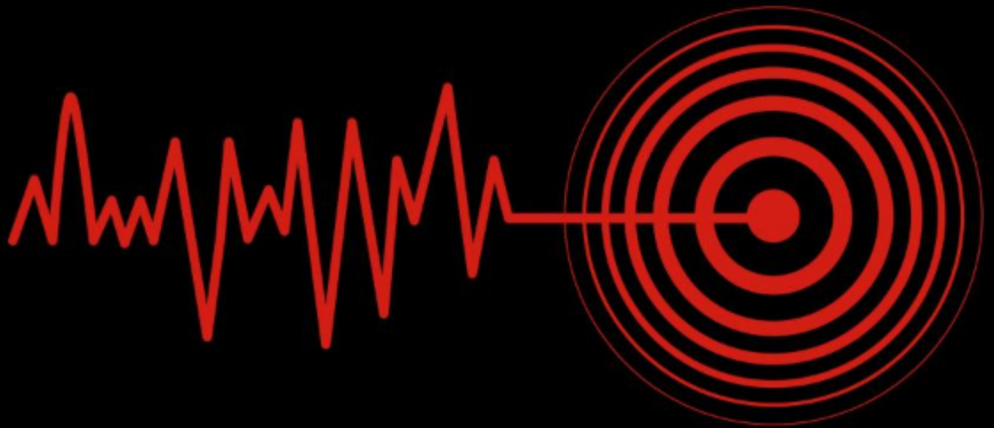
Class 11th
Science
PHYSICS
MOTION IN A PLANE

CHAPTER OVERVIEW

- VECTORS
- PROJECTILE MOTION
- RELATIVE MOTION

Physical Quantities

- **Scalars**
  - Magnitude
- **Vectors**
  - Magnitude
  - Direction
  - obeys laws of vector algebra



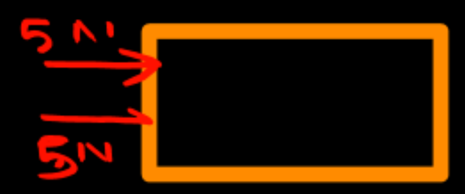
Next Toppers

Identify the Vector quantities

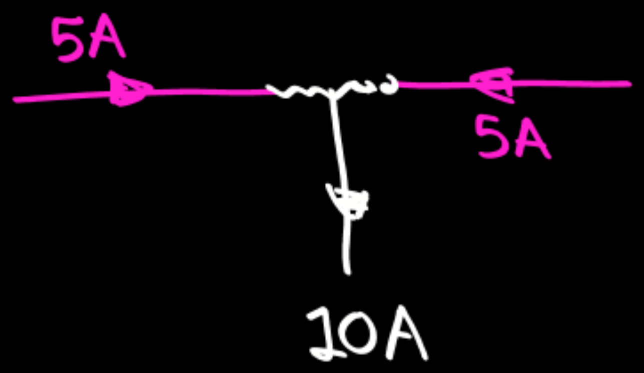
- S Mass
- S Speed
- S Temperature
- V Velocity $= \frac{disp.}{t}$
- S Time
- V Force
- S Volume
- \# Electric Current

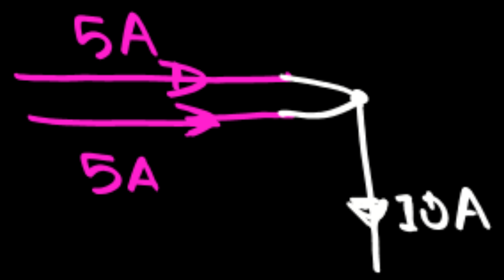
Scalar

$F_{net} = 5 - 5 = 0$



$F_{net} = 5 + 5 = 10N$





Represention of vector

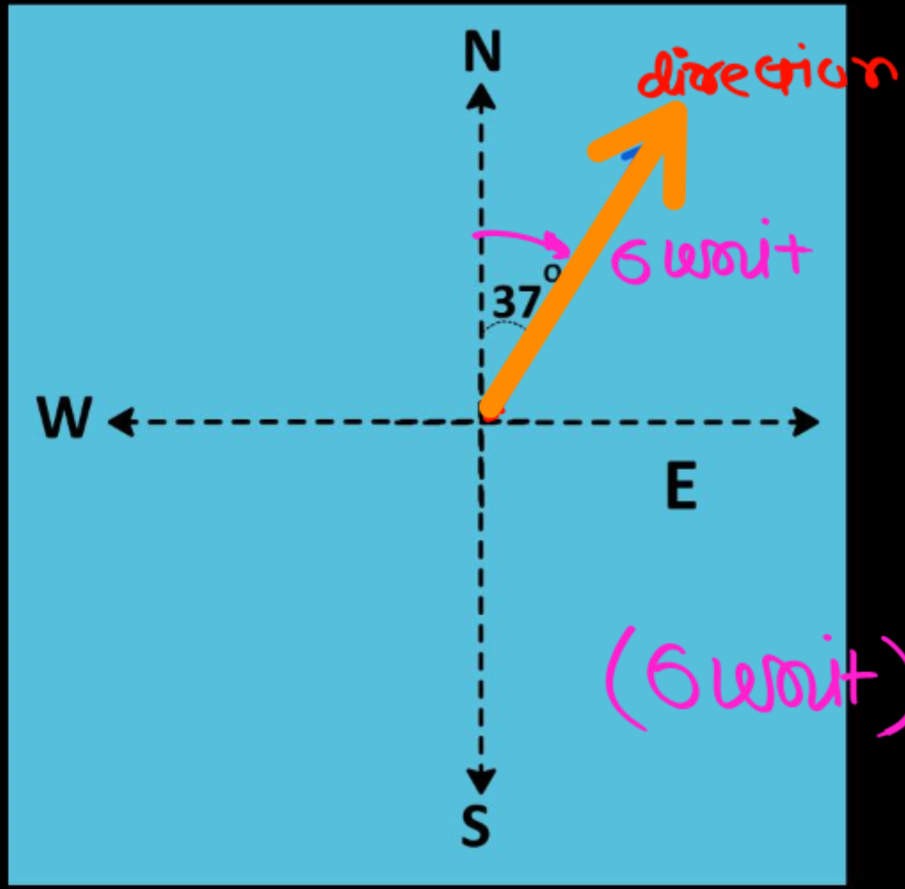
Algebraic

PQ = Magnitude . direction

$$\vec{A} = |\vec{A}| \hat{A}$$

$$\vec{A} = A \hat{A}$$

Geometric



(6 unit) (37° East of North)

PQ = (Magnitude) (direction)

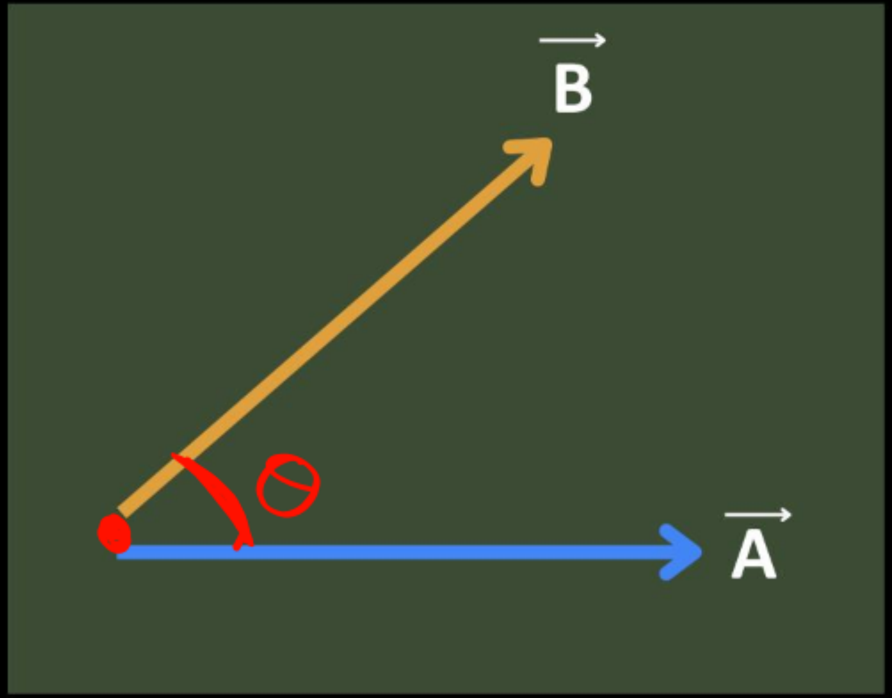
↓

length

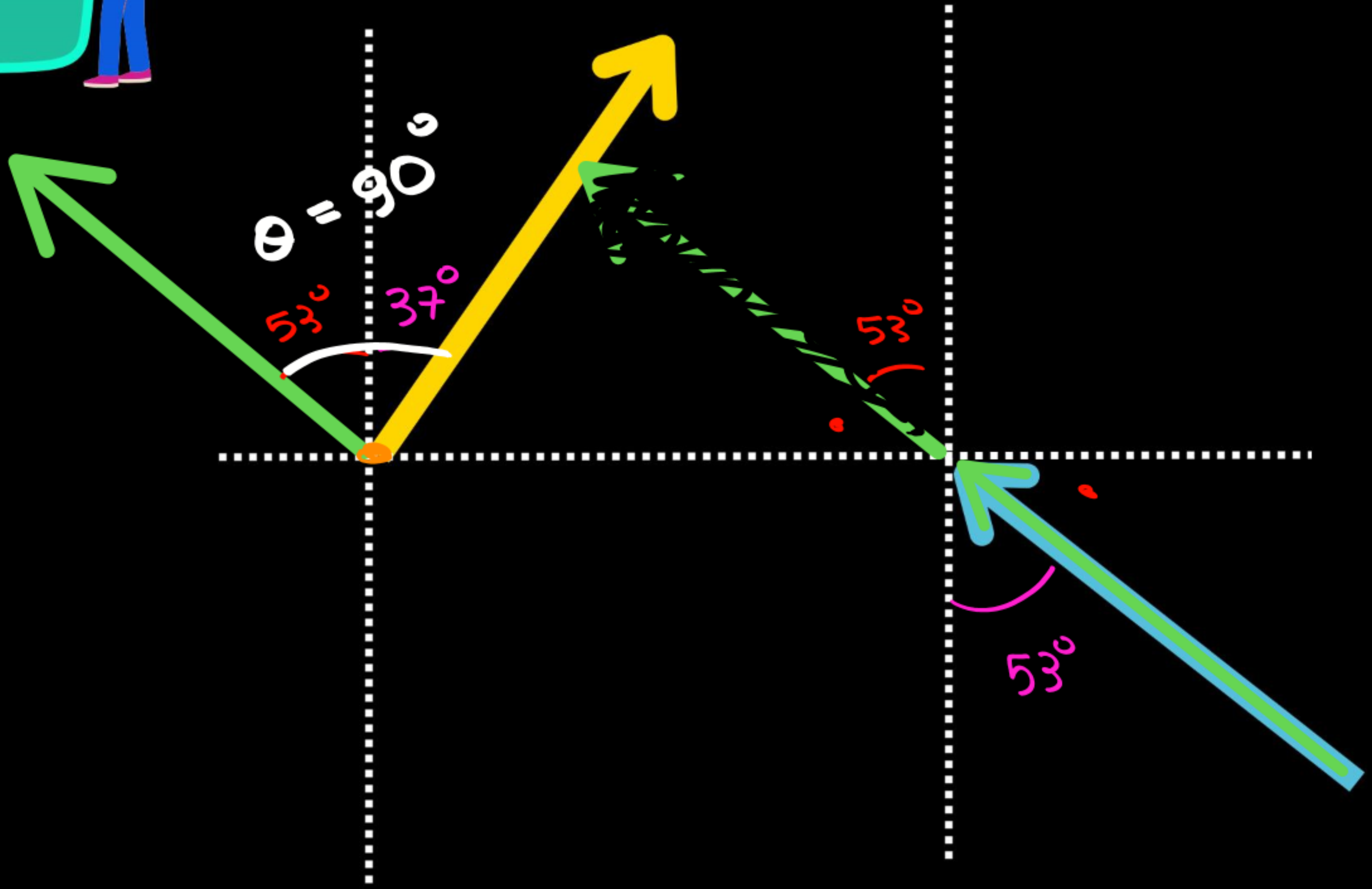
2N

4N

Angle between vectors



→ we can move vector without changing its direction & length

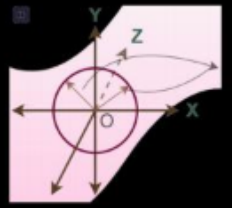


Types of Vector

Collinear	Lie on the same line
Parallel	Vectors along lines parallel to each other
Negative	Equal in magnitude but opposite in direction
Equal	Same magnitude and along the same direction
Null or zero	Zero magnitude

Null

Unit Vector

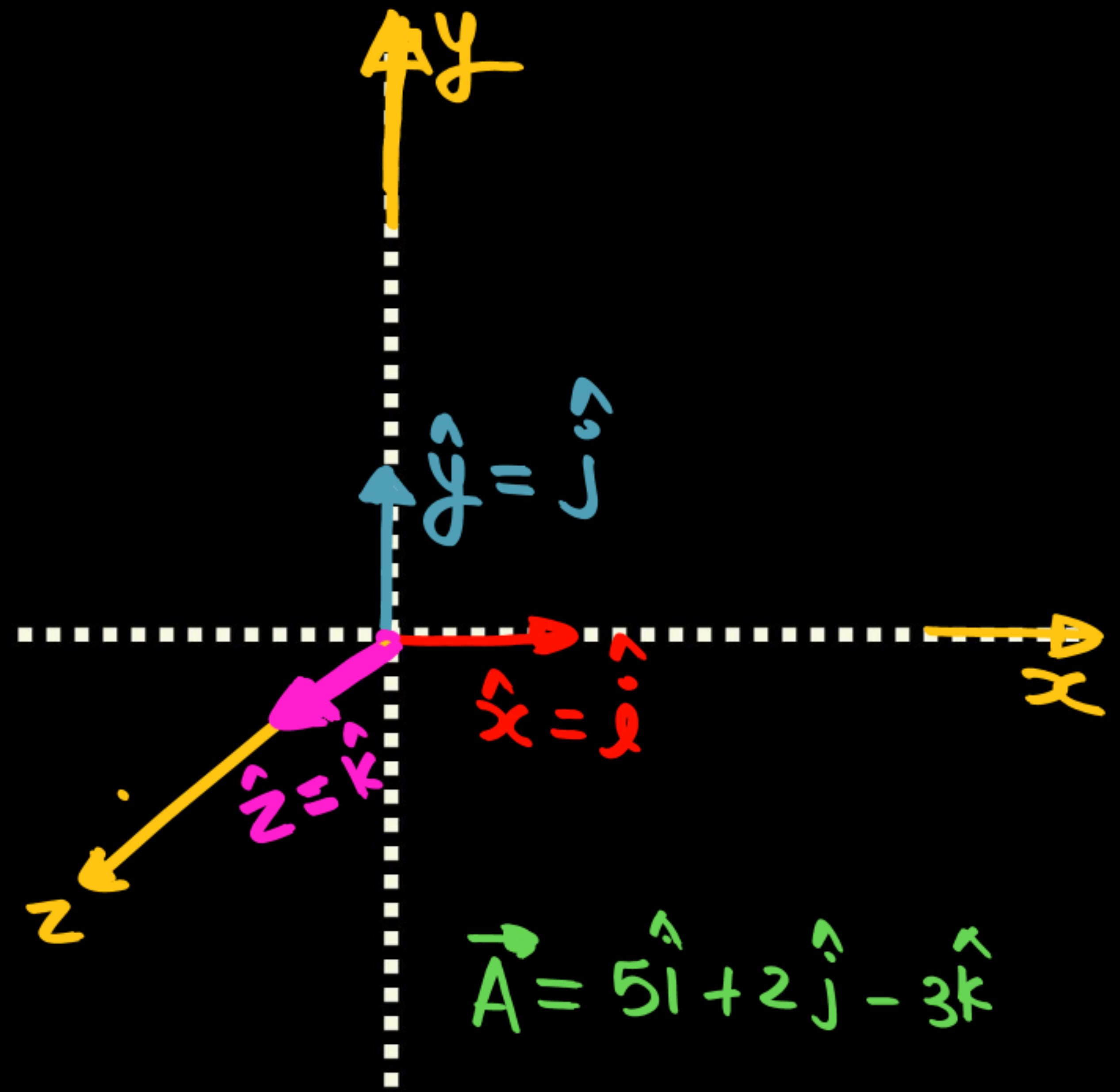


Vector divided by its own magnitude is a vector with unit magnitude and direction along the parent vector.

Magnitude = 1 unit

$$\vec{A} = |\vec{A}|\,\hat{A} \Rightarrow \boxed{\hat{A} = \frac{\vec{A}}{|\vec{A}|}}$$

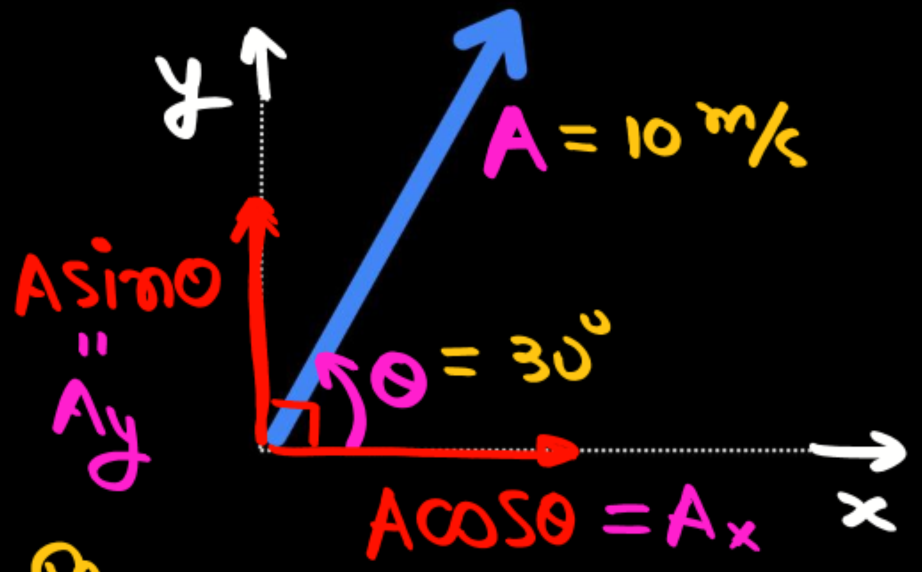
$\hat{A}$ → unit vector



$$\vec{A} = 5\hat{i} + 2\hat{j} - 3\hat{k}$$

Resolution of vector

2D



Eg.

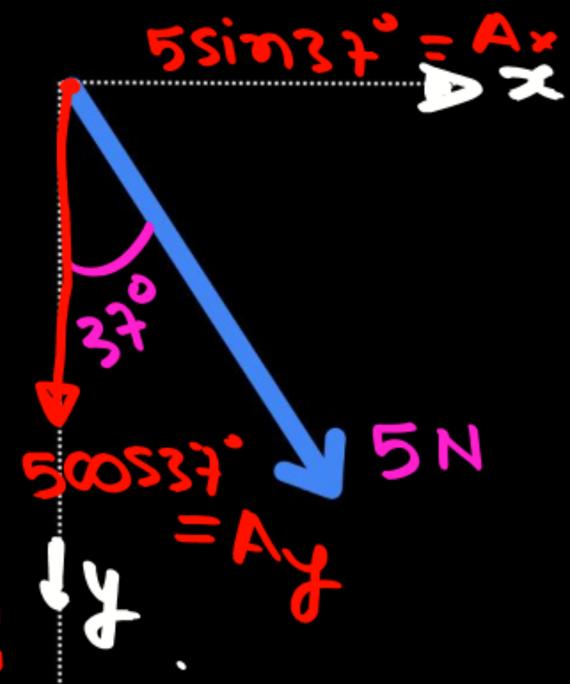
$10\sin 30° = 5\ m/s$

$10\cos 30° = 5\sqrt{3}\ m/s$

$A_x = 5\sqrt{3}\ m/s$

$A_y = 5\ m/s$

Q.



