1/a, $A = \begin{bmatrix} 2 & -1 & -1 \\ 3 & -2 & -3 \\ -1 & 1 & 2 \end{bmatrix}$

1. SÉ:

$$0 = \det(A - \lambda I) = \det \begin{bmatrix} {}^{+}2-\lambda & -1 & -1 \\ {}^{-}3 & -2-\lambda & -3 \\ {}^{+}-1 & 1 & 2-\lambda \end{bmatrix} =$$

1. oszlop

$$\downarrow$$

$$= (2-\lambda)\cdot \det \begin{bmatrix} -2-\lambda & -3 \\ 1 & 2-\lambda \end{bmatrix} - 3 \cdot \det \begin{bmatrix} -1 & -1 \\ 1 & 2-\lambda \end{bmatrix} - 1 \cdot \det \begin{bmatrix} -1 & -1 \\ -2-\lambda & -3 \end{bmatrix}$$

$$= (2-\lambda)\cdot\Big(\underbrace{(-2-\lambda)(2-\lambda)}_{-(2+\lambda)(2-\lambda) = -(4-\lambda^2)} + 3\Big) - 3\cdot\big(-(2-\lambda)+1\big) - \big(3 + (-2-\lambda)\big) =$$

$$= -(2-\lambda)(4-\lambda^2) + 3(2-\lambda) +$$

$$+3(2-\lambda) - 3$$

$$-3 + (2+\lambda)$$

$$= -(2-\lambda)(4-\lambda^2) + \overbrace{6(2-\lambda) - 6}^{6(2-\lambda-1) = 6(1-\lambda)} + (2+\lambda) =$$

$$= -(2-\lambda)(4-\lambda^2) + 6(1-\lambda) + 2 + \lambda =$$

$$= -8 + 2\lambda^2 + 4\lambda - \lambda^3 + 6 - 6\lambda + 2 + \lambda =$$

$$= -\lambda^3 + 2\lambda^2 - \lambda$$

$$p(\lambda) = \boxed{-\lambda^3 + 2\lambda^2 - \lambda = 0}$$

$$-\lambda(\lambda^2 - 2\lambda + 1) = 0$$

$$-\lambda(\lambda - 1)^2 = 0$$

$$\lambda = 0 \qquad \lambda = 1$$

$$a(0) = 1 \qquad a(1) = 2$$

0 sajátérték

algebrai mult.

$\lambda = 0$-hoz tart. sajátvektorok:

$$(A - 0 \cdot I)x = 0$$

$$Ax = 0$$

- GJ
- behelyettesítés

mo.: $x_3 = -x_1$, $x_2 = 3x_1$, $x_1 \in \mathbb{R}$

$$\rightarrow \text{so.: } x = \begin{bmatrix} x_1 \\ 3x_1 \\ -x_1 \end{bmatrix} = x_1 \cdot \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} \quad (x_1 \in \mathbb{R} \setminus \{0\})$$

$$W_0 = \left\{ x_1 \cdot \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} \;\middle|\; x_1 \in \mathbb{R} \right\} = \operatorname{Span}\left(\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} \right)$$

$$\dim(W_0) = 1 = g(0)$$

$\lambda = 1$-hez sv:

$$(A - 1 \cdot I)x = 0$$

$$\left(\begin{bmatrix} 2 & -1 & -1 \\ 3 & -2 & -3 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \cdot x = 0$$

$$\begin{bmatrix} 1 & -1 & -1 \\ 3 & -3 & -3 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \xrightarrow{\text{HF}} \text{mo:}$$

$$x_1 = x_2 + x_3$$

$$x_2, x_3 \in \mathbb{R}$$

Sajátvektorok: $\left\{ \begin{pmatrix} x_2 + x_3 \\ x_2 \\ x_3 \end{pmatrix} \;\middle|\; x_2, x_3 \in \mathbb{R} \right\} \setminus \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$

sajátérték

$$W_1 = \mathrm{Span}\left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right) \Rightarrow g(1) = 2$$

$$\begin{pmatrix} x_2 + x_3 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_2 \\ 0 \end{pmatrix} + \begin{pmatrix} x_3 \\ 0 \\ x_3 \end{pmatrix} = x_2 \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + x_3 \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

Sajátbázis:

$\lambda = 0 \quad a(0) = 1 = g(0) = 1$

$\lambda = 1 \quad a(1) = 2 = g(1) = 2$

$+ \quad 3$

$\Rightarrow \exists \; SB$

$$\underbrace{\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}}_{W_0}, \underbrace{\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}_{W_1}$$