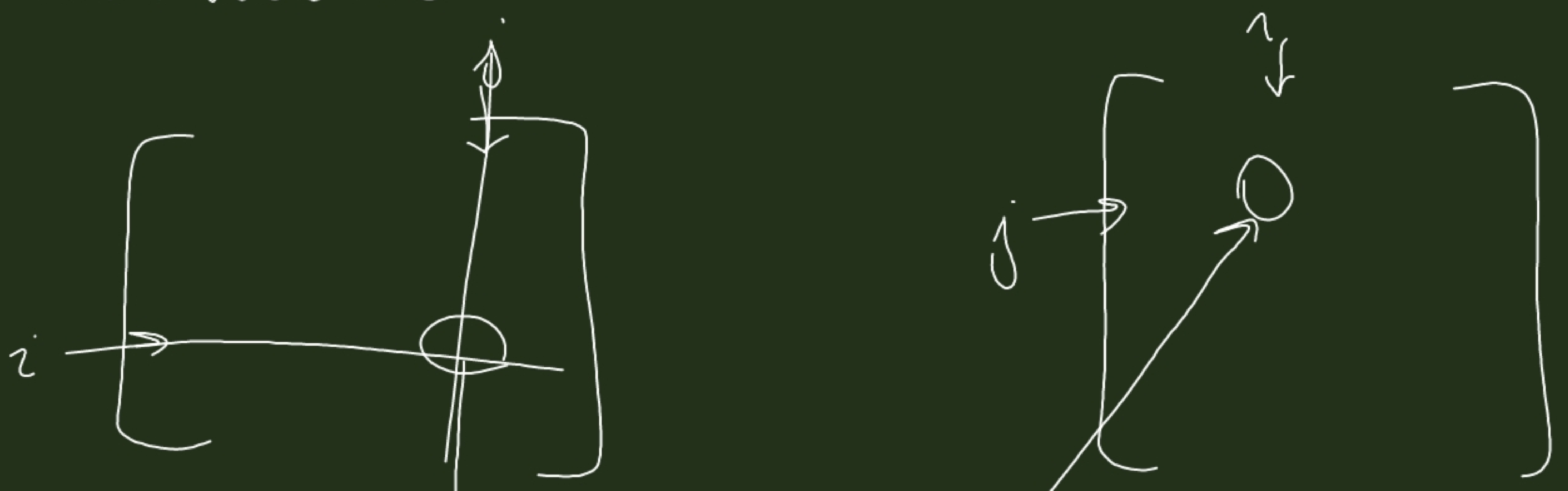
Inverz számolása (ism.)



$$(-1)^{i+j}\cdot\det(A_{ij})$$

19. fejezet, feladatok

1, HF

2, a, $A = \begin{bmatrix} 5 & 2 & -3 \\ 3 & 1 & -2 \\ 2 & -3 & -4 \end{bmatrix}$ $\quad A^{-1} = ?$

$$\left(\begin{array}{ccc|ccc} 5 & 2 & -3 & 1 & 0 & 0 \\ 3 & \boxed{1} & -2 & 0 & 1 & 0 \\ 2 & -3 & -4 & 0 & 0 & 1 \end{array}\right) \cdot \frac{1}{1} \xrightarrow[(3)+3\cdot(2)]{(1)-2\cdot(2)} \left(\begin{array}{ccc|ccc} -1 & 0 & \boxed{1} & 1 & -2 & 0 \\ \frac{1}{1}\,3 & \underline{1} & -2 & 0 & 1 & 0 \\ 11 & 0 & -10 & 0 & 3 & 1 \end{array}\right) \xrightarrow[(3)+10\cdot(1)]{(2)+2\cdot(1)}$$

