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ASSIGNMENT - 2

(1) R_0^1

Let us consider Rotation matrix

$$\begin{array}{cccc} & \text{Column 1} & \text{Column 2} & \text{Column 3} \\ & \downarrow & \downarrow & \downarrow \end{array}$$

$$R_{z,\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

~~Since~~ let us take Column 1 and 2.

we know that for two vectors to be orthogonal there dot product should be 'zero'.

(column 1) . (column 2)

$$\begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} \cdot \begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix} = -\cos\theta\sin\theta + \sin\theta\cos\theta + 0 = 0$$

$$(\text{Column 2}) \cdot (\text{column3}) = \begin{pmatrix} -\sin\theta \\ \cos\theta \\ 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = 0+0+0 = 0$$

$$(\text{Column 3}) \cdot (\text{Column 1}) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} = 0+0+0 = 0$$

Hence columns of a Rotation matrix are orthogonal.

(2) R_0^1

let us take $R_{z,\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$|R_0^1| = |R_{z,\theta}| = \cos\theta(\cos\theta) + \sin\theta(\sin\theta) + 0 = 1$$

Hence $\det(R_0^1) = 1$

(5) To show: $RS(a)R^T = S(Ra)$

Proof: An important property possessed by the skew symmetric matrice is linearity i.e for any vector a and b, and scalars α and β, we have

$$S(\alpha a + \beta b) = \alpha S(a) + \beta S(b)$$

Another important property of $S(a)$ is that for any vector $P = (P_x, P_y, P_z)^T$

$$S(a)P = a \times P \quad \text{--- (1)}$$

If $R \in SO(3)$ and a, b are vectors, it can be shown by direct calculation that

$$R(a \times b) = Ra \times Rb \quad \text{--- (2)}$$

(→ this equation is true since R is an orthogonal matrix)

For any $R \in SO(3)$ and any $b \in \mathbb{R}^3$, it can be shown from equation (1) & (2) that,

