Félév anyaga: 1–10 : Matematikai alapok

11–23 : Lineáris algebra

24–26 : Függvények!

Kvíz: keddenként (10 perc, 4 pont)

Algebrai és gyökös kifejezések

$a^3 - b^3 = \ldots\ldots$

1) $a, b \in \mathbb{R}$

$\in$ – eleme; $\mathbb{R}$ → valós számok

$$\underbrace{a^2 + a\cdot b + b^2}_{(a+b)^2 - ab} = 3\cdot\left(\frac{a+b}{2}\right)^2 + \left(\frac{a-b}{2}\right)^2$$

HF...

$$3\cdot\left(\frac{a+b}{2}\right)^2 + \left(\frac{a-b}{2}\right)^2 = 3\cdot\frac{(a+b)^2}{4} + \frac{(a-b)^2}{4} =$$

$$= \frac{3}{4}\cdot\left(a^2+2ab+b^2\right) + \frac{1}{4}\cdot\left(a^2-2ab+b^2\right) = a^2 + \overbrace{\left(2\cdot\frac{3}{4} - 2\cdot\frac{1}{4}\right)}^{1}ab + b^2 =$$

$$= a^2 + ab + b^2 \checkmark$$

Tfh. $a-b=2$, $a+b=\sqrt{5}$, $a^3-b^3=?$

$$a^3-b^3=(a-b)(a^2+ab+b^2)=$$

$$=(a-b)\cdot\left(3\cdot\left(\frac{a+b}{2}\right)^2+\left(\frac{a-b}{2}\right)^2\right)=2\cdot\left(3\left(\frac{\sqrt{5}}{2}\right)^2+\left(\frac{2}{2}\right)^2\right)=$$

$$=2\cdot\left(3\cdot\frac{5}{4}+1\right)=\cancel{2}\cdot\frac{15+4}{\cancel{4}\,2}=\frac{19}{2}$$

3/b) $\forall\, a \neq b,\ a, b \in \mathbb{R}$:

minden, „alle"

$$\frac{a\cdot(a-b)}{a^3+a^2b+ab^2+b^3\cdot(\ )}+\frac{b(a+b)}{a^3-a^2b+ab^2-b^3\cdot(\ )}+$$

$$+\frac{1\cdot(a^2+b^2)}{a^2-b^2\cdot(\ )}-\frac{1(a^2-b^2)}{a^2+b^2\cdot(\ )}-\frac{a^2+3b^2}{a^4-b^4}=0.$$

közös nevező: a^4-b^4

$$(a^3+a^2b+ab^2+b^3)(a-b)=a^4-b^4$$

$$(a^3-a^2b+ab^2-b^3)(a+b)=a^4-b^4$$

$$(a^2-b^2)(a^2+b^2)=a^4-b^4$$

b. o.:

