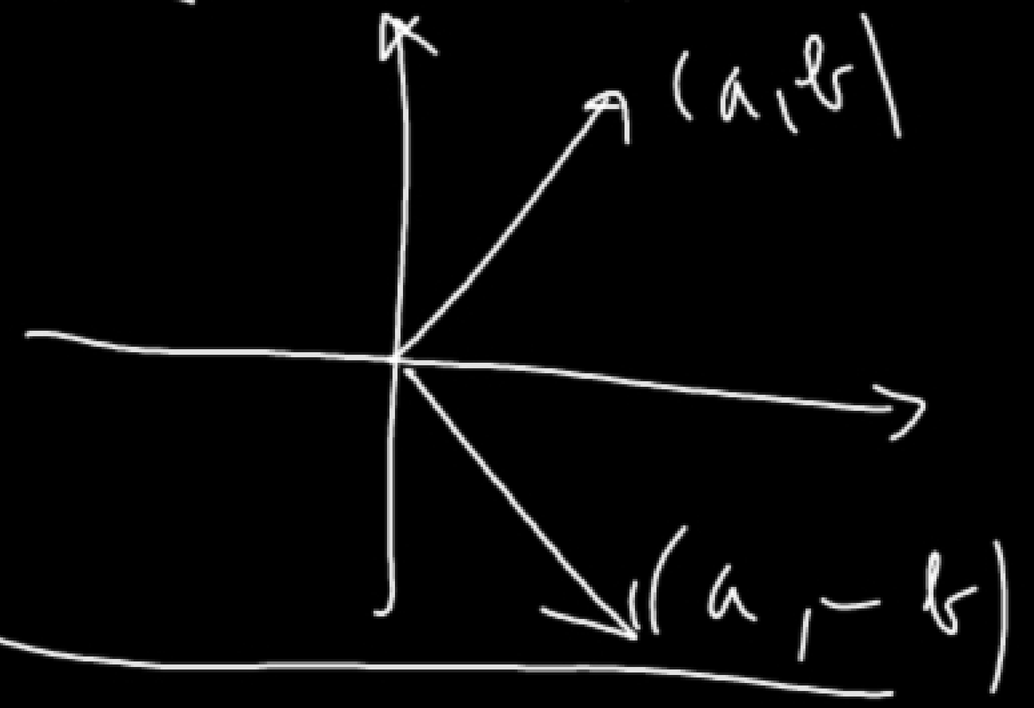
1. $z_1 = 8 - i$, $z_2 = 3 - 2i$

$$\overline{a + b\cdot i} = a - b\cdot i$$



$$\left(\frac{z_1}{z_2}\right)^2 \cdot \overline{(z_1 - z_2)} = (3+4i)(5-i) = \underline{\underline{19 + 17i}}$$

$$z_1 - z_2 = 5 + i(-1 - (-2)) = 5 + i$$

$$\overline{z_1 - z_2} = 5 - i$$

$$(-i)\cdot 2i = -2\cdot i^2 = 2$$

$$\frac{z_1}{z_2} = \frac{8-i}{3-2i}\cdot\frac{3+2i}{3+2i} = \frac{(8-i)(3+2i)}{9+4} = \frac{24 - 3i \overbrace{+2} + 16i}{13} =$$

$$= \frac{26 + 13i}{13} = 2 + i, \quad (2+i)^2 = 4 + 4i + \underbrace{i^2}_{-1} = 3 + 4i$$

b) $z^3 - 3z^2 + 4z - 2 = 0$

	1	−3	4	−2
1	1	−2	2	0

$$(z-1)(z^2 - 2z + 2) = 0$$

$\underline{\underline{z = 1}}$

$z^2 - 2z + 2 = 0$

$$z_{1,2} = \frac{2 \pm \sqrt{4 - 8}}{2} = \frac{2 \pm \sqrt{-4}}{2} =$$

$\sqrt{4}\cdot\sqrt{-1} = 2i$

$$= \frac{2 \pm 2i}{2} = \underline{\underline{1 \pm i}}$$

2.

