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ASSIGNMENT - 3

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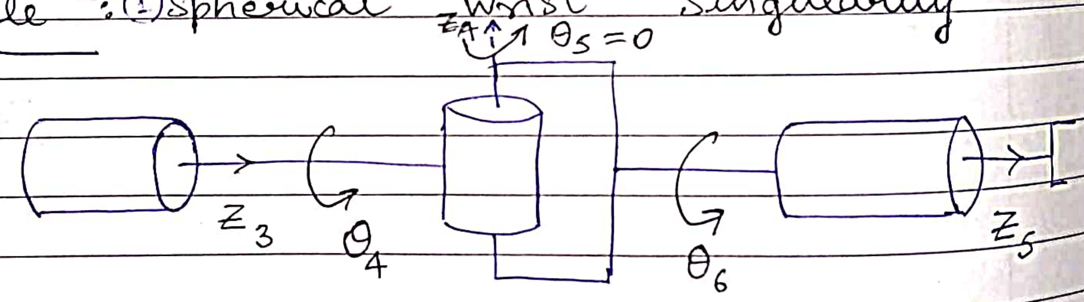
(1) Singular Configurations

Configurations from which certain directions of motion are Unattainable, Bounded end-effector velocities may correspond to unbounded joint velocities often on boundary of manipulator workspace.

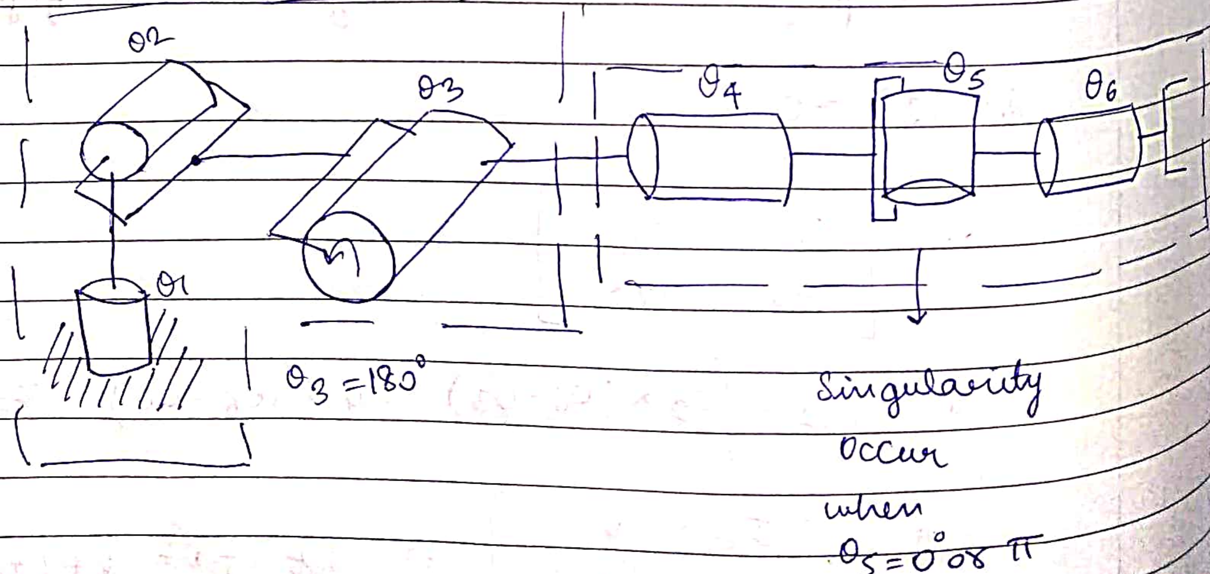
Square matrix is singular when its determinant = 0.

q is singular iff $\det(J(q)) = 0$

Example : (1) Spherical Wrist Singularity



(2)



Singularity occur when $\theta_5 = 0°$ or π

Determining Singular Configuration

→ Separate singularities into arm singularity and wrist singularity.

→ analogous to Kinematic decoupling.

→ assumption ① 6 DOF manipulator
② we can break J into block form

A 6 DOF manipulator that has a 3 axis arm and a spherical wrist

$$J = \left[\begin{array}{ccc|ccc} z_0 \times (O_6 - O_0) & z_1 \times (O_6 - O_1) & z_2 \times (O_6 - O_2) & z_3 \times (O_6 - O_3) & z_4 \times (O_6 - O_4) & z_5 \times (O_6 - O_5) \\ \hline z_0 & z_1 & z_2 & z_3 & z_4 & z_5 \end{array}\right]$$

- The manipulator is a singular configuration if

$$\det(J) = 0$$

$$J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$$

if $J_{12} = \begin{bmatrix} z_3 \times (O_6 - O_3) & z_4 \times (O_6 - O_4) & z_5 \times (O_6 - O_5) \end{bmatrix} = 0$

$$J = \begin{bmatrix} J_{11} & 0 \\ J_{21} & J_{22} \end{bmatrix} \Rightarrow \det(J) = \det(J_{11}) \det(J_{22})$$

For a spherical wrist, $O_3 = O_4 = O_5 = 0$
and $O_6 = 0$

$$\Rightarrow J = \begin{bmatrix} z_0\,(O_6 - O_0) & z_1 \times (O_6 - O_1) & z_2 \times (O_6 - O_2) & 0 & 0 & 0 \\ z_0 & z_1 & z_2 & z_3 & z_4 & z_5 \end{bmatrix}$$

$$J = \begin{bmatrix} J_{11} & 0 \\ J_{21} & J_{22} \end{bmatrix}$$

$$\det(J) = \det(J_1)\,\det(J_2)$$

<u>condition for Singularity</u> $\det(J_1) = 0$
or
$\det(J_2) = 0$

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[3] DH (Denavit Hartenberg Representation)

