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Financial Econometrics (Equity Assignment)

Part I:

Equally Weighted Portfolio Calculation: The equally weighted portfolio was created by averaging the returns of the three stocks (Harris, Urban, and Maya). This provides a diversified portfolio with balanced exposure across all three stocks.

| Investment Options | Mean Return (%) | Standard Deviation (%) |
|----------------------------|-----------------|------------------------|
| S&P 500 | 0.754708 | 4.337642 |
| Harris | 0.583083 | 5.798418 |
| Urban | 0.825542 | 8.243906 |
| Maya | 1.078458 | 12.59934 |
| Equally Weighted Portfolio | 0.829028 | 6.223702 |

Comparison and Contrast of Investments

1. **Mean Return:**

- **Maya** has the highest mean return at **1.0785%** per month, followed by the **equally weighted portfolio** at **0.8290%**.
- **Harris** has the lowest mean return at **0.5831%**.
- **S&P 500** has a moderate return of **0.7547%**, slightly below the equally weighted portfolio but above Harris.

2. **Standard Deviation (Risk):**

- **Maya** has the highest standard deviation (12.5993%), indicating that it is the most volatile and thus the riskiest individual stock.
- **Urban** also shows significant volatility at **8.2439%**, making it riskier than both Harris and the S&P 500.
- **Harris** has a relatively lower standard deviation at **5.7984%**, while the **S&P 500** has the lowest risk at **4.3376%**.
- The **equally weighted portfolio** has a standard deviation of **6.2237%**, which is lower than Urban and Maya, demonstrating the benefit of diversification in reducing overall portfolio risk.

Noticeable Differences

1. **Return vs. Risk Trade-off:**

- Maya offers the highest return but also the highest risk, making it suitable for investors willing to tolerate significant volatility for the potential of higher returns.
- The S&P 500 provides a more stable return with the lowest risk, ideal for conservative investors.

2. **Diversification Benefit:**

- The equally weighted portfolio's standard deviation (6.2237%) is lower than Maya and Urban individually. This illustrates how combining assets can reduce overall portfolio volatility through diversification, even if individual stocks are volatile.

3. **Suitability for Different Investors:**

- **Risk-Averse Investor (e.g., a Retiree):** The S&P 500 and Harris are more suitable choices due to their lower volatility.
- **Risk-Tolerant Investor (e.g., a Young Manager):** Maya or the equally weighted portfolio could be attractive for their higher returns, as the investor can accept more risk in pursuit of higher returns.

The **equally weighted portfolio** provides a balanced choice, capturing a higher return than the S&P 500 with moderate risk, demonstrating how diversification can enhance risk-adjusted returns.



| Investment Option | Mean Return (%) | Standard Deviation (%) |
|-------------------|-----------------|------------------------|
| Harris | 0.5831 | 5.7984 |
| Urban | 0.8255 | 8.2439 |
| Maya | 1.0785 | 12.5993 |

- **For a Young Manager (High Risk Tolerance):**
 - **Maya** would be recommended due to its **highest mean return (1.0785%)**. Although it has the highest standard deviation, a young investor can tolerate more volatility in exchange for potential higher returns.
- **For a Retiree (Low Risk Tolerance):**
 - **Harris** would be the best option among the stocks. It has the **lowest risk (5.7984%)** but still offers a positive return. Given that risk is a major concern, Harris aligns better with a retiree's goal of maintaining stability over chasing high returns.

Recommendation After Including the S&P 500 Index

| Investment Option | Mean Return (%) | Standard Deviation (%) |
|--------------------|-----------------|------------------------|
| S&P 500 | 0.7547 | 4.3376 |
| Harris | 0.5831 | 5.7984 |
| Urban | 0.8255 | 8.2439 |
| Maya | 1.0785 | 12.5993 |

- **For the Young Manager:**

- The **S&P 500** provides moderate returns at a much lower risk than Maya.
- However, if the goal is to maximize returns despite the risk, **Maya** still remains a viable choice for higher returns.
- **Final Recommendation:** The young manager could choose **Maya** if seeking maximum returns, or the **S&P 500** if they prefer some moderation in risk.

