1. singular configration:-

While the manipulator is moving, there are some configratins for which the velocity component of three directions are linearly dependent.

So, for some configration, we can't independently set the velocity and solve for joint variables and $\dot{q}_s$.

This is the reasult of decrese in rank of the jakobian matrix. If the rank of jacobian is less than it's maximum value, it is in a singular configration.

→ to find singular configratins, we can calculate the ~~derivative of~~ rank of jacobian.

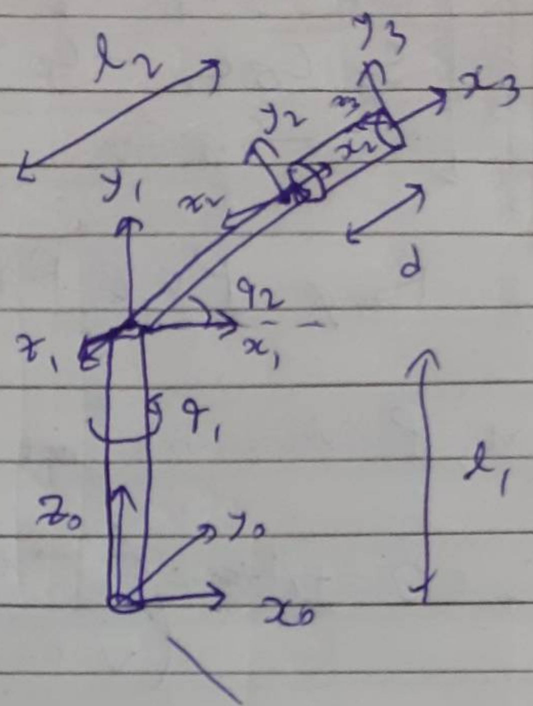
→ if the colomns of jacobian are linearly dependent, it is in singular configration.

⇒ for a nxn jacobian matrix, if det(J) is near zero, it is near singular configratin.

→ for other dimentions, we need to check if coloumns are approx. linearly dependent or not.

(for example $\begin{bmatrix} 1 & 1.9 \\ 2 & 3.9 \\ 3 & 5.9 \end{bmatrix}$ is close to singular config.)

4. ⇒ for RRP - standford manipulator,



→ D-H Parameters table.

Link	Link params. θ_i	d_i	a_i	α_i
1	q_1^*	l_1	0	$\pi/2$
2	q_2^*	0	l_2	0
3	0	0	d^*	0

⇒ using A2_q8.ipynb code to compare position of end effector,

→ for $q_1 = q_2 = 0$, $d = 1$

from A3_q3 code, $[2, 0, 1]^T$; from A2-q8, $[2, 0, 1]^T$

→ for $q_1 = \pi/2$, $q_2 = 0$, $d = 1$

from A3_q3 code, $[0, 2, 1]^T$; from A2-q8, $[0, 2, 1]^T$

→ for , $q_1 = q_2 = \pi/4$, $d = 0.1$.

