$e^{x} \cdot e^{-x} = 1$

Solve the following D.E.'s :

① $3y^2 y' + y^3 = e^{-x}$ ⤳ Bernoulli ⤳ $\frac{dv}{dx} + v = e^{-x}$

Ans: $y^3 = e^{-x}(x+C)$

② $y' = (1-y)\cos x$; $y(\pi) = 2$ → Ans: $y = 1 + e^{-\sin x}$

③ $\frac{dy}{dx} = \frac{3x(x-y)}{y^2}$

Ans: $3y(1-Cx^4) = 4x$

④ Solve the exact eqn: $2xy - 9x^2 + (2y+x^2+1)\frac{dy}{dx} = 0$; $y(0) = -3$.

Ans: $y^2 + (x^2+1)y - 3x^2 = 6$.

⑤ For what points (x_0, y_0) does the following IVP have a unique solution in some interval containing x_0 ?

$$y' = \frac{10}{3} x y^{2/5} \quad ; \quad y(x_0) = y_0 .$$

—— x ——

$y' = f(x,y)$ Here $f(x,y) = \frac{10}{3} x y^{2/5}$

$$\frac{\partial f}{\partial y} = \frac{10}{3} \cdot x \cdot \frac{2}{5} y^{-3/5}$$

$$= \frac{4}{3} x y^{-3/5}$$

$y = 0$ is a problem point

$(x_0, 0) \to$ No unique solution

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③ $\frac{dy}{dx} = \frac{3x(x-y)}{y^2}$

$$\frac{dy}{dx} = \frac{3x}{y}\left(\frac{x}{y} - \frac{y}{y}\right) = \frac{3x}{y}\left(\frac{x}{y} - 1\right)$$

⤳ This is a homogeneous

Set $y = vx$ $(\to v = y/x)$ $\quad y' = v + xv'$

$$v + x\frac{dv}{dx} = \frac{3}{v}\left(\frac{1}{v} - 1\right)$$

$$x\frac{dv}{dx} = \frac{3}{v^2} - \frac{3}{v} - v = \frac{3 - 3v - v^3}{v^2}$$

$$-\int \frac{v^2\,dv}{-3+3v+v^3} = \int \frac{dx}{x} + C$$

$$v^3 + 3v - 3 = t$$
$$(3v^2 + 3)\,dv = dt.$$

$$-\frac{1}{3}\int \frac{3v^2+3-3}{-3+3v+v^3}\,dv = \log|x| + C$$

(incomplete)

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$$2xy - 9x^2 + (2y + x^2 + 1)\frac{dy}{dx} = 0 \; ; \; y(0) = -3.$$

$$\underbrace{(2xy - 9x^2)}_{M}dx + \underbrace{(2y + x^2 + 1)}_{N}dy = 0$$

Check that this is exact → $\frac{\partial M}{\partial y} = 2x$ $\quad \frac{\partial N}{\partial x} = 2x$ ✓

$$F(x,y) = \int (2xy - 9x^2)\,dx + g(y)$$
$$= x^2 y - 3x^3 + g(y)$$

$\frac{\partial F}{\partial y} = (2y + x^2 + 1)$ so $x^2 + g'(y) = 2y + x^2 + 1$

$$g'(y) = 2y + 1$$
$$\therefore\ g(y) = y^2 + y + C_0.$$

∴ The general solution is

$$x^2 y - 3x^3 + y^2 + y = \underbrace{C - C_0}_{C_1}$$

Using $y(0) = -3$:

$$(0)^2 \cdot -3 - 3\cdot(0)^3 + 9 - 3 = C_1$$
$$\therefore\ C_1 = 6.$$

∴ The particular solution is $x^2 y - 3x^3 + y^2 + y - 6 = 0.$

$$y^2 + y(x^2+1) - 3x^3 - 6 = 0$$