Task: 1

Let us consider rotation matrix with $\frac{\pi}{4}$ about z-axis.

$$R = \begin{bmatrix} c(\pi/4) & -s(\pi/4) & 0 \\ s(\pi/4) & c(\pi/4) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

For this matrix to be orthogonal

$$RR^T = I.$$

$$\begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ -1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2}+\frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2}+\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

This means all its column are orthogonal to each other.

Task : 2 $\det(R_0^1) = 1$

→ Determinant of any matrix gives the amount by which area formed by unit vectors in R^2 space is scaled after transformation.

→ In-case of rotation matrix it purely rotates all basis vector equally without scaling.

$\therefore$ Determinant $= 1$

$\Downarrow$

Area formed by unit vectors before and after transformation remains the same.

Task: 5 $\quad R\,S(a)\,R^T = S(Ra)$

Proof: $\quad R\,S(a)\,R^T$

$$= R\cdot(a \times R^T)$$

$$= (R\cdot a \times R\cdot R^T)$$

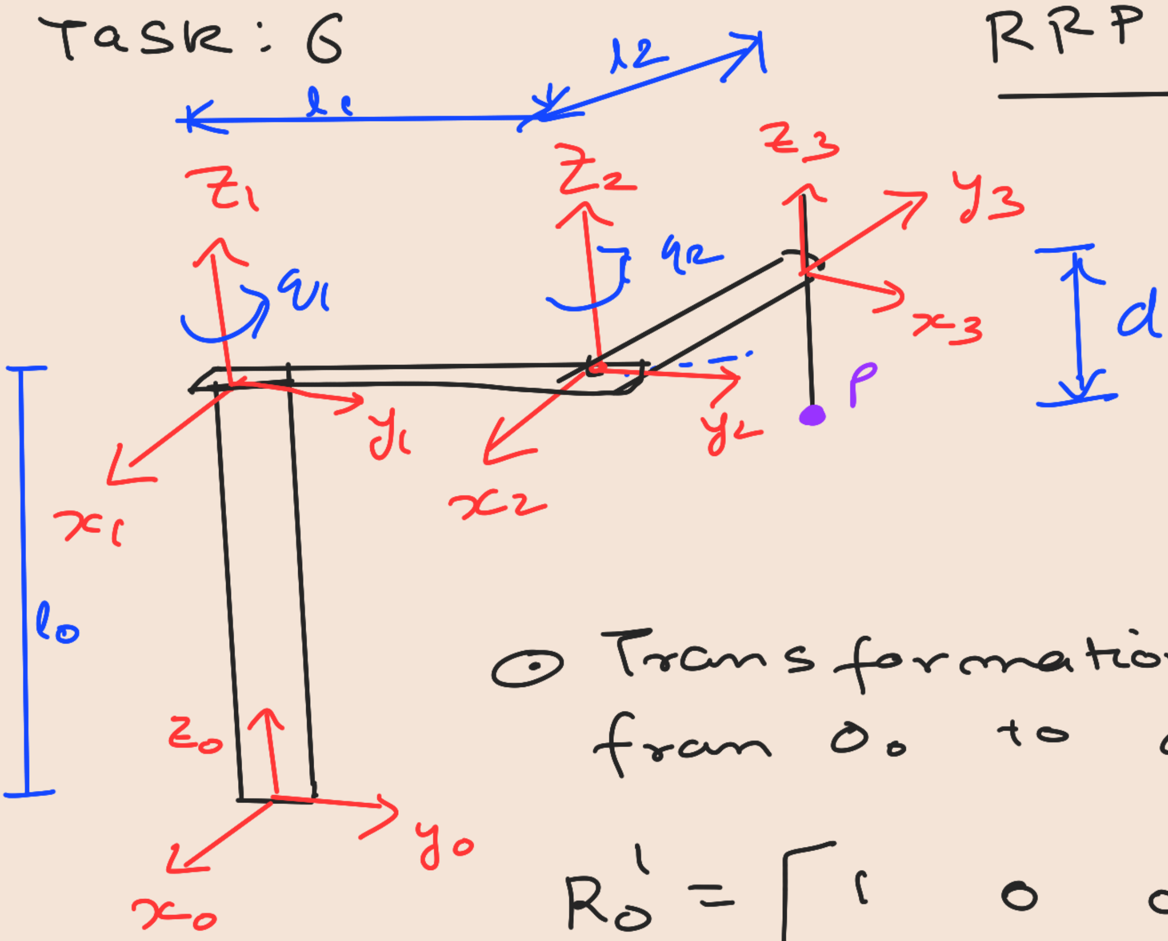
$$= (R\cdot a \times I) \quad [RR^T = I]$$

