Tasks:

1. Review the discussion on singularities, decoupling of singularities, and various examples of singularities and singular configurations in the textbook. Describe in 3-4 sentences in your own words what is a singular configuration and how do you find singular configurations. Also, can you detect if a particular configuration is close to a singular configuration using the Manipulator Jacobian?

Singular configurations are the ones that decrease the rank of a Jacobian matrix, which is a function of configuration q. These are special configurations because they determine the configurations where the direction of motions are unattainable, unbounded joint velocities and torques may be obtained or where a unique solution to an inverse kinematic problem is not possible. For a particular manipulator Jacobian with 6 links, we get a singular configuration only if its determinant comes out to be zero. Also, if the z_3, z_4 and z_5 corresponding to the spherical wrist are linearly dependent, we get a singular configuration.

2. Read the definition of DH parameters in the textbook including the summary of steps. Pay particular attention to the end-effector frame and wrist as that was not discussed in class.

3. Write a python subroutine that takes in as inputs the number of links and the DH parameters in table/matrix form, and returns the (a) complete manipulator Jacobian, (b) the end-effector position, and (c) end-effector velocity. If you need any other inputs (such as information about the nature of joints (R/P), incorporate this as an additional input to the python code. However, the code is to be set up in a way that if this information is not provided, the default assumption of all joints being revolute joints is to be assumed.

4. Apply the above code to the two common RRP configurations of Stanford manipulator and SCARA manipulator. Verify that the results obtained using the code match with the expressions derived earlier (by yourself and in the textbook). You may choose a few configurations (numerical values) to verify your results.

Stanford Manipulator

The T matrix comes out as follows:

$$T = A_1 A_2 A_3$$

$$T = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & 0 & S_2 & d_3 S_2 \\ S_2 & 0 & -C_2 & -d_3 C_2 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_1 C_2 & -S_1 & C_1 S_2 & d_3 C_1 S_2 - d_2 S_1 \\ S_1 C_2 & C_1 & S_1 S_2 & d_3 S_1 S_2 + d_2 C_1 \\ -S_2 & 0 & C_2 & d_3 C_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Scara Manipulator

The A- matrices as follows :-

$$T = A_1 A_2 A_3$$

$$= \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & s_2 & 0 & a_2 c_2 \\ s_2 & -c_2 & 0 & a_2 s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} c_{12} & s_{12} & 0 & a_2 c_{12} + a_1 c_1 \\ -s_{12} & -c_{12} & 0 & a_2 s_{12} + a_1 s_1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} c_{12} & s_{12} & 0 & a_2 c_{12} + a_1 c_1 \\ -s_{12} & -c_{12} & 0 & a_2 s_{12} + a_1 s_1 \\ 0 & 0 & -1 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. Solve problems 3-7 in the textbook and also verify your hand-derived answers using the code in Task 3.

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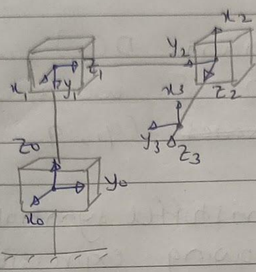
Task 5:-

