$$|f(x)-2| = \left|\frac{-x^2-4x+13}{x^3+2x-5}\right| = \frac{|-x^2-4x+13|}{|x^3+2x-5|} \le$$

Δ egyenlőtlenség

$$|a+b| \le |a|+|b|$$

$$\le \frac{|-x^2|+|-4x|+13}{|x^3+2x-5|} = \frac{x^2+4x+13}{x^3+2x-5} < \varepsilon$$

$x > \sqrt[3]{10} > 0$

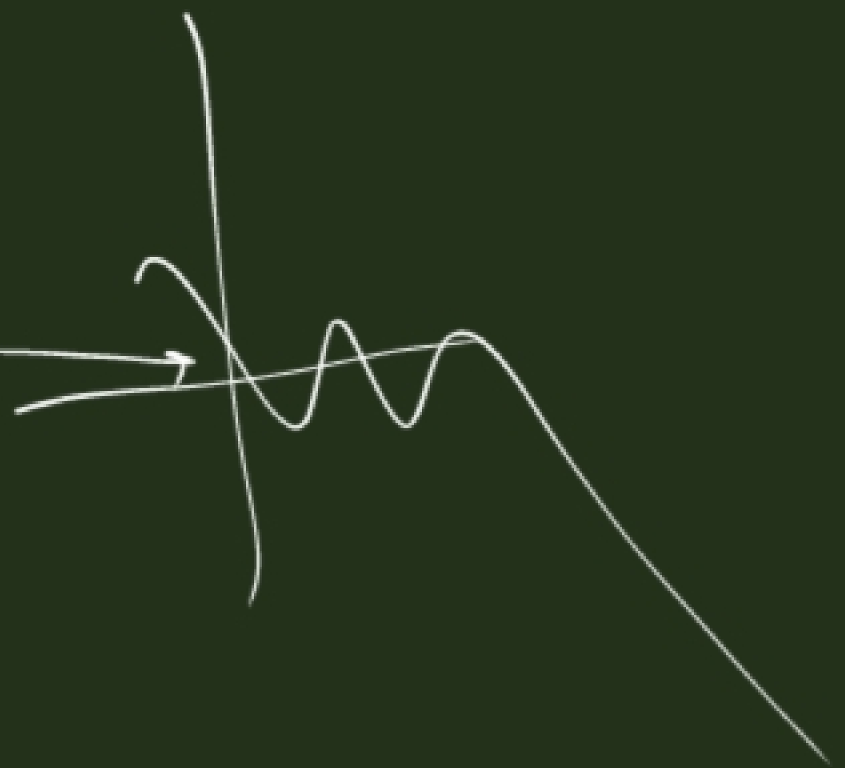
$$x^3+2x-5 > 0 \qquad (x > R)$$

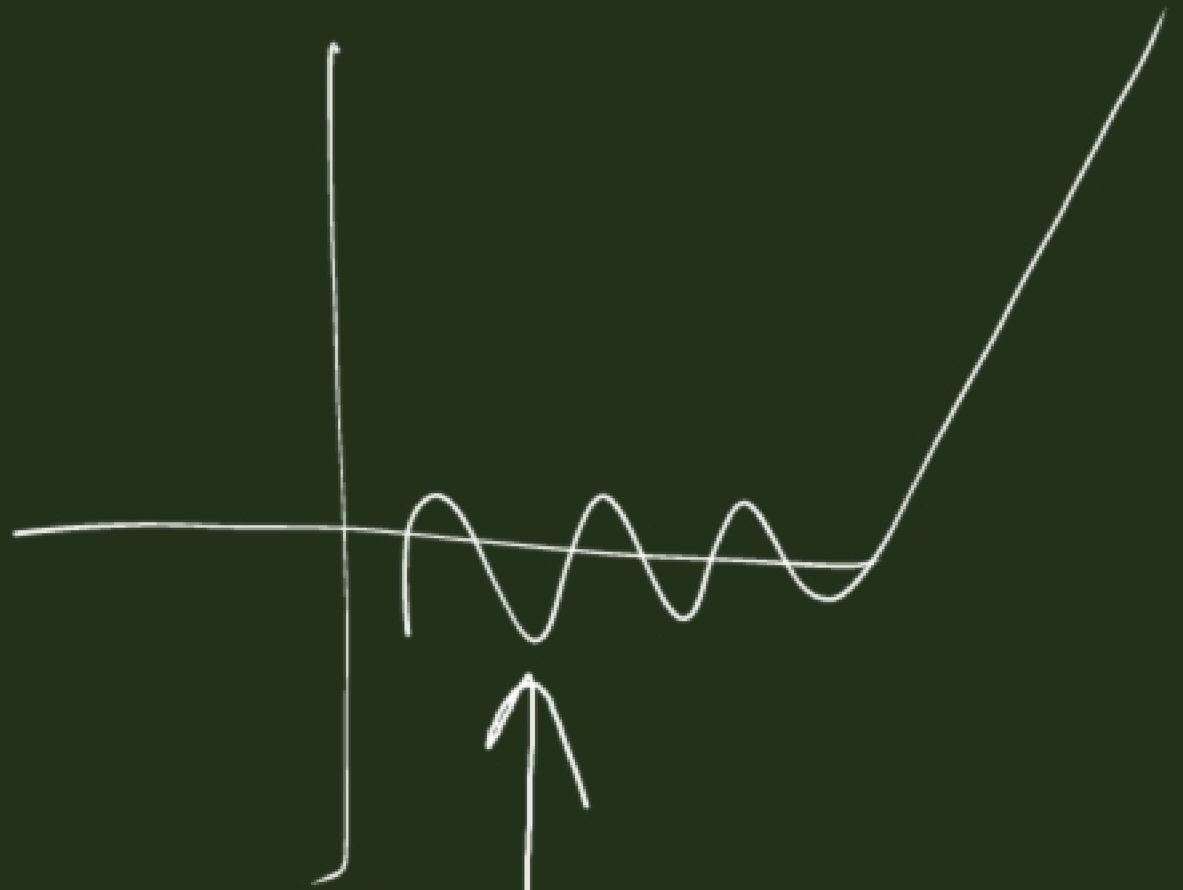
$$x^3+\underbrace{2x}_{>0}-5 > x^3-5 = \frac{1}{2}x^3 + \underbrace{\frac{1}{2}x^3-5}_{>0} \ge \frac{1}{2}x^3 > 0$$

$\left(5 < 5x^2\right)$ $x>1$

$\frac{1}{2}x^3-5 > 0 \iff \boxed{x > \sqrt[3]{10}}$

$$\left(\frac{\cdots\cdots}{\Big|\underbrace{-x^3+2x-5}_{\xrightarrow[x\to+\infty]{}\;-\infty}\Big|}\right.$$





$$|-x^3+2x-5| = |(-1)(x^3-2x+5)| = |\underbrace{x^3-2x+5}_{\to\,+\infty}|$$

$$x^3-2x+5>0$$

$$(x>R)$$

? 22. javító)