$$= S(R\cdot a)\cdot I$$

$$= S(R\cdot a)$$

Task: 6

RRP SCARA



- Transformation from frame O_0 to O_1

$$R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ l_0 \end{bmatrix}$$

$$\therefore H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

⊙ Transformation from O_1 to O_2

$$R_1^2 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_1^2 = \begin{bmatrix} l_1 cq_1 \\ l_1 sq_1 \\ 0 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} cq_1 & -sq_1 & 0 & l_1 cq_1 \\ sq_1 & cq_1 & 0 & l_1 sq_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

⊙ Transformation from $O_2 - O_3$

$$R_2^3 = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_2^3 = \begin{bmatrix} l_2 cq_2 \\ l_2 sq_2 \\ 0 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} cq_2 & -sq_2 & 0 & l_2 cq_2 \\ sq_2 & cq_2 & 0 & l_2 sq_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 & 0 & d & 1 \end{bmatrix}^T$$

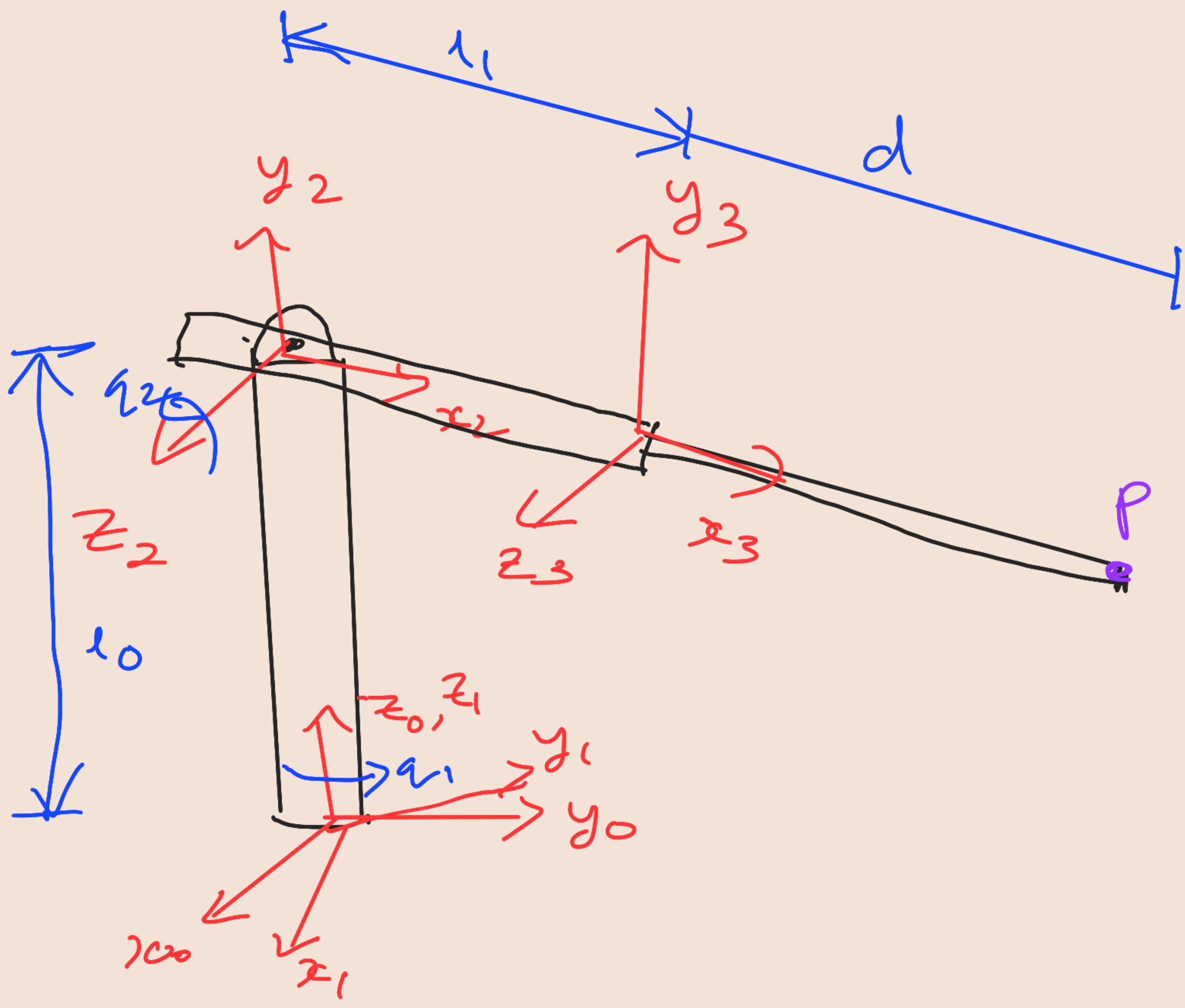
$$\boxed{P_0 = H_0^1 \cdot H_1^2 \cdot H_2^3 \; P_3}$$