- **For the Retiree:**

- The **S&P 500** becomes the preferable option, with the **lowest risk** (4.3376%) and a reasonable return (0.7547%).
- **Final Recommendation:** **S&P 500** is now the recommended option as it offers both stability and a more favorable return than Harris.

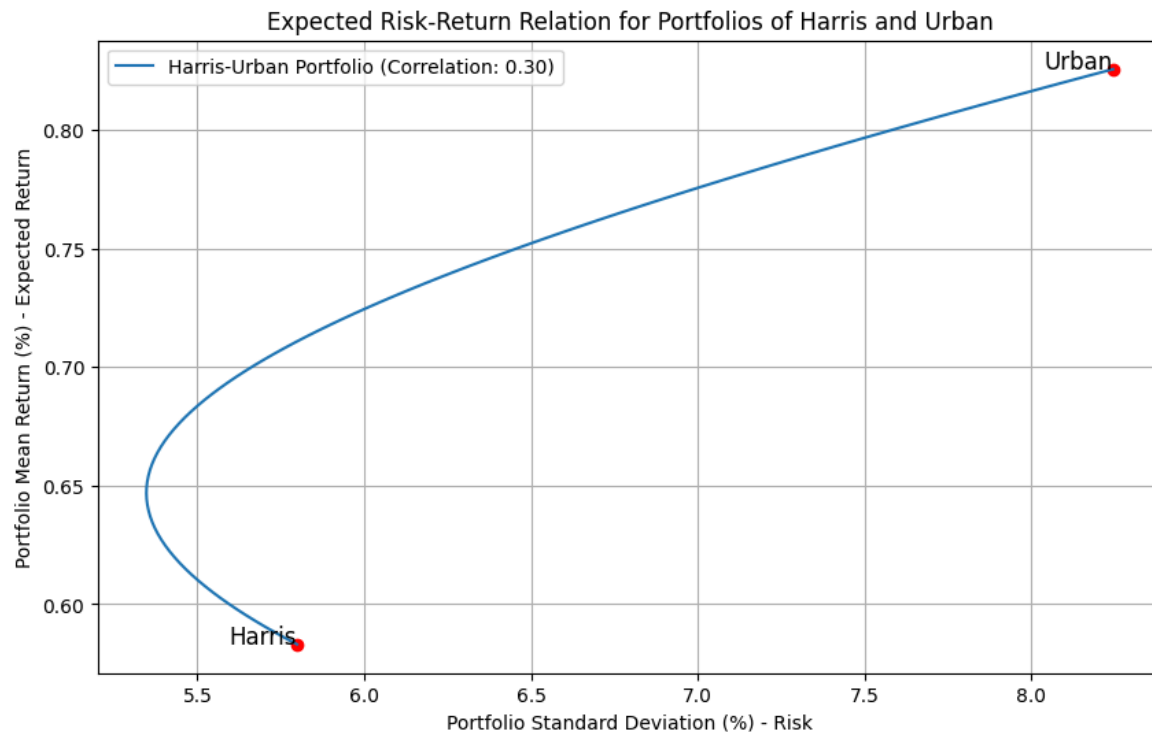
PART II:

Correlation Matrix:

| | Harris | Urban | Maya | S&P 500 | Equally Weighted Portfolio |
|---------|----------|----------|----------|----------|----------------------------|
| Harris | 1 | 0.303819 | 0.080084 | 0.293137 | 0.498743 |
| Urban | 0.303819 | 1 | 0.228829 | 0.495461 | 0.6903 |
| Maya | 0.080084 | 0.228829 | 1 | 0.449022 | 0.80071 |
| S&P 500 | 0.293137 | 0.495461 | 0.449022 | 1 | 0.612799 |

| | | | | | |
|----------------------------|----------|--------|---------|----------|---|
| Equally Weighted Portfolio | 0.498743 | 0.6903 | 0.80071 | 0.612799 | 1 |
|----------------------------|----------|--------|---------|----------|---|

