18. fejezet, feladatok

méret: 3 egyenlet, 3 ism.
(3 x 3-as)

1/a/
$$\begin{cases} \phantom{4x_1+{}} x_2 - 3x_3 = -5 \\ 4x_1 + 5x_2 - 2x_3 = 10 \\ 2x_1 + 3x_2 - x_3 = 7 \end{cases}$$

$$A = \begin{bmatrix} 0 & 1 & -3 \\ 4 & 5 & -2 \\ 2 & 3 & -1 \end{bmatrix} \qquad b = \begin{bmatrix} -5 \\ 10 \\ 7 \end{bmatrix}$$

Gauss-Jordan módszer

$$\begin{array}{ccc|c} x_1 & x_2 & x_3 & b \\ \hline 0 & \boxed{1} & -3 & -5 \\ 4 & 5 & -2 & 10 \\ 2 & 3 & -1 & 7 \end{array} \xrightarrow[(3)-3\cdot(1)]{(2)-5\cdot(1)} \begin{array}{ccc|c} 0 & \boxed{1} & -3 & -5 \\ 4 & 0 & 13 & 35 \\ 2 & 0 & 8 & 22 \end{array}$$

$$\begin{array}{ccc|c} 0 & \underline{1} & -3 & -5 \\ 4 & 0 & 13 & 35 \\ \boxed{2} & 0 & 8 & 22 \end{array} \xrightarrow{(3):2} \begin{array}{ccc|c} 0 & \underline{1} & -3 & -5 \\ 4 & 0 & 13 & 35 \\ \boxed{1} & 0 & 4 & 11 \end{array} \xrightarrow{(2)-4\cdot(3)} \begin{array}{ccc|c} 0 & \underline{1} & -3 & -5 \\ 0 & 0 & -3 & -9 \\ \boxed{1} & 0 & 4 & 11 \end{array}$$

$$\left[\begin{array}{ccc|c} 0 & 1 & -3 & -5 \\ 0 & 0 & \boxed{-3} & -9 \\ 1 & 0 & 4 & 11 \end{array}\right] \xrightarrow{(2):-3} \left[\begin{array}{ccc|c} 0 & 1 & -3 & -5 \\ 0 & 0 & \boxed{1} & 3 \\ 1 & 0 & 4 & 11 \end{array}\right]$$

jelölt elemek száma

$$r = 3$$

$$\Downarrow$$

$$\exists! \text{ mo.}$$

$$\xrightarrow[(3)-4\cdot(2)]{(1)+3\cdot(2)} \left[\begin{array}{ccc|c} 0 & 1 & 0 & 4 \\ 0 & 0 & \boxed{1} & 3 \\ 1 & 0 & 0 & -1 \end{array}\right] \longrightarrow \text{STOP}$$

$$
\begin{array}{ccc|c}
x_1 & x_2 & x_3 & \\
0 & 1 & 0 & 4 \\
0 & 0 & 1 & 3 \\
1 & 0 & 0 & -1
\end{array}
\longrightarrow
\begin{array}{l}
x_2 = 4 \\
x_3 = 3 \\
x_1 = -1
\end{array}
$$

kötött ism.: x_1, x_2, x_3

szabad ism.: $\emptyset$

c, $\text{rang}(A) = 3$

d, $X = \begin{pmatrix} -1 \\ 4 \\ 3 \end{pmatrix}$,

e, $M = \{(-1, 4, 3)\}$,

f, hom. egy. $\quad A \cdot X = 0 \quad (\text{rang}(A) = 3 \Rightarrow \exists!\ \text{mo.})$

$\rightarrow$ egyetlen mo.: $x_1 = 0,\ x_2 = 0,\ x_3 = 0$

vektoros alak: $X = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$M_h = \{(0,0,0)\} = \text{Ker}(A)$, $\dim M_h = 0$

1/b)
$$\begin{cases} -3x_1 + x_2 + x_3 - x_4 - 2x_5 = 2 \\ 2x_1 - x_2 + x_5 = 0 \\ -x_1 + x_2 + 2x_3 + x_4 - x_5 = 8 \\ x_2 + x_3 + 2x_4 = 6 \end{cases}$$

a, méret: 4×5, $A = \begin{bmatrix} -3 & 1 & 1 & -1 & -2 \\ 2 & -1 & 0 & 0 & 1 \\ -1 & 1 & 2 & 1 & -1 \\ 0 & 1 & 1 & 2 & 0 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ 0 \\ 8 \\ 6 \end{bmatrix}$