$$RS(a)R^T b = R(a \times R^T b)$$

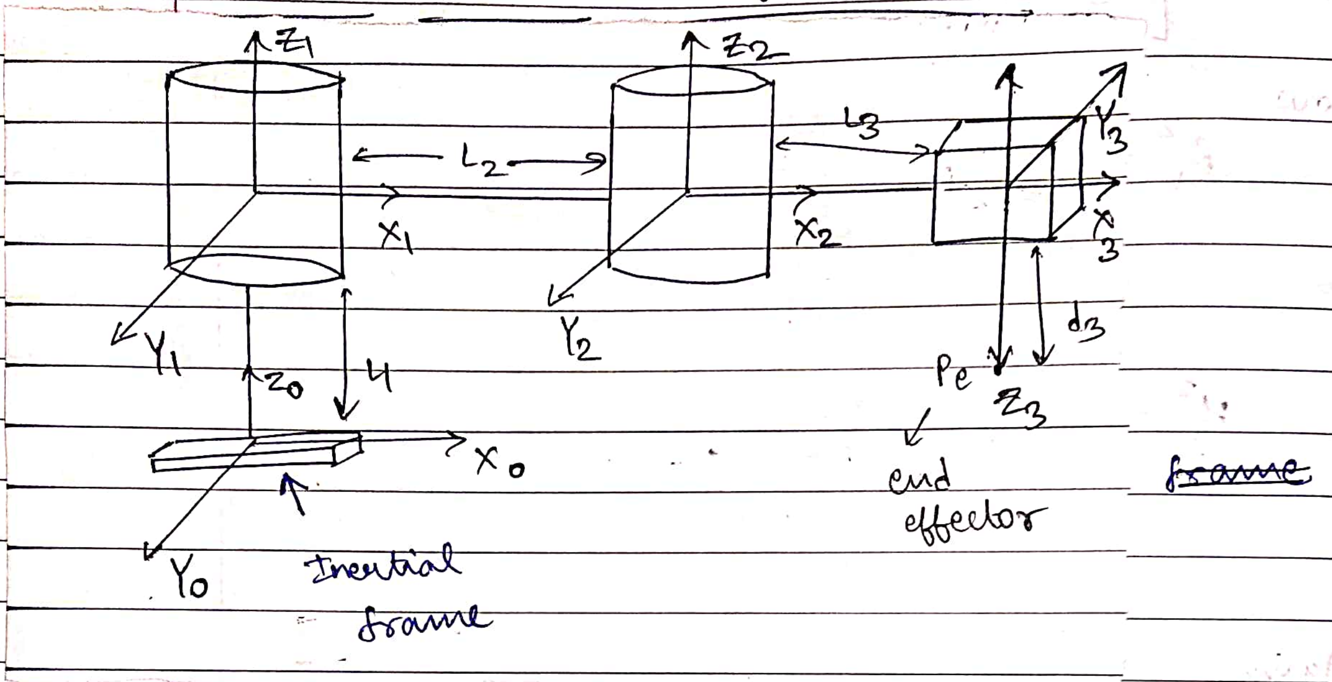
$$= (Ra) \times (RR^T b)$$

$$= (Ra) \times b \quad \text{since } R \text{ is orthogonal}$$

$$= S(Ra)b$$

Hence, $\boxed{RS(a)R^T = S(Ra)}$

(6) RRP Scara Configuration



Consider the above shown RRP SCARA configuration robot.

From Homogeneous transformation, we know that

$$\Rightarrow \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$\Rightarrow H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix}$$

$$\Rightarrow H_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix}$$

Now

$$R_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_1^2 = \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & L_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Again, $H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix}$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$, Since axes are rotated about x_3 axis by $180°$

and $d_2^3 = \begin{bmatrix} L_3 \\ 0 \\ 0 \end{bmatrix}$

$\Rightarrow$

$$\begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ d_3 \\ 1 \end{bmatrix}, \quad H_2^3 = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 P_3 = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ d_3 \\ 1 \end{bmatrix} = \begin{bmatrix} L_3 \\ 0 \\ -d_3 \\ 1 \end{bmatrix}$$

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$$H_0^1 H_1^2 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & L_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2 & -\cos\theta_1\sin\theta_2 - \sin\theta_1\cos\theta_2 & 0 & L_2\cos\theta_1 \\ \sin\theta_1\cos\theta_2 + \cos\theta_1\sin\theta_2 & -\sin\theta_2\sin\theta_1 + \cos\theta_1\cos\theta_2 & 0 & L_2 S\theta_1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_2+\theta_1) & 0 & L_2\cos\theta_1 \\ \sin(\theta_1+\theta_2) & +\cancel{\sin}\cos(\theta_1+\theta_2) & 0 & L_2\sin\theta_1 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Hence $H_0^1 H_1^2 H_2^3 P_3 = \begin{bmatrix} L_3 C(\theta_1+\theta_2) + L_2\cos\theta_1 \\ S(\theta_1+\theta_2) L_3 + L_2 S\theta_1 \\ -d_3 + L_1 \\ 1 \end{bmatrix}$