using A3-q3 code, $\begin{bmatrix} 0.55 \\ 0.55 \\ 1.778 \end{bmatrix}$

using A2_q8 code, $\begin{bmatrix} 0.55 \\ 0.55 \\ 1.778 \end{bmatrix}$

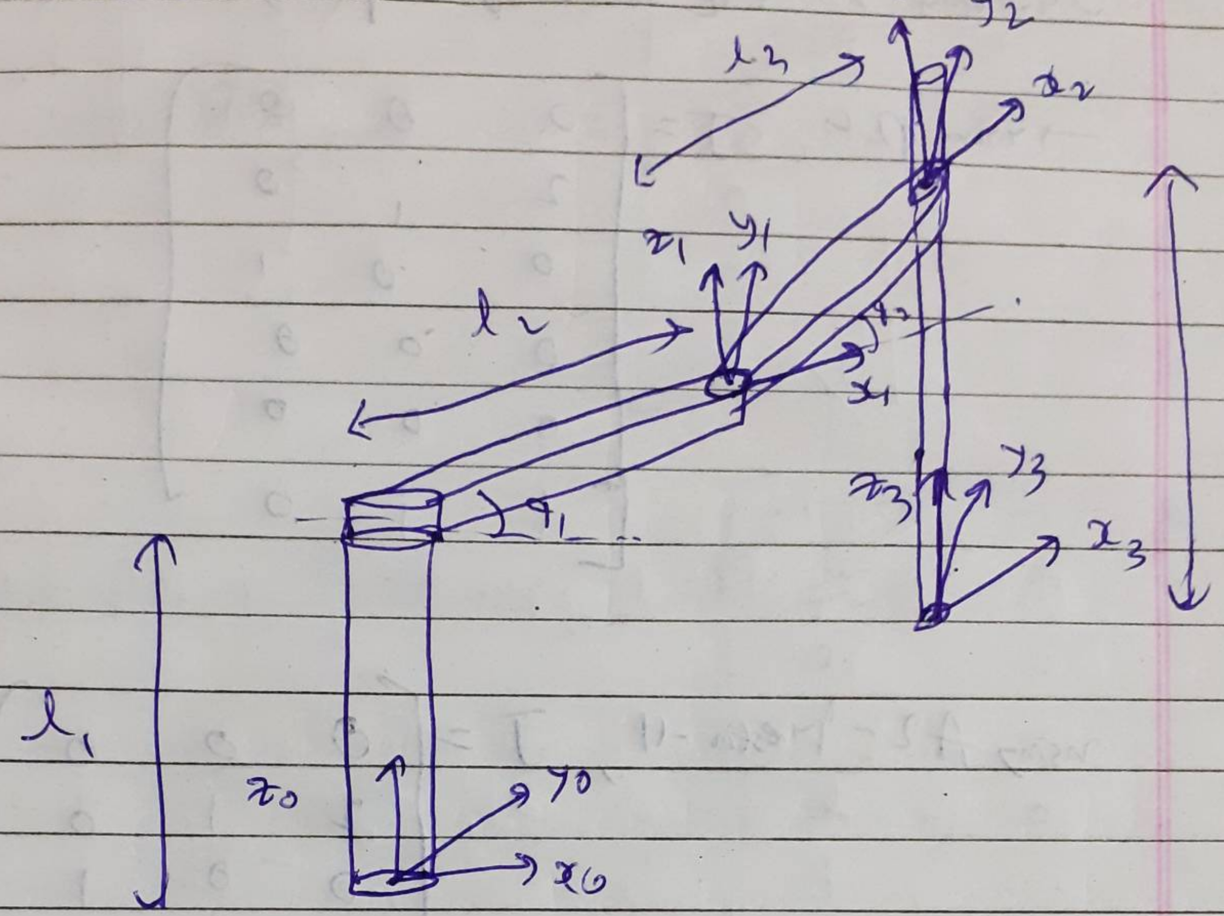
⇒ So, we can see that the results are same for D-H parmeters method and ~~using~~ by manually assigning the axis as shown in question 8 - assignment-2.

⇒ Rotation of end effector for $q_1 = q_2 = \pi/4$, $d = 0.1$

using A3_q3 (D-H params), $\begin{bmatrix} 0.5 & -0.5 & 0.71 \\ 0.5 & -0.5 & -0.71 \\ 0.71 & 0.71 & 0 \end{bmatrix}$

using A2_q8 (manual multiplication), $\begin{bmatrix} 0.5 & -0.5 & 0.71 \\ 0.5 & -0.5 & -0.71 \\ 0.71 & 0.71 & 0 \end{bmatrix}$

⟹ RRP SCARA :-



⟹ D-H Param table

Link	θ_i	d_i	a_i	α_i
1	q_1^*	l_1	l_2	0
2	q_2^*	0	l_3	0
3	0	d^*	0	0

⟹ for $q_1 = q_2 = 0$, $d = -0.1$

using D-H params, position of end effector = $[2, 0, 0.9]^T$

using Assignment-2, Q-6's ans = $[2, 0, 0.9]^T$

Jacobian for the same joint params,

→ using D-H, $J = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

using A2 - Question - 11, $J = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