$A_x = 5\sin 37° = 3\ N$

$A_y = 5\cos 37° = 4\ N$

* $\vec{A} = A_x\hat{i} + A_y\hat{j}$

✓ $\vec{A} = 3\hat{i} + 4\hat{j}$

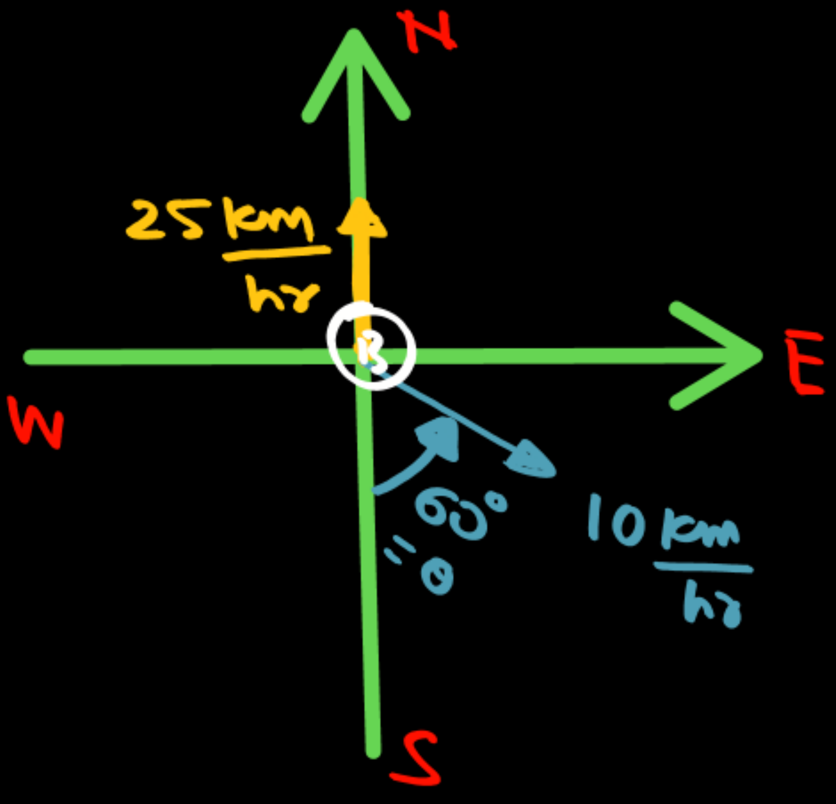
* $|\vec{A}| = \sqrt{A_x^2 + A_y^2} = 5\ N$

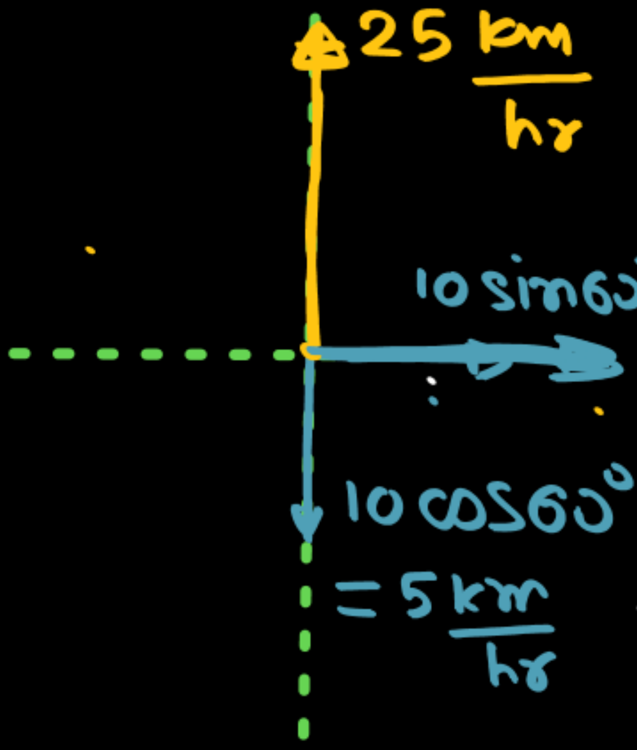
* $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

$\hat{A} = \dfrac{3\hat{i} + 4\hat{j}}{5}$

Example 3.3

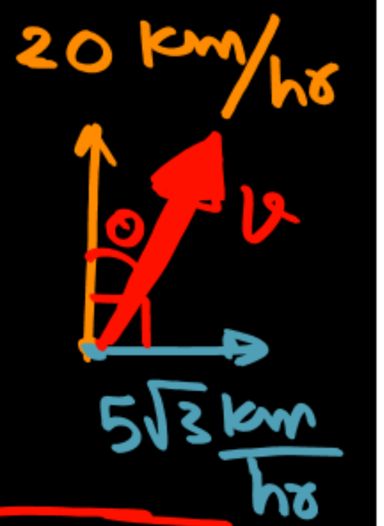
A motorboat is racing towards north at 25 km/h and the water current in that region is 10 km/h in the direction of 60° east of south. Find the resultant velocity of the boat.





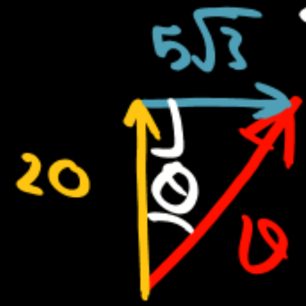
$10 \sin 60^\circ = 5\sqrt{3}$ km/hr

$10 \cos 60^\circ = 5$ km/hr



$|\vec{v}| = \sqrt{(20)^2 + (5\sqrt{3})^2}$

$\tan\theta = \dfrac{5\sqrt{3}}{20} = \sqrt{3}/4$

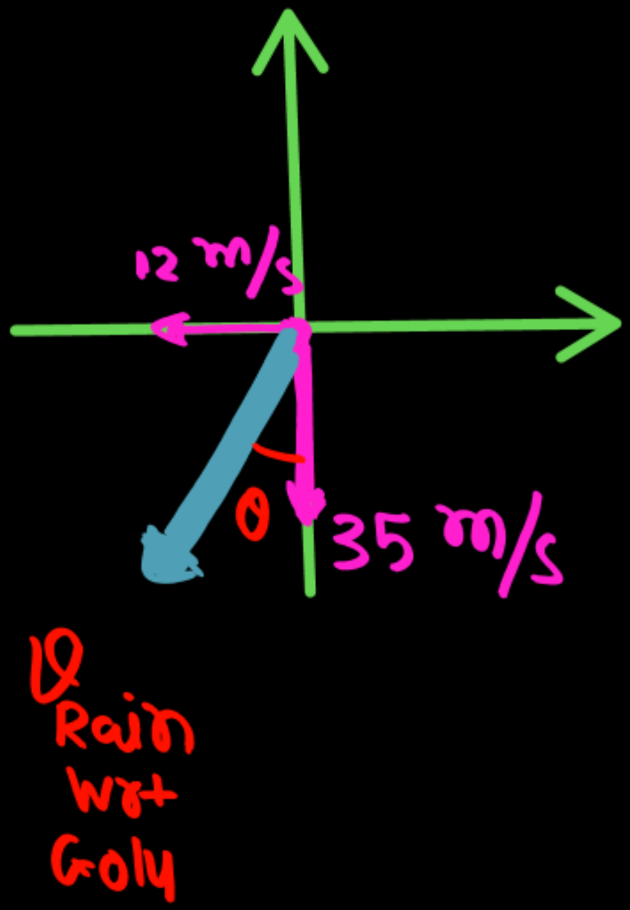


Example 3.1

Rain is falling vertically with a speed of 35 m s[1]. Winds starts blowing after sometime with a speed of 12 m s[1] in east to west direction. In which direction should a boy waiting at a bus stop hold his umbrella?

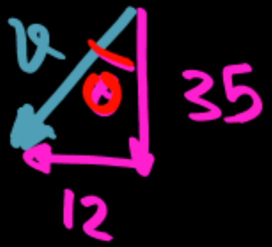
Golu bhai

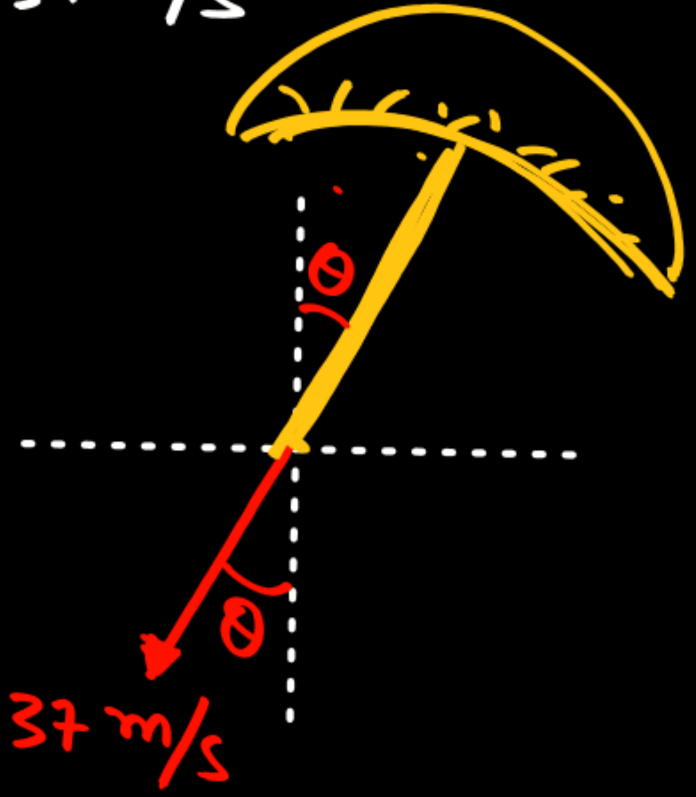
Ratan



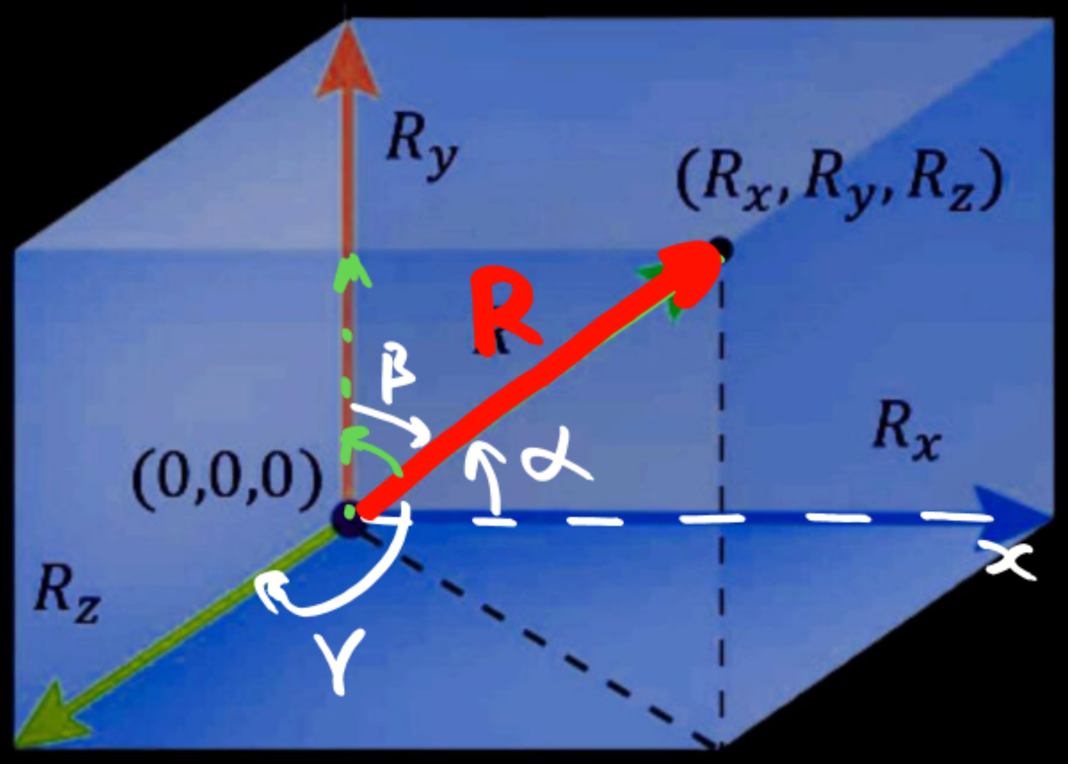
$$v = \sqrt{(35)^2+(12)^2} = 37 \text{ m/s}$$

$$\tan\theta = \frac{12}{35}$$





3D



$$\vec{R} = R_x\hat{i} + R_y\hat{j} + R_z\hat{k}$$

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2 + R_z^2} = R$$

$$\cos\alpha = \frac{R_x}{R} = \frac{R_x}{\sqrt{R_x^2 + R_y^2 + R_z^2}}$$

$$\cos\beta = \frac{R_y}{R} = \frac{R_y}{\sqrt{R_x^2 + R_y^2 + R_z^2}}$$

$$\cos\gamma = \frac{R_z}{R} = \frac{R_z}{\sqrt{R_x^2 + R_y^2 + R_z^2}}$$

Vector Algebra

$\vec{A} + \vec{B} = ?$

$\vec{A} - \vec{B} = ?$

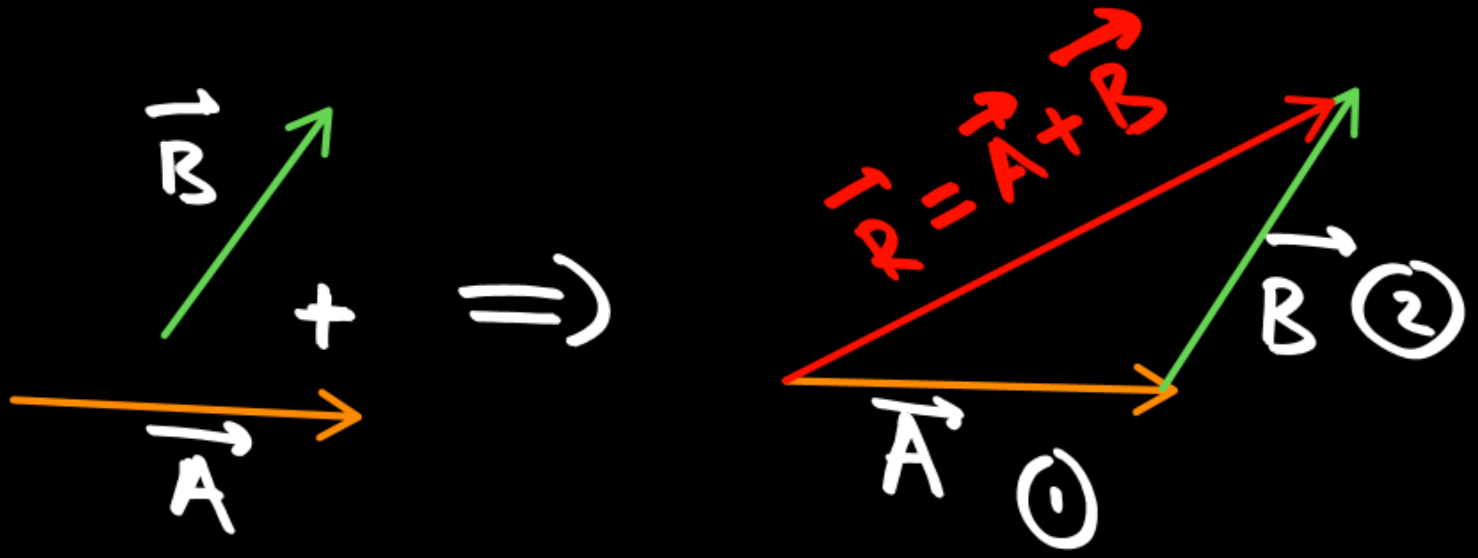
$\vec{A} + (-\vec{B}) = ?$

Vector Addition

- Triangle Law
- Polygon Law
- Parallelogram Law

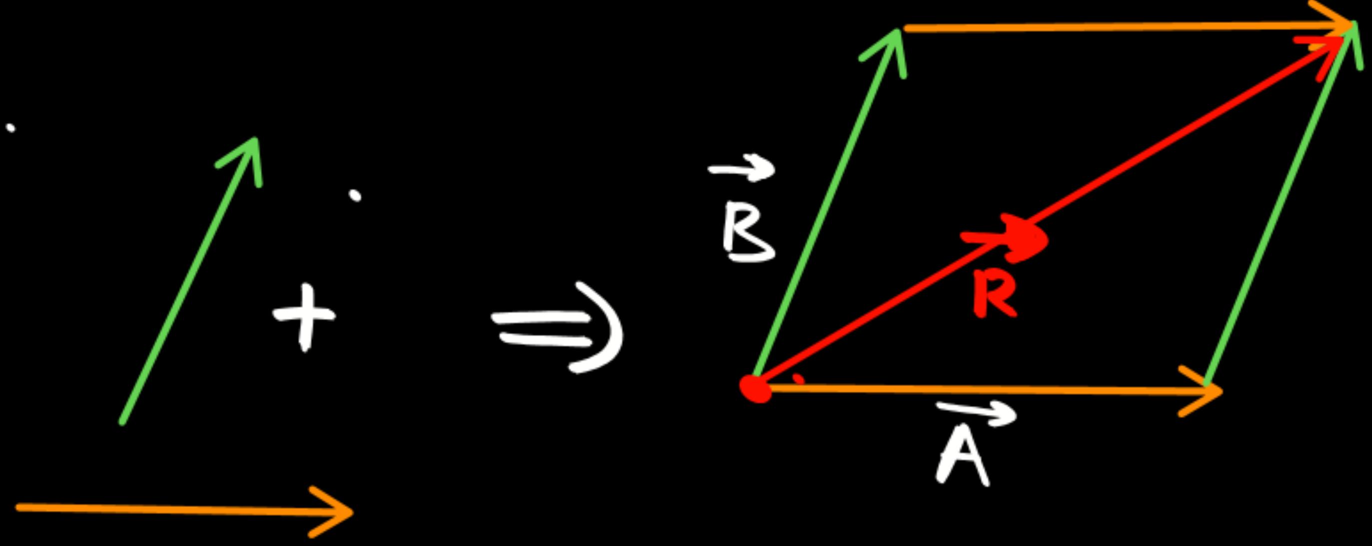
1. Triangle law of vector addition

When **two Vectors** are aligned head to tail, their "vector sum" or **"resultant vector"** is represented by the **third side of the completed triangle** in the opposite order

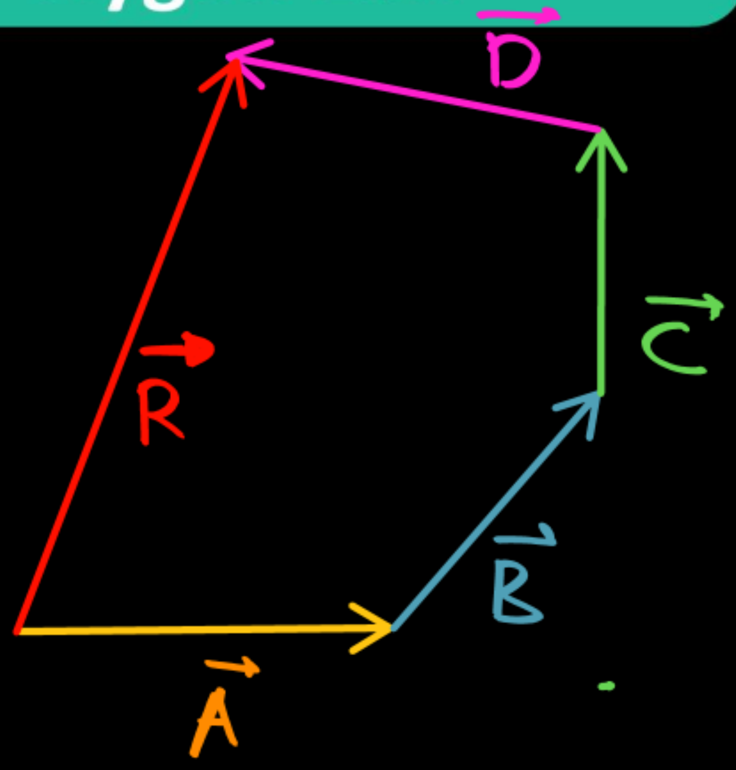


2. Parallelogram Law

If two vectors are oriented **coinitial**, representing **two adjacent sides of a parallelogram**, then their **resultant is represented by the included diagonal** of the completed parallelogram.