$\det(A) = ?$

$\rightarrow A_1 \quad \det(A_1) = \frac{1}{1} \cdot \det(A)$

$$\left(\begin{array}{ccc|ccc} -1 & 0 & \underline{1} & 1 & -2 & 0 \\ 1 & \underline{1} & 0 & 2 & -3 & 0 \\ \boxed{1} & 0 & 0 & 10 & -17 & 1 \end{array}\right) \xrightarrow{\substack{(1)+(3) \\ (2)-(3)}} \left(\begin{array}{ccc|ccc} 0 & 0 & \underline{1} & 11 & -19 & 1 \\ 0 & \underline{1} & 0 & -8 & 14 & -1 \\ \underline{1} & 0 & 0 & 10 & -17 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|ccc} 0 & 0 & 1 & 11 & -19 & 1 \\ 0 & 1 & 0 & -8 & 14 & -1 \\ 1 & 0 & 0 & 10 & -17 & 1 \end{array}\right) \xrightarrow[\text{Sorcsere}]{(1) \leftrightarrow (3)} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 10 & -17 & 1 \\ 0 & 1 & 0 & -8 & 14 & -1 \\ 0 & 0 & 1 & 11 & -19 & 1 \end{array}\right)$$

$$A^{-1} = \begin{bmatrix} 10 & -17 & 1 \\ -8 & 14 & -1 \\ 11 & -19 & 1 \end{bmatrix}.$$

? HF: Miért a sorokat kell kicserélni?

$$A x_{1} = e_1$$
$$A x_{2} = e_2$$
$$A x_{3} = e_3$$

6, $A = \begin{bmatrix} 1 & 1 & -1 \\ 2 & 0 & 3 \\ -3 & 1 & -7 \end{bmatrix}$ $\quad A^{-1} = ? \quad \det(A) = ?$

$$\left[\begin{array}{ccc|ccc} 1 & \boxed{1} & -1 & 1 & 0 & 0 \\ 2 & 0 & 3 & 0 & 1 & 0 \\ -3 & 1 & -7 & 0 & 0 & 1 \end{array}\right] \xrightarrow{(3)-(1)} \left[\begin{array}{ccc|ccc} 1 & \underline{1} & -1 & 1 & 0 & 0 \\ \boxed{2} & 0 & 3 & 0 & 1 & 0 \\ -4 & 0 & -6 & -1 & 0 & 1 \end{array}\right]$$

A_1, $\frac{1}{1}\det(A) = \cdot\det(A_1)$

$\det(A) = 1\cdot\det(A_1)$

$$\underbrace{\begin{array}{ccc} 1 & \underline{1} & -1 \\ \boxed{2} & 0 & 3 \\ -4 & 0 & -6 \end{array}}_{A_1}\left|\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{array}\right. \xrightarrow{(2):2} \underbrace{\begin{array}{ccc} 1 & \underline{1} & -1 \\ \boxed{1} & 0 & 3/2 \\ -4 & 0 & -6 \end{array}}\left|\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1/2 & 0 \\ -1 & 0 & 1 \end{array}\right. \begin{array}{l} (1)-(2) \\ \xrightarrow{(3)+4\cdot(2)} \end{array}$$

A_2, $\det(A_2) = \frac{1}{2} \cdot \det(A_1)$

$\rightarrow \det(A_1) = 2 \cdot \det(A_2)$

$\left(\det(A) = 1 \cdot \det(A_1) = \right.$

$\left. = 1 \cdot 2 \cdot \det(A_2)\right)$

$$\downarrow$$

$$\begin{array}{ccc} 0 & \underline{1} & -5/2 \\ \underline{1} & 0 & 3/2 \\ 0 & 0 & 0 \end{array}\left|\begin{array}{ccc} 1 & -1/2 & 0 \\ 0 & 1/2 & 0 \\ -1 & 2 & 1 \end{array}\right. \longrightarrow \nexists A^{-1}$$

$\rightarrow \det(A_2) = 0$

$\det(A) = 0 \; \left(\operatorname{rang}(A) = 2\right)$

$\rightarrow \det(A) = 1 \cdot 2 \cdot \det(A_2) = 0$

3, 4 HF