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On the Entropy of a Discrete Black Hole

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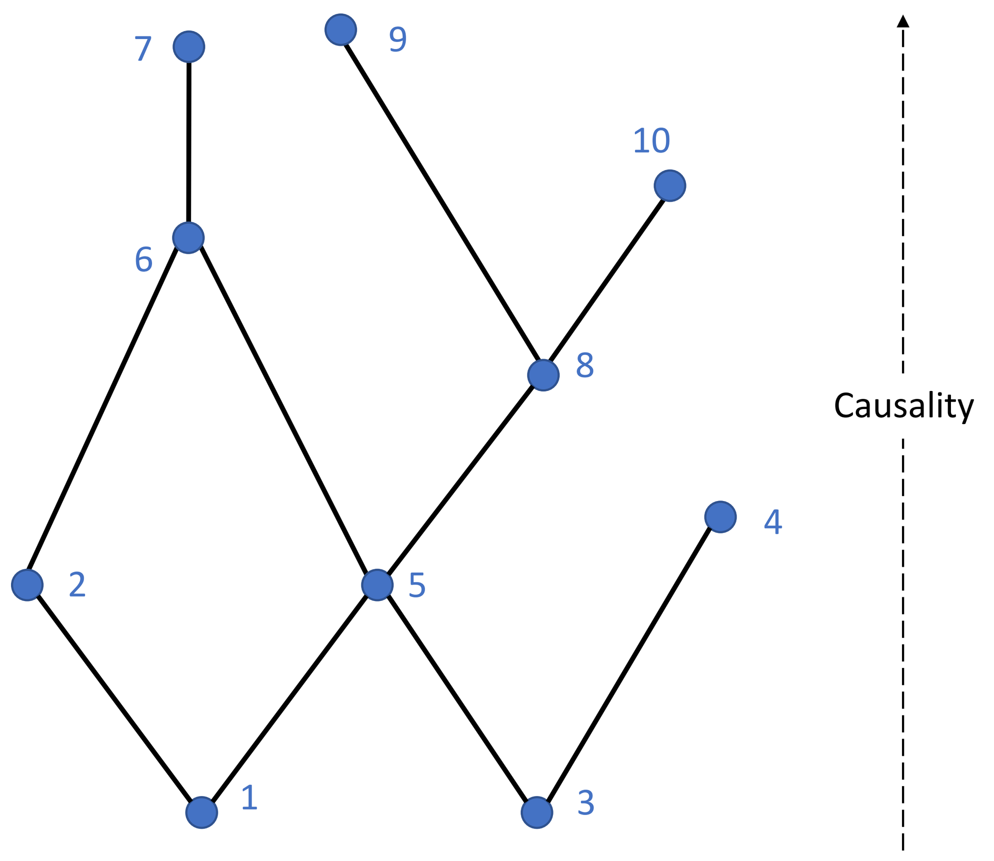
The Causal Set Approach to Quantum Gravity

GEOMETRY = ORDER + NUMBER!

Causal structure and the discreteness scale uniquely define spacetime (HKMM theorem[1,2]).

Causal Set Theory[3] assumes that the continuous spacetime emerges as a macroscopic approximation to an underlying causal set, a discrete partially ordered set of events C with an order relation $\prec$. A causal set (causet) satisfies ($\forall\, x, y, z \in C$):

- *Transitivity:* $x \prec y \wedge y \prec z \Rightarrow x \prec z$.
- *Acyclicity:* $x \prec y_1 \prec y_2 \prec \ldots \prec y_N \neq x$.
- *Local finiteness:* $|\{z|\ x \prec z \prec y\}|$ is finite.



