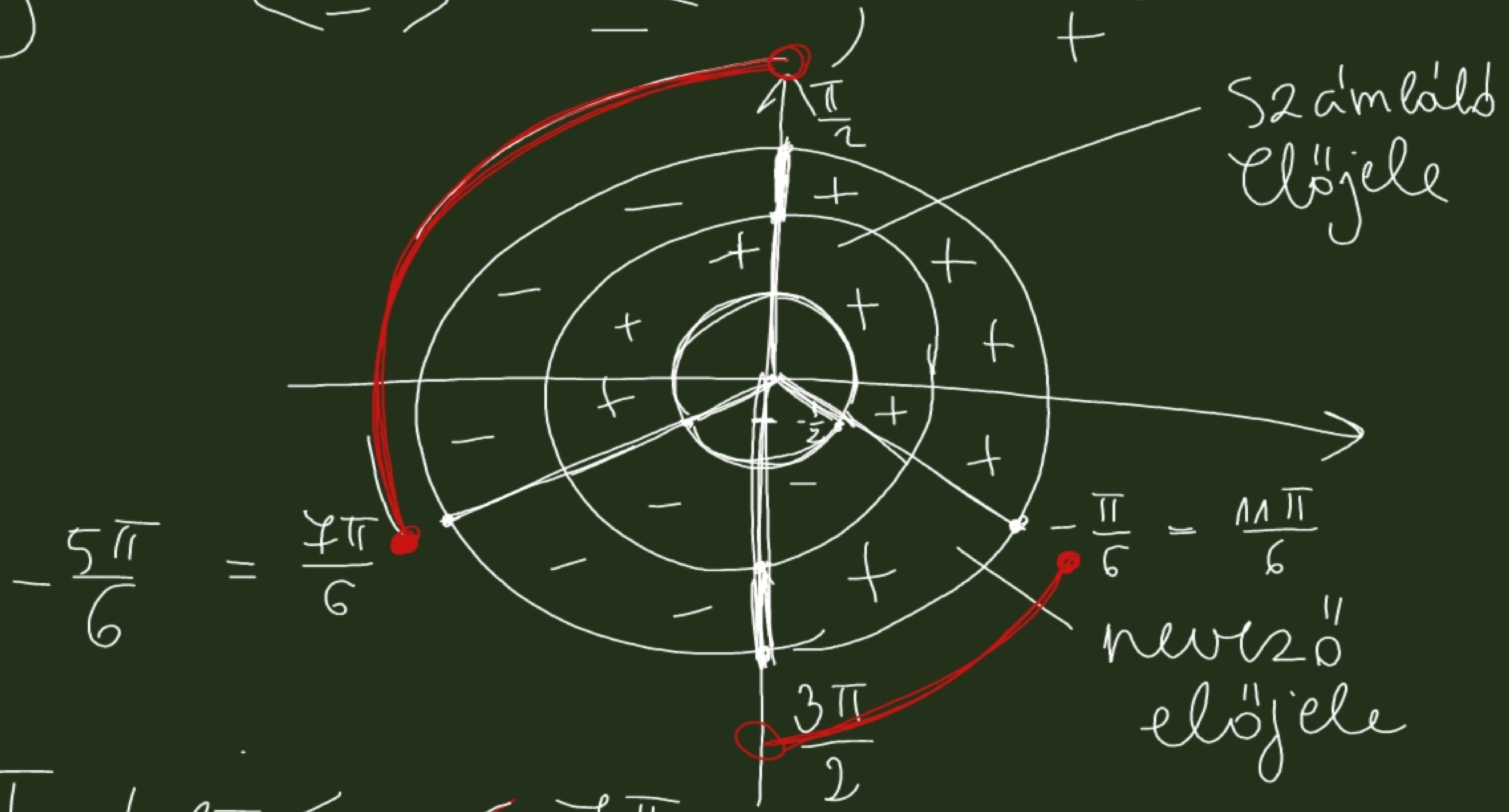
5. feladatsor, $\boxed{7/c}$ Kikötés: $\cos(x) \neq 0 \iff \frac{\pi}{2} + k\cdot\pi \neq x$