DH parameters (for each joint): θ, d, a, α

Transformation matrix :

$$A = Rot_{z,\theta} \cdot trans_{z,d} \; Trans_{x,a} \; Rot_{x,\alpha}$$

$$= \begin{bmatrix} \cos\theta & -\sin\theta\cos\alpha & \sin\theta\sin\alpha & a\cos\theta \\ \sin\theta & \cos\theta\cos\alpha & -\cos\theta\sin\alpha & a\sin\theta \\ 0 & \sin\alpha & \cos\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(a) Complete manipulator jacobian

Let no. of links be n

So $A_0^1 = \begin{bmatrix} C_\theta & -S_\theta C_\alpha & S_\theta S_\alpha & aC_\theta \\ S_\theta & C_\theta C_\alpha & -C_\theta S_\alpha & aS_\theta \\ 0 & S_\alpha & C_\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$ for θ, d, a, α for I link.

$A_1^2 = \begin{bmatrix} C_\theta & -S_\theta C_\alpha & S_\theta S_\alpha & aC_\theta \\ S_\theta & C_\theta C_\alpha & -C_\theta S_\alpha & aS_\theta \\ 0 & S_\alpha & C_\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$ for θ, d, a, α of II link ...

Similarly $A_2^3, A_3^4 \ldots A_{n-1}^n$ can be calculated.

$$z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$z_1 = \begin{bmatrix} S_\theta S_\alpha \\ -C_\theta S_\alpha \\ C_\alpha \\ \cancel{0} \end{bmatrix}$$ for θ, d, a, α of I link

$$z_2 = \begin{bmatrix} S_\theta S_\alpha \\ -C_\theta S_\alpha \\ C_\alpha \\ \cancel{0} \end{bmatrix}$$ for θ, d, a, α of II link

....

$$t_1 = \begin{bmatrix} aC_\theta \\ aS_\theta \\ d \end{bmatrix}$$ for θ, d, a, α of I link

$$t_2 = \begin{bmatrix} aC_\theta \\ aS_\theta \\ d \end{bmatrix}$$ for θ, d, a, α of II link ...

<u>Final transformation matrix</u>

$$T = A_0^1 \times A_1^2 \times A_2^3 \times A_3^4 \times \dots A_{n-1}^n$$

So Jacobian, $J =$

$$\begin{bmatrix} z_0 \times (t_n - t_0) & \dots & z_{n-1} \times (t_n - t_{n-1}) \\ z_0 & \dots & z_{n-1} \end{bmatrix}$$

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let us take $n=6$ (for RRR configuration)

$$J = \begin{bmatrix} z_0 \times (t_6 - t_0) & z_1 \times (t_6 - t_1) & z_2 \times (t_6 - t_2) & z_3 \times (t_6 - t_3) & z_4 \times (t_6 - t_4) & z_5 \times (t_6 - t_5) \\ z_0 & z_1 & z_2 & z_3 & z_4 & z_5 \end{bmatrix}$$

(b) End effector position (x, y)

$$x = A_{41} \quad , \quad y = A_{42} \quad \text{—— (A)}$$

where A_{41} = 4^{th} column 1^{st} row element of final transformation matrix 'T'.
A_{42} = 4^{th} column 2^{nd} row element of final transformation matrix 'T'.

Eg: $x = a\cos\theta$, $y = a\sin\theta$ (for a 1 link manipulator)

(c) end effector velocity

differentiate equation (A) wrt time

$$\dot{x} = -a\sin\theta\,\dot{\theta} \quad , \quad \dot{y} = a\cos\theta\,\dot{\theta}$$

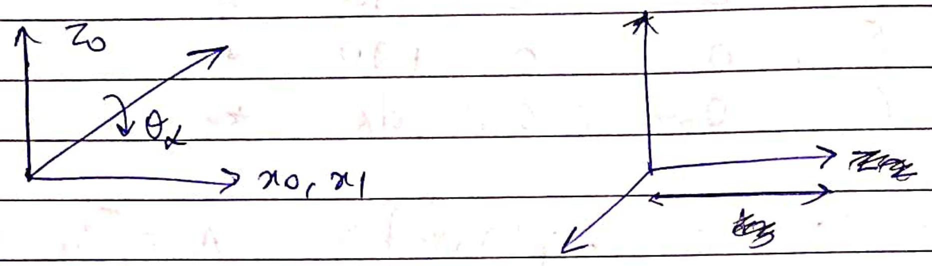
or find by Jacobian $\begin{bmatrix} \dot{x} \\ \omega \end{bmatrix} = J \begin{bmatrix} \dot{q} \end{bmatrix}$ $\left\{ \begin{bmatrix} J_v \\ J_w \end{bmatrix} \right\}$

