$P(x) = x^5 - 3x^3 - 2x^2 + 7x + 21$, melyik jó NRA

- $7x \quad (x \geq 5)$ ✗ nem nagyságrendőrző!
- $\frac{x^5}{100} \quad (x \geq 6)$ ✓ $P(x) \geq \underbrace{x^5}_{\frac{1}{2}x^5 \quad \frac{1}{2}x^5} - (3x^3 + 2x^2) \geq \frac{1}{2}x^5 \geq \frac{1}{100}x^5$
  $x \geq \dots$
- $x^5 \quad (x \geq 7)$ ✗ $P(x) \geq x^5$ nem igaz
  $P(x) = x^5 - \dots$
- nem létezik ✗

7. feladatsor

Kvantorok: $\forall$, $\exists$

$\boxed{5}$ $\frac{1}{n} < 0{,}01 \quad (n \in \mathbb{N}^+)$

$\left(\frac{1}{n} < 0{,}01 \iff n > 100 \right)$

a, $\forall n \in \mathbb{N}^+ : \frac{1}{n} < 0{,}01$

Hamis, ellenpélda: $n = 1$

tagadás: $\exists n \in \mathbb{N}^+ : \frac{1}{n} \geq 0{,}01$ Igaz

b, $\exists n \in \mathbb{N}^+ : \frac{1}{n} < 0{,}01$

Igaz, mert $n := 101$, $n = 1000$

tagadás: $\forall n \in \mathbb{N}^+ : \frac{1}{n} \geq 0{,}01$

c) $\forall n \geq N : \frac{1}{n} < 0{,}01 \quad (N \in \mathbb{N}^+)$ → nyitott kijelentés

$n \in \mathbb{N}$

$\forall n \in [N, +\infty)$

d) $\exists N \in \mathbb{N}^+ : \forall n \geq N : \frac{1}{n} < 0{,}01$

küszöb

Igaz, mert $N := 2.000.000$ $(N := 101)$.

tagadás:

$n \in [N, +\infty)$

$\forall N \in \mathbb{N}^+ : \exists n \geq N : \frac{1}{n} \geq 0{,}01$

5] a, Van olyan n term. szám, hogy $\frac{n^2}{10n-7} > 100$.

$$\boxed{\exists n \in \mathbb{N} : \frac{n^2}{10n-7} > 100}$$

Igaz, mert ⊛

tag.: $\forall n \in \mathbb{N} : \frac{n^2}{10n-7} \leq 100$

$$10n \underbrace{-7}_{\leq 0} \leq 10n$$

⊛ $\frac{n^2}{10n-7} \geq \ldots\ldots > 100$

$$\frac{n^2}{10n-7} \geq \frac{n^2}{10n} = \frac{n}{10} > 100 \rightsquigarrow n := 1001.$$

$$\frac{n^2}{10n-7} > \frac{n}{10} > 100 \Rightarrow \frac{n^2}{10n-7} > 100$$

6, Minden n term. számra $\frac{n^2}{10n-7} > 100$.

$$\forall n \in \mathbb{N} : \frac{n^2}{10n-7} > 100$$

tag.: $\exists n \in \mathbb{N} : \frac{n^2}{10n-7} \leq 100$ Igaz, $n := 1$

c, Van olyan term. szám, hogy minden nála nagyobb n term. számra $\dfrac{n^2}{10n-7} > 100$.

$$\exists N \in \mathbb{N} : \forall n > N : \frac{n^2}{10n-7} > 100.$$ Igaz, $N := 1001$ jó példa.

tag.: $\forall N \in \mathbb{N} : \exists n > N : \dfrac{n^2}{10n-7} \leq 100.$

7./b) Minden elég nagy n term. számra

$$f(n) := \frac{2n^3+3}{n^5-3n^4-7n^3+2n^2-10n+1} < 0{,}05.$$

$$\exists N \in \mathbb{N} : \forall n > N : f(n) < 0{,}05$$

Igaz, $N := \ldots$

$$f(n) \sim \frac{1}{n^2}$$

$f(n)$ NRF :

$n \geq 1$

$$\frac{2n^3 + 3}{n^5 - 3n^4 - 7n^3 + 2n^2 - 10n + 1} \leq \frac{5n^3}{\frac{1}{2} n^5}$$

$$2n^3 + 3 \leq 5n^3$$

$$3 \leq n^3$$

$$n \geq 1$$

$$n^5 - 3n^4 - 7n^3 \underbrace{+ 2n^2}_{\geq 0} - 10n \underbrace{+ 1}_{\geq 0} \geq n^5 - (\underbrace{3n^4 + 7n^3 + 10n}_{\substack{\leq 20n^4 \\ n \geq 1}}) \geq$$

$$\geq n^5 - 20n^4 = \frac{1}{2} n^5 + \underbrace{\frac{1}{2} n^5 - 20n^4}_{\substack{\frac{1}{2} n^4 (n - 40) \geq 0 \\ n \geq 40}} \geq \frac{1}{2} n^5 \quad (n \geq 40)$$

$$f(n) \le \boxed{\frac{10}{n^2} < 0{,}05 = \frac{5}{100}} \quad (n \geq 40) \qquad n > N$$

$$n^2 > \frac{1000}{5} = 200$$

$$n > \sqrt{200} = \underbrace{\sqrt{2}}_{>1{,}4} \cdot 10 > 14 \to \text{kell: } N > 14$$

$$N := 40$$

$\boxed{7/c \text{ HF}}$

$\boxed{\text{8. fejezet}}$

$\boxed{1}$ $a, b, x, y \in \mathbb{R}$

$\Rightarrow, \Leftarrow, \Leftrightarrow$

a) $a+b=0 \Longleftrightarrow \underbrace{a^2+b^2=-2ab}$

$$a^2+2ab+b^2=0$$

$$\Updownarrow$$

$$(a+b)^2=0 \Longleftrightarrow a+b=0$$

b)

$a+b=1 \Longleftrightarrow a^2+b^2=1-2ab$ hamis

$a^2+2ab+b^2=1 \Leftrightarrow (a+b)^2=1 \Leftrightarrow a+b=\pm 1$

$\Longrightarrow$ igaz $a+b=1 \Rightarrow (a+b)^2=1 \Rightarrow a^2+b^2=1-2ab$

$\Longleftarrow$ hamis

c, $x=-1 \Longleftrightarrow x^2+x=0$ hamis ($\Rightarrow$ igaz)

$x(x+1)=0$

3| holnap

9. fejezet /2.

a, $x^2=25 \Longleftarrow x=5$

c, $a^2+b^2=0 \Longrightarrow a \cdot b=0$ ($\Leftarrow$ hamis, $a=0, b=5$)

e, $x^3-x^2-x+1=0 \Longleftarrow x=1$

$x^2(x-1)-(x-1)=0$

$(x-1)(x^2-1)=0$

i) $|x| = x \iff x \geq 0$

$$|x| = \begin{cases} x & (x \geq 0) \\ -x & (x < 0) \end{cases}$$

j) $\sin(2x) = tg(x) \Longleftarrow x = \frac{\pi}{4} + k \cdot \frac{\pi}{2} \quad (k \in \mathbb{Z})$

$$2\sin(x) \cdot \cos(x) = \frac{\sin(x)}{\cos(x)}$$

$$\sin(x) \cdot \left(2\cos^2(x) - 1\right) = 0$$

$\sin(x) = 0$

$x = k \cdot \pi$

$2\cos^2(x) - 1 = 0$

$x = \frac{\pi}{4} + k \cdot \frac{\pi}{2} \quad (k \in \mathbb{Z})$