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Chapter 1

Multi objective evolutionary algorithms applied to the Protein Structure Prediction Problem

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1.1 Introduction

1.2 Protein Structure Prediction

Proteins are macromolecules made out of twenty different amino acids, also referred to as residues. An amino acid has a peptide backbone and a distinctive side chain group. The peptide bond is defined by an amino group and a carboxyl group connected to an alpha carbon to which a hydrogen and side chain group are attached.

Amino acids are combined to form sequences which are considered the primary structure of the peptides or proteins. The secondary structure is the locally ordered structure brought via hydrogen bounding mainly within the peptide backbone. The most common secondary structure elements in proteins are the alpha helix and the beta sheet. The tertiary structure is the global folding of a single polypeptide chain.

Under specific conditions, the protein sequence folds into a unique native 3-d structure. Each possible protein fold has an associated energy. The *thermodynamic hypothesis* states that the native structure of a protein is the one for which the free energy achieves the global minimum. Based on this hypothesis, many methods that search for the protein native structure define an approximation of the protein energy and use optimization algorithms that look for the protein fold that minimizes this energy. These approaches mainly differ in the type of energy approximation employed and in the characteristics of the protein modeling.

1.2.1 The HP Model

The Protein Structures are very complex. Using a representation close to the real would be impossible for current computers to process the information in a reasonable time. Having this in mind, Lau and Dill [15] created a model called *Hydrophobic-Hydrophilic* Model (HP Model), to represent the proteins using simplifications. The model can be used either to represent proteins in a 2D space or 3D space.

The HP model considers two types of residues: hydrophobic (H) residues and hydrophilic or polar (P) residues. A protein is considered a sequence of these two types of residues, which are located in regular lattice models forming self-avoided paths. Given a pair of residues, they are considered neighbors if they are adjacent either in the chain (connected neighbors) or in the lattice but not connected in the chain (topological neighbors).

The total number of topological neighboring positions in the lattice (z) is called the lattice coordination number.

For the HP model, an energy function that measures the interaction between topological neighbor residues is defined as $\epsilon_{HH} = -1$ and $\epsilon_{HP} = \epsilon_{PP} = 0$. The HP problem consists of finding the solution that minimizes the total energy. In the linear representation of the sequence, hydrophobic residues are

tables, similar to the proposed method by [24], however, using the HP model for the representation and solution evaluation.

Unger and Moulton [25] described a genetic algorithm (GA) that uses heuristic-based operators for crossover and mutation for the HP model. The algorithm outperformed many variants of Monte Carlo methods for different instances. Although the good results, the GA was unable to find the optimal solution for the longest instances considered.

The multimeme algorithm (MMA) proposed by [?] is a GA combined with a set of local searches. The algorithm for each different instance or individuals in the population, selects the local search method that best fits. Originally used to find solutions for the functional model protein, the algorithm was later improved with fuzzy-logic-based local searches, leading the algorithm to find improved results in the PSP problem.

The chain growth algorithm as the pruned-enriched Rosenbluth method (PERM) [13] is based on growing the sequence conformation by adding individuals particles aiming to increase good configurations and eliminating bad ones.

The ant colony optimization (ACO) [22, 23] is an algorithm that incorporates the use of a modeling step. In this approach, the artificial ants build conformations for a given HP protein sequence, apply a local search to improve the results and maintain a probability value based on the quality of the found solutions, the so called pheromone trail.

1.3 Multi-objective Optimization

Evolutionary Algorithm (EA) is an optimization and search technique, highly parallel, inspired by the Darwinian principle of natural selection and genetic reproduction. The nature principles that inspire the EAs are simple. According to the theory of Charles Darwin, the principle of natural selection favors individuals with high fitness, therefore, with high probability of reproduction. Individuals with more descendants have more chance to perpetuate their genetic code in future generations. The genetic code is what gives the identity of each individual and is represented in the chromosomes. These principles are used in the construction of computational algorithms, that search for better solutions given a specific problem by the evolution of a population of solutions coded in artificial chromosomes – data structures used to represent a feasible solution for a given problem in the algorithm execution [19].