1. $A = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 4 & -3 \\ 2 & 2 & -1 \end{bmatrix}$

se'... $P(\lambda) = \det \begin{array}{ccc} + & - & + \end{array} \begin{bmatrix} 2-\lambda & 1 & -1 \\ 3 & 4-\lambda & -3 \\ 2 & 2 & -1-\lambda \end{bmatrix} = (2-\lambda)\cdot \det \begin{bmatrix} 4-\lambda & -3 \\ 2 & -1-\lambda \end{bmatrix} -$

$-1\cdot \det \begin{bmatrix} 3 & -3 \\ 2 & -1-\lambda \end{bmatrix} - 1\cdot \det \begin{bmatrix} 3 & 4-\lambda \\ 2 & 2 \end{bmatrix} = (2-\lambda)\cdot\left((4-\lambda)(-1-\lambda)+6\right) -$

$\underbrace{-\left(\underbrace{3(-1-\lambda)+6}_{-3\lambda+3}\right) - \left(\underbrace{6-2(4-\lambda)}_{2\lambda-2}\right)}_{3(\lambda-1)-2(\lambda-1)=\lambda-1} = (2-\lambda)\left(\underbrace{(4-\lambda)(-1-\lambda)+6}_{-4-4\lambda+\lambda+\lambda^2+6=}\right)+\lambda-1 =$

