# Assignment-3

Due date: Wednesday, 18 Oct.

1. Compute the column space $C(A)$ and null space $N(A)$ of the following matrixes:

(i) $A = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 0 & 0 & 3 \\ 1 & 2 & 3 \end{bmatrix}$.

(iii) $C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

Also write down a basis for $C(A)$, $N(A)$ in each case.

2. The trace of a matrix is defined to be the sum of its diagonal elements.

eg. trace of $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = 1+4 = 5$. Trace of $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} = 1+3+4 = 8$.

(a) Consider the set of matrices in $M_3(\mathbb{R})$ with trace $= 0$. i.e., $W = \{ A \in M_3(\mathbb{R}) : \text{Trace}(A) = 0 \}$.

Show that $W$ is a subspace of $M_3(\mathbb{R})$.

(b) Write down a basis for $W$. What is its dimension?

3. Compute the matrix of the following linear transformations:

(a) $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, using the basis $\{(1,2),(1,1)\}$ of $\mathbb{R}^2$.

$(x,y) \mapsto (x+2y,\ 3x+7y)$

(b) $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$, using the standard basis for $\mathbb{R}^2, \mathbb{R}^3$.

$(x,y) \longmapsto (x+y,\ y,\ 3x)$

4. Consider a pentagon in $\mathbb{R}^2$ with vertices $(1,1)$, $(3,1)$, $(4,2)$, $(2,4)$ and $(1,3)$. For each of the following transformations on the plane, find the $3\times 3$ matrix that describes the transformation with respect to homogeneous coordinates, and use it to find the image of the pentagon:

a) reflection across the line $y = x$

b) translation by the fixed vector $(3, -2)$