Hasse diagram of a generic causet

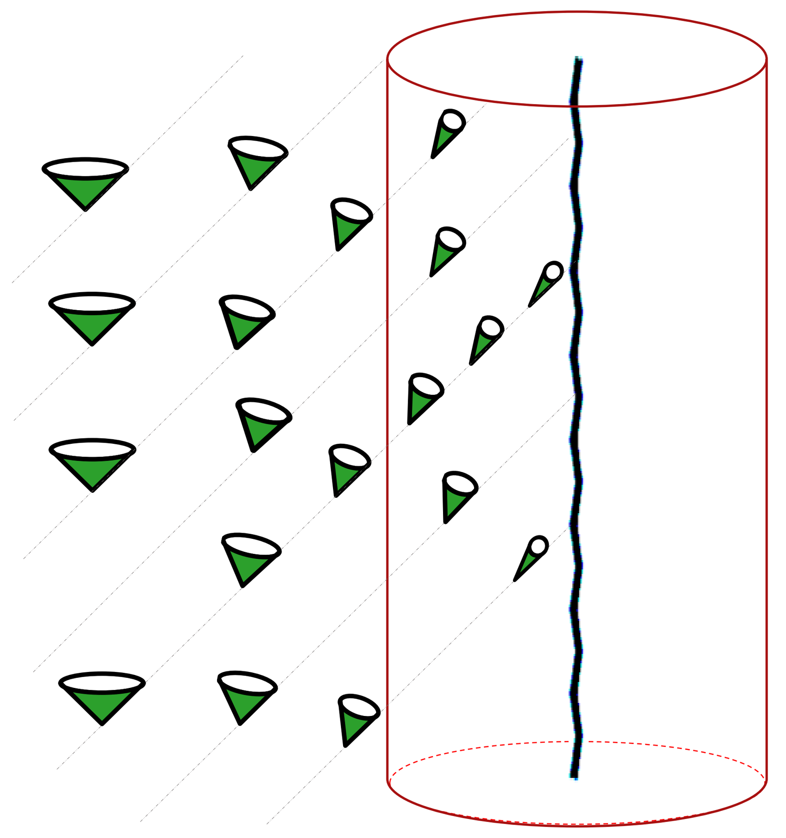
How to study it? **Poisson Sprinkling!**

To embed a causet in a spacetime region of volume V:

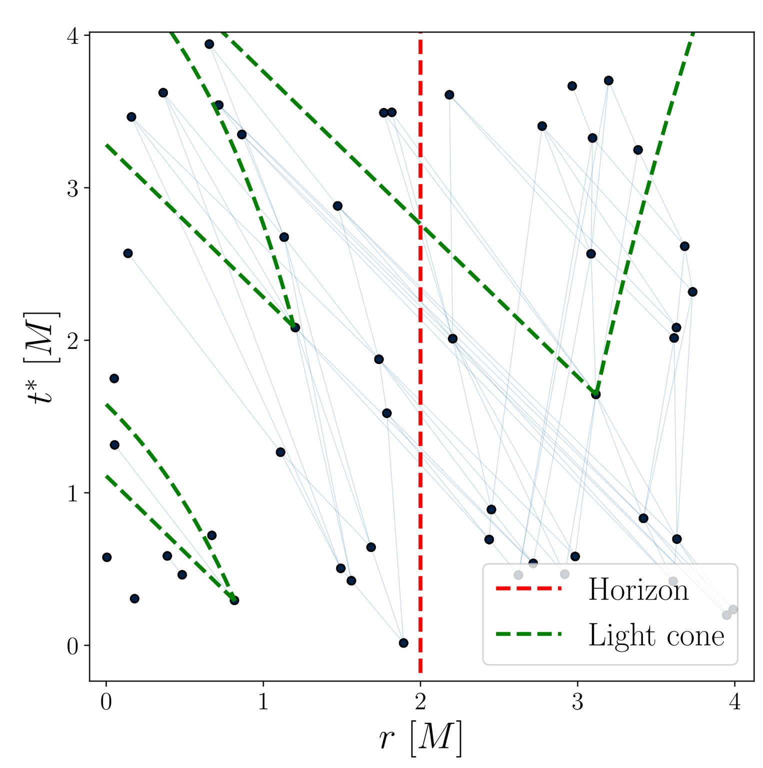
1. Pick the sprinkling density $\rho = \ell^{-4}$.
2. Distribute $N = \mathrm{Poiss}(\rho V)$ points uniformly such that the probability of having n points in a volume V is
$$\mathrm{Poiss}(n; V) = \frac{(\rho V)^n}{n!} e^{-\rho V}.$$
Note: $dV(x) \propto \sqrt{-g_{\mu\nu}(x)}$.
3. Connect the elements according to the causal structure of the spacetime region.

Black Holes: the Testing Arena of Quantum Gravity

Black holes arise from **General Relativity** and combined with **Quantum Mechanics** yield a full set of **Thermodynamic** laws.



(2+1)D black hole. Inside the horizon, light cones converge on the singularity, the only possible future.



Causal set of a (1+1)D Schwarzschild black hole. Connections form only within the light cones.

