Q1 Show that column of the Rotation matrix R_0^1 are orthogonal

$$R_0^1 = \begin{bmatrix} i_1 \cdot i_0 & j_1 \cdot i_0 & k_1 \cdot i_0 \\ i_1 \cdot j_0 & j_1 \cdot j_0 & k_1 \cdot j_0 \\ i_1 \cdot k_0 & j_1 \cdot k_0 & k_1 \cdot k_0 \end{bmatrix}$$

Now for checking orthogonality. Either Row vectors to be taken on Column vectors

$$(i_1 i_0 \cdot j_1 i_0 + i_1 \cdot j_0 \cdot j_1 \cdot j_0 + i_1 k_0 \cdot j_1 k_0)$$

by using commutative properties we can rearrange them h

$$= i_1 \cdot j_1 \cdot i_0 \cdot i_0 + i_1 \cdot j_1 \cdot j_0 \cdot j_0 + i_1 \cdot j_1 \cdot k_0 \cdot k_0$$

now $i_1 \cdot j_1 = 0$ as they are perpendicular to each other

$= 0$ similarly other pair of column vectors & Row vectors will also result in the same

Q2 Show that $\det(R_0^1) = 1$

As we now that $(R_0^1)^{-1} = (R_0^1)^T$ and $\det(R) = \det(R^T)$

$$I = R_0^1 \times (R_0^1)^{-1}$$

$$I = R_0^1 \times R_0^{1T}$$

applying det on both side

$$\det(I) = \det(R_0^1 \times R_0^{1T}) = 1 = \det(R_0^1) \cdot \det((R_0^1)^T)$$

$$1 = \det(R_0^1)^2$$

$$1 = \det(R_0^1)$$

Q5

Show that $R\,S(a)\,R^T = S(Ra)$

Suppose $R = R_{z,\theta} = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R^T = R_{z,\theta}^T = \begin{bmatrix} c\theta & s\theta & 0 \\ -s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Suppose $a = [1\ 0\ 0]^T$

$S(a) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$

$R.(a) = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

$R(a) = [c\theta\ \ s\theta\ \ 0]^T$

$S(Ra) = \begin{bmatrix} 0 & 0 & s\theta \\ 0 & 0 & -c\theta \\ -s\theta & c\theta & 0 \end{bmatrix}$

$S(a)\,R^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} c\theta & s\theta & 0 \\ -s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

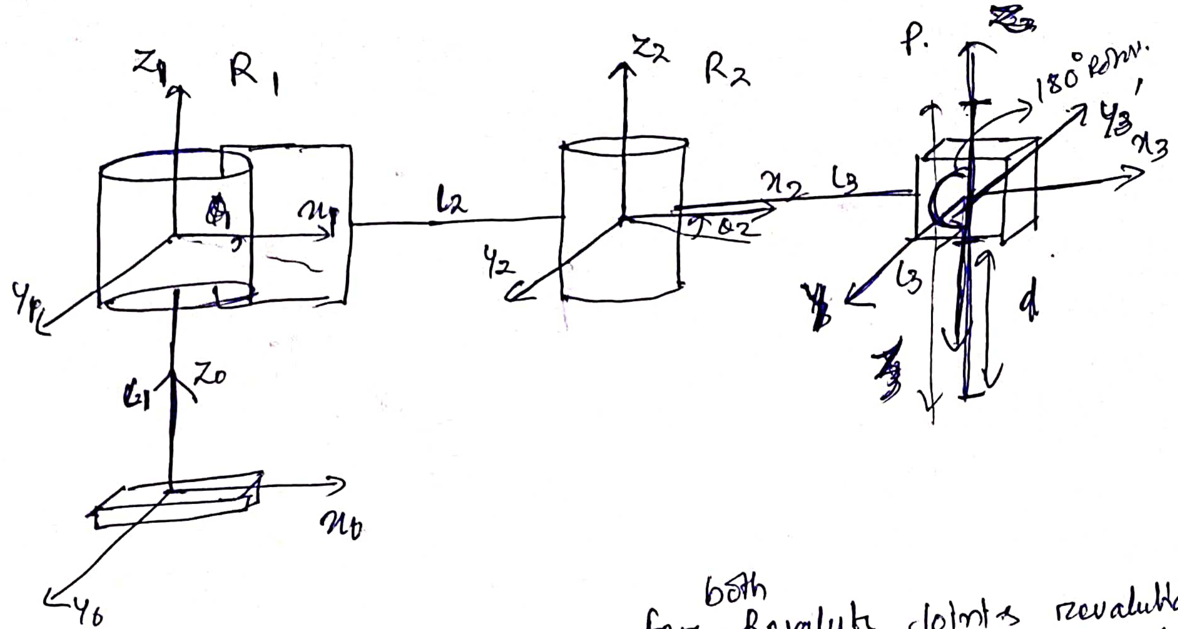
$$S(a)\,R^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ -S\theta & C\theta & 0 \end{bmatrix}$$

