$px^2 - x + 1$ pontosan két gyök

$D = 1 - 4p > 0$

$p < \frac{1}{4}$

$p \neq 0$

$D = 1 - 0 = 1 > 0$

$x_{1,2} = \frac{-b \pm \sqrt{D}}{2 \cdot a}$

3. fejezet: algebrai átalakítások,
abszolútértékes feladatok
gyökös feladatok

$\boxed{3/1}$ b,
$$\frac{x^4+5x^2+4}{x^4-16} = \frac{y^2+5y+4}{y^2-16} = \frac{\cancel{(y+4)}(y+1)}{(y-4)\cancel{(y+4)}} = \frac{y+1}{y-4} =$$

$(y-(-4))$

$\pm 4$

$x^2 = y$

$$= \frac{x^2+1}{x^2-4}$$

$$y^2+5y+4 = \underset{y-(-1)}{(y+1)}\underset{y-(-4)}{(y+4)}$$

$$y_{1,2} = \frac{-5 \pm \sqrt{25-16}}{2} \begin{cases} -1 \\ -4 \end{cases}$$

$$y^2+5y+4 = (y+1)(y+4)$$

c) $$\frac{2x^2-13x-7}{\underset{(2x)^3+1^3}{8x^3+1}} = \frac{\cancel{(2x+1)}(x-7)}{\cancel{(2x+1)}(4x^2-2x+1)} = \frac{x-7}{4x^2-2x+1}$$

$$4\cdot 7^2-2\cdot 7+1 \cong 183$$

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

$$D=4-16<0 \ !$$

e) $$\frac{2}{x^2-1}-\frac{3}{x^3-1} = \frac{2(x^2+x+1)-3(x+1)}{(x-1)(x+1)(x^2+x+1)} = \frac{2x^2-x-1}{(x-1)(x+1)(x^2+x+1)} = \frac{\cancel{(x-1)}\overbrace{(2x+1)}^{2\left(x+\frac{1}{2}\right)}}{\cancel{(x-1)}(x+1)\underbrace{(x^2+x+1)}_{D<0}}$$

$$x^2-1=(x-1)(x+1)$$

$$x^3-1=(x-1)(x^2+x+1)$$

2) a) $$\frac{\sqrt{x^2+1}-\sqrt{2}}{x^3-1}\cdot\frac{\sqrt{x^2+1}+\sqrt{2}}{\sqrt{x^2+1}+\sqrt{2}} =$$

$$\left((\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b}) = a-b\right)$$

$$= \frac{x^2+1-2}{(x^3-1)(\sqrt{x^2+1}+\sqrt{2})} = \frac{\overbrace{x^2-1}^{=(x-1)(x+1)}}{\underbrace{(x^3-1)}_{(x-1)(\dots)}(\sqrt{x^2+1}+\sqrt{2})} = \dots$$

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$$b)\ \frac{x^2+x-6}{\underbrace{\sqrt{\sqrt{x}-\sqrt{2}+1}}_{\sqrt{a}}-\underbrace{1}_{\sqrt{b}}}\cdot\frac{\sqrt{\sqrt{x}-\sqrt{2}+1}+1}{\sqrt{\sqrt{x}-\sqrt{2}+1}+1}=$$

$$c)\ \frac{x^2-64}{\sqrt[3]{x}-2}$$

$$=\frac{(x^2+x-6)\left(\sqrt{\sqrt{x}-\sqrt{2}+1}+1\right)}{\underbrace{\sqrt{x}-\sqrt{2}+\cancel{1}}_{a}-\underbrace{\cancel{1}}_{b}}=\frac{(x^2+x-6)\cdot\left(\sqrt{\sqrt{x}-\sqrt{2}+1}+1\right)}{\sqrt{x}-\sqrt{2}}\cdot\frac{\sqrt{x}+\sqrt{2}}{\sqrt{x}+\sqrt{2}}=$$

$$=\frac{(x^2+x-6)\left(\sqrt{\sqrt{x}-\sqrt{2}+1}+1\right)\cdot\left(\sqrt{x}+\sqrt{2}\right)}{x-2}=$$

