Sai University
Linear Algebra Test-2

PLEASE WRITE YOUR NAME HERE:

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1. This examination is **90 minutes** in length and the maximum score is 20 marks.
2. **Please submit this question paper and any rough sheets used, along with your answer sheet.**

1. (4 marks) Consider the linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ given by $T(x_1, x_2, x_3, x_4) = (x_1 + x_2, x_2 + x_3, 2x_4)$. Compute the matrix of this linear transformation using the standard basis for $\mathbb{R}^4$ and $\mathbb{R}^3$.

2. (3 marks)

   (a) Define the null space of a linear transformation $T : V \to W$ of vector spaces V and W.

   (b) Let $A = \begin{pmatrix} 2 & 5 & 1 \\ -1 & -7 & -5 \\ 3 & 4 & -2 \end{pmatrix}$. Let $v_1 = \begin{pmatrix} 6 \\ -2 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 2 \\ -1 \\ 1. \end{pmatrix}$ Determine whether v_1 or v_2 are in the null space of the given matrix A.

3. (3 marks) Express the vector $(6, 1, -8)$ as a linear combination of the following basis of $\mathbb{R}^3$:

$$\left\{(3,\ 1,\ 1), (-1,\ 2,\ 1), \left(-\frac{1}{2},\ -2,\ \frac{7}{2}\right)\right\}.$$

4. (4 marks) Consider the set

$$W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 - 2x_2 = 0\}.$$

   (a) Show that W is a subspace of $\mathbb{R}^3$.

   (b) Write down a basis of W.

5. (6 marks) Answer the following in brief:

   (a) Give an example of a subspace of $M_2(\mathbb{R})$, the vector space of 2×2 matrices with entries in $\mathbb{R}$ (other than $\{\bar{0}\}$ and $M_2(\mathbb{R})$.)

   (b) A certain linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ has Ker $T = \{0\}$.

      i. What is the dimension of Range(T)?

      ii. Is T surjective? Justify briefly.

   (c) Consider the vector $v = (1, 2, 3) \in \mathbb{R}^3$. Write down a non-zero vector in $\mathbb{R}^3$ that is orthogonal to v. What is the norm of v?

   (d) Let $A = \begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$. The column space of A is a subspace of which vector space?

*****END OF TEST*****