Real world problems commonly have multiple objectives to minimize/maximize and are present in most knowledge areas. To optimize multi objective problems, two or more objectives are considered which usually are conflicting. For these problems it is impossible to find one unique solution. A set of solutions is reached evaluating the Pareto dominance relation [2] between

the solutions. The main goal is to find the solutions that are non-dominated by any other. A solution dominates other, if and only if, it was better in at least one of the objectives, without being worst in any of the objectives. The set of non-dominated solutions constitutes the Pareto Front. Finding the real Pareto Front is a NP-hard problem [10], this way, the objective is to find a good approximation to this front.

Multi-Objective Evolutionary Algorithms (MOEAS) are extensions of EAs to multi objective problems that apply the concepts of Pareto dominance to create different strategies to evolve and diversify the solutions. In this work two MOEAs were used: NSGAII [8] and IBEA [27].

1.3.1 Non-dominated sorting Genetic Algorithm II

The main characteristic of this algorithm is a strong elitism mechanism, classifying at each generation every solution in different fronts according with the non-dominance relation (line 15 of Algorithm 1). After the classification, solutions from the first front are non-dominated by any other solution. Solutions from the second front are dominated only by the solutions of the first front, and so on. For solutions of the same front, the algorithm uses a Crowding Distance operator to calculate how distant are the neighbors of a given solution (line 19 of Algorithm 1). Solutions with high values of Crowding Distance have priority, because they will contribute more to the population's diversity. The binary tournament selects solutions from the small front with the higher values of Crowding Distance. A new population is generated using the crossover and mutation operators (line 25 of Algorithm 1).

1.3.2 IBEA (Indicator-Based Evolutionary Algorithm)

In the multi-objective optimization context, optimizing consists in finding a front with a good approximation to the true Pareto front. However, there is no general definition about what is the true Pareto front. This way, indicators have been used to evaluate the quality of an approximation front. The *hypervolume* is an example of indicator to the evaluation and comparison of fronts.

The IBEA is an algorithm that considers the optimization by the use of quality indicators. The indicator is the way used to evaluate the non-dominated set of solutions [9]. To use the IBEA it is necessary to define which indicator will be used to associate each ordered pair of solutions to a scalar value. One of the most used indicators is the *hypervolume* due to its capacity of evaluating the convergence and diversity at the same time of the search process [14].

$$F(x_i) = \sum_{x_j \in (P - x_i)} -e^{\frac{-I_{Hy}(x_j, x_i)}{k}} \quad (1.1)$$

Algorithm 1 NSGAI

```

1:  $N \leftarrow$  Population Size
2:  $T \leftarrow$  Max evaluations
3:  $P_0 \leftarrow \text{CreatePopulation}(N)$ ;
4:  $\text{CalculateFitness}(P_0)$ ;
5:  $\text{FastNonDominatedSort}(P_0)$ ;
6:  $Q_0 \leftarrow 0$ 
7: while  $Q_0 < N$  do
8:    $\text{Parents} \leftarrow \text{BinaryTournament}(P_0)$ ;
9:    $\text{Children} \leftarrow \text{CrossoverMutation}(\text{Parents})$ ;
10:   $Q_0 \leftarrow \text{Children}$ 
11: end while
12:  $\text{CalculateFitness}(Q_0)$ ;
13:  $t \leftarrow 0$ 
14: while  $t < T$  do
15:    $R_t \leftarrow P_t \cup Q_t$ ;
16:    $\text{Fronts} \leftarrow \text{FastNonDominatedSort}(R_t)$ ;
17:    $P_{t+1} \leftarrow 0$ 
18:    $i \leftarrow 0$ 
19:   while  $P_{t+1} + \text{Front}_i < N$  do
20:      $\text{CrowdingDistanceAssignment}(\text{Front}_i)$ ;
21:      $P_{t+1} \leftarrow P_{t+1} \cup \text{Front}_i$ 
22:      $i \leftarrow i + 1$ 
23:   end while
24:    $\text{CrowdingDistanceSort}(\text{Front}_i)$ ;
25:    $P_{t+1} \leftarrow P_{t+1} \cup \text{Front}_i[1 : (N - P_{t+1})]$ 
26:    $\text{Parents} \leftarrow \text{BinaryTournament}(P_{t+1})$ ;
27:    $Q_{t+1} \leftarrow \text{CrossoverMutation}(\text{Parents})$ ;
28:    $t \leftarrow t + 1$ 
29: end while
30: return  $P \leftarrow$  Set of non-dominated solutions.