From Entropy to Horizon Molecules

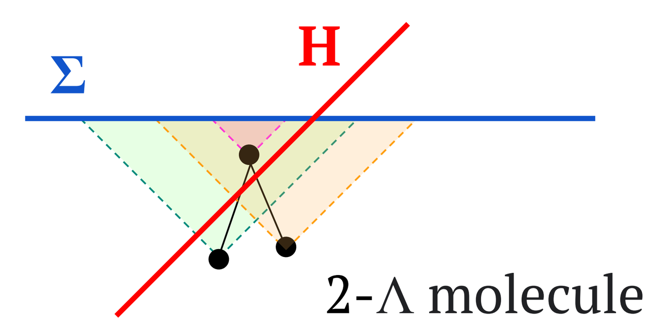
"If your theory is found to be against the second law of thermodynamics, I can give you no hope; there is nothing for it but to collapse in deepest humiliation."
Arthur Eddington, 1915.

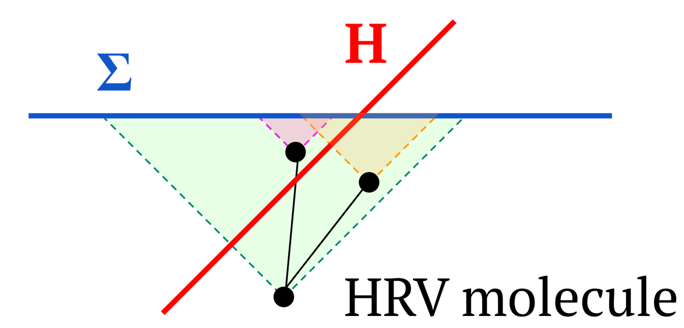
The entropy[4,5] of a black hole with event horizon area A is given by

$$S_{BH} = \frac{1}{4}\frac{A}{\ell_p^2}, \qquad \ell_p = \sqrt{\frac{\hbar G}{c^3}} \sim 10^{-35}\ \mathrm{m}.$$

Where is the black hole entropy? *On* the horizon[6], embodied by "horizon molecules". In analogy to statistical physics, we can define

$$S = -N_{mol} \sum p_n \ln(p_n)$$





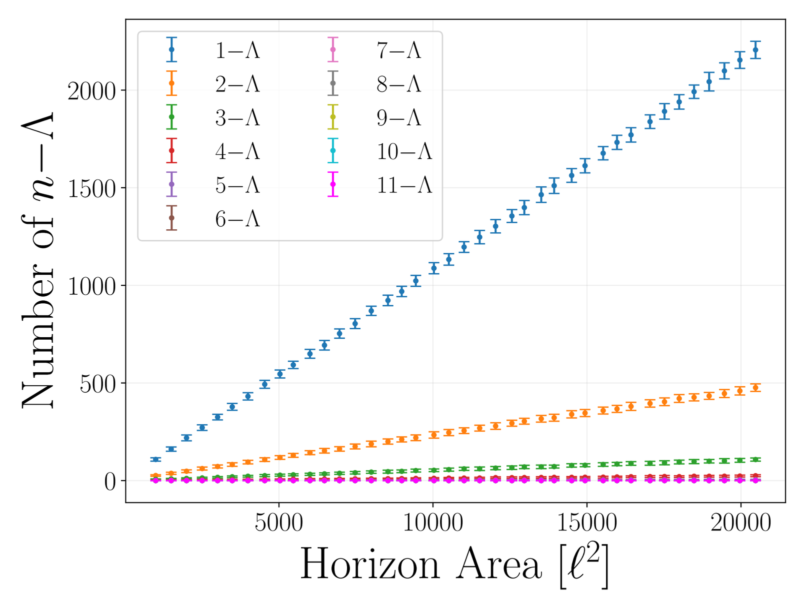
Aims

- *First-ever* study of causal sets in **Schwarzschild spacetime**[7] (750k elements)!
- Test different molecule models of black hole entropy.
- Compare our simulations with analytic results[8] in flat spacetime (for Rindler horizon).

