Assignment-2

1. Proving that R_0^1 is orthogonal using a generalized 3D rotation matrix.

→ for rotation about x, y, z axis with the angles ϕ, θ, ψ, the Rotation matrix is,

$$R_0^1 = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

where, $R_{11} = \cos\theta \cos\psi$

$R_{21} = \cos\theta \sin\psi$

$R_{31} = -\sin\theta$

→ $R_{12} = -\cos\phi \sin\psi + \sin\theta \sin\phi \cos\psi$

$R_{22} = \cos\phi \cos\psi + \sin\phi \sin\theta \sin\psi$

$R_{32} = \sin\phi \cos\theta$

→ $R_{13} = \sin\phi \sin\psi + \cos\phi \sin\theta \cos\psi$

$R_{23} = -\sin\phi \cos\psi + \cos\phi \sin\theta \sin\psi$

$R_{33} = \cos\phi \cos\theta$

(1)

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$\Rightarrow$ Now, to prove that R' is orthogonal we need to prove that,

$$R_{11} \cdot R_{12} + R_{21} \cdot R_{22} + R_{31} \cdot R_{32} = 0,$$
$$R_{11} \cdot R_{13} + R_{21} \cdot R_{23} + R_{31} \cdot R_{33} = 0$$
and $$R_{12} \cdot R_{13} + R_{22} \cdot R_{23} + R_{32} \cdot R_{33} = 0$$

$\rightarrow R_{11} \cdot R_{12} + R_{21} \cdot R_{22} + R_{31} \cdot R_{32}$

$$= -\cos\phi\cos\theta\sin\psi\cos\psi + \sin\phi\sin\theta\cos\theta\cos^2\psi + \cos\phi\cos\theta\sin\psi\cos\psi + \sin\phi\sin\theta\cos\theta\sin^2\psi - \sin\phi\sin\theta\cos\theta$$

$$= \sin\phi\sin\theta\cos\theta(\sin^2\psi + \cos^2\psi) - \sin\phi\sin\theta\cos\theta$$
$$= 0$$

$\rightarrow R_{11} \cdot R_{13} + R_{21} \cdot R_{23} + R_{31} \cdot R_{33}$

$$= \sin\phi\cos\theta\sin\psi\cos\psi + \cos\phi\sin\theta\cos\theta\cos^2\psi - \sin\phi\cos\theta\sin\psi\cos\psi + \cos\phi\sin\theta\cos\theta\sin^2\psi - \cos\phi\sin\theta\cos\theta$$

$$= \cos\phi\sin\theta\cos\theta(\sin^2\psi + \cos^2\psi) - \cos\phi\sin\theta\cos\theta$$

$$= 0$$

$\rightarrow R_{12} \cdot R_{13} + R_{22} \cdot R_{23} + R_{32} \cdot R_{33}$

$= -\sin\phi\cos\phi\sin^2\psi - \cancel{\sin\phi}\cos^2\phi\sin\theta\cos\theta \overset{\sin\psi\cos\psi}{\cancel{\cos\psi}} +$
$\sin\phi\cos\phi\sin^2\theta\cos^2\psi + \sin^2\phi\sin\theta\sin\psi\cos\psi$

$- \sin\phi\cos\phi\cos^2\psi + \cos^2\phi\sin\theta\sin\psi\cos\psi +$
$\sin\phi\cos\phi\sin^2\theta\sin^2\psi - \sin^2\phi\sin\theta\sin\psi\cos\psi$

$+ \sin\phi\cos\phi\cos^2\theta$

$= -\sin\phi\cos\phi(\sin^2\psi + \cos^2\psi) + \sin\phi\cos\phi\sin^2\theta(\sin^2\psi + \cos^2\psi)$
$+ \sin\phi\cos\phi\cos^2\theta$

$= -\sin\phi\cos\phi + \sin\phi\cos\phi\sin^2\theta + \sin\phi\cos\phi\cos^2\theta$

$= -\sin\phi\cos\phi(1 - \sin^2\theta - \cos^2\theta)$

$= 0$

So, R' is orthogonal matrix.

$\Rightarrow$ geometrically it means that the basis vectors $(\hat{i}, \hat{j}, \hat{k})$ remain orthogonal after rotation. Which is obvious.

2. any 3D Rotation matrix can be seen as a composition of three rotations about the axis x, y and z.