where $\dot{x} = v$ = linear velocity
ω = angular vel of end effector

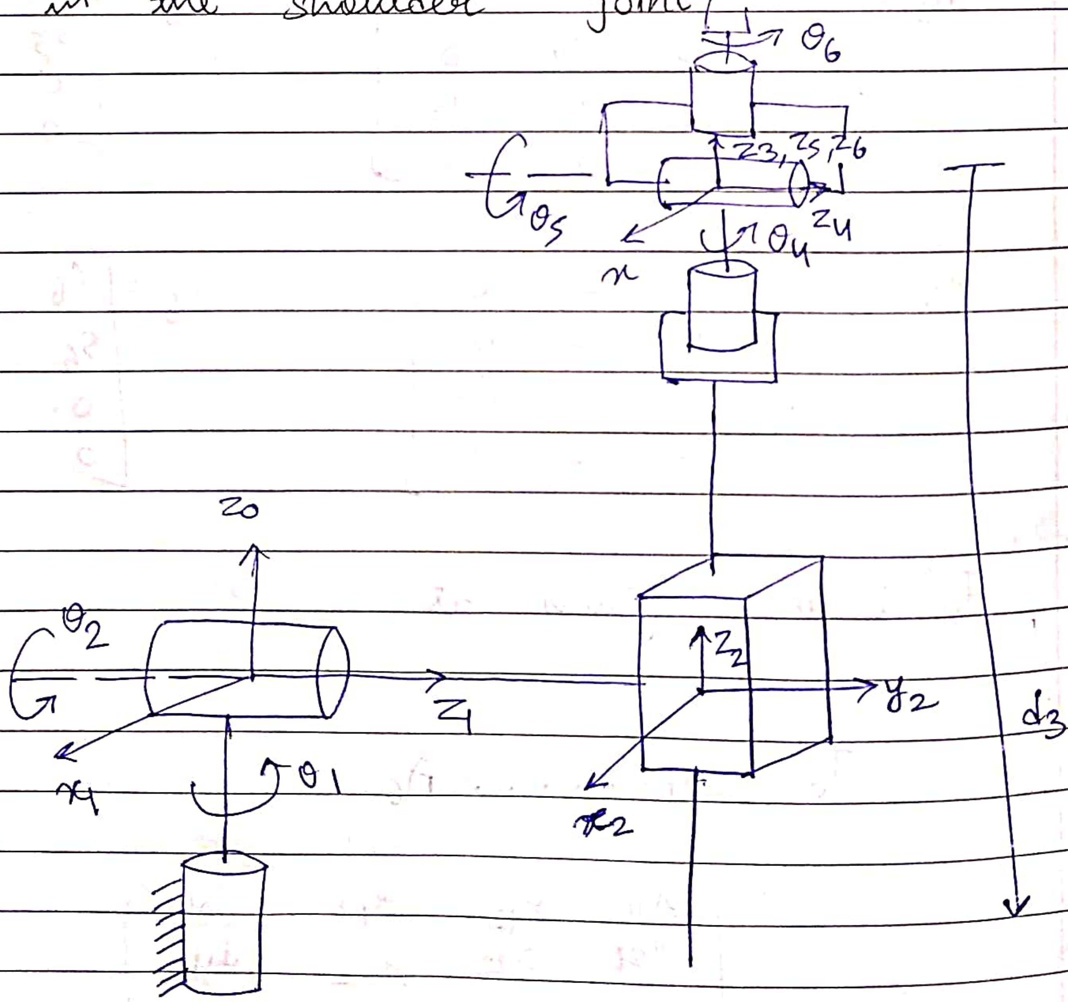
(4)

Forward Kinematics for Stanford Manipulator

DH convention [RRP]



This manipulator is an example of a spherical (RRP) manipulator with a spherical wrist. This manipulator has an offset in the shoulder joint.



Link	d_i	a_i	α_i	θ_i
1	0	0	-90	*
2	d_2	0	+90	*
3	*	0	0	0
4	0	0	-90	*
5	0	0	+90	*
6	0	0	d_6	*

$$A_1 = \begin{bmatrix} C_1 & 0 & -S_1 & 0 \\ S_1 & 0 & C_1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_4 = \begin{bmatrix} C_4 & 0 & -S_4 & 0 \\ S_4 & 0 & C_4 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C_2 & 0 & S_2 & 0 \\ S_2 & 0 & -C_2 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_5 = \begin{bmatrix} C_5 & 0 & S_5 & 0 \\ S_5 & 0 & -C_5 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A_6 = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

T_0^6 is then given as

$$T_0^6 = A_1 \cdots\cdots A_6$$

$$\therefore \quad = \begin{bmatrix} r_{11} & r_{12} & r_{13} & d_x \\ r_{21} & r_{22} & r_{32} & d_y \\ r_{31} & r_{32} & r_{33} & d_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where

$$r_{11} = c_1 \left[c_2 (c_4 c_5 c_6 - s_4 s_6) - s_2 s_5 c_6 \right] - s_1 (s_4 c_5 c_6 + c_4 s_6)$$

$$r_{21} = s_1 \left[c_2 (c_4 c_5 c_6 - s_4 s_6) - s_2 s_5 c_6 \right] + c_1 (s_4 c_5 c_6 + c_4 s_6)$$

$$r_{31} = -s_2 (c_4 c_5 c_6 - s_4 s_6) - c_2 s_5 c_6$$

$$r_{12} = c_1 \left[-c_2 (c_4 c_5 s_6 + s_4 c_6) + s_2 s_5 s_6 \right] - s_1 (-s_4 c_5 s_6 + c_4 c_6)$$

$$r_{22} = s_1 \left[-c_2 (c_4 c_5 s_6 + s_4 c_6) + s_2 s_5 s_6 \right] + c_1 (-s_4 c_5 s_6 + c_4 c_6)$$