$$R\,S(a)\,R^T = \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ -S\theta & C\theta & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & +S\theta \\ 0 & 0 & -C\theta \\ -S\theta & C\theta & 0 \end{bmatrix}$$

So we can see both $S(Ra) = R\,S(a)\,R^T$

Q6 RRP SCARA CONFIGURATION:



for both Revolute Joints revolution/rotation is with Z axis as axis of Rotation.

For Prismatic Joints translation is along the Z axis.

As per homogenous transformation.

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix}$$

with Translation L_1 & Rotation θ_1 (Z axis)

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \Rightarrow \quad R_0^1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation L_2 & Rotation θ_2

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \Rightarrow R_1^2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 \\ S\theta_2 & C\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_1^2 = \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & L_2 \\ S\theta_2 & C\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation L_3 & Rotation of 180° about x_3 axis.

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} \Rightarrow R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad d_2^3 = \begin{bmatrix} L_3 \\ 0 \\ 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 \\ 0 \\ d \\ 1 \end{bmatrix} \qquad H_2^3 = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now solving by

$$H_2^3 P_3 = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ d \\ 1 \end{bmatrix} = \begin{bmatrix} L_3 \\ 0 \\ -d \\ 1 \end{bmatrix}$$

Translation L_2 & Rotation θ_2

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \Rightarrow R_1^2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_1^2 = \begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_2 \\ s\theta_2 & c\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation L_3 & Rotation of $180°$ about x_3 axis.

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} \Rightarrow R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad d_2^3 = \begin{bmatrix} L_3 \\ 0 \\ 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} 0 \\ 0 \\ d \\ 1 \end{bmatrix} \qquad H_2^3 = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now solving by

$$H_2^3 P_3 = \begin{bmatrix} 1 & 0 & 0 & L_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ d \\ 1 \end{bmatrix} = \begin{bmatrix} L_3 \\ 0 \\ -d \\ 1 \end{bmatrix}$$

$$H_2^1 H_2^3 P_3 \Rightarrow \begin{bmatrix} CQ_2 & -SQ_2 & 0 & L_2 \\ SQ_2 & CQ_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_3 \\ 0 \\ -d \\ 1 \end{bmatrix} = \begin{bmatrix} CQ_2 L_3 + L_2 \\ SQ_2 L_3 \\ -d \\ 1 \end{bmatrix}$$

