Assignment - 3

18110131

1.) Singularity : When Jacobian matrix attains some nullity (i.e. whole col^n 0's decrease in rank) at certain values of q_0 — These are important to know because at singular configurations it becomes difficult to get certain directions & may lead to unstability or undesired motion. $\det J(q) = 0 \Rightarrow$ Singularity configuration

For $n=6$ with Arm 3 DOF & Wrist → Spherical (3DOF)

We get $J = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}$

J_{11} → Velocity J for Arm J_{12} → Velocity J for spherical

J_{21} → Angular J Arm J_{22} → Angular J spherical

Since, origins of spherical at same point as end effector (assumed) then $J_{12} = 0$

$\therefore J = \begin{bmatrix} J_{11} & 0 \\ J_{21} & J_{22} \end{bmatrix}$

For singularity, $\det(J) = 0 = \det(J_{11})\det(J_{22})$

$\therefore$ either $\det(J_{11}) = 0$ or $\det(J_{22}) = 0$ then singular

$\det(J_{22}) = 0$ when any 2 3 axes of spherical joint coincide.

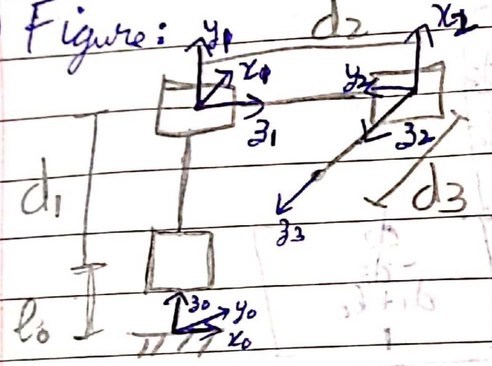
$\det(J_{11})$ depends on value of q

$S_{q_1} = \sin q_1$

$C_{q_1} = \cos q_1$

4.) Results are same as previous code & solution. For SCARA, I went with Textbook convention of rotating z down ($\alpha_2 = 180°$) unlike in previous solution where I took d as negative for prismatic. Results are same though which was required.

5.) Figure:



DH Parameters:

	Type	a	α	d	θ
0-1	P	0	$\pi/2$	$l_0 + d_1$	$\pi/2$
1-2	P	0	$-\pi/2$	d_2	$\pi/2$
2-3	P	0	0	d_3	0

$$H_0^1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & l_0 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

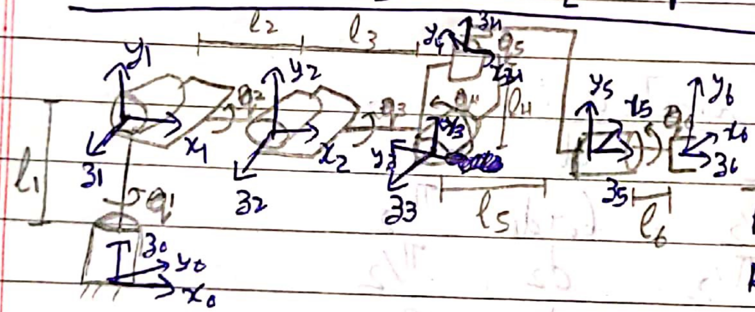
$$H_2^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore H_0^3 = H_0^1 H_1^2 H_2^3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_1+l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -1 & -d_3 \\ 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_0^3 = \begin{bmatrix} 0 & -1 & 0 & d_2 \\ 0 & 0 & -1 & -d_3 \\ 1 & 0 & 0 & d_1+l_0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^3 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} d_2 \\ -d_3 \\ d_1+l_0 \\ 1 \end{bmatrix}$$

6.)



	a	α	d	θ
R	0	$\pi/2$	l_1	q_1
R	l_2	0	0	q_2
R	l_3	0	0	$q_3+\frac{\pi}{2}$
R	l_4	$-\frac{\pi}{2}$	0	$q_4-\frac{\pi}{2}$
R	l_5	$\pi/2$	$-l_4$	$q_5+\pi/2$
R	l_6	0	0	q_6