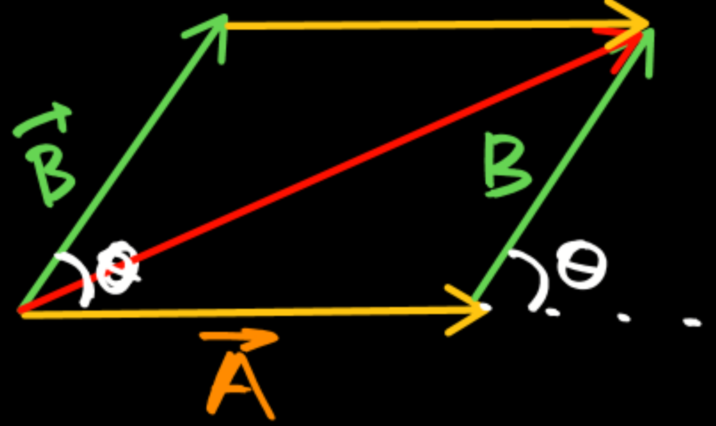


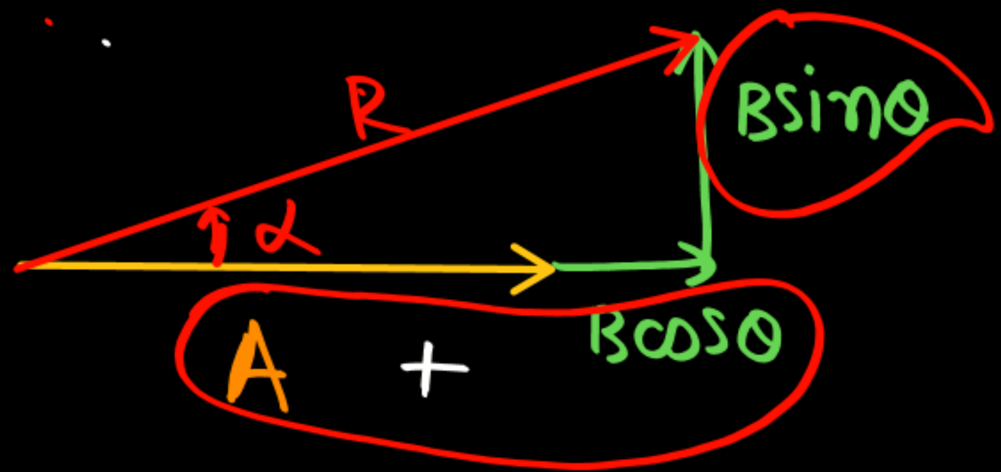
3. Polygon Law



$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$

Analytical Method





$R_x = A + B\cos\theta$

$R_y = B\sin\theta$

$$|\vec{R}| = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(A + B\cos\theta)^2 + (B\sin\theta)^2}$$

$$= \sqrt{A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta}$$

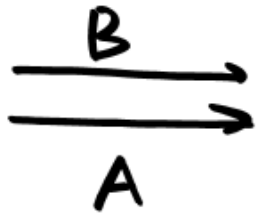
$$(\because \sin^2\theta + \cos^2\theta = 1)$$

$$\boxed{|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}}$$

$$\boxed{\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}}$$

$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

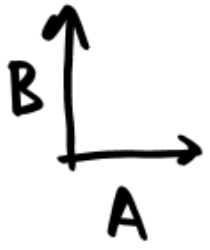
Q.



$\theta = 0$

$|\vec{R}| = ?$

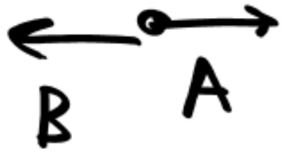
Q.



$\theta = 90^\circ$

$|\vec{R}| = ?$

Q.



$\theta = 180^\circ$

$|\vec{R}| = ?$

Vector Algebra

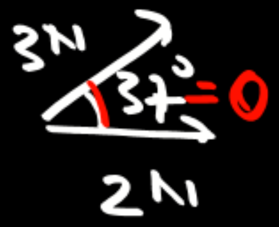
$\vec{A}$ $\vec{B}$

Multiplication of Vectors

Scalar/Dot Product

$$\vec{A}\cdot\vec{B} = (AB\cos\theta) = R$$

Q.



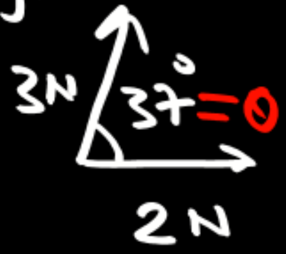
$$\vec{A}\cdot\vec{B} = (2)(3)\cos 37^\circ$$
$$= 6\times\frac{4}{5} = \frac{24}{5}\,N$$

Vector/Cross Product

$$\vec{A}\times\vec{B} = (AB\sin\theta)\,\hat{n} = \vec{R}$$

$\hat{n}$: ⊥ to A, ⊥ to B

Q.



$$|\vec{A}\times\vec{B}| = (2)(3)\sin 37^\circ$$
$$= 6\times 3/5 = \frac{18}{5}\,N$$

~~2. Cross~~ 1. Dot Product

$\vec{A} = 1\hat{i} - 2\hat{j} + 3\hat{k}$

$\vec{B} = 1\hat{i} + 1\hat{j} + 0\hat{k}$

$\hat{i}\cdot\hat{i} = 1 = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k}$

$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

$\vec{A}\cdot\vec{B} = (1\cdot 1) + (-2\cdot 1) + (3\cdot 0)$

$= 1 + (-2) + 0$

$= -2$

scalar

cross

$\hat{i}\ \hat{j}\ \hat{k}\ \hat{i}$

$\hat{i}\times\hat{j} = \hat{k}$ $\quad \hat{k}\times\hat{i} = \hat{j}$ $\quad \hat{k}\times\hat{j} = -\hat{i}$

$\hat{j}\times\hat{k} = \hat{i}$ $\quad \hat{i}\times\hat{k} = -\hat{j}$

2. ~~Dot~~ cross Product

$$\vec{A} = \hat{i} - 2\hat{j} + 3\hat{k}$$

$$\vec{B} = \hat{i} + \hat{j} + 0\hat{k}$$

$$\hat{i} \times \hat{j} = \hat{k}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{i} \times \hat{i} = 0 = \hat{j} \times \hat{j} = \hat{k} \times \hat{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2 & 3 \\ 1 & 1 & 0 \end{vmatrix}$$

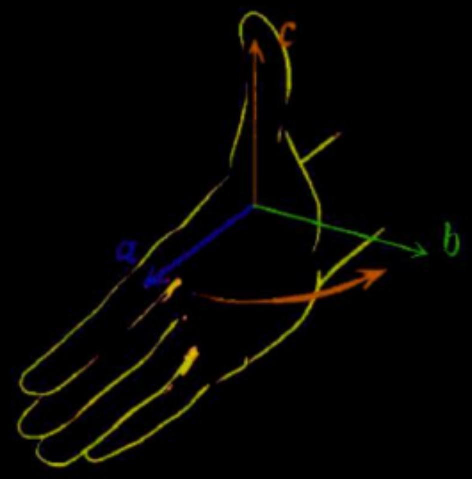
$$= \hat{i}(0-3) - \hat{j}(0-3) + \hat{k}(1-(-2))$$

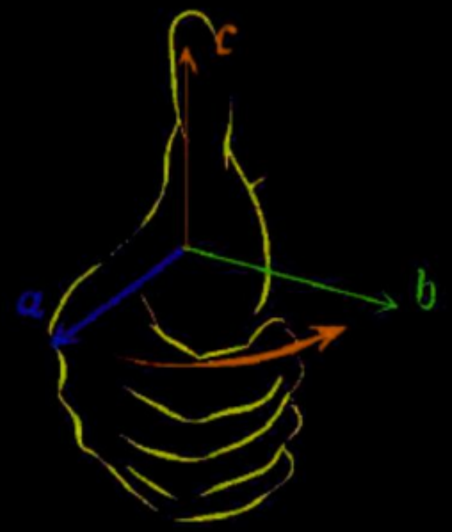
$$\vec{R} = -3\hat{i} + 3\hat{j} + 3\hat{k}$$

vector

Right hand thumb rule

- Draw a and b such that they are coinitial.
- Place the stretched right palm perpendicular to the plane of a and b such that the fingers are along a and when the fingers are closed, they go towards b.
- The direction in which thumb points gives the direction of resultant





MOTION IN A PLANE

- A 2D motion can be interpreted as combination of two simultaneous and independent 1D motions along any two mutually perpendicular axes.
- The net motion is the vector superposition of the motion along x and y axes.
- The two independent 1D motion have time in common.

y x

1D

x	y
u_x	u_y
a_x	a_y
v_x	v_y
$t_x = t$	$t_y = t$

1D

Example 3.4

The position of a particle is given by $\vec{r} = 3.0t\,\hat{i} + 2.0t^2\,\hat{j} + 5.0\,\hat{k}$ where t is in seconds and the coefficients have the proper units for r to be in metres. (a) Find v(t) and a(t) of the particle. (b) Find the magnitude and direction of v(t) at t = 1.0 s.

$$\vec{r} = 3t\,\hat{i} + 2t^2\,\hat{j} + 5\hat{k}$$

$x = 3t$	$y = 2t^2$	$z = 5$
$v_x = \frac{dx}{dt} = 3$	$\frac{dy}{dt} = 4t$	$\frac{dz}{dt} = 0$
$v_x = 3$	$v_y = 4t$	$v_z = 0$
$a_x = \frac{dv_x}{dt} = 0$	$a_y = 4$	$\frac{dv_z}{dt} = 0$

$$\Rightarrow \vec{v} = 3\hat{i} + 4t\,\hat{j}$$

$$\vec{v}_{(t=1\,sec)} = 3\hat{i} + 4\hat{j}$$

$$\Rightarrow \vec{a} = 4\hat{j}$$

$\tan\theta = \frac{4}{3}$

A particle moves from the point $(2.0\hat{i}+4.0\hat{j})$m at t = 0 with an initial velocity $(5.0\hat{i}+4.0\hat{j})\text{ms}^{-1}$. It is acted upon by a constant force which produces a constant acceleration $(4.0\hat{i}+4.0\hat{j})\text{ms}^{-2}$. What is the distance of the particle from the origin at time 2s?

(a) 15 m

(b) $20\sqrt{2}m$

(c) 5 m

(d) $10\sqrt{2}m$

$x_i = 2$ m | $y_i = 4$ m

$u_x = 5$ m/s | $u_y = 4\hat{j}$

$a_x = 4$ m/s^2 | $a_y = 4$ m/s^2

$\boxed{a = \text{const.} \Rightarrow \text{EOM}}$

$\boxed{t = 2\text{ sec}}$

$\Delta x = u_x t + \frac{1}{2} a_x t^2$

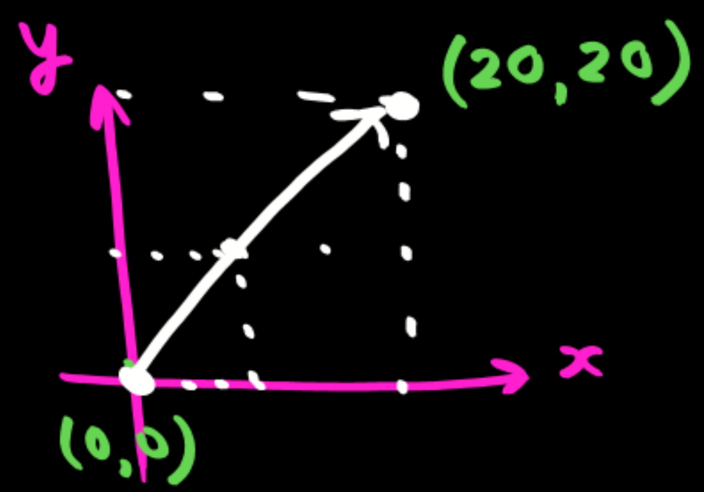
$x_F - 2 = 10 + 8$

$x_F = 20$ m

$\Delta y = u_y t + \frac{1}{2} a_y t^2$

$y_F - 4 = 8 + 8$

$y_F = 20$ m



$r = \sqrt{(20)^2 + (20)^2}$
$= 20\sqrt{2}$ m

Question

A particle moves from the point $(2.0\hat{i}+4.0\hat{j})\text{m}$ at t = 0 with an initial velocity $(5.0\hat{i}+4.0\hat{j})\text{ms}^{-1}$. It is acted upon by a constant force which produces a constant acceleratio $(4.0\hat{i}+4.0\hat{j})\text{ms}^{-2}$. What is the distance of the particle from the origin at time 2s? [11 Jan, 2019 (Shift-II)]

(a) 15 m

(b) $20\sqrt{2}m$

(c) 5 m

(d) $10\sqrt{2}m$

Ans. $20\sqrt{2}m$

The co-ordinates of a moving particle at any time 't' are given by $x = \alpha t^3$ and $y = \beta t^3$. The speed of the particle at time 't' is given by **[2003]**

(a) $3t\sqrt{\alpha^2+\beta^2}$

(b) $3t^2\sqrt{\alpha^2+\beta^2}$ ✓

(c) $t^2\sqrt{\alpha^2+\beta^2}$

(d) $\sqrt{\alpha^2+\beta^2}$

$x = \alpha t^3$

$V_x = \frac{dx}{dt} = 3\alpha t^2$

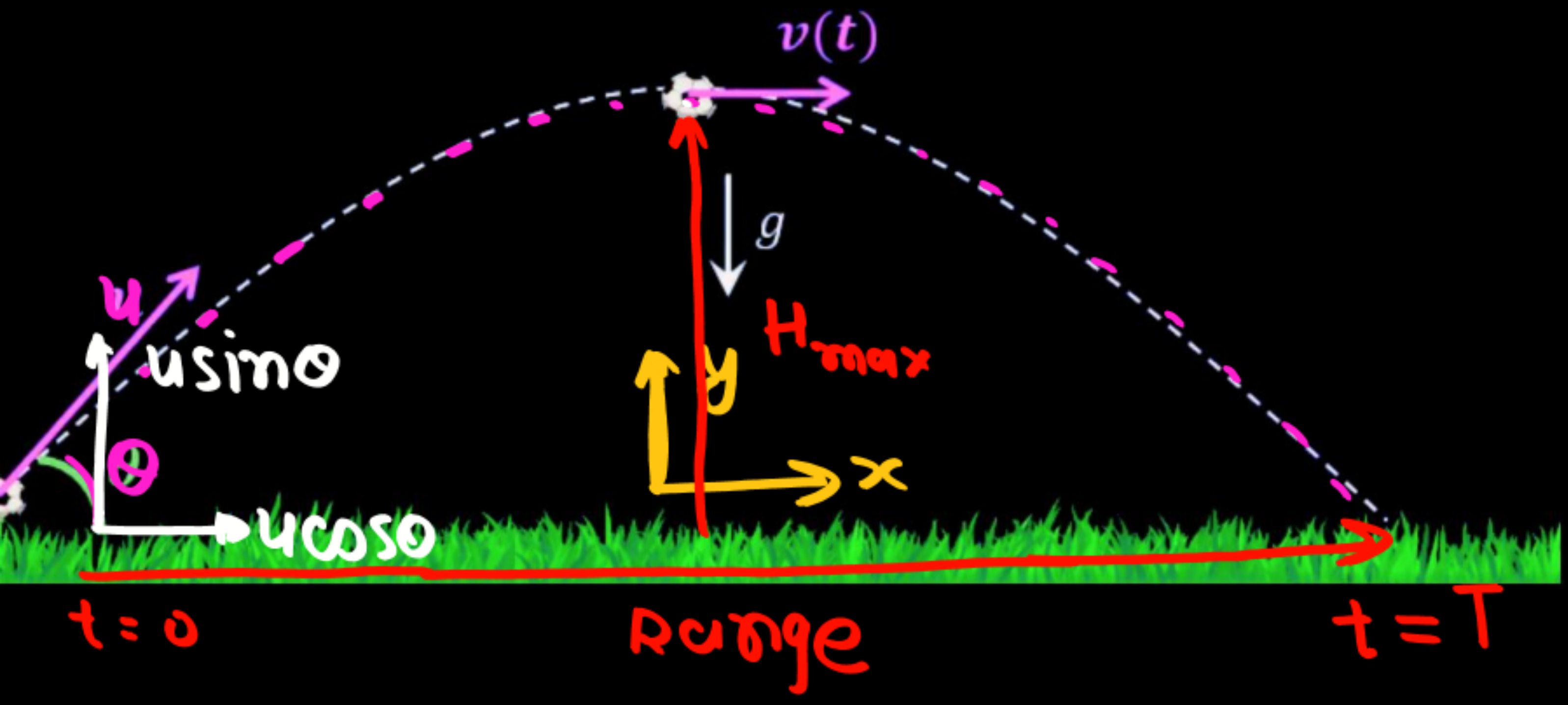
$y = \beta t^3$

$V_y = \frac{dy}{dt} = 3\beta t^2$

$\Rightarrow V_{net} = \sqrt{V_x^2 + V_y^2}$

$= 3t^2\sqrt{\alpha^2+\beta^2}$

PROJECTILE MOTION



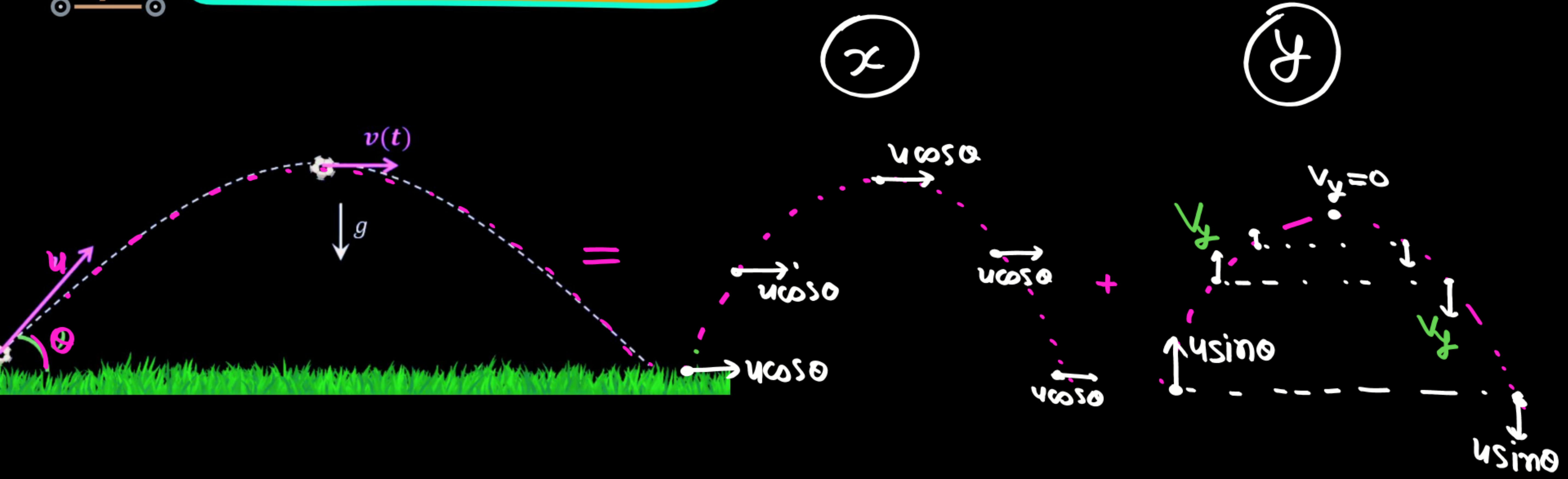
x	y
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = 0$	$a_y = g$
$V_x = u_x = u\cos\theta$ $= \text{const.}$	$V_y = u_y + a_y t$
	$V_y = u\sin\theta - gt$

h and Range <<< radius of earth.