$$H_0^1 H_1^2 H_2^3 P_3$$

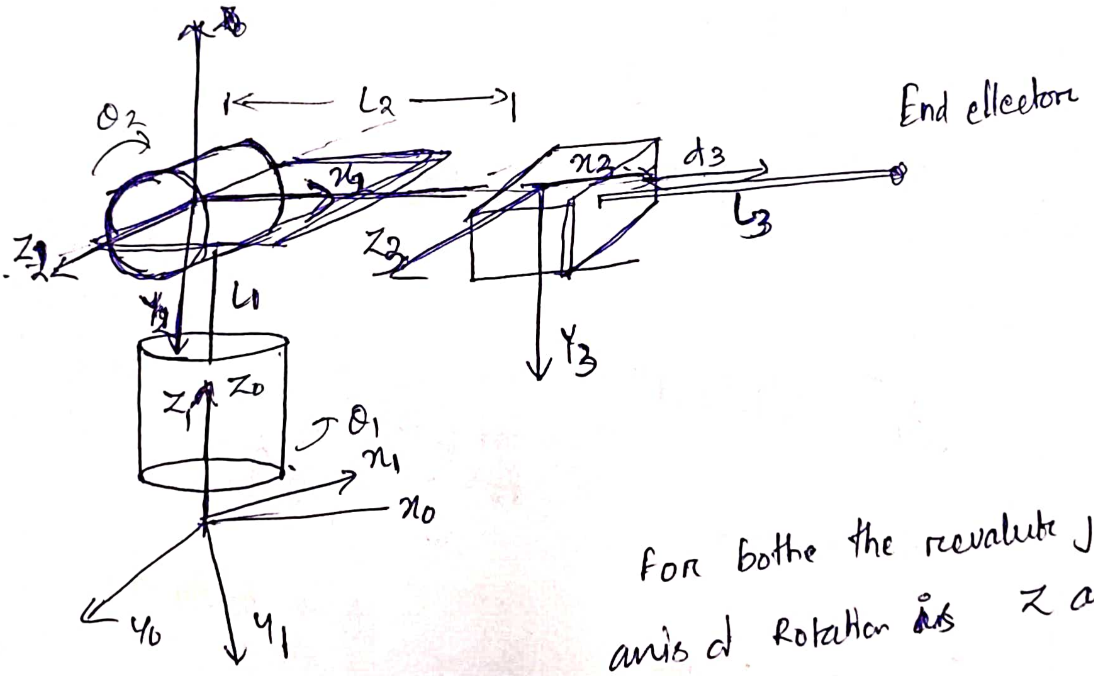
$$= \begin{bmatrix} CQ_1 & -SQ_1 & 0 & 0 \\ SQ_1 & CQ_1 & 0 & 0 \\ 0 & 0 & 1 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} CQ_2 L_3 + L_2 \\ SQ_2 L_3 \\ -d \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} CQ_1 (CQ_2 L_3 + L_2) - SQ_1 SQ_2 L_3 \\ SQ_1 (CQ_2 L_3 + L_2) + CQ_1 (SQ_2 L_3) \\ -d + L_1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} L_3 CQ_1 CQ_2 - SQ_1 SQ_2 L_3 + CQ_1 L_2 \\ L_3 (SQ_1 CQ_2 + CQ_1 SQ_2) + L_2 SQ_1 \\ L_1 - d \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 \begin{bmatrix} P_3 \\ 1 \end{bmatrix} = \begin{bmatrix} l_3 C(\theta_1+\theta_2) + C\theta_1 l_2 \\ l_3 S(\theta_1+\theta_2) + S\theta_1 l_2 \\ l_1 - d \\ 1 \end{bmatrix}$$

Q8 STANDFORD TYPE RRP CONFIGURATION.



For bothe the revalute joints anis d Rotation is Z anis

for the Prismatic joint the anis d Motion is alon X anis.

As per homogenas trasformation

$$P_0 = H_0^1 H_1^2 H_2^3 P_3$$

Rotation is abaut Z_0 anis & Zero traslah

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 & 0 \\ S\theta_1 & C\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \Rightarrow$$

1st Rotation about x_1 axis is 90° &
2nd Rotation about z_2 axis by θ_2
Translation is L_3

$$d_1^2 = \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 \\ S\theta_2 & C\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 \\ 0 & 0 & -1 \\ S\theta_2 & C\theta_2 & 0 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ S\theta_2 & C\theta_2 & 0 & L_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

There's no Rotation only translation

$$H_2^3 = \begin{bmatrix} R_2^3 & d_2^3 \\ 0 & 1 \end{bmatrix} \Rightarrow R_2^3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_2^3 = \begin{bmatrix} l_2 + d \\ 0 \\ 0 \end{bmatrix}$$

$$P_3 = \begin{bmatrix} l_3 + d \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad H_2^3 = \begin{bmatrix} 1 & 0 & 0 & l_2 + d \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_2^3 P_3 \Rightarrow \begin{bmatrix} 1 & 0 & 0 & l_2 + d \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_3 + d \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} l_3 + l_2 + d \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