⇒ for $q_1 = \pi/4$, $q_2 = \pi/4$, $d = 0.5$,

Position of end effector :-

using D-H params, $\begin{bmatrix} 0.707 \\ 1.707 \\ 1.5 \end{bmatrix}$

using Assignment - 2, Question - 6, $\begin{bmatrix} 1/\sqrt{2} \\ 1+\frac{1}{\sqrt{2}} \\ 1.5 \end{bmatrix}$

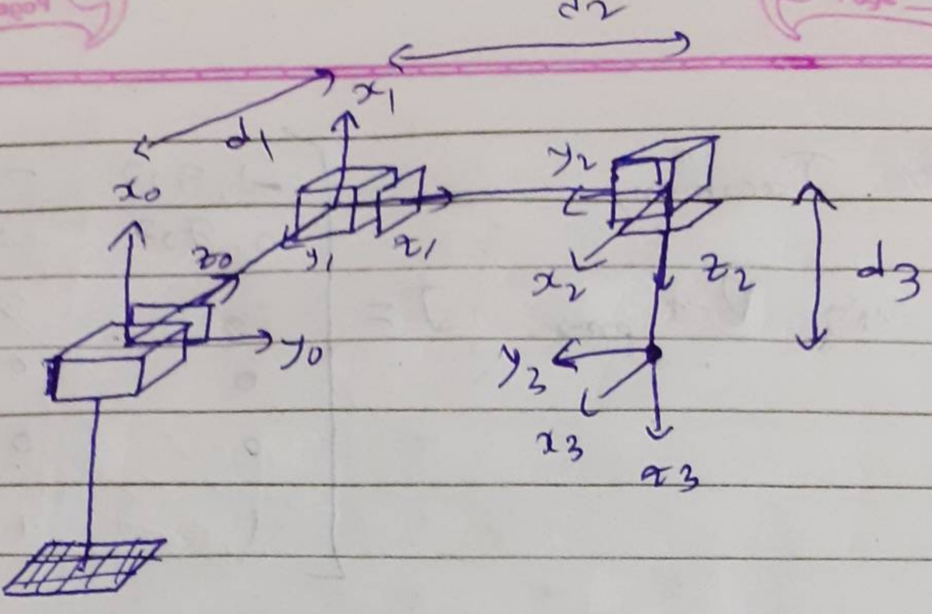
⇒ for Jacobian,

using D-H paras. $J = \begin{bmatrix} -1.707 & -1 & 0 \\ 0.707 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

using Asig-2 question-11, $J = \begin{bmatrix} -1-\frac{1}{\sqrt{2}} & -1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

⇒ so, for RRP-SCARA, the ~~value~~ values got from D-H. convention are same as that from using manual axis and using transformations. (using Assignment-2 question-6 & 11)

5.



→ D-H parameters table:-

Link	θ_i	d_i	a_i	α_i
1	0	d_1^*	0	$-\pi/2$
2	$\pi/2$	d_2^*	0	$-\pi/2$
3	0	d_3^*	0	0

$$\therefore A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_0^3 = A_1 A_2 A_3$

$$= \begin{bmatrix} 0 & 0 & -1 & -d_3 \\ 0 & -1 & 0 & d_2 \\ -1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