3-link Cartesian manipulator



So, we get DH parameters as follows

Link	a	α	d	θ
1	0	-90	*	0
2	0	-90	*	-90
3	0	0	*	0

* ⇒ Variable

So, the A matrices are

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

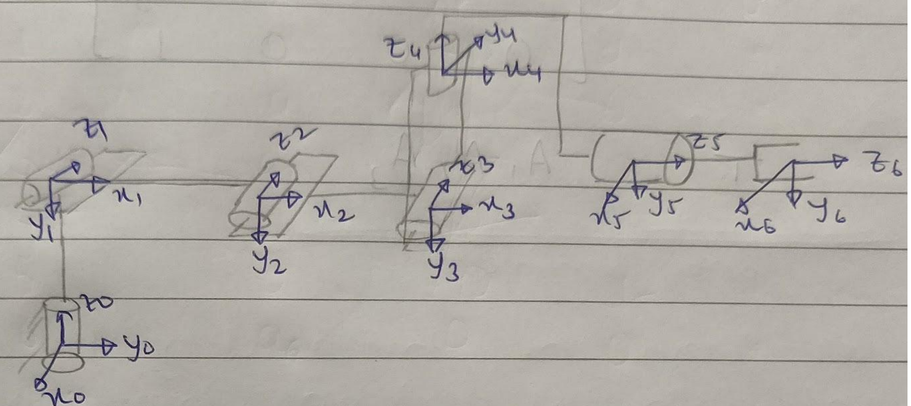
$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3^* \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = A_1 A_2 A_3$$

6. Solve problems 3-8 in the textbook and also verify your hand-derived answers using the code in Task 3.

Task 6



The DH parameters are as follows:-

Link	a	α	d	θ
1	0	-90	d_1	*
2	a_2	0	0	*
3	a_3	0	0	*
4	0	90	d_4	*
5	0	-90	d_5	*
6	0	0	d_6	*

The A matrices are as follows.

$$A_1 = \begin{bmatrix} C\theta_1 & 0 & -S\theta_1 & 0 \\ S\theta_1 & 0 & C\theta_1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2 C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_2 S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & a_3 C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & a_3 S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

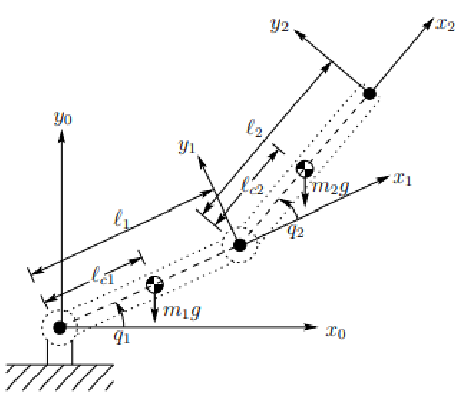
$$A_4 = \begin{bmatrix} C\theta_4 & 0 & S\theta_4 & 0 \\ S\theta_4 & 0 & -C\theta_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} C\theta_5 & 0 & -S\theta_5 & 0 \\ S\theta_5 & 0 & C\theta_5 & 0 \\ 0 & -1 & 0 & d_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

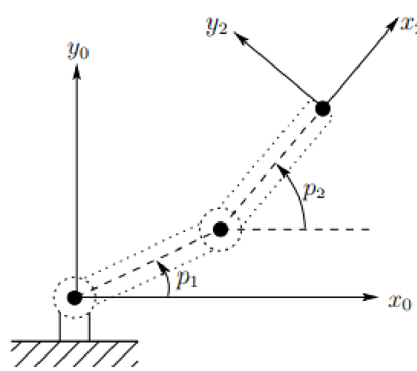
$$A_6 = \begin{bmatrix} C\theta_6 & -S\theta_6 & 0 & \cancel{a_6 C\theta_6}\, 0 \\ S\theta_6 & C\theta_6 & 0 & \cancel{a_6 S\theta_6}\, 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{T = A_1 A_2 A_3 A_4 A_5 A_6}$$

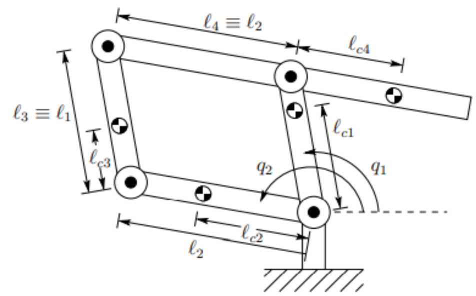
7. Compare the three different configurations for the 2R manipulator (direct drive, remotely-driven, and 5-bar parallelogram arrangement) and explain the key differences and advantages of each arrangement.