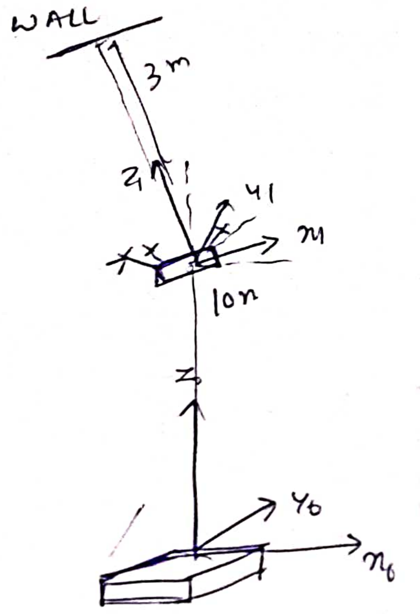
$$H_1^2 H_2^3 P_3 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\theta_2 & c\theta_2 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_3 + l_2 + d \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$H_1^2 H_2^3 P_3 = \begin{bmatrix} (L_3+L_2+d)\, CQ_2 \\ 0 \\ (L_3+L_2+d)\, SQ_2 + L_1 \\ 1 \end{bmatrix} \Rightarrow H_0^1 H_1^2 H_2^3 P_3 = \begin{bmatrix} CQ_1 & -SQ_1 & 0 & 0 \\ SQ_1 & CQ_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} (L_3+L_2+d)\, CQ_2 \\ 0 \\ (L_3+L_2+d)\, SQ_2 + L_1 \\ 1 \end{bmatrix}$$

$$\therefore \quad L_2 + L_3 + d = K$$

$$P_0 = \begin{bmatrix} K\, CQ_1\, CQ_2 \\ K\, SQ_1\, CQ_2 \\ K\, SQ_2 + L_1 \\ 1 \end{bmatrix}$$

Q9 Drone Problem



So $P_0 = H_0^1 P_1$ Here there is both rotation & translation.

$$P_1 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$$

$$H_0^1 = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix}$$

$$R_0^1 = R_{x30} R_{z60}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -1/2 \\ 0 & 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 1/2 & -\frac{\sqrt{3}}{2} & 0 \\ \sqrt{3}/2 & 1/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 \\ \frac{\sqrt{3}}{4} & 1/4 & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$$d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

Q4

$$H_0^1 = \begin{bmatrix} \frac{1}{2} & -\sqrt{3}/2 & 0 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \frac{\sqrt{3}}{2} & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} 1/2 & -\sqrt{3}/2 & 0 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} 0 & -\frac{3}{2} & 3\sqrt{3}/2 + 10 & 1 \end{bmatrix}^T$$

Q10 different types of gear boxes typically used with motors in robotics application.

So basically it is a mechanical device utilized to increase the output torque or change the RPM speed of motor.

Bevel Gear box:

These type of arrangement/gearbox uses bevel gears, which are used in right angle application. Again it also involves straight bevel gears used in low speed & Spiral bevel gears with curved & oblique teeths used in applications requiring high perform & high speed.

Advantages of Bevel Gears!

- Right angle configuration.
- Durable.

Disadvantages of Bevel gears:

- Axes must be able to support forces.
- Poorly cut teeth result in excessive vibration & noise during operation

Helical gear box:

These type of gear box uses Helical gears, which allow gradual contact between each of the helical gear teeth. Thus allowing smooth & quiet operation, It has high horse power application

Advantages:

- can be meshed in parallel or cross orientation.
- Smooth & quiet operation
- Efficient
- High Horse power

Disadvantages of the Used Gears.

- Resultant thrust along axis of gear
- Additives to Lubrication

Spur Gear box!