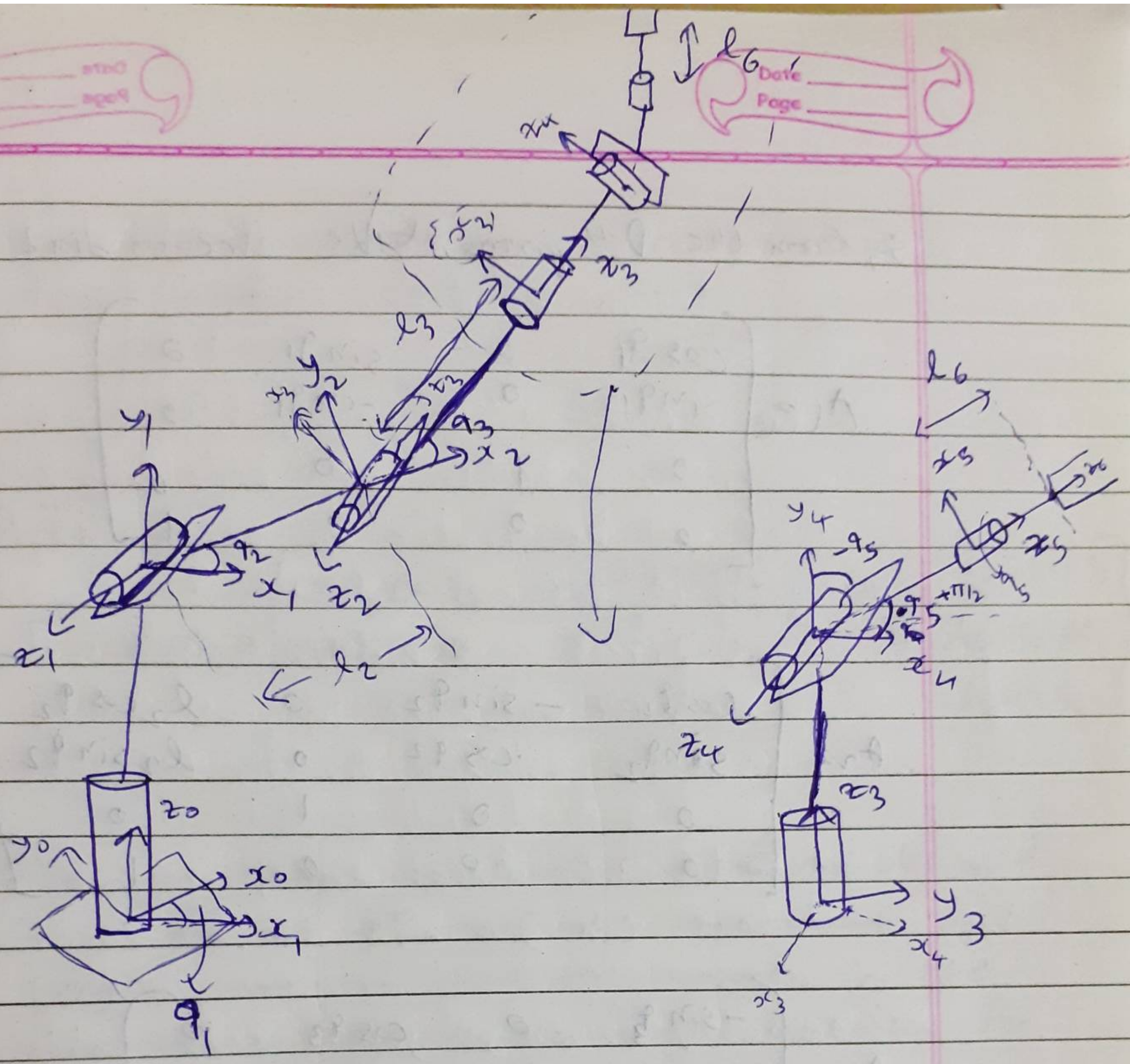
$\Rightarrow$ to check the above matrix with the code in question-3,

take, $d_1 = 1$, $d_2 = 2$, $d_3 = 3$ in D-H params.

$$T_0^1(1,2,3) = \begin{bmatrix} 0 & 0 & -1 & -3 \\ 0 & -1 & 0 & 2 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

which mathes with what we derived by hand.

6.



