ASSIGNMENT - 2

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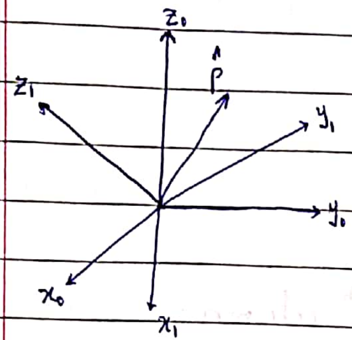
1) We consider the axis transformation $x_0 y_0 z_0$ to $x_1 y_1 z_1$.



We get the rotation matrix:

$$R_0^1 = \begin{bmatrix} \hat{i}_1 \cdot \hat{i}_0 & \hat{j}_1 \cdot \hat{i}_0 & \hat{k}_1 \cdot \hat{i}_0 \\ \hat{i}_1 \cdot \hat{j}_0 & \hat{j}_1 \cdot \hat{j}_0 & \hat{k}_1 \cdot \hat{j}_0 \\ \hat{i}_1 \cdot \hat{k}_0 & \hat{j}_1 \cdot \hat{k}_0 & \hat{k}_1 \cdot \hat{k}_0 \end{bmatrix}$$

For R_0^1 ~~to be orthogonal &~~ to have orthogonal columns, we have, the relation,

$$R_0^1 \, R_0^{1\,T} = I \quad \ldots$$

We use this condition of orthogonality of R_0^1 to prove that columns of R_0^1 are orthogonal

$\therefore$ Solving LHS, we get;

$$R_0^1 R_0^{1\,T} = \begin{bmatrix} \hat{i}_1 \cdot \hat{i}_0 & \hat{j}_1 \cdot \hat{i}_0 & \hat{k}_1 \cdot \hat{i}_0 \\ \hat{i}_1 \cdot \hat{j}_0 & \hat{j}_1 \cdot \hat{j}_0 & \hat{k}_1 \cdot \hat{j}_0 \\ \hat{i}_1 \cdot \hat{k}_0 & \hat{j}_1 \cdot \hat{k}_0 & \hat{k}_1 \cdot \hat{k}_0 \end{bmatrix} \begin{bmatrix} \hat{i}_1 \cdot \hat{i}_0 & \hat{i}_1 \cdot \hat{j}_0 & \hat{i}_1 \cdot \hat{k}_0 \\ \hat{j}_1 \cdot \hat{i}_0 & \hat{j}_1 \cdot \hat{j}_0 & \hat{j}_1 \cdot \hat{k}_0 \\ \hat{k}_1 \cdot \hat{i}_0 & \hat{k}_1 \cdot \hat{j}_0 & \hat{k}_1 \cdot \hat{k}_0 \end{bmatrix} = I$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} (\hat{i}_1 \cdot \hat{i}_0)^2 + (\hat{j}_1 \cdot \hat{i}_0)^2 + (\hat{k}_1 \hat{i}_0)^2 & (\hat{i}_1 \cdot \hat{i}_0)(\hat{i}_1 \cdot \hat{j}_0) + (\hat{j}_1 \cdot \hat{i}_0)(\hat{j}_1 \hat{j}_0) + (\hat{k}_1 \hat{i}_0)(\hat{k}_1 \hat{j}_0) & (\hat{i}_1 \hat{i}_0)(\hat{i}_1 \hat{k}_0) + (\hat{j}_1 \hat{i}_0)(\hat{j}_1 \hat{k}_0) + (\hat{k}_1 \hat{i}_0)(\hat{k}_1 \hat{k}_0) \\ (\hat{i}_1 \hat{j}_0)(\hat{i}_1 \hat{i}_0) + (\hat{j}_1 \hat{j}_0)(\hat{j}_1 \hat{i}_0) + (\hat{k}_1 \hat{j}_0)(\hat{k}_1 \hat{i}_0) & (\hat{i}_1 \hat{j}_0)^2 + (\hat{j}_1 \hat{j}_0)^2 + (\hat{k}_1 \hat{j}_0)^2 & (\hat{i}_1 \hat{j}_0)(\hat{i}_1 \hat{k}_0) + (\hat{j}_1 \hat{j}_0)(\hat{j}_1 \hat{k}_0) + (\hat{k}_1 \hat{j}_0)(\hat{k}_1 \hat{k}_0) \\ (\hat{i}_1 \hat{k}_0)(\hat{i}_1 \hat{i}_0) + (\hat{j}_1 \hat{k}_0)(\hat{j}_1 \hat{i}_0) + (\hat{k}_1 \hat{k}_0)(\hat{k}_1 \hat{i}_0) & (\hat{i}_1 \hat{k}_0)(\hat{i}_1 \hat{j}_0) + (\hat{j}_1 \hat{k}_0)(\hat{j}_1 \hat{j}_0) + (\hat{k}_1 \hat{k}_0)(\hat{k}_1 \hat{j}_0) & (\hat{i}_1 \hat{k}_0)^2 + (\hat{j}_1 \hat{k}_0)^2 + (\hat{k}_1 \hat{k}_0)^2 \end{bmatrix}$$