These types of Gear box uses Spur gears. These are some of the easyer to manufacture gears used. And no constraints in radius and it pays a part of other gears aswell.

Advantages!

- Cost-effective.
- High Gear Ratio
- Compact
- High torque output

Disadvantages of spur gears

- Noisy
- Prone to wear and tear.

WORM GEAR BOX!

These type of Gear box uses worm gears. These gears are able to withstand high shock loads, Low in noise Level & maintainence free. but are less efficient than other gear types. Worm gears can be used in right angle configuration. The worm can turn the gear with ease. however, the gear cannot turn the worm. The prevention of the gear to move the worm can be used as a braking system.

Advantages!

- High precision
- Right angled configuration.
- Braking system.
- Low noise.
- Maintenance free.

Disadvantages of worm Gears

- Limitations
- Non reversable
- Low efficiency.

Planetary Gear box:-

Planetary Gears are used in this case & thus having a resemblance to solar system. The components of Planetary gear include a sun gear, ring gear, & planetary gears.

Advantages of Planetary gears

- High power density.
- compact
- High efficiency levels in power transmission
- Greater Stability
- Load distribution among planetary gears

Disadvantages of planetary Gears.

- High bearing loads
- complex design
- Inaccessibility.

Motors in drone application are. basically BLDC motors. which has electronic speed control so they dont need gears for speed change.

Q13 MANIPULATOR JACOBIAN FOR THE ~~SCARA~~ RRR CONFIGURATION

$$J = \begin{bmatrix} J_1 & J_2 & J_3 \end{bmatrix}$$ as RRR so all are Revolute Joints.

$$J = \begin{bmatrix} Z_0 \times (O_3 - O_0) & Z_1 \times (O_3 - O_1) & Z_2 \times (O_3 - O_1) \\ Z_0 & Z_1 & Z_2 \end{bmatrix}$$

$$O_3 - O_0 = R_0^0 d_0^3 = d_0^3 = d_0^1 + R_0^1 d_1^2 + R_0^1 R_1^2 d_2^3$$

$$O_3 - O_1 = R_0^1 d_1^3 = R_0^1 [\, d_1^2 + R_1^2 d_2^3]$$

$$O_3 - O_2 = R_0^2 d_2^3 = R_0^1 R_1^2 d_2^3$$

$$Z_0 = R_0^0 \hat{k}$$
$$Z_1 = R_0^1 \hat{k}$$
$$Z_2 = R_0^2 \hat{k}$$

$$d_0^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 \\ S\theta_2 & C\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$d_0^3 = \begin{bmatrix} l_1 C\theta_1 \\ l_1 S\theta_1 \\ 0 \end{bmatrix} + \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 C\theta_2 \\ l_2 S\theta_2 \\ 0 \end{bmatrix}$$

$$d_0^3 = \begin{bmatrix} l_1 C\theta_1 \\ l_1 S\theta_1 \\ 0 \end{bmatrix} + \begin{bmatrix} l_2 C\theta_2 C\theta_1 - l_2 S\theta_1 S\theta_2 \\ l_2 C\theta_2 S\theta_1 + l_2 C\theta_1 S\theta_2 \\ 0 \end{bmatrix}$$

$$d_0^3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 \\ S\theta_2 & C\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$d_0^3 = \begin{bmatrix} l_1 C\theta_1 \\ l_1 S\theta_1 \\ 0 \end{bmatrix} + \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 C\theta_2 \\ l_2 S\theta_2 \\ 0 \end{bmatrix}$$

$$d_0^3 = \begin{bmatrix} l_1 C\theta_1 \\ l_1 S\theta_1 \\ 0 \end{bmatrix} + \begin{bmatrix} l_2 C\theta_2 C\theta_1 - l_2 S\theta_1 S\theta_2 \\ l_2 C\theta_2 S\theta_1 + l_2 C\theta_1 S\theta_2 \\ 0 \end{bmatrix}$$

$$O_3 - O_0 = d_0^3 = \begin{bmatrix} l_1 C\theta_1 + l_2 C(\theta_1+\theta_2) \\ l_1 S\theta_1 + l_2 S(\theta_1+\theta_2) \\ 0 \end{bmatrix}$$