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

→ so, generalised rotation matrix,

$$R_1^0 = R_x(\phi) \cdot R_y(\theta) \cdot R_z(\psi) \quad \text{--- (2)}$$

$$\therefore \det(R_0^1) = \det(R_x(\phi)) \cdot \det(R_y(\theta)) \cdot \det(R_z(\psi)) \quad \text{--- (3)}$$

so, $\det(R_x(\phi))$

$$= 1[\cos^2\phi + \sin^2\phi]$$

$$= 1$$

$$\det(R_y(\theta)) = 1[\cos\theta\cos\theta - (-\sin\theta\sin\theta)]$$

$$= 1$$

$$\det(R_z(\psi)) = \cancel{1[\cos\theta}\, c$$

$$= 1[\cos\psi\cos\psi - (-\sin\psi\sin\psi)]$$

$$= 1$$

from eq^n ③,

$$\det(R_0') = 1 \cdot 1 \cdot 1$$

$$= \underline{1}$$

⇒ Geometrically it means that the factor by which the volume of the parallelepiped changes after Rotation is 1.

→ So, no streaching or chang in length in rotation.

5.

→ we need to prove,

$$R\,S(a)\,R^T = S(Ra)$$

→ for $R = R_x$,

$$R_x(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}$$

$$\therefore\ S(a)\,R_x^T = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & +\sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -a_z\cos\phi - a_y\sin\phi & -a_z\sin\phi + a_y\cos\phi \\ a_x & +a_x\sin\phi & -a_x\cos\phi \\ -a_y & a_x\cos\phi & a_x\sin\phi \end{bmatrix}$$

$$\therefore\ R_x\left(S(a)\cdot R_x^T\right) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix}\begin{bmatrix} 0 \\ a_z \\ -a_y \end{bmatrix}$$

$$\begin{bmatrix} 0 & -a_z\cos\phi - a_y\sin\phi & -a_z\sin\phi + a_y\cos\phi \\ a_x & a_x\sin\phi & -a_x\cos\phi \\ -a_y & a_x\cos\phi & a_x\sin\phi \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -a_z\cos\phi - a_y\sin\phi & a_y\cos\phi - a_z\sin\phi \\ a_z\cos\phi + a_y\sin\phi & 0 & -a_x \\ a_x\sin\phi + a_y\cos\phi & a_x & 0 \end{bmatrix}$$

$$= S\left(\begin{bmatrix} a_x \\ a_y\cos\phi - a_z\sin\phi \\ a_y\sin\phi + a_z\cos\phi \end{bmatrix}\right)$$

$$= S\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & c & -s \\ 0 & s & c \end{bmatrix}\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}\right)$$

$$= S(R \cdot a)$$

$$\Rightarrow \text{for } R = R_y = \begin{bmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{bmatrix}$$

$$\therefore S(a) R_y^T = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}\begin{bmatrix} c & 0 & -s \\ 0 & 1 & 0 \\ s & 0 & c \end{bmatrix}$$

$$= \begin{bmatrix} +s a_y & -a_z & c a_y \\ c a_z + s a_x & 0 & -s a_z - c a_x \\ -c a_y & a_x & +s a_y \end{bmatrix}$$

$$\Rightarrow R_y S(a) R_y^T = \begin{bmatrix} 0 & Sa_x - Ca_z & a_y \\ -Sa_x + Ca_x & 0 & -Ca_x - Sa_x \\ -a_y & Ca_x + Sa_x & 0 \end{bmatrix}$$

$$= S\left(\begin{bmatrix} Ca_x + Sa_z \\ a_y \\ -Sa_x + Ca_z \end{bmatrix}\right)$$

$$= S\left(\begin{bmatrix} C & 0 & S \\ 0 & 1 & 0 \\ -S & 0 & C \end{bmatrix}\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}\right)$$

$$= S(R_y \cdot a)$$

$$\Rightarrow \text{for } R = R_x = \begin{bmatrix} C & -S & 0 \\ S & C & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S(a) \cdot R_x^T = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}\begin{bmatrix} C & S & 0 \\ -S & C & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -Sa_z & -Sa_z & a_y \\ Ca_z & Sa_z & -a_x \\ -Ca_y - Sa_x & -Sa_y + Ca_x & 0 \end{bmatrix}$$

$$\therefore R_z \cdot S(a) R_z^T = \begin{bmatrix} 0 & -a_z & Sa_x + Ca_y \\ a_z & 0 & -Ca_x + Sa_y \\ -Sa_x - Ca_y & Ca_x - Sa_y & 0 \end{bmatrix}$$