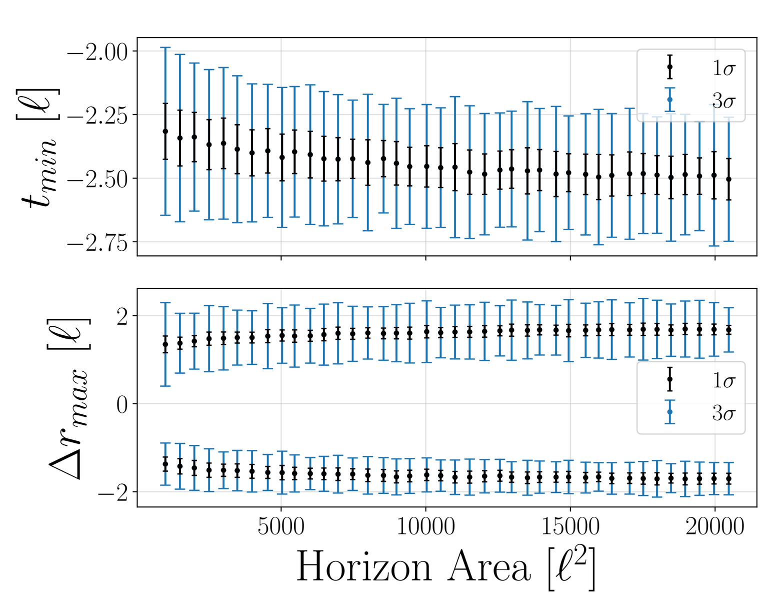
Further Work

- Discretise more complex black holes.
- Investigate other entropy models (SMI[9], SSEE[10]).
- Explore curvature features and corrections in causets[9,11].

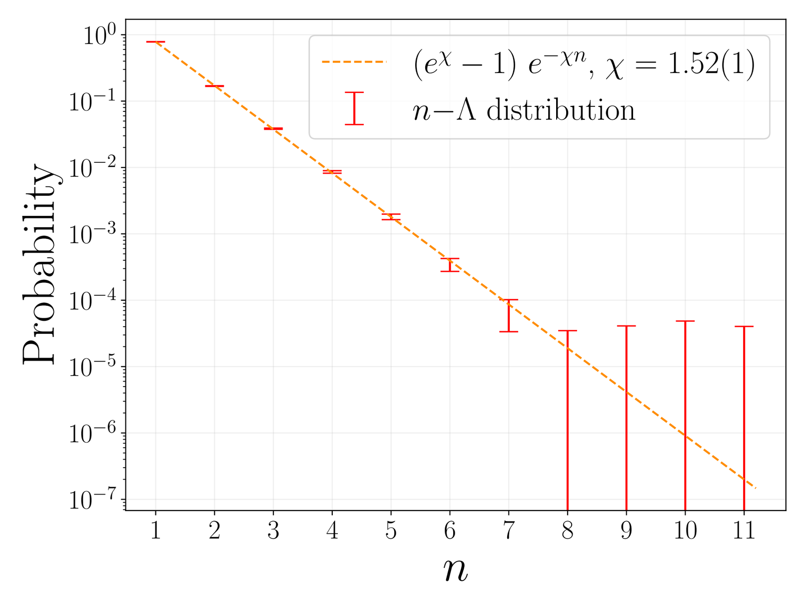
Results



Desired scaling $N_{n\Lambda} \propto A$.



Λs are localised on the horizon.



Distribution of n-Λs $p_n \propto e^{-\chi n}$.

For the n-Λs, we found that the entropy scales as

$$S_\Lambda = 0.0929(6)\frac{A}{\ell^2}.$$

Setting $S_\Lambda = S_{BH}$ gives a discreteness scale

$$\ell = 0.610(2)\ell_p.$$

Writing $p_n \propto e^{-E_n/k_B T}$ we obtain, as for the energy of n-atomic molecules in 3D space,

$$E_n \approx \frac{3n}{2} k_B T.$$

We confirm analytical calculations[8] for the scaling of single-link molecules when accounting for curvature corrections. Extending these to Λs yields

$$\Delta S_\Lambda = 0.173(1)\frac{\sqrt{A}}{\ell}.$$

For HRVs, we analytically found $N_{HRV} = 0.0742\ A/\ell^2$. This result was confirmed to within the uncertainty. The entropy and discreteness scale for HRVs are

$$S_{HRV} = 0.0098(3)\frac{A}{\ell^2}, \quad \ell = 0.198(3)\ell_p.$$

References

[1] Hawking SW, King AR, McCarthy PJ. 1976. J. Math. Phys. 17:174-181.
[2] Malament DB. 1977. J. Math. Phys. 18:1399-1404.
[3] Bombelli L, et al. 1987. Phys. Rev. Lett. 59:521-524.
[4] Bekenstein JD. 1972. Lett. Nuovo Cimento 4:737–740.
[5] Hawking SW. 1975. Commun.Math. Phys. 43:199–220.
[6] Dou D, Sorkin RS. 2003. Found.Phys. 33:279-296.
[7] He S, Rideout D. 2009. Class.Quant.Grav. 26:125015.
[8] Barton C, et al. 2019. Phys. Rev. D 100:126008.
[9] Benincasa DMT. 2013. PhD Thesis. Imperial College London.
[10] Sorkin RS, Yazdi YK. 2018. Class. Quantum Grav. 35:074004.
[11] Roy M, Sinha D, Surya S. 2013. Phys. Rev. D 87:044046.

Our Code

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