# SOLUTIONS TO ASSIGNMENT 3

1. Compute the column space $C(A)$ and null space $N(A)$ of the following matrixes:
(12 marks)

(i) $A = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$ (ii) $A = \begin{bmatrix} 0 & 0 & 3 \\ 1 & 2 & 3 \end{bmatrix}$,

(iii) $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Also compute a basis for $N(A)$, $C(A)$.

(i) $C(A) = \text{span}\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \end{pmatrix} \right\}$ Basis: $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right\}$

$N(A)$ : $\text{span}\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$ Basis: $\left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$

(ii) $C(A) = \text{span}\left\{ \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 3 \\ 3 \end{pmatrix} \right\}$ Basis: $\left\{ \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$

$N(A) = \text{span}\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \right\}$ Basis: $\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} \right\}$

iii) $C(A) = \left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$ Basis: $\left\{ \begin{pmatrix} 0 \\ 0 \end{pmatrix} \right\}$

$N(A) = \mathbb{R}^3$ Basis: $\left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$

2. (a) Consider the set of matrices in $M_3(\mathbb{R})$ with trace = 0. i.e., $W = \{ A \in M_3(\mathbb{R}) : \text{Trace}(A) = 0 \}$.

[6 marks)

Show that W is a subspace of $M_3(\mathbb{R})$.

(b) Write down a basis for W. What is its dimension?

(6 marks)

Soln

(a)

i) $\bar{0} \in W$ : $\bar{0}$ here is $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ which has trace = 0.

ii) If A and B have trace 0,

i.e., if $A = \begin{bmatrix} a_1 & * & * \\ * & a_2 & * \\ * & * & a_3 \end{bmatrix}$ $B = \begin{bmatrix} b_1 & * & * \\ * & b_2 & * \\ * & * & b_3 \end{bmatrix}$

so that $a_1 + a_2 + a_3 = 0$, $b_1 + b_2 + b_3 = 0$

Clearly, trace of $A+B$ is $(a_1+b_1) + (a_2+b_2) + (a_3+b_3)$
$= 0$.

$\therefore A, B \in W \Rightarrow A+B \in W$.

iii) If $c \in \mathbb{R}$ and $A \in W$ } trace $(cA) = c\,\text{trace}(A)$
$= c \cdot 0$
$= 0$. $\therefore cA \in W$

This shows that W is a subspace of $M_3(\mathbb{R})$.

(b) One basis is

$$\left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \right\}$$

dimension is 8.

3. Compute the matrix of the following linear transformations:

(a) $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, using the basis $\{(1,2),(1,1)\}$ of $\mathbb{R}^2$.

(3 marks) $(x,y) \mapsto (x+2y, 3x+7y)$

(b) $T: \mathbb{R}^2 \longrightarrow \mathbb{R}^3$, using the standard basis for $\mathbb{R}^2, \mathbb{R}^3$.

(3 marks) $(x,y) \longmapsto (x+y, y, 3x)$

Soln. a) $\begin{pmatrix} 12 & 7 \\ -7 & -4 \end{pmatrix}$ b) $\begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 3 & 0 \end{pmatrix}$

4. i) The matrix is given by

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$ and the image coordinates

(3 marks) are obtained by multiplying

$$\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 4 & 2 & 1 \\ 1 & 1 & 2 & 4 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 & 2 & 4 & 3 \\ 1 & 3 & 4 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$\therefore$ The transformed pentagon vertices are : $(1,1)$, $(1,3)$, $(2,4)$, $(4,2)$, $(3,1)$.

ii) The matrix is given by

(3 marks) $$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$ and the image coordinates

are obtained as follows :

$$\begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & 4 & 2 & 1 \\ 1 & 1 & 2 & 4 & 3 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 6 & 7 & 5 & 4 \\ -1 & -1 & 0 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$\therefore$ The translated pentagon has vertices

$(4,-1)$, $(6,-1)$, $(7,0)$, $(5,2)$, $(4,1)$