

ODE-Assignment 4

1. Determine the period and frequency of the simple harmonic motion of a body of mass 0.75kg at the end of a spring with spring constant 48N/m . (Note: Simple harmonic motion is the same as free, undamped motion.)
2. A body of mass 250g is attached to the end of a spring that is stretched 25cm by a force of 9N . At time $t = 0$, the body is pulled 1m to the right, stretching the spring and set in motion with an initial velocity of 5m/s to the left. Find the position function, amplitude and period of motion of the body.
3. Consider a mass-spring system where mass $m = 1$, damping constant $c = 4$, spring constant $k = 4$ and external force $F_E = 10 \cos(3t)$, all in our standard units. Determine the position of the mass at any time.
4. A block of mass of 0.1 kg stretches a spring 0.05 m . Assume there is no damping and the gravitational constant is $g = 9.8\text{ m/sec}^2$.
 - (a) Suppose the mass is set in motion from its equilibrium position with a downward velocity of 0.1 m/sec , determine the position of the mass at any time.
 - (b) What's the amplitude of the motion of the mass?
 - (c) When does the mass first return to its equilibrium position?
5. Find the Laplace transform of the following functions:
 - (a) $f(t) = (t - 2)(t^2 - 2)$
 - (b) $f(t) = 3 \cos^2 t$
 - (c) $f(t) = e^{(3t + \frac{1}{2})} e^{4t}$
 - (d) $f(t) = \cos(2t + \frac{\pi}{6})$
6. Find the inverse transform of the following functions:
 - (a) $F(s) = \frac{s - 2}{s^2 - 25}$
 - (b) $F(s) = \frac{1}{s + 10} - \frac{5}{s^4}$
 - (c) $F(s) = \frac{s + 2}{s^3 + 2s}$