```

For the IBEA fitness calculation (Equation 1.1), k is a parameter commonly used with a value of 0.05. The value for $F(x_i)$ corresponds to a quality loss measure of the approximation to the Pareto front if the solution x_i was removed of the population [9], based on the value of the quality indicator I_{Hy} , in this case, the *hypervolume*. Based on the fitness calculation described above, the basic IBEA algorithm consists in iteratively do the selection (line 10 of Algorithm 2), crossover, mutation (line 11 of Algorithm 2) and environment selection, removing the worst individual from the population and updating the values of fitness of the remaining individuals (lines 4 to 8 of Algorithm 2).

¹ *Hypervolume*: Proposed quality indicators used in the study of [28], denoted as the "size of the covered search space". This indicator has two important advantages in relation to others [26]: 1 - Sensitive to any kind of improvement in the approximation set in relation

Algorithm 2 IBEA

```

1:  $N \leftarrow$  Population Size
2:  $T \leftarrow$  Max Evaluations
3:  $k \leftarrow$  Scale factor of Fitness
4:  $P \leftarrow$  CreatePopulation( $N$ );
5:  $m \leftarrow 0$ 
6: CalculateFitness( $P$ );
7: while  $m \geq T$  or other stop criterion is reached do
8:    $\bar{P} \leftarrow$  BinaryTournament( $P$ );
9:    $P \leftarrow$  CrossoverMutation( $\bar{P}$ );
10:   $m \leftarrow m + 1$ 
11:  while Size( $P$ ) >  $N$  do
12:     $x^* \leftarrow$  WorstIndividualByFitness();
13:    RemoveFromPopulation( $x^*$ ,  $P$ );
14:    CalculateFitness( $P$ );
15:  end while
16: end while
17: return  $P \leftarrow$  Set of non-dominated solutions

```

1.4 Proposed method

Two multi-objective approaches were designed in this chapter, using the MOEAs (NSGAII and IBEA) described on subsection 1.3. The relative representation was chosen to represent the chromosomes. Integer vectors are used whereas the genes specifies which direction, relative to the previous residue, should be placed the next residue. The genes can assume only three values (0,1,2): 0 indicates that next residue should be placed on right of the previous one, 1 indicates that the next residue should be placed on left from the previous and 2 indicates that the next residue should be placed in front of the previous one. Figure 1.2 shows an example of a hypothetical chromosome and the path generated by it in the 2D lattice.

The first approach consisted on applying two well-known state of art MOEAs (IBEA and NSGAII) to the PSP using the HP-2D model. The genetic operators used by IBEA and NSGAII algorithms, in this approach, were only the single point crossover, in the case of crossover, and bit flip mutation for mutation operator. It was decided to use only the single point crossover and the bit flip mutation because this combination of operators presented better results in previous experiments within the PSP problem and the HP-2D model.

In the case of the second approach the IBEA and NSGAII algorithms

to other set. 2 - As result of 1, the indicator guarantee that for any approximation set A that has high values of hypervolume, also has all the solutions of the true Pareto front.

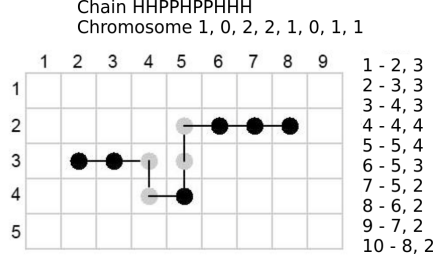


FIGURE 1.2: Example of a conformation generated by a chromosome with relative representation.

were modified in order to improve their results when compared with the first approach. The modifications implemented will be described next:

- **Pool of operators:** The use of traditional operators usually does not guide the search to prominent regions of the search space of the HP-2D model. In order to supplement the MOEAs, a pool of operators were designed based on the literature. For every mating the crossover and mutation operators are selected randomly from the pool of operators and then applied. Also the crossover and mutation operators are always applied differently from the first approach which uses a probability of occurrence. The pool of operators will be described next:
 - Single Point Crossover (1x): A single point on both parent individuals is selected. All data beyond that point in either individual will be swapped between the two parent individuals. Resulting in two distinct child individuals [12].
 - Two Points Crossover (2X): Two points are selected on both parent individuals. Everything between the two points is swapped between the parents, building two new distinct individuals [12].
 - Multi Points Crossover (MPX): The MPX operator is similar to 2X, but the number of points, c , is a function of the sequence length, n , given by $c = \text{int}(n \times 0.1)$ [6]. The MPX operator is usually used to promote structural diversity by performing a random shuffle between individuals, although not as thorough as a uniform crossover.
 - Bit Flip Mutation (BFM): The BFM operator selects one random gene from a parent individual and changes it to other value. Resulting in one new individual [12].
 - Local Move Mutation (LMM): The LMM operator swaps the directions between two randomly chosen consecutive genes. This operator introduces a corner movement [3]. Figure 1.4 presents an example of application of this operator.

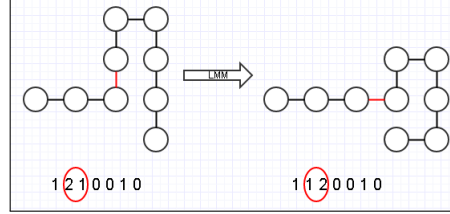


FIGURE 1.3: Example of application of the LMM operator. The genes from the red circle of leftmost figure were swapped resulting in the rightmost figure.

- Loop Move Mutation (LOMM): This operator is similar with LMM however exchanges directions between genes that are five positions apart on the sequence creating a loop movement. Both LMM and LOMM are useful to generate modifications on compact structures [7].

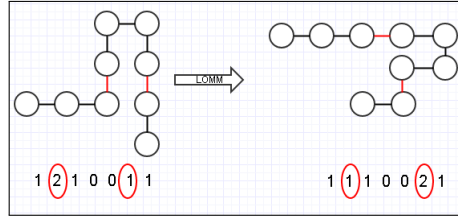


FIGURE 1.4: Example of application of the LOMM operator. The genes from the red circle of leftmost figure were swapped resulting in the rightmost figure.

- Segment Mutation (SM): This operator changes a random number of consecutive genes (from two to seven) into new random directions. This operator introduces large conformational changes and has a high probability of creating collisions, in order to avoid too much invalid solutions the repair mechanism is applied on the generated child [7]. Figure 1.4 demonstrate an application of this operator.
- Opposite Mutation (OM): This operator changes a random number of consecutive genes to its inverse position. In the case of the relative representation to the HP-2D model, only left and right can be mapped to its inverse. Figure 1.4 presents an example of application of this operator.
- **Backtrack Initialization:** Traditionally the initial population of solutions are generated randomly in the presented MOEAs. The random based generation of the solutions has a great potential of generating a

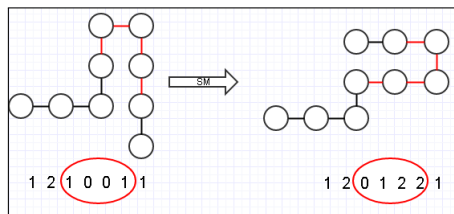


FIGURE 1.5: Example of application of the SM operator. The genes from the red circle of leftmost figure were swapped by random genes resulting in the rightmost figure.

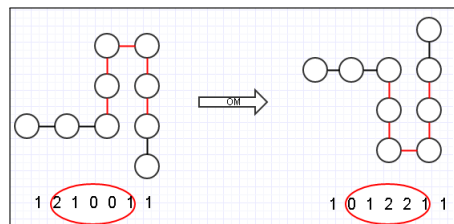


FIGURE 1.6: Example of application of the OM operator. The genes from the red circle of leftmost figure were swapped by random genes resulting in the rightmost figure.

large number of invalid solutions for the PSP problem within the HP-2D model. Solutions that are not self-avoiding walk (SAW) are said to contain collisions. If the initial population is fully generated randomly the evolutionary algorithms will spend time processing invalid solutions until getting good results. In order to subdue this problem a backtrack strategy should be applied. In this approach 20 percent of the initial population was generated using the backtrack initialization.