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c, $\dfrac{x^2-64}{\sqrt[3]{x}-2} \cdot \dfrac{\sqrt[3]{x^2}+\sqrt[3]{8x}+(\sqrt[3]{8})^2}{\sqrt[3]{x^2}+\sqrt[3]{8x}+(\sqrt[3]{8})^2} =$

$2 = \sqrt[3]{8}$

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

$a=\sqrt[3]{x}, \quad b=\sqrt[3]{y}$

$$x-y=(\sqrt[3]{x}-\sqrt[3]{y})(\sqrt[3]{x^2}+\sqrt[3]{xy}+\sqrt[3]{y^2})$$

$$= \frac{(x^2-64)\cdot(\sqrt[3]{x^2}+\sqrt[3]{8x}+4)}{x-8} = \ldots$$

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Abszolútértékes feladatok

3/b, $|2x-7|+|2x+7| = x+15$

$$|2x-7| = \begin{cases} -(2x-7) & (2x-7<0) \\ 2x-7 & (2x-7 \geq 0) \end{cases} = \begin{cases} -2x+7 & \left(x<\frac{7}{2}\right) \\ 2x-7 & \left(x \geq \frac{7}{2}\right) \end{cases}$$

$$|2x+7| = \begin{cases} -2x-7 & \left(x<-\frac{7}{2}\right) \\ 2x+7 & \left(x \geq -\frac{7}{2}\right) \end{cases}$$

Esetszétválasztás:

I. II. III.

$-\frac{7}{2}$ $\frac{7}{2}$

I. $x < -\frac{7}{2} \iff x \in \left(-\infty, -\frac{7}{2}\right) =: H_1$

$$-2x+\cancel{7}-2x-\cancel{7} = x+15$$

$$-5x = 15$$

$$x = -3 \notin H_1$$

II. $x \in \left[-\frac{7}{2}, \frac{7}{2}\right) =: H_2$

$$-\cancel{2x}+7+\cancel{2x}+7 = x+15 \longrightarrow x = -1 \in H_2$$

III. $x \in \left[\frac{7}{2}, +\infty\right) =: H_3$

$$2x-\cancel{7}+2x+\cancel{7} = x+15 \longrightarrow x = 5 \in H_3$$

Válasz:

$$M = \{-1, 5\}$$

3/e) $|x^2-9|+|x^2-4|=5$ | esetszétválasztás: HF

$y := x^2 \quad (y \geq 0)$

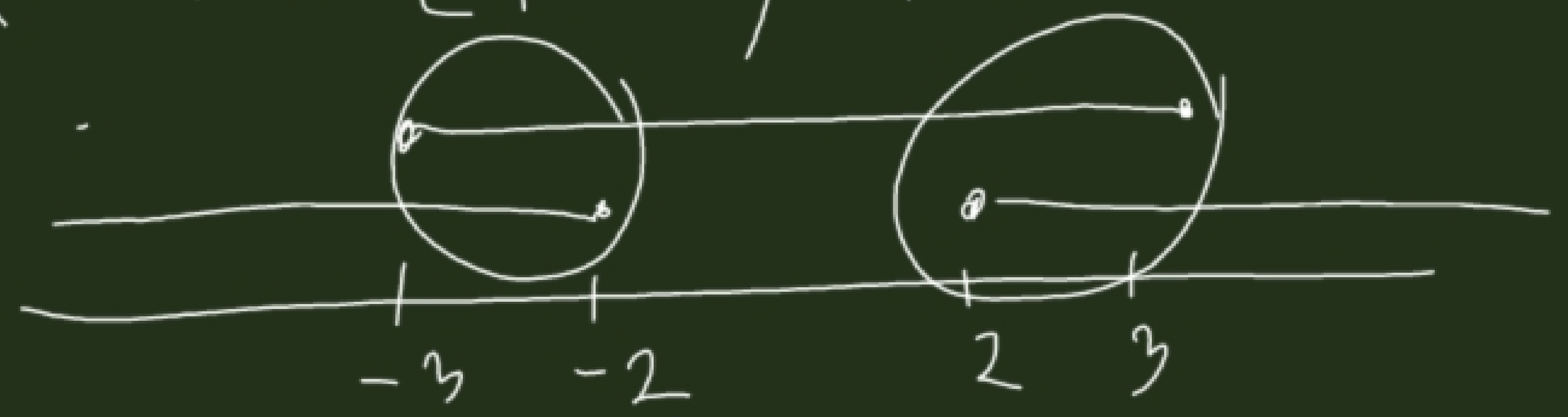
$|y-9|+|y-4|=5 \xrightarrow[\text{esetválasztás}]{\text{HF}} y \in [4,9]$