We observe the corresponding values of the matrix terms. In order to prove column orthogonality, we need to multiply $R_0^{1\,T} R_1^0$ to get required terms. $R_1^0 R_0^{1\,T}$ gives all terms that prove row orthogonality. $\therefore$ We have;

$$R_0^{1\,T} R_1^0 = \begin{bmatrix} \hat{i}_1 \cdot \hat{i}_0 & \hat{i}_1 \cdot \hat{j}_0 & \hat{i}_1 \cdot \hat{k}_0 \\ \hat{j}_1 \cdot \hat{i}_0 & \hat{j}_1 \cdot \hat{j}_0 & \hat{j}_1 \cdot \hat{k}_0 \\ \hat{k}_1 \cdot \hat{i}_0 & \hat{k}_1 \cdot \hat{j}_0 & \hat{k}_1 \cdot \hat{k}_0 \end{bmatrix} \begin{bmatrix} \hat{i}_1 \cdot \hat{i}_0 & \hat{j}_1 \cdot \hat{i}_0 & \hat{k}_1 \cdot \hat{i}_0 \\ \hat{i}_1 \cdot \hat{j}_0 & \hat{j}_1 \cdot \hat{j}_0 & \hat{k}_1 \cdot \hat{j}_0 \\ \hat{i}_1 \cdot \hat{k}_0 & \hat{j}_1 \cdot \hat{k}_0 & \hat{k}_1 \cdot \hat{k}_0 \end{bmatrix} = I$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} (\hat{i}_1 \hat{i}_0)^2 + (\hat{i}_1 \hat{j}_0)^2 + (\hat{i}_1 \hat{k}_0)^2 & (\hat{i}_1 \hat{i}_0)(\hat{j}_1 \hat{i}_0) + (\hat{i}_1 \hat{j}_0)(\hat{j}_1 \hat{j}_0) + (\hat{i}_1 \hat{k}_0)(\hat{j}_1 \hat{k}_0) & (\hat{i}_1 \hat{i}_0)(\hat{k}_1 \hat{i}_0) + (\hat{i}_1 \hat{j}_0)(\hat{k}_1 \hat{j}_0) + (\hat{i}_1 \hat{k}_0)(\hat{k}_1 \hat{k}_0) \\ (\hat{j}_1 \hat{i}_0)(\hat{i}_1 \hat{i}_0) + (\hat{j}_1 \hat{j}_0)(\hat{i}_1 \hat{j}_0) + (\hat{j}_1 \hat{k}_0)(\hat{i}_1 \hat{k}_0) & (\hat{j}_1 \hat{i}_0)^2 + (\hat{j}_1 \hat{j}_0)^2 + (\hat{j}_1 \hat{k}_0)^2 & (\hat{j}_1 \hat{i}_0)(\hat{k}_1 \hat{i}_0) + (\hat{j}_1 \hat{j}_0)(\hat{k}_1 \hat{j}_0) + (\hat{j}_1 \hat{k}_0)(\hat{k}_1 \hat{k}_0) \\ (\hat{k}_1 \hat{i}_0)(\hat{i}_1 \hat{i}_0) + (\hat{k}_1 \hat{j}_0)(\hat{i}_1 \hat{j}_0) + (\hat{k}_1 \hat{k}_0)(\hat{i}_1 \hat{k}_0) & (\hat{k}_1 \hat{i}_0)(\hat{j}_1 \hat{i}_0) + (\hat{k}_1 \hat{j}_0)(\hat{j}_1 \hat{j}_0) + (\hat{k}_1 \hat{k}_0)(\hat{j}_1 \hat{k}_0) & (\hat{k}_1 \hat{i}_0)^2 + (\hat{k}_1 \hat{j}_0)^2 + (\hat{k}_1 \hat{k}_0)^2 \end{bmatrix}$$