$$H_0^3 = \begin{bmatrix} c^2q_1 - sq_1 sq_2 & -2cq_1 sq_1 & 0 & l_1 cq_1 + l_2 c(q_1+q_2) \\ 2cq_1 sq_1 & cq_1 cq_2 - s^2 q_1 & 0 & l_1 sq_1 + l_2 s(q_1+q_2) \\ 0 & 0 & 1 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} l_1 cq_1 - d(sq_1 sq_2 - c^2 q_1) + l_2 c(q_1+q_2) \\ d\, s(q_1+q_2) + l_1 s(q_1) + l_2 s(q_1+q_2) \\ l_0 \\ 1 \end{bmatrix}$$

Task : 8 Stanford Manipulator

RRP



① Transformation from $o_0 \to o_1$

$$R_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 \\ sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} cq_1 & -sq_1 & 0 & 0 \\ sq_1 & cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

⊙ Transformation from $O_1 \to O_2$

To reach O_2 1st rotate $90°$ about y_1

2nd rotate $90°$ about new z

$$R_1^2 = \begin{bmatrix} c(\frac{\pi}{2}) & 0 & s(\pi/2) \\ 0 & 1 & 0 \\ -s(\frac{\pi}{2}) & 0 & c(\frac{\pi}{2}) \end{bmatrix} \begin{bmatrix} c(\pi/2) & -s(\pi/2) & 0 \\ s(\pi/2) & c(\pi/2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_0 \end{bmatrix}$$

$$\therefore H_1^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

⊙ Transfermation from $O_2 \to O_3$

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} cq_2 & -sq_2 & 0 & l_2 cq_2 \\ sq_2 & cq_2 & 0 & l_2 sq_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} d & 0 & 0 & 1 \end{bmatrix}^T$$