$$H_0^1 = \begin{bmatrix} Cq_1 & 0 & Sq_1 & 0 \\ Sq_1 & 0 & -Cq_1 & 0 \\ 0 & 1 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_1^2 = \begin{bmatrix} Cq_2 & -Sq_2 & 0 & l_2 Cq_2 \\ Sq_2 & Cq_2 & 0 & l_2 Sq_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad H_2^3 = \begin{bmatrix} -Sq_3 & -Cq_3 & 0 & -l_3 Sq_3 \\ Cq_3 & -Sq_3 & 0 & l_3 Cq_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_3^4 = \begin{bmatrix} Sq_4 & 0 & Cq_4 & l_4 Sq_4 \\ -Cq_4 & 0 & Sq_4 & -l_4 Cq_4 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_4^5 = \begin{bmatrix} -Sq_5 & 0 & Cq_5 & -l_5 Sq_5 \\ Cq_5 & 0 & Sq_5 & l_5 Cq_5 \\ 0 & 1 & 0 & -l_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H_5^6 = \begin{bmatrix} Cq_6 & -Sq_6 & 0 & l_6 Cq_6 \\ Sq_6 & Cq_6 & 0 & l_6 Sq_6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 H_2^3 H_3^4 H_4^5 H_5^6 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The results match with that of code

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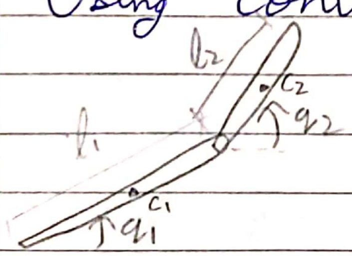
7.) 3 Types of Configuration for

i) Direct drive : Actuators shafts connected rigidly to the end of link (point of actuation). Any force on the link will directly affect the actuator shaft, Shaft may break as well. The first link ~~as~~ has to ~~o~~ take on extra load due to the motor attached on the link itself. Angle q_2 will be w.r.t. (link 1)

ii) Remotely Actuated : (Example belt drive) Safety of actuator shaft, no unnecessary load on the first link, angle q_2 will be w.r.t. ground (negligible influence of link 1 rotation)

iii) 5 bar linkage :
Independent actuation of q_1 & q_2, dynamics equations are decoupled by using this which makes the complex equation ~~eq~~ simpler.

8.) Using convention of miniproject :



We know that,

$$v_{C_1} = \begin{bmatrix} -l_{1/2} \, s q_1 \\ l_{1/2} \, c q_1 \\ 0 \end{bmatrix} \dot{q}_1 \qquad v_{C_2} = \begin{bmatrix} -l_1 s q_1 & -\frac{l_2}{2} s q_2 \\ l_1 c q_1 & \frac{l_2}{2} c q_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\omega_1 = \dot{q}_1 \hat{k} \qquad \omega_2 = \dot{q}_2 \hat{k}$$

Kinetic energ eq^n:

$$K = \frac{1}{2} \sum m_i v_{ci}^T v_{ci} + \frac{1}{2} \sum \omega_i^T I_i \omega_i$$

$$\cancel{K = \frac{1}{2} \dot{q}^T \sum m_i [}$$

$$K = \frac{1}{2} \dot{q}^T D(q) \dot{q}$$

where $\dot{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$ &

$$D(q) = \begin{bmatrix} m_1 \frac{l_1^2}{4} + m_2 l_1^2 + I_1 & m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1) \\ m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1) & m_2 \frac{l_2^2}{4} + I_2 \end{bmatrix}$$

Potential energy:

& $$V = m_1 g \frac{l_1}{2} s_{q_1} + m_2 g \left[l_1 s_{q_1} + \frac{l_2}{2} s_{q_2} \right]$$

For Langrange's Eq^n: $L = K - V$

& $$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = \tau_k$$

$$\Rightarrow \sum d_{kj}(q) \ddot{q}_j + \sum_{i,j} c_{ij,k}(q) \dot{q}_i \dot{q}_j + \phi_k(q) = \tau_k$$

where d_{kj} = k,j^{th} element of $D(q)$

$$c_{ij,k} = \frac{1}{2}\left[\frac{\partial d_{kj}}{\partial q_i} + \frac{\partial d_{ki}}{\partial q_j} - \frac{\partial d_{ij}}{\partial q_k} \right]$$