$\longleftarrow$ 1

$\therefore$ For column orthogonality, we have,

$(\hat{i}_1\cdot\hat{i}_0)(\hat{j}_1\cdot\hat{i}_0) + (\hat{i}_1\cdot\hat{j}_0)(\hat{j}_1\cdot\hat{j}_0) + (\hat{i}_1\cdot\hat{k}_0)(\hat{j}_1\cdot\hat{k}_0) = 0$ $\quad\cdots$ C_1 & C_2 orthogonal

$(\hat{j}_1\cdot\hat{i}_0)(\hat{k}_1\cdot\hat{i}_0) + (\hat{j}_1\cdot\hat{j}_0)(\hat{k}_1\cdot\hat{j}_0) + (\hat{j}_1\cdot\hat{k}_0)(\hat{k}_1\cdot\hat{k}_0) = 0$ $\quad\cdots$ C_2 & C_3 orthogonal

$(\hat{k}_1\cdot\hat{i}_0)(\hat{i}_1\cdot\hat{i}_0) + (\hat{k}_1\cdot\hat{j}_0)(\hat{i}_1\cdot\hat{j}_0) + (\hat{k}_1\cdot\hat{k}_0)(\hat{i}_1\cdot\hat{k}_0) = 0$ $\quad\cdots$ C_3 & C_1 orthogonal.

$\therefore$ We get all the above equations from the uppertriangular part of the matrices in equation ①.

$\therefore$ It is proved that the columns of R_0' are orthogonal.

2) To prove: $\det(R_0') = 1$

$\therefore$ We have:- $R_0' = \begin{bmatrix} \hat{i}_1\cdot\hat{i}_0 & \hat{j}_1\cdot\hat{i}_0 & \hat{k}_1\cdot\hat{i}_0 \\ \hat{i}_1\cdot\hat{j}_0 & \hat{j}_1\cdot\hat{j}_0 & \hat{k}_1\cdot\hat{j}_0 \\ \hat{i}_1\cdot\hat{k}_0 & \hat{j}_1\cdot\hat{k}_0 & \hat{k}_1\cdot\hat{k}_0 \end{bmatrix}$

$\therefore$ Calculating, we get;

$$\det(R_0') = \hat{i}_1\cdot\hat{i}_0\left[(\hat{j}_1\cdot\hat{j}_0)(\hat{k}_1\cdot\hat{k}_0) - (\hat{k}_1\cdot\hat{j}_0)(\hat{j}_1\cdot\hat{k}_0)\right] - \hat{j}_1\cdot\hat{i}_0\left[(\hat{i}_1\cdot\hat{j}_0)(\hat{k}_1\cdot\hat{k}_0) - (\hat{k}_1\cdot\hat{j}_0)(\hat{i}_1\cdot\hat{k}_0)\right]$$
$$+\ \hat{k}_1\cdot\hat{i}_0\left[(\hat{i}_1\cdot\hat{j}_0)(\hat{j}_1\cdot\hat{k}_0) - (\hat{i}_1\cdot\hat{k}_0)(\hat{j}_1\cdot\hat{j}_0)\right]$$
$$= (\hat{i}_1\cdot\hat{i}_0)(\hat{j}_1\cdot\hat{j}_0)(\hat{k}_1\cdot\hat{k}_0) - (\hat{i}_1\cdot\hat{i}_0)(\hat{k}_1\cdot\hat{j}_0)(\hat{j}_1\cdot\hat{k}_0) - (\hat{j}_1\cdot\hat{i}_0)(\hat{i}_1\cdot\hat{j}_0)(\hat{k}_1\cdot\hat{k}_0) + (\hat{j}_1\cdot\hat{i}_0)(\hat{k}_1\cdot\hat{j}_0)(\hat{i}_1\cdot\hat{k}_0)$$
$$+ (\hat{k}_1\cdot\hat{i}_0)(\hat{i}_1\cdot\hat{j}_0)(\hat{j}_1\cdot\hat{k}_0) - (\hat{k}_1\cdot\hat{i}_0)(\hat{i}_1\cdot\hat{k}_0)(\hat{j}_1\cdot\hat{j}_0)$$