For both approaches the same objective functions were used and will be explained at next:

- **Energy value:** This is the main objective and consists in the energy of given protein conformation. The goal is to minimize the energy value and it is calculated as described in section X. This objective guides the search progress towards regions that the energy associated to the protein conformations are minimal, thus, achieving protein conformations which are closest to native structure of a protein.
- **Minimize the distance between the two farthest residues:** This is a secondary objective and it was inspired by the related work [11]. The motivation for this objective is that more compact conformations tend to have more hydrophobic contacts which means a lesser energy value.

The distance between two residues is calculated using the Euclidean distance.

The relative representation is subject to generation of invalid solutions using the HP-2D model. A solution is considered invalid when it solution does not perform a self-avoiding walk (SAW) as mentioned before. In other words, an invalid solution is when a given residue collides with another already placed on the lattice. A simple mechanism for repairing these situations was developed, and the code can be seen in the Algorithm 3.

Algorithm 3 Mechanism to repair infeasible solutions

1. Obtains the direction that next residue should be placed.
 2. Verifies if this direction will cause collision.
 3. If a collision is detected, a new direction is used.
 4. Repeat the steps 2 and 3, until is possible to place the next residue, or if all directions were tested and cause collisions.
 5. If it was possible to place the next residue, the mechanism achieved success, if not, the solution is considered infeasible and it will be penalized in the evaluation process.
-

This mechanism was implemented because in previous experiments it was observed that the number of infeasible solutions very high. It is necessary to mention that even with the mechanism to repair solutions, there are still infeasible solutions because the mechanism cannot always repair them. Thus, infeasible solutions are penalized by adding the number of collisions to the energy value. This mechanism is used by the evaluation process of both approaches described before (MOEAs without any modifications and the MOEAs supported by the backtracking initialization and the pool of operators).

To evaluate and compare the performance of multi-objective algorithms, quality indicators are commonly used. In this study it was used the hypervolume indicator, which considers the volume of the search space dominated by the known pareto front [29] of an algorithm. Higher hypervolume value means that the quality is better than one lesser hypervolume value.

1.5 Experiments

This section presents the set of experiments designed to evaluate the performance of the approaches introduced in section 1.4. The HP sequences used in the experiments are shown in table 1.1. Those instances have been used in previous works such as [1, 22, 25, 5, 21, 23, 16]. The values presented in table 1.1 correspond to the sequence identifier, the size of aminoacid sequence, the best known solutions ($H(x^*)$) for the HP-2D model and the sequence itself.

TABLE 1.1: HP instances used in the experiments. The search space of each instance is 2^n where n is the size of the instance.

inst.	size	$H(\mathbf{x}^*)$	sequence
<i>sq1</i>	20	-9	<i>HPHPPHHPHHPHPPHPH</i>
<i>sq2</i>	24	-9	<i>HHPPHPPHPPHPPHPPHPPHH</i>
<i>sq3</i>	25	-8	<i>PPHPPHHP⁴HHP⁴HHP⁴HH</i>
<i>sq4</i>	36	-14	<i>P³HHPPHHP⁵H⁷PPHHP⁴HHPPHPP</i>
<i>sq5</i>	48	-23	<i>PPHPPHHPHHP⁵H¹⁰P⁶</i> <i>HHPPHHPHPPH⁵</i>
<i>sq6</i>	50	-21	<i>HHPHHPHHPH⁴PHP³HP³HP⁴</i> <i>HP³HP³HPH⁴{PH}⁴H</i>
<i>sq7</i>	60	-36	<i>PPH³PH⁸P³H¹⁰PHP³</i> <i>H¹²P⁴H⁶PHHPHP</i>
<i>sq8</i>	64	-42	<i>H¹²PHPH{PPHH}²PPH{PPHH}²</i> <i>PPH{PPHH}²PPHPHPH¹²</i>

It is worthwhile to mention the sequences used in this chapter were randomly generated. Hence they do not fold to a single conformation, as natural proteins, because they are not products of natural selection [4].

