Assignment 3

Mechanics of Robots — 2020

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Assignment 3

T1.

(a) Singular configuration :-

Jacobian is the function of configuration q, those configuration for which the rank of J decreases are known as singular configuration.

(b) How to find singular configuration :-

Configuration q is singular if $\det J(q) = 0$

Any set of singular configurations is the union of non-singular configuration satisfying $\det J_v = 0$ and $\det J_{\omega_v} = 0$.

(c) Sketch if a particular configuration is close to singular configuration using manipulator Jacobian.

T2.

(a) Steps :-

① Select the joint axes as $z_0, \ldots, z_{n-1}$

② Select the origin anywhere on z_i axis

③ Select origin O_i where z_i and z_{i+1} intersect. If not exactly then choose any convenient position along z_i.

④ Establish x_i along common normal between z_i and z_i through O_i, or in the direction normal to (z_{i-1}, z_i) plane if z_{i-1} and z_i intersect.

⑤ Establish y_i considering right hand frame

⑥ Create a table of link parameters $\theta_i, d_i, a_i, \alpha_i$

θ_i is distance along x_{i-1} from x_i to intersection of x_i and z_{i-1}

d_i is distance along z_{i-1} from O_{i-1} to the intersection of x_i and z_{i-1}

a_i is angle from z_{i-1} and z_i measured about x_i

α_i is angle over x_{i-1} and x_i measured about z_{i-1}

⑦ Form A_i and then $T_n^0 = A_1 \ldots A_n$

T3

Python Code

Input is numbers of joint and DH parameters

will be joint (R/P) dataset 1

Output is (a) Complete transformation Jacobian

(b) end-effector position

(c) end-effector velocity

Code:

https://colab.research.google.com/drive/1Ka3ldfBfNwKzKGXBdNwdNc3NGJ1ywv-drive

T4.

(a)(i) This code is FOR cases and time-bound simulation and verify results

DH parameters for FRP robot

Joint	α_i	a_i	d_i	θ_i
1	0	0	d_1	q_1^*
2	$-\pi/2$	d_2	q_2^*	0
3	$\pi/2$	0	0	q_3^*

DH parameters if Simplified kinematics

Joint	α_i	a_i	d_i	θ_i
1	0	$-\pi/2$	d_1	q_1^*
2	$-\pi/2$	d_2	q_2^*	0
3	$\pi/2$	0	0	0

Code for (i)

https://colab.research.google.com/drive/1Ka3ldfBfNwKzKGXBdNwdNc3NGJ1ywv-drive

Code for (ii)

https://colab.research.google.com/drive/1Ka3ldfBfNwKzKGXBdNwdNc3NGJ1ywv-drive

T5

Problem 3-7

3 link cartesian manipulator

Link	α_i	a_i	d_i	θ_i
1	$-\pi/2$	d_1^*	$-\pi/2$	0
2	$-\pi/2$	d_2^*	$-\pi/2$	0
3	0	d_3^*	0	0

$$A_i = \begin{bmatrix} c_\theta & -s_\theta c_\alpha & s_\theta s_\alpha & a c_\theta \\ s_\theta & c_\theta c_\alpha & -c_\theta s_\alpha & a s_\theta \\ 0 & s_\alpha & c_\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* variable

$$A_1^0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2^1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = A_1^0 A_2^1 A_3^2$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3^0 = \begin{bmatrix} 0 & 0 & 1 & d_3 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

position of end effector would be (d_3, d_1, d_2) in the general frame

Complete transformation jacobian is J

$$J_v = \begin{bmatrix} \frac{\partial P_x}{\partial q_1} & \frac{\partial P_x}{\partial q_2} & \frac{\partial P_x}{\partial q_3} \\ \frac{\partial P_y}{\partial q_1} & \frac{\partial P_y}{\partial q_2} & \frac{\partial P_y}{\partial q_3} \\ \frac{\partial P_z}{\partial q_1} & \frac{\partial P_z}{\partial q_2} & \frac{\partial P_z}{\partial q_3} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$J_\omega = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ — all three are prismatic joints hence direct contribution

$$J = \begin{bmatrix} J_v \\ J_\omega \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

