20. fejezet 1/a, HF: 21 fej. 1/a

20./21. /1/b

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

SÉ: $P_A(\lambda) = \det(A - \lambda I) = \det \begin{bmatrix} 1-\lambda & -1 & 1 \\ 1 & 1-\lambda & -1 \\ 0 & -1 & 2-\lambda \end{bmatrix} =$

$$= (1-\lambda) \cdot \det \begin{bmatrix} 1-\lambda & -1 \\ -1 & 2-\lambda \end{bmatrix} - \det \begin{bmatrix} -1 & 1 \\ -1 & 2-\lambda \end{bmatrix} =$$

$$= (1-\lambda)\big((1-\lambda)(2-\lambda) - 1\big) - \big(-(2-\lambda) + 1\big) =$$

$$= (1-\lambda)^2 \cdot (2-\lambda) - (1-\lambda) + (2-\lambda) - 1 =$$

$$= (1-\lambda)^2 \cdot (2-\lambda) - \cancel{(1-\lambda)} + \cancel{(1-\lambda)} = (1-\lambda)^2 (2-\lambda) = 0$$

$$(1-\lambda)^2(2-\lambda)=0 \quad \boxed{\begin{matrix}\lambda_1 = 1 \\ \lambda_2 = 2\end{matrix}}_{SE} \qquad \begin{matrix} a(1) = 2 \\ a(2) = 1 \end{matrix}$$

$$(1-\lambda)(1-\lambda)(2-\lambda)$$

$\boxed{SV}$

$$\lambda_1 = 1 : \quad (A\cdot x = 1\cdot x)$$

$$(A - 1\cdot I)\cdot x = 0$$

$$\begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ 0 & -1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \longrightarrow \left\{ \begin{matrix} -x_2 + x_3 = 0 \\ x_1 - x_3 = 0 \\ -x_2 + x_3 = 0 \end{matrix} \right. \qquad \begin{matrix} x_2 = x_3 \\ x_1 = x_3 \\ x_3 \in \mathbb{K} \end{matrix}$$

$$M_h = \left\{ x_3 \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \in \mathbb{K}^3 \;\middle|\; x_3 \in \mathbb{K} \right\} = W_1 = \operatorname{Span}\left(\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right)$$

Sajátvektorok: $W_1 \setminus \{(0,0,0)\}$

$$g(1) = \dim(W_1) = 1$$

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

$$\boxed{\lambda_2 = 2}$$

$$(A - 2 \cdot I) \cdot x = 0$$

$$\begin{cases} -x_1 - x_2 + x_3 = 0 \\ x_1 - x_2 - x_3 = 0 \\ -x_2 = 0 \end{cases}$$

$$\rightarrow \left.\begin{matrix} -x_1 + x_3 = 0 \\ x_1 - x_3 = 0 \end{matrix}\right\} \rightarrow x_1 = x_3$$

$$-x_2 = 0 \rightarrow x_2 = 0$$

$$\boxed{v : \begin{pmatrix} x_1 \\ 0 \\ x_1 \end{pmatrix}, \quad x_1 \in \underset{\{\mathbb{R}, \mathbb{C}\}}{\overset{}{\mathbb{K}}}, \; x_1 \neq 0} \rightarrow W_2 = \mathrm{Span}((1, 0, 1))$$

$$g(2) = 1$$

	$\lambda_1 = 1$	$\lambda_2 = 2$
$a(\lambda)$	2	1
$g(\lambda)$	1	1

$a(\lambda)$: $+$ 3 ✓

$g(\lambda)$: $\times$ ✓

$\Downarrow$

$\nexists$ SB., A nem diagonalizálható.

1/c HF

$\underline{1(d)}$ $A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

SÉ: $\det(A-\lambda I) = \dots = (1-\lambda)(\lambda^2 - 2\lambda + 5) = 0$

$\lambda_1 = 1$

$\lambda_{2,3} = \dfrac{2 \pm \sqrt{4-20}}{2} = \dfrac{2 \pm \sqrt{-16}}{2} = 1 \pm 2i$