$\therefore$ We try an alternate approach.

We have, due to orthogonality of R_0',

$$R_0'\, R_0'^{T} = I$$

$$\therefore \quad \det(R_0'\, R_0'^{T}) = \det(I).$$

$$\det(R_0')\det(R_0'^{T}) = 1 \qquad \cdots \det(AB) = \det(A)\det(B)$$

$$\det(R_0')\det(R_0') = 1 \qquad \cdots \det(A^T) = \det(A)$$

$$\det(R_0')^2 = 1$$

$$\det(R_0') = \pm 1$$

$\therefore$ Taking the current convention (right hand, counterclockwise = +ve). we get $\det(R_0') = 1$ and hence we can eliminate the case of $\det(R_0') = -1$

$\therefore$ Due to definition of convention,

$$\det(R_0') = 1$$

$\therefore$ Proved.

5) We have a rotation matrix R & the operator S. We have,

① $S(\alpha a + \beta b) = \alpha S(a) + \beta S(b)$ ... $a, b \in \mathbb{R}^3$ are vectors $\alpha, \beta \in$ Constan

② $S(a)\,p = a \times p$ ... $a, p \in \mathbb{R}^3$ are vectors.

③ $R(a \times b) = Ra \times Rb$ ... $a, b \in \mathbb{R}^3$ are vectors, $R \in SO(3)$

∴ For $R \in SO(3)$ & $a \in \mathbb{R}^3$, we have,

$$R\,S(a)\,R^T = R\,S(a)\,R^T b * \frac{1}{b}$$

$$= R(a \times R^T b) * \frac{1}{b} \quad \text{... from ②.}$$

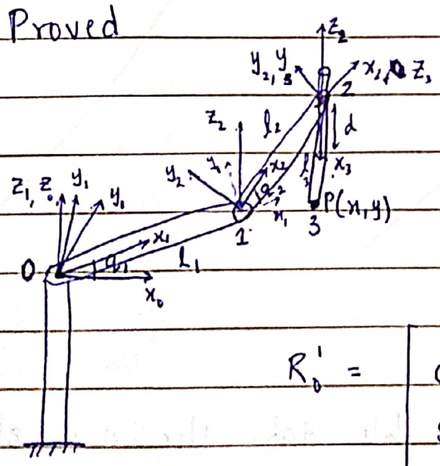
$$= (Ra \times RR^T b) * \frac{1}{b} \quad \text{.... from ③.}$$

$$= (Ra \times b) * \frac{1}{b} \quad \text{... } RR^T = I$$

$$= S(Ra)\, b * \frac{1}{b} \quad \text{.... from ②.}$$

$$R\,S(a)\,R^T = S(Ra)$$

∴ Proved

6) 

RRP SCARA

We take the axis change as in the diagram. We write down the R & d matrices for each transformation.

$$R_0^1 = \begin{bmatrix} c_{q_1} & -s_{q_1} & 0 \\ s_{q_1} & c_{q_1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} c_{q_2} & -s_{q_2} & 0 \\ s_{q_2} & c_{q_2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} c_{\pi/2} & 0 & s_{\pi/2} \\ 0 & 1 & 0 \\ -s_{\pi/2} & 0 & c_{\pi/2} \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} l_3 + d \\ 0 \\ 0 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} C_{q_1} & -S_{q_1} & 0 & l_1 \\ S_{q_1} & C_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