The configuration used for the MOEAs was defined based on the sequence length. For smaller sequences it was used a smaller population size and for larger sequences it was used a larger population size. The same is true in the case of the stop condition (max evaluations). Table 1.2 presents the population size and max evaluations configurations used for each amino-acid sequence. In the case of the first approach the probability of crossover/mutation occurrence was fixed, for all sequences, in 0.9 and 0.01 respective. The second approach does not uses a probability since the operators are always applied to generate new individuals. The auxiliary population size used by the IBEA algorithm was fixed in 200 for all sequences.

1.5.1 Comparison between the modified/traditional versions of the MOEAs

As mentioned in the end of section 1.4 the hypervolume indicator was used in order to compare the MOEAs performance. The hypervolume results are presented on table 1.3. The hypervolume average and standard deviation, of 30 independent executions, are presented. The average values highlighted with a bold font are the highest values. Looking to table 1.3 is possible to notice, except for *sq1*, that for all sequences the M-IBEA (modified version of the IBEA with backtrack and pool of operators) obtained a higher hypervolume average than the other algorithms. In the case of *sq1* the IBEA without modifications obtained a higher value compared with the others. It is also possible to see,

TABLE 1.2: Population size and max evaluations configurations for each sequence

Sequences	Size	Population Size	Max Evaluations
sq1	20	100	25000
sq2	24	100	25000
sq3	25	500	250000
sq4	36	500	250000
sq5	48	1000	2500000
sq6	50	1000	2500000
sq7	60	2500	2500000
sq8	64	2500	2500000

comparing only the NSGAII versions, that the modified version M_NSGAII obtained better results, except for *sq1*. In general, the MOEAs with backtrack and pool of operators presented an improvement in relation to the traditional MOEAs.

1.5.2 Comparison with previous works

This section presents the comparison of the results obtained by the MOEAs with other approaches from the previous works described on section 1.2.1, and is only concerned with the first objective. (Energy of given conformation), since the other works are single-objective. Table 1.4 presents the best results, in terms of energy, found by the modified MOEAs and also the best results obtained by the previous works.

For the first 3 sequences *sq1*, *sq2* and *sq3* the modified MOEAs (M_NSGAII and M_IBEA) obtained the same results that the previous works obtained. In the case of *sq4* both M_IBEA and M_NSGAII obtained a value of -13 and all the previous works obtained the optimum value of -14. For sequence *sq5* four previous works and M_IBEA have achieved the optimum value -23. However M_NSGAII and the other previous works obtained a lesser value of -22. In the case of sequence *sq6* all algorithms obtained the optimum value of -21. For sequence *sq7* the M_IBEA obtained -35 as the EDA [20]. However the best value found for *sq7*, -36, were obtained by NewACO [23] and PERM [13]. For the last sequence *sq8* the M_IBEA obtained the optimum value of -42 which is the same obtained by EDA [20] and NewACO [23]. All other approaches obtained lesser values for sequence *sq8*.

TABLE 1.3: Results of hypervolume average/standard deviation of the MOEAs

Instance	Hypervolume Average (Std D)			
	NSGAII	M_NSGAII	IBEA	M_IBEI
sq1	0.742827 (0.106315)	0.720864 (0.131351)	0.789712 (0.067660)	0.786571 (0.099424)
sq2	0.680572 (0.083445)	0.712275 (0.137226)	0.719960 (0.080727)	0.737086 (0.095299)
sq3	0.671171 (0.129417)	0.709898 (0.124201)	0.716438 (0.148112)	0.738017 (0.155638)
sq4	0.702280 (0.0689832)	0.740153 (0.075271)	0.751755 (0.092427)	0.785728 (0.055607)
sq5	0.707654 (0.082611)	0.758128 (0.062315)	0.733464 (0.128757)	0.807637 (0.039620)
sq6	0.667771 (0.132218)	0.774017 (0.063231)	0.728699 (0.080679)	0.821177 (0.048124)
sq7	0.784483 (0.063257)	0.792843 (0.033062)	0.801778 (0.067111)	0.810351 (0.054576)
sq8	0.677464 (0.041287)	0.705798 (0.053048)	0.7450656 (0.036454)	0.811439 (0.050087)