$$r_{32} = s_2 (c_4 c_5 s_6 + s_4 c_6) + c_2 s_5 s_6$$

$$r_{13} = c_1 (c_2 c_4 s_5 + s_2 c_5) - s_1 s_4 s_5$$

$$r_{23} = s_1 (c_2 c_4 s_5 + s_2 c_5) + c_1 s_4 s_5$$

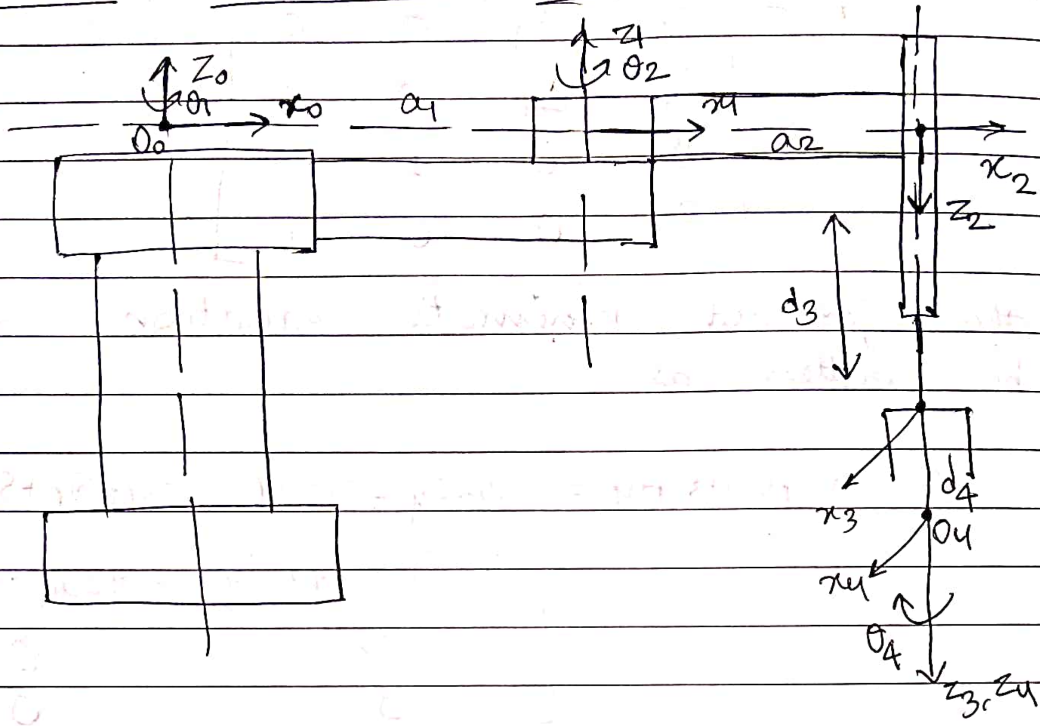
$$r_{33} = -s_2 c_4 s_5 + c_2 c_5$$

$$d_x = c_1 s_2 d_3 - s_1 d_2 + d_6 (c_1 c_2 c_4 s_5 + c_1 c_5 s_2 - s_1 s_4 s_5)$$

$$d_y = s_1 s_2 d_3 + c_1 d_2 + d_6 (c_1 s_4 s_5 + c_2 c_4 s_1 s_5 + c_5 s_1 s_2)$$

$$d_z = c_2 d_3 + d_6 (c_2 c_5 - c_4 s_2 s_5)$$

Forward Kinematics for SCARA manipulator



we establish x_0 axis in the plane of the page as shown. This is completely arbitrary and only affects the zero configuration of the manipulator, that is, the position of the manipulator when $\theta_1 = 0$

$$A_1 = \begin{bmatrix} c_1 & -s_1 & 0 & a_1 c_1 \\ s_1 & c_1 & 0 & a_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} c_2 & s_2 & 0 & a_2 c_2 \\ s_2 & -c_2 & 0 & a_2 s_2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

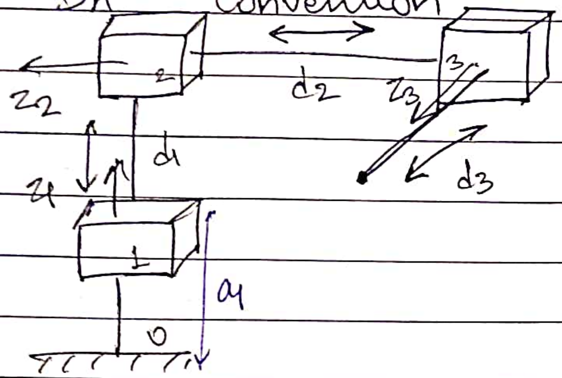
$$A_4 = \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The forward kinematic equation can therefore be written as

$$T_0^4 = A_1 A_2 A_3 A_4 = \begin{bmatrix} C_{12}C_4 + S_{12}S_4 & -C_{12}S_4 + S_{12}C_4 & 0 & a_1C_1 + a_2C_{12} \\ S_{12}C_4 - C_{12}S_4 & -S_{12}S_4 - C_{12}C_4 & 0 & a_1S_1 + a_2S_{12} \\ 0 & 0 & -1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(5) Problem 3-7

Consider 3 link cartesian manipulator as shown in figure. Derive Forward Kinematic equation using DH convention