Earth surface is assumed to be flat over the range of projectile.

Air resistance is ignored

PROJECTILE MOTION



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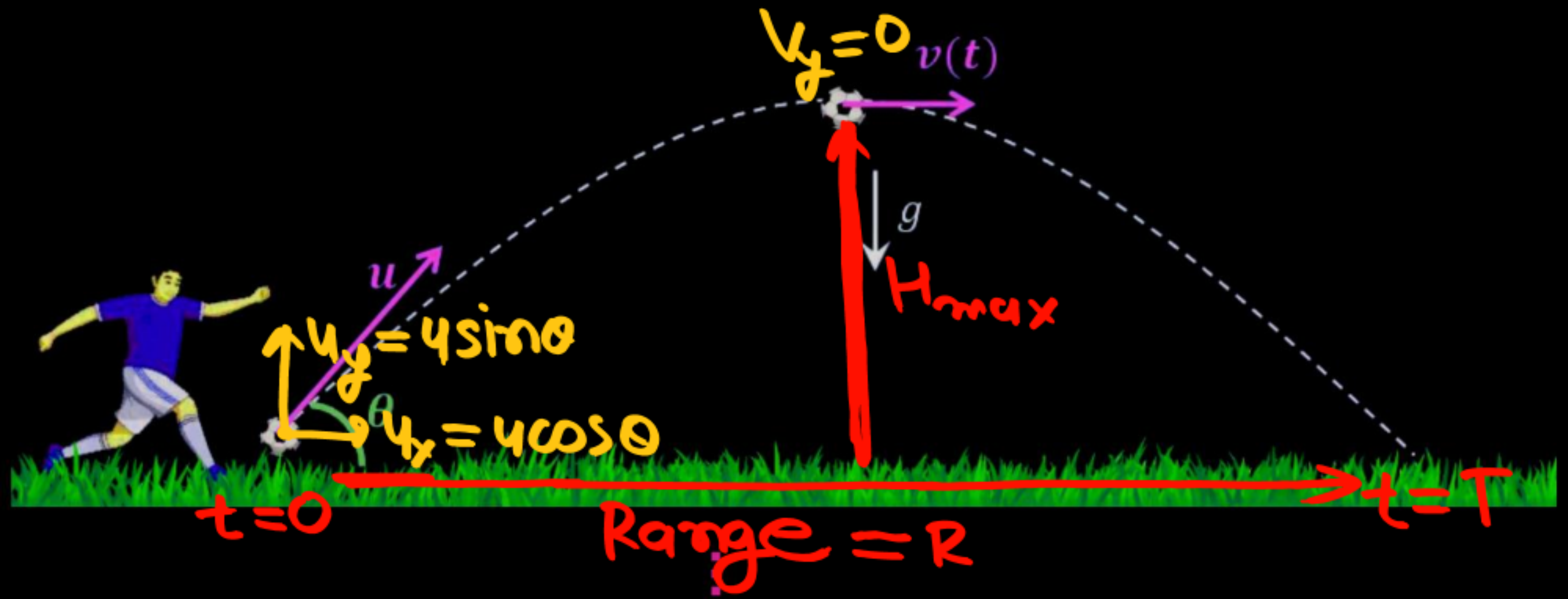
MOTION IN A PLANE



$2\sin\theta \cdot \cos\theta = \sin 2\theta$

Height (H)

y-axis :- 3rd EOM

$$v_y^2 - u_y^2 = 2a_y H_{max}$$

$$0 - u_y^2 = -2g H_{max}$$

$$\boxed{H_{max} = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g}}$$

Time of Flight (T)

$S_y = 0$ y-axis 2nd EOM

$$y = u_y t + \frac{1}{2}a_y t^2$$

$$0 = u_y \cancel{T} - \frac{1}{2} g T^{\cancel{2}}$$

$$\boxed{T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}}$$

Range (R)

x-axis
u_x = cont.

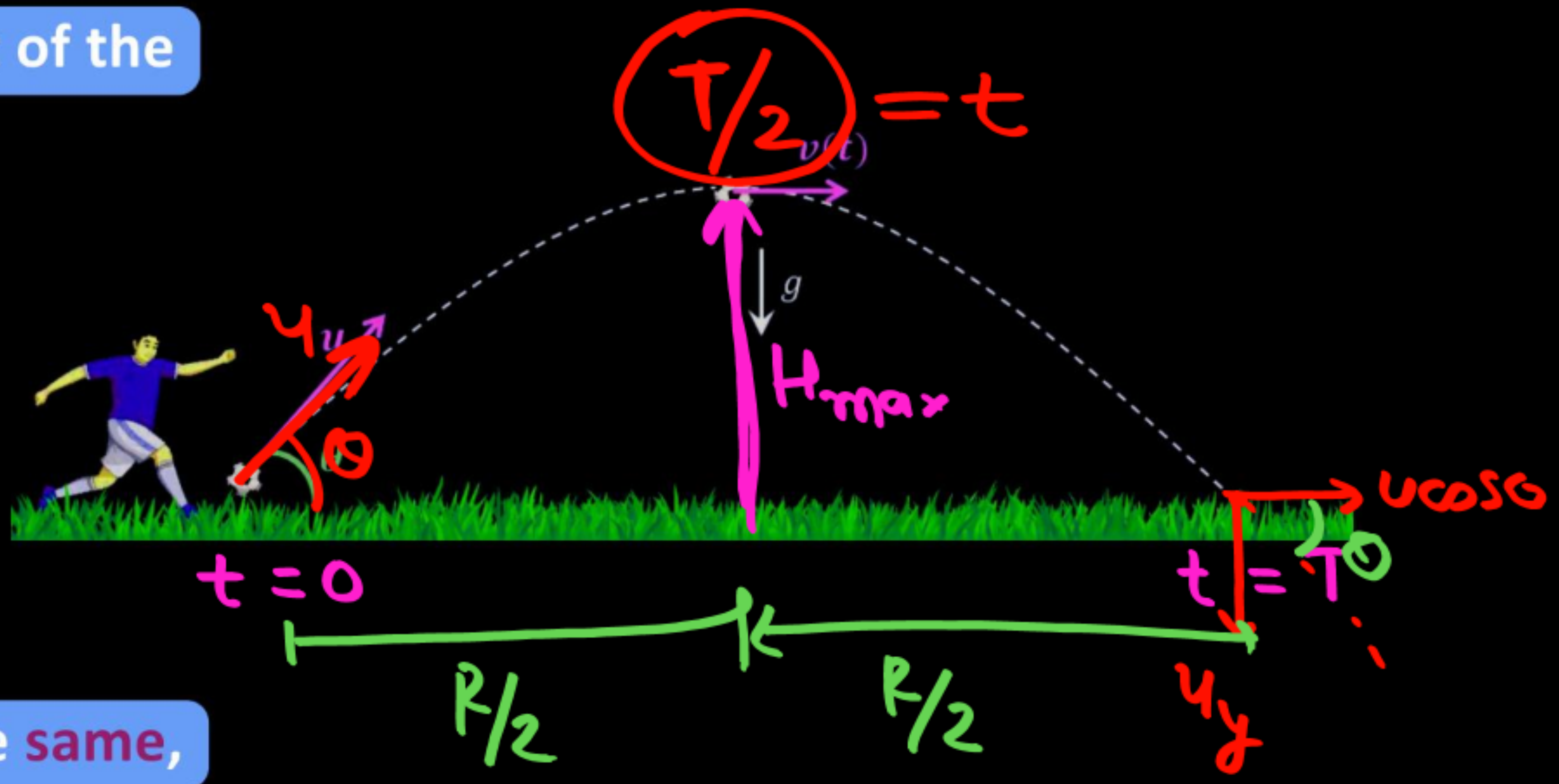
$$x = u_x t$$

$$R = u_x\left(\frac{2u_y}{g}\right) = \frac{2u\cos\theta \cdot u\sin\theta}{g}$$

$$\boxed{R = \frac{2u_x u_y}{g}} \quad \boxed{= \frac{u^2\sin 2\theta}{g}}$$

Symmetry in Projectile Motion

- Maximum height occurs halfway through the flight of the projectile.
- Launch angle is symmetric with Landing angle.
- Time of ascent = Time of descent
- Path is symmetric about vertical line at R/2.
- On the same horizontal level vertical velocity is the same, only the direction is reversed.



$$H_{max} = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g}$$

$$T = \frac{2u_y}{g}$$

$$R = \frac{u^2\sin 2\theta}{g} = \frac{2u_x u_y}{g}$$

sp. case

① $R \to Max$

$$\sin 2\theta = 1$$

$$2\theta = 90^\circ$$

$$\boxed{\theta = 45^\circ}$$

$$\boxed{\underset{(\theta=45^\circ)}{R_{max}} = \frac{u^2}{2g}}$$

$$\boxed{H_{max} = \frac{u^2}{2g}\left(\frac{1}{2}\right) = \frac{u^2}{4g} = \frac{R_{max}}{4}}$$

sp. case

② Range = H_{max}

$$\frac{2\,\cancel{u}\cos\theta\,\cancel{u}\sin\theta}{\cancel{g}} = \frac{\cancel{u^2}\sin^{\cancel{2}}\theta}{2\cancel{g}}$$

$$4 = \frac{\sin\theta}{\cos\theta}$$

$$\boxed{\tan\theta = 4}$$

$$H_{max} = \frac{u_y^2}{2g}$$

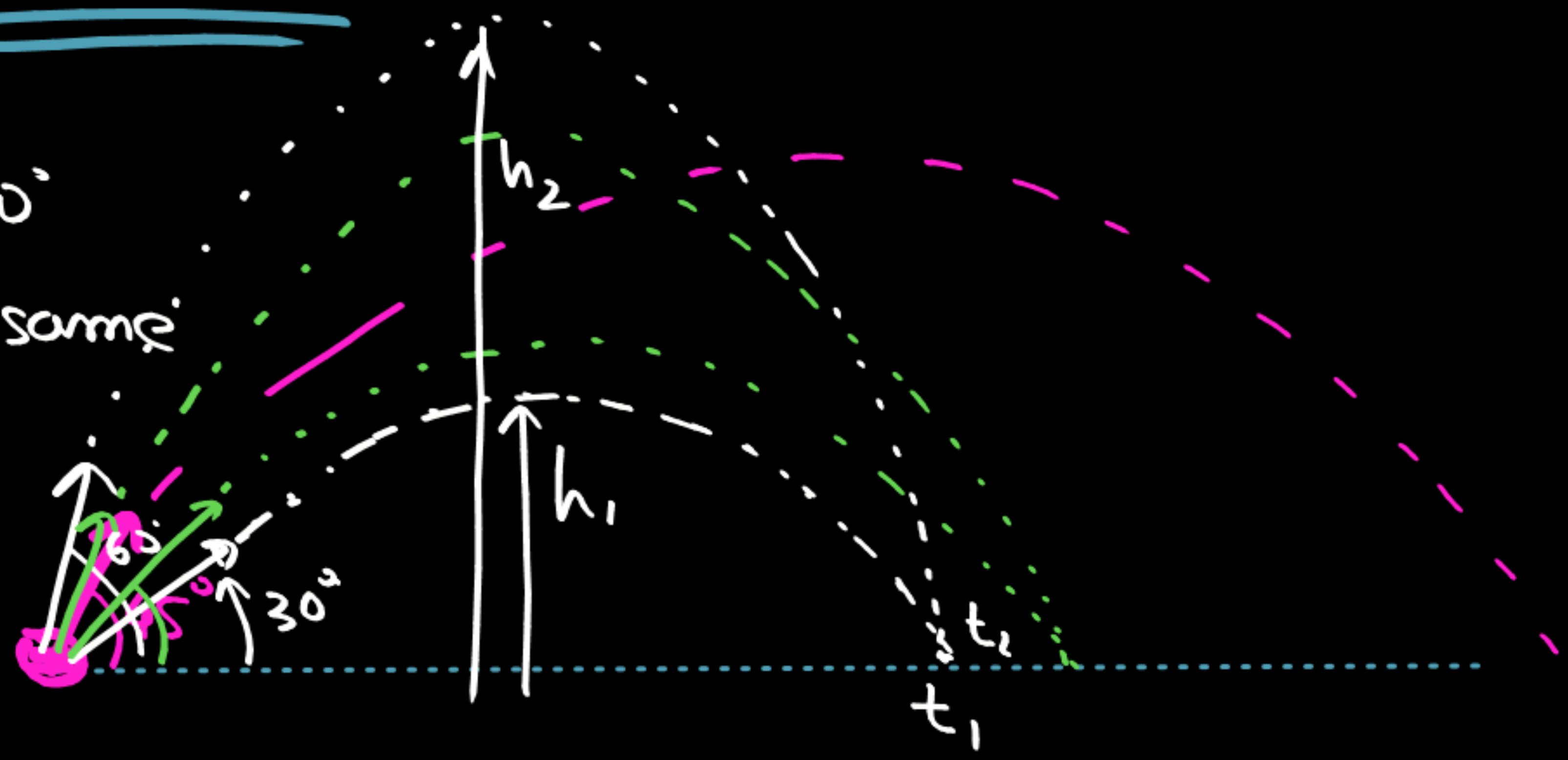
$$T = \frac{2u_y}{g}$$

$$R = \frac{u^2 \sin 2\theta}{g} = \frac{2u_x u_y}{g}$$

KHAS BAAT

$\theta_1 + \theta_2 = 90^\circ$

$\Rightarrow$ Range = same



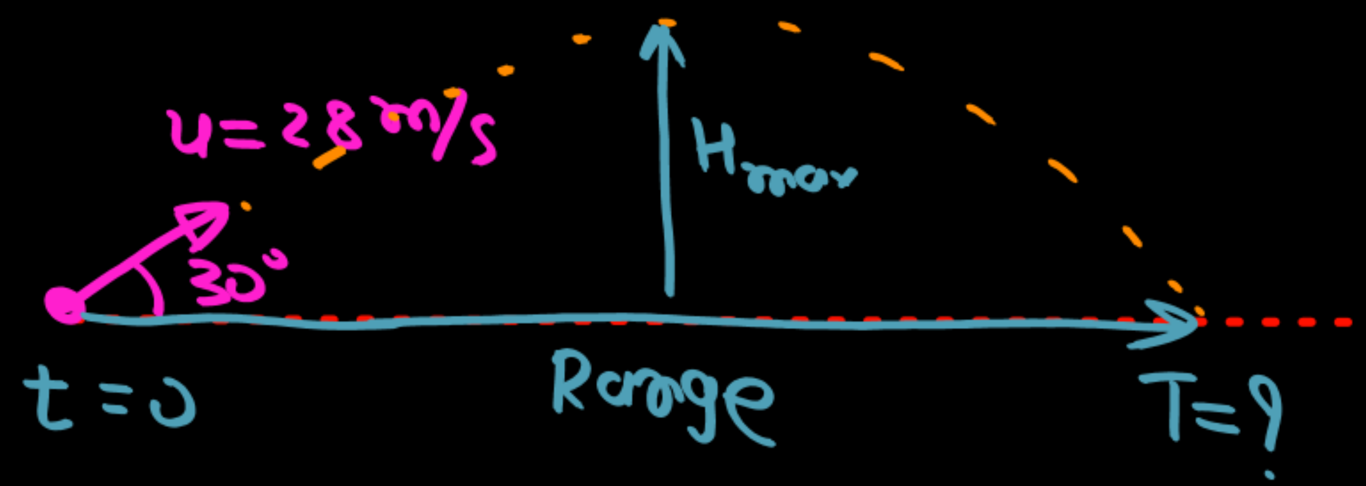
$$\left.\begin{array}{l}\theta_1 = 37^\circ \\ \theta_2 = 53^\circ\end{array}\right\} R_1 = R_2$$

$$\left.\begin{array}{l}\overset{h_1}{\theta_1} = 30^\circ \xrightarrow{} t_1 \\ \underset{h_2}{\theta_2} = 60^\circ \xrightarrow{} t_2\end{array}\right\} R_1 = R_2$$

- Range $= R = 4\sqrt{h_1 h_2}$
- $t_1 \cdot t_2 = \frac{2R}{g}$

Example 3.8

A cricket ball is thrown at a speed of 28 m s^{-1} in a direction 30° above the horizontal. Calculate (a) the maximum height, (b) the time taken by the ball to return to the same level, and (c) the distance from the thrower to the point where the ball returns to the same level.



$$\Rightarrow H_{max} = \frac{u_y^2}{2g} = \frac{(28)^2(\sin 30)^2}{2g}$$

$$\Rightarrow T = \frac{2u_y}{g} = \frac{2(28)\sin 30^\circ}{g}$$

$$\Rightarrow R = \frac{(28)^2 \sin(60^\circ)}{g}$$

The maximum height reached by a projectile is 64 m. If the initial velocity is halved, the new maximum height of the projectile is ___________ m.

$$H_{max} = 64 \text{ m} = \frac{u^2 \sin^2\theta}{2g}$$

$$u \rightarrow u/2$$

$$H'_{max} = \frac{\left(\frac{u}{2}\right)^2 \sin^2\theta}{2g} = \frac{u^2 \sin^2\theta}{4 \times 2g} = \frac{64}{4} = 16 \text{ m}$$

Question

The maximum height reached by a projectile is 64 m. If the initial velocity is halved, the new maximum height of the projectile is __________ m. [05 April, 2024 (Shift-II)]

Ans. 16

The angle of projection for a projectile to have same horizontal range and maximum height is:

(a) $\tan^{-1}(2)$

(b) $\tan^{-1}(4)$

(c) $tan^{-1}(\frac{1}{4})$

(d) $tan^{-1}(\frac{1}{2})$

$H_{max} = \text{Range} \Rightarrow \tan\theta = 4$

$\Rightarrow \theta = \tan^{-1}(4)$

Question

The angle of projection for a projectile to have same horizontal range and maximum height is:

[08 April, 2024 (Shift-II)]

