Assignment - 3

Due date : Tuesday April 23.

I. Solve the following reducible second order ODEs.

(i) $yy'' + (y')^2 = 0$

(ii) $y'' = (x + y')^2$

II. Find a general solution of the following homogeneous equation:

$$xy'' - (2x+1)y' + (x+1)y = 0$$

given that $y_1 = e^x$ is one solution.

III. Find the Wronskian of the following functions:

(i) $f_1(x) = e^{-3x}$, $f_2(x) = \cos 2x$, $f_3(x) = \sin 2x$

(ii) $f(x) = 2\cos x + 3\sin x$, $g(x) = 3\cos x - 2\sin x$

IV. Show that $y_1(x) = 1$, $y_2(x) = \sqrt{x}$ are solutions of $y y'' + (y')^2 = 0$, but that their sum $y = y_1 + y_2$ is not a solution.

[Note: This example shows that the principle of superposition does not hold for non-linear differential equations]

V. Find a general solution of the following differential equations:

1) $2y'' + 3y' = 0$.

2) $9y'' - 12y' + 4y = 0$.

3) $y'' + 8y' + 25y = 0$.

4) $y^{(3)} + 27y = 0$.

VI. Recall the second order Euler equation

$$ax^2y'' + bxy' + cy = 0 \quad -① $$

from class. [A] Prove that if the roots r_1 and r_2 of the D.E obtained by the substitution $v = \log x$:

$$a\frac{d^2y}{dv^2} + (b-a)\frac{dy}{dv} + cy = 0$$

are such that r_1 and r_2 are real and distinct, then a general solution of ① is given by $y(x) = c_1x^{r_1} + c_2x^{r_2}$.

[B] What if $r_1 = r_2 = \alpha$, a repeated real root? What would a general solution look like in this case?