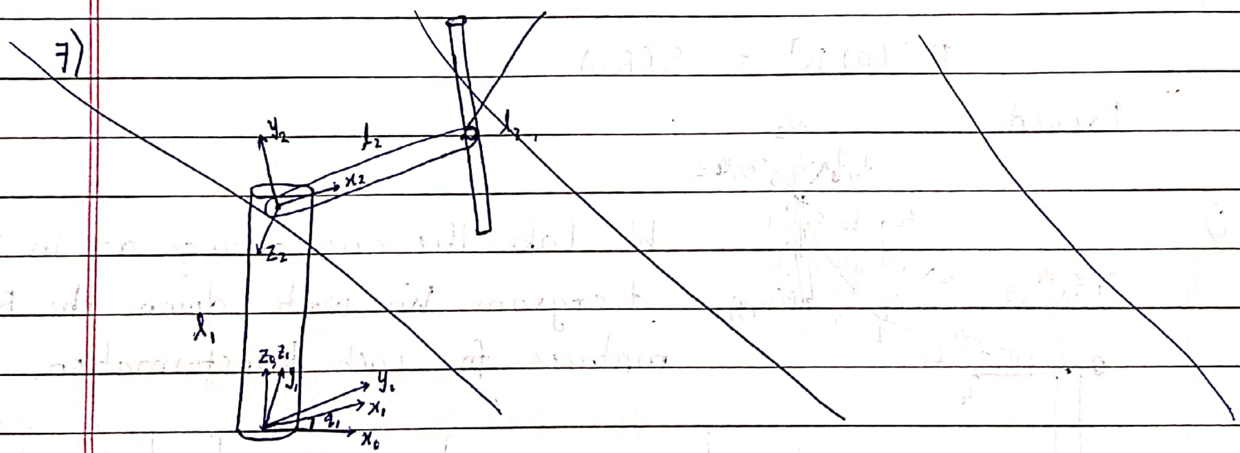
$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} C_{q_2} & -S_{q_2} & 0 & l_2 \\ S_{q_2} & C_{q_2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} C_{\pi/2} & 0 & S_{\pi/2} & l_3+d \\ 0 & 1 & 0 & 0 \\ -S_{\pi/2} & 0 & C_{\pi/2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad P_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

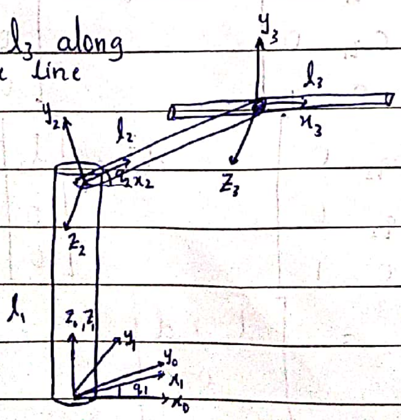
$\therefore$ We have;

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \, H_1^2 \, H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

7)



8) l_2 & l_3 along same line



$\therefore$ We take the axis change as shown in the diagram. We write down the R & d matrices for each axis transformation. In the diagram, the links l_3 is along link l_2 through a prismatic joint $\therefore$ We have;

$$R_0^1 = \begin{bmatrix} Cq_1 & -Sq_1 & 0 \\ Sq_1 & Cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\pi/2 & -S\pi/2 \\ 0 & S\pi/2 & C\pi/2 \end{bmatrix} \begin{bmatrix} Cq_2 & -Sq_2 & 0 \\ Sq_2 & Cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} \qquad P_3 = \begin{bmatrix} l_3 + d \\ 0 \\ 0 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} Cq_1 & -Sq_1 & 0 & 0 \\ Sq_1 & Cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