& $$\phi_k = \frac{\partial V}{\partial q_k}$$

For $k=1$ $\quad \phi_1 = m_1 g \frac{l_1}{2} c_{q_1} + m_2 g l_1 c_{q_1}$

$$\phi_2 = m_2 g \frac{l_2}{2} c_{q_2}$$

$$c_{111} = 0 \qquad c_{121} = c_{211} = 0$$

$C_{221} = -m_2 l_1 \frac{l_2}{2} \sin(q_2 - q_1)$

$C_{112} = m_2 l_1 \frac{l_2}{2} \sin(q_2 - q_1)$

$C_{212} = C_{122} = 0 \qquad C_{222} = 0$

$\therefore$ We have,

$$\left(m_1 \frac{l_1^2}{4} + m_2 l_1^2 + I_1\right)\ddot{q}_1 + m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1)\ddot{q}_2 - m_2 l_1 \frac{l_2}{2} \sin(q_2 - q_1)\dot{q}_2^2 + \left(\frac{m_1}{2} + m_2\right) l_1 g c q_1 = \tau_1$$

$$\left(m_2 \frac{l_2^2}{2} + I_2\right)\ddot{q}_2 + m_2 l_1 \frac{l_2}{2} \cos(q_2 - q_1)\ddot{q}_1 + m_2 l_1 \frac{l_2}{2} \sin(q_2 - q_1)\dot{q}_1^2 + m_2 g \frac{l_2}{2} c q_2 = \tau_2$$

For spring like action replace,

$\tau_1 \to \tau_1 + \tau_{1s}$ & $\tau_2 \to \tau_2 + \tau_{2s}$

$\tau_{1s} = k l_1 [(l_1 s q_1 + l_2 s q_2) c q_1 - (l_1 c q_1 + l_2 c q_2) s q_1]$

$\tau_{2s} = k l_2 [(l_1 s q_1 + l_2 s q_2) c q_2 - (l_1 c q_1 + l_2 c q_2) s q_2]$

10.) To derive equations of motions from $D(q)$ & $V(q)$ & Input Torques (if any).

We know that, $K = \frac{1}{2} \dot{q}^T D(q) \dot{q} = \frac{1}{2} \sum d_{ij}(q) \dot{q}_i \dot{q}_j$

$L = K - V$

$\frac{dL}{d\dot{q}_k} = \sum_i d_{ki} \dot{q}_i$

$$\therefore \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) = \sum_i d_{ki}\ddot{q}_i + \sum_i \frac{d}{dt} d_{ki}\, \dot{q}_i$$

$$= \sum_i d_{ki}\ddot{q}_i + \sum_{i,j} \frac{\partial d_{ki}}{\partial q_j} \dot{q}_i \dot{q}_j$$

$$\& \quad \frac{\partial L}{\partial q_k} = \frac{1}{2}\sum_{i,j} \frac{\partial d_{ij}}{\partial q_k} \dot{q}_i \dot{q}_j - \frac{\partial V}{\partial q_k}$$

$\therefore$ From Lagrange's Eq^n we have

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = \tau_k$$

$$\sum_i d_{ki}\ddot{q}_i + \sum_{i,j} \frac{\partial d_{ki}}{\partial q_j} \dot{q}_i \dot{q}_j - \frac{1}{2}\sum_{i,j} \frac{\partial d_{ij}}{\partial q_k} \dot{q}_i \dot{q}_j + \frac{\partial V}{\partial q_k} = \tau_k$$

We know that $d_{ki} = d_{kj}$ (Symmetry)

$$\therefore \sum \frac{\partial d_{ki}}{\partial q_j} \dot{q}_i \dot{q}_j = \frac{1}{2}\sum \left(\frac{\partial d_{ki}}{\partial q_j} + \frac{\partial d_{kj}}{\partial q_i}\right) \dot{q}_i \dot{q}_j$$

$$\therefore \sum d_{ki}\ddot{q}_i + \underbrace{\frac{1}{2}\sum_{i,j}\left[\frac{\partial d_{ki}}{\partial q_j} + \frac{\partial d_{kj}}{\partial q_i} - \frac{\partial d_{ij}}{\partial q_k}\right]}_{c_{ijk}} \dot{q}_i \dot{q}_j + \underbrace{\frac{\partial V}{\partial q_k}}_{\phi_k} = \tau_k$$

$$\therefore \underline{\sum_i d_{ki}\ddot{q}_i + \sum_{i,j} c_{ijk}\dot{q}_i \dot{q}_j + \phi_k = \tau_k}$$

$$D(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

$g(q) = [\phi_1 \cdots \phi_n]^T$, $\tau = [\tau_1, \cdots \tau_n]^T$

& For $C(q,\dot{q})$:

Element at row $= k$ & column j:

$$C_{kj} = \sum_{i=1}^{n} c_{ijk}\dot{q}_i$$

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