Solution

	a_i	α_i	d_i	θ_i
0-1	a_1	0	0	0
1-2	0	90	d_1	0
2-3	0	90	$-d_2$	-90
3-4	0	0	d_3	0

Now

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T = A_1 \times A_2 \times A_3 \times A_4$$

$$= \begin{bmatrix} 0 & 0 & -1 & a_1 - d_3 \\ 0 & -1 & 0 & d_2 \\ -1 & 0 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad \text{—— (A)}$$

verify with code

Put $a_1 = 1m$, $d_1 = 0.5m$, $d_2 = 0.6m$, $d_3 = 0.8m$

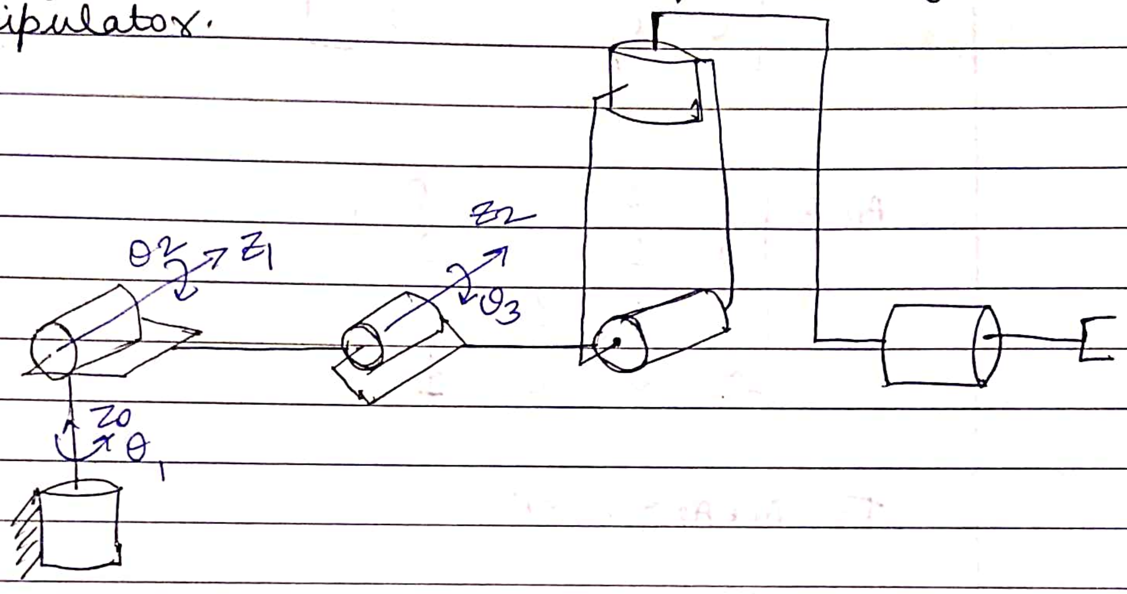
T_{from} equation no. (A) is

$$T = \begin{bmatrix} 0 & 0 & -1 & 0.2 \\ 0 & -1 & 0 & 0.6 \\ -1 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

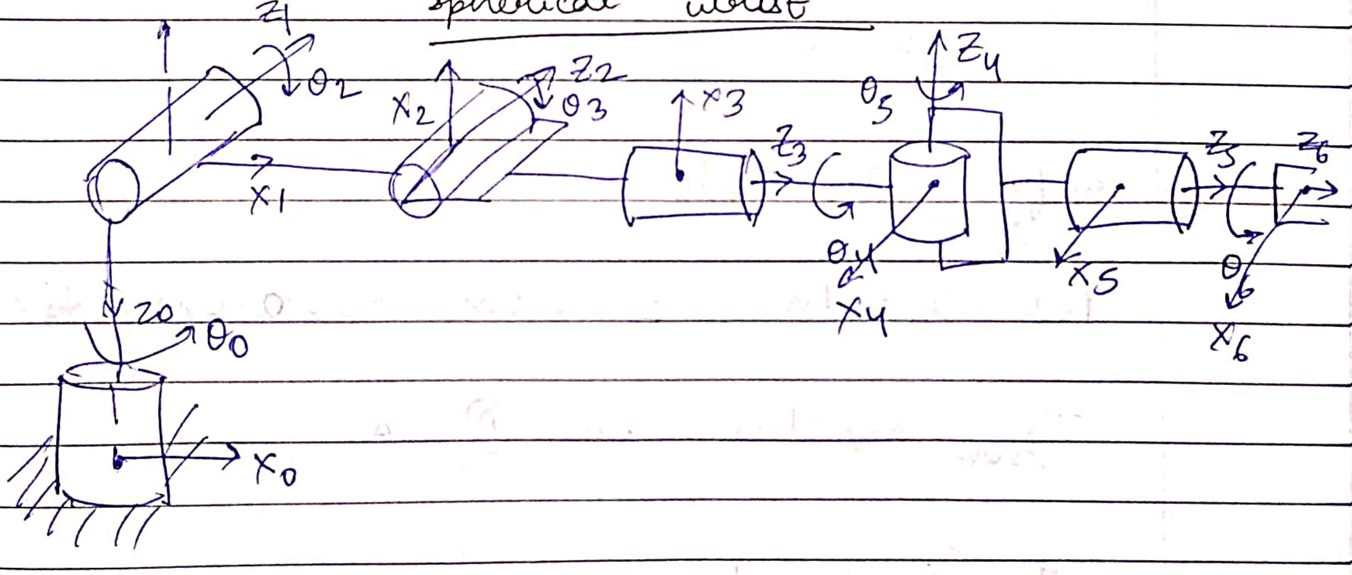
The same is obtained with Python code.

Problem 3-8

(6) Attach a spherical wrist to the 3 link articulated manipulator as shown in fig. Derive forward Kinematic equation for this manipulator.



