Solutions

1) Worked out in Lecture 6.

$$A^{-1} = \begin{bmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ 7/2 & 3/2 & 1/2 \end{bmatrix}$$

2.

$$\begin{bmatrix} 1 & 0 & -13/16 & 0 \\ 0 & 1 & -11/2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

3. The linear system is given by

$$x_1 + x_2 - x_4 = 0$$

$$x_1 + 2x_2 - x_5 = 0$$

$$2x_3 - 4x_4 - 2x_5 = 0.$$

4. $$\begin{pmatrix}1&-2&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\underbrace{\begin{pmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}1&0&2&0\\0&1&3&0\\0&0&1&0\\0&0&0&1\end{pmatrix}}_{\parallel}$$

$$= \begin{pmatrix}1&-2&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\begin{pmatrix}1&0&2&0\\0&0&1&0\\0&1&3&0\\0&0&0&1\end{pmatrix}$$

$$= \begin{pmatrix}1&0&0&0\\0&0&1&0\\0&1&3&0\\0&0&0&1\end{pmatrix}$$

5. No solutions. Observe that $R_3 \mapsto R_3 - 2R_1$ would convert the given augmented matrix to: $$\begin{bmatrix}1&0&1&2\\0&2&0&3\\0&0&0&-2\end{bmatrix}$$

The last row then corresponds to the equation $0x_1 + 0x_2 + 0x_3 = -2$, which is impossible.

6. $L\bar{y} = \bar{v}$, $U\bar{x} = \bar{y}$.

7. False.

(Observe that the given row operation is $R_3 \mapsto -\frac{1}{2}R_3 + R_2$.

An (elementary) row replacement has to be of the form $R_i \mapsto R_i + cR_j$ where i and j are distinct natural numbers corresponding to rows of a matrix. In particular, R_i must not have any coefficient other than 1.)