$$\frac{2\cdot\sin(x)+1}{2\cdot\cos(x)} \leq 0 \iff \frac{+/0}{-}\ ,\quad \frac{-\;0}{+}$$

$$2\cdot\sin(x)+1 \geq 0$$

$$\sin(x) \geq -\frac{1}{2}$$

$$\cos(x) > 0$$



megoldás: $\frac{\pi}{2} + k\cdot 2\pi < x \leq \frac{7\pi}{6} + k\cdot 2\pi$

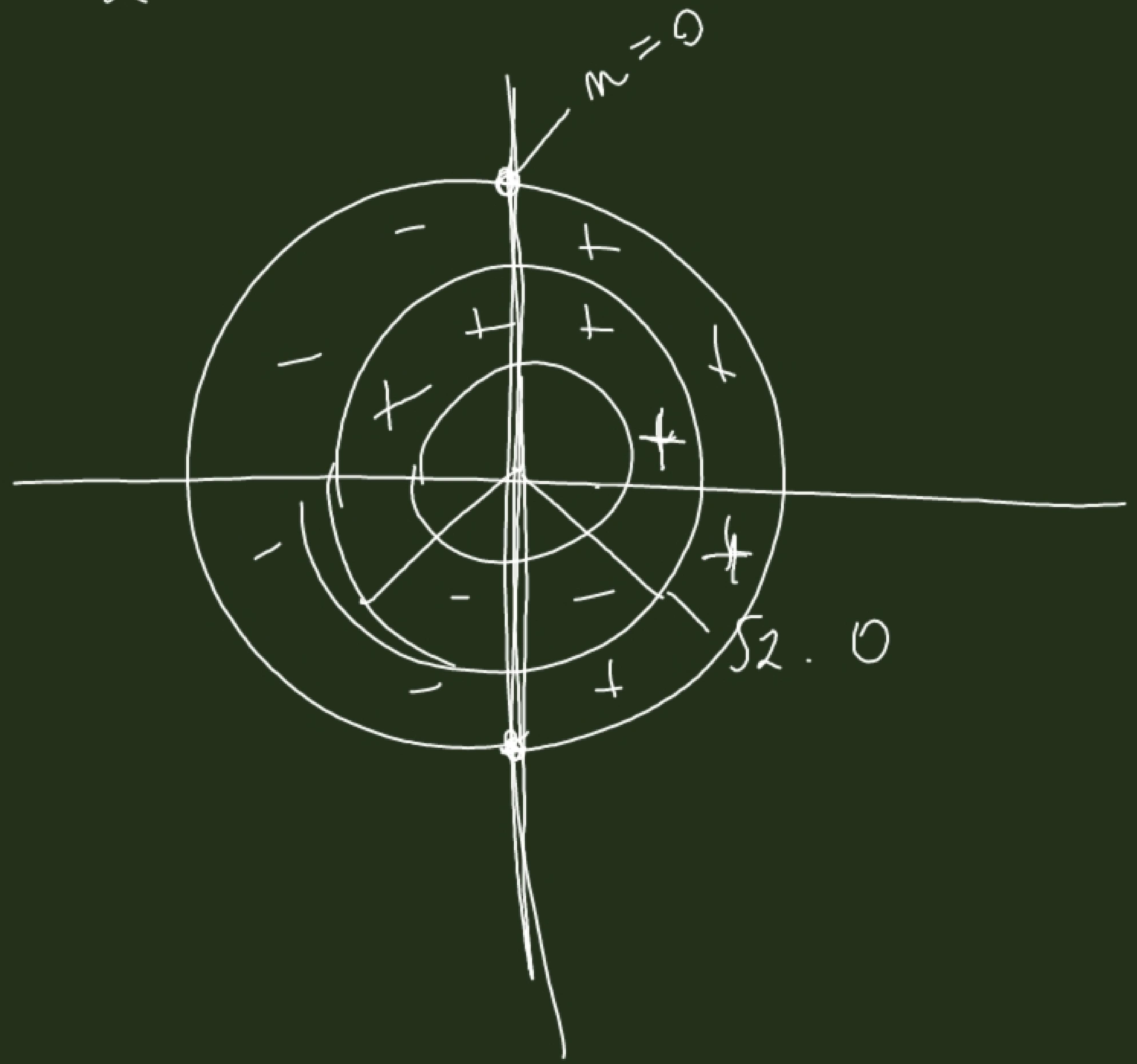
vagy: $\frac{3\pi}{2} + k\cdot 2\pi < x \leq \frac{11\pi}{6} + k\cdot 2\pi$ $\quad (k \in \mathbb{Z})$