(a) $\tan^{-1}(2)$

(b) $\tan^{-1}(4)$

(c) $\tan^{-1}(\frac{1}{4})$

(d) $\tan^{-1}(\frac{1}{2})$

Ans. $\tan^{-1}(4)$

The range of the projectile projected at an angle of 15° with horizontal is 50 m. If the projectile is projected with same velocity at an angle of 45° with horizontal, then its range will be:

(a) 50 m

(b) $50\sqrt{2}\,m$

(c) 100 m

(d) $100\sqrt{2}\,m$

$$R_1 = 50 = \frac{u^2 \sin(2\times 15)}{g}$$

$$R_2 = \frac{u^2 \sin(2\times 45^\circ)}{g}$$

$$\frac{R_2}{R_1} = \frac{\sin(90)}{\sin 30^\circ} = \frac{1}{(1/2)}$$

$$R_2 = R_1 \times 2 = 100\,m$$

Question

The range of the projectile projected at an angle of 15° with horizontal is 50 m. If the projectile is projected with same velocity at an angle of 45° with horizontal, then its range will be:

[10 April, 2023 (Shift-I)]

(a) 50 m

(b) $50\sqrt{2}\,m$

(c) 100 m

(d) $100\sqrt{2}\,m$

Ans. (c) 100 m

Two projectiles are projected at 30 deg and 60 deg with the horizontal with the same speed. The ratio of the maximum height attained by the two projectiles respectively is:

(a) $2:\sqrt{3}$

(b) $\sqrt{3}:1$

(c) $1:3$

(d) $1:\sqrt{3}$

Question

Two projectiles are projected at 30 deg and 60 deg with the horizontal with the same speed. The ratio of the maximum height attained by the two projectiles respectively is: [10 April, 2023 (Shift-II)]

(a) $2:\sqrt{3}$

(b) $\sqrt{3}:1$

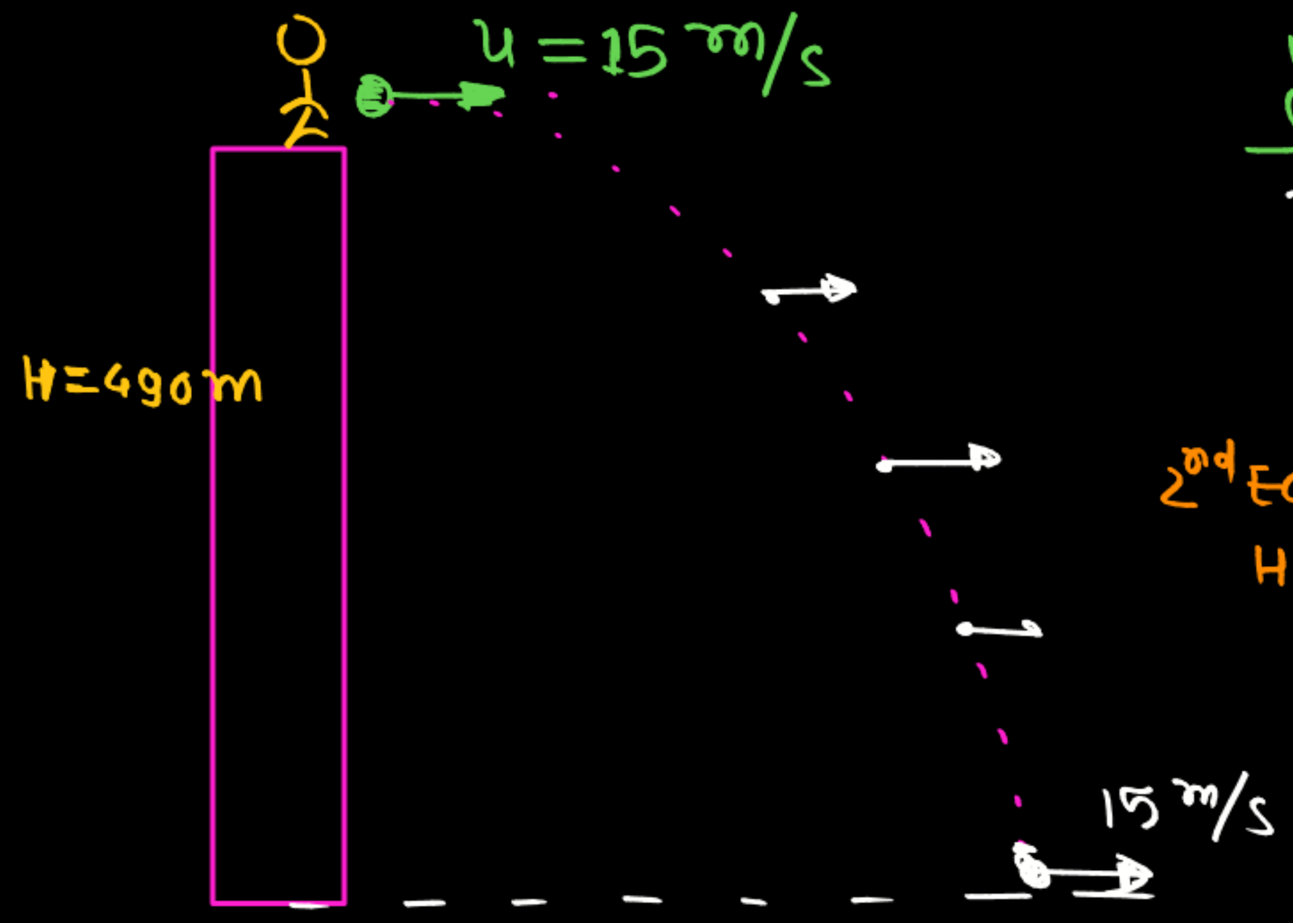
(c) 1 : 3

(d) $1:\sqrt{3}$

Ans. (c) 1 : 3

Example 3.7

A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of 15 m s^{-1}. Neglecting air resistance, find the time taken by the stone to reach the ground, and the speed with which it hits the ground. (Take g = 9.8m s^{-2}) .



y-axis

$u_y = 0$

$a_y = g$

2nd EOM :-

$H = 0 + (g) t^2$

x-axis

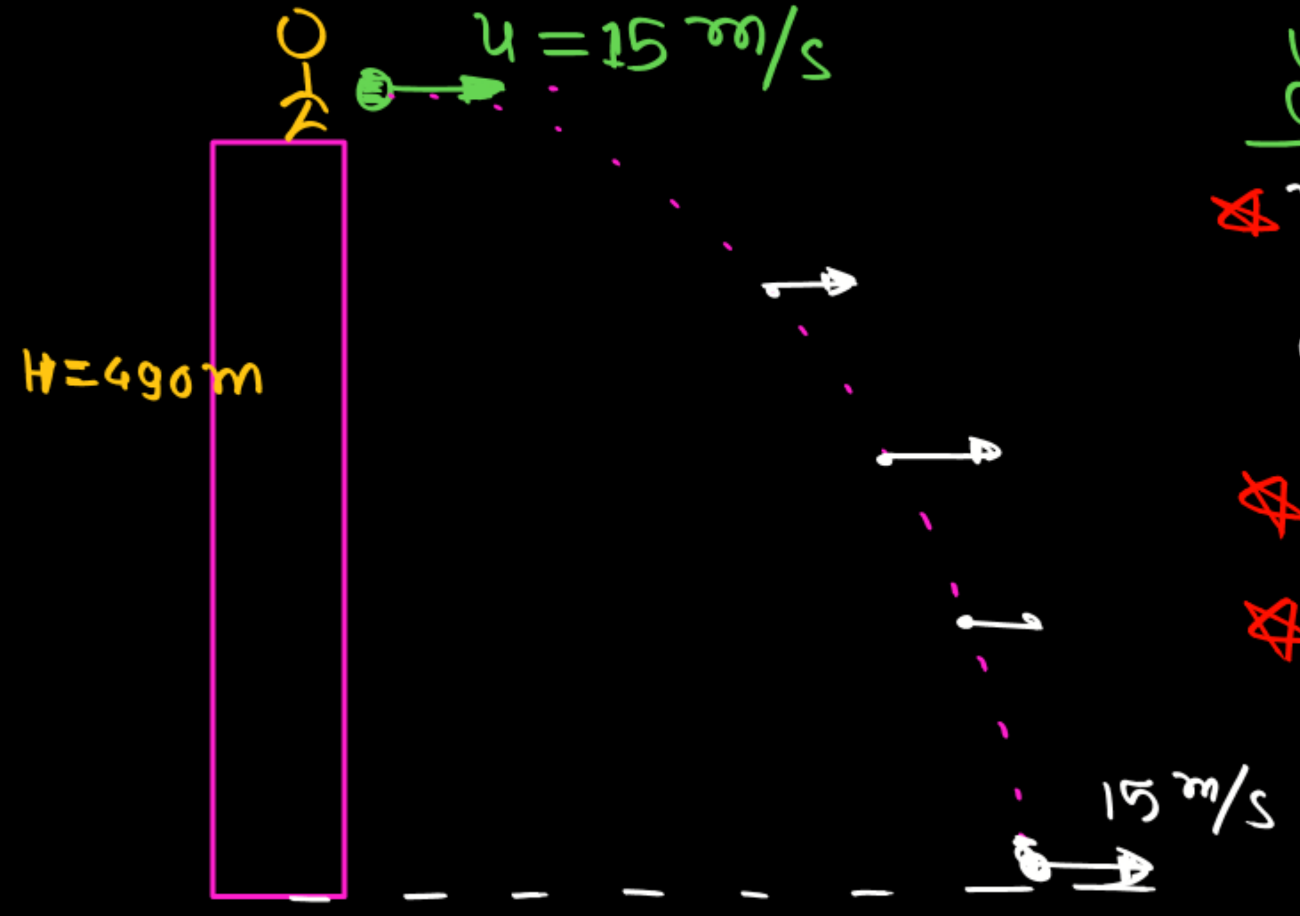
$u_x = 15$ m/s

$a_x = 0$

$\boxed{V_x = 15 \text{ m/s} = \text{const.}}$

Example 3.7

A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of 15 m s^{-1}. Neglecting air resistance, find the time taken by the stone to reach the ground, and the speed with which it hits the ground. (Take g = 9.8m s^{-2}) .



y-axis

$u_y = 0$

$a_y = g$

$T = ?$

$v_{net} = ? = \sqrt{V_x^2 + V_y^2}$

2nd EOM

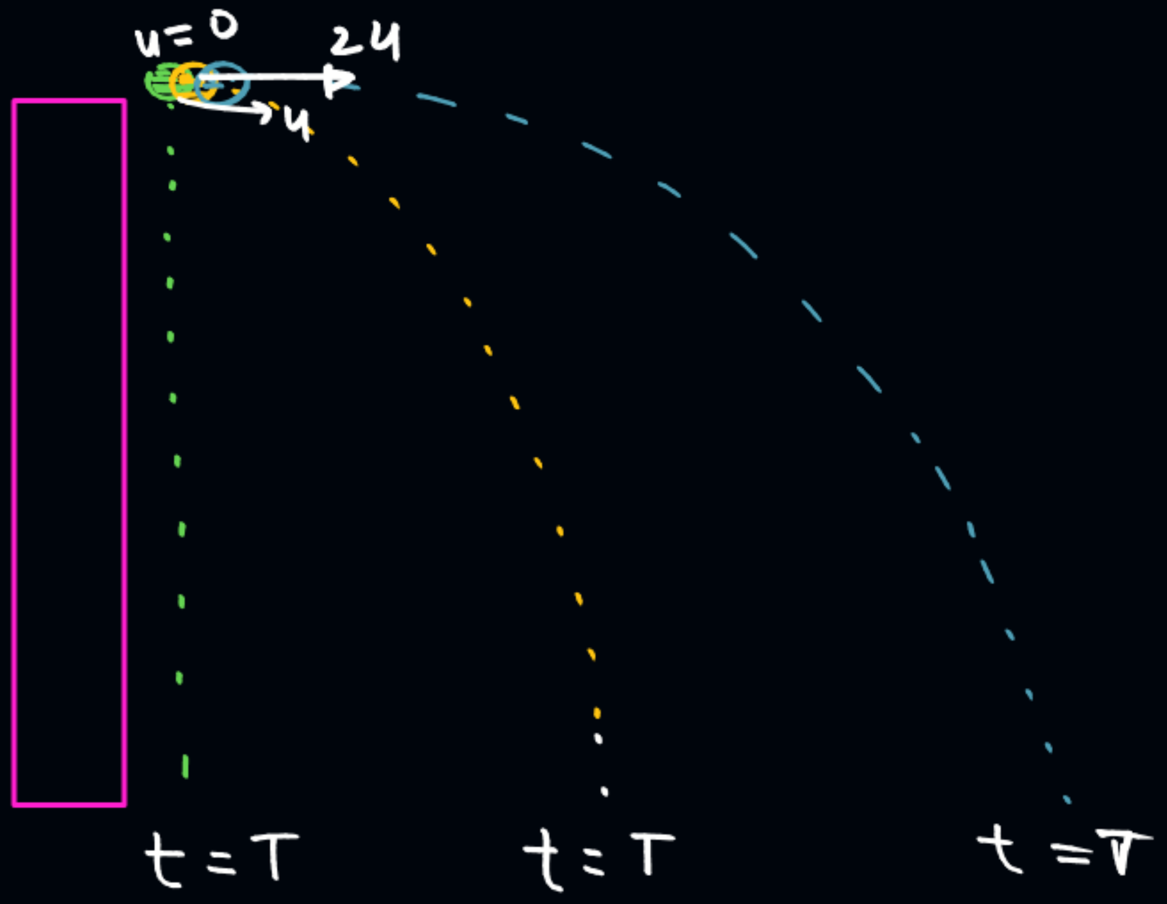
$-490 = 0 - \frac{1}{2}(9.8)t^2 \Rightarrow \boxed{t = \quad}$

3rd EOM

$v_{y_F}^2 - \cancel{v_{y_i}^2} = 2(-9.8)(-490)$

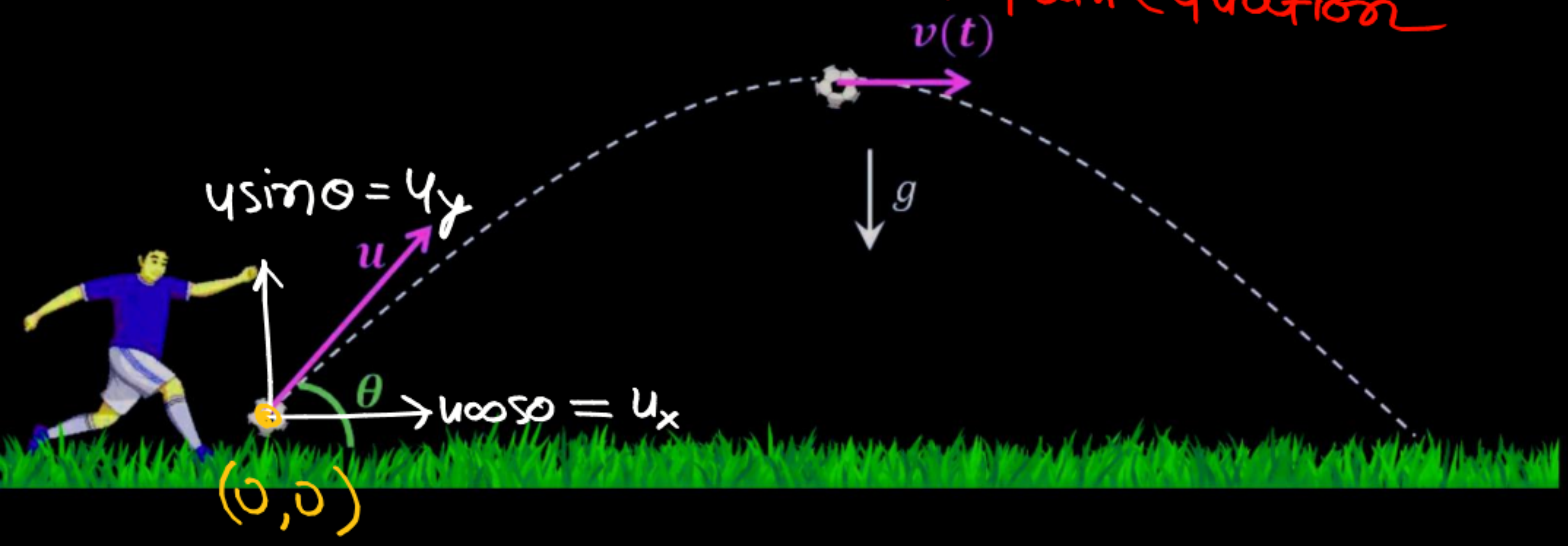
$\boxed{v_y = \quad}$

Horizontal projectile Motion



Equation of Trajectory

→ Relation b/t x & y
→ path equation



x	y
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = 0$	$a_y = g$

$\boxed{2^{nd}\ \text{EOM}}$

$$x = u_x t$$

$$\boxed{t = \frac{x}{u_x}}$$

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$y = u_y\left(\frac{x}{u_x}\right) - \frac{g}{2}\left(\frac{x^2}{u_x^2}\right)$$

$$y = \frac{(u\sin\theta)\,x}{(u\cos\theta)} - \frac{g x^2}{2u^2\cos^2\theta}$$

$$\boxed{y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta}}$$

EOT :-

$$\boxed{y = x\tan\theta - \frac{g x^2}{2u^2\cos^2\theta}}$$

$$\text{Range} = \frac{2u_x u_y}{g} = \frac{2(u\cos\theta)(u\sin\theta)}{g}$$

$$y = x\tan\theta - \frac{x^2\,(\sin\theta)}{\frac{2(u\cos\theta)(u\sin\theta)}{g}\cos\theta}$$

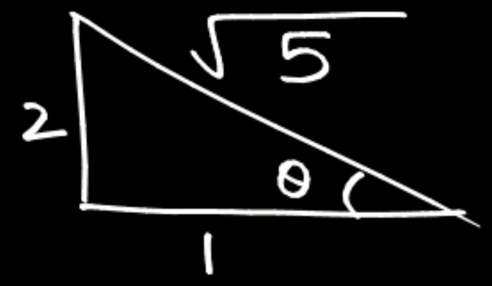
$$y = x\tan\theta - \frac{x^2\tan\theta}{R} \Rightarrow \boxed{y = x\tan\theta\left(1 - \frac{x}{R}\right)}$$

The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then ($g = 10\ \text{ms}^{-2}$): **[12 April 2019 I]**

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

$$y = 2x - 9x^2$$

$$\tan\theta = \frac{2}{1} \Rightarrow$$



$$\cos\theta = \frac{1}{\sqrt{5}}$$

$$\frac{g}{2u^2\cos^2\theta} = 9$$

$$\Downarrow$$

$u = ✓$

Question

A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$ where K is a constant. The general equation for its path is:

(a) $y = x^2$ + constant

(b) $y^2 = x$ + constant

(c) $y^2 = x^2$ + constant

(d) xy = constant

$\Rightarrow \vec{v} = \boxed{ky}\hat{i} + \boxed{kx}\hat{j}$

$\boxed{ky} = V_x$, $\boxed{kx} = V_y$

$V_x = \frac{dx}{dt} = ky$ | $V_y = \frac{dy}{dt} = kx$

$\Rightarrow \frac{dy/\cancel{dt}}{dx/\cancel{dt}} = \frac{\cancel{k}x}{\cancel{k}y}$

$\frac{dy}{dx} = \frac{x}{y}$

$\int y \cdot dy = \int x \cdot dx$

$\frac{y^2}{2} = \frac{x^2}{2} + C$

$y^2 = x^2 + 2C \rightarrow$ const.

