17. fejezet

(1.) $x_1 := (3, 0, -2, 4)$

$x_2 := (2, 1, -1, 3)$

$x_3 := (-1, 4, 2, 0)$

$x_4 := (-1, 1, 1, -1)$

$x_1, x_2, x_3, x_4 \in \mathbb{R}^4$

$$\boxed{W := \operatorname{Span}(x_1, x_2, x_3, x_4)}$$

$\hookrightarrow$ bázis = ?

$\dim(W) = ?$

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x_1, x_2, x_3, x_4 (G) W-ben

(F) ?

x_1, x_2, x_3, x_4 (F) ?

$$a \cdot x_1 + b x_2 + c x_3 + d \cdot x_4 = 0 \qquad (a, b, c, d \in \mathbb{R})$$

$$a \cdot \begin{pmatrix} 3 \\ 0 \\ -2 \\ 4 \end{pmatrix} + b \cdot \begin{pmatrix} 2 \\ 1 \\ -1 \\ 3 \end{pmatrix} + c \cdot \begin{pmatrix} -1 \\ 4 \\ 2 \\ 0 \end{pmatrix} + d \cdot \begin{pmatrix} -1 \\ 1 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\rightarrow \begin{cases} 3a + 2b - c - d = 0 \\ b + 4c + d = 0 \\ -2a - b + 2c + d = 0 \\ 4a + 3b - d = 0 \end{cases} \rightarrow d = 4a + 3b$$

$$\begin{cases} -a-b-c=0 \\ 4a+4b+4c=0 \\ 2a+2b+2c=0 \end{cases} \to c=-a-b$$

$c=-a-b$ behelyettesítése:

$$\begin{cases} 0\cdot a+0b=0 \\ 0\cdot a+0\cdot b=0 \end{cases} \to a,b\in\mathbb{R} \text{ (tetszőleges)}$$

$ax_1+bx_2+cx_3+dx_4=0$ összes mo.: $\begin{cases} d=4a+3b \\ c=-a-b \\ a,b\in\mathbb{R} \end{cases} \Rightarrow$ $\exists$ nemtrivi mo.

$a=0,\ b=1$

$c=-1,\ d=3$

$\Downarrow$

elhagyjuk x_4-et

$\mathrm{Span}(x_1, x_2, x_3) = W \quad (x_1, x_2, x_3 \text{ G})$

x_1, x_2, x_3 F ?

$a \cdot x_1 + b \cdot x_2 + c \cdot x_3 = 0 \longrightarrow$ összes mo.: $\boxed{\begin{array}{l} b = -\frac{4}{3}a \\ c = \frac{1}{3}a \end{array} \quad a \in \mathbb{R}}$

$$\begin{cases} 3a + 2b - c = 0 \\ b + 4c = 0 \\ -2a - b + 2c = 0 \\ 4a + 3b = 0 \end{cases}$$

$\longrightarrow c = 3a + 2b$

$\downarrow$ behely.

$$\begin{cases} 12a + 9b = 0 \\ 4a + 3b = 0 \\ 4a + 3b = 0 \end{cases} \longrightarrow b = -\frac{4}{3}a$$

$\downarrow$

$c = 3a - \frac{8}{3}a = \frac{1}{3}a$

Ism.:

x_1, x_2, x_3 F

$$\Updownarrow$$

$$\boxed{a \cdot x_1 + b \cdot x_2 + c \cdot x_3 = 0} \Leftrightarrow a = b = c = 0$$

x_1, x_2, x_3 Ö

$$\Updownarrow$$

$ax_1 + bx_2 + cx_3 = 0$ -nak van $\underbrace{\text{nemtriviális}}_{a \neq 0 \text{ vagy } b \neq 0 \text{ vagy } c \neq 0}$ mo-a

$a \cdot x_1 + b x_2 + c x_3 = 0$ -nak $\exists$ nemtriv mo-a,

pl.: $a = 1$, $b = -\frac{4}{3}$, $c = \frac{1}{3}$

$\Longrightarrow$ elhagyjuk x_3-at

$\mathrm{Span}(x_1, x_2) = W$, x_1, x_2 F $\Longrightarrow$ x_1, x_2 B -a

W-nek

$\rightarrow \dim W = 2$.

x_1, x_2 F ?

$a \cdot x_1 + b x_2 = 0$

$$\begin{cases} 3a + 2b = 0 \\ \quad\;\; b = 0 \\ -2a - b = 0 \\ 4a + 3b = 0 \end{cases}$$

$\longrightarrow$ Összes mo.:

$a = 0$
$b = 0$

$\Longrightarrow$ x_1, x_2 F

$\boxed{3}$ HF

$\boxed{2}$ $x_1, x_2, x_3, x_4, x_5 \in \mathbb{R}^4$

$\boxed{B = F + G}$

a, x_1, x_2 lehet-e B $\mathbb{R}^4$-ben?

Nem, mert $2 < 4$

b, x_1, x_2, x_3, x_4, x_5 B-e $\mathbb{R}^4$-ben?

$B = F + G$

$5 > 4 \Rightarrow$ nem F (Biz: szorg.)

c, HF