(P, Q pol.)

eml. $\dfrac{P(x)}{Q(x)} \le 0$ (2. fejezet)

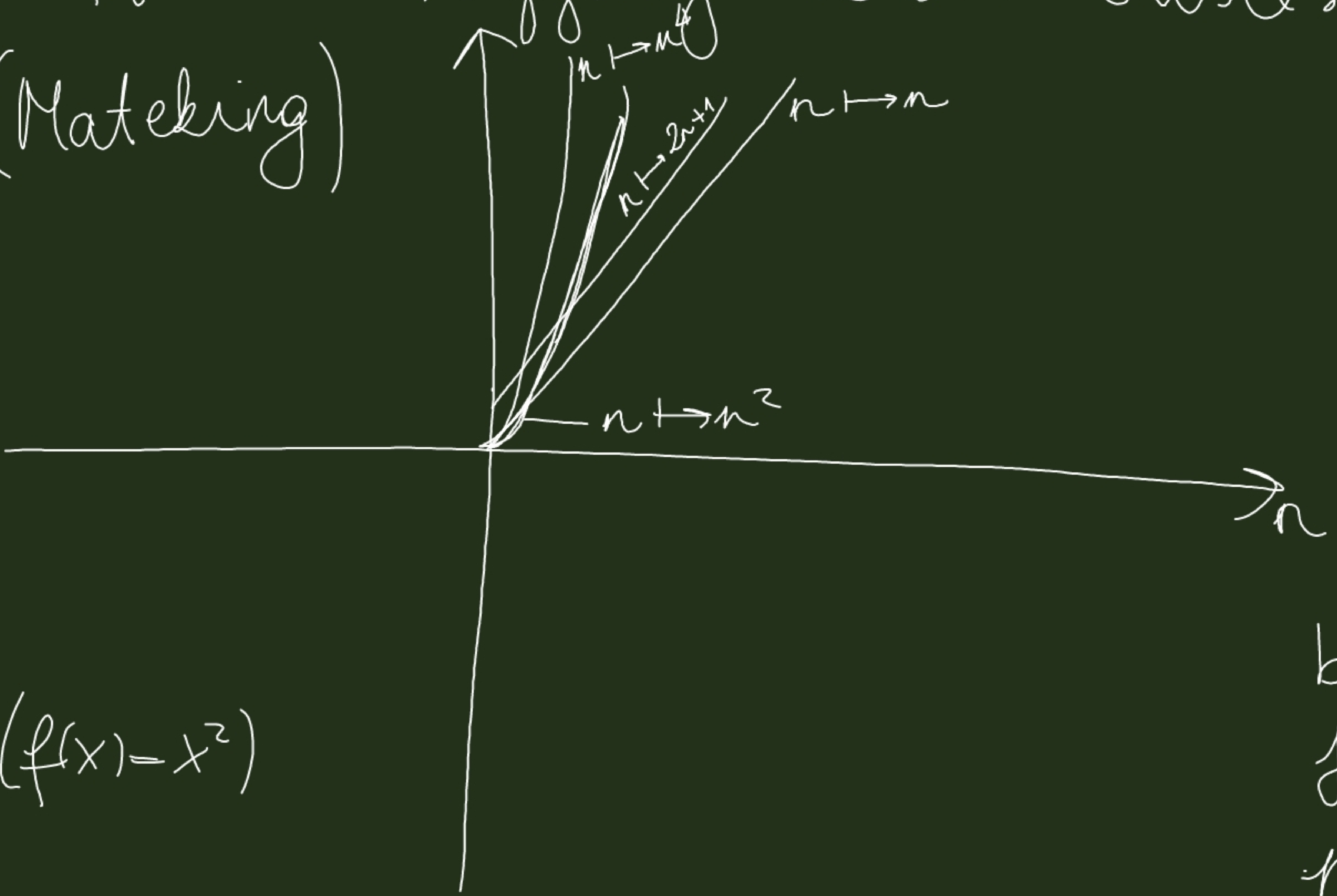
x		x_1		x_2	
P	$+$	0	$-$	$-$	$+$
Q	$-$	$-$	$-$	0	$+$
tört	$-$	0	$+$	↯	$+$