Elbow manipulator with spherical wrist



	α	θ	d	a
1	90	θ_1	0	0
2	0	θ_2	0	a_1
3	-90	θ_3	0	0
4	-90	θ_4	d_4	0
5	-90	θ_5	0	0
6	0	θ_6	d_6	0

$$A_1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & a\,\cancel{\cos\theta_1}^{\,0} \\ \sin\theta_1 & 0 & -\cos\theta_1 & a\,\cancel{\sin\theta_1}^{\,0} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} C_{\theta_2} & -S_{\theta_2} & 0 & a_1 C_{\theta 2} \\ S_{\theta 2} & C_{\theta 2} & 0 & a_1 S_{\theta 2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C_{\theta_3} & 0 & -S_{\theta 3} & 0 \\ S_{\theta_3} & 0 & C_{\theta 3} & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} C_{\theta 4} & 0 & -S_{\theta 4} & 0 \\ S_{\theta 4} & 0 & C_{\theta 4} & 0 \\ 0 & -1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} C_{\theta 5} & 0 & -S_{\theta 5} & 0 \\ S_{\theta 5} & 0 & C_{\theta 5} & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} C_{\theta_6} & -S_{\theta 6} & 0 & 0 \\ S_{\theta 6} & C_{\theta 6} & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Final transformation matrix

$$T = A_1 \times A_2 \times A_3 \times A_4 \times A_5 \times A_6$$

$$= \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & 0 \\ S\theta_1 & 0 & -C\theta_1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_1 \cos\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_1 S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times A_3 A_4 . A_5 . A_6$$

$$= \begin{bmatrix} C\theta_1 C\theta_2 & -C\theta_1 S\theta_2 & S\theta_1 & a_1 C\theta_1 C\theta_2 \\ S\theta_1 C\theta_2 & -S\theta_1 S\theta_2 & -C\theta_1 & a_1 S\theta_1 C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_1 S\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3 & 0 & -S\theta_3 & 0 \\ S\theta_3 & 0 & C\theta_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_4 . A_5 . A_6$$

$$= \begin{bmatrix} C\theta_1 C\theta_2 C\theta_3 - C\theta_1 S\theta_2 S\theta_3 & -S\theta_1 & -C\theta_1 C\theta_2 S\theta_3 - C\theta_1 S\theta_2 C\theta_3 & a_1 C\theta_1 C\theta_2 \\ S\theta_1 C\theta_2 C\theta_3 - S\theta_1 S\theta_2 S\theta_3 & C\theta_1 & -S\theta_1 C\theta_2 S\theta_3 - S\theta_1 S\theta_2 C\theta_3 & a_1 S\theta_1 C\theta_2 \\ S\theta_2 C\theta_3 + C\theta_2 S\theta_3 & 0 & -S\theta_3 S\theta_2 + C\theta_2 C\theta_3 & a_1 S\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} . A_4 . A_5 . A_6$$

Verify with code

$\theta_1 = 20^\circ$, $\theta_2 = 20^\circ$ $\theta_3 = 15^\circ$ $\theta_4 = 25^\circ$

$\theta_5 = 10^\circ$ $\theta_6 = 10^\circ$

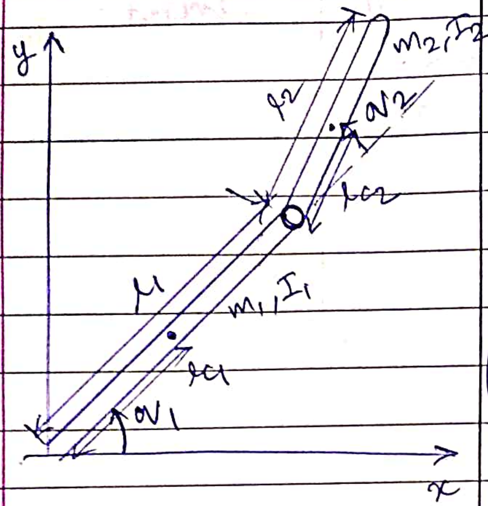
Put $n=6$, $\theta_0 = 20^\circ, \alpha_0 = 90, a_0 = 0, d_0 = 0, \theta_1 = 20, \alpha_1 = 0$
$a_1 = 1, d_1 = 0, \theta_2 = 15, \alpha_2 = -90, a_2 = 0, d_2 = 0$
$\theta_3 = 25, \alpha_3 = -90, a_3 = 0, d_3 = 0.5, \theta_4 = 10, \alpha_4 = -90,$
$a_4 = 0, d_4 = 0, \theta_5 = 10, \alpha_5 = 0, a_5 = 0, d_5 = 0.5$

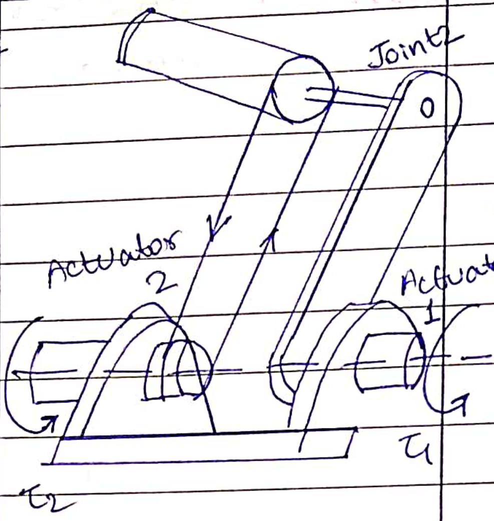
T from code

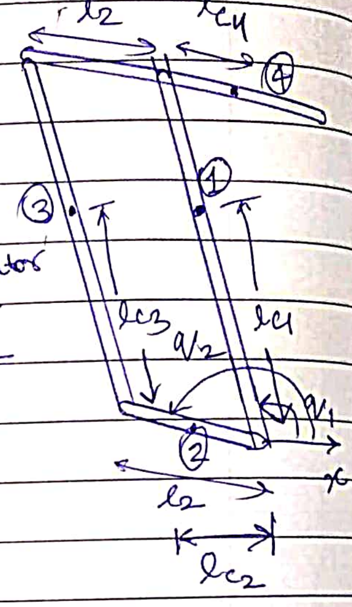
$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.34 & -0.93 & -0.0 & 0 \\ 0 & 0 & 0 & 0 \\ -0.45 & -0.21 & -0.996 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(7) DIRECT DRIVE | REMOTELY DRIVEN | 5-BAR PARALLELOGRAM

