

3/c $\quad \dfrac{x-1}{x+1} > \dfrac{3x+4}{1-2x}$

0-ra rendezés!

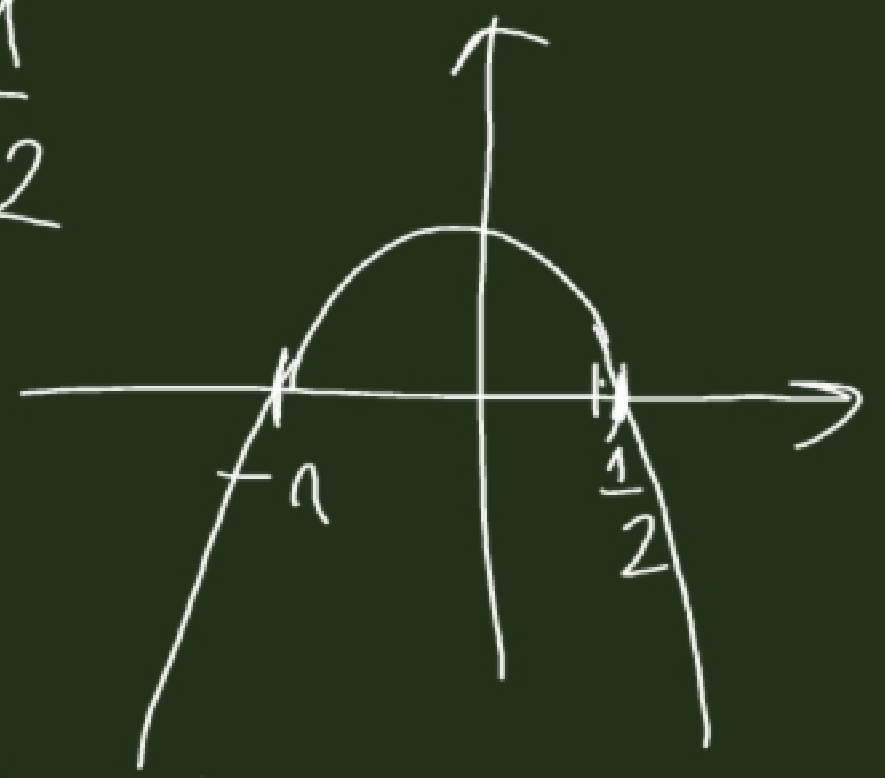
kikötés: $x \neq -1$, $x \neq \dfrac{1}{2}$

$\vdots$

$$\frac{-5x^2-4x-5}{(x+1)(1-2x)} > 0$$

n.: főegyüttható: $-2 < 0$

gyökök: -1, $\dfrac{1}{2}$



$$\frac{5x^2+4x+5}{(x+1)(1-2x)} < 0$$

Sz.: $D = 16 - 100 < 0$

$\rightarrow$ Sz. > 0

$$\Updownarrow$$

$$(x+1)(1-2x) < 0$$

$M = (-\infty, -1) \cup \left(\dfrac{1}{2}, +\infty\right)$ a megoldáshalmaz

4/c) Adjuk meg azokat a $p \in \mathbb{R}$ paramétereket,

amikre $\boxed{(p^2-1)x^2 + 2(p-1)x + 1 > 0}$ igaz $\forall x \in \mathbb{R}$.

$$\overset{p=2}{3x^2 + 2x + 1 > 0}, \quad \overset{p=3}{(9-1)x^2 + 2(3-1)x + 1 > 0}, \ldots$$

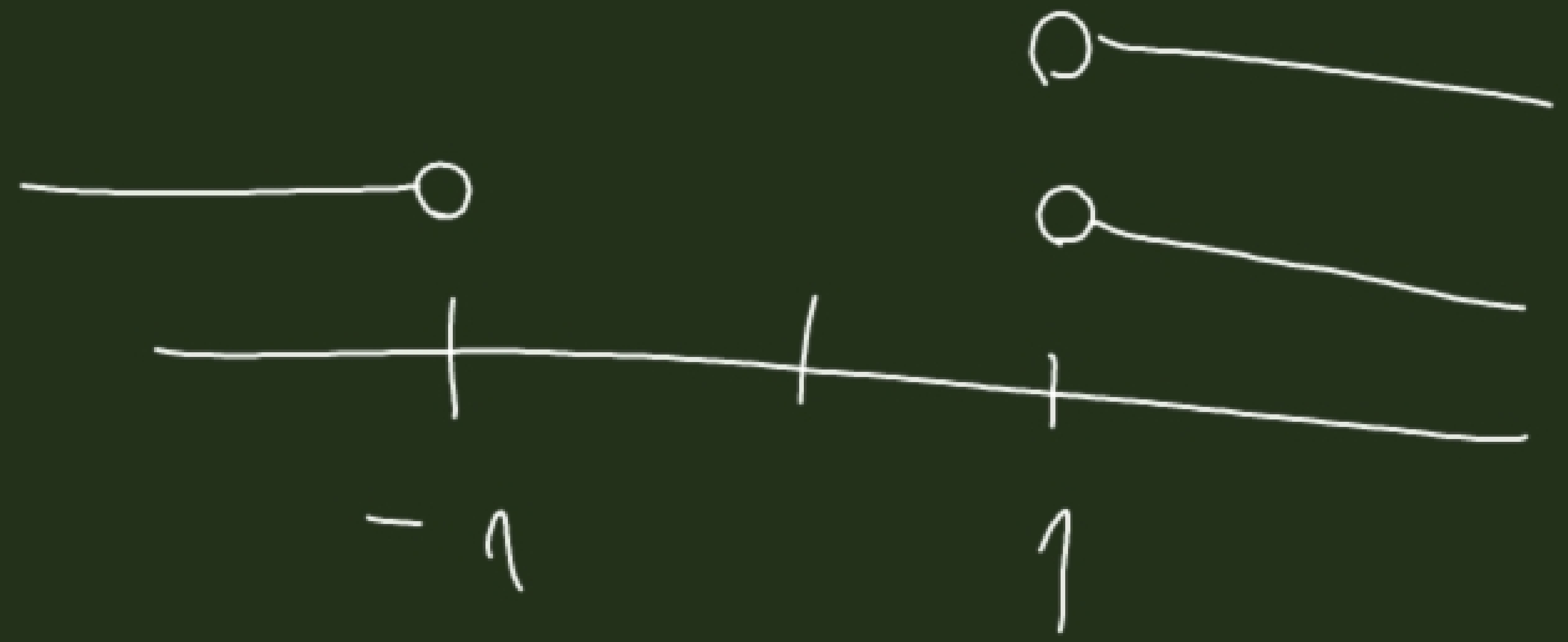
nincs benne: $(4-1)x^2 + 2(3-1)x + 1 > 0$