Direct Driven elbow manipulator



Remotely driven elbow manipulator



5-bar linkage

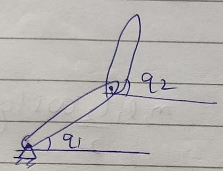
For a directly driven system, the joint variable q_2 depends on the joint variable q_1. In this type of system, there is no backlash, hysteresis or loss of motion. This increases the overall reliability of the system.

However, in the case of a remotely driven manipulator, q_2 is affected by the driving motor and not q_1. For the latter case, we can eliminate the Coriolis forces which simplify the dynamics. However, the centrifugal forces remain in both the systems coupling the two joints. Since this system is remotely actuated, the inertia of the links is reduced. There is still the disadvantage of backlash and hysteresis.

For a 5-bar linkage, we get a closed kinematic chain system. When solving for dynamic equations, we can adjust the link lengths and masses in a way that makes the inertia matrix diagonal and constant. This eliminates the Coriolis and centrifugal forces altogether. Further, we can adjust the two joint variables independently without worrying about their interactions with each other.

8. Complete the derivation of the dynamic equations of the 2R manipulator discussed in class and compare your results with those in the mini-project. Remark on any discrepancies or observations.

Task 8 // **Example:-** Elbow Manipulator



Links are uniform slender bars

$$v_{c_1} = \begin{bmatrix} -\frac{l_1}{2}\sin q_1 \\ \frac{l_1}{2}\cos q_1 \\ 0 \end{bmatrix} \dot{q}_1$$

$$v_{c_2} = \begin{bmatrix} -l_1 \sin q_1 & -\frac{l_2}{2}\sin q_2 \\ l_1 \cos q_1 & \frac{l_2}{2}\cos q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\omega_1 = \dot{q}_1 \hat{k} \quad , \quad \omega_2 = \dot{q}_2 \hat{k}$$

$$K = \frac{1}{2} \sum_{i=1}^{n} m_i v_{c_i}^T v_{c_i} + \frac{1}{2} \sum_{i=1}^{n} \omega_i^T I_i \omega_i$$

$$v_{c_i} = J_{v_{c_i}}(q)\dot{q} \; ,$$

$$\omega_i = R_i^T J_{\omega_i}(q)\dot{q}$$

$$K = \frac{1}{2} \dot{q}^T \sum_{i=1}^{n} \left[m_i J_{v_{c_i}}(q)^T J_{v_{c_i}}(q) + J_{\omega_i}(q)^T I_i R_i(q)^T J_{\omega_i}(q) \right] \dot{q}.$$

$$K = \frac{1}{2} \dot{q}^T D(q) \dot{q}$$

→ Always symmetric positive definite matrix

In this example,

$$D(q) = \begin{bmatrix} \frac{m_1 l_1^2}{4} + m_2 l_1^2 + I_1 & m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1) \\ m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1) & m_2 \frac{l_2^2}{4} + I_2 \end{bmatrix}$$

Computing the christoffel symbols

~~C_{ijk} =~~

$$C_{111} = \frac{1}{2} \left[\frac{\partial_{11}}{\partial q_1} \right] = 0$$

$$C_{121} = C_{211} = \frac{1}{2} \left[\frac{\partial d_{11}}{\partial q_2} + \frac{\partial d_{12}}{\partial q_1} - \frac{\partial d_{12}}{\partial q_1} \right]$$

$$c_{121} = c_{211} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_2} = 0$$

$$c_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2}\frac{\partial \cancel{d_{22}}^{\,0}}{\partial q_1} = \frac{\partial d_{12}}{\partial q_2} = -m_2 l_1 \frac{l_2}{2} \sin(q_2 - q_1)$$

$$c_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial \cancel{d_{11}}^{\,0}}{\partial q_2} = + m_2 l_1 \frac{l_2}{2} \sin(q_2 - q_1)$$

$$c_{212} = c_{122} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0 , \cancel{c_{222}}$$

$$c_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

Potential energy $\Rightarrow V = m_1 g \frac{l_1}{2} \sin q_1 + m_2 g \left(l_1 \sin q_1 + \frac{l_2}{2} \sin q_2 \right)$

$$\phi_1 = \frac{\partial V}{\partial q_1} , \quad \phi_2 = \frac{\partial V}{\partial q_2}$$