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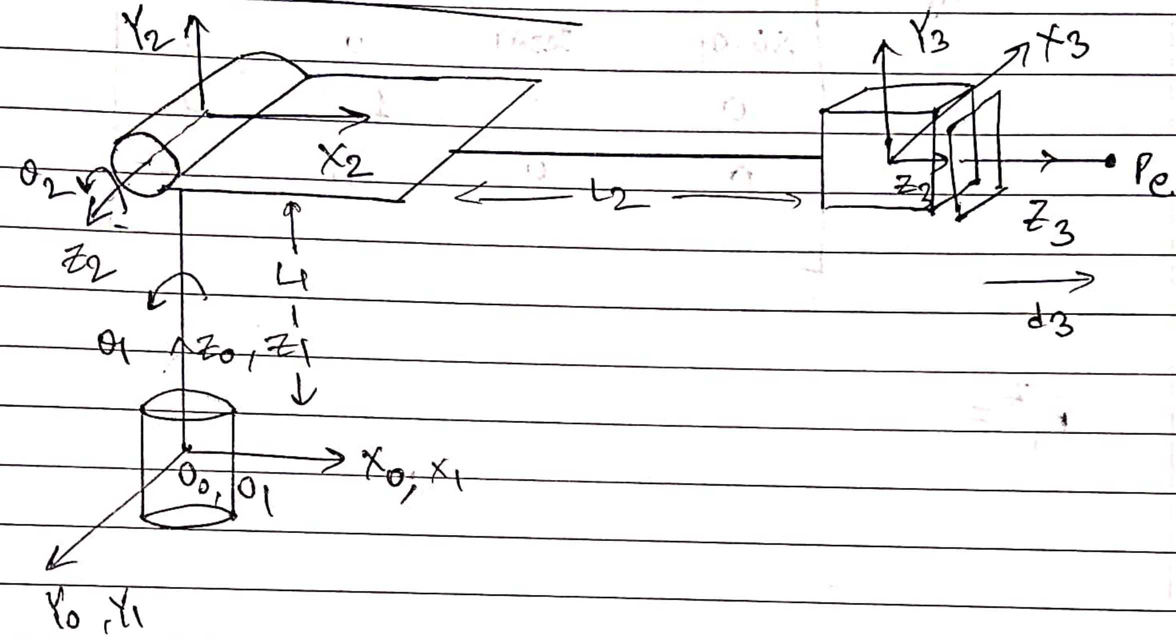
(8) RRP Stanford type



Consider above shown RRP stanford configuration

From Homogeneous transformation equations.

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \; H_1^2 \; H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 90 & -\sin 90 \\ 0 & \sin 90 & \cos 90 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ 0 & 0 & -1 \\ \sin\theta_2 & \cos\theta_2 & 0 \end{bmatrix}, \quad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} L_2 + d_3 \\ 0 \\ 0 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} 0 & 0 & -1 & L_2 + d_3 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now $\begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ d_3 \\ 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \, H_1^2 \, H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ +\sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ -0 & 0 & -1 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & L_2+d_3 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