T6

Problem 3-8

Elbow manipulator with spherical wrist

Joint	α_i	a_i	d_i	θ_i
1	$\pi/2$	a_1	0	q_1^*
2	$\pi/2$	0	0	q_2^*
3	$\pi/2$	0	0	q_3^*
4	$-\pi/2$	a_2	d_1	q_4^*
5	$\pi/2$	0	0	q_5^*
6	$\pi/2$	0	d_2	q_6^*

$$A_i = \begin{bmatrix} c_\theta & -s_\theta c_\alpha & s_\theta s_\alpha & a c_\theta \\ s_\theta & c_\theta c_\alpha & -c_\theta s_\alpha & a s_\theta \\ 0 & s_\alpha & c_\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* variable

$$A_1^0 = \begin{bmatrix} c_\theta & 0 & s_\theta & a_1 c_\theta \\ s_\theta & 0 & -c_\theta & a_1 s_\theta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2^1 = \begin{bmatrix} c_\theta & 0 & s_\theta & a_1 c_\theta \\ s_\theta & 0 & -c_\theta & a_1 s_\theta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_3^2 = \begin{bmatrix} c_\theta & 0 & s_\theta & a_1 c_\theta \\ s_\theta & 0 & -c_\theta & a_1 s_\theta \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^3 = \begin{bmatrix} c_\theta & 0 & -s_\theta & a_2 c_\theta \\ s_\theta & 0 & c_\theta & a_2 s_\theta \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_5^4 = \begin{bmatrix} c_\theta & 0 & s_\theta & 0 \\ s_\theta & 0 & -c_\theta & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_6^5 = \begin{bmatrix} c_\theta & -s_\theta c_\alpha & s_\theta s_\alpha & a c_\theta \\ s_\theta & c_\theta c_\alpha & -c_\theta s_\alpha & a s_\theta \\ 0 & s_\alpha & c_\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6^0 = A_1^0 A_2^1 A_3^2 A_4^3 A_5^4 A_6^5$$

$$= \begin{bmatrix} c_{q_1} c_{q_2} c_{q_3} & c_{q_1} c_{q_2} s_{q_3} & s_{q_1} & a_1 c_{q_1} \\ s_{q_1} c_{q_2} c_{q_3} & s_{q_1} c_{q_2} s_{q_3} & -c_{q_1} & a_1 s_{q_1} \\ s_{q_2} c_{q_3} & s_{q_2} s_{q_3} & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{q_4} & 0 & -s_{q_4} & a_2 c_{q_4} \\ s_{q_4} & 0 & c_{q_4} & a_2 s_{q_4} \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{q_5} & 0 & s_{q_5} & 0 \\ s_{q_5} & 0 & -c_{q_5} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{q_6} & -s_{q_6} c_\alpha & s_{q_6} s_\alpha & a c_{q_6} \\ s_{q_6} & c_{q_6} c_\alpha & -c_{q_6} s_\alpha & a s_{q_6} \\ 0 & s_\alpha & c_\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{q_1} c_{q_2} c_{q_3} & c_{q_1} c_{q_2} s_{q_3} & s_{q_1} & a_1 c_{q_1} \\ s_{q_1} c_{q_2} c_{q_3} & s_{q_1} c_{q_2} s_{q_3} & -c_{q_1} & a_1 s_{q_1} \\ s_{q_2} c_{q_3} & s_{q_2} s_{q_3} & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{q_4} & 0 & -s_{q_4} & a_2 c_{q_4} \\ s_{q_4} & 0 & c_{q_4} & a_2 s_{q_4} \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{q_5} & 0 & s_{q_5} & 0 \\ s_{q_5} & 0 & -c_{q_5} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{q_6} & -s_{q_6} c_\alpha & s_{q_6} s_\alpha & a c_{q_6} \\ s_{q_6} & c_{q_6} c_\alpha & -c_{q_6} s_\alpha & a s_{q_6} \\ 0 & s_\alpha & c_\alpha & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6^0 = A_1^0 A_2^1 A_3^2 A_4^3$$

$$= \begin{bmatrix} c_{q_1} c_{q_2} c_{q_3} & c_{q_1} c_{q_2} s_{q_3} & s_{q_1} & a_1 c_{q_1} \\ s_{q_1} c_{q_2} c_{q_3} & s_{q_1} c_{q_2} s_{q_3} & -c_{q_1} & a_1 s_{q_1} \\ s_{q_2} c_{q_3} & s_{q_2} s_{q_3} & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_{q_4} & 0 & -s_{q_4} & a_2 c_{q_4} \\ s_{q_4} & 0 & c_{q_4} & a_2 s_{q_4} \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c_{q_1} c_{q_2} c_{q_3} c_{q_4} - s_{q_1} s_{q_4} & -c_{q_1} c_{q_2} c_{q_3} s_{q_4} - s_{q_1} c_{q_4} & -c_{q_1} c_{q_2} s_{q_3} & a_1 c_{q_1} + a_2 c_{q_1} c_{q_2} c_{q_3} \\ s_{q_1} c_{q_2} c_{q_3} c_{q_4} + c_{q_1} s_{q_4} & -s_{q_1} c_{q_2} c_{q_3} s_{q_4} + c_{q_1} c_{q_4} & -s_{q_1} c_{q_2} s_{q_3} & a_1 s_{q_1} + a_2 s_{q_1} c_{q_2} c_{q_3} \\ s_{q_2} c_{q_3} c_{q_4} & -s_{q_2} c_{q_3} s_{q_4} & -s_{q_2} s_{q_3} & d_1 + a_2 s_{q_2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

