$$W := \{(x, y, z) \in \mathbb{R}^3 \mid \boxed{x \cdot y = z}\} \subset \mathbb{R}^3$$

nem altér

$((0,0,0) \in W \quad \checkmark)$

$a = (x_1, y_1, z_1) \in W$

$b = (x_2, y_2, z_2) \in W$

$a + b \in W$?, $\forall \lambda \in \mathbb{R}$ $\lambda \cdot a \in W$?

$(x_1 + x_2)(y_1 + y_2) \overset{?}{=} z_1 + z_2$

$\Uparrow$

$x_1 y_1 = z_1$ és $x_2 y_2 = z_2$

$a \in W$

$\Updownarrow$

$x_1 \cdot y_1 = z_1$

$\lambda \cdot a = (\lambda x_1, \lambda y_1, \lambda z_1) \in W$

$\Updownarrow$

$\lambda \cdot x_1 \cdot \lambda \cdot y_1 = \lambda z_1$

$\lambda^2 x_1 y_1 = \lambda z_1 \quad (\lambda \neq 0)$

$\Updownarrow$

$\Longrightarrow$ $\forall \lambda \in \mathbb{R}$: $\lambda \cdot x_1 y_1 = z_1$

nem igaz $\Rightarrow$ W nem altér

$\{(0,0,0)\} \subset \mathbb{R}^3$ altér?

Igen $\left(\text{Triviális altér: } \mathbb{R}^3, \{(0,0,0)\} \right)$

$V, \{0_V\}$

$W = \{ (x,y) \in \mathbb{R}^2 \mid x \geq 0, y \geq 0 \}$

$\lambda \cdot (x,y) = (\lambda x, \lambda y) \in W \iff \lambda \geq 0$

$(1,2) \in W$

$(-2) \cdot (1,2) = (-2,-4) \notin W$

$\mathbb{K}^{m \times n}$

$$\begin{array}{cc} & n \\ m & \begin{bmatrix} \quad & \quad & \quad \\ & & \\ & & \end{bmatrix} \end{array}$$

$$\dim(\mathbb{K}^{m \times n}) \stackrel{?}{=} \begin{array}{l} m \\ m \cdot n \\ n \\ m+n \\ 2 \end{array} \qquad \cdots\cdots$$

$$\mathrm{Span}(x_1, \dots, x_k) = \left\{ \sum_{i=1}^{k} \lambda_i x_i \;\middle|\; \lambda_i \in \mathbb{R} \right\}$$

2) $u = (1, 2, -1) \in \mathbb{R}^3, \quad v = (6, 4, 2) \in \mathbb{R}^3$

a) $-2u + 3v = (-2 \cdot 1 + 3 \cdot 6, \; -2 \cdot 2 + 3 \cdot 4, \; (-2) \cdot (-1) + 3 \cdot 2) =$

$= (16, 8, 8)$

b) $\mathrm{Span}(u, v) = \{ \lambda \cdot u + \eta \cdot v \in \mathbb{R}^3 \mid \lambda, \eta \in \mathbb{R} \}$

$= \{ \lambda \cdot (1, 2, -1) + \eta \cdot (6, 4, 2) \mid \lambda, \eta \in \mathbb{R} \} =$

$= \{ (\lambda + 6\eta, \; 2\lambda + 4\eta, \; -\lambda + 2\eta) \mid \lambda, \eta \in \mathbb{R} \}$

c) $x = (9, 2, 7) \overset{?}{\in} \mathrm{Span}(u, v)$

c, $X = (9,2,7) \in \{(\lambda + 6\eta, 2\lambda + 4\eta, -\lambda + 2\eta) \mid \lambda, \eta \in \mathbb{R}\}$

$\lambda, \eta = ?$

$$\begin{cases} \lambda + 6\eta = 9 \\ 2\lambda + 4\eta = 2 \\ -\lambda + 2\eta = 7 \end{cases} \xrightarrow{HF} \begin{array}{l} \lambda = -3 \\ \eta = 2 \end{array}$$

d, HF

3/ a) $S_5 = \{ (x-y, 3x, 2x+y) \in \mathbb{R}^3 \mid x, y \in \mathbb{R} \} \subseteq \mathbb{R}^3$

altér

$(x^2, 2x+y, 3x)$

$S_5 = \text{Span}(\dots)$?

$$\begin{pmatrix} x-y \\ 3x \\ 2x+y \end{pmatrix} = x \cdot \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + y \cdot \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \Longrightarrow S_5 = \text{Span}\{(1,3,2), (-1,0,1)\}$$

$\Downarrow$

S_5 altér

b) $S_3 = \{(x,y,z) \in \mathbb{R}^3 \mid 2x - 3y + z = 0\}$

$S_3 = \text{Span}(\dots\dots)$?

$$2x - 3y + z = 0 \quad \Rightarrow \quad z = -2x + 3y$$

$$(x,y,z) = \underbrace{(x,y,-2x+3y)}_{x,y \in \mathbb{R}}$$

$$S_3 = \{(x,y,-2x+3y) \mid x,y \in \mathbb{R}\} = \text{Span}\left((1,0,-2),(0,1,3)\right\}$$

$$\begin{pmatrix} x \\ y \\ -2x+3y \end{pmatrix} = x \cdot \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} + y \cdot \begin{pmatrix} 0 \\ 1 \\ 3 \end{pmatrix}$$

4) a) $W_1 = \left\{ (x,y,z) \in \mathbb{R}^3 \;\middle|\; \underbrace{\begin{bmatrix} 2 & -3 & 5 \end{bmatrix} \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0}_{} \right\} \overset{?}{=} \operatorname{Span}(\ldots)$

$$2x - 3y + 5z = 0$$

$$\rightarrow y = \frac{2}{3}x + \frac{5}{3}z$$

$(x,y,z) = \left(x, \frac{2}{3}x + \frac{5}{3}z, z\right) \quad (x,z \in \mathbb{R})$

$\rightarrow W_1 = \operatorname{Span}\left(\left(1, \frac{2}{3}, 0\right), \left(0, \frac{5}{3}, 1\right)\right)$

↳ HF!!!!