$$= S\left(\begin{bmatrix} Ca_x - Sa_y \\ Sa_x + Ca_y. \\ a_z \end{bmatrix}\right)$$

$$= S\left(\begin{bmatrix} C & -S & 0 \\ S & C & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} a_x \\ a_y. \\ a_z \end{bmatrix}\right)$$

$$= S(R_z \cdot a)$$

$\Rightarrow$ So we have,

$$R_x S(a) R_x^T = S(R_x a)$$

$$R_y S(a) R_y^T = S(R_y a)$$

$$R_z S(a) R_z^T = S(R_z a)$$

$\Rightarrow$ for ~~a~~ any generalised rotation,

$$R = R_x R_y R_z$$

$$\therefore \text{LHS} = R\,S(a)\,R^T$$

$$= R_x R_y R_z S(a) (R_x R_y R_z)^T$$

$$= R_x R_y R_z\, S(a)\, R_z^T R_y^T R_x^T$$

$$= R_x R_y \left(R_z S(a) R_z^T\right) R_y^T R_x^T$$

$$= R_x R_y\, S(R_z a)\, R_y^T R_x^T$$

$$= R_x\, S(R_y R_z a)\, R_x^T$$

$$= S(R_x R_y R_z a)$$

$$= S(Ra)$$

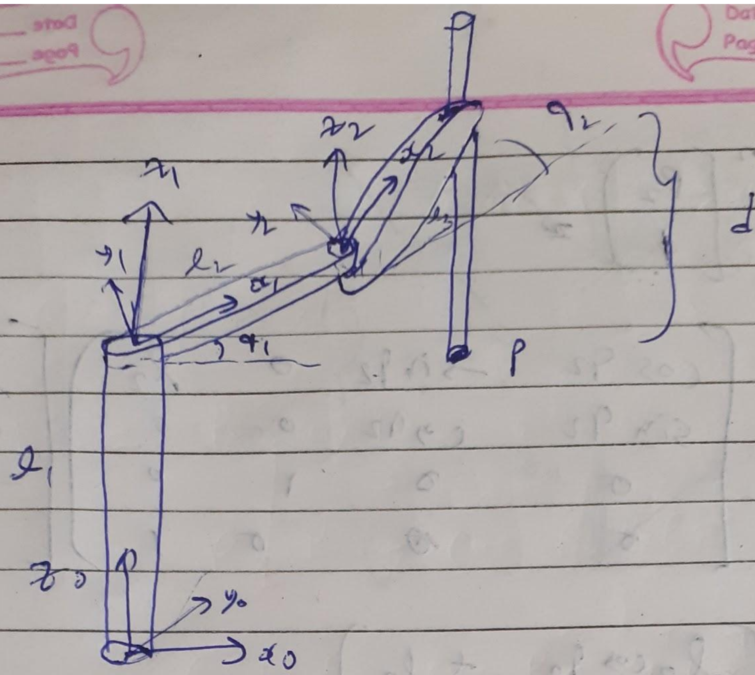
hence, $R\,S(a)\,R^T = S(Ra)$ is proved

$\Rightarrow$ from the book:-

$$(R\,S(a)\,R^T)b = R\cdot(a\times R^T b) = Ra\times RR^T b = Ra\times b = S(Ra)\,b$$

$\therefore$ $\boxed{R\,S(a)\,R^T = S(Ra)}$ (here b is a non-zero vector) $b\in\mathbb{R}^3$

6.



→ here, $P_2 = \begin{bmatrix} l_3 \\ 0 \\ -d \end{bmatrix}$

$$R_1^2 = \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 \\ \sin q_2 & \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad d_1^2 = \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad d_0^1 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$$

so, $\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 \begin{bmatrix} P_2 \\ 1 \end{bmatrix}$

$$\rightarrow H_1^2 \begin{bmatrix} P_2 \\ \phi \end{bmatrix}$$

$$= \begin{bmatrix} \cos q_2 & -\sin q_2 & 0 & l_2 \\ \sin q_2 & \cos q_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_3 \\ 0 \\ -d \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} l_3 \cos q_2 + l_2 \\ l_3 \sin q_2 \\ -d \\ 1 \end{bmatrix}$$

$$\rightarrow H_0^1 \left(H_1^2 \begin{bmatrix} P_2 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 & 0 \\ \sin q_1 & \cos q_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} l_3 \cos q_2 + l_2 \\ l_3 \sin q_2 \\ -d \\ 1 \end{bmatrix}$$

$$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} l_3 \cos q_1 \cos q_2 + l_2 \cos q_1 - l_3 \sin q_1 \sin q_2 \\ l_3 \sin q_1 \cos q_2 + l_2 \sin q_1 + l_3 \cos q_1 \sin q_2 \\ -d + l_1 \\ 1 \end{bmatrix}$$

