Jacobi method

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1 Method

We want to solve the equation

$$Ax = \lambda x \tag{1.1}$$

Using

2 The frobenius norm

$$||A||_F = \sqrt{\sum_{i,j} a_{i,j}^2} \tag{2.1}$$

During the similarity transformation

$$B = S^T AS \tag{2.2}$$

The frobenius norm is conserved.
For the $2x2$ case

$$a_{pp}^2 + a_{qq}^2 + 2a_{pq}^2 = b_{pp}^2 + b_{qq}^2 + 2b_{pq} = b_{pp}^2 + b_{qq}^2 \tag{2.3}$$

Since they are symmetric. Now

$$off(B) = ||B||_F^2 - \sum_i^m b_{ii}^2 \tag{2.4}$$

$$= ||A||_F^2 - \sum_{i=1}^n a_{ii}^2 + 2app^2 + a_{qq}^2 - b_{pp}^2 - b_{qq}^2 \tag{2.5}$$

$$= ||A||_F^2 - \sum_{i=1}^n a_{ii}^2 - 2a_{pq}^2 \tag{2.6}$$

$$= off(A) - 2a_{pq}^2 \tag{2.7}$$

And thus

$$off(B) \leq off(A) \tag{2.8}$$

3 Other

$$b_{pq} = 0 = a_{pq}(c^2 - s^2) + (a_{pp} - a_{qq})cs \tag{3.1}$$

$$\tau = \frac{a_{qq} - a_{pp}}{2a_{pq}} \tag{3.2}$$

$$\tau\theta = \frac{s}{c} \tag{3.3}$$

Which gives

$$t^2 + 2t\tau - 1 = 0 \tag{3.4}$$

$$t = -\tau \pm \sqrt{1 + \tau^2} \tag{3.5}$$

We have best convergence when the absolute value of θ is less than equal to $\pi/4$. We would then choose the smaller ofthe roots. Assume $\tau \geq 0$, then we're choosing

$$t = -\tau + \sqrt{1 + \tau^2} \tag{3.6}$$

This may cause problems due to subtraction of very equal numbers. What we do is multiply the equation over and under with $(\tau + \sqrt{1 + \tau^2})$

4 Estimate of floating point operations

Referencing to the jacobi implemented by morten.

Method	Flops
off-diag	n^2
Rotation part (Withouth eigenvectors)	$6n$ (standard claim in txtbooks)

Tabell 4.1: Flops of the different jacobi.