$x^2 \in [4,9]$

$x^2 \geq 4$ és $x^2 \leq 9$

$x \in (-\infty,-2] \cup [2,+\infty) \qquad x \in [-3,3]$

$M = \left((-\infty,-2] \cup [2,+\infty)\right) \cap [-3,3] = [-3,-2] \cup [2,3]$



3/f $\quad \underbrace{|x-2|}_{\geq 0} < \underbrace{3}_{\geq 0}$

négyzetre emelés:

$$\left(|x-2|\right)^2 < 9$$

$$x^2-4x+4 < 9$$

$$x^2-4x-5 < 0$$

$$(x-5)(x+1) < 0$$

$\rightarrow$ gyökök: $5, -1$

$M: x \in (-1, 5)$

HF: esetszétválasztás

$|x-2| < 3 \Leftrightarrow -3 < x-2 < 3$

3/g HF

$\underbrace{|2x-1|}_{\geq 0} < \underbrace{|x-1|}_{\geq 0}$

Gyökös feladatok

6/a, $\sqrt{x+1} - \sqrt{9-x} = \sqrt{2x-12}$

kikötés:

$x+1 \geq 0 \quad : x \geq -1$

$9-x \geq 0 \quad x \leq 9$

$2x-12 \geq 0 \quad x \geq 6$

$\rightarrow H := [6, 9]$

$$\underbrace{\sqrt{x+1}}_{\geq 0} = \underbrace{\sqrt{9-x} + \sqrt{2x-12}}_{\geq 0} \quad / (\cdot)^2$$

$$\cancel{x} + 1 = 9 - \cancel{x} + \cancel{2}x - 12 + 2 \cdot \sqrt{(9-x)(2x-12)}$$

$$\underbrace{2\cancel{4}}_{\geq 0} = \cancel{2} \cdot \underbrace{\sqrt{(9-x)(2x-12)}}_{\geq 0} \quad / (\cdot)^2$$

$$2x^2 - 30x + 112 = 0$$

$$x^2 - 15x + 56 = 0$$

$$(x-7)(x-8) = 0$$

$$x = 7, 8 \in \mathbb{N} \checkmark$$

$$M := \{7; 8\}$$

$\boxed{6/B}$ $\underbrace{\sqrt{x^2+4x}}_{\geq 0} > 2-x$

kik.:

$x^2+4x = x(x+4) \geq 0$

$x \in \underbrace{(-\infty,-4] \cup [0,+\infty)}_{=:H}$


Esetszétválasztás:

I. $2-x<0$ $\left(x \in \underbrace{(2,+\infty)}_{=:H_1}\right)$

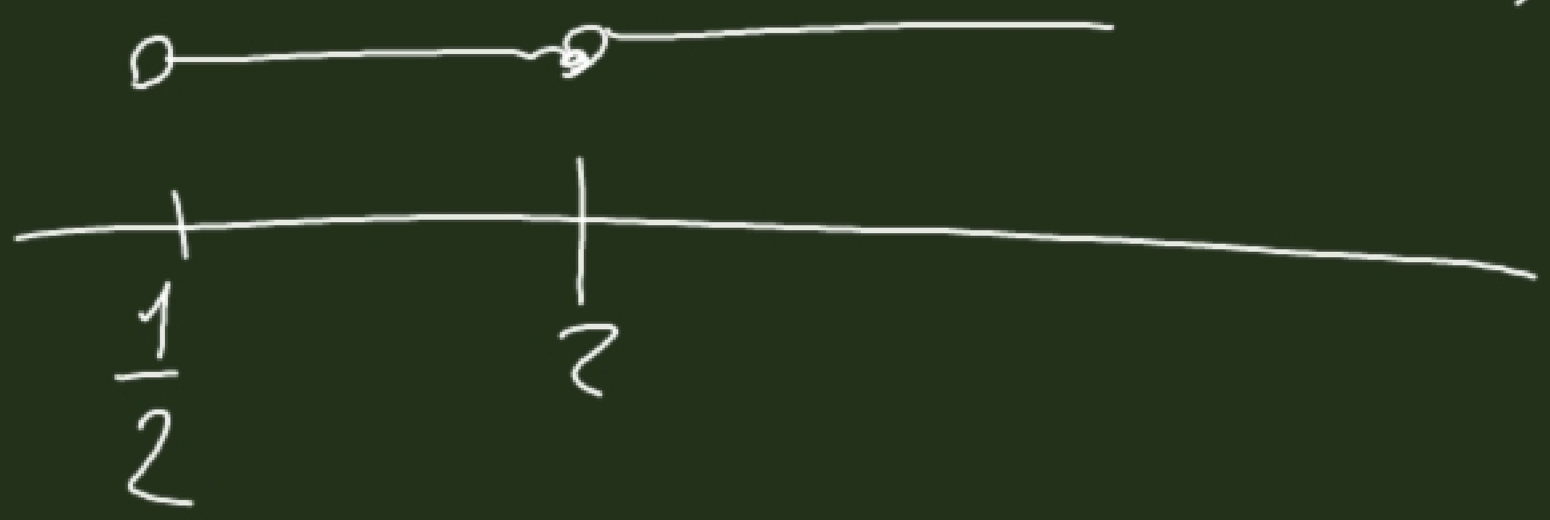
$\underbrace{\sqrt{x^2+4x}}_{\geq 0} > \underbrace{2-x}_{<0}$ $\forall x \in H_1$-re igaz

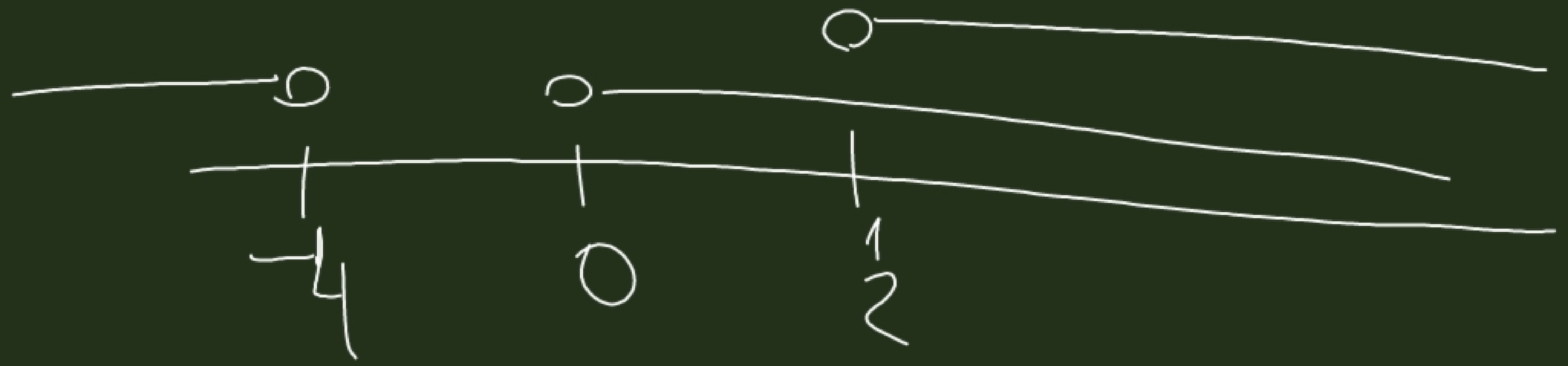
II. $x \in (-\infty, 2] =: H_2$

$\underbrace{\sqrt{x^2+4x}}_{\geq 0} > \underbrace{2-x}_{\geq 0} \xrightarrow[\text{HF}]{\text{négyzetre em.}} x \in \left(\frac{1}{2},+\infty\right) \cap H_2 = \left(\frac{1}{2}, 2\right]$

$$\left(H_1 \cup \left(\tfrac{1}{2}, 2\right]\right) \cap H =$$

$$= \left((2,+\infty) \cup \left(\tfrac{1}{2}, 2\right]\right) \cap \left((-\infty,-4) \cup (0,+\infty)\right) = \underline{\underline{\left(\tfrac{1}{2},+\infty\right)}}$$




6/n

HF