T7

(a) Three different configuration of 3R manipulator

(i) Direct Drive

(ii) In the case of direct drive robot, problems of backlash, friction and compliance due to gears are eliminated

(iii) Dynamics of motor may be much more complex

(iv) This method is used to eliminate the complication of transmission and mechanical backlash associated with other devices.

(v) Remotely Driven

(vi) In this case, there is complication of transmission but no problem the motors. Dynamics of motor motion

(vii) This single method of each joint can be easily calculated using this method. But the calculations of dynamics gets slightly complicated

(b) 3 bar parallelogram arrangement

(i) In this type of arrangement, instead of 3 links and 3 joints, and two grounded joints are attached to a crank.

(ii) Controlling these two motors would help in moving the end-effector anywhere

(iii) This type of arrangement provides a higher mechanical advantage and hence contain higher load inputs.

T8

Derivation of dynamic modelling of 2R manipulator, discussed in class and compare your results with manipulator. Hence write jacobian

$$T_1 = \begin{bmatrix} c_{q_1} & -s_{q_1} & 0 & 0 \\ s_{q_1} & c_{q_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_2 = \begin{bmatrix} c_{q_2} & -s_{q_2} & 0 & l_1 c_{q_2} \\ s_{q_2} & c_{q_2} & 0 & l_1 s_{q_2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\omega_1 = \dot{\theta}_1^2 \quad \omega_2 = \dot{\theta}_2^2$$

$$v_1 = \frac{1}{2} \frac{\partial T_1}{\partial q_1} q_1 + \frac{1}{2} \frac{\partial T_1}{\partial q_2} q_2 = 0$$

$$v_2 = \frac{1}{2} \frac{\partial T_2}{\partial q_1} q_1 + \frac{1}{2} \frac{\partial T_2}{\partial q_2} q_2 = \begin{bmatrix} -l_1 s_{q_2} \\ l_1 c_{q_2} \end{bmatrix}$$

$$\omega_1 = \dot{\theta}_1^2 \quad \omega_2 = \dot{\theta}_2^2$$

$$K = \frac{1}{2} \sum_{i=1}^{2} \left[m_i v_i^T v_i + \omega_i^T I_i \omega_i \right]$$

$$K = \frac{1}{2} \sum_{i=1}^{2} \left[m_i \begin{bmatrix} -l_1 s_{q_2} & l_1 c_{q_2} \end{bmatrix} \begin{bmatrix} -l_1 s_{q_2} \\ l_1 c_{q_2} \end{bmatrix} + \begin{bmatrix} 0 & 0 & \dot{\theta}_1^2 \\ 0 & 0 & \dot{\theta}_2^2 \\ \dot{\theta}_1^2 & \dot{\theta}_2^2 & 0 \end{bmatrix} I_i \right]$$

In 2R elbow manipulator,

$$D(q) = \begin{bmatrix} m_1 l_1^2 + m_2 l_1^2 + I_2 & m_2 l_1 l_2 \cos(q_2) \\ m_2 l_1 l_2 \cos(q_2) & m_2 l_2^2 + I_2 \end{bmatrix}$$

Computing Christoffel symbols

$$C_{111} = \frac{1}{2} \left[\frac{\partial D_{11}}{\partial q_1} + \frac{\partial D_{11}}{\partial q_1} - \frac{\partial D_{11}}{\partial q_1} \right]$$

$$C_{112} = \frac{1}{2} \left[\frac{\partial D_{11}}{\partial q_2} + \frac{\partial D_{12}}{\partial q_1} - \frac{\partial D_{12}}{\partial q_1} \right]$$

$$= \frac{1}{2} \left[\frac{\partial D_{11}}{\partial q_2} + \frac{1}{2} \frac{\partial}{\partial q_1} \left(-m_2 l_1 l_2 \cos(q_2) \right) \right]$$

$$= 0$$

$$C_{121} = C_{211} = \frac{1}{2} \left[\frac{\partial D_{11}}{\partial q_2} + \frac{\partial D_{12}}{\partial q_1} - \frac{\partial D_{21}}{\partial q_1} \right]$$

$$= \frac{1}{2} \frac{\partial D_{11}}{\partial q_2} - \frac{1}{2} \frac{\partial}{\partial q_1} \left(-m_2 l_1 l_2 \cos(q_2) \right)$$