→ D-H Params:-

Link	θ_i	d_i	a_i	α_i
1	q_1^*	0	0	$\pi/2$
2	q_2^*	0	l_2	0
3	$q_3^* + \pi/2$	0	0	$\pi/2$
4	$q_4^* + \pi/2$	l_3	0	$\pi/2$
5	$q_5^* + \pi/2$	0	0	$\pi/2$
6	q_6^*	l_6	0	0

So, from the DH params table we can find

$$A_1 = \begin{bmatrix} \cos q_1 & 0 & \sin q_1 & 0 \\ \sin q_1 & 0 & -\cos q_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & l_2 \cos q_2 \\ \sin q_2 & \cos q_2 & 0 & l_2 \sin q_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -\sin q_3 & 0 & \cos q_3 & 0 \\ \cos q_3 & 0 & \sin q_3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} -\sin q_4 & 0 & \cos q_4 & 0 \\ \cos q_4 & 0 & \sin q_4 & 0 \\ 0 & 1 & 0 & l_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} -\cos q_5 & 0 & -\sin q_5 & 0 \\ -\sin q_5 & 0 & \cos q_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} \cos q_6 & -\sin q_6 & 0 & 0 \\ \sin q_6 & \cos q_6 & 0 & 0 \\ 0 & 0 & 1 & l_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\Rightarrow T_0^1 = A_1$

$$T_0^2 = A_1 A_2 = \begin{bmatrix} \cos q_1 \cos q_2 & -\cos q_1 \sin q_2 & \sin q_1 & l_2 \cos q_1 \cos q_2 \\ \sin q_1 \cos q_2 & -\sin q_1 \sin q_2 & -\cos q_1 & l_2 \sin q_1 \cos q_2 \\ \sin q_2 & \cos q_2 & 0 & l_2 \sin q_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

→ as the expresion in terms of joint variables and l_2, l_3, l_6 will be complicated, ~~[illegible]~~ and lengthy, we can check the answers by the code and by these matrix by substituting for $q_i = 0$ (for $i = 1$ to 6) ~~and $l_2 = 2$, $l_3 = 3$, $l_6 = 6$~~.

so, $$T_0^6(0,0,0,0,0,0) = \begin{bmatrix} 0 & 0 & 1 & l_2 + l_3 + l_6 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

→ to check above T_0^6 with code in Task-3, we can take $l_2 = 2$, $l_3 = 3$, $l_6 = 6$ and we get,

$$T_0^6(\text{from code}) = \begin{bmatrix} 0 & 0 & 1 & 11 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

7. → There are three ways to actuate the 2R manipulator, one of them is direct drive.

⇒ In directly driven joints, there are no gears. so, there is no backlash. responce time will be quicker as there are no gears in between.

— as direct drive doesn't ~~incl~~ include gear box, it will have less cost. It is just a motor attached directly to the joint.

→ to use direct drive, we need a motor that can give high torque for lower RPM.

⇒ remotely-driven joint uses some sort of gears or other mechanism to drive the joints while the motor is in the base frame.

→ This is easier to impliment as the arms ~~need not~~ will not be having an additional weight of motr to lift.

→ No additional torque is applied by the motor to the privious join like directly driven joints.

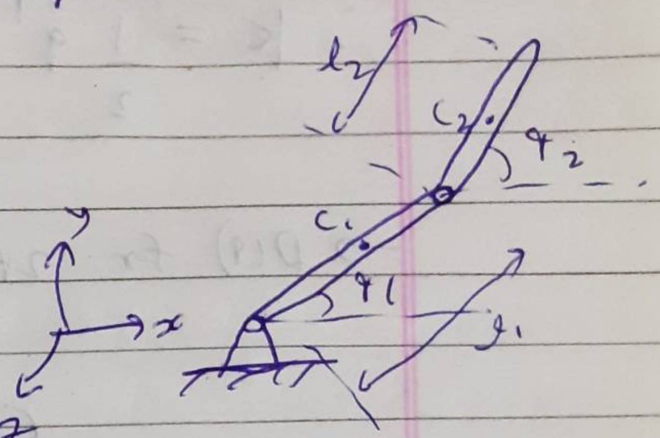
⇒ in 5-bar parallelogram arrangement, both motors can be on the ground and the links are arranged such that it can achive workspace similer to 2R manipulator.