$= \lambda^2-3\lambda+2 = (\lambda-1)(\lambda-2)$

$$= (2-\lambda)(\lambda-1)(\lambda-2) + (\lambda-1) =$$

$$= (\lambda-1)\Big(\underbrace{1-(\lambda-2)^2}_{a^2-b^2}\Big) = (\lambda-1)\big(1-(\lambda-2)\big)\big(1+\lambda-2\big) =$$

$$= (\lambda-1)^2(-\lambda+3) = -(\lambda-3)(\lambda-1)^2 = 0$$

$\lambda_1 = 1 \quad a(1) = 2$

$\lambda_2 = 3, \ a(3) = 1$

($\lambda_1, \lambda_2 \in \mathbb{R} \Rightarrow$ ugyanazok a válaszok $\mathbb{R}$ és $\mathbb{C}$ felett)

Sajátvektorok:

$\boxed{\lambda_1 = 1}$ olyan $x \neq 0$ vektorokat keresünk, amikre:

$$(A - 1 \cdot I)x = 0$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 3 & 3 & -3 \\ 2 & 2 & -2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow x_1 + x_2 - x_3 = 0$$

$$x_3 = x_1 + x_2, \quad x_1, x_2 \in \mathbb{R}$$

$$\left(\begin{array}{l} x_1 + x_2 - x_3 = 0 \longrightarrow x_3 = x_1 + x_2 \\ \qquad\qquad\qquad\qquad \hookrightarrow (2)\checkmark, \ (3)\checkmark \\ 3x_1 + 3x_2 - 3x_3 = 0 \\ 2x_1 + 2x_2 - 2x_3 = 0 \end{array}\right)$$

$(\text{mw.: } x_3 = x_1 + x_2)$

sajátvektorok: $\begin{pmatrix} x_1 \\ x_2 \\ x_1 + x_2 \end{pmatrix} = x_1 \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + x_2 \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, $x_1, x_2 \in \mathbb{R}$

(kivéve: $x_1 = x_2 = 0$)

$W_1 = \text{Span}\left(\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right)$, $g(1) = 2$

$$\boxed{\lambda = 3}$$

$$(A - 3 \cdot I) \cdot X = 0$$

$$\begin{bmatrix} -1 & 1 & -1 \\ 3 & 1 & -3 \\ 2 & 2 & -4 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \text{mo:} \quad \begin{matrix} x_2 = 3x_1 \\ x_3 = 2x_1 \\ x_1 \in \mathbb{R} \end{matrix}$$

$$\begin{cases} -x_1 + x_2 - x_3 = 0 \\ 3x_1 + x_2 - 3x_3 = 0 \\ 2x_1 + 2x_2 - 4x_3 = 0 \end{cases} \longrightarrow x_2 = x_1 + x_3 = x_1 + 2x_1 = \boxed{3x_1 = x_2}$$

$$3x_1 + (x_1 + x_3) - 3x_3 = 0$$

$$4x_1 - 2x_3 = 0$$

$$\boxed{x_3 = 2x_1}$$

$$\rightarrow (3):$$

$$2x_1 + 6x_1 - 8x_1 = 0 \checkmark$$

Sajátvektorok: $\begin{pmatrix} x_1 \\ 3x_1 \\ 2x_1 \end{pmatrix} = x_1 \cdot \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \quad (x_1 \in \mathbb{R} \setminus \{0\})$

$\underline{W_3 = \text{Span}\left(\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}\right)}, \quad g(3) = 1$

diagonalizálhatóság:

$a(1) + a(3) = 2 + 1 = 3$ ✓

$a(1) = g(1)$ ✓

$a(3) = g(3)$ ✓

$\Rightarrow \exists$ SB $\mathbb{R}^3$-ban

$\Rightarrow$ A diagonalizálható $\mathbb{R}$ felett diagonalizáló mátrix:

$$C = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$\in W_1$ $\in W_3$

diagonális alak:

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

② $W = \{(a,b,c,d) \in \mathbb{R}^4 \mid a - b + c + d = 0\} \subset \mathbb{R}^4$

a, OB, ONB W-ben?

