

Assignment 2

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Name: Murkute Nikhil

Roll no: 18110104

1. columns of rotation matrix are orthogonal

$$\text{let, } R_i \text{ is rotation matrix} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

we know, $R_i R_i^T = I$

$$\begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix} \begin{pmatrix} r_{11} & r_{21} & r_{31} \\ r_{12} & r_{22} & r_{32} \\ r_{13} & r_{23} & r_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} r_{11}^2 + r_{12}^2 + r_{13}^2 & r_{11}r_{21} + r_{12}r_{22} + r_{13}r_{23} & r_{11}r_{31} + r_{12}r_{32} + r_{13}r_{33} \\ r_{12}r_{21} + r_{22}r_{31} + r_{32}r_{13} & r_{12}^2 + r_{22}^2 + r_{32}^2 & r_{12}r_{31} + r_{22}r_{32} + r_{32}r_{13} \\ r_{13}r_{21} + r_{23}r_{31} + r_{33}r_{13} & r_{13}r_{21} + r_{23}r_{32} + r_{33}r_{12} & r_{13}^2 + r_{23}^2 + r_{33}^2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

as

$$r_{11}r_{21} + r_{12}r_{22} + r_{13}r_{23} = \text{row}_1 \cdot \text{row}_2 = \text{column}_1 \cdot \text{column}_2 = 0$$

$$r_{12}r_{21} + r_{22}r_{31} + r_{32}r_{13} = \text{row}_1 \cdot \text{row}_3 = \text{column}_1 \cdot \text{column}_3 = 0$$

$$r_{13}r_{21} + r_{23}r_{31} + r_{33}r_{12} = \text{row}_2 \cdot \text{row}_3 = \text{column}_2 \cdot \text{column}_3 = 0$$

as the dot products is zero, so, rows and columns of rotation matrix are orthogonal.

Similarly, if we assume $R = (\bar{R}_1 \bar{R}_2 \dots \bar{R}_n)$

\bar{R}_i is vector for column i and similarly others

we have,

$$R^T R = I$$

$$(\bar{R}_1 \bar{R}_2 \dots \bar{R}_n)^T (\bar{R}_1 \bar{R}_2 \dots \bar{R}_n) = I$$

$$\begin{pmatrix} \bar{R}_1^T \bar{R}_1 & \bar{R}_1^T \bar{R}_2 & \dots & \bar{R}_1^T \bar{R}_n \\ \vdots & & & \\ \bar{R}_n^T \bar{R}_1 & \bar{R}_n^T \bar{R}_2 & \dots & \bar{R}_n^T \bar{R}_n \end{pmatrix} = I$$

As the dot product of all the columns $\bar{R}_i \cdot \bar{R}_j = R_i R_j^T = 0$
all the columns are orthogonal.

2.

$$\det(R_o^T) = 1$$

we have

$$R_o^T (R_o^T)^T = R_o^T R_o^0 = I$$

$$\det(a \cdot b) = \det(a) \cdot \det(b)$$

$$\det(R_o^T) = \det(R_o^T)^T$$

Using these equations,

$$\det(R_o^T \cdot (R_o^T)^T) = \det(R_o^T) \cdot \det((R_o^T)^T)$$

$$\det(I) = [\det(R_o^T)]^2$$

$$[\det(R_o^T)]^2 = 1$$

$$\det(R_o^T) = 1$$

5.

various co-ordinates frame

P_o for RRP SCARA robot

$$P_2 = \begin{bmatrix} l_2 \\ 0 \\ d \end{bmatrix}$$

$$R_o^1 = \begin{bmatrix} C_{q_1} & -S_{q_1} & 0 \\ S_{q_1} & C_{q_1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_o^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} C_{q_2} & -S_{q_2} & 0 \\ S_{q_2} & C_{q_2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_o \\ 1 \end{bmatrix} = H_o^1 H_1^2 \begin{bmatrix} P_2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_o \\ 1 \end{bmatrix} = \begin{bmatrix} R_o^1 & d_o^1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_1^2 & d_1^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} P_2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_o \\ 1 \end{bmatrix} = \begin{bmatrix} C_{q_1} & -S_{q_1} & 0 & 0 \\ S_{q_1} & C_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{q_2} & -S_{q_2} & 0 & 0 \\ S_{q_2} & C_{q_2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ 0 \\ d \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{q_1}C_{q_2} - S_{q_1}S_{q_2} & -C_{q_1}S_{q_2} - S_{q_1}C_{q_2} & 0 & 0 \\ S_{q_1}C_{q_2} + C_{q_1}S_{q_2} & -S_{q_1}S_{q_2} + C_{q_1}C_{q_2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 \\ 0 \\ d \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} P_o \\ 1 \end{bmatrix} = \begin{bmatrix} (l_2+d)(C_{q_1}C_{q_2}) & (l_2+d)(-S_{q_1}S_{q_2}) & 0 & 0 \\ (l_2+d)(S_{q_1}C_{q_2} + C_{q_1}S_{q_2}) & (l_2+d)(-S_{q_1}S_{q_2} + C_{q_1}C_{q_2}) & 0 & 0 \\ (l_2+d)S_{q_2} + d_1 & (l_2+d)C_{q_2} & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$P_o = \begin{bmatrix} 0 \\ -3/2 \\ 10 + 3\sqrt{3}/2 \end{bmatrix}$$



8.

