

1/a, $A = \begin{bmatrix} 2 & -1 & -1 \\ 3 & -2 & -3 \\ -1 & 1 & 2 \end{bmatrix}$

① SE:

$$0 = \det(A - \lambda I) = \det \begin{bmatrix} 2-\lambda & -1 & -1 \\ 3 & -2-\lambda & -3 \\ -1 & 1 & 2-\lambda \end{bmatrix} =$$

1. oszlop

$$\downarrow$$

$$= (2-\lambda) \cdot \det \begin{bmatrix} -2-\lambda & -3 \\ 1 & 2-\lambda \end{bmatrix} - 3 \cdot \det \begin{bmatrix} -1 & -1 \\ 1 & 2-\lambda \end{bmatrix} - 1 \cdot \det \begin{bmatrix} -1 & -1 \\ -2-\lambda & -3 \end{bmatrix}$$

$$= (2-\lambda) \cdot \left(\underbrace{(-2-\lambda)(2-\lambda) + 3}_{-(2+\lambda)(2-\lambda) = -(4-\lambda^2)} - 3 \cdot (- (2-\lambda) + 1) - (3 + (-2-\lambda)) \right) =$$

$$= -(2-\lambda)(4-\lambda^2) + 3(2-\lambda) +$$

$$+ 3(2-\lambda) - 3$$

$$- 3 + (2+\lambda)$$

$$6(2-\lambda-1) = 6(1-\lambda)$$

$$= -(2-\lambda)(4-\lambda^2) + 6(2-\lambda) - 6 + (2+\lambda) =$$

$$= -(2-\lambda)(4-\lambda^2) + 6(1-\lambda) + 2 + \lambda =$$

$$= -8 + 2\lambda^2 + 4\lambda - \lambda^3 + 6(-6\lambda) + 2(+\lambda) =$$

$$= -\lambda^3 + 2\lambda^2 - \lambda$$

$$p(\lambda) = -\lambda^3 + 2\lambda^2 - \lambda = 0$$

$$-\lambda(\lambda^2 - 2\lambda + 1) = 0$$

$$-\lambda(\lambda - 1)^2 = 0$$

$$\lambda = 0 \quad \lambda = 1$$

$$a(0) = 1 \quad a(1) = 2$$



0 sajátérték

algebrai mult.

$\lambda = 0$ - hoz tart. Sajátvektorok:

$$(A - 0 \cdot I)x = 0$$

$$Ax = 0$$

• \mathbb{C}^3

• behelyettesítés

$$\text{mo.: } x_3 = -x_1, \quad x_2 = 3x_1, \quad x_1 \in \mathbb{Q}$$

$$\rightarrow \text{so.: } x = \begin{bmatrix} x_1 \\ 3x_1 \\ -x_1 \end{bmatrix} = x_1 \cdot \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} \quad (x_1 \in \mathbb{Q} \setminus \{0\})$$

$$W_0 = \left\{ x_1 \cdot \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} \mid x_1 \in \mathbb{Q} \right\} = \text{Span} \left(\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix} \right)$$

$$\dim(W_0) = 1 = g(0)$$

$\lambda = 1$ - her so:

$$(A - 1 \cdot I)x = 0$$

$$\left(\begin{bmatrix} 2 & -1 & -1 \\ 3 & -2 & -3 \\ -1 & 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \cdot x = 0$$

$$\begin{bmatrix} 1 & -1 & -1 \\ 3 & -3 & -3 \\ -1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

→ mo:
HF $x_1 = x_2 + x_3$
 $x_2, x_3 \in \mathbb{Q}$

Sajátvektorok: $\left\{ \begin{pmatrix} x_2 + x_3 \\ x_2 \\ x_3 \end{pmatrix} \mid x_2, x_3 \in \mathbb{Q} \right\} \setminus \left\{ \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$

$W_1 = \text{Span} \left(\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \right) \Rightarrow g \left(\begin{pmatrix} 1 \\ 1 \end{pmatrix} \right) = 2$

← sajátérték

$$\begin{pmatrix} x_2 + x_3 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_2 \\ x_2 \\ 0 \end{pmatrix} + \begin{pmatrix} x_3 \\ 0 \\ x_3 \end{pmatrix} = x_2 \cdot \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + x_3 \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

Sajátbázis:

$$\lambda = 0$$

$$\lambda = 1$$

$$a(0) = 1 = g(0) = 1$$

$$\frac{a(1) = 2}{+ 3} = g(1) = 2$$

$\Rightarrow \exists$ SB

$$\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

w_0 w_1

The diagram shows three column vectors: $\begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, and $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$. Below the first vector is the label w_0 with an arrow pointing up to its third component (-1). Below the second and third vectors is the label w_1 with two arrows pointing up to their third components (0 and 1 respectively).