HW-01 Probability Assignment (1)

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1.1 $P(J) = 0.20$, $P(S) = 0.30$, $P(J \cap S) = 0.08$ → Given

(a) P(J at bank given that S was there) = ?

i.e $P(J/S) = \frac{P(J \cap S)}{P(S)} = \frac{0.08}{0.30} = \boxed{0.267}$

(b) P(J at bank given that S was 'not' at bank) = ?

i.e $P(J/S') = \frac{P(J \cap S')}{P(S')}$

$P(S') = 1 - P(S) = 1 - 0.30 = 0.70$

$P(J) = P(J \cap S) + P(J \cap S')$

$0.20 = 0.08 + P(J \cap S')$ $\therefore P(J \cap S') = 0.12$

$P(J/S') = \frac{P(J \cap S')}{P(S')} = \frac{0.12}{0.70} = \boxed{0.171}$

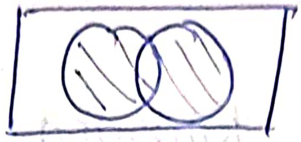
(c) P(both are at bank given that one is there) i.e

i.e $P(J \cap S / J \cup S) = \frac{P(J \cap S)}{P(J \cup S)}$

$\therefore P(J \cup S) = P(J) + P(S) - P(J \cap S) = 0.20 + 0.30 - 0.08$

$= 0.42$

$\therefore P(J \cap S / J \cup S) = \frac{P(J \cap S)}{P(J \cup S)} = \frac{0.08}{0.42} = \boxed{0.1904}$



1.2. Given: $P(H) = 0.80, P(S) = 0.90, P(H \cup S) = 0.91$

(a) ~~$P(H \cap S')$~~ $P(H \cap S') = ?$

$P(H \cap S') = P(H) - P(H \cap S)$

Now, $P(H \cap S) = P(A) + P(B) - P(H \cup S) = 0.80 + 0.90 - 0.91$
$= 0.79.$

$\therefore P(H \cap S') = P(H) - P(H \cap S) = 0.80 - 0.79 = \boxed{0.01}$

(b) $P(H' \cap S) = P(S) - P(H \cap S) = 0.90 - 0.79 = \boxed{0.11}$

(c) $P(H' \cap S') = 1 - P(H \cup S) = 1 - 0.91 = \boxed{0.09}$

1.3. $P(J) = 0.20, P(S) = 0.30, P(J \cap S) = 0.08.$

For events to be independent

$P(J \cap S) = P(J) \times P(S)$

But here we can clearly see $0.08 \neq 0.06$

Thus, event Jerry at Bank and Saisan at bank are not independant.

1.4. (a) $P(A)$ = sum is 6. $\therefore$ [(1,5)(2,4),(3,3)(4,2)(5,1)] $= \frac{5}{36}$

$P(B)$ = second die is 5 $= 6/36$

$P(A \cap B) = 1/36.$

$\because P(A \cap B) \neq P(A) \times P(B)$ NOT independent events

(b) $P(C)$ = sum is 7 $\therefore$ [(6,1)(2,5),(3,4)(4,3),(5,2),(1,6)] $= 6/36$

$P(D)$ = first die shows 5 $\therefore$ 6/36

$P(C \cap D) = 1/36$

$\because P(C \cap D) = P(C) \times P(D)$ Events ARE independent

1.5 $P(T) = 0.60$, $P(N) = 0.1$, $P(O/T) = 0.30$,
$P(O/A) = 0.2$, $P(O/N) = 0.1$, $\rightarrow$ Given

$\therefore P(A) = 1 - [P(T) + P(N)] = 1 - [0.60 + 0.10]$

$P(A) = 0.7$

1) $P(O) = P(O/T) \cdot P(T) + P(O/N) \cdot P(N) + P(O/A) \cdot P(A)$

$= [0.30 \times 0.60] + [0.1 \times 0.1] + [0.2 \times 0.7]$

$= 0.18 + 0.01 + 0.14 = \underline{\underline{0.23}}$

2) $P(T/O) = \dfrac{P(O/T) \cdot P(T)}{P(O)} = \dfrac{0.3 \times 0.6}{0.23} = \underline{\underline{0.783}}$

Bayes formula.

1.6. ① $P(\text{did not survive}) = \dfrac{1490}{2201} = 0.677$.

② $P(\text{first class}) = \dfrac{325}{2201} = 0.148$

③ $P(\text{first class / passenger survived}) = \dfrac{203}{711} = 0.286$

④ $P(\text{survived AND first class}) \neq P(\text{survived}) \times P(\text{first class})$

$\therefore$ Classes are NOT independent

⑤ $P(\text{first class AND child / survived}) = \dfrac{6}{711} = 0.0084$

⑥

⑥ $P(\text{adult}/\text{survived}) = \frac{654}{711} = 0.92$

⑦ $P(\text{first class and adult}/\text{survived}) \neq$
$P(\text{first class}/\text{survived}) \times P(\text{adult}/\text{survived})$

∴ The events are not independent

1.7 Confusion Matrix

~~Ac~~	Predict AI	Predict Human	Total
Actual: AI.	TP 970	FN 30	1000
Actual: Human	FP 70	TN 930.	1000
Total	1040.	960	2000

$$\text{Accuracy} = \frac{TP+TN}{\text{Total}} = \frac{970+930}{1000} = 0.95.$$

$$\text{Precision} = \frac{TP}{TP+FP} = \frac{970}{970+70} = 0.933.$$

$$\text{Recall} = \frac{TP}{TP+FN} = \frac{970}{970+30} = 0.97$$

$$\text{F1-score} = \cancel{\frac{1}{\text{precision}} + \frac{1}{\text{recall}}} = \frac{2}{1/\text{precision} + 1/\text{recall}}$$

$$= 0.951$$