I'll choose **Harris and Urban** for this analysis:



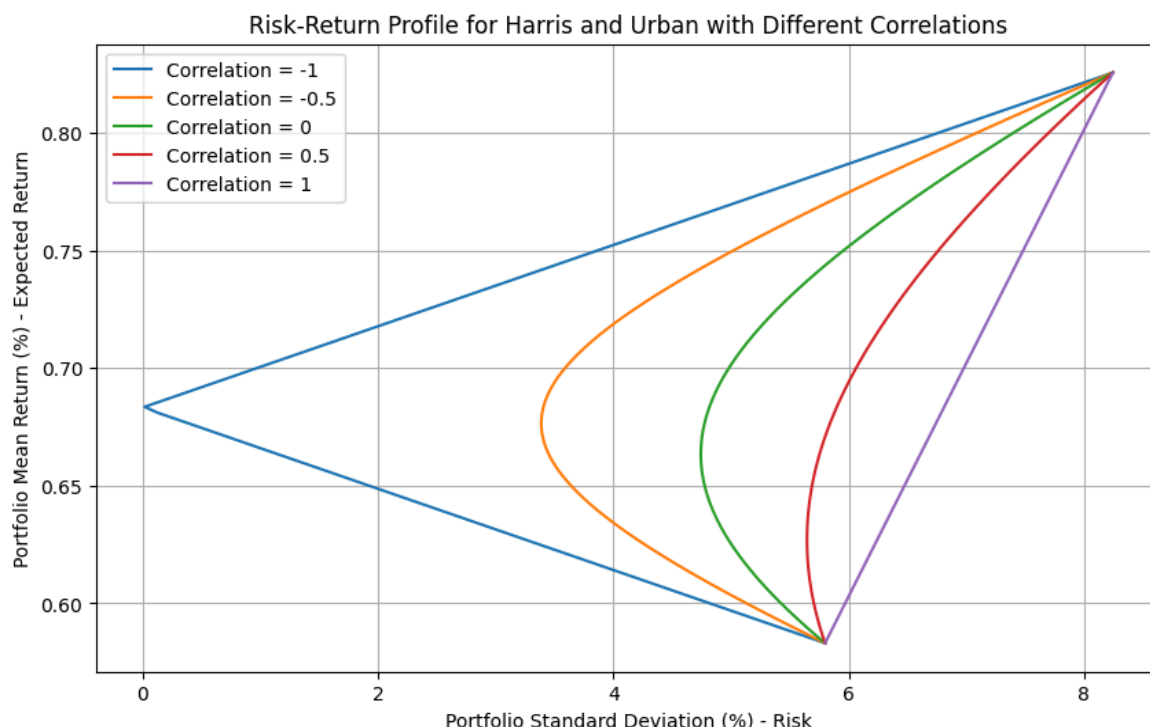
Curve Shape and Diversification Benefit:

- The curve bends **downward at the start** (near Harris), showing that by including a small proportion of Urban with Harris, we can achieve a **higher return with a relatively low increase in risk**. This is because of the **moderate positive correlation** (0.30) between Harris and Urban, which allows for some diversification benefit.
- The **lower left point on the curve** (closest to Harris) represents the **minimum risk portfolio** between Harris and Urban. This point is optimal for risk-averse investors looking to balance risk and return.

Impact of Correlation:

- With a correlation of 0.30, the curve is moderately curved, indicating that while there is some benefit to diversifying between these two assets, it is not as strong as it would be if the correlation were lower. If the correlation were closer to zero or negative, the curve would bend more sharply downward, resulting in **greater diversification benefits** and a **larger reduction in risk** for the same level of return. If the correlation were close to 1, the curve would be nearly a straight line, showing that adding Urban to a Harris portfolio does not significantly reduce risk (limited diversification).