$$\frac{2\sin(x) + 1}{2 \cdot \cos(x)} \le 0$$



6. fejezet: Nagyságrend-őrző becslések

(Mateking)



(nagyságrend $\rightsquigarrow$ lépésszám, futásidő, memória ...)

$(f(x) = x^2)$

bemenet hossza: n

futási idő:

n, $2n+1$, n^2, n^4

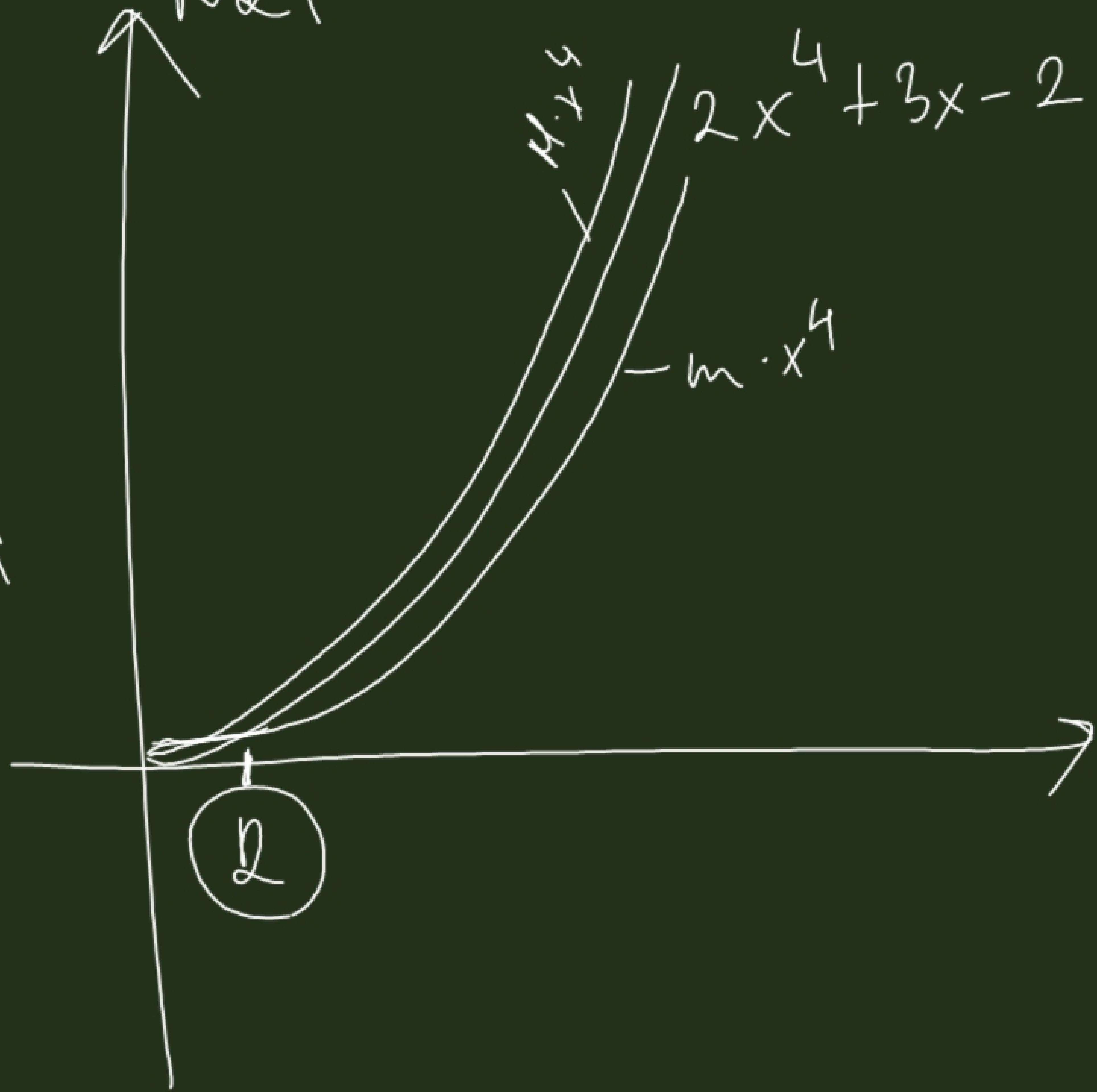
$$m \cdot x^4 \le 2x^4 + 3x - 2 \le M \cdot x^4 \quad (x > R)$$