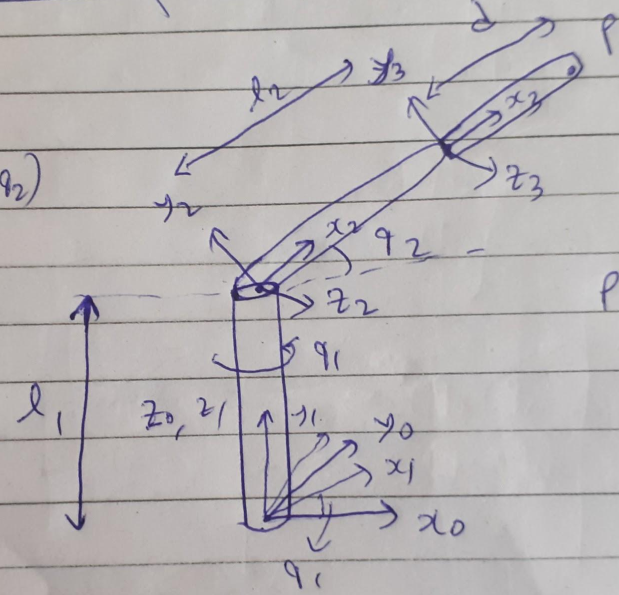
$$\therefore P_0 = \begin{bmatrix} l_3 \cos(q_2 + q_1) + l_2 \cos q_1 \\ l_3 \sin(q_2 + q_1) + l_2 \sin q_1 \\ -d + l_1 \end{bmatrix}$$

8. $R_0^1 = R_z(q_1)$

$R_1^2 = R_x(\pi/2) R_z(q_2)$

$d_0^1 = [0 \ 0 \ 0]^T$

$d_1^2 = [0 \ 0 \ l_1]^T$

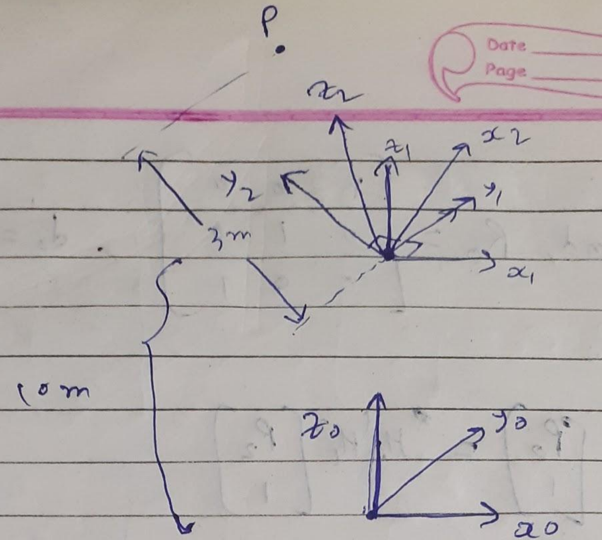


$$P_3 = \begin{bmatrix} d \\ 0 \\ 0 \end{bmatrix}$$

$R_2^3 = I$

$d_2^3 = [l_2 \ 0 \ 0]^T$

9.



$\Rightarrow$ here the coordinate system $x_0 - y_0 - z_0$ is fixed to the ground.

$\rightarrow$ $x_1 - y_1 - z_1$ is just a 10m translation on z_0 direction.

$\rightarrow$ $x_2 - y_2 - z_2$ is the result of the rotation of 30° about x_1 axis and the rotation of 60° about new z_1 axis.

