Assignment 2

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1.

columns of rotation matrix are orthogonal

let, R_1 is rotation matrix $= \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$

we know, $R_1 R_1^T = I$

$$\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \begin{pmatrix} r_{11} & r_{21} & r_{31} \\ r_{12} & r_{22} & r_{32} \\ r_{13} & r_{23} & r_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} r_{11}^2 + r_{12}^2 + r_{13}^2 & r_{11} r_{21} + r_{12} r_{22} + r_{13} r_{23} & r_{11} r_{31} + r_{12} r_{32} + r_{13} r_{33} \\ r_{12} r_{11} + r_{22} r_{12} + r_{32} r_{13} & r_{12}^2 + r_{22}^2 + r_{32}^2 & r_{12} r_{31} + r_{22} r_{32} + r_{32} r_{33} \\ r_{13} r_{11} + r_{23} r_{12} + r_{33} r_{13} & r_{13} r_{21} + r_{23} r_{22} + r_{33} r_{23} & r_{13}^2 + r_{23}^2 + r_{33}^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

as

$r_{11} r_{21} + r_{12} r_{22} + r_{13} r_{23} = row_1 \cdot row_2 = column_1 \cdot column_2 = 0$

$r_{13} r_{11} + r_{23} r_{12} + r_{33} r_{13} = row_1 \cdot row_3 = column_1 \cdot column_3 = 0$

$r_{13} r_{21} + r_{23} r_{22} + r_{33} r_{23} = row_2 \cdot row_3 = column_2 \cdot column_3 = 0$

as the dot products is zero, so, rows and columns of rotation matrix are orthogonal.

Similary, if we assume $R = (\bar{R}_1 \ \bar{R}_2 \ \cdots \ \bar{R}_n)$

$\bar{R}_1$ is vector for column 1 and similarly others

we have,

$$R^T R = I$$

$$(\bar{R}_1 \ \bar{R}_2 \ \cdots \ \bar{R}_n)^T (\bar{R}_1 \ \bar{R}_2 \ \cdots \ \bar{R}_n) = I$$

$$\begin{pmatrix} R_1^T R_1 & R_1^T R_2 & \cdots & R_1^T R_n \\ \vdots & & & \\ R_n^T R_1 & R_n^T R_2 & \cdots & R_n^T R_n \end{pmatrix} = I$$

As the dot product of all the columns $\bar{R}_i \cdot \bar{R}_j = R_1 R_j^T = 0$
all the columns are orthogonal.

2.

$\det(R_0^1) = 1$

we have

$R_0^1 (R_0^1)^T = R_0^1 R_1^0 = I$

$\det(a \cdot b) = \det(a) \cdot \det(b)$

$\det(R_0^1) = \det(R_0^1)^T$

Using these equations,

$\det(R_0^1 \cdot (R_0^1)^T) = \det(R_0^1) \cdot \det((R_0^1)^T)$

$\det(I) = [\det(R_0^1)]^2$

$[\det(R_0^1)]^2 = 1$

$\det(R_0^1) = 1$

5.

$R\, s(a) R^T = s(Ra)$

for any $R \in SO(3)$ and any $b \in R^3$

$R\, s(a)\, R^T b = R(a \times R^T b)$ ..... $S(a)\, P = a \times P$

$= (Ra) \times (R R^T b)$ .... $R(a \times b) = Ra \times Rb$

$= (Ra) \times (b)$ .... $R R^T = I$

$R\, s(a)\, R^T b = s(Ra)\, b$ .... $a \times b = s(a)\, b$

as this expression is valid for all b
hence,

$R\, s(a)\, R^T = s(Ra)$

6.

various co-ordinates frame

P_0 for RRP scara robot

$P_2 = \begin{bmatrix} l_2 \\ 0 \\ d \end{bmatrix}$

$R_0^1 = \begin{bmatrix} Cq_1 & -Sq_1 & 0 \\ Sq_1 & Cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_1^2 = \begin{bmatrix} Cq_2 & -Sq_2 & 0 \\ Sq_2 & Cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 \begin{bmatrix} P_2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} Cq_1 & -Sq_1 & 0 & 0 \\ Sq_1 & Cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Cq_2 & -Sq_2 & 0 & 0 \\ Sq_2 & Cq_2 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ 0 \\ d \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} Cq_1 Cq_2 - Sq_1 Sq_2 & -Cq_1 Sq_2 - Sq_1 Cq_2 & 0 & 0 \\ Sq_1 Cq_2 + Cq_1 Sq_2 & -Sq_1 Sq_2 + Cq_1 Cq_2 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ 0 \\ d \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} l_2 (Cq_1 Cq_2 - Sq_1 Sq_2) \\ l_2 (Sq_1 Cq_2 + Cq_1 Sq_2) \\ d + l_1 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} l_2 (Cq_1 Cq_2 - Sq_1 Sq_2) \\ l_2 (Sq_1 Cq_2 + Cq_1 Sq_2) \\ d + l_1 \end{bmatrix}$$