Final equations:-

$$\boxed{\begin{aligned} d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{221}\dot{q}_1^2 + \phi_1 &= \tau_1 \\ d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_2^2 + \phi_2 &= \tau_2 \end{aligned}}$$

Here, $V = m_1 g \frac{l_1}{2} \sin q_1 + m_2 g \left(l_1 \sin q_1 + \frac{l_2}{2} \sin q_2 \right)$

Substituting all values, we get

(I) $\left(m_1 \frac{l_1^2}{4} + m_2 l_1^2 + I_1 \right) \ddot{q}_1 + \left(m_2 \frac{l_1 l_2}{2} \cos(q_2 - q_1) \right) \ddot{q}_2$
$- m_2 \frac{l_1 l_2}{2} \sin(q_2 - q_1) \dot{q}_1^2 + m_1 g \frac{l_1}{2} c q_2 + m_2 g l_1 c q_1 = \tau_1$

(II) $m_2 \frac{l_1 l_2}{2} \cos(q_2 - q_1) \ddot{q}_1 + \left(m_2 \frac{l_2^2}{4} + I_2 \right) \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} s(q_2 - q_1) \dot{q}_2^2$
$+ m_2 g \frac{l_2}{2} \sin(q_2) = \tau_2$

Here, $I_1 = \frac{m_1 l_1^2}{12}$, $I_2 = \frac{m_2 l_2^2}{12}$

(I) $\left(\frac{m_1 l_1^2}{3} + m_2 l_1^2 \right) \ddot{q}_1 + \left(m_2 \frac{l_1 l_2}{2} c(q_2 - q_1) \right) \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} s(q_2 - q_1) \dot{q}_1^2$
$+ m_1 g \frac{l_1}{2} c q_2 + m_2 g l_1 c q_1 = \tau_1$

(II) $m_2 \frac{l_1 l_2}{2} c(q_2 - q_1) \ddot{q}_1 + m_2 \frac{l_2^2}{3} \ddot{q}_2 + m_2 \frac{l_1 l_2}{2} s(q_2 - q_1) \dot{q}_2^2$
$+ m_2 g \frac{l_2}{2} s(q_2) = \tau_2$.

We observe that the terms $\dot{q}_1 \dot{q}_2$ do not appear in the above form in contrast to the equations derived in mini-project. The reason being that links here are remotely driven.

9. Review derivations of dynamics equations of motion for the other two configurations of the 2R manipulator discussed in the textbook.

10. Summarize neatly in your own handwriting, the key steps to derive equations of motion when you are already provided D(q) and V(q).

Task 10 :-

Deriving eqns of motion given $D(q)$ & $V(q)$

① Based on size of matrix $D(q)$, we know number of links, n. So, we assume the ~~generalized coordinates~~ joint variables $(q_1, q_2 \dots q_n)$

② We derive the Christoffel symbols of first kind with the following expression:

$$C_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} + \frac{\partial d_{ij}}{\partial q_k}\right]$$

③ Calculate partial derivatives of potential energy $V(q)$ w.r.t. each of the q_i. i.e.

$$\phi_k = \cancel{\partial P}\ \frac{\partial V}{\partial q_k}$$

Here, $i, j, k \Rightarrow 1, 2, \dots n$

④ Now, we write an equation of motion for each link separately as:-

$$\boxed{\tau_k = \sum_j d_{kj}(q)\ddot{q}_j + \sum_{ji} C_{ijk}\dot{q}_i\dot{q}_j + \phi_k(q)}$$

11. Write a code that uses symbolic computation and differentiation to derive the equations of motion for any robot given D(q) and V(q).