(Brief) Solutions to practice problems - I

1) a) 45
   b) -8
   c) 0

2) 25, 30

3) No. $\det(A-5I) = -20$

4) $$x^6 - 4x^5 - 12x^4 = x^4(x^2 - 4x - 12)$$
$$= x^4 (x+2)(x-6)$$

$\therefore$ The eigenvalues are $\underbrace{0, 0, 0, 0}_{\text{coming from } x^4 = (x-0)^4.}, -2, 6.$

5) $\lambda_1 = 3$, with eigenvector $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$\lambda_2 = -7$, with eigenvector $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

5. $A = \begin{pmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{pmatrix} \rightsquigarrow$ eigenvalues: $9, 2, 2$.

eigenvectors: $(1,1,1)$, $(-3,0,1)$, $(1,2,0)$ OR $(1,1,1)$, $(0,6,1)$, $(1,2,0)$

Computing eigenvalues:

Char. poly:

$\det(A - \lambda I) = \lambda^3 - 13\lambda^2 + 40\lambda - 36$.

Find one root by trial-and-error: $\lambda = 2$ $\therefore (\lambda - 2)$ divides the polynomial. Factorize using long division:

$$
\begin{array}{r|l}
 & \lambda^2 - 11\lambda + 18 \\ \hline
\lambda - 2 & \lambda^3 - 13\lambda^2 + 40\lambda - 36 \\
 & \lambda^3 - 2\lambda^2 \\ \hline
 & \quad -11\lambda^2 + 40\lambda - 36 \\
 & \quad -11\lambda^2 + 22\lambda \\ \hline
 & \qquad 18\lambda - 36 \\
 & \qquad 18\lambda - 36 \\ \hline
 & \qquad\quad 0
\end{array}
$$

Therefore, we can write the characteristic polynomial as $(\lambda - 2)(\lambda^2 - 11\lambda + 18)$. Further factorize $\lambda^2 - 11\lambda + 18$ to get $(\lambda - 9)(\lambda - 2)$.

$\therefore$ The eigenvalues of A are $9, 2, 2$.