Assignment-2

Due on Monday, 25 March

1. Assume that Lake B has a volume of $1640\,km^3$ and that its rate of inflow from Lake A and outflow to Lake C are both $410\,km^3/yr$.
Suppose that at $t = 0$ years, the pollutant concentration of Lake B due to past industrial pollution (which has now ceased) is 5 times that of Lake A. If the outflow is perfectly mixed Lake water, how long will it take to reduce the pollution concentration in Lake B to twice that of Lake A?

2. Use appropriate substitution techniques to give a general solution for the following differential equations:

(i) $$xy\frac{dy}{dx} = y^2 + x\sqrt{4x^2+y^2}$$

(ii) $$x^2y' + 2xy = 5y^4$$

(iii) $y' = (x+y)^2$

3. Verify that the given D.E. is exact, and then solve it:

$$\left(x^3 + \frac{y}{x}\right) dx + (y^2 + \log x)\, dy = 0 .$$

4. Show that the substitution $v = \log y$ transforms the differential equation

$$\frac{dy}{dx} + P(x)\, y = Q(x)\, y \log y$$

into the linear equation

$$\frac{dv}{dx} + P(x) = Q(x)\, v(x).$$

Use this idea to solve:

$$x \frac{dy}{dx} - 4x^2 y + 2y \log y = 0.$$

5. Show that the general solution to the logistic equation

$$\frac{dP}{dt} = kP(M-P) \quad ; \quad P(0) = P_0,$$

is given by $P(t) = \dfrac{MP_0}{(M-P_0)e^{-kMt} + P_0}$.