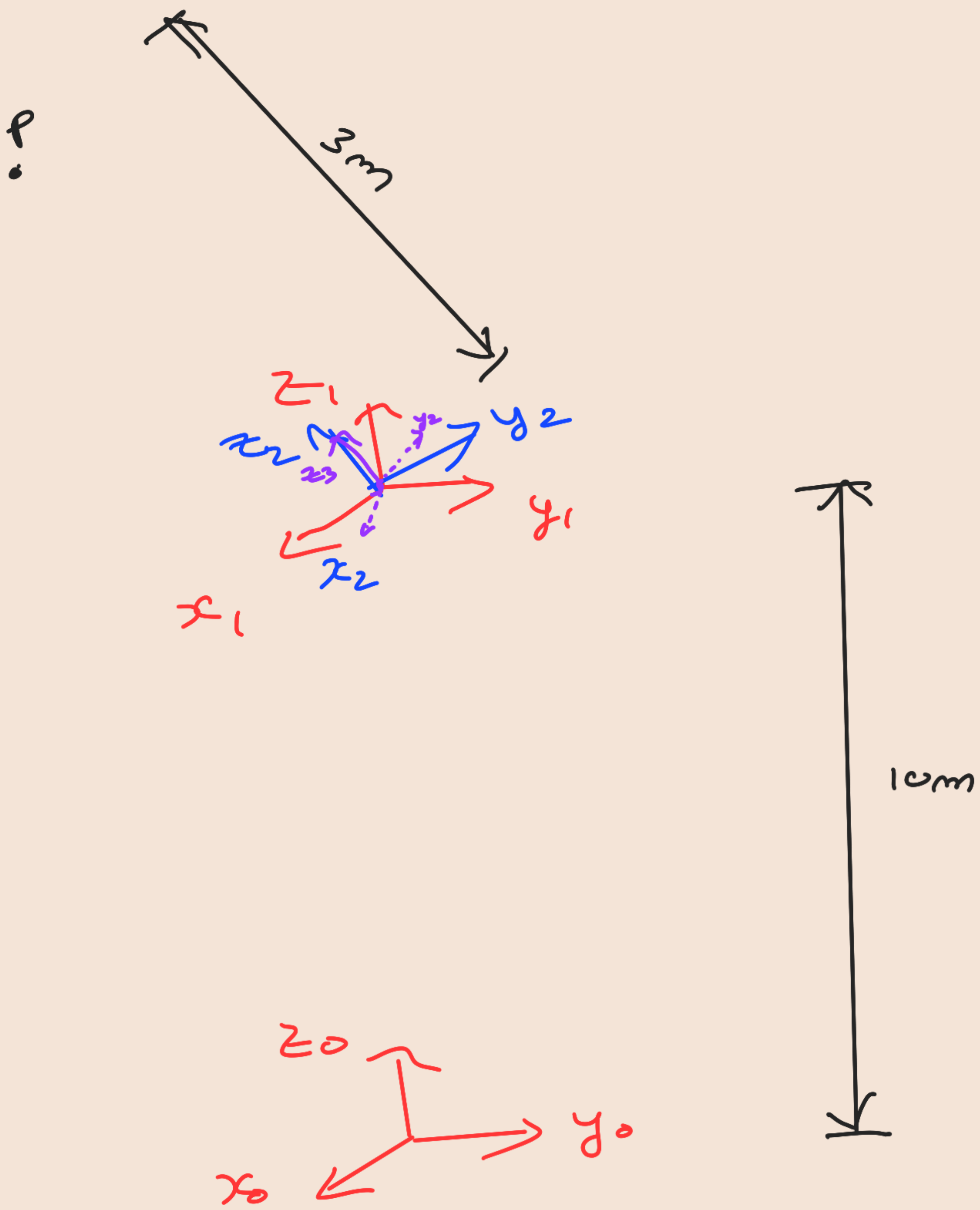
$$\boxed{\therefore P_0 = H_0^1 H_1^2 H_2^3 P_3}$$

$$H_0^3 = \begin{bmatrix} -cq_2 sq_1 & sq_1 sq_2 & cq_1 & -l_2 cq_2 sq_1 \\ cq_1 cq_2 & -cq_1 sq_2 & sq_1 & l_2 cq_1 cq_2 \\ sq_2 & cq_2 & 0 & l_0 + l_2 sq_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} -d cq_2 sq_1 - l_2 cq_2 sq_1 \\ d\, cq_1 cq_2 + l_2\, cq_1 cq_2 \\ l_0 + d sq_2 + l_2 sq_2 \\ 1 \end{bmatrix}$$

Task : 9

→ Took off from base station travelled straight up 10m.



$$H_0^1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c(\pi/6) & -s(\pi/6) & 0 \\ 0 & s(\pi/6) & c(\pi/6) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 = \begin{bmatrix} c(\pi/3) & -s(\pi/3) & 0 & 0 \\ s(\pi/3) & c(\pi/3) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 & 0 & 3 & 1 \end{bmatrix}^T$$

$$P_0 = H_0^1 \, H_1^2 \, H_2^3 \, P_3$$

$$P_0 = \begin{bmatrix} 0 & -1.5 & 12.598 & 1 \end{bmatrix}^T$$

Task: 10 Types of Gearbox used in motor in robotic application can be broadly classified into groups of:

(a) Gearbox from basic type of gears (spur, helical etc)

(b) Planetary gearbox

(c) Harmonic drives.

(d) Ball screw based Design.

⇒ Gearbox from basic type

Pros: Simple to design & assemble

cons: Backlash, failure to impact load, Short life, Bulky design

⇒ Planatary gearbox: Most commonly used due to less expensive.

Pros: compact design.

cons: Backlash, backdrivability

⇒ Harmonic drives :

Pros: Compact design & no backlash

cons: Less efficiency, poor backdrivability & costly.

⇒ Ball screw based design :

usage depends on application.