Q.7 links

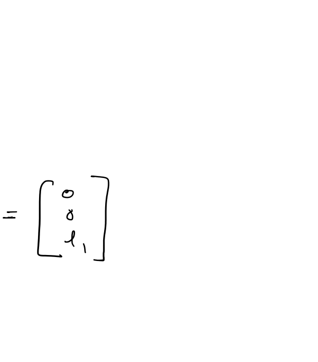
https://colab.research.google.com/drive/1z2Qw6dJQ02neOC5dvT1dDft-YJaT9U0i?usp=sharing

Q.8 links

https://colab.research.google.com/drive/1nHQc9eZHR8r1Vxdj5GJlY9j24zbwSYH9?usp=sharing

8.

P_0 for RRP stanford type robot



$P_2 = \begin{bmatrix} l_2 + d \\ 0 \\ 0 \end{bmatrix}$

$R_0^1 = \begin{bmatrix} Cq_1 & -Sq_1 & 0 \\ Sq_1 & Cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\pi/2 & -S\pi/2 \\ 0 & S\pi/2 & C\pi/2 \end{bmatrix} \begin{bmatrix} Cq_2 & -Sq_2 & 0 \\ Sq_2 & Cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $d_1^2 = \begin{bmatrix} 0 \\ 0 \\ l_1 \end{bmatrix}$

$= \begin{bmatrix} Cq_2 & -Sq_2 & 0 \\ 0 & 0 & -1 \\ Sq_2 & Cq_2 & 0 \end{bmatrix}$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 \begin{bmatrix} P_2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} R_0^1 & d_0^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} Cq_1 & -Sq_1 & 0 & 0 \\ Sq_1 & Cq_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Cq_2 & -Sq_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ Sq_2 & Cq_2 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 + d \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

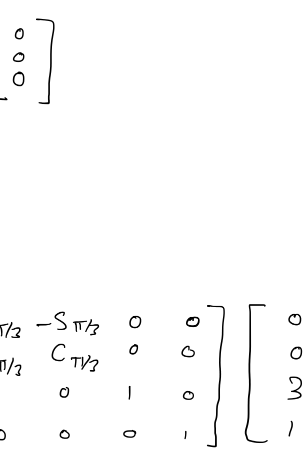
$$= \begin{bmatrix} Cq_1 Cq_2 & -Cq_1 Sq_2 & Sq_1 & 0 \\ Sq_1 Cq_2 & -Sq_1 Sq_2 & -Cq_1 & 0 \\ Sq_2 & Cq_2 & 0 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 + d \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} (l_2 + d)(Cq_1 Cq_2) \\ (l_2 + d)(Sq_1 Cq_2) \\ (l_2 + d)\, Sq_2 + l_1 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} (l_2 + d)(Cq_1 Cq_2) \\ (l_2 + d)(Sq_1 Cq_2) \\ (l_2 + d)\, Sq_2 + l_1 \end{bmatrix}$$

9.

drone



$P_2 = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$

$R_0^1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\pi/6 & -S\pi/6 \\ 0 & S\pi/6 & C\pi/6 \end{bmatrix}$ $d_0^1 = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$

$R_1^2 = \begin{bmatrix} C\pi/3 & -S\pi/3 & 0 \\ S\pi/3 & C\pi/3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = H_0^1 H_1^2 \begin{bmatrix} P_2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\pi/6 & -S\pi/6 & 0 \\ 0 & S\pi/6 & C\pi/6 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\pi/3 & -S\pi/3 & 0 & 0 \\ S\pi/3 & C\pi/3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\pi/6 & -S\pi/6 & 0 \\ 0 & S\pi/6 & C\pi/6 & 10 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -3\, S\pi/6 \\ 3\, C\pi/6 + 10 \\ 1 \end{bmatrix}$$

$$P_0 = \begin{bmatrix} 0 \\ -3/2 \\ 10 + 3\sqrt{3}/2 \end{bmatrix}$$

10.

Different gearboxes

1. Planetory gearbox :→ These type of gearboxes are compact and are broadly used in power trains. These gearboxes make good usage of high gear meshing producing higher efficiencies.

2. Harmonic Drives :→ It is a zero backlash, lightweight strain wave gearbox. It has high applications in aerospace. These gearboxes have larger diameters compared to lengths. Have effeciencies lower than planetory gearbox.

3. Cycloidal Drives :→ These gearboxes have high robustness and torsional stiffness hence are mainly used in boats cranes and some large equipments. This cycloidal motion is produced due to eccentric input and provides high reduction capacity.