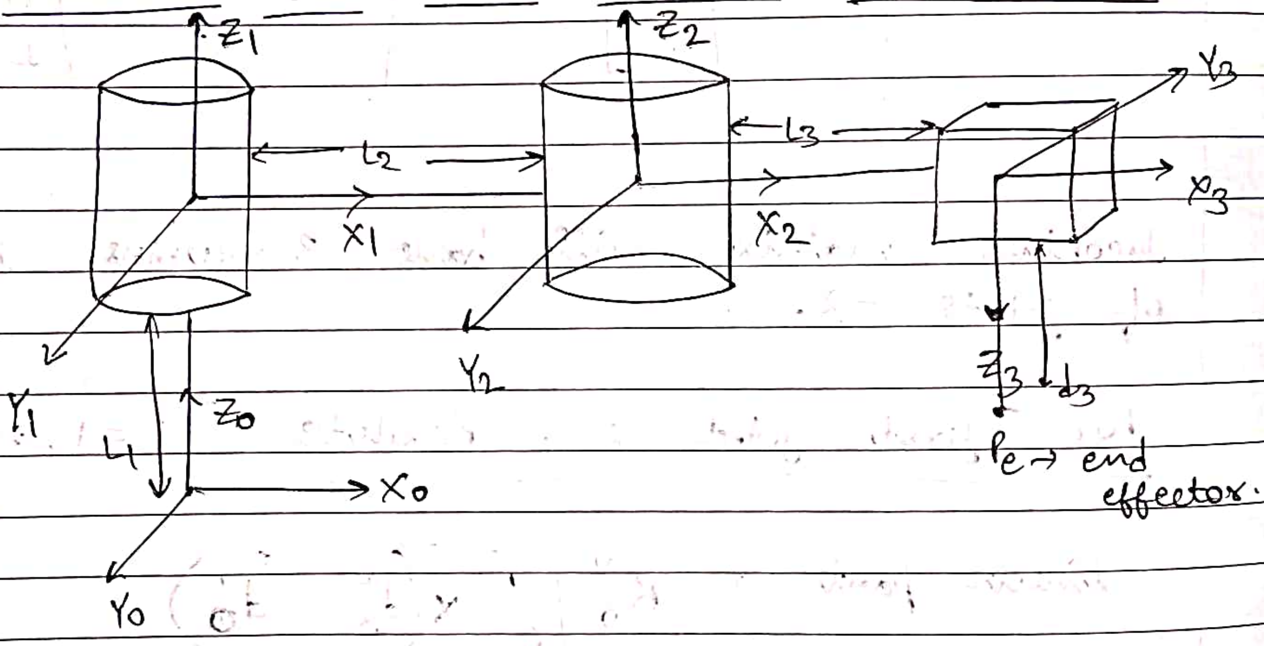
$$= \begin{bmatrix} \cos\theta_1\cos\theta_2 & -\cos\theta_1\sin\theta_2 & \sin\theta_1 & 0 \\ \sin\theta_1\cos\theta_2 & -\sin\theta_1\sin\theta_2 & -\cos\theta_1 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & L_2+d_3 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sin\theta_1 & -\cos\theta_1\sin\theta_2 & -\cos\theta_1\cos\theta_2 & (L_2+d_3)\cos\theta_1\cos\theta_2 \\ -\cos\theta_1 & -\sin\theta_1\sin\theta_2 & -\sin\theta_1\cos\theta_2 & (L_2+d_3)\sin\theta_1\cos\theta_2 \\ 0 & \cos\theta_2 & -\sin\theta_2 & \sin\theta_2(L_2+d_3)+L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ d_3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -d_3\cos\theta_1\cos\theta_2 + (L_2+d_3)\cos\theta_1\cos\theta_2 \\ -d_3\sin\theta_1\cos\theta_2 + (L_2+d_3)\sin\theta_1\cos\theta_2 \\ -d_3\sin\theta_2 + \sin\theta_2(L_2+d_3)+L_1 \\ 1 \end{bmatrix}$$

Ans So $\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} L_2 \cos\theta_1 \cos\theta_2 \\ L_2 \sin\theta_1 \cos\theta_2 \\ L_2 \sin\theta_2 + L_1 \\ 1 \end{bmatrix}$

(11) Jacobian for RRP SCARA Configuration



we know that

$$\begin{bmatrix} v_0^n \\ \omega_0^n \end{bmatrix} = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} \dot{q}$$

$$\Rightarrow \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = J \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

	Prismatic	Revolute
Linear	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d_n^0 - d_{i-1}^0)$
Rotational	$\begin{bmatrix}0\\0\\0\end{bmatrix}$	$R_{i-1}^{0}\begin{bmatrix}0\\0\\1\end{bmatrix}$

Jacobian matrix will have 3 columns as no. of joints = 3.

For first joint i.e. Revolute $i=1, n=3$

linear part $\rightarrow$ $R_0^0\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d_3^0 - d_0^0)$

Rotational part $\rightarrow$ $R_0^0\begin{bmatrix}0\\0\\1\end{bmatrix}$

For second joint i.e. Revolute $i=2, n=3$

linear part $\rightarrow$ $R_1^0\begin{bmatrix}0\\0\\1\end{bmatrix}\times(d_3^0 - d_1^0)$

Rotational part $\rightarrow$ $R_1^0\begin{bmatrix}0\\0\\1\end{bmatrix}$

for third joint i.e Prismatic $i=3, n=3$

linear part $R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Rotational part $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

