Assignment - 2

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1. $$R_0^1 = \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} \hat{i}_1 \cdot \hat{i}_0 & \hat{j}_1 \cdot \hat{i}_0 & \hat{k}_1 \cdot \hat{i}_0 \\ \hat{i}_1 \cdot \hat{j}_0 & \hat{j}_1 \cdot \hat{j}_0 & \hat{k}_1 \cdot \hat{j}_0 \\ \hat{i}_1 \cdot \hat{k}_0 & \hat{j}_1 \cdot \hat{k}_0 & \hat{k}_1 \cdot \hat{k}_0 \end{bmatrix} \end{array}$$

Columns are C_1, C_2 & C_3.

To Prove :

$\Rightarrow$ $C_1 \cdot C_2 = C_2 \cdot C_3 = C_3 \cdot C_1 = 0$

let we write R_0^1 as:

$$R_0^1 = \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$$

then

$$R_0^{1\,T} = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

For the rotation matrix: $R_0^1 \cdot R_0^{1T} = R_0^{1T} \cdot R_0^1 = I$

$\Rightarrow$ $R_0^{1T} \cdot R_0^1 = I$

$$\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} \cdot \begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a_1^2 + b_1^2 + c_1^2 & a_1a_2 + b_1b_2 + c_1c_2 & a_1a_3 + b_1b_3 + c_1c_3 \\ a_1a_2 + b_1b_2 + c_1c_2 & a_2^2 + b_2^2 + c_2^2 & a_2a_3 + b_2b_3 + c_2c_3 \\ a_1a_3 + b_1b_3 + c_1c_3 & a_2a_3 + b_2b_3 + c_2c_3 & a_3^2 + b_3^2 + c_3^2 \end{bmatrix} = I$$

$C_1C_2 = a_1a_2 + b_1b_2 + c_1c_2 = 0$

$C_2C_3 = a_2a_3 + b_2b_3 + c_2c_3 = 0$

$C_1C_3 = a_1a_3 + b_1b_3 + c_1c_3 = 0$

Hence columns are orthogonal.

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12. Show that $\det(R_0^1) = 1$

We know that for any matrix:

$$\det(R) = \det(R^T)$$

also $$\det(R \cdot R^T) = \det(R) \cdot \det(R^T)$$

for rotation matrix:

$$R \cdot R^T = I$$

$$\det(R \cdot R^T) = \det(I)$$

$$\det(R) \cdot \det(R^T) = 1$$

$$\because \det(R^T) = \det(R)$$

$$[\det(R)]^2 = 1$$

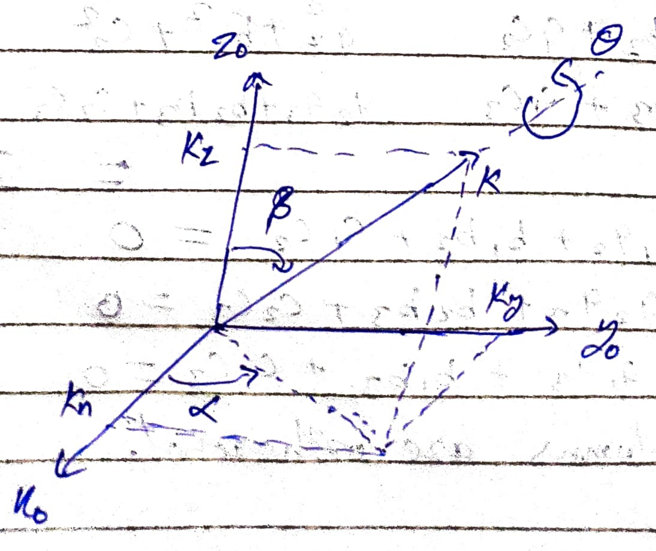
$$\underline{\underline{\det(R) = 1}}$$

13. Done.

14. Rotation about an arbitrary axis k.

$$k = [k_x, k_y, k_z]^T \quad \text{(in frame } O_0 x_0 y_0 z_0\text{)}$$

Rotation matrix $= R_{k,\theta}$ $\Rightarrow$ rotation of θ degree about axis k.