NRA NRF

$m, M \in \mathbb{R}$ ⟸ ezeket keressük

$R \in \mathbb{R}$ ⟸

$\mathbb{R}$ ← valós számok halmaza



$M \cdot x^5$

1) NRF

a, $4x^5 \underbrace{-3x^4-2x^2-5-x}_{\leq 0} \leq 4x^5$ $\quad (x \geq 0) \rightarrow \boxed{\begin{matrix} M=4 \\ R=0 \end{matrix}}$

$-x \leq 0$ igaz, ha $x \geq 0$ $\quad (x \geq 1)$

b, $2x^3 \underbrace{-3x^2}_{\leq 0} + 6x + 7 \leq 2x^3 + 6x + 7 \leq 2x^3 + 6x^3 + 7x^3 =$

$6x \leq 6x^3 \quad (x \geq 1)$

$7 \leq 7x^3 \quad (x \geq 1)$

$= 15x^3 \quad (x \geq 1) \rightarrow \boxed{\begin{matrix} M=15, \\ R=1 \end{matrix}}$

c, $6x^5 + \underbrace{7x^4}_{\le 7x^5} + \underbrace{10x^3}_{\le 10x^5} + \underbrace{x^2}_{\le x^5} + \underbrace{2x}_{\le 2x^5} + \underbrace{3}_{\le 3x^5} \le 29x^5 \quad (x \ge 1)$

$$\boxed{M = 29,\ k = 1}$$

2, NKA

a, $6x^5 + \underbrace{7x^4 + 10x^3 + x^2 + 2x + 3}_{\ge 0} \ge 6x^5 \quad (x \ge 0)$

$$\boxed{m = 6,\ k = 0}$$

b, $2x^3-3x^2\underbrace{+6x+7}_{\geq 0} \underset{x\geq 0}{\geq} 2x^3-3x^2 \geq 2x^3-3x^3=-x^3$:(

ötlet: többet vonjunk le $3x^2$-nél

$$3x^2 \underset{x\geq 1}{\leq} 3x^3$$

még egyszer:

$$\underbrace{2x^3}_{x^3+x^3}-3x^2 = x^3 + \underbrace{x^3-3x^2}_{\substack{\geq 0 \\ \text{cél}}} = x^3 + \underbrace{x^2}_{\geq 0}\underbrace{(x-3)}_{\substack{\geq 0 \\ x \geq 3}} \geq x^3 \quad (x \geq 3)$$

$m=1,\ k=3$

c) $4x^5 - 3x^4 - 2x^2 - 5 = 4x^5 - (3x^4 + 2x^2 + 5) \geq$

$x \geq 10$

$(x \geq 1)$ $\geq 4x^5 - 10x^4 = 3x^5 + \underbrace{x^5 - 10x^4} = 3x^5 + \underbrace{x^4}_{\geq 0}\underbrace{(x-10)}_{\geq 0} \geq 3x^5$