másodfokú pol. mindig pozitív $\iff$ feh. > 0 és $D < 0$

$$D = 4(p-1)^2 - 4(p^2-1) = 4(\cancel{p^2} - 2p + 1) - \cancel{4p^2} + 4 = \underline{-8p + 8}$$

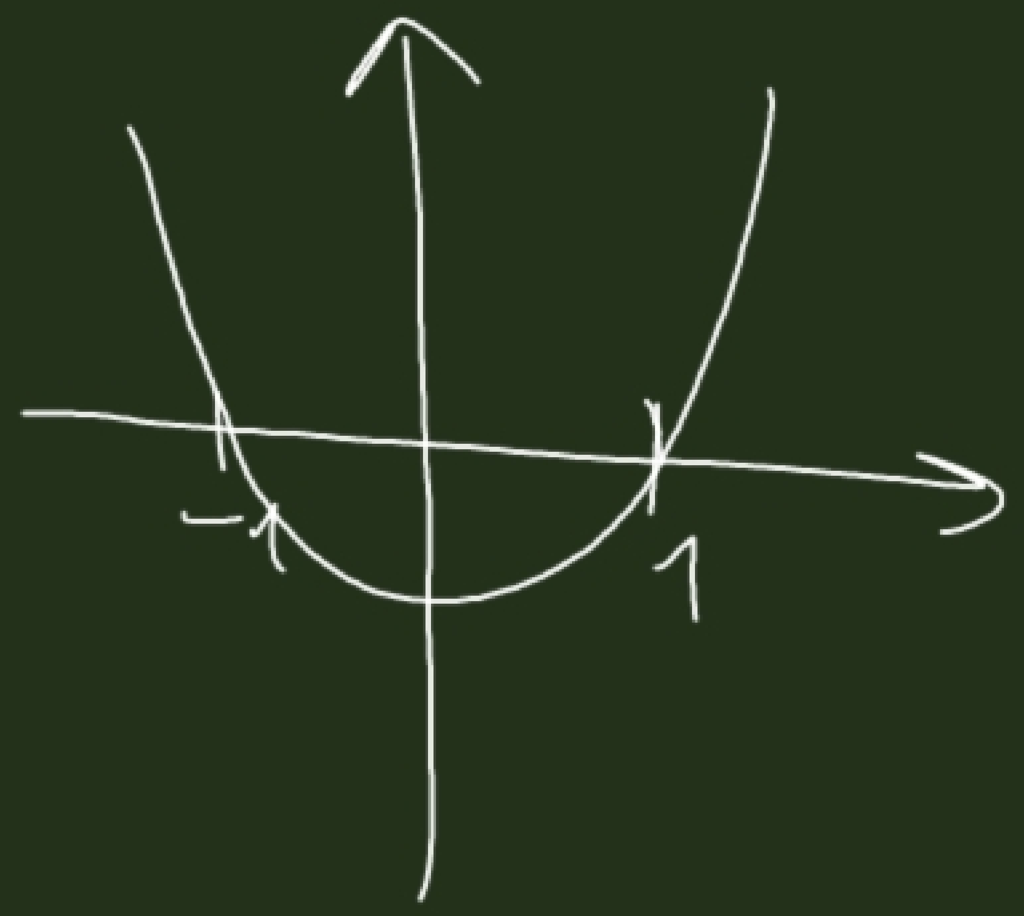
Olyan p-t keresünk, amire

$$p^2-1>0 \quad \text{és} \quad -8p+8<0$$



$$\Updownarrow \qquad\qquad\qquad \Updownarrow$$

$$p\in(-\infty,-1)\cup(1,+\infty) \qquad\qquad p\in(1,+\infty)$$



$$p\in\Big((-\infty,-1)\cup(1,+\infty)\Big)\bigcap\,(1,+\infty)=(1,+\infty)$$

$p^2 - 1 = 0$ esetek:

① $p = -1$: $0 \cdot x^2 + 2(-1-1)x + 1 > 0$

$-4x + 1 > 0$ ← nem teljesül $\forall x$-re

② $p = 1$: $0 \cdot x^2 + 2 \cdot 0 \cdot x + 1 > 0$

$1 > 0$ ← ez igaz $\forall x$-re

$\Rightarrow M: \; p \in [1, +\infty)$

9 HF