8, Gf

$$\left(\begin{array}{ccccc|c} -3 & 1 & \boxed{1} & -1 & -2 & 2 \\ 2 & -1 & 0 & 0 & 1 & 0 \\ -1 & 1 & 2 & 1 & -1 & 8 \\ 0 & 1 & 1 & 2 & 0 & 6 \end{array}\right) \xrightarrow[(4)-(1)]{(3)-2\cdot(1)} \left(\begin{array}{ccccc|c} -3 & 1 & \boxed{1} & -1 & -2 & 2 \\ 2 & -1 & 0 & 0 & \boxed{1} & 0 \\ 5 & -1 & 0 & 3 & 3 & 4 \\ 3 & 0 & 0 & 3 & 2 & 4 \end{array}\right)$$

$$\xrightarrow[(4)-2\cdot(2)]{\substack{(1)+2\cdot(2)\\(3)-3\cdot(2)}} \left(\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 0 & 2 \\ 2 & -1 & 0 & 0 & \boxed{1} & 0 \\ \boxed{-1} & 2 & 0 & 3 & 0 & 4 \\ -1 & 2 & 0 & 3 & 0 & 4 \end{array}\right) \xrightarrow{(3):-1}$$

$$\left(\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 0 & 2 \\ 2 & -1 & 0 & 0 & 1 & 0 \\ \boxed{1} & -2 & 0 & -3 & 0 & -4 \\ -1 & 2 & 0 & 3 & 0 & 4 \end{array}\right) \xrightarrow[\substack{(2)-2\cdot(3) \\ (4)+(3)}]{(1)-(3)} \begin{array}{cccccc} x_1 & x_2 & x_3 & x_4 & x_5 & \\ 0 & 1 & 1 & 2 & 0 & \vert\; 6 \\ 0 & 3 & 0 & 6 & 1 & \vert\; 8 \\ 1 & -2 & 0 & -3 & 0 & \vert\; -4 \\ 0 & 0 & 0 & 0 & 0 & \vert\; 0 \end{array}$$

$r = 3$

$$\begin{cases} x_2 + x_3 + 2x_4 = 6 \\ 3x_2 + 6x_4 + x_5 = 8 \\ x_1 - 2x_2 - 3x_4 = -4 \end{cases} \longrightarrow \begin{array}{l} x_3 = 6 - x_2 - 2x_4 \\ x_5 = 8 - 3x_2 - 6x_4 \\ x_1 = -4 + 2x_2 + 3x_4 \end{array}$$

$x_2, x_4 \in \mathbb{R}$

kötött ism.: x_1, x_3, x_5

szabad ism.: x_2, x_4

c) $\operatorname{rang}(A) = 3$

d)
$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} -4 + 2x_2 + 3x_4 \\ x_2 \\ 6 - x_2 - 2x_4 \\ x_4 \\ 8 - 3x_2 - 6x_4 \end{pmatrix} = \begin{array}{c} x_B \\ \shortparallel \\ \begin{pmatrix} -4 \\ 0 \\ 6 \\ 0 \\ 8 \end{pmatrix} \end{array} + x_2 \cdot \begin{array}{c} v_1 \\ \shortparallel \\ \begin{pmatrix} 2 \\ 1 \\ -1 \\ 0 \\ -3 \end{pmatrix} \end{array} + x_4 \cdot \begin{array}{c} v_2 \\ \shortparallel \\ \begin{pmatrix} 3 \\ 0 \\ -2 \\ 1 \\ -6 \end{pmatrix} \end{array}$$

$x_2, x_4 \in \mathbb{Q}$

e) $\mathcal{M} = \{ x_B + x_2 \cdot v_1 + x_4 \cdot v_2 \mid x_2, x_4 \in \mathbb{Q} \}$

f, $\underline{Ax = 0}$ megoldásai:

$$\mathcal{M}_h = \{ x_2 \cdot v_1 + x_4 \cdot v_2 \mid x_2, x_4 \in \mathbb{R} \} = \mathrm{Span}(v_1, v_2)$$

$$\dim(\mathcal{M}_h) = 2$$

c, $\begin{cases} 2x_1 + 3x_2 - x_3 + 2x_4 = -1 \\ x_1 + 4x_2 - 4x_3 + 3x_4 = 2 \\ 4x_1 + x_2 + 5x_3 = 1 \end{cases}$

a, méret: 3×4

$A =$

$b =$

b, GJ:

$$\left(\begin{array}{cccc|c} -5 & 0 & -8 & 1 & -2 \\ 0 & 0 & 0 & 0 & 4 \\ 4 & 1 & 5 & 0 & 1 \end{array}\right)$$

?

$\longrightarrow \nexists$ mo.

$$0 \cdot x_1 + 0 x_2 + 0 x_3 + 0 x_4 = 4$$

$$M = \emptyset, \quad M_h = \{ x_1 \underbrace{(1\ -4\ 0\ 5)}_{=: v_1} + x_3 \underbrace{(0\ -5\ 1\ 8)}_{=: v_3} \mid x_1, x_3 \in \mathbb{R} \}$$

$\dim(M_h) = 2$, M_h bázisa: v_1, v_3