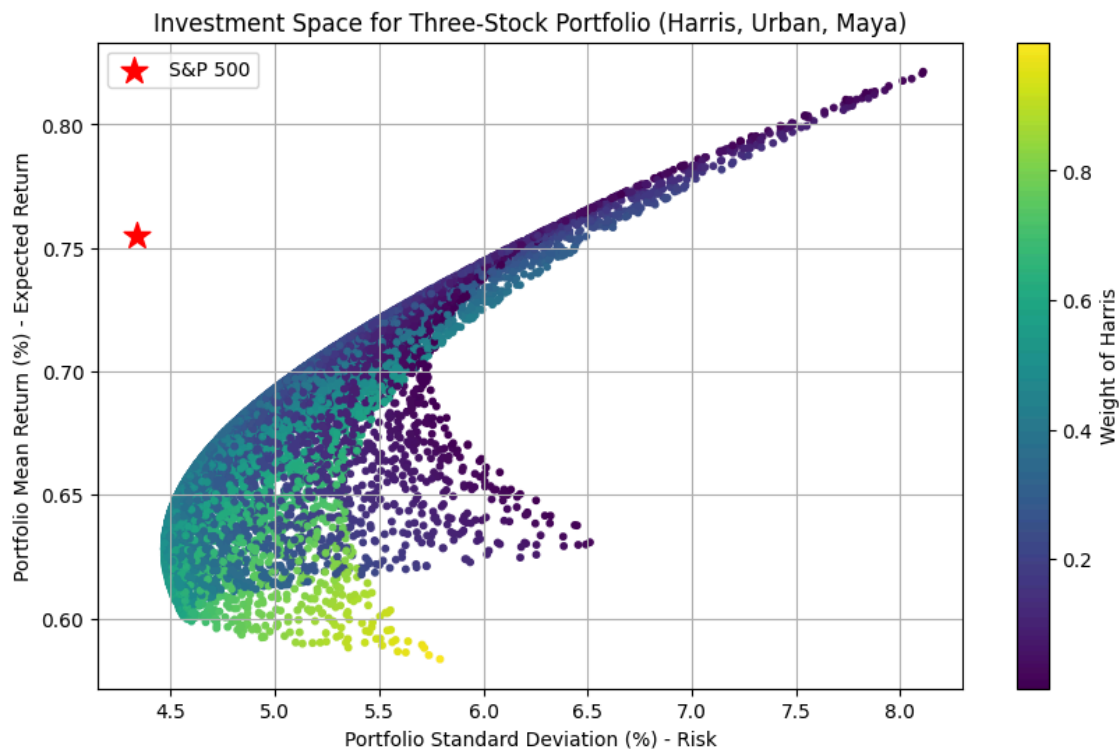
- Highly Negative Correlation (close to -1): A strong concave shape, maximizing the reduction in risk.
- Moderate Positive/Negative Correlations (e.g., -0.5 to 0.5): A more balanced concave shape, as seen in the previous graph for 0.3.
- High Positive Correlation (close to +1): The curve will become increasingly linear, showing minimal diversification benefits.



- **Correlations between Stocks:**
 - ❖ Harris and Urban have a moderate correlation (0.30), which is why the portfolio benefits from some diversification. The higher the correlation, the more the returns of Harris and Urban move in the same direction, and thus, the smaller the diversification benefit.
 - ❖ Low correlations between assets are ideal for reducing overall portfolio risk while achieving a reasonable return.
- **Insight from the Correlation with the S&P 500:**
 - ❖ The **equally weighted portfolio's correlation with the S&P 500 (0.61)** shows that it captures some of the overall market's performance. This makes it a **balanced option for investors** who want exposure to the market without committing entirely to high-volatility stocks like Maya.
 - ❖ The equally weighted portfolio benefits from diversification and offers a **middle ground** between mimicking the S&P 500 and maximizing individual stock returns.