Question

A particle is moving with a velocity $\vec{v} = K(y\hat{i} + x\hat{j})$ where K is a constant. The general equation for its path is: [9 Jan, 2019 (Shift-I)]

(a) $y = x^2$ + constant

(b) $y^2 = x$ + constant

(c) $y^2 = x^2$ + constant

(d) xy = constant

Ans. (c) $y^2 = x^2$ + constant

Question

Motion of a particle in x - y plane is described by a set of following equations $x = 4\sin\left(\frac{\pi}{2} - \omega t\right) m$ and $y = 4\sin(\omega t)\text{m}$. The path of the particle will be:

(a) Circular

(b) Helical

(c) Parabolic

(d) Elliptical

$\Rightarrow y = 4\sin\omega t$

$\Rightarrow \sin\omega t = y/4$

$x = 4\sin(90^\circ - \omega t)$

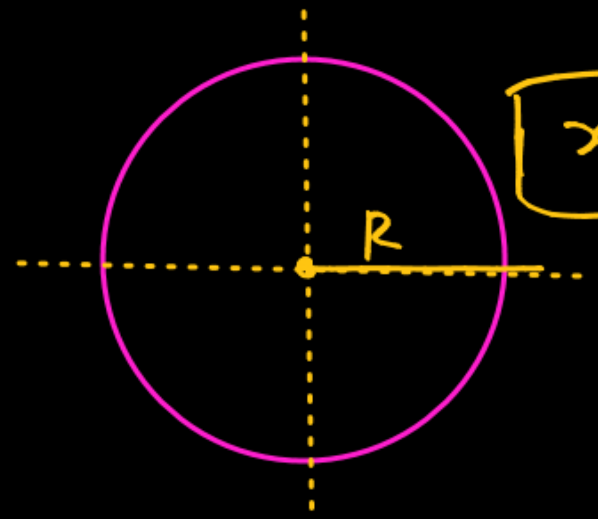
$x = 4\cos\omega t$

$\Rightarrow \cos\omega t = x/4$

B: $\Rightarrow \sin^2 \omega t + \cos^2 \omega t = 1$

$$\frac{y^2}{4^2} + \frac{x^2}{4^2} = 1$$

$$y^2 + x^2 = 4^2$$



$\boxed{x^2 + y^2 = R^2}$

Question

Motion of a particle in x - y plane is described by a set of following equations $x = 4\sin\left(\frac{\pi}{2} - \omega t\right) m$ and $y = 4\sin(\omega t)$m .The path of the particle will be: [28 June, 2022 (Shift-I)]

(a) Circular

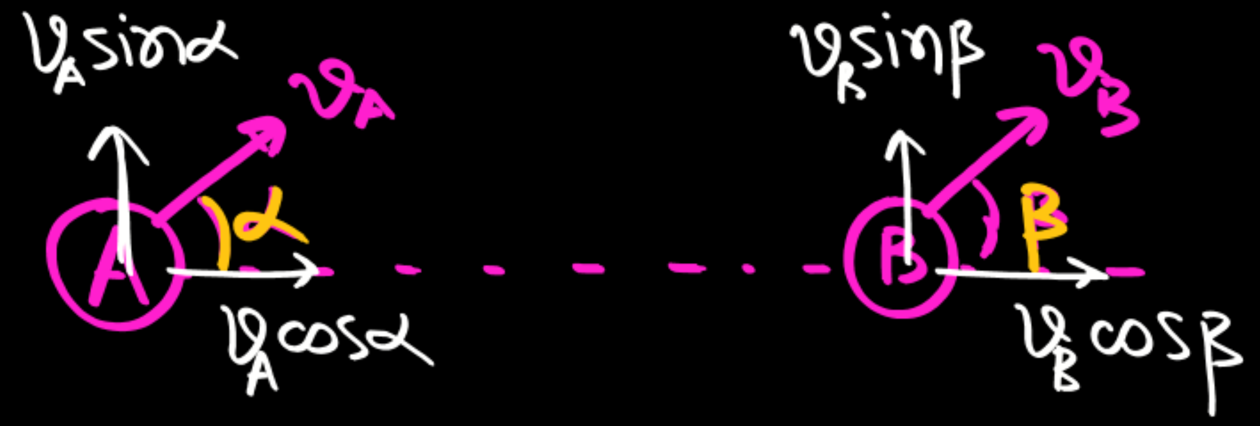
(b) Helical

(c) Parabolic

(d) Elliptical

Ans. Circular

Relative Motion



$\vec{v}_A = v_A\cos\alpha\,\hat{i} + v_A\sin\alpha\,\hat{j}$

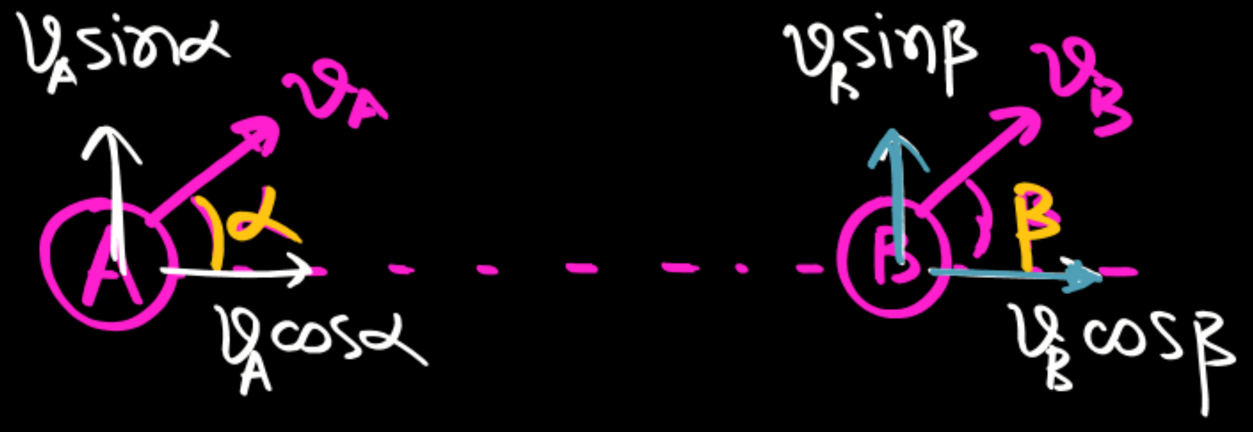
$\vec{v}_B = v_B\cos\beta\,\hat{i} + v_B\sin\beta\,\hat{j}$

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$ = Velocity of B wrt A

$= (v_B\cos\beta - v_A\cos\alpha)\,\hat{i}$

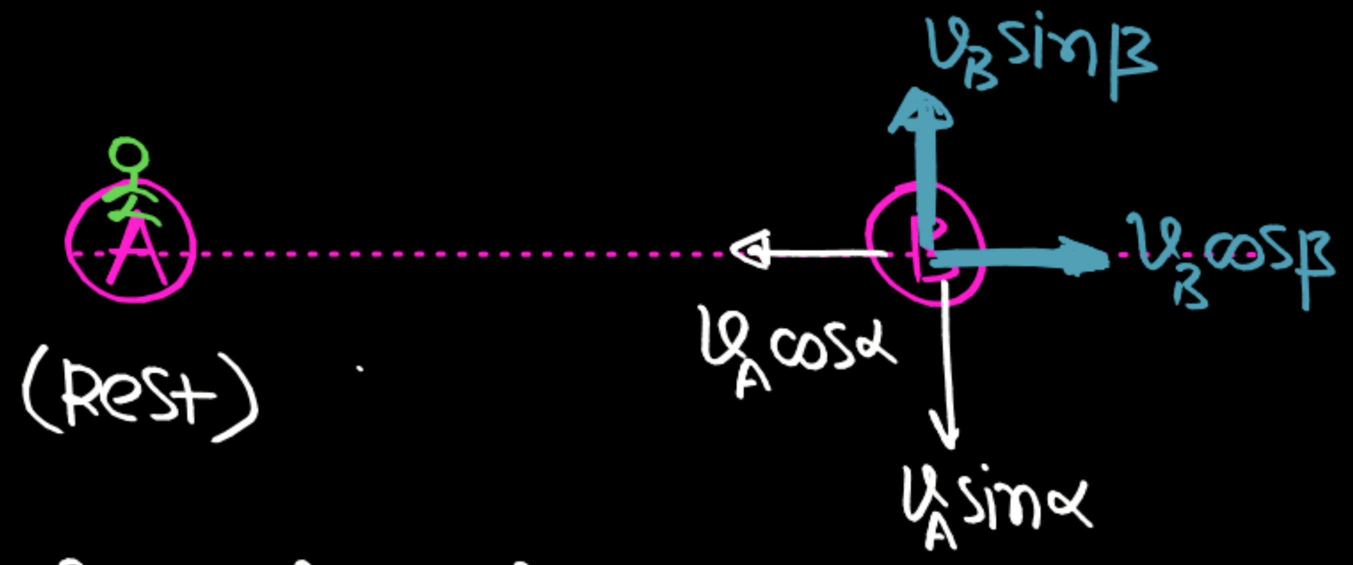
$+ (v_B\sin\beta - v_A\sin\alpha)\,\hat{j}$

Relative Motion



$\overrightarrow{v}_{BA} = \overrightarrow{v}_B - \overrightarrow{v}_A$ = velocity of B wrt A

wrt A → Rest



wrt A

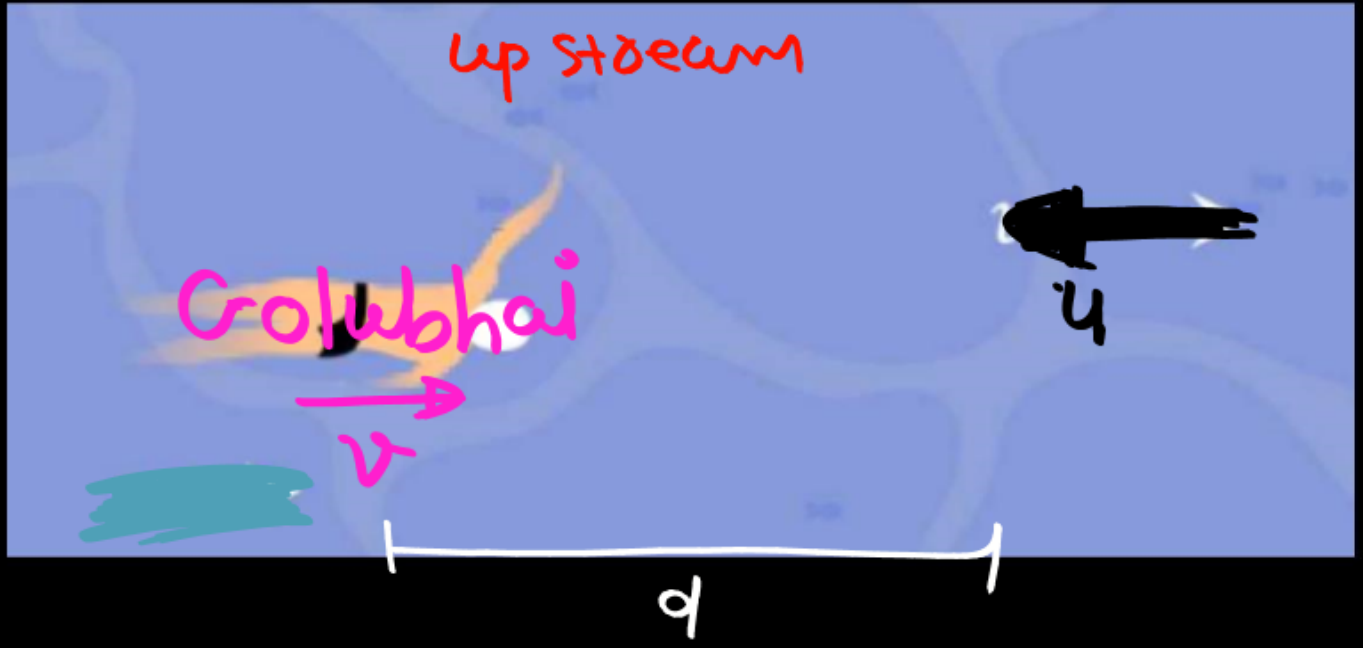
$$\begin{cases} v_{B(x)} = v_B \cos\beta - v_A \cos\alpha \\ v_{B(y)} = v_B \sin\beta - v_A \sin\alpha \end{cases}$$

River -Man Problem

v = velocity of Golubhai in still water

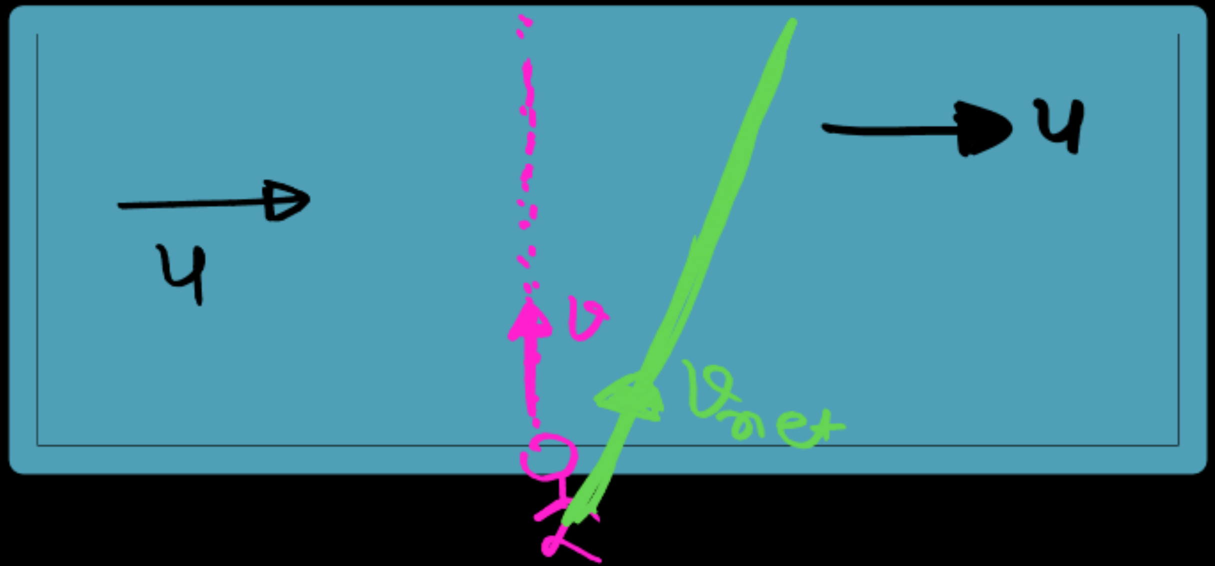
u = velocity of river wrt ground

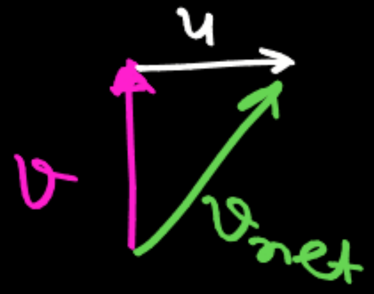
$$t = \frac{d}{v+u}$$



$$t = \frac{d}{v-u}$$

Crossing The River



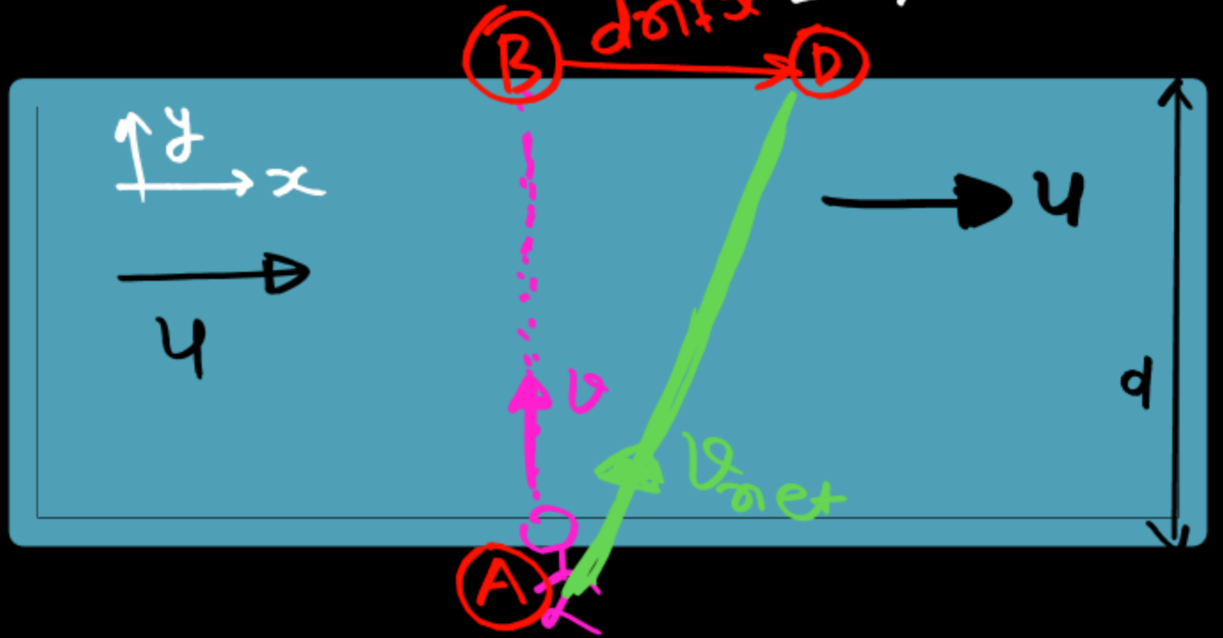


v = Golubhai चाहता है = $v_{\text{Golu wrt River}}$

v_{net} = Velocity Golubhai wrt ground = चला जाता है।

Crossing The River

t_{min}



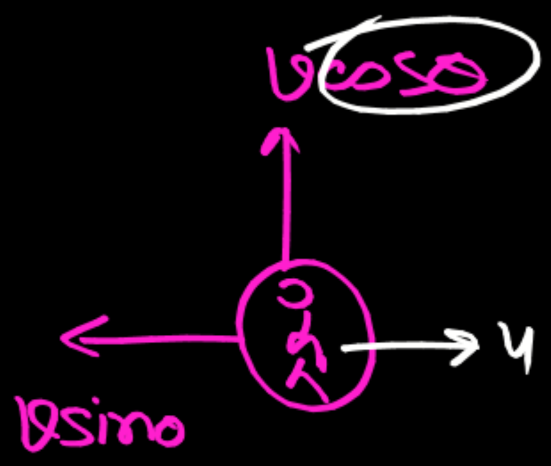
x	y
u	v
$\cancel{drift} = u_x t$	$y = d$

$$\boxed{t = \frac{d}{v}}$$

$$\boxed{drift = u\left(\frac{d}{v}\right)}$$

Crossing the River:

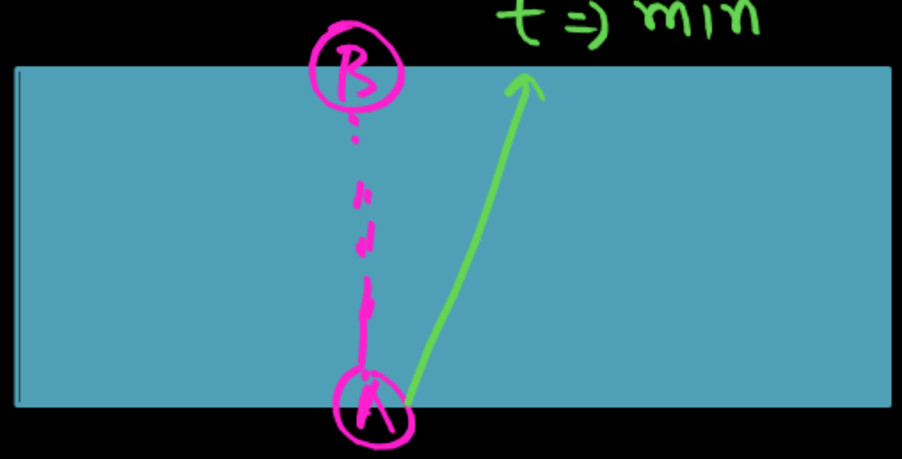
Minimum Time



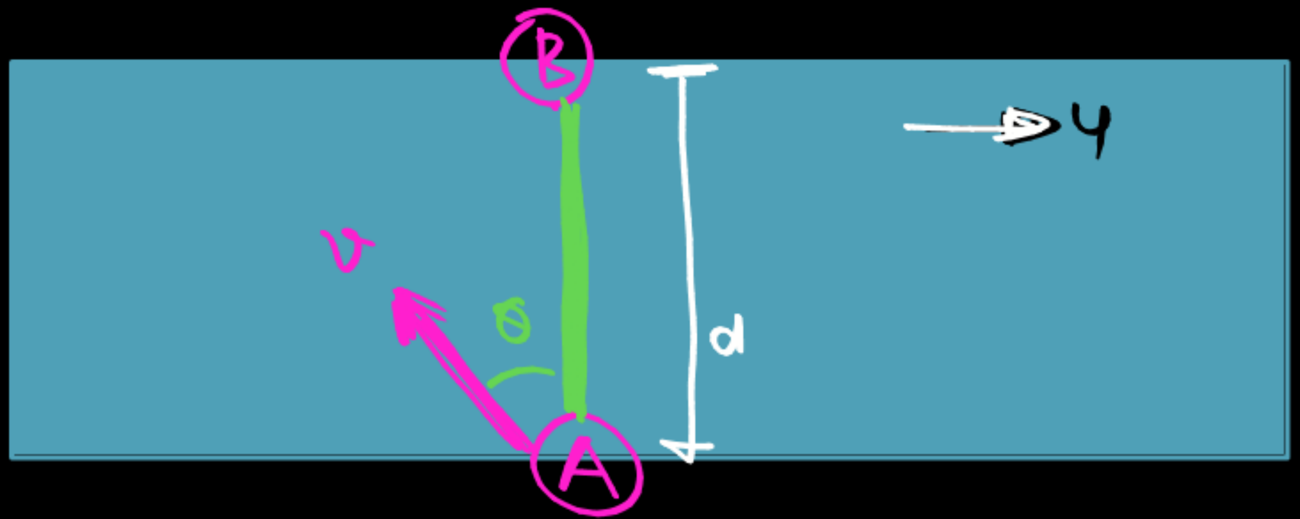
$$t = \frac{d}{v\cos\theta}$$

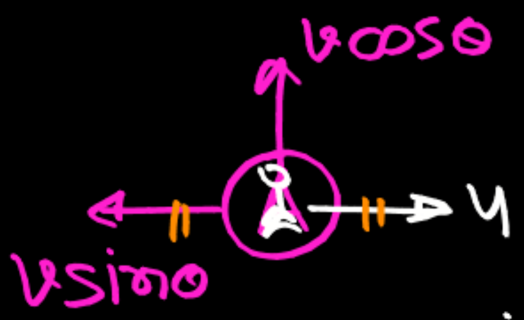
$$\cos\theta = 1$$

$$\theta = 0^\circ$$



Crossing the River without Drift





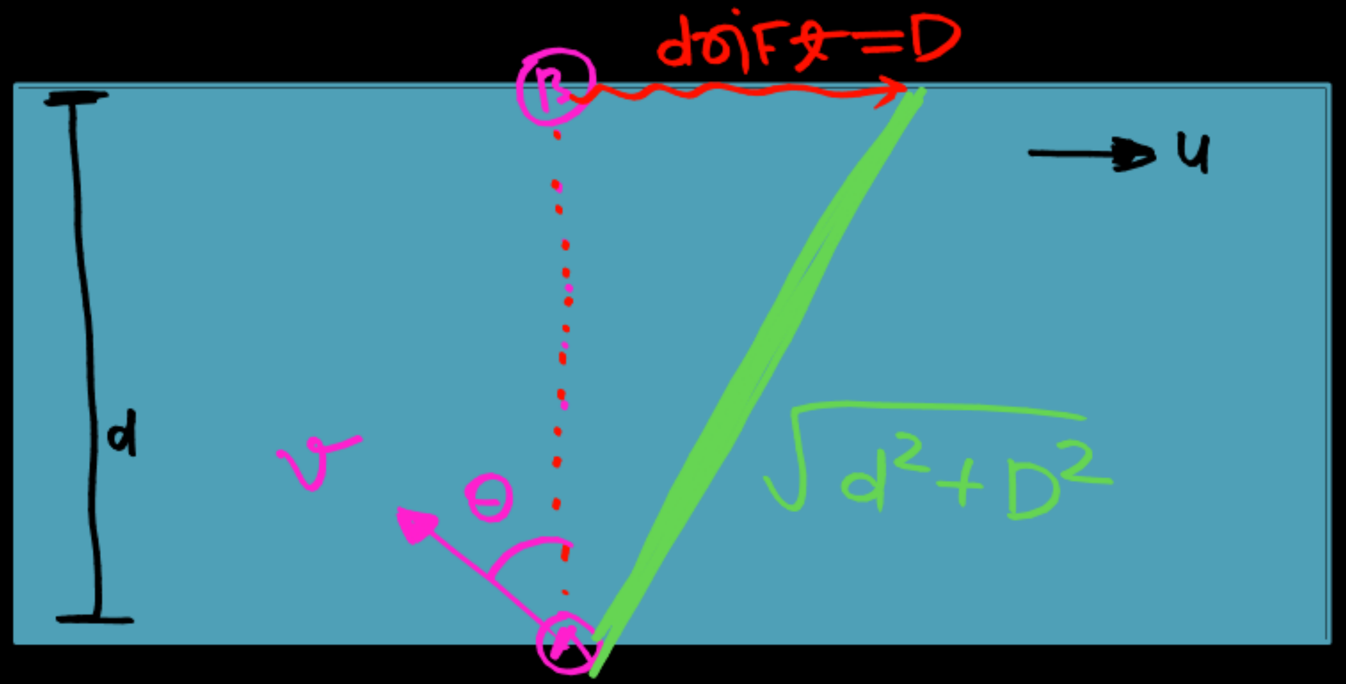
condition :-

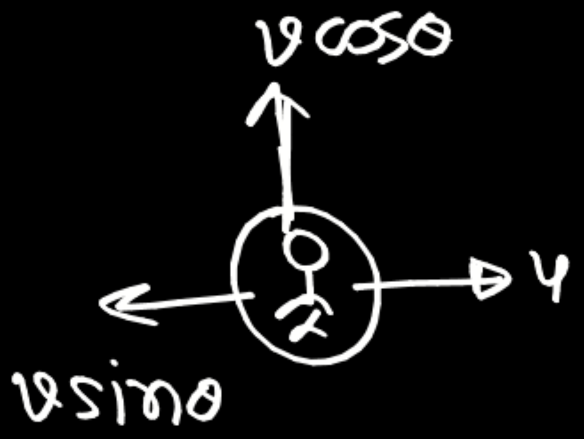
$\Rightarrow u = v\sin\theta$

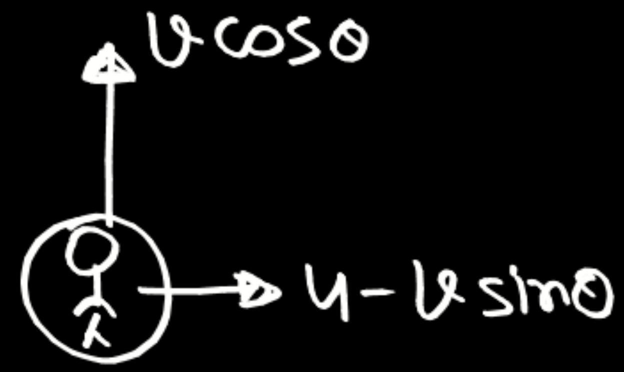
$$\boxed{\sin\theta = \frac{u}{v}}$$

$$\boxed{t_{A\to B} = \frac{d}{v\cos\theta}}$$

$u > v$







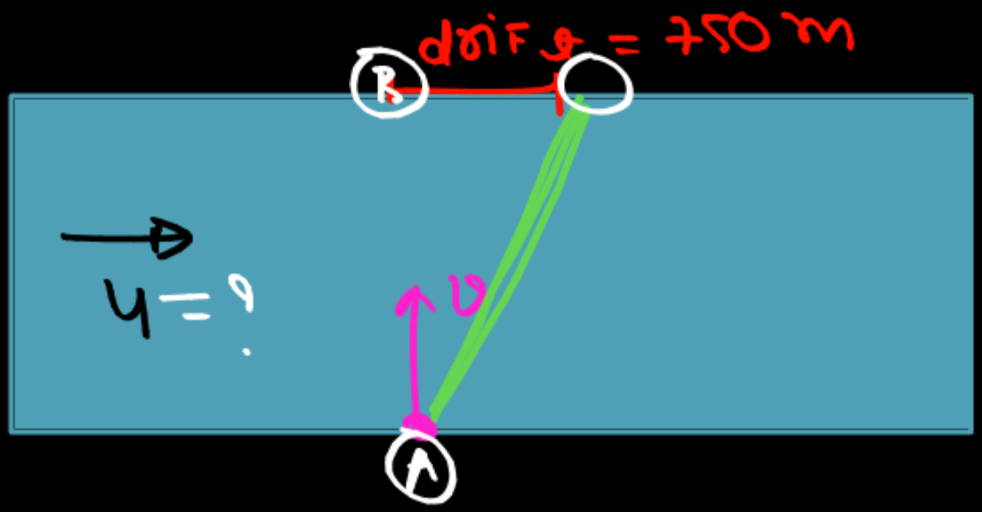
$$t_{A\to B} = \frac{d}{v\cos\theta}$$

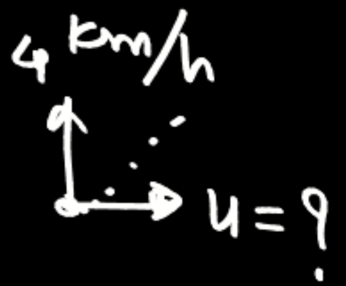
$$\text{drift} = v_x t$$

$$\text{drift} = (u - v\sin\theta)\left(\frac{d}{v\cos\theta}\right)$$

The speed of a swimmer is 4 km h^{-1} in still water. If the swimmer makes his strokes normal to the flow of river of width 1 km, he reaches a point 750 m down the stream on the opposite bank. The speed of the river water is __________ km h^{-1}. [31 Jan, 2023 (Shift-I)]

$= v$ $= d$ $\frac{3}{4}$ km





$$t = \frac{d}{v} = \frac{1}{4} \text{ hr}$$

$$\text{drift} = (u)(t)$$

$$\left(\frac{3}{\cancel{4}}\right) = (u)\left(\frac{1}{\cancel{4}}\right)$$

$$\boxed{u = 3 \text{ km/h}}$$

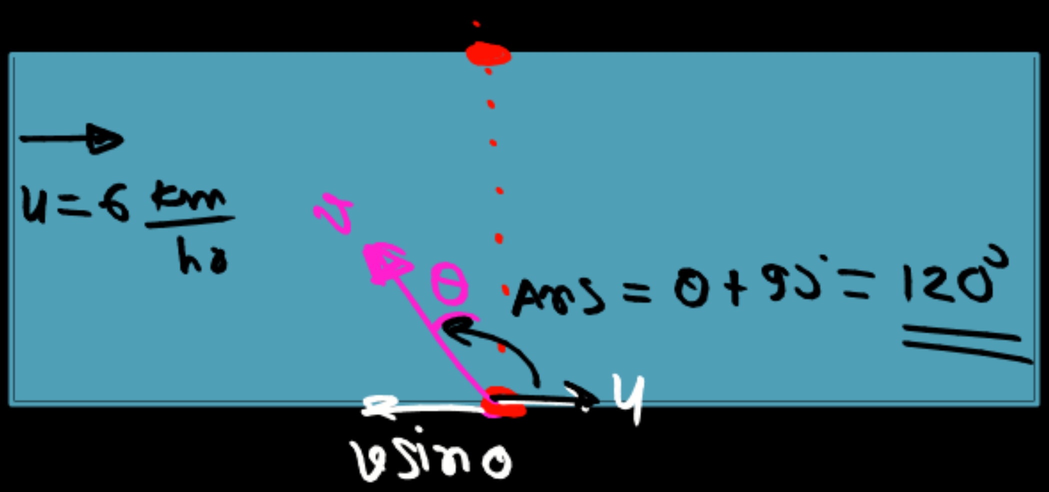
The speed of a swimmer is 4 km h^{-1} in still water. If the swimmer makes his strokes normal to the flow of river of width 1 km, he reaches a point 750 m down the stream on the opposite bank.

The speed of the river water is ___3___ km h^{-1}. [31 Jan, 2023 (Shift-I)]

Ans. 3

A swimmer can swim with velocity of 12 km/h in still water. Water flowing in a river has velocity 6 km/h. The direction with respect to the direction of flow of river water he should swim in order to reach the point on the other bank just opposite to his starting point is ___________°.

(Round off to the Nearest Integer) (Find the angle in degrees)



No drift condition :-

$v\sin\theta = u$

$\sin\theta = \frac{6}{12} = \frac{1}{2}$

$\boxed{\theta = 30°}$

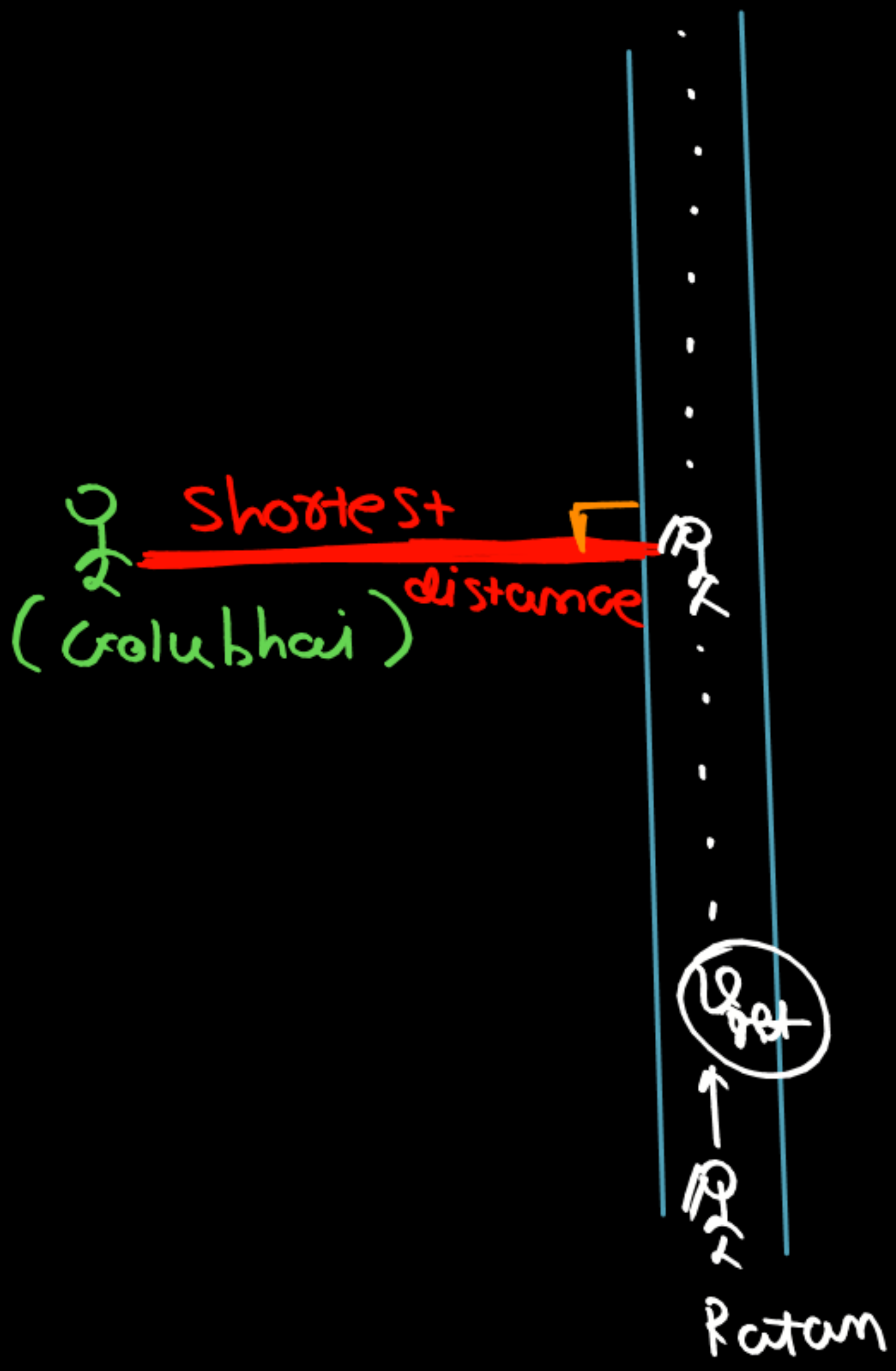
Question

A swimmer can swim with velocity of 12 km/h in still water. Water flowing in a river has velocity 6 km/h. The direction with respect to the direction of flow of river water he should swim in order to reach the point on the other bank just opposite to his starting point is ___________°.