→ This can be thought as an extention to remotely driven joint or just anothe way to drive the joint. so, it has the advantages same as the remotely driven joints has.

→ It has a workspace similer to the serial manipulator but it is smaller in size due to constraints on other bars.

8. → Dynamics eq^n of 2R manipulator :-

$$v_{c1} = \begin{bmatrix} -\frac{l_1}{2}\sin q_1 \\ \frac{l_1}{2}\cos q_1 \\ 0 \end{bmatrix} \dot{q}_1$$



$$v_{c2} = \begin{bmatrix} -l_1\sin q_1 & -\frac{l_2}{2}\sin q_2 \\ l_1\cos q_1 & \frac{l_2}{2}\cos q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$w_1 = \dot{q}_1 \hat{k}$, $w_2 = \dot{q}_2 \hat{k}$

→ kinetic energy , $K = \frac{1}{2}\sum_{i=1}^{n} m_i v_{ci}^T v_{ci} + \frac{1}{2}\sum_{i=1}^{n} w_i^T I_i w_i$

where, $v_{ci} = J_{vci}(q)\cdot\dot{q}$, $w_i = R_i^T J_{wi}(q)\dot{q}$

$$\therefore K = \frac{1}{2}\dot{q}^T \sum_{i=1}^{n}\left[m_i J_{vci}(q)^T J_{vci}(q) + J_{wi}(q)^T R_i(q) . I_i R_i(q)^T J_{wi}(q)\right]\dot{q}$$

$\therefore K = \frac{1}{2} \dot{q}^T D(q) \dot{q}$

$\rightarrow$ $D(q)$ for 2R manipulator,

$$D(q) = \begin{bmatrix} m_1 \frac{l_1^2}{4} + m_2 l_1^2 + I_1 & m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1) \\ m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1) & m_2 \frac{l_2^2}{4} + I_2 \end{bmatrix}$$

$\rightarrow$ christoffel symbols,

$$C_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

$$\therefore C_{111} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_1} = 0$$

$$C_{121} = C_{211} = \frac{1}{2} \frac{\partial d_{11}}{\partial q_2} = 0$$

$$C_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2} \frac{\partial d_{22}}{\partial q_1} = -m_2 \frac{l_1 l_2}{2} \sin(q_2 - q_1)$$

$\therefore\ C_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = m_2 \frac{l_1 l_2}{2} \sin(q_2 - q_1)$

$C_{212} = C_{122} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0$

$C_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$

$\Rightarrow$ Potential energy,

$V = m_1 g \frac{l_1}{2} \sin q_1 + m_2 g\left(l_1 \sin q_1 + \frac{l_2}{2} \sin q_2\right)$

$\therefore\ \phi_1 = \frac{\partial V}{\partial q_1} = m_1 g \frac{l_1}{2} \cos q_1 + m_2 g l_1 \cos q_1$

$\phi_2 = \frac{\partial V}{\partial q_2} = m_2 g \frac{l_2}{2} \cos q_2$

$\Rightarrow$ so, final eq^ns are,

$d_{11} \ddot{q}_1 + d_{12} \ddot{q}_2 + C_{221} \dot{q}_2^2 + \phi_1 = \tau_1$

$$\therefore \left(m_1 \frac{l_1^2}{4} + m_2 l_2^2 + \frac{1}{12} m l_1^2\right) \ddot{q}_1 +$$

$$m_2 \frac{l_1 l_2}{2} \cos(q_2 - q_1) \ddot{q}_2 - m_2 \frac{l_1 l_2}{2} \sin(q_2 - q_1) \dot{q}_1^2 +$$

$$\frac{1}{2} m_1 g l_1 \cos q_1 + m_2 g l_1 \cos q_1 = \tau_1 \quad \text{--- (1)}$$