$$= 0$$

$$C_{221} = \frac{1}{2} \left[\frac{\partial D_{12}}{\partial q_2} + \frac{\partial D_{22}}{\partial q_1} - \frac{\partial D_{21}}{\partial q_1} \right]$$

$$= \frac{1}{2} \left[\frac{\partial}{\partial q_2} \left(m_2 l_1 l_2 \cos(q_2) \right) - \frac{1}{2} \frac{\partial}{\partial q_1} \left(-m_2 l_1 l_2 \cos(q_2) \right) \right]$$

$$= m_2 l_1 l_2 \sin(q_2)$$

$$C_{122} = C_{212} = \frac{1}{2} \left[\frac{\partial D_{12}}{\partial q_2} + \frac{\partial D_{22}}{\partial q_1} - \frac{\partial D_{22}}{\partial q_1} \right]$$

$$= \frac{1}{2} \left[\frac{\partial D_{12}}{\partial q_2} \right]$$

$$= 0$$

$$C_{222} = \frac{1}{2} \left[\frac{\partial D_{22}}{\partial q_2} + \frac{\partial D_{22}}{\partial q_2} - \frac{\partial D_{22}}{\partial q_2} \right]$$

$$= 0$$

we have potential energy as

$$V = m_2 g_0 l_1 \cos q_1 + m_2 g \left(l_1 \cos q_1 + l_2 \cos q_2 \right)$$

$$\theta_1 = \frac{\partial V}{\partial q_1} = m_2 \frac{\partial}{\partial q_1} \left(l_1 \cos q_1 \right) + m_2 g \left(l_1 \cos q_1 \right)$$

$$\theta_2 = \frac{\partial V}{\partial q_2} = m_2 g \frac{\partial}{\partial q_2} \left(l_2 \cos q_2 \right)$$

we have,

$$\frac{1}{2} \dot{\theta}_1 D_1(q) \dot{\theta}_1 + \dot{\theta}_1^T C_1(q) \dot{\theta}_1 \dot{\theta}_1 + \dot{\theta}_1^T C_2(q) \dot{\theta}_2 \dot{\theta}_2 = \tau_1 \qquad k = 1, 2$$

$$\tau_1 = D_1(q) \ddot{\theta}_1 + \dot{\theta}_1^T C_1(q) \dot{\theta}_1 \dot{\theta}_1 + \dot{\theta}_1^T C_2(q) \dot{\theta}_2 \dot{\theta}_2 + \theta_1(q)$$

$$\tau_2 = D_2(q) \ddot{\theta}_2 + \dot{\theta}_1^T C_1(q) \dot{\theta}_1 \dot{\theta}_1 + \dot{\theta}_1^T C_2(q) \dot{\theta}_2 \dot{\theta}_2 + \dot{\theta}_1^T C_2(q) \dot{\theta}_2 \dot{\theta}_2 + \dot{\theta}_2^T C_2(q) \dot{\theta}_2 \dot{\theta}_2 + \theta_2(q)$$

$$\boxed{\tau_k = D_k \ddot{\theta}_k + \dot{\theta}_k^T C_k \dot{\theta}_k + \theta_k(q)}$$

T9.

(a) Derivation of equation of motion from $D(q)$ and $V(q)$

(i) we know $D(q)$ matrix and its elements as d_{ij} hence, use the Christoffel symbols

$$C_{ijk} = \frac{1}{2} \left[\frac{\partial d_{ij}}{\partial q_k} + \frac{\partial d_{ik}}{\partial q_j} - \frac{\partial d_{jk}}{\partial q_i} \right]$$

(ii) we have $V(q)$ function and computing variable $\theta_k(q)$

$$\theta_k(q) = \frac{\partial V}{\partial q_k}$$

(iii) Putting above values in euler-lagrange's eqn above,

$$\frac{1}{2} \dot{\theta}_1 D_1(q) \dot{\theta}_1 + \dot{\theta}_1^T C_1(q) \dot{\theta}_1 \dot{\theta}_1 + \dot{\theta}_1^T C_2(q) \dot{\theta}_2 \dot{\theta}_2 = \tau_k$$

(iv) More compact to write as

$$D(q) \ddot{\theta} + C(\dot{\theta}, \dot{\theta}) \dot{\theta} + g(\theta) = \tau \qquad \text{... all equations together in matrix form}$$

T11.

Code that uses symbolic computation and differentiation to derive the kinematics and produce any one joint $D(q)$ and $V(q)$

Code:

https://colab.research.google.com/drive/1Ka3ldfBfNwKzKGXBdNwdNc3NGJ1ywv-drive