$$\frac{a\cdot(a-b)+b\cdot(a+b)+(a^2+b^2)-(a^2-b^2)-(a^2+3b^2)}{a^4-b^4}=0$$

4/b) $a, b, c \in \mathbb{R}$, $a+b+c=0 \Rightarrow a^3+b^3+c^3 = 3abc$

$$a+b+c=0 \rightarrow \boxed{c = -(a+b)}$$

$$a^3+b^3+(-(a+b))^3 = a^3+b^3-(a+b)^3 =$$

$$= a^3+b^3-(a^3+3a^2b+3ab^2+b^3) =$$

$$= -3a^2b-3ab^2 = 3ab(\underbrace{-a-b}_{c}) = 3abc$$

1

1 1

1 2 1

1 3 3 1

1 4 6 4 1

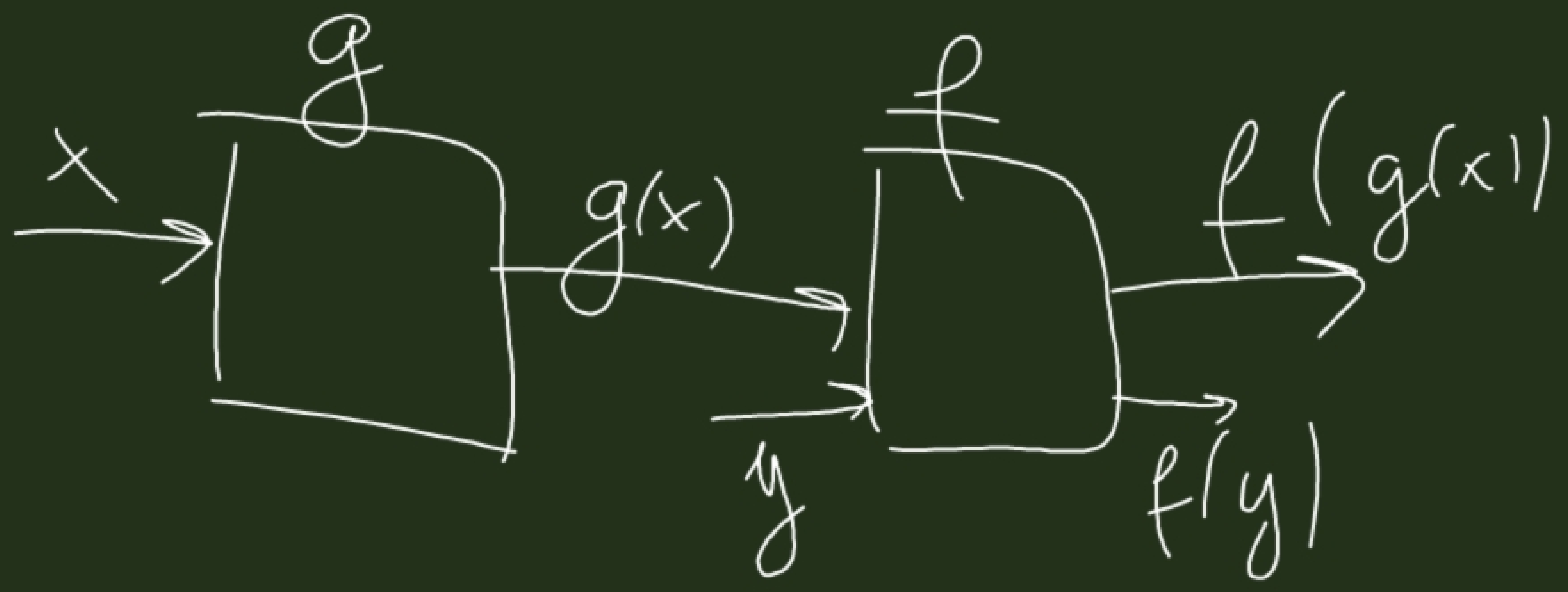
Pascal-Δ

б) HF

9. $f(x) := \dfrac{1-x}{1+x}$ $\left(x \in \mathbb{R}\setminus\{-1\}\right)$, $g(x) := \dfrac{1+x}{1-x}$ $\left(x \in \mathbb{R}\setminus\{1\}\right)$

Biz. be: $f(g(x)) \cdot g(f(x)) + 1 = 0$.

$$f(g(x)) = (f \circ g)(x) = f\left(\frac{1+x}{1-x}\right) = \frac{1 - \frac{1+x}{1-x}}{1 + \frac{1+x}{1-x}} = \ldots = -x$$



$$\frac{1-\frac{1+x}{1-x}}{1+\frac{1+x}{1-x}} = \frac{\frac{\cancel{1}-x-(\cancel{1}+x)}{1-x}}{\frac{1-\cancel{x}+1+\cancel{x}}{1-x}} = \frac{\frac{-2x}{1-x}}{\frac{2}{1-x}} =$$

$$= \frac{-\cancel{2}x}{\cancel{1-x}} \cdot \frac{\cancel{1-x}}{\cancel{2}} = -x$$

kon.: $g(f(x)) \overset{HF}{=} \frac{1}{x} \longrightarrow -x \cdot \frac{1}{x} + 1 = 0$ ✓