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$$R_{k,\theta} = R_{z,\alpha} R_{y,\beta} R_{z,\theta} R_{y,-\beta} R_{z,-\alpha}$$

for unit vector k:

$$\sin\alpha = \frac{k_y}{\sqrt{k_x^2 + k_y^2}}, \quad \cos\alpha = \frac{k_x}{\sqrt{k_x^2 + k_y^2}}$$

$$\sin\beta = \sqrt{k_x^2 + k_y^2}, \quad \cos\beta = k_z$$

$$R_{k,\theta} = \begin{bmatrix} k_x^2 V\theta + C\theta & k_x k_y V\theta - k_z S\theta & k_x k_z V\theta + k_y S\theta \\ k_x k_y V\theta + k_z S\theta & k_y^2 V\theta + C\theta & k_y k_z V\theta - k_x S\theta \\ k_x k_z V\theta - k_y S\theta & k_y k_z V\theta + k_x S\theta & k_z^2 V\theta + C\theta \end{bmatrix}$$

where $V\theta = \text{vers}\,\theta = 1 - C\theta$

Calculation of θ and k vectors for Rotation matrix:

⇒ Components of rotation matrix R are r_{ij}

then $\theta = \cos^{-1}\left(\frac{Tr(R) - 1}{2}\right)$

for 3×3 matrix $\theta = \cos^{-1}\left(\frac{r_{11} + r_{22} + r_{33} - 1}{2}\right)$

Tr = trace of R

$$k = \frac{1}{2\sin\theta}\begin{bmatrix} r_{32} - r_{23} \\ r_{13} - r_{31} \\ r_{21} - r_{12} \end{bmatrix}$$

$$R_{k,\theta} = R_{-k,-\theta}$$

15. $R S(a) R^T = S(Ra)$

Left hand side:

$\Rightarrow \quad R S(a) R^T$

we know that $S(a) b = a \times b$

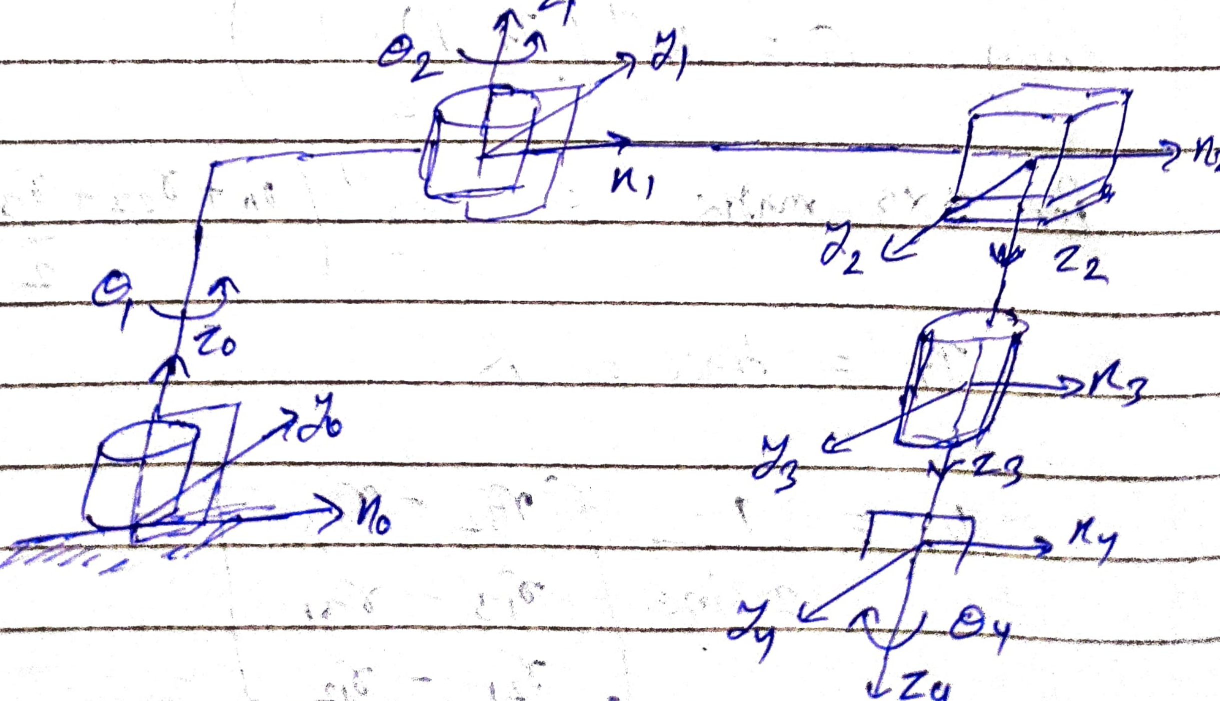
also for orthogonal matrix R:

$$R.(a \times b) = Ra \times Rb$$

$$\Rightarrow R S(a) R^T = R.(a \times R^T)$$
$$= Ra \times R.R^T$$
$$= Ra \times I$$
$$= S(Ra).I$$
$$R S(a) R^T = S(Ra)$$

16. SCARA manipulator:

Coordinate frames:



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(10). Gearboxes in Robots:

(1. Planetary Gearheads: These are used in high precision motion control applications that require high torque to volume ratio, high torsional stiffness and low backlash.

(2. Strain wave Gearing (Harmonic gearing): These are very light weight with zero backlash and high gear ratio. It has coaxial input and output shaft. Good for repetable tasks.

(3. Cycloidal drive:
These are used to reduce speed. These has compact size with zero backlash.

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11. For the SCARA Diagram in Q.6:

$$O_1 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}, \quad O_2 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ 0 \end{bmatrix}$$

$$O_3 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ d_3 \end{bmatrix}, \quad O_4 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ d_3 - d_4 \end{bmatrix}$$

$$J = \begin{bmatrix} z_0 \times (O_4 - O_0) & z_1 \times (O_4 - O_1) & z_2 & z_3 \times (O_4 - O_3) \\ z_0 & z_1 & 0 & z_3 \end{bmatrix}$$

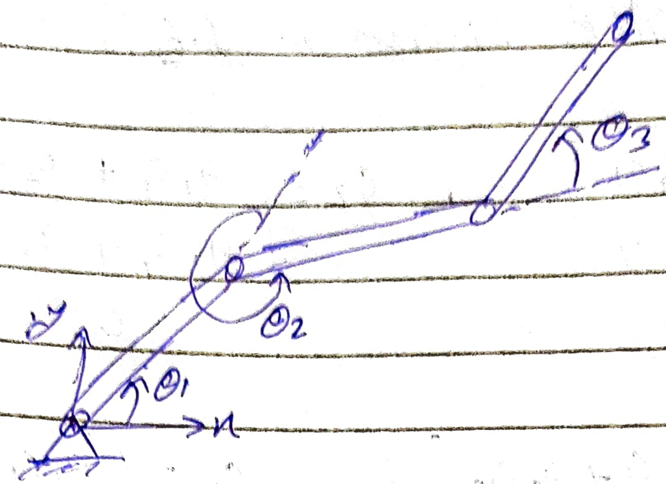
$\because$ z_3 and $(O_4 - O_3)$ are parallel

$z_3 \times (O_4 - O_3) = 0$

$\Rightarrow$ $z_0 = z_1 = k = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad z_2 = z_3 = -k = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$

$$\Rightarrow J = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} & -a_2 s_{12} & 0 & 0 \\ a_1 c_1 + a_2 c_{12} & a_2 c_{12} & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & -1 \end{bmatrix}$$

Q13.



$$O_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad O_1 = \begin{bmatrix} a_1 c_1 \\ a_1 s_1 \\ 0 \end{bmatrix}$$

$$O_2 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} \\ a_1 s_1 + a_2 s_{12} \\ 0 \end{bmatrix}, \quad O_3 = \begin{bmatrix} a_1 c_1 + a_2 c_{12} + a_3 c_{123} \\ a_1 s_1 + a_2 s_{12} + a_3 s_{123} \\ 0 \end{bmatrix}$$

$$J = \begin{bmatrix} z_0 \times (O_3 - O_0) & z_1 \times (O_3 - O_1) & z_2 \times (O_3 - O_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$

where $z_0 = z_1 = z_2 = k = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$J = \begin{bmatrix} -a_1 s_1 - a_2 s_{12} - a_3 s_{123} & -a_2 s_{12} - a_3 s_{123} & -a_3 s_{123} \\ a_1 c_1 + a_2 c_{12} + a_3 c_{123} & a_2 c_{12} + a_3 c_{123} & a_3 c_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$