Now

$$R_0^0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow \text{Identity matrix}$$

for d_3^0

$$H_3^0 = \begin{bmatrix} R_3^0 & d_3^0 \\ 0 & 1 \end{bmatrix}$$

and $H_3^0 = H_0^1 H_1^2 H_2^3 = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 & L_2\cos\theta_1 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 & L_2 S\theta_1 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$\begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & \sin(\theta_1+\theta_2) & 0 & L_3 C(\theta_1+\theta_2) + L_2\cos\theta_1 \\ \sin(\theta_1+\theta_2) & -\cos(\theta_1+\theta_2) & 0 & L_3 S(\theta_1+\theta_2) + L_2 S\theta_1 \\ 0 & 0 & -1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$$\Rightarrow d_3^0 = \begin{bmatrix} L_3 \cos(\theta_1+\theta_2) + L_2 \cos\theta_1 \\ L_3 \sin(\theta_1+\theta_2) + L_2 \sin\theta_1 \\ L_1 \end{bmatrix}$$

$$H_1^0 = \begin{bmatrix} R_1^0 & d_1^0 \\ 0 & 1 \end{bmatrix}$$

$$d_1^0 = \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix}$$

~~For~~ First joint :

linear $\rightarrow \underset{3\times1}{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}} \times \underset{3\times1}{\begin{bmatrix} L_3 \cos(\theta_1+\theta_2) + L_2 \cos\theta_1 \\ L_3 \sin(\theta_1+\theta_2) + L_2 \sin\theta_1 \\ L_1 \end{bmatrix}}$

Rotational $\rightarrow \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} -L_3 \sin(\theta_1+\theta_2) - L_2 \sin\theta_1 \\ +L_3 \cos(\theta_1+\theta_2) + L_2 \cos\theta_1 \\ 0 \end{bmatrix}$$

Second joint

linear $\rightarrow \begin{bmatrix} \cancel{\cos\theta_2} & 0 \\ \cancel{\sin\theta_2} & 0 \\ & 1 \end{bmatrix} \times \begin{bmatrix} L_3 \cos(\theta_1+\theta_2) + L_2 \cos\theta_1 \\ L_3 \sin(\theta_1+\theta_2) + L_2 \sin\theta_1 \\ 0 \end{bmatrix}$

$\begin{bmatrix} -L_3 \sin(\theta_1+\theta_2) - L_2 \sin\theta_1 \\ +L_3 \sin(\theta_1+\theta_2) + L_2 \sin\theta_1 \\ 0 \end{bmatrix}$ Rotational $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Third joint

$$R_2^0 = R_0^1 R_1^2$$

$$= \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Ans $\rightarrow$

$$J = \begin{bmatrix} -L_3S(\theta_1+\theta_2) - L_2\sin\theta_1 & -L_3S(\theta_1+\theta_2) - L_2S\theta_1 & 0 \\ L_3\cos(\theta_1+\theta_2) + L_2\cos\theta_1 & L_3S(\theta_1+\theta_2) + L_2S\theta_1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