P_o for RRP Stanford type robot

$$P_2 = \begin{bmatrix} l_2 + d \\ 0 \\ 0 \end{bmatrix}$$

$$R_o^1 = \begin{bmatrix} C_{q_1} & -S_{q_1} & 0 \\ S_{q_1} & C_{q_1} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_o^1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

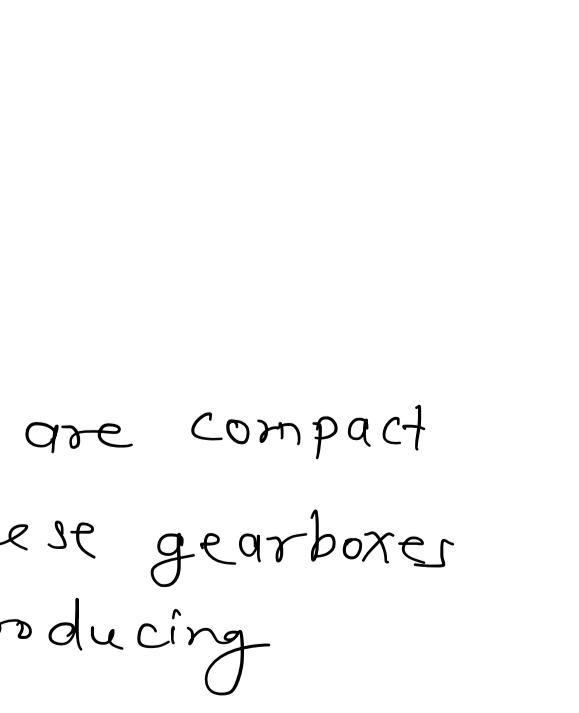
$$R_1^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C_{\pi/4} & -S_{\pi/4} \\ 0 & S_{\pi/4} & C_{\pi/4} \end{bmatrix} \begin{bmatrix} C_{q_2} & -S_{q_2} & 0 \\ S_{q_2} & C_{q_2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad d_1^2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} C_{q_2} & -S_{q_2} & 0 \\ S_{q_2} & C_{q_2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_{\pi/4} & -S_{\pi/4} & 0 \\ S_{\pi/4} & C_{\pi/4} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 + d \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} C_{q_2}C_{\pi/4} - S_{q_2}S_{\pi/4} & -C_{q_2}S_{\pi/4} - S_{q_2}C_{\pi/4} & 0 \\ S_{q_2}C_{\pi/4} + C_{q_2}S_{\pi/4} & -S_{q_2}S_{\pi/4} + C_{q_2}C_{\pi/4} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} l_2 + d \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} C_{q_2}C_{\pi/4} - S_{q_2}S_{\pi/4} & -C_{q_2}S_{\pi/4} - S_{q_2}C_{\pi/4} & 0 \\ S_{q_2}C_{\pi/4} + C_{q_2}S_{\pi/4} & -S_{q_2}S_{\pi/4} + C_{q_2}C_{\pi/4} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} (l_2+d)(C_{q_2}C_{\pi/4}) & (l_2+d)(-S_{q_2}S_{\pi/4}) & 0 \\ (l_2+d)(S_{q_2}C_{\pi/4} + C_{q_2}S_{\pi/4}) & (l_2+d)(-S_{q_2}S_{\pi/4} + C_{q_2}C_{\pi/4}) & 0 \\ (l_2+d)S_{\pi/4} + l_1 & (l_2+d)C_{\pi/4} & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$P_o = \begin{bmatrix} 0 \\ -3/2 \\ 10 + 3\sqrt{3}/2 \end{bmatrix}$$



10.

Different gearboxes

① Planetary gearbox: These type of gearboxes are compact and are broadly used in power trains. These gearboxes make good usage of high gear meshing producing higher efficiencies.

② Harmonic Drives: It is a zero backlash, lightweight strain wave gearbox. It has high applications in aerospace. These gearboxes have larger diameters compared to lengths. Have efficiencies lower than planetary gearbox.

③ Cycloidal Drives: These gearboxes have high robustness and torsional stiffness hence are mainly used in boats, cranes and some large equipments. This cycloidal motion is produced due to eccentric input and provides high reduction capacity.