$$A = \begin{bmatrix} 2 & -1 & 3 \\ 2 & 0 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 2 \end{bmatrix}$$

a)

$$\underbrace{(\underbrace{A}_{2\times 3} \cdot \underbrace{B^T}_{3\times 2})^{-1}}_{2\times 2} \cdot \underbrace{(A - B)}_{2\times 3} = \underbrace{\begin{bmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \end{bmatrix}}_{2\times 3}$$

$$A - B = \begin{bmatrix} -1 & -2 & 5 \\ 0 & -4 & 0 \end{bmatrix}$$

$$A \cdot B^T = \begin{bmatrix} 2 & -1 & 3 \\ 2 & 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 3 & 2 \\ 1 & 4 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 6 \\ 2 & 8 \end{bmatrix}$$

$$A \cdot B^T = \begin{bmatrix} -1 & 6 \\ 2 & 8 \end{bmatrix}$$

$$(A \cdot B^T)^{-1} = \frac{1}{\det(A \cdot B^T)} \cdot \begin{bmatrix} 8 & -6 \\ -2 & -1 \end{bmatrix} = \frac{1}{-20} \cdot \begin{bmatrix} 8 & -6 \\ -2 & -1 \end{bmatrix} =$$

$$\left(= \begin{bmatrix} -\frac{2}{5} & \frac{3}{10} \\ \frac{1}{10} & \frac{1}{20} \end{bmatrix} \right)$$

$$(A\cdot B^T)^{-1}\cdot(A-B) = -\frac{1}{20}\cdot\begin{bmatrix} 8 & -6 \\ -2 & -1 \end{bmatrix}\cdot\begin{bmatrix} -1 & -2 & 5 \\ 0 & -4 & 0 \end{bmatrix} =$$

$$= -\frac{1}{20}\cdot\begin{bmatrix} -8 & 4 & 40 \\ 2 & 8 & -10 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{2}{5} & -\frac{1}{5} & -2 \\ -\frac{1}{10} & -\frac{2}{5} & \frac{1}{2} \end{bmatrix}$$

b) $\det(A^T \cdot B) \underset{\text{Sarrus-szabály}}{=} 20 + 260 - (60 + 220) = 0$

$$A^T \cdot B = \underset{3 \times 2}{\begin{bmatrix} 2 & 2 \\ -1 & 0 \\ 3 & 2 \end{bmatrix}} \cdot \underset{2 \times 3}{\begin{bmatrix} 3 & 1 & -2 \\ 2 & 4 & 2 \end{bmatrix}} = \begin{array}{c} \begin{array}{ccc} + & - & + \end{array} \\ \begin{bmatrix} 10 & 10 & 0 \\ -3 & -1 & 2 \\ 13 & 11 & -2 \end{bmatrix} \end{array}$$

$$\det(A^T B) = 10 \cdot \det\begin{bmatrix} -1 & 2 \\ 11 & -2 \end{bmatrix} - 10 \cdot \det\begin{bmatrix} -3 & 2 \\ 13 & -2 \end{bmatrix} =$$

$$= 10 \cdot (-20) - 10 \cdot (-20) = \underline{\underline{0}}$$

$$A^T B = \begin{bmatrix} 10 & 10 & 1 \\ -3 & -1 & 2 \\ 13 & 11 & -2 \end{bmatrix}$$

$$(A^T \cdot B)^{-1} = \frac{1}{\det} \cdot \underbrace{\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}}_{\tilde{A}}$$

$$+1 \cdot \det \begin{bmatrix} -1 & 2 \\ 11 & -2 \end{bmatrix} = -20$$

$$-1 \cdot \det \begin{bmatrix} 10 & 1 \\ 11 & -2 \end{bmatrix} = 31$$

$$+1 \cdot \det \begin{bmatrix} 10 & 1 \\ -1 & 2 \end{bmatrix} = 21$$

③ $H := \{(x, y, z, u) \in \mathbb{R}^4 \mid x \cdot y \cdot z \cdot u \geq 0\} \subset \mathbb{R}^4$

- $\lambda \cdot \overbrace{(x, y, z, u)}^{\in H} = (\lambda x, \lambda y, \lambda z, \lambda u)$

$$\underbrace{\lambda^4}_{\geq 0} \cdot \underbrace{x \cdot y \cdot z \cdot u}_{\geq 0} \geq 0 \Rightarrow \lambda \cdot (x, y, z, u) \in H$$

- $\underbrace{(x_1, y_1, z_1, u_1)}_{a_1}, \underbrace{(x_2, y_2, z_2, u_2)}_{a_2} \in H$

$$a_1 + a_2 = (x_1 + x_2, y_1 + y_2, z_1 + z_2, u_1 + u_2)$$

$$(x_1 + x_2)(y_1 + y_2)(z_1 + z_2) \cdot (u_1 + u_2) \not\geq 0$$

ellenpélda:

$\underline{a_1 := (1, 1, 1, 0), a_2 := (0, -2, 1, 1)} \Rightarrow a_1 + a_2 \notin H$