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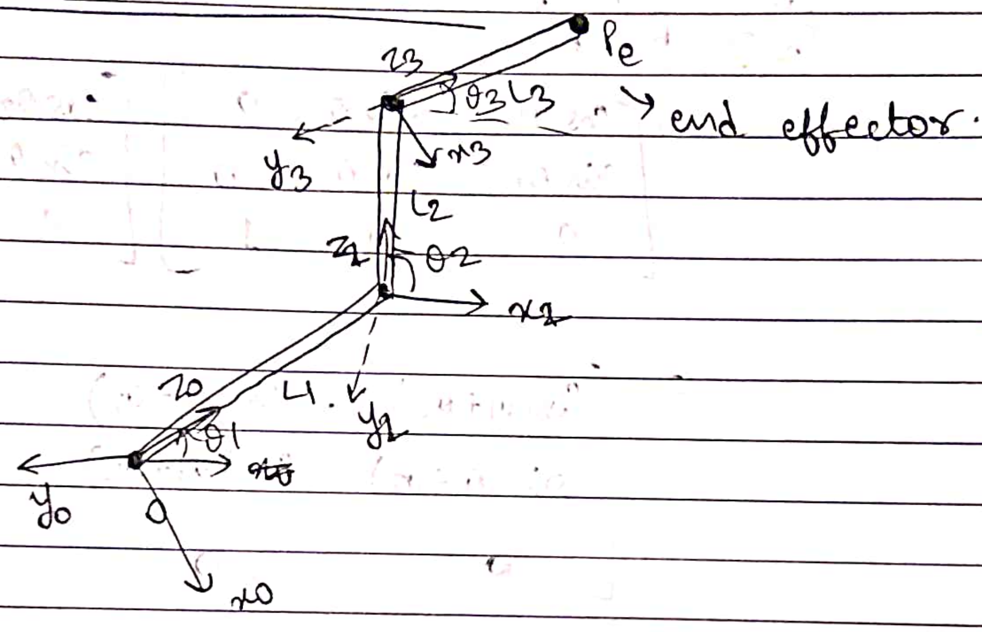
(13) RRR Configuration



Consider above RRR Configuration
y_0, y_1, y_2, y_3 axis are perpendicular to paper and are parallel to each other.

end effector Position w.r.t. 0 frame. $\rightarrow \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \, H_1^2 \, H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$ — (2)

$\therefore \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ L_3 \\ 1 \end{bmatrix}$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$H_0^1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_2^3 = \begin{bmatrix} 0 \\ 0 \\ L_2 \end{bmatrix}$$

$$H_0^1 H_1^2 H_2^3 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & l_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{— } (1)$$

From eq^n (1) & eq^n (2)

we will find $\begin{bmatrix} P_0 \\ 1 \end{bmatrix}$ ~~use~~

Now for Jacobian

Revolute

linear $\quad R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left(d_n^0 - d_{i-1}^0 \right)$

Prismatic : $\quad R_{i-1}^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = J \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_n \end{bmatrix}$$

For First joint

$i=1$, $n=3$

linear $R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_0^0)$

Prismatic $R_0^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

For Second joint

$i=2$, $n=3$

linear $R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_1^0)$

Prismatic $R_1^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

For III joint

$i=3$, $n=3$

linear $R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times (d_3^0 - d_2^0)$

Prismatic $R_2^0 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

(9) Rotation matrix for location of drone wrt ground frame

$$R_{eq} = R_{30°,x} \times R_{60°,z}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 \\ 0 & \sin 30 & \cos 30 \end{bmatrix} \begin{bmatrix} \cos 60 & -\sin 60 & 0 \\ \sin 60 & \cos 60 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Position of drone

$$\cancel{\begin{bmatrix} P_{drone} \\ 1 \end{bmatrix} = \begin{bmatrix} R_{eq} & d_0^f \\ 0 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} P_{lidar} \\ 1 \end{bmatrix} = \begin{bmatrix} R_{eq} & d_0^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_2 \\ 1 \end{bmatrix} \quad — (1)$$

$$d_0^2 = \begin{bmatrix} d_0^1 \end{bmatrix} \begin{bmatrix} d_1^2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} , \quad \begin{bmatrix} P_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

Hence P_{lidar} can be found from equation (1).

Question 10
Ans)

GEARBOX
A gearbox is a mechanical device used to increase the output torque or change the speed (RPM, rotation per minute) of a motor. The motor's shaft is attached to one end of the gearbox and through the internal configuration of gears of a gearbox, it provides a given output torque and speed determined by the gear ratio. Geared motors are used in robotics where high torque is needed.
Few gearboxes that are widely used in the field of robotics, as per the requirement, are-
Twin-Motor Gearbox

- This is a ready to assemble gearbox that uses two electric motors to independently control each of the output shafts.
- It is strong enough only to power up small robots.
- Low cost.
- The gears and gear case are accurately moulded from tough plastic for smooth and efficient operation.
- Two gear ratios, 207:1 or 58:1 can be selected.
  It might fail to Synchronize the two motors connected at some time.