12/c

Ötlet: $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})=x-y$ $\quad (x,y>0,\ x\neq y)$

$$\left(\sqrt{x}-\frac{\sqrt{x\cdot y}+\underset{\sqrt{y}\cdot\sqrt{y}}{y}}{\sqrt{x}+\sqrt{y}}\right)\cdot\left(\frac{\sqrt{x}}{\sqrt{x}+\sqrt{y}}+\frac{\sqrt{y}}{\sqrt{x}-\sqrt{y}}+\frac{2\cdot\sqrt{x\cdot y}}{x-y}\right)=$$

$$=\left(\sqrt{x}-\frac{\sqrt{y}\,\cancel{(\sqrt{x}+\sqrt{y})}}{\cancel{\sqrt{x}+\sqrt{y}}}\right)\cdot\left(\frac{\sqrt{x}(\sqrt{x}-\sqrt{y})+\sqrt{y}(\sqrt{x}+\sqrt{y})+2\sqrt{x\cdot y}}{x-y}\right)=$$

$$=\cancel{(\sqrt{x}-\sqrt{y})}\cdot\left(\frac{\overbrace{\cancel{x+2\cdot\sqrt{xy}+y}}^{(\sqrt{x}+\sqrt{y})^{\cancel{2}}}}{x-y=\cancel{(\sqrt{x}+\sqrt{y})}\cancel{(\sqrt{x}-\sqrt{y})}}\right)=\sqrt{x}+\sqrt{y}$$

13 tipp: $a := \sqrt[6]{x}$, $b := \sqrt[6]{y}$ $\left(\sqrt[3]{x} = a^2,\ \sqrt[3]{y} = b^2 \right)$

HF

$$E(x,y) := \ldots\ldots$$

19/b) $P(x) := 2x^3 - 4x^2 - 18x$; $x_0 := 3$

Igazoljuk, hogy x_0 gyöke P-nek.

$$P(x_0) = P(3) = 2\cdot 3^3 - 4\cdot 3^2 - 18 = 2\cdot 27 - 4\cdot 9 - 18 =$$
$$= 54 - 36 - 18 = \underline{\underline{0}}$$

Emeljük ki a 3-hoz tartozó gyöktényezőt: $P(x) = (x-3)\cdot Q(x)$, Q egy polinom

trükkös verzió (megoldókulcs): $P(x) - P(3) =$

$$\overset{P(x)}{=} 2\cdot x^3 - 4x^2 - \cancel{18} -$$
$$-\left(2\cdot 3^3 - 4\cdot 3^2 - \cancel{18}\right) =$$
$$= 2\cdot\left(x^3 - 3^3\right) - 4\left(x^2 - 3^2\right) = (x-3)(\dots)$$

Horner-táblázat:

$P(x) = 2x^3 - 4x^2 - 18$

	x^3	x^2	x^1	x^0
e.h.	2	−4	0	−18
$x_0 = 3$	2	2	6	0
	x^2	x^1	x^0	

$0 \rightarrow x_0$ gyöke P-nek

$\rightarrow P(x) = (x-3)(2x^2 + 2x + 6)$

$\boxed{19/c}$ $P(x) := 2x^4 \overbrace{-5x^3}^{+(-5)\cdot x^3} - 6x^2 + 3x + 2, \quad x_0 := -1$

	x^4	x^3	....		
e.h.	2	−5	−6	3	2
−1	2	−7	1	2	0
	x^3	...			

$\rightarrow -1$ gyöke P-nek

$-2-5=-7$

$$P(x) = \underbrace{(x-(-1))}_{x+1}(2x^3 - 7x^2 + x + 2)$$