TABLE 1.4: Comparison with the previous works

inst	M_IBEI	M_NSGAII	EDA	GA	MMA	ACO	NewACO	PERM
sq1	-9	-9	-9	-9	-9	-9	-9	-9
sq2	-9	-9	-9	-9	-9	-9	-9	-9
sq3	-8	-8	-8	-8	-8	-8	-8	-8
sq4	-13	-13	-14	-14	-14	-14	-14	-14
sq5	-23	-22	-23	-22	-22	-23	-23	-23
sq6	-21	-21	-21	-21		-21	-21	-21
sq7	-35	-34	-35	-34		-34	-36	-36
sq8	-42	-39	-42	-37		-32	-42	-38

1.6 Conclusion

MOEAs are evolutionary algorithms that try to address the challenge of optimization of multiple objectives at the same time. They have been presenting good results with many areas of science. At this chapter two well known MOEAs were applied in order to address the PSP problem using the HP-2D model. Two multi-objective approaches were presented: the first approach utilizes the IBEA and NSGAII algorithms as is, without any modification; the second approach consisted on modifying IBEA and NSGAII, adding backtrack initialization and a pool of operators, in order to enhance the results. Given the experiments results it became clear that the second approach was able to explore better the search space than the first approach. Also it was possible to check that the multi-objective approach using the IBEA algorithm, modified with backtrack and a pool of operators, obtained competitive results when compared with the previous works. However the multi-objective within NSGAII algorithm did not presented satisfactory results.



Bibliography

- [1] Ugo Bastolla, Helge Frauenkron, Erwin Gerstner, Peter Grassberger, and Walter Nadler. Testing a new monte carlo algorithm for protein folding. *arXiv preprint cond-mat/9710030*, 1997.
- [2] Henri Joseph Léon Baudrillart. *Manuel d'économie politique*. Guillaumin et cie, 1872.
- [3] Andrea Bazzoli and Andrea GB Tettamanzi. A memetic algorithm for protein structure prediction in a 3d-lattice hp model. In *Applications of Evolutionary Computing*, pages 1–10. Springer, 2004.
- [4] HS Chan and E Bornberg-Bauer. Perspectives on protein evolution from simple exact models. *Applied bioinformatics*, 1(3):121–144, 2001.
- [5] Carlos Cotta. Protein structure prediction using evolutionary algorithms hybridized with backtracking. In *Artificial Neural Nets Problem Solving Methods*, pages 321–328. Springer, 2003.
- [6] Fábio L Custódio, Hélio JC Barbosa, and Laurent E Dardenne. Investigation of the three-dimensional lattice hp protein folding model using a genetic algorithm. *Genetics and Molecular Biology*, 27(4):611–615, 2004.
- [7] Fábio Lima Custódio, Helio JC Barbosa, and Laurent Emmanuel Dardenne. A multiple minima genetic algorithm for protein structure prediction. *Applied Soft Computing*, 15:88–99, 2014.
- [8] Kalyanmoy Deb, Amrit Pratap, Sameer Agarwal, and TAMT Meyarivan. A fast and elitist multiobjective genetic algorithm: Nsga-ii. *Evolutionary Computation, IEEE Transactions on*, 6(2):182–197, 2002.
- [9] Elliackin Messias do Nascimento Figueiredo, Teresa Bernarda Orientadora Ludermir, and Carmelo José Albanez Coorientador Bastos Filho. Algoritmo baseado em enxame de partículas para otimização de problemas com muitos objetivos. 2013.
- [10] Carlos M Fonseca, Joshua D Knowles, Lothar Thiele, and Eckart Zitzler. A tutorial on the performance assessment of stochastic multiobjective optimizers. In *Third International Conference on Evolutionary Multi-Criterion Optimization (EMO 2005)*, volume 216, page 240, 2005.