6- Speed Gearbox

- Gearbox can be assembled for six different gear ratios. Select one according to application before assembling. Gear ratio can be altered from 11.6:1 to 1300.9:1 by gear combinations. 3 volt RE-260 motor. Gear types include 8T, 36/14T, and a clutch.
- Gear box accepts 130 or 140 motor types. Other parts include a crossed horn and output shaft. Use cam for vertical movement or change to rectilinear movement.
- Grease included for lubrication, plus a hex wrench and small philips screws.

4 – Speed Gearbox

- By altering the combinations of four gears, four different gear ratios can be obtained: 126:1, 441:1, 1543:1, 5402:1.
- Higher speed setting is suitable for car model, and so on.
- Lower speed setting is suitable for tracked vehicle model, and so on.
- Included motor operates on 3 volts.

Worm Gearbox

- Can produce extremely low speeds of rotation and is used for gear reduction.
- Allows 2 gear ratios, 216.1:1 and 336.1:1 to be selected.
- Gear case is injection moulded and gears are polyacetal resin for reducing mechanical noise.
- Accepts 140 or 260 type motors.
- Included motor operates on 3-4.5 volts.

Planetary Gearbox

- A highly versatile system for reduction of high RPM electric motors to high-torque low RPM applications.
- Often used in precision instruments because of the reliability and accuracy of the unit.
- RC-260 motor operates on 3 volts.
- Combining the gearboxes in various ways enables 8 different gear ratios (4:1, 5:1, 16:1, 20:1, 25:1, 80:1, 100:1, 400:1). At a gear ration of 400:1 and voltage of 3V, the gearbox produces a torque of about 15 kg.

High Power Gearbox

- Designed for high torque output rather than speed.
- Suitable for constructing models such as tanks, buggies, tracked vehicles, and so on. Allows 2 gear ratios, 41.7:1 and 64.8:1 to be selected.
- Gear case is injection moulded and gears are polyacetal resin for reducing mechanical noise. Accepts 140 or 260 type motors. Included motor operates on 3-4.5 volts.

Universal Gearbox

- The Universal Gear Box works well in a variety of applications and can be mounted several ways. A hex shaft and a threaded shaft are included to give the builder flexibility in design.
- The plastic gears and metal gear box are all high quality.

Mini-Motor Low Speed Gearbox

- This type 030 motor features a low electricity consumption rate and does not emit electronic noise.

- Gearbox features 7 different gear ratios (71.4:1, 149.9:1, 314.9:1, 661.2:1) which can be easily adjusted by changing the final gear attachment position 3mm hex shaft and gearbox attachment holes are compatible with other Tamiya craft items.
- 2 different gearbox attachment positions are available.
- The gearbox has a transparent gear case to show the internal mechanisms.

Mini Motor Multi Ratio Gearbox

- This gearbox features a wide range of low to high gear ratios (12 Speeds).
- Type 030 motor with a transparent gear case to show the internal mechanisms.

Vertical Shaft Worm Drive Gear Box

- This gearbox utilizes a 30:1 ratio worm-drive reduction for applications that require extremely slow and smooth rotational motion.
- The Vertical Shaft Worm-Drive Gearbox is perfect for turn-tables, time-lapse systems and low-speed applications that require high precision and torque.

Having a top-quality drone motor will give longer flight times, exceptional flight stability and more precise flying. It will be quieter, have a better cooling system and will last longer.
Selection of motor entirely depends on the application's requirement.
This means that we must determine end results such as how big of a load are you moving and how fast do you want it to move, and then translate these into requirements such as output torque and speed.
It should have enough power to lift the craft off the ground. So, we need to know the total drone weight, including the frame and the camera.
Choosing the correct combination of a motor and a gearbox for a given application is very important.
Without appropriate motor-gearbox combos, robot does not function as quickly and effectively as intended, and may have a tendency to burn out motors.