$O_3 - O_1 = R_0^1 d_1^3 =$ same as above as $d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$R_0^1 \left(d_1^2 + R_1^2 d_2^3\right) = \begin{bmatrix} l_1 C\theta_1 + l_2 C(\theta_1+\theta_2) \\ l_1 S\theta_1 + l_2 S(\theta_1+\theta_2) \\ 0 \end{bmatrix}$$

$O_3 -$

$$O_3 - O_2 = R_0^2 d_2^3 = R_0^1 R_1^2 d_2^3$$

$$= \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 \\ S\theta_2 & C\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 C\theta_2 \\ l_2 S\theta_2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} l_2 C\theta_2 C\theta_1 - l_2 S\theta_2 S\theta_1 \\ l_2 C\theta_2 S\theta_1 + l_2 S\theta_2 C\theta_1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} l_2 C(\theta_1+\theta_2) \\ l_2 S(\theta_1+\theta_2) \\ 0 \end{bmatrix}$$

$$z_0 = R_0^0 \hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad z_1 = R_0^1 \hat{k} = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$z_2 = R_0^1 R_1^2 \hat{k} = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 \\ S\theta_2 & C\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$z_2 = R_0^1 R_1^2 \hat{k} = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$z_0 \times (O_3 - O_0) = [0 \ 0 \ 1]^T \times \begin{bmatrix} l_1 c\theta_1 + l_2 c(\theta_1+\theta_2) \\ l_1 s\theta_1 + l_2 s(\theta_1+\theta_2) \\ 0 \end{bmatrix} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ l_1 c\theta_1 + l_2 c(\theta_1+\theta_2) & l_1 s\theta_1 + l_2 s(\theta_1+\theta_2) & 0 \end{bmatrix}$$

$$= (-l_1 s\theta_1 - l_2 s(\theta_1+\theta_2))\hat{i} + (l c\theta_1 + l_2 c(\theta_1+\theta_2))\hat{j}$$

$$z_1 \times (O_3 - O_1) = [0 \ 0 \ 1]^T \times \begin{bmatrix} l_1 c\theta_1 + l_2 c(\theta_1+\theta_2) \\ l_1 s\theta_1 + l_2 s(\theta_1+\theta_2) \\ 0 \end{bmatrix} = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ l_1 c\theta_1 + l_2 c(\theta_1+\theta_2) & l_1 s\theta_1 + l_2 s(\theta_1\theta_2 & 0 \end{bmatrix}$$

$$z_1 \times (O_3 - O_1) = \begin{bmatrix} -l_1 s\theta_1 - l_2 s(\theta_1+\theta_2) \\ l_1 c\theta_1 + l_2 c(\theta_1+\theta_2) \\ 0 \end{bmatrix} = z_0 \times (O_3 - O_0)$$

$$Z_2 \times (O_3 - O_2) = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ l_2 C(\theta_1+\theta_2) & l_2 S(\theta_1+\theta_2) & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -l_2 S(\theta_1+\theta_2) \\ l_2 C(\theta_1+\theta_2) \\ 0 \end{bmatrix}$$

Replacing in the equation.

$$J = \begin{bmatrix} -l_1 S\theta_1 - l_2 S(\theta_1+\theta_2) & -l_1 S\theta_1 - l_2 S(\theta_1+\theta_2) & -l_2 S(\theta_1+\theta_2) \\ l_1 C\theta_1 + l_2 C(\theta_1+\theta_2) & l_1 C\theta_1 + l_2 C(\theta_1+\theta_2) & l_2 C(\theta_1+\theta_2) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Q 11.

MANIPULATOR JACOBIAN FOR THE SCARA CONFIG.

$$J = [\, J_1 \quad J_2 \quad J_3 \,]$$

as RRP
so two Revalute joint & one Prismatic joint.