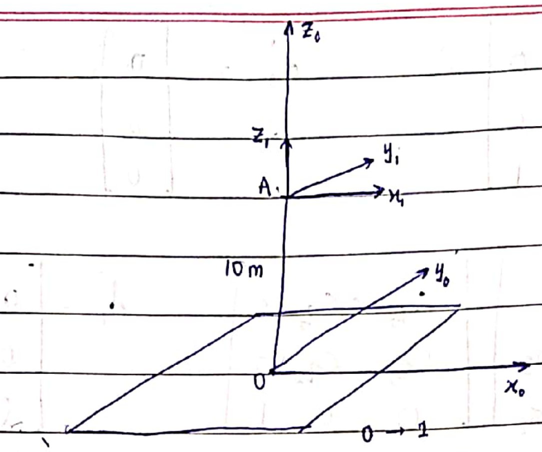
$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} Cq_2 & -Sq_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ Sq_2 & Cq_2 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

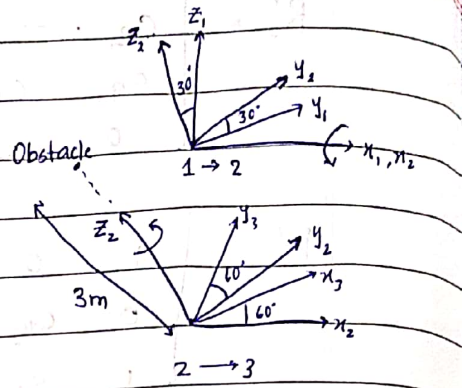
$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & l_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad P_3 = \begin{bmatrix} l_3 + d \\ 0 \\ 0 \end{bmatrix}$$

∴ We have;

$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

9)





∴ Based on the axis definition above, we define the R & d matrices. For clarity, we split 1→2 & 2→3 as shown.

$$R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 10 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_{\pi/6} & -S_{\pi/6} \\ 0 & S_{\pi/6} & C_{\pi/6} \end{bmatrix} \qquad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_2^3 = \begin{bmatrix} C_{\pi/3} & -S_{\pi/3} & 0 \\ S_{\pi/3} & C_{\pi/3} & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_2^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$ ∵ Obstacle is located 3 m above the drone, i.e. in z direction.

∴ We write H matrices :-

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sqrt{3}/2 & -1/2 & 0 \\ 0 & 1/2 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ \sqrt{3}/2 & 1/2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ We have;-

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 & -0.866 & 0 & 10 \\ 0.75 & 0.433 & -0.5 & 0 \\ 0.433 & 0.25 & 0.866 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ -1.5 \\ 2.5981 \\ 1 \end{bmatrix}$$

$\therefore P_0 = (10, -1.5, 2.5981)$.

10) The main types of gearboxes found are of two catgories:-
① Industrial gearboxes. ② Automobile gearboxes.

The industrial gearboxes are primarily used in applications of robotics:-

a) Helical gearboxes: It is a compact & consumes low-power. It has a angle, which enables more teeth to interact in the same direction. Application: Conveyors, Crushers etc.

b) Coaxial helical inline gearbox: Used for heavy duty applications. Has high degree of specification, allowing to maximize loads & transmission ratios.

c) Bevel Helical gearbox: Curved set of teeth located on cone shaped surface close to rim of unit. Provides rotatory motion between non-parallel shafts. eg: quarries, mining, conveyors.

g) Harmonic gear drive: Do not have any backlash, are lightweight, A non-rigid, thin cylindrical cup engages with a fixed solid ~~circular~~ elastomaterring with internal gear teeth.
h) Cycloid gear drive: They have high robustness & torsional stiffness. Eccentric input creates wobbly cycloidal motion, converted back to rotation of output shaft.

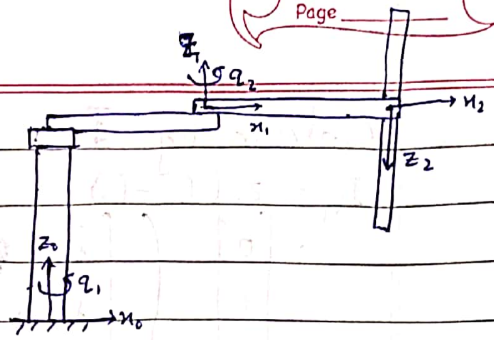
d) Skew bevel helical gearbox: Rigid, monolithic structure, usable in heavy loads & other applications. Highly customizable & provide mechanical advantage on correct motor shaft output.
e) Worm reduction gearbox: Heavy duty operations, used when there is need for increased speed reduction between non-intersecting crossed axis shafts. Worm wheel has larger diameter & causes the wheel with which it is meshed with to move due to screw like movement.
f) Planetary gearbox: Has a central sun gear surrounded by four planet gears held together by outer ring gear with internal teeth. Can achieve high torque in small space & has greater endurance & accuracy. Used in 3D printing & Robotics.