(1)







(2)

Direct drive:

$$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + c_{121}\dot{q}_1\dot{q}_2 + c_{211}\dot{q}_2\dot{q}_1 + c_{221}\dot{q}_2^2 + \phi_1 = \tau_1$$

$$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + c_{112}\dot{q}_1^2 + \phi_2 = \tau_2$$

where

$$\phi_1 = \frac{\partial V}{\partial q_1} = (m_1 l_{c1} + m_2 l_1) g c_{q_1} + m_2 l_{c2} g c_{q_1 q_2}$$

$$\phi_2 = \frac{\partial V}{\partial q_2} = m_2 l_{c2} c_{q_1 q_2}$$

$$D(q) = m_1 J_{v_{c1}}^T J_{v_{c1}} + m_2 J_{v_{c2}}^T J_{v_{c2}} + \begin{bmatrix} I_1 + I_2 & I_2 \\ I_2 & I_2 \end{bmatrix}$$

Remotely driven:

$$d_{11}\ddot{p}_1 + d_{12}\ddot{p}_2 + c_{221}\dot{p}_2^2 + \phi_1 = \tau_1$$

$$d_{21}\ddot{p}_1 + d_{22}\ddot{p}_2 + c_{112}\dot{p}_1^2 + \phi_2 = \tau_2$$

where

$$\phi_1 = (m_1 l_{c1} + m_2 l_1) g \cos p_1$$

$$\phi_2 = m_2 l_{c2} g \cos p_2$$

$$D(p) = \begin{bmatrix} m_1 l_{c1}^2 + m_2 l_1^2 + I_1 & m_2 l_1 l_{c2} \cos(p_2 - p_1) \\ m_2 l_1 l_{c2} \cos(p_2 - p_1) & m_2 l_{c2}^2 + I_2 \end{bmatrix}$$

5-bar parallelogram:

$$d_{11}\ddot{q}_1 + \phi_1(q_1) = \tau_1$$

$$d_{22}\ddot{q}_2 + \phi_2(q_2) = \tau_2$$

where

$$\phi_1 = g \cos q_1 (m_1 l_{c1} + m_3 l_{c3} + m_4 l_1)$$

$$\phi_2 = g \cos q_2 (m_2 l_{c2} + m_3 l_2 - m_4 l_{c4})$$

$$D(q) = \sum_{i=1}^{4} m_i J_{v_c}^T J_{v_c} + \begin{bmatrix} I_1 + I_3 & 0 \\ 0 & I_2 + I_4 \end{bmatrix}$$

(3) Planar Elbow manipulator	Planar elbow manipulator with remotely driven link.	This one ~~it~~ forms closed Kinematic chain.

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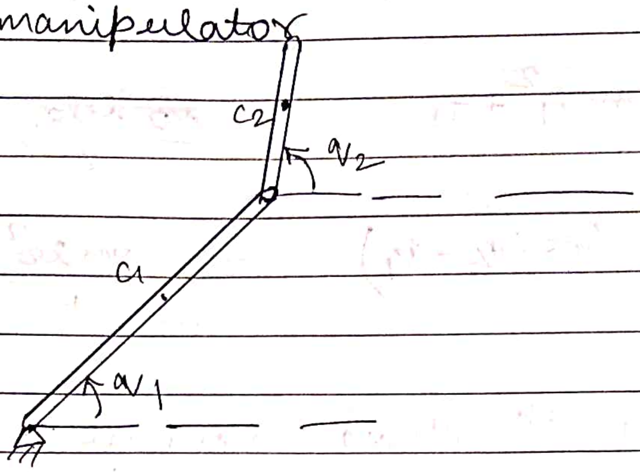
(8) Derivation for dynamic equations of 2R manipulator



$$v_{c_1} = \begin{bmatrix} -\frac{l_1}{2} s q_1 \\ \frac{l_1}{2} c q_1 \\ 0 \end{bmatrix} \dot{q}_1 , \quad v_{c_2} = \begin{bmatrix} -l_1 s q_1 & -\frac{l_2}{2} s q_{12} \\ l_1 c q_1 & \frac{l_2}{2} c q_{12} \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

Now $\omega_1 = \dot{q}_1 \hat{k}$, $\omega_2 = \dot{q}_2 \hat{k}$

$$K = \frac{1}{2} \sum_{i=1}^{n} m_i v_{c_i}^T v_{c_i} + \frac{1}{2} \sum_{i=1}^{n} \omega_i^T I_i \omega_i$$

$$v_{c_i} = J_{v_{c_i}}(q) \dot{q}$$

Now $\omega_i = R_i^T J_{\omega_i}(q) \dot{q}$

$$K = \frac{1}{2} \dot{q}^T \sum_{i=1}^{n} \left[m_i J_{v_{c_i}}(q)^T J_{v_{c_i}}(q) + J_{\omega_i}(q)^T R_i(q) I_i R_i(q)^T J_{\omega_i}(q) \right] \dot{q}$$

