

COM-440, Introduction to Quantum Cryptography, Fall 2025

Exercise Solution # 12

1. Message authentication codes

- (a) Suppose that Alice and Bob have a preshared key $k \leftarrow \text{Gen}(1^\lambda)$.

When one of them needs to send a message m through CAC to the other one, the sender can send m together with $\sigma \leftarrow \text{Tag}_k(m)$, and the receiver on receiving a message m together with σ , checks whether $\text{Ver}_k(m, \sigma) = \text{"accept"}$, and accepts the message m if so.

In this way, Eve cannot make the receiver accept a message m that Alice and Bob never intended to send. This establishes a classical authenticated channel.

- (b) Consider the following MAC scheme based on a PRF.

- $\text{Gen}(1^\lambda)$ samples a key k uniformly at random from $\{0, 1\}^{m(\lambda)}$ where $m(\lambda)$ is the key length of the PRF.
- $\text{Tag}_k(m)$ outputs a tag $\sigma = F_\lambda(k, m)$.
- $\text{Ver}_k(m, \sigma)$ outputs “accept” if and only if $\sigma = F_\lambda(k, m)$.

From the construction, we can see that the scheme satisfies correctness:

$$\Pr_{k \leftarrow \text{Gen}(1^\lambda)} [\text{Ver}_k(m, \text{Tag}_k(m)) = \text{"accept"}] = 1 .$$

Moreover, for every quantum polynomial-time \mathcal{A} ,

$$\Pr_{\substack{G \leftarrow_U \{0,1\}^n \rightarrow \{0,1\}^\ell \\ m, \sigma \leftarrow \mathcal{A}^{G(\cdot)}(1^\lambda)}} [\mathcal{A} \text{ did not query } m \wedge \sigma = G(m)] = \frac{1}{2^\ell} ,$$

because the function value of a uniformly random function G on a not queried position is uniformly random from the range.

Since PRF family cannot be distinguished from the uniformly random function G , we have that

$$\Pr_{\substack{k \leftarrow \text{Gen}(1^\lambda) \\ m, \sigma \leftarrow \mathcal{A}^{\text{Tag}_k(\cdot)}(1^\lambda)}} [\mathcal{A} \text{ did not query } m \wedge \text{Ver}_k(m, \sigma) = \text{"accept"}] \leq \text{negl}(\lambda) + \frac{1}{2^\ell} = \text{negl}(\lambda) ,$$

for $\ell(\lambda) = \Omega(\lambda)$. (If $\ell(\lambda) = o(\lambda)$, we can do parallel repetition to the PRF to increase the output length. Therefore, without loss of generality, we can assume $\ell(\lambda) = \Omega(\lambda)$.)

So the construction satisfies both correctness and security.

2. Bit commitment

- (a) Suppose for contradiction that the scheme is not hiding. Then there exists a computationally bounded Alice that on input $r^b \oplus F(k, 0^n)$, outputs b with probability more than $\frac{1}{2} + \frac{1}{\text{poly}(\lambda)}$, where r^b denotes the all zero string 0^ℓ .

Then we can construct an adversary for the underlying PRF as follows: it samples $b \leftarrow \{0, 1\}$ and runs Alice to get r . Then it makes a query to the oracle on 0^n to get a output y , and then runs Alice on input $v = r^b \oplus y$. If Alice returns b , then the adversary outputs 1; otherwise, the adversary outputs 0.

It is easy to see that the above adversary outputs 1 with probability more than $\frac{1}{2} + \frac{1}{\text{poly}(\lambda)}$ if given access to the PRF, and outputs 1 with probability exactly $\frac{1}{2}$ if given access to a uniformly random function