W elemei:

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} a \\ a+c+d \\ c \\ d \end{pmatrix} = a \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} + c \cdot \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} + d \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$$a - b + c + d = 0 \rightarrow b = a + c + d$$

$$\Rightarrow W = \mathrm{Span}\Big(\underbrace{(1,1,0,0),\ (0,1,1,0),\ (0,1,0,1)}_{\text{G és F is}}\Big)$$

$\mathcal{B}: \underbrace{(1,1,0,0)}_{=:b_1}, \underbrace{(0,1,1,0)}_{=:b_2}, \underbrace{(0,1,0,1)}_{=:b_3}$

GM:

$u_1 := (1,1,0,0)$

$\boxed{u_2}: b_2 - \frac{\langle b_2, u_1\rangle}{\langle u_1, u_1\rangle} \cdot u_1 = (0,1,1,0) - \frac{1}{2}\cdot(1,1,0,0) = \boxed{\left(-\frac{1}{2}, \frac{1}{2}, 1, 0\right)}$

$\langle b_2, u_1\rangle = \langle (0,1,1,0), (1,1,0,0)\rangle = 0\cdot 1 + 1\cdot 1 + 1\cdot 0 + 0\cdot 0 = 1$

$\langle u_1, u_1\rangle = \left(\|u_1\|^2\right) = 2$

$u_2 := (-1, 1, 2, 0)$

$$u_3 := b_3 - \frac{\langle b_3, u_1\rangle}{\langle u_1, u_1\rangle}\cdot u_1 - \frac{\langle b_3, u_2\rangle}{\langle u_2, u_2\rangle}\cdot u_2 =$$

$$\langle b_3, u_1\rangle = \langle (0,1,0,1), (1,1,0,0)\rangle = 1, \quad \langle u_1, u_1\rangle = 2$$

$$\langle b_3, u_2\rangle = \langle (0,1,0,1), (-1,1,2,0)\rangle = 1, \quad \langle u_2, u_2\rangle = 6$$

$$\frac{\langle b_3, u_1\rangle}{\langle u_1, u_1\rangle}\cdot u_1 = \frac{1}{2}\cdot(1,1,0,0)$$

$$\frac{\langle b_3, u_2\rangle}{\langle u_2, u_2\rangle}\cdot u_2 = \frac{1}{6}\cdot(-1,1,2,0)$$

$$= b_3 - \frac{1}{2}u_1 - \frac{1}{6}u_2 = \frac{1}{6}\cdot(6b_3 - 3u_1 - u_2) =$$

$$= \frac{1}{6}\cdot\left(6\cdot(0,1,0,1) - 3\cdot(1,1,0,0) - (-1,1,2,0)\right) =$$

$$= \frac{1}{6}\cdot(-2, 2, -2, 6) \Rightarrow \boxed{u_3 := (-1, 1, -1, 3)}$$

OB W-ben: u_1, u_2, u_3

$$(1,1,0,0), \quad (-1,1,2,0), \quad (-1,1,-1,3)$$

$$\|u_1\| = \sqrt{2} \qquad \|u_2\| = \sqrt{6} \qquad \|u_3\| = \sqrt{12}$$

ONB:

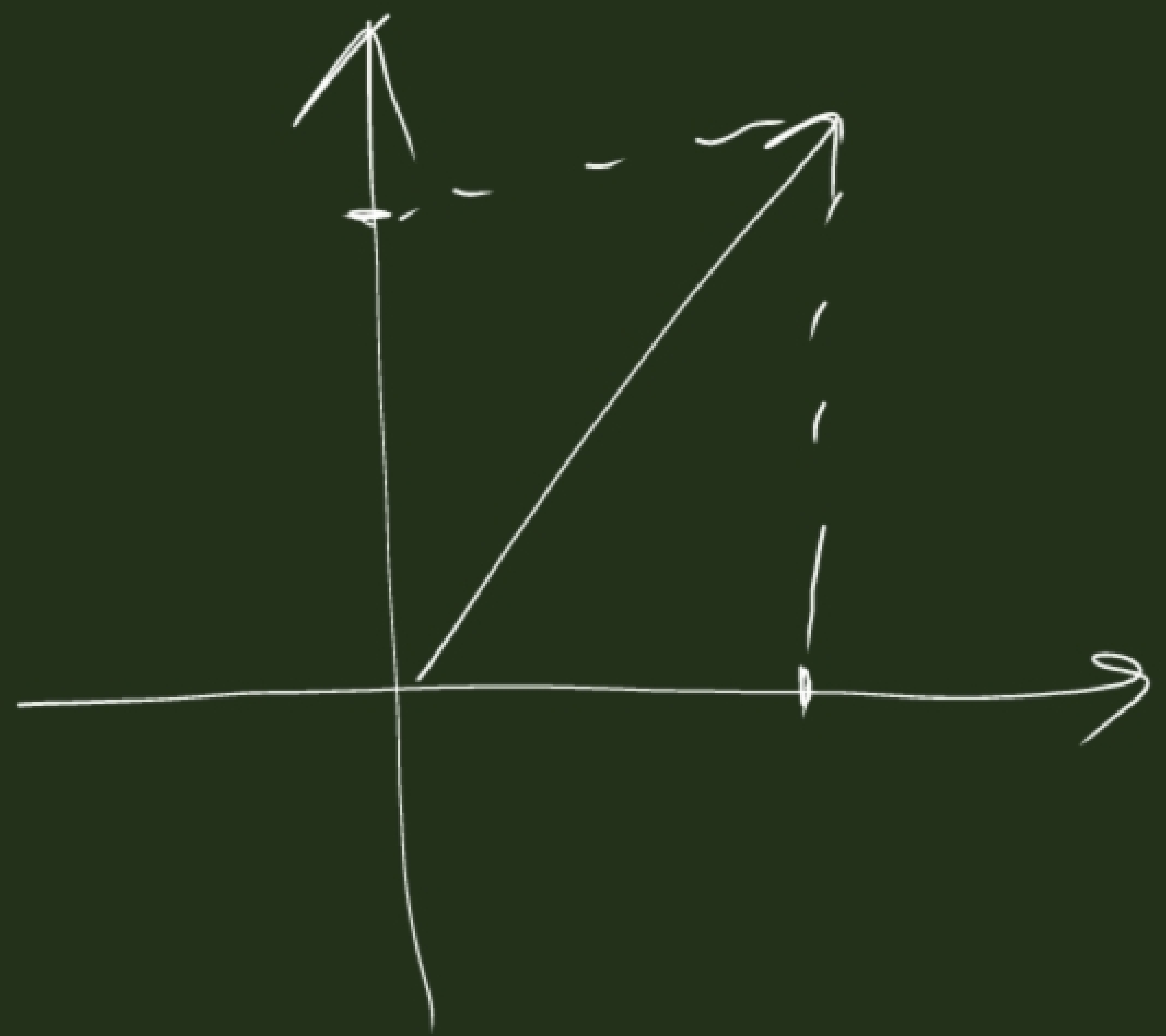
$$e_1 := \frac{u_1}{\|u_1\|} = \frac{1}{\sqrt{2}} \cdot u_1, \quad e_2 := \frac{1}{\sqrt{6}} \cdot u_2, \quad e_3 := \frac{1}{\sqrt{12}} \cdot u_3$$

b, $x = (4,0,4,-4) = P(x) + Q(x)$ (W szerint) (felbontási tétel)

$$P(x) = \frac{\langle x, u_1\rangle}{\langle u_1, u_1\rangle} \cdot u_1 + \frac{\langle x, u_2\rangle}{\langle u_2, u_2\rangle} \cdot u_2 + \frac{\langle x, u_3\rangle}{\langle u_3, u_3\rangle} \cdot u_3$$

$$Q(x) = x - P(x)$$

(u_1, u_2, u_3 a W-ben OB)



Jjm.:

- ortogonális vektorrendszer:

$$x_1, \dots, x_k \text{ OR } \iff \forall i \neq j : \langle x_i, x_j \rangle = 0$$

$$\boxed{\|x\| := \sqrt{\langle x, x \rangle} = \sqrt{x_1^2 + \dots + x_n^2} \qquad x = (x_1, \dots, x_n) \in \mathbb{R}^n}$$

- $x_1, \dots, x_k$ ONR $\iff$ OR és $\forall i : \|x_i\| = 1.$

normálás: $x \in V$

$$x^0 := \frac{x}{\|x\|} \leftarrow x\text{-szel párh.}, \quad \|x^0\| = 1.$$