

## Assignment - 2

Problem 1 a) Compute the inverse of the following matrix using Gaussian elimination:

$$A = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 5 & 3 \\ 2 & 6 & -1 \end{pmatrix}$$

b) Without performing any further row operations, solve the following system of linear equations:

$$x_1 + 2x_2 - 2x_3 = 1$$

$$x_1 + 5x_2 + 3x_3 = 0$$

$$2x_1 + 6x_2 - x_3 = 1$$

Note: Notice that the coefficients in this system form the matrix  $A$  of part a). So this question is asking you to solve this system by calculating  $A^{-1}b$ , and not using REF + back substitution.

Problem 2 a) Find the LU decomposition

of  $A = \begin{pmatrix} 1 & 4 & 3 \\ -2 & -6 & 0 \\ -3 & -10 & 0 \end{pmatrix}$ .

b) Using the LU decomposition of  $A$ , solve the linear system

$$x_1 + 4x_2 + 3x_3 = 1$$

$$-2x_1 - 6x_2 = -1$$

$$-3x_1 - 10x_2 = 2$$

c) Use your calculations in part a) to compute  $L^{-1}$ .

### Problem 3

a) Write down the row operations corresponding to the following elementary matrices:

$$(i) \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -5 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

In each part below, write down the elementary matrix of the given size corresponding to the given elementary row operation:

(i)  $5 \times 5$  matrix,  
 $R_5 \mapsto R_5 + 3R_2$

(ii)  $4 \times 4$  matrix,  
 $R_2 \leftrightarrow R_3$

(iii)  $3 \times 3$  matrix,  
 $R_3 \mapsto \sqrt{2}R_3$

(iv)  $2 \times 2$  matrix  
 $R_2 \mapsto R_2 - 2R_1$