$$\boxed{K = \frac{1}{2} \dot{q}^T D \dot{q}}$$

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Now

$$D(q) = \begin{bmatrix} \frac{m_1 l_1^2}{4} + m_2 l_1^2 + I_1 & \frac{m_2 l_1 l_2}{2} \cos(q_2 - q_1) \\ \frac{m_2 l_1 l_2}{2} \cos(q_2 - q_1) & \frac{m_2 l_2^2}{4} + I_2 \end{bmatrix}$$

d_{ij} is i^{th} row j^{th} column element of $D(q)$

Computing the ~~ct~~ christoffel's symbols

$$C_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

$(c_{ijk} = c_{jik})$

$$C_{111} = \frac{1}{2}\frac{\partial d_{11}}{\partial q_1} = 0$$

$$C_{121} = C_{211} = \frac{1}{2}\left[\frac{\partial d_{11}}{\partial q_2} + \frac{\partial d_{12}}{\partial q_1} - \frac{\partial d_{12}}{\partial q_1}\right] = \frac{1}{2}\left[\frac{\partial d_{11}}{\partial q_2}\right] = 0$$

$$C_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = \frac{\partial d_{12}}{\partial q_2}$$

$$= -\frac{m_2 l_1 l_2}{2} \sin(q_2 - q_1)$$

$$C_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2}$$

$$= \frac{m_2 l_1 l_2}{2} \sin(q_2 - q_1)$$

$$C_{212} = C_{122} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1} = 0$$

$$C_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2} = 0$$

Potential energy, $V = \frac{m_1 g l_1}{2}\sin q_1 + m_2 g\left(l_1 \sin q_1 + \frac{l_2}{2}\sin q_2\right)$

$$\phi_1 = \frac{\partial V}{\partial q_1}, \quad \phi_2 = \frac{\partial V}{\partial q_2}$$

Final equations

$$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + C_{221}\dot{q}_1^2 + \phi_1 = \tau_1$$

$$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + C_{112}\dot{q}_1^2 + \phi_2 = \tau_2$$

substituting values, we get

$$\left(\frac{m_1 l_1^2}{4} + m_2 l_1^2 + I_1\right)\ddot{q}_1 + \left(\frac{m_2 l_1 l_2 \cos(q_2 - q_1)}{2}\right)\ddot{q}_2 + \left(\frac{-m_2 l_1 l_2 \sin(q_2 - q_1)}{2}\right)\dot{q}_1^2 + \frac{m_1 g l_1 \cos q_1}{2} + m_2 g l_1 \cos q_1 = \tau_1$$

$$I_1 = \frac{m_1 l_1^2}{12}$$

we get

$$\boxed{\tau_1 = \frac{m_1 l_1^2}{3}\ddot{q}_1 + m_2 l_1^2 \ddot{q}_1 + \frac{m_2 l_1 l_2}{2}\ddot{q}_2 \cos(q_2 - q_1) - \frac{m_2 l_1 l_2}{2}\dot{q}_2^2 \sin(q_2 - q_1) + \frac{m_1 g l_1}{2}\cos q_1 + m_2 g l_1 \cos q_1}$$

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for τ_2

put $I_2 = \frac{m_2 l_2^2}{12}$

$$\Rightarrow \boxed{\tau_2 = \frac{m_2 l_2^2}{3}\ddot{q}_2 + \frac{m_2 l_1 l_2}{2}\ddot{q}_1 \cos(q_2 - q_1) + \frac{m_2 l_1 l_2}{2}\dot{q}_1^2 \sin(q_2 - q_1) + \frac{m_2 g l_2}{2}\cos q_2}$$

Hence these are the same results derived for 2R manipulator in miniproject.

(10) Steps to derive equation of motion given $D(q)$ and $V(q)$:

Step 1 : Find the components of $D(q)$ i.e. d_{ij}

d_{ij} is i^{th} row j^{th} column element of $D(q)$

Step 2 : Compute ~~cryst~~offel's symbols (C_{ijk})

$$C_{ijk} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k}\right]$$

$$C_{111} = \frac{1}{2}\left(\frac{\partial d_{11}}{\partial q_1}\right) = 0 \qquad (C_{ijk} = C_{jik})$$

$$C_{121} = C_{211} = \frac{1}{2}\left[\frac{\partial d_{11}}{\partial q_2} + \frac{\partial d_{12}}{\partial q_1} - \frac{\partial d_{12}}{\partial q_1}\right]$$

$$C_{221} = \frac{\partial d_{12}}{\partial q_2} - \frac{1}{2}\frac{\partial d_{22}}{\partial q_1}$$

$$C_{112} = \frac{\partial d_{21}}{\partial q_1} - \frac{1}{2}\frac{\partial d_{11}}{\partial q_2}$$

$$C_{212} = C_{122} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_1}$$

$$C_{222} = \frac{1}{2}\frac{\partial d_{22}}{\partial q_2}$$

Step3 : Compute Potential energy, V

Eg:

$$V = \frac{m_1 g l_1}{2} Sq_1 + m_2 g\left(l_1 Sq_1 + \frac{l_2}{2} Sq_2\right)$$ for 2R planar manipulator

Step4 : ~~use~~ calculate ϕ_1 & ϕ_2

$$\phi_1 = \frac{\partial V}{\partial q_1}, \quad \phi_2 = \frac{\partial V}{\partial q_2}$$

Step5: write final equations for τ_1 & τ_2 using general Euler-Lagrange's equation

i.e. $D(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau$

For a 2R manipulator

$$d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + C_{221}\dot{q}_1^2 + \phi_1 = \tau_1$$

$$d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + C_{112}\dot{q}_1^2 + \phi_2 = \tau_2$$