23. fejezet

$$\left(u_1, u_2, u_3 \text{ OR } \checkmark \right)$$

1, C, $u_1 = (1, 1, 1, 1)$

$u_2 = (1, -1, -1, 1)$

$u_3 = (-1, 0, 0, 1)$

Bontsuk fel az $x = (2, 1, 3, 1) \in \mathbb{R}^4$ -t a

$W := \mathrm{Span}(u_1, u_2, u_3)$ altér szerinti párh. és mer. komp.!

$$P(x) = \langle x, u_1 \rangle \cdot u_1 + \langle x, u_2 \rangle \cdot u_2 + \langle x, u_3 \rangle \cdot u_3$$

! ez csak ONR esetén működik!

$$Q(x) = x - P(x)$$

→ 1. lépés : NORMÁLÁS

$$e_1 := \frac{u_1}{\|u_1\|}, \quad e_2 := \frac{u_2}{\|u_2\|}, \quad e_3 := \frac{u_3}{\|u_3\|}$$

$$e_1 = \frac{1}{2}\cdot(1,1,1,1), \; e_2 = \frac{1}{2}\cdot(1,-1,-1,1), \quad e_3 := \frac{1}{\sqrt{2}}\cdot(-1,0,0,1)$$

$$\left(= \left(\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)\right) \left(\|u_1\| = \sqrt{\langle u_1, u_1\rangle} = \right)$$

$$P(x) = \langle x, e_1\rangle \cdot e_1 + \langle x, e_2\rangle \cdot e_2 + \langle x, e_3\rangle \cdot e_3$$

$$\langle x, e_1\rangle = \left\langle (2,1,3,1), \frac{1}{2}\cdot(1,1,1,1)\right\rangle = \frac{1}{2}\cdot 7 = \frac{7}{2}$$

$$\langle x, e_1\rangle \cdot e_1 = \frac{7}{2}\cdot\frac{1}{2}(1,1,1,1) = \boxed{\frac{7}{4}\cdot(1,1,1,1)}$$

$$\langle x, e_2\rangle = \frac{1}{2}\cdot(-1) = -\frac{1}{2}$$

$$\langle x, e_2\rangle \cdot e_2 = -\frac{1}{2}\cdot\frac{1}{2}\cdot(1,-1,-1,1) = \boxed{-\frac{1}{4}(1,-1,-1,1)}$$

$$\langle x, e_3\rangle = \frac{1}{\sqrt{2}}\cdot(-1) = -\frac{1}{\sqrt{2}}$$

$$\langle x, e_3\rangle \cdot e_3 = -\frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\cdot(-1,0,0,1) = \boxed{-\frac{1}{2}\cdot(-1,0,0,1)}$$

$$e_1 = \frac{1}{2}\cdot(1,1,1,1)$$

$$e_2 = \frac{1}{2}\cdot(1,-1,-1,1)$$

$$e_3 = \frac{1}{\sqrt{2}}\cdot(-1,0,0,1)$$

$$x = (2,1,3,1)$$

$$\rightarrow P(x) = \frac{7}{4}\cdot(1,1,1,1) - \frac{1}{4}\cdot(1,-1,-1,1) - \overset{\frac{2}{4}}{\frac{1}{2}}\cdot(-1,0,0,1) =$$

$$= \frac{1}{4}\Big((7,7,7,7) - (1,-1,-1,1) - (-2,0,0,2)\Big) =$$

$$= \frac{1}{4}\cdot(8,8,8,4) = \underline{\underline{(2,2,2,1)}}, \quad Q(x) = x - P(x) = \underline{\underline{(0,-1,1,0)}}$$

2. $b_1=(1,1,1,1)$, $b_2=(3,3,-1,-1)$, $b_3=(-2,0,6,8)$

b_1, b_2, b_3 (F) $\longrightarrow$ u_1, u_2, u_3 OR

Gram-Schmidt eljárás:

$u_1 := b_1 = (1,1,1,1)$

u_2: b_2 merőleges komponense a $\mathrm{Span}(u_1)$-re vonatkozóan

$$b_2 - \frac{\langle b_2, u_1\rangle}{\langle u_1, u_1\rangle}\cdot u_1 = b_2 - \frac{\langle b_2, u_1\rangle}{\|u_1\|^2}\cdot u_1 =$$

$$= b_2 - \left\langle b_2, \frac{u_1}{\|u_1\|}\right\rangle \cdot \frac{u_1}{\|u_1\|}$$

$$b_2 - \frac{1}{\|u_1\|^2}\langle b_2, u_1\rangle \cdot u_1 = (3,3,-1,-1) - \frac{1}{4}\cdot 4\cdot(1,1,1,1) =$$

$$b_2 = (3,3,-1,-1) \qquad = (2,2,-2,-2)$$

$$u_1 = b_1 = (1,1,1,1)$$

$$\underline{u_2} := C\cdot(2,2,-2,-2) \longrightarrow u_2 := (1,1,-1,-1)$$

$$\boxed{u_3}$$

$$b_3 - \left(\frac{\langle b_3, u_1\rangle}{\|u_1\|^2}\cdot u_1 + \frac{\langle b_3, u_2\rangle}{\|u_2\|^2}\cdot u_2\right) \overset{HF}{=} (-1,1,-1,1)$$

$\Rightarrow$ O.B : $u_1 = (1,1,1,1)$, $u_2 = (1,1,-1,-1)$, $u_3 = (-1,1,-1,1)$

HF: OB?

ONB : $e_1 := \frac{1}{\|u_1\|} \cdot u_1$, $e_2 = \frac{1}{\|u_2\|} \cdot u_2$, $e_3 := \frac{1}{\|u_3\|} \cdot u_3$

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3. $\mathrm{Span}(e_1, e_2, e_3)$

OB : u_1, u_2, u_3

ONB : e_1, e_2, e_3

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4. HF