

# Linear Algebra

## ASSIGNMENT-1

DUE DATE : 4 September 2023.

Submit it in my office between 10 a.m.  
to 1 p.m.

### Problem 1

Describe whether the following augmented matrices are in row echelon form, reduced row echelon form, or neither:

a) 
$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

b) 
$$\begin{bmatrix} 0 & 1 & 0 & 0 & -9 & 4 \\ 0 & 0 & 1 & 0 & 3 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

c) 
$$\begin{bmatrix} 1 & 3 & 0 & 1 & -1 \\ 0 & 2 & 1 & 2 & 2 \\ 0 & 0 & 4 & 3 & 2 \\ 1 & 0 & 0 & 3 & 0 \end{bmatrix}$$

d) 
$$\begin{bmatrix} 1 & -4 & 8 & 1 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

e) 
$$\begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$$

f) 
$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$g) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Problem 2 Convert the following matrix into reduced row echelon form. Clearly indicate the row operations used in each step:

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{bmatrix}$$

Problem 3 Solve the linear system whose augmented matrices are given below, by converting them to either row echelon or reduced row echelon form:

$$1) \begin{bmatrix} 1 & -2 & -1 & 3 \\ 3 & -6 & -2 & 2 \end{bmatrix}$$

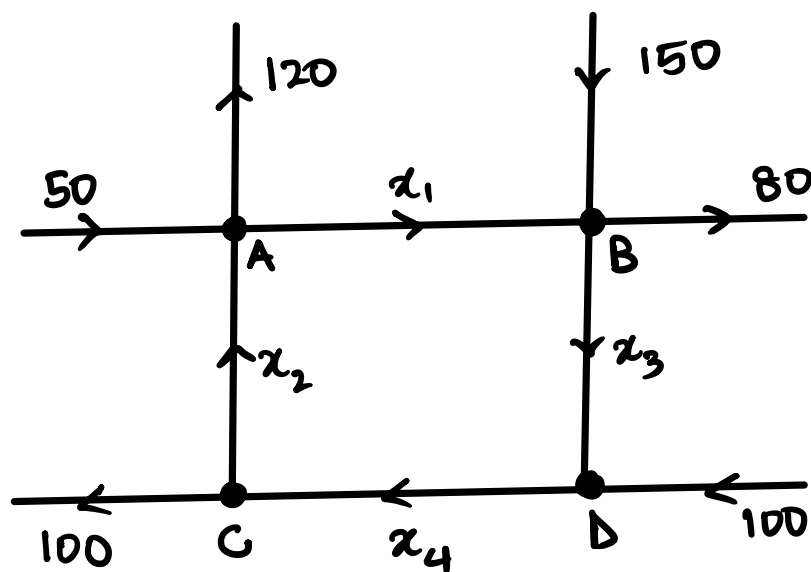
$$2) \begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix}$$

Problem 4 Balance the following chemical reaction by converting it into a problem of solving a linear system:



(Note: there is more than one correct solution!)

Problem 5: Find the minimum value for  $x_4$  in the following system of one-way roads. The assumption is that each node traffic going into each node is the same as the traffic going out of it.



Problem 6 Choose  $h$  and  $k$  so that the following linear system

$$x_1 + h x_2 = 2$$

$$4x_1 + 8x_2 = k$$

has

- a) No solution
- b) Unique solution
- c) Infinitely many solutions.

Justify your choices in each case.