$\mathbb{R}$ felett sé-kek: $\lambda_1 = 1$ $\quad a(1) = 1$

$\mathbb{C}$ felett: $\lambda_1 = 1,\ \lambda_2 = 1+2i,\ \lambda_3 = 1-2i$

$a(\lambda_1) = a(\lambda_2) = a(\lambda_3) = 1$

$P_A(\lambda) = (\lambda-1)(\lambda-(1+2i))(\lambda-(1-2i))$

$\lambda_1 = 1 \overset{\boxed{SV}}{:} \quad (A - I)x = 0$

$$\begin{bmatrix} 0 & -1 & -1 \\ 1 & 0 & 0 \\ 3 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \text{mo-}\frac{1}{2}: \begin{pmatrix} 0 \\ -x_3 \\ x_3 \end{pmatrix}, \quad x_3 \in \mathbb{K}$$

$$-x_2 - x_3 = 0 \longrightarrow x_2 = -x_3$$

$$x_1 = 0$$

$$3x_1 = 0$$

($\mathbb{C}$ felett

$\begin{pmatrix} 0 \\ -i \\ i \end{pmatrix}$ SV-a $\lambda_1 = 1$-nek)

$$\boxed{\lambda_2 = 1+2i}$$

$$\left(A - (1+2i)I\right) \cdot x = 0$$

$$\begin{bmatrix} -2i & -1 & -1 \\ 1 & -2i & 0 \\ 3 & 0 & -2i \end{bmatrix} \cdot \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} -2i \cdot x_1 - x_2 - x_3 = 0 \\ x_1 - 2i \cdot x_2 = 0 \longrightarrow \boxed{x_1 = 2i \cdot x_2} \\ 3x_1 - 2i \cdot x_3 = 0 \longrightarrow 3x_1 = 2i \cdot x_3 \end{cases}$$

$$6i \cdot x_2 = 2i \cdot x_3 \Longrightarrow \boxed{3x_2 = x_3}$$

$$\Longrightarrow \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2i \cdot x_2 \\ x_2 \\ 3x_2 \end{pmatrix} = x_2 \cdot \begin{pmatrix} 2i \\ 1 \\ 3 \end{pmatrix}$$

SV-de: $x_2 \cdot \begin{pmatrix} 2i \\ 1 \\ 3 \end{pmatrix}$, $x_2 \in \mathbb{C}$, $x_2 \neq 0 \rightarrow W_{1+2i} = \mathrm{Span}\left((2i, 1, 3)\right)$

$g(1+2i) = 1$

$x_2 \in \mathbb{R}$:

$\begin{pmatrix} 2i \\ 1 \\ 3 \end{pmatrix}, 2 \cdot \begin{pmatrix} 2i \\ 1 \\ 3 \end{pmatrix}, \ldots$

$x_2 \in \mathbb{C}$

$\begin{pmatrix} 2i \\ 1 \\ 3 \end{pmatrix}, 2 \cdot \begin{pmatrix} 2i \\ 1 \\ 3 \end{pmatrix}, \ldots, \begin{pmatrix} -2 \\ i \\ 3i \end{pmatrix}$

$\lambda_3 = 1-2i$: $W_{1-2i} = \mathrm{Span}\left((-2i, 1, 3) \right)$

$\boxed{SB}$ $\mathbb{C}$ felett

λ	1	$1+2i$	$1-2i$	
$a(\lambda)$	1	1	1	3 ✓
$g(\lambda)$	1	1	1	
	✓	✓	✓	

$\Rightarrow \mathbb{C}$ felett $\exists$ SB, és A diag.

$\mathbb{R}$ felett

λ	1
$a(\lambda)$	1
$g(\lambda)$	1

$\rightarrow 1 \neq 3$ $\times$

$\Downarrow$

$\mathbb{R}$ felett

$\nexists$ SB.

$\Downarrow$

$\mathbb{R}$ felett A

nem diag.

$\mathbb{C}$ felett SB: $\underbrace{(0,1,-1)}_{\in W_1}, \underbrace{(2i,1,3)}_{\in W_{1+2i}}, \underbrace{(-2i,1,3)}_{\in W_{1-2i}}$

A diag: $\exists C: \det(C) \neq 0$ és $C^{-1} \cdot A \cdot C = D$

$$C = \begin{array}{c} \begin{bmatrix} 0 & 2i & -2i \\ 1 & 1 & 1 \\ -1 & 3 & 3 \end{bmatrix} \\ \begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ W_1 & W_{1+2i} & W_{1-2i} \end{array} \end{array} \qquad D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1+2i & 0 \\ 0 & 0 & 1-2i \end{bmatrix}$$

HF: $C^{-1} \cdot A \cdot C = D$