Revalute join $J_{\theta} = \begin{bmatrix} Z_0 \times (O_3 - O_0) & Z_1 \times (O_3 - O_1) & Z_2 \\ Z_0 & Z_1 & 0 \end{bmatrix}$

$$O_3 - O_0 = R_0^0 d_0^3 = d_0^3 = d_0^1 + R_0^1 d_1^2 + R_0^1 R_1^2 d_2^3$$

$$O_3 - O_1 = R_0^1 d_1^3 = R_0^1 [\, d_1^2 + R_1^2 d_2^3 \,]$$

$$Z_0 = R_0^0 \hat{k}$$

$$Z_1 = R_0^1 \hat{k}$$

$$Z_2 = R_0^2 \hat{k} = \cancel{\ } + \cancel{\ } = R_0^1 R_1^2$$

$$d_0^3 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix} + \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 \\ S\theta_2 & C\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix} + \begin{bmatrix} l_2 C\theta_1 \\ l_2 S\theta_1 \\ 0 \end{bmatrix} + \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_3 C\theta_2 \\ l_3 S\theta_2 \\ 0 \end{bmatrix}$$

$$d_0^3 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix} + \begin{bmatrix} l_2 c\theta_1 \\ l_2 s\theta_1 \\ 0 \end{bmatrix} + \begin{bmatrix} l_3 (c\theta_1 c\theta_2 - s\theta_2 s\theta_1) \\ l_3 (c\theta_2 s\theta_1 + s\theta_2 c\theta_1) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix} + \begin{bmatrix} l_2 c\theta_1 \\ l_2 s\theta_1 \\ 0 \end{bmatrix} + \begin{bmatrix} l_3\, c(\theta_1+\theta_2) \\ l_3\, s(\theta_1+\theta_2) \\ 0 \end{bmatrix}$$

$$O_3 - O_0 = d_0^3 = \begin{bmatrix} 0 + l_2 c\theta_1 + l_3\, c(\theta_1+\theta_2) \\ 0 + l_2 s\theta_1 + l_3\, s(\theta_1+\theta_2) \\ l_1 + 0 + 0 \end{bmatrix} = \begin{bmatrix} l_2 c\theta_1 + l_3\, c(\theta_1+\theta_2) \\ l_2 s\theta_1 + l_3\, s(\theta_1+\theta_2) \\ l_1 \end{bmatrix}_{3\times 1}$$

$$O_3 - O_1 = R_0^1 \left[d_1^2 + R_1^2 d_2^3 \right]$$

$$= \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 \\ s\theta_2 & c\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$O_3 - O_1 = \begin{bmatrix} c\theta_1 & -s\theta_1 & 0 \\ s\theta_1 & c\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 + l_3 c\theta_2 \\ l_3 s\theta_2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} l_2 c\theta_1 + l_3 c\theta_2 c\theta_1 - s\theta_1 l_2 s\theta_2 \\ l_2 s\theta_1 + l_3 c\theta_2 s\theta_1 + l_3 s\theta_2 c\theta_1 \\ 0 \end{bmatrix} = \begin{bmatrix} l_2 c\theta_1 + l_3\, c(\theta_1+\theta_2) \\ l_2 s\theta_1 + l_3\, s(\theta_1+\theta_2) \\ 0 \end{bmatrix}_{3\times 1}$$

$$d_0^2 = \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix} + \begin{bmatrix} L_2 C\theta_1 \\ L_2 S\theta_1 \\ 0 \end{bmatrix} + \begin{bmatrix} l_3 (C\theta_1 C\theta_2 - S\theta_2 S\theta_1) \\ L_3 (C\theta_2 S\theta_1 + S\theta_2 C\theta_1) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix} + \begin{bmatrix} L_2 C\theta_1 \\ L_2 S\theta_1 \\ 0 \end{bmatrix} + \begin{bmatrix} l_3 C(\theta_1 + \theta_2) \\ l_3 S(\theta_1 + \theta_2 \\ 0 \end{bmatrix}$$