$(1, 1, 1, 0), \quad (0, a, 1, 1) \qquad (1+0)(1+a)(1+1) \cdot (1+0) < 0$

④ $W = \{ (y+z+2u,\ x,\ x-y-u,\ x+y-2z+u) \in \mathbb{Q}^4 \mid$

$x, y, z, u \in \mathbb{Q},\ 2x+y = z+u \} \subset \mathbb{Q}^4$ altér

a) $2x+y = z+u \longrightarrow u = 2x+y-z$

W elemei: $\begin{pmatrix} y+z+2(2x+y-z) \\ x \\ x-y-(2x+y-z) \\ x+y-2z+2x+y-z \end{pmatrix} = \begin{pmatrix} 4x+3y-z \\ x \\ -x-2y+z \\ 3x+2y-3z \end{pmatrix}$ $(x, y, z \in \mathbb{Q})$

W elemei:

$$\begin{pmatrix} 4x \\ x \\ -x \\ 3x \end{pmatrix} + \begin{pmatrix} 3y \\ 0 \\ -2y \\ 2y \end{pmatrix} + \begin{pmatrix} -z \\ 0 \\ z \\ -3z \end{pmatrix} = x \cdot \underbrace{\begin{pmatrix} 4 \\ 1 \\ -1 \\ 3 \end{pmatrix}}_{=:v_1} + y \cdot \underbrace{\begin{pmatrix} 3 \\ 0 \\ -2 \\ 2 \end{pmatrix}}_{=:v_2} + z \cdot \underbrace{\begin{pmatrix} -1 \\ 0 \\ 1 \\ -3 \end{pmatrix}}_{=:v_3}$$

$$W = \mathrm{Span}(v_1, v_2, v_3) \implies v_1, v_2, v_3 \text{ G-a } W\text{-nek}$$

F - e?

$$\begin{cases} 4x + 3y - z = 0 \\ \boxed{x = 0} \\ -x - 2y + z = 0 \\ 3x + 2y - 3z = 0 \end{cases}$$

$$\begin{cases} 3y - z = 0 \\ -2y + z = 0 \\ 2y - 3z = 0 \end{cases} \quad -2z = 0 \quad \boxed{z = 0} \quad \to \boxed{y = 0}$$

$\Longrightarrow v_1, v_2, v_3$ F $\Longrightarrow v_1, v_2, v_3$ B W-ben,

$$\boxed{\dim(W) = 3}$$

$$\operatorname{rang}(x_1, \dots, x_k) = \dim\left(\operatorname{Span}(x_1, \dots, x_k)\right)$$

$$\operatorname{rang}(A) = \dim\left(O(A)\right) = \dim\left(S(A)\right)$$

(GJ: kötött ism.)

b, $\underbrace{\operatorname{Span}\big((0,0,0,0),(1,0,-1,3)\big)}$ altér-e W-ben

$$\operatorname{Span}(v_1,v_2,v_3)$$

$$\| $$

$$\operatorname{Span}\big(\underbrace{(1,0,-1,3)}_{=:v}\big)$$

$x \cdot v_1 + y \cdot v_2 + z \cdot v_3 = v$

$$\begin{cases} 4x + 3y - z = 1 \\ \boxed{x = 0} \\ -x - 2y + z = -1 \\ 3x + 2y + 3z = 3 \end{cases}$$

$$\begin{cases} 3y - z = 1 \quad / \cdot (-1) \longrightarrow -3y + z = -1 \\ -2y + z = -1 \qquad\qquad -2y + z = -1 \\ 2y + 3z = 3 \end{cases}$$

$\longrightarrow \boxed{y = 0}$

$\boxed{z = -1}$

$2 \cdot 0 + 3 \cdot (-1) \neq 3$

$\Longrightarrow v \notin W \Longrightarrow \operatorname{Span}(v)$ nem altér W-ben

$v \in W \Longrightarrow \operatorname{Span}(v)$ altér W-ben

5.

Skalár alak: $x_1 = 2$, $x_2 = 3$, $x_3 = x_4 + x_5$, $x_4, x_5 \in \mathbb{R}$

vektoros:

$$x = \begin{pmatrix} 2 \\ 3 \\ x_4 + x_5 \\ x_4 \\ x_5 \end{pmatrix} = \underbrace{\begin{pmatrix} 2 \\ 3 \\ 0 \\ 0 \\ 0 \end{pmatrix}}_{x^B} + x_4 \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \cancel{+ x_5 \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}}$$

x_4, x_5 szabad

$\text{rang}(A) = 3$