$\rightarrow$ also $P_2 = [0 \; 0 \; 3]^T$ from the given lidar observation.

$$\therefore R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30° & -\sin 30° \\ 0 & \sin 30° & \cos 30° \end{bmatrix} \begin{bmatrix} \cos 60° & -\sin 60° & 0 \\ \sin 60° & \cos 60° & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & -\sqrt{3}/2 & 0 \\ 3/4 & \sqrt{3}/4 & -\frac{1}{2} \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 \end{bmatrix}, \quad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

and, $R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$

$\therefore \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 \begin{bmatrix} P_2 \\ 1 \end{bmatrix}$

so, $H_1^2 \begin{bmatrix} P_2 \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\sqrt{3}/2 & 0 & 0 \\ 3/4 & \sqrt{3}/4 & -1/2 & 0 \\ \sqrt{3}/4 & 1/4 & \sqrt{3}/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ -1.5 \\ 2.5981 \\ 1 \end{bmatrix}$

$\rightarrow \begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 \begin{bmatrix} 0 \\ -1.5 \\ 2.5981 \\ 1 \end{bmatrix}$

$$\therefore \begin{pmatrix} P_0 \\ 1 \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -1.5 \\ 2.5981 \\ 1 \end{bmatrix}$$

$$\Rightarrow \cancel{\begin{bmatrix} 0 \\ -1.5 \\ 24.481 \\ 1 \end{bmatrix}} \quad = \begin{bmatrix} 0 \\ -1.5 \\ 12.5981 \\ 1 \end{bmatrix}$$

$$\therefore P_0 = (0, -1.5, \overset{12.5981}{\cancel{24.481}})\,m$$

10. Types of gearboxes:

- Planetary gearbox:
  - It has good accuracy, it's precise, optimal for the performance, and there are many mounting options available. These types of gears are typically used in automobiles, marine application, etc where there is a need for higher torque and/or low speed.
  - But it's more expensive because of its complex design, it may need cooling for high speed applications.
- Helical gearbox:
  - It has a compact size, it is a low power consumer, used in heavy industries maybe because of its strength, silent operation, longer lifetime.
  - But it is less energy efficient, so more power loss. The reason could be more teeth touching each other.

Typically, a gearbox is not used along with a motor in a drone application because usually gearbox is used for higher torque and lower speed but in a drone application we want higher thrust thus more speed. We might be able to use overdrive gear (gearbox with gear ratio less than one) in a drone but that might cause some trouble as torque provided is decreasing. So, usually, drone applications don't use gearboxes!

11. for RRP SCARA,

$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad \begin{pmatrix} O_1 \\ 1 \end{pmatrix} = H_0^1 \begin{bmatrix} l_2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore \quad O_1 = \begin{bmatrix} l_2 \cos q_1 \\ l_2 \sin q_1 \\ l_1 \end{bmatrix}$$

$$\begin{pmatrix} O_2 \\ 1 \end{pmatrix} = H_0^1 H_1^2 \begin{bmatrix} l_3 \\ 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow O_2 = \begin{bmatrix} l_3 \cos(q_2 + q_1) + l_2 \cos q_1 \\ l_3 \sin(q_2 + q_1) + l_2 \sin q_1 \\ l_1 \end{bmatrix}$$

$$O_3 = P_0 = \begin{bmatrix} l_3 \cos(q_2 + q_1) + l_2 \cos q_1 \\ l_3 \sin(q_2 + q_1) + l_2 \sin q_1 \\ -d + l_1 \end{bmatrix} \quad \text{(from que-6)}$$

→ as there are three joint variables (q_1, q_2, d), Jacobian will be of 6x3 dimentions

$$J = \begin{bmatrix} z_0 \times (O_2 - O_0) & z_1 \times (O_2 - O_1) & z_2 \\ z_0 & z_1 & 0 \end{bmatrix}$$

here $z_i = R_0^i \cdot \hat{k}$

so, $z_0 = R_0^0 \cdot \hat{k}$

$= I \cdot \hat{k}$

$= [0 \ 0 \ 1]^T$

$z_1 = R_0^1 \cdot \hat{k}$

$$= \begin{bmatrix} \cos q_1 & -\sin q_1 & 0 \\ \sin q_1 & \cos q_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$z_2 = R_0^2 \hat{k}$

$$= \begin{bmatrix} \cos(q_1+q_2) & -\sin(q_1+q_2) & 0 \\ \sin(q_1+q_2) & \cos(q_1+q_2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

11. cont... So, $z_0 \times (o_3 - o_0) = \begin{bmatrix} -l_3\sin(q_1+q_2) - l_2\sin q_1 \\ +l_3\cos(q_1+q_2) + l_2\cos q_1 \\ 0 \end{bmatrix}$

