Practice Problems

1. Write down the 3×3 matrices corresponding to the following, using homogeneous coordinates)

a) Translation by the vector $(3, -\frac{1}{2})$

b) Shear in the direction of the y-axis by a factor of $\sqrt{2}$

c) Rotation anti-clockwise by 60°

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2. Write down a basis for the following subspaces:

1) The set of all diagonal matrices in $M_3(\mathbb{R})$

i.e., $\left\{ \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} : a, b, c \in \mathbb{R} \right\}$

2) $\{ (x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 + x_2 - x_3 = 0 \}$

3. Which of the following are subspaces W of the given vector space V?

(a) $V = \mathbb{P}^2$, the space of polynomials with coefficients in $\mathbb{R}$ of degree at most 2.

W : set of polynomials in V with constant term $= 0$.

(b) $V = \mathbb{R}^2$.

$W = \{ (x_1, x_2) \in \mathbb{R}^2 : x_1 \geq 0, x_2 \leq 0 \}$

(c) $V = \mathbb{R}^2$

$W = \{ (x_1, x_2) \in \mathbb{R}^2 : x_1 + x_2 \geq 0 \}$

(d) $V = \mathbb{R}^3$

$W = \{ (x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 = 0 \}$.

If W in any of the above is a subspace, can you write down a basis?

4. Find the determinant:

a) $A = \begin{bmatrix} 5 & 4 \\ -2 & -2 \end{bmatrix}$

b) $A = \begin{bmatrix} -1 & 5 & 2 \\ 3 & 1 & 6 \\ -5 & 2 & 4 \end{bmatrix}$

c) $A = \begin{bmatrix} 12 & 13 & 25 \\ -5 & 6 & 1 \\ 3 & -22 & -19 \end{bmatrix}$

d) $A = \begin{bmatrix} 11 & 22 & 5 \\ 5 & 10 & 3 \\ 6 & 12 & -45 \end{bmatrix}$

Remark: For c) and d) above, did you use the formula? There is a much quicker way to arrive at the answer!

4. Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be defined by

$$T(x, y) = (4x - 3y,\ 4x - 2y).$$

i) Find the matrix of T using the standard basis of $\mathbb{R}^2$

ii) Find the matrix of T using the basis $\{(1,1), (0,-1)\}$ of $\mathbb{R}^2$.

NOTE: Recall that you need to use the same basis for domain and co-domain, if they are the same vector space.

5. Suppose $T: \mathbb{R}^3 \to \mathbb{R}^3$ is a linear trans. given by
$$T(x_1, x_2, x_3) = \begin{bmatrix} 2 & -2 & 4 \\ 0 & 0 & 1 \\ 4 & -3 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Find the matrix of this transformation using the basis $\{\underbrace{(1,3,1)}_{v_1}, \underbrace{(0,1,1)}_{v_2}, \underbrace{(1,1,0)}_{v_3}\}$.

6. Let $W = \text{Span}\{\overset{u_1}{(1,2,1)}, \overset{u_2}{(1,0,-1)}\}$.

Find the orthogonal projection of $\bar{x} = (0,1,2)$ onto W.

7. Find a least-squares solution for the inconsistent linear system

$$Ax = b$$

where $A = \begin{bmatrix} 1 & -2 \\ 3 & -2 \\ 2 & 1 \end{bmatrix}$ $\quad b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

8. Find eigenvalues and eigenvectors of

a) $A = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$

b) $A = \begin{pmatrix} -2 & -4 & 2 \\ -2 & 1 & 2 \\ 4 & 2 & 5 \end{pmatrix}$

9.

Let $A = \begin{pmatrix} 2 & -1 & 1 \\ 1 & 3 & -2 \\ 0 & 1 & -2 \end{pmatrix}$. Compute the QR-factorization of A.