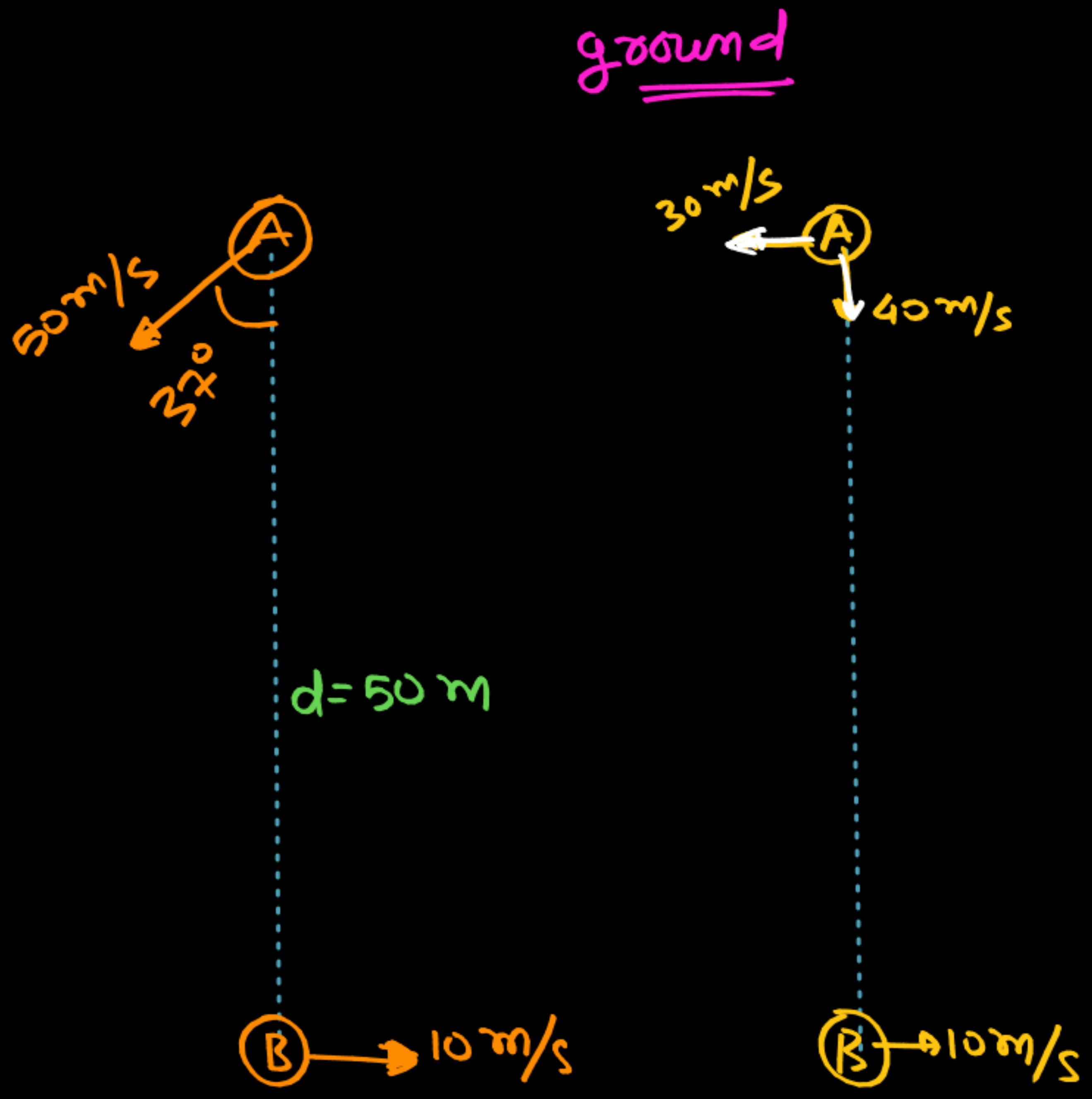
(Round off to the Nearest Integer) (Find the angle in degrees) [16 March, 2021 (Shift-II)]

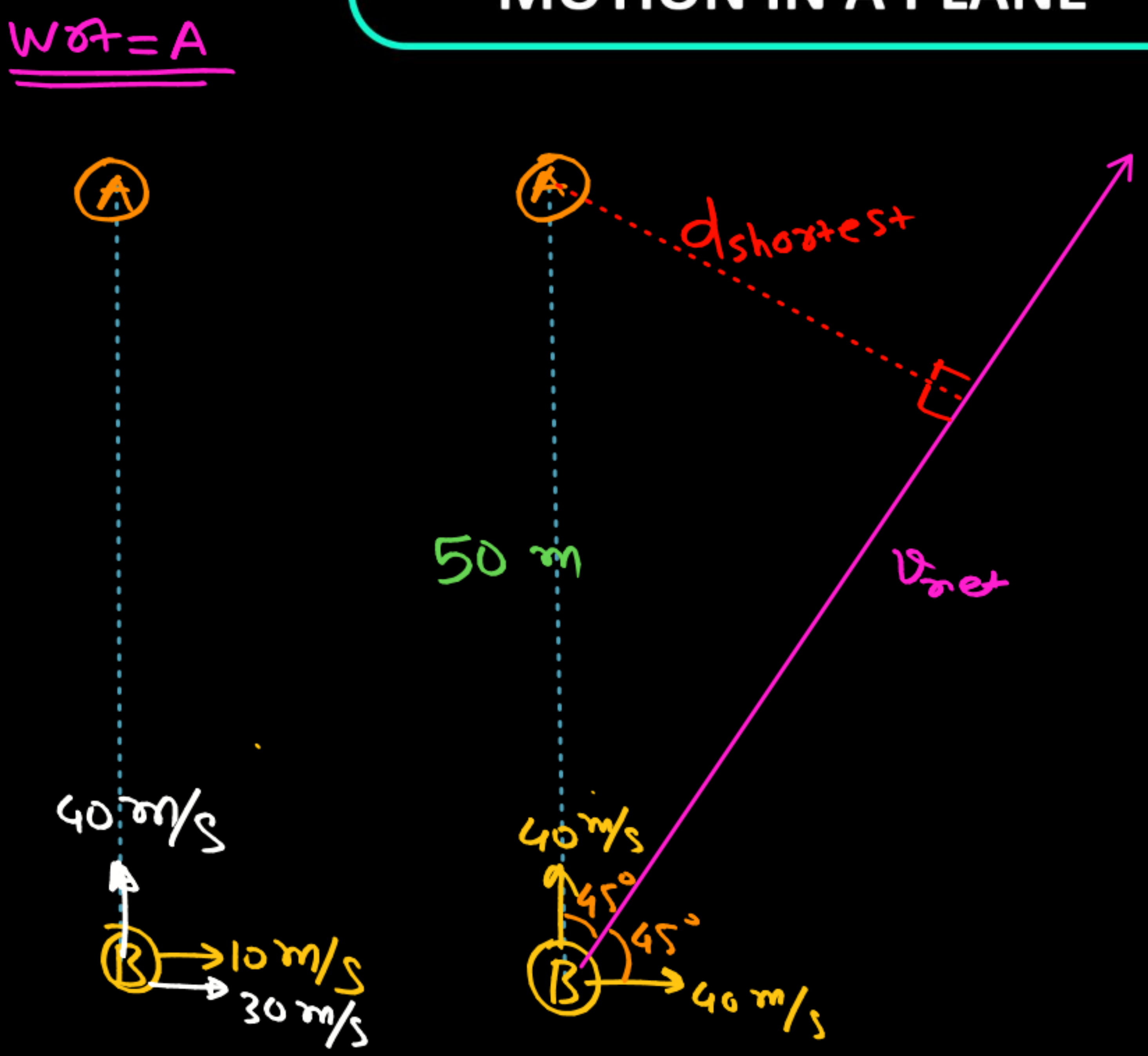
Ans. 120

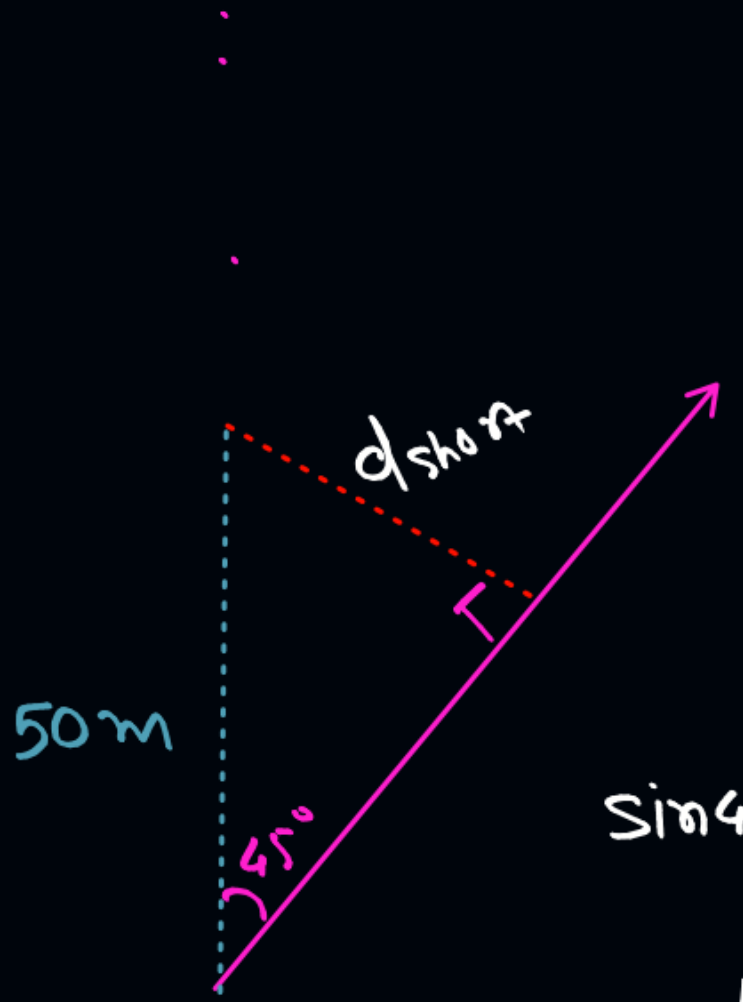
shortest distance b/t two bodies



1. Resolution of velocities
2. कहना किसी एक के Respect
3. V_{net} & its direction
4. wrt wale में $\perp^r$ circle
5. by using trigonometry


ground
50 m/s
37°
A
d = 50 m
B
10 m/s
30 m/s
A
40 m/s
B
10 m/s


w.r.t = A
A
40 m/s
B
10 m/s
30 m/s
A
d shortest
50 m
v rel
40 m/s
45°
45°
B
40 m/s



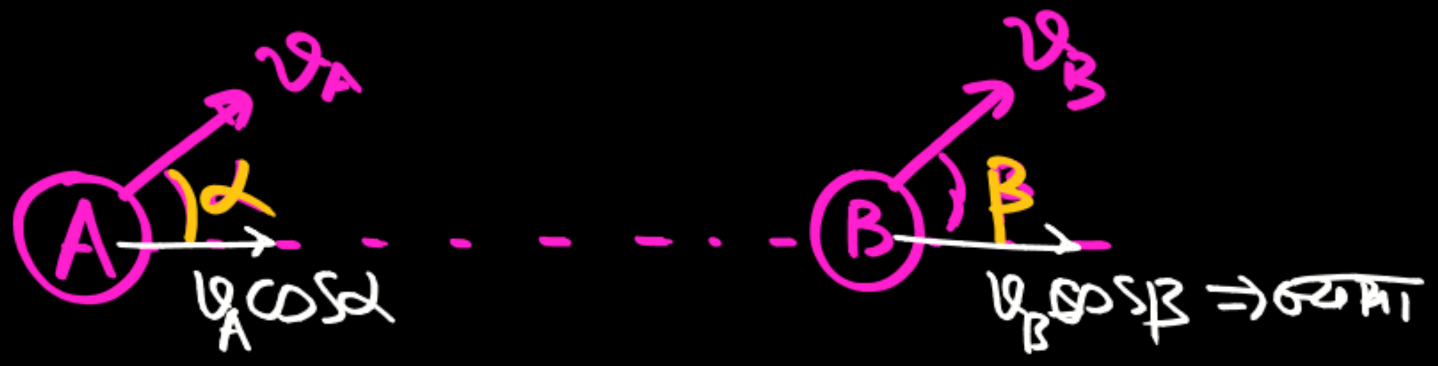
$$\sin 45^\circ = \frac{d_{short}}{50}$$

$$\frac{1}{\sqrt{2}} = \frac{d_{short}}{50}$$

$$\boxed{d_{short} = 25\sqrt{2}\ m}$$

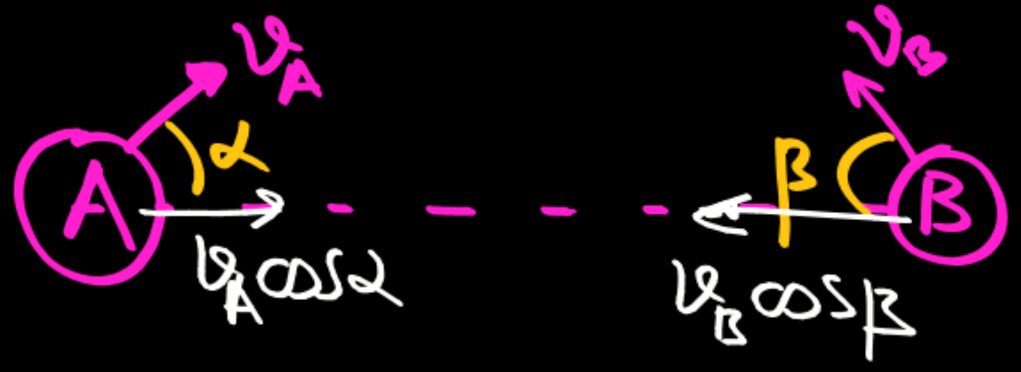
Relative Motion

Velocity of separation

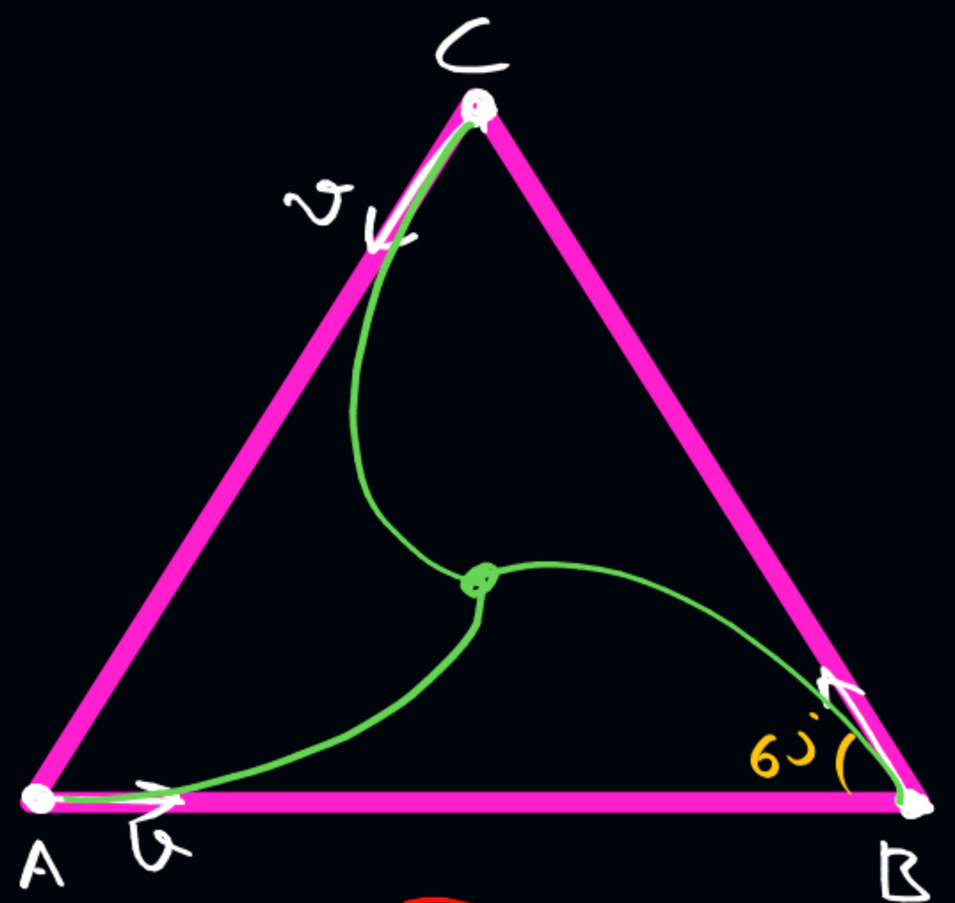


$$v_{sep.} = v_B \cos\beta - v_A \cos\alpha$$

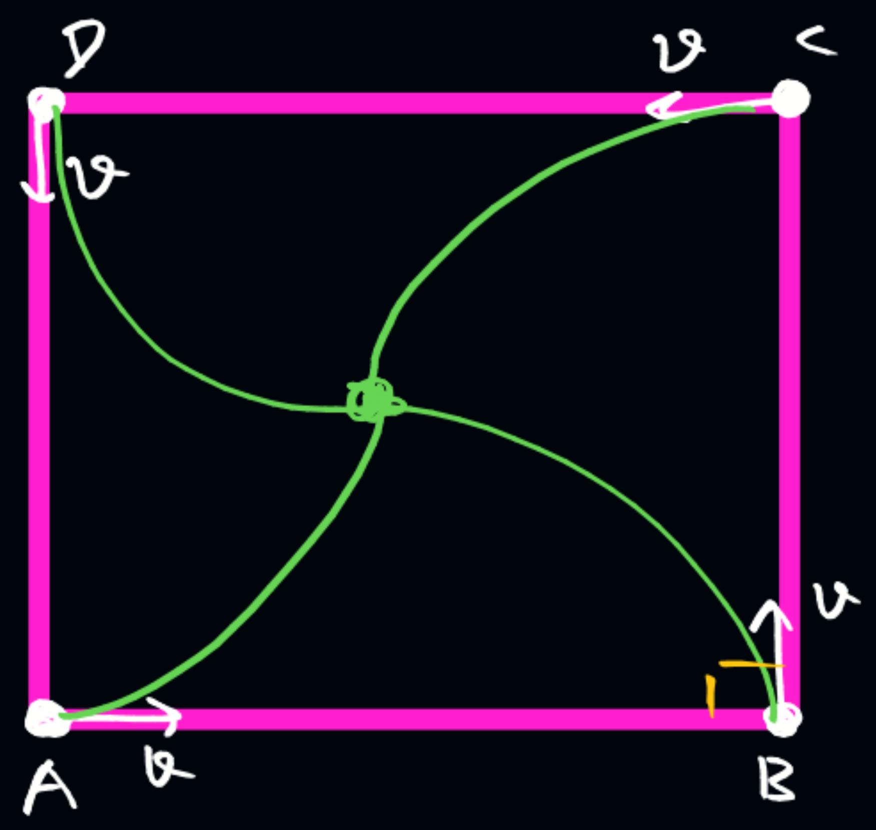
Velocity of Approach



$$v_{app.} = v_A \cos\alpha + v_B \cos\beta$$

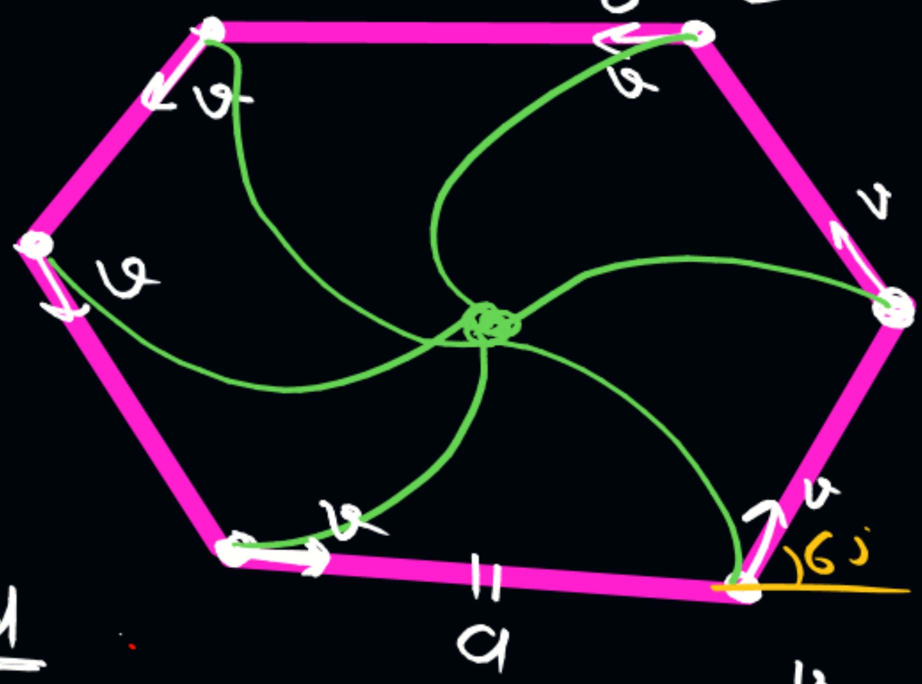


HW



HW

Regular (uniform) Hexagon



Method

$v_{app.} = (v - v/2) = \frac{v}{2}$

$$t = \frac{a}{v_{app.}} = \frac{a}{v/2} = \frac{2a}{v}$$

STAY CONNECTED
KEEP LEARNING
TOGETHER WE CAN, WE WILL

THANK YOU !