PART III:

1. **Investment Space:** The **cloud of points** represents the full investment space of all possible portfolios combining Harris, Urban, and Maya stocks. Each point represents a portfolio with a specific combination of returns and risks.
2. **Efficient Frontier:** The **efficient frontier** is the outer edge of this investment space. These points represent the portfolios that offer the **highest return for each level of risk**. Investors typically choose points on the efficient frontier, as these provide the optimal risk-return balance.



For the two hypothetical investors, here are potential strategies based on risk tolerance:

1. **Risk-Averse Investor:** Choose a portfolio near the **lower left end of the efficient frontier**. This will have a **lower risk** (standard deviation) and a more conservative return. Harris has the lowest expected return of **0.5831** and the lowest standard deviation of **5.7984**, so portfolios in this region would generally have **higher weights in Harris** and possibly some diversification with **Maya** or **Urban** to minimize risk.
2. **Risk-Tolerant Investor:** Select a portfolio on the **upper right end of the efficient frontier**. This will have a **higher risk** but a higher expected return, suited to an investor comfortable with volatility. In this case, **Urban** has the highest expected return of **0.8255** and the highest standard deviation of **8.2439**. So, portfolios in the upper right would likely have **Urban** as a dominant component.

The **S&P 500 index** is marked by a **red star (*)** on the graph, representing its position relative to the three-stock combinations of Harris, Urban, and Maya.

1. **Portfolio Standard Deviation - Risk:** The S&P 500 has a standard deviation of approximately 4.34%. This is significantly lower than the standard deviations of most portfolios formed by Harris, Urban, and Maya, suggesting that the S&P 500 is relatively **less risky compared to these three-stock portfolios**.
2. **Portfolio Mean Return - Expected Return:** The S&P 500 has a mean return of approximately 0.75%. This places it on the upper end of the vertical axis, indicating a **relatively high expected return compared to many of the three-stock combinations**.

This suggests that the S&P 500 offers a combination of **relatively high returns with lower risk**, making it an attractive investment option compared to the portfolios formed by Harris, Urban, and Maya. However, it's important to consider that past performance is not always indicative of future results, and diversification strategies might still provide value depending on individual investment goals and risk tolerance.

PART IV:

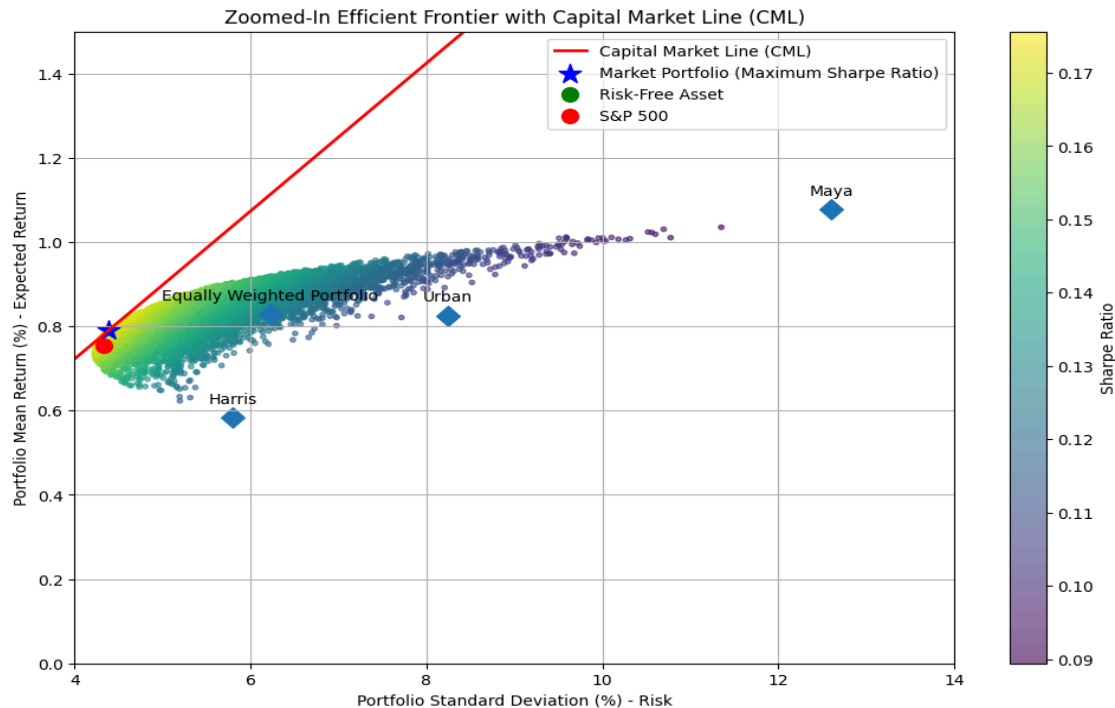
Diversification Benefits:

Adding a U.S. government bond with a 0.45% monthly return (approximately 5.5% annually) to your portfolio provides the following benefits:

1. **Reducing Portfolio Risk:** The bond's zero volatility stabilizes your portfolio.
2. **Creating New Portfolio Options:** Combining this bond with the market portfolio creates the CML, offering diverse investment choices tailored to various risk preferences.

Capital Market Line (CML) Shape: The CML is a straight line extending from the risk-free rate on the y-axis to the market portfolio. Here's what it signifies:

- **Slope of the CML:** The slope represents the Sharpe Ratio of the market portfolio. Let's calculate it using the data provided:
 - Risk-Free Rate: 5.5% annually (0.45% monthly)
 - Market Portfolio Return: Suppose this is RMR_M annually.
 - Standard Deviation (Risk) of Market Portfolio: Suppose this is σ_M annually.
- **Line Steepness:** The CML's slope shows the return per unit of risk. A steeper line indicates better risk-return trade-off.



For a Risk-Averse Investor:

- **Recommendation:** Choose a portfolio close to the risk-free rate on the CML by combining a larger portion of the bond and a smaller portion of the market portfolio. This provides **stable returns with minimal risk**, suitable for a low-risk investor.

For a Risk-Tolerant Investor:

- **Recommendation:** Opt for a portfolio higher up the CML, potentially leveraging to invest more in the market portfolio. This approach seeks higher returns, fitting a **risk-tolerant investor** willing to accept more volatility.

PART V:

| | Alpha | Beta |
|--------|----------|----------|
| Harris | 0.286109 | 0.393495 |
| Urban | 0.111898 | 0.945588 |
| Maya | 0.090011 | 1.309708 |

Based on the Betas: **Maya** is the most risky stock with a **beta of 1.3097**. This indicates that Maya is more volatile than the market, meaning it will typically experience larger fluctuations in response to market movements. **Harris** is the least risky stock with a **beta of 0.3935**. This suggests Harris is significantly less sensitive to market movements compared to both Urban and Maya, providing greater stability relative to the market.

Comparison with Standard Deviations:

Standard Deviation as a Measure of Total Risk:

- Using standard deviation, **Maya** is also the most risky, with a standard deviation of 12.5993%.
- **Harris** remains the least risky based on standard deviation, with a value of 5.7984%.
- Both beta and standard deviation identify **Maya** as the most risky and **Harris** as the least risky.

Beta vs. Standard Deviation: Beta specifically measures **market-related (systematic) risk**—how much the stock moves in response to market changes. Standard deviation measures **total risk** (both market and firm-specific risks). While both metrics align in this case, beta emphasizes that Maya is more sensitive to market volatility, while Harris has lower exposure to market-driven fluctuations.

Suitability for Hypothetical Investors:

1. **Young Manager (High Risk Tolerance):**

- **Recommendation:** The young manager, who can handle volatility, may benefit from Maya's higher beta, as it offers potential for higher returns in line with the market's positive swings. Maya's higher beta is suitable given the manager's ability to endure market-driven fluctuations.