$3x^4 + 2x^2 + 5 \leq 3x^4 + 2x^4 + 5x^4 = 10x^4 \quad (x \geq 1)$

$\Rightarrow 4x^5 - 3x^4 - 2x^2 - 5 \geq 3x^5 \quad (x \geq 10) \to m = 3, \; k = 10$

HF: $4x^5 = 2x^5 + 2x^5 \to m = 2, \; k = 5$

3/a) NRF és NRA az alábbi függvényre:

$$f(x) := \frac{3x^4 + 2x^3 + 5x^2 + 7x + 6}{5x^2 - 3x - 10} =: \frac{P(x)}{Q(x)}$$

$$m \cdot x^2 \leq f(x) \leq M \cdot x^{2} \quad = 4-2$$

$$\frac{m_1}{M_2} \cdot x^2 \leq \frac{P(x)}{Q(x)} \leq \frac{M_1 x^4}{m_2 \cdot x^2} = M \cdot x^2$$

$$(x \geq \max(R_1, R_2)) \quad M = \frac{M_1}{m_2}$$

$$m_1 x^4 \leq P(x) \leq M_1 x^4, \quad m_2 \cdot x^2 \leq Q(x) \leq M_2 x^2$$

$$(x \geq R_1) \qquad (x \geq R_2)$$

P(x) becslései:

NRA: $3x^4 \underbrace{+2x^3+5x^2+7x+6}_{\geq 0} \geq 3x^4 \quad (x \geq 0)$

NRF: $3x^4+2x^3+5x^2+7x+6 \leq 23x^4 \quad (x \geq 1)$

Q(x) becslése:

NRA: $5x^2-3x-10 \geq 5x^2-\underbrace{(3x+10)}_{\leq 13x} \overset{x\geq 1}{\geq} 5x^2-13x = 4x^2+\underbrace{x^2-13x}=$

$= 4x^2+\underbrace{x}_{\substack{\geq 0 \\ (x\geq 0)}}\underbrace{(x-13)}_{\substack{\geq 0 \\ (x\geq 13)}} \geq 4x^2 \quad (x \geq 13)$

$x \geq 1$

NRF: $5x^2-3x-10 \leq 5x^2 \quad (x \geq 0)$

$$3x^4 \le P(x) \le 23x^4 \qquad (x \geq 1)$$

$$4x^2 \le Q(x) \le 5x^2 \qquad (x \geq 13)$$

$$Q(x) \le 5x^2$$

$$\frac{1}{Q(x)} \geq \frac{1}{5x^2}$$

$$\Rightarrow \qquad \frac{3}{5}x^2 \le \frac{P(x)}{Q(x)} \le \frac{23}{4}x^2 \qquad (x \geq 13)$$

$$\frac{P(x)}{Q(x)} = P(x) \cdot \frac{1}{Q(x)}$$

7. fejezet : Logika

3

a) $x = 0$ és $y = 0 \Longrightarrow x^2 + y^2 = 0$

IGAZ, mert : $x^2 + y^2 = 0^2 + 0^2 = 0$

megfordítás:

$x = 0$ és $y = 0 \Longleftarrow x^2 + y^2 = 0$

IGAZ

b) $x \cdot y = x \cdot z \Longrightarrow y = z$

HAMIS ($x \cdot y = x \cdot z$ $\quad x \cdot (y - z) = 0$ $\begin{cases} y = z \\ x = 0 \end{cases}$

$x \cdot y = x \cdot z \Longleftarrow y = z$

IGAZ

$\boxed{3/C}$ $x > y^2 \Longrightarrow x > 0$

$x = 0,\ y = 0$

IGAZ: $x > y^2 \geq 0 \Longrightarrow x > 0$

megfordítás:

$$x > y^2 \Longleftarrow x > 0$$

HAMIS, mert $x = 1,\ y = 2$