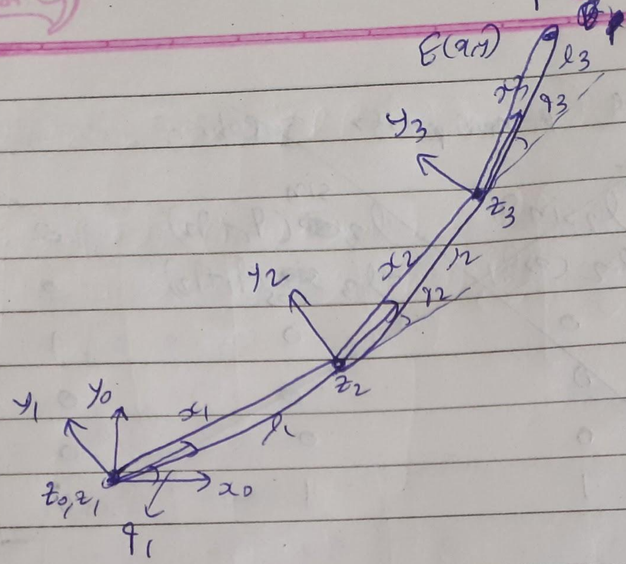
$$z_1 \times (o_3 - o_1) = \begin{bmatrix} -l_3\sin(q_1+q_2) \\ +l_3\cos(q_1+q_2) \\ 0 \end{bmatrix}$$

$$z_2 \times (o_3 - o_2) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

→ So, the manipulator Jacobian,

$$J = \begin{bmatrix} -l_3\sin(q_1+q_2) - l_2\sin q_1 & -l_3\sin(q_1+q_2) & 0 \\ +l_3\cos(q_1+q_2) + l_2\cos q_1 & +l_3\cos(q_1+q_2) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

13.



$\Rightarrow$ as this is planer bot, the ~~z~~ z-axis is not changing

$\therefore \quad z_1 = z_2 = z_3 = z_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\Rightarrow$ also, $o_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$, $o_1 = R_0^1 \begin{bmatrix} l_1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} l_1 \cos q_1 \\ l_1 \sin q_1 \\ 0 \end{bmatrix}$

$$O_2 = O_1 + R_0^2 \begin{bmatrix} l_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} l_1 \cos q_1 \\ l_1 \sin q_1 \\ 0 \end{bmatrix} + \begin{bmatrix} l_2 \cos(q_1+q_2) \\ l_2 \sin(q_1+q_2) \\ 0 \end{bmatrix}$$

$$\therefore \; O_2 = \begin{bmatrix} l_1 \cos q_1 + l_2 \cos(q_1+q_2) \\ l_1 \sin q_1 + l_1 \sin(q_1+q_2) \\ 0 \end{bmatrix}$$

$$O_3 = O_1 + R_0^3 \begin{bmatrix} l_3 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} l_1 \cos q_1 + l_2 \cos(q_1+q_2) + l_3 \cos(q_1+q_2+q_3) \\ l_1 \sin q_1 + l_2 \sin(q_1+q_2) + l_3 \sin(q_1+q_2+q_3) \\ 0 \end{bmatrix}$$

$$\rightarrow \; z_0 \times (O_3 - O_0) = \begin{bmatrix} -l_1 \sin q_1 - l_2 \sin(q_1+q_2) - l_3 \sin(q_1+q_2+q_3) \\ +l_1 \cos q_1 + l_2 \cos(q_1+q_2) + l_3 \cos(q_1+q_2+q_3) \\ 0 \end{bmatrix}$$

$$z_1 \times (O_3 - O_1) = \begin{bmatrix} -l_2 \sin(q_1+q_2) - l_3 \sin(q_1+q_2+q_3) \\ +l_2 \cos(q_1+q_2) + l_3 \cos(q_1+q_2+q_3) \\ 0 \end{bmatrix}$$

$$z_2 \times (O_3 - O_2) = \begin{bmatrix} -l_3 \, \overset{\sin}{\cancel{\cos}} (q_1+q_2+q_3) \\ +l_3 \, \cancel{\sin} \cos(q_1+q_2+q_3) \\ 0 \end{bmatrix}$$

→ So, Jacobian for RRR planer manipulator,

$$J = \begin{bmatrix} -l_1 \sin q_1 - l_2 \sin(q_1+q_2) - l_3 \sin(q_1+q_2+q_3) & -l_2 \sin(q_1+q_2) - l_3 \sin(q_1+q_2+q_3) & -l_3 \sin(q_1+q_2+q_3) \\ +l_1 \cos(q_1) + l_2 \cos(q_1+q_2) + l_3 \cos(q_1+q_2+q_3) & +l_2 \cos(q_1+q_2) + l_3 \cos(q_1+q_2+q_3) & +l_3 \cos(q_1+q_2+q_3) \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$