$$O_3 - O_0 = d_0^3 = \begin{bmatrix} 0 + L_2 C\theta_1 + L_3 C(\theta_1 + \theta_2) \\ 0 + L_2 S\theta_1 + l_3\ S(\theta_1 + \theta_2) \\ L_1 + 0 + 0 \end{bmatrix} = \begin{bmatrix} L_2 C\theta_1 + L_3 C(\theta_1 + \theta_2) \\ L_2 S\theta_1 + L_3 S(\theta_1 + \theta_2) \\ L_1 \end{bmatrix}_{3\times 1}$$

$$O_3 - O_1 = R_0^1 [\ d_1^2 + R_1^2 d_2^3]$$

$$= \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} L_2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 \\ S\theta_2 & C\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_3 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$O_3 - O_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} L_2 + L_3 C\theta_2 \\ L_3 S\theta_2 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} L_2 C\theta_1 + L_3 C\theta_2 C\theta_1 - S\theta_1 L_2 S\theta_2 \\ L_2 S\theta_1 + L_3 C\theta_2 S\theta_1 + L_3 S\theta_2 C\theta_1 \\ 0 \end{bmatrix} = \begin{bmatrix} L_2 C\theta_1 + L_3 C(\theta_1 + \theta_2) \\ L_2 S\theta_1 + L_3 S(\theta_1 + \theta_2) \\ 0 \end{bmatrix}_{3\times 1}$$

$$Z_0 = R_0^0 \hat{k} = I \hat{k} = \begin{bmatrix} 0 \\ 0 \\ \hat{k} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Z_1 = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \hat{k} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \hat{k} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Z_{\theta 2} = R_0^2 \hat{k} = \begin{bmatrix} C\theta_1 & -S\theta_1 & 0 \\ S\theta_1 & C\theta_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 \\ S\theta_2 & C\theta_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \hat{k} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Z_{\theta 2} = \begin{bmatrix} C(\theta_1+\theta_2) & -S(\theta_1+\theta_2) & 0 \\ S(\theta_1+\theta_2) & C(\theta_1+\theta_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \hat{k} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$Z_0 \times (O_3 - O_0) = \begin{matrix} i & j & \hat{k} \\ 0 & 0 & 1 \end{matrix} \begin{bmatrix} i & j & \hat{k} \\ 0 \; a & 0 \; b & 1 \; c \\ \overset{d}{L_2 C\theta_1 + L_3 C(\theta_1+\theta_2)} & \overset{e}{L_2 S\theta_1 + L_3 S(\theta_1+\theta_2)} & \overset{f}{L_1} \end{bmatrix}$$

$\cancel{L_2 C\theta_1 + L}$

$$i \begin{vmatrix} 0 & 1 \\ L_2 S\theta_1 + L_3 S(\theta_1+\theta_2) & \cancel{L_2 S\theta_1 + L_3 S(\theta_1+\theta_2)}\ L_1 \end{vmatrix} + j \begin{vmatrix} 0 & 1 \\ L_2 C\theta_1 + L_3 C(\theta_1+\theta_2) & L_1 \end{vmatrix} + \hat{k}\, 0$$

$$= \begin{bmatrix} -L_2 S\theta_1 + L_3 S(\theta_1+\theta_2) \\ L_2 C\theta_1 + L_3 C(\theta_1+\theta_2) \\ 0 \end{bmatrix}$$

$$Z_1 \times (O_3 - O_1) = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & 1 \\ L_2 C\theta_1 + l_3 C(\theta_1+\theta_2) & L_2 S\theta_1 + L_3 S(\theta_1+\theta_2) & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -l_2 S\theta_1 + l_3 S(\theta_1+\theta_2) \\ l_2 C\theta_1 + l_3 C(\theta_1+\theta_2) \\ 0 \end{bmatrix}$$

Now replacing all in the eqn we get

$$J = \begin{bmatrix} -l_2 S\theta_1 + L_3 S(\theta_1+\theta_2) & -L_2 S\theta_1 + l_3 S(\theta_1+\theta_2) & 0 \\ l_2 C\theta_1 + L_3 (\theta_1+\theta_2) & l_2 C\theta_1 + l_3 C(\theta_1+\theta_2 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$$