2. **Retiree (Low Risk Tolerance):**

- **Recommendation:** With the lowest beta, Harris minimizes market-related risk, making it ideal for a retiree seeking stability over aggressive growth. Harris's low market sensitivity and positive alpha mean it can provide steady returns with minimal exposure to market volatility, aligning with the retiree's risk profile.

Economic Meaning of Alpha:

Alpha represents the excess return an asset provides relative to the market return, adjusted for its risk. This is a critical measure for assessing the performance of individual stocks compared to the market.

- **Harris:** Harris's alpha of 0.286109 indicates that it outperforms the market return by approximately 28.61%, adjusted for its lower beta of 0.393495. This suggests that Harris has a substantial positive alpha, making it an attractive investment as it provides additional returns beyond what its beta predicts.
- **Urban:** Urban's alpha of 0.111898 shows it outperforms the market return by around 11.19%, considering its beta of 0.945588. Urban's alpha is positive, implying that it offers additional returns above the expected market return based on its risk level.
- **Maya:** Maya's alpha of 0.090011 indicates it surpasses the market return by approximately 9.00%, adjusted for its higher beta of 1.309708. Despite having the highest beta among the three stocks, Maya still provides a positive alpha, suggesting it delivers extra returns beyond the market's movement.

PART VI:

| | Beta | Expected Monthly Return (%) | Annual Expected Return (%) | Annual Realized Return (%) |
|--|------|-----------------------------|----------------------------|----------------------------|
|--|------|-----------------------------|----------------------------|----------------------------|

| | | | | |
|--------|--------|---------|----------|----------|
| Harris | 0.3935 | 0.64675 | 8.043108 | 7.226023 |
| Urban | 0.9456 | 0.9228 | 11.65328 | 10.36837 |
| Maya | 1.3097 | 1.10485 | 14.09428 | 13.73797 |

1. **Expected Returns** (CAPM-based): These expected returns reflect each stock's **market risk exposure** (beta), with Maya expected to yield the highest return due to its higher beta and Harris the lowest due to its lower beta.
2. **Realized Returns** (Actual Historical Performance): All realized returns are close to but slightly below their expected values, indicating that each stock has performed relatively in line with its CAPM-based expected return, though each has slightly underperformed.

How Realized Returns Compare with Expected Returns

- **Harris:** The realized return of **7.23%** is close to but slightly below the expected return of **8.04%**.
- **Urban:** The realized return of **10.37%** is also below the expected return of **11.65%**.
- **Maya:** The realized return of **13.74%** is just under the expected return of **14.09%**.

Interpretation of Differences and Market Implications

- **If Realized Returns Differ from Expected Returns:**
 - **Slight Underperformance:** The minor difference between realized and expected returns suggests slight underperformance relative to CAPM predictions.
 - **Market Impact:** If this trend of underperformance persists, market participants may reassess the stock's price, potentially adjusting it downward to align with the lower realized returns. This would bring the stock's price to a level where future expected returns better match realized performance.
- **CAPM Assumptions and Realized Performance:**
 - **CAPM relies on market beta** to estimate returns, assuming the stock's sensitivity to the market is the primary driver of risk.
 - Real-world deviations, as seen here, may arise due to **firm-specific factors** or **market conditions** not captured by beta alone, which can lead to small but consistent gaps between realized and expected returns.

For Investors

1. **Young Manager (Higher Risk Tolerance):** Maya remains an attractive choice with the highest expected and realized returns, despite slight underperformance. The young manager can accept Maya's higher volatility, aiming for the return potential it offers based on its higher beta.
2. **Retiree (Lower Risk Tolerance):** Harris remains suitable, with the lowest expected and realized returns, both of which align with lower volatility. Harris's lower beta makes it less sensitive to market movements, fitting the retiree's preference for stability.