- [11] Paulo HR Gabriel, Vinícius V de Melo, and Alexandre CB Delbem. Algoritmos evolutivos e modelo hp para predição de estruturas de proteínas. *Revista de Controle e Automação*, 23(1):25–37, 2012.
- [12] John H Holland. Adaptation in natural and artificial systems: An introductory analysis with applications to biology, control, and artificial intelligence. 1975.
- [13] Hsiao-Ping Hsu, Vishal Mehra, Walter Nadler, and Peter Grassberger. Growth algorithms for lattice heteropolymers at low temperatures. *The Journal of chemical physics*, 118(1):444–451, 2003.
- [14] Hisao Ishibuchi, Noritaka Tsukamoto, and Yusuke Nojima. Evolutionary many-objective optimization. In *Genetic and Evolving Systems, 2008. GEFS 2008. 3rd International Workshop on*, pages 47–52. IEEE, 2008.
- [15] Kit Fun Lau and Ken A Dill. A lattice statistical mechanics model of the conformational and sequence spaces of proteins. *Macromolecules*, 22(10):3986–3997, 1989.
- [16] Neal Lesh, Michael Mitzenmacher, and Sue Whitesides. A complete and effective move set for simplified protein folding. In *Proceedings of the seventh annual international conference on Research in computational molecular biology*, pages 188–195. ACM, 2003.
- [17] Gang Li, Jingfa Liu, Zhaoxia Liu, and Yu Zheng. Genetic algorithm with pull moves for folding 2d model proteins. *International Journal of Digital Content Technology and its Applications*, 6(21):412, 2012.
- [18] Cheng-Jian Lin and Shih-Chieh Su. Protein 3 d hp model folding simulation using a hybrid of genetic algorithm and particle swarm optimization. *International Journal of Fuzzy Systems*, 13(2):140–147, 2011.
- [19] Marco Aurélio Cavalcanti Pacheco. Algoritmos genéticos: princípios e aplicações. *ICA: Laboratório de Inteligência Computacional Aplicada. Departamento de Engenharia Elétrica. Pontifícia Universidade Católica do Rio de Janeiro. Fonte desconhecida*, 1999.
- [20] Roberto Santana, Pedro Larrañaga, Jose Lozano, et al. Protein folding in simplified models with estimation of distribution algorithms. *Evolutionary Computation, IEEE Transactions on*, 12(4):418–438, 2008.
- [21] Roberto Santana, Pedro Larranaga, and José A Lozano. Protein folding in 2-dimensional lattices with estimation of distribution algorithms. In *Biological and Medical Data Analysis*, pages 388–398. Springer, 2004.
- [22] Alena Shmygelska, Rosalia Aguirre-Hernandez, and Holger H Hoos. An ant colony optimization algorithm for the 2d hp protein folding problem. In *Ant Algorithms*, pages 40–52. Springer, 2002.

- [23] Alena Shmygelska and Holger H Hoos. An improved ant colony optimisation algorithm for the 2d hp protein folding problem. In *Advances in Artificial Intelligence*, pages 400–417. Springer, 2003.
- [24] Christiane Regina Soares Brasil, Alexandre Cláudio Botazzo Delbem, and Daniel Rodrigo Ferraz Bonetti. Investigating relevant aspects of moeas for protein structures prediction. In *Proceedings of the 13th annual conference on Genetic and evolutionary computation*, pages 705–712. ACM, 2011.
- [25] Ron Unger and John Moult. Genetic algorithms for protein folding simulations. *Journal of molecular biology*, 231(1):75–81, 1993.
- [26] Eckart Zitzler, Dimo Brockhoff, and Lothar Thiele. The hypervolume indicator revisited: On the design of pareto-compliant indicators via weighted integration. In *Evolutionary multi-criterion optimization*, pages 862–876. Springer, 2007.
- [27] Eckart Zitzler and Simon Künzli. Indicator-based selection in multiobjective search. In *Parallel Problem Solving from Nature-PPSN VIII*, pages 832–842. Springer, 2004.
- [28] Eckart Zitzler and Lothar Thiele. Multiobjective optimization using evolutionary algorithms: a comparative case study. In *Parallel problem solving from nature PPSN V*, pages 292–301. Springer, 1998.
- [29] Eckart Zitzler, Lothar Thiele, Marco Laumanns, Carlos M Fonseca, and Viviane Grunert Da Fonseca. Performance assessment of multiobjective optimizers: an analysis and review. *Evolutionary Computation, IEEE Transactions on*, 7(2):117–132, 2003.