

14. feladat) Feladatok

1) HF

2) $K := \{ (x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1 \} \subset \mathbb{R}^2$

(geometriailag: )

Altér-e \mathbb{R}^2 -ben?

$(0,0) \notin K \Rightarrow K$ nem altér



$$\underbrace{(-2)}_{\in \mathbb{N}} \cdot \underbrace{\begin{pmatrix} x \\ y \end{pmatrix}}_{\in \mathbb{R}^2} = \underbrace{\begin{pmatrix} -2x & -2y \end{pmatrix}}_{\substack{\text{wr} \\ < 0 \quad \text{wr} \\ < 0}} \notin \mathbb{R}^2$$

3) $N := \{ (x, y) \in \mathbb{R}^2 \mid x \geq 0, y \geq 0 \}$

? skálártartomány: \mathbb{R}_0^+ \rightarrow elbukt?

Tétel \Rightarrow $\begin{matrix} x, y > 0 \\ N \text{ nem altér} \\ \mathbb{R}^2\text{-ben} \end{matrix}$

3/ Alter-e \mathbb{Q}^3 -raum?

a, b, HF

$$c) S_3 := \{(x, y, z) \in \mathbb{Q}^3 \mid 2x - 3y + z = 0\} \quad (\text{Subraum d. } \mathbb{Q}^3)$$

Tipp: Alter

$$a_1, a_2 \in S_3 \quad \left(\begin{array}{l} a_1 = (x_1, y_1, z_1), \quad a_2 = (x_2, y_2, z_2) \\ 2x_1 - 3y_1 + z_1 = 0 \quad 2x_2 - 3y_2 + z_2 = 0 \end{array} \right)$$

$$\textcircled{1} a_1 + a_2 \stackrel{?}{\in} S_3$$

$$a_1 + a_2 = (x_1 + x_2, y_1 + y_2, z_1 + z_2) \stackrel{?}{\in} S_3$$

$$\stackrel{?}{=} 2(x_1 + x_2) - 3(y_1 + y_2) + (z_1 + z_2) = \underbrace{2x_1 - 3y_1 + z_1}_{=0} + \underbrace{2x_2 - 3y_2 + z_2}_{=0} = 0 \quad \checkmark$$

$$\textcircled{2} \lambda \in \mathbb{Q}: \lambda \cdot a_1 \stackrel{?}{\in} S_3$$

$$\lambda \cdot (x_1, y_1, z_1) = (\lambda x_1, \lambda y_1, \lambda z_1) \cdot 2 \cdot \lambda x_1 - 3 \cdot \lambda y_1 + \lambda z_1 = \lambda \underbrace{(2x_1 - 3y_1 + z_1)}_{=0} = 0 \quad \checkmark$$

S_3 alter

$$d, S_4 = \{(x, y, z) \in \mathbb{R}^3 \mid 2x - 3y + z = 5\}$$

Nem akár, mert $(0, 0, 0) \notin S_3$.

$$e, S_5 = \{(x-y, 3x, 2x+y) \in \mathbb{R}^3 \mid x, y \in \mathbb{R}\}$$

$$\begin{pmatrix} x-y \\ x \\ 2x+y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \cdot x + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \cdot y$$

$$\boxed{\text{MF}} \quad x_1, x_2, y_1, y_2 \in \mathbb{R} \quad \overset{?}{\in} S_5$$

$$\left(\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} x_1 + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} y_1 \right) + \left(\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} x_2 + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} y_2 \right)$$

$$\lambda \left(\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} x_1 + \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} y_1 \right) \overset{?}{\in} S_5$$