~~ethm~~

$\Rightarrow$ for $k = 2$,

$$d_{21} \ddot{q}_1 + d_{22} \ddot{q}_2 + C_{12} \dot{q}_1^2 + \phi_2 = \tau_2$$

$$\therefore \frac{1}{2} m_2 l_1 l_2 \cos(q_2 - q_1) \ddot{q}_1 + \left(\frac{1}{4} m_2 l_2^2 + \frac{1}{12} m_2 l_2^2\right) \ddot{q}_2 +$$

$$\frac{1}{2} m_2 l_1 l_2 \sin(q_2 - q_1) \dot{q}_1^2 + \frac{1}{2} m_2 g l_2 \cos(q_2) = \tau_2$$

$$\therefore \frac{1}{2} m_2 l_1 l_2 \cos(q_2 - q_1) \ddot{q}_1 + \frac{1}{3} m_2 l_2^2 \ddot{q}_2 + \frac{1}{2} m_2 l_1 l_2 \sin(q_2 - q_1) \dot{q}_1^2$$

$$+ \frac{1}{2} m_2 g l_2 \cos q_2 = \tau_2 \quad \text{--- (2)}$$

$\rightarrow$ eqn (1) & (2) are exactly same as that of miniproject.

10.8. ~~A~~ for given matrix $D(q)$,

the kinetic energy,

$$K = \frac{1}{2} \sum_{i,j}^{n} d_{ij}(q)\, \dot{q}_i \dot{q}_j$$

$$= \frac{1}{2} \dot{q}^T D(q) \dot{q}$$

so, $L = K - V$

$$= \frac{1}{2} \sum_{i,j}^{n} d_{ij}(q)\, \dot{q}_i \dot{q}_j - V(q)$$

(assuming potential energy is depending on q only and not $\dot{q}$)

$$\rightarrow \frac{\partial L}{\partial \dot{q}_k} = \sum_{j} d_{kj}(q)\, \dot{q}_j$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) = \sum_{j} d_{kj}(q)\, \ddot{q}_j + \sum_{j} \frac{d}{dt}\left(d_{kj}(q)\right) \dot{q}_j$$

$$= \sum_{j} d_{kj}(q)\, \ddot{q}_j + \sum_{i,j} \frac{\partial d_{kj}}{\partial q_i} \dot{q}_i \dot{q}_j$$

$$\rightarrow \frac{\partial L}{\partial q_k} = \frac{1}{2} \sum_{ij} \frac{\partial d_{ij}}{\partial q_k} \dot{q}_i \dot{q}_j - \frac{\partial V}{\partial q_k}$$

$\therefore$ using Langrangian's eq^n,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = \tau_k \quad , \quad k = 1, 2, \ldots, n$$

$$\therefore \sum_j d_{kj} \ddot{q}_j + \sum_{i,j} \left[\frac{\partial d_{kj}}{\partial q_i} - \frac{1}{2} \frac{\partial d_{ij}}{\partial q_k} \right] \dot{q}_i \dot{q}_j + \frac{\partial V}{\partial q_k} = \tau_k \qquad \text{--- (1)}$$

$$\rightarrow \text{by symmetry}, \sum_{i,j} \left(\frac{\partial d_{kj}}{\partial q_i} \right) \dot{q}_i \dot{q}_j = \frac{1}{2} \sum_{i,j} \left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} \right] \dot{q}_i \dot{q}_j$$

So, using the symmetry, we can simplify the eq^n (1) to the following,

$$\sum_j d_{kj}\ddot{q}_j + \sum_{i,j}\frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]\dot{q}_i\dot{q}_j$$

$$+ \frac{\partial V}{\partial q_k} = \tau_k$$

taking $c_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$

$$\boxed{\sum_j d_{kj}(q)\,\ddot{q}_j + \sum_{i,j} c_{ijk}(q)\,\dot{q}_i\dot{q}_j + \phi_k(q) = \tau_k}$$

where, $\phi_k(q) = \frac{\partial V}{\partial q_k}$

→ in matrix form,

$$\boxed{D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau}$$