Usually gearboxes are not present in conjunction with drone motors to allow direct control to the BLDC motor, however in some cases drone motors use a series of two gears to increase angular velocity by using the gear ratio. Gearboxes however are not often used in drone applications since the weight of the drone increases & there are wastages in energy transmission & control implementation time may increase.

11) We refer to the diagram in the next page for solving this question.
∴ We calculate J_ω:

$J_\omega = [\,\rho_1 z_0 \quad \rho_2 z_1 \quad \rho_3 z_2\,]$ ∴ RRP ⇒ $\rho_1 = \rho_2 = 1$, $\rho_3 = 0$.

$$\therefore z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad z_2 = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$



$$\therefore J_\omega = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$

$\therefore$ Calculating J_v:

$J_{v_1} = z_0(O_3 - O_0)$

$$J_{v_1} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \left[H_0^3 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right]$$

$J_{v_2} = z_1(O_3 - O_1)$

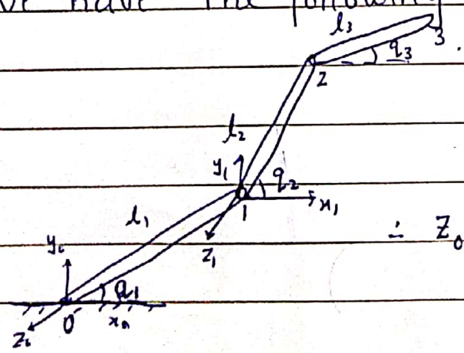
$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left(H_0^3 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} l_2 \cos q_1 \\ l_2 \sin q_1 \\ l_1 \end{bmatrix} \right).$$

$J_{v_3} = z_2(O_3 - O_2)$.

$$= \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \left(H_0^3 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - H_0^2 \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$\therefore J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} J_{v_1} & J_{v_2} & J_{v_3} \\ J_{\omega_1} & J_{\omega_2} & J_{\omega_3} \end{bmatrix}$$

13) We have the following diagram for axes:-



$\therefore$ We derive J_ω:

$J_\omega = [\rho_1 z_0 \quad \rho_2 z_1 \quad \rho_3 z_2]$

RRR $\Rightarrow \rho_1 = \rho_2 = \rho_3 = 1$

$$\therefore z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad z_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore J_\omega = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Calculating J_v:-

$$J_{v_1} = z_0 (O_3 - O_0)$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left(\begin{bmatrix} l_1 \cos q_1 + l_2 \cos q_2 + l_3 \cos q_3 \\ l_1 \sin q_1 + l_2 \sin q_2 + l_3 \sin q_3 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$J_{v_2} = z_1 (O_3 - O_1)$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left(\begin{bmatrix} l_1 \cos q_1 + l_2 \cos q_2 + l_3 \cos q_3 \\ l_1 \sin q_1 + l_2 \sin q_2 + l_3 \sin q_3 \\ 0 \end{bmatrix} - \begin{bmatrix} l_1 \cos q_1 \\ l_1 \sin q_1 \\ 0 \end{bmatrix} \right)$$

$$J_{v_3} = z_2 (O_3 - O_2)$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \left(\begin{bmatrix} l_1 \cos q_1 + l_2 \cos q_2 + l_3 \cos q_3 \\ l_1 \sin q_1 + l_2 \sin q_2 + l_3 \sin q_3 \\ 0 \end{bmatrix} - \begin{bmatrix} l_1 \cos q_1 + l_2 \cos q_2 \\ l_1 \sin q_1 + l_2 \sin q_2 \\ 0 \end{bmatrix} \right)$$