Pros: High efficiency speed reduction backdrivable, tolerant to impact load.

cons: Compactness issue.

In drone we want high RPM to make it fly & gearbox are generally used to reduce speed & amplify torques; which is against the requirements of drone.

so No gearbox in drone.

Task : 11 Manipulator Jacobian

for RRP SCARA

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$$

$$J_v = \begin{bmatrix} \frac{\partial x}{\partial q_1} & \frac{\partial x}{\partial q_2} & \frac{\partial x}{\partial d} \\ \frac{\partial y}{\partial q_1} & \frac{\partial y}{\partial q_2} & \frac{\partial y}{\partial d} \\ \frac{\partial z}{\partial q_1} & \frac{\partial z}{\partial q_2} & \frac{\partial z}{\partial d} \end{bmatrix}$$

$$[x, y, z] = d_{03}$$

$$J_v = \begin{bmatrix} -l_1 s q_1 - l_2 s(q_1+q_2) & -l_2 s(q_1+q_2) & 0 \\ l_1 c q_1 + l_2 c(q_1+q_2) & l_2 c(q_1+q_2) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$J_\omega = \begin{bmatrix} \hat{\omega}_1^b & \hat{\omega}_2^b & \hat{\omega}_3^b \end{bmatrix}$$

where $\hat{\omega}_1^b = R_0^1 \cdot (T_0^1([1:3], 3))$

$\Downarrow$

3rd column of R_0^1

$$\hat{\omega}_1^b = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

$$\hat{\omega}_2^b = R_0^1 R_1^2 \cdot (T_0^2 [1:3], 3)$$

$$= \begin{bmatrix} -sq_1 & cq_1 & 0 \end{bmatrix}^T$$

$$\hat{\omega}_3^b = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$$

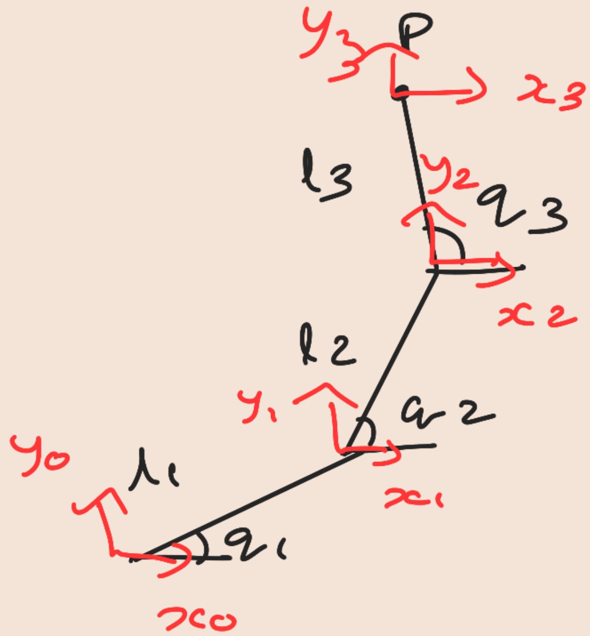
$\Downarrow$

Prismatic Joint

$$\therefore J_\omega = \begin{bmatrix} 0 & -sq_1 & 0 \\ 0 & cq_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$\therefore J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}_{6\times 3}$$

Task : 13 RRR planar robot



$$x_p = l_1 cq_1 + l_2 cq_2 + l_3 cq_3$$

$$y_p = l_1 sq_1 + l_2 sq_2 + l_3 sq_3$$

$$z_p = 0$$

$$\therefore J_v = \begin{bmatrix} -l_1 sq_1 & -l_2 sq_2 & -l_3 sq_3 \\ l_1 cq_1 & l_2 cq_2 & l_3 cq_3 \\ 0 & 0 & 0 \end{bmatrix}$$

$$J\omega = \begin{bmatrix} \hat{\omega}_1^b & \hat{\omega}_2^b & \hat{\omega}_3^b \end{bmatrix}$$

$$\hat{\omega}_1^b = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

$$\hat{\omega}_2^b = R_0^1 \cdot \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\hat{\omega}_3^b = R_0^1 R_1^2 \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T$$

$$\therefore J\omega = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\therefore J = \begin{bmatrix} J\omega \\ J\omega \end{bmatrix}$$

Link: https://colab.research.google.com/drive/1pffxVM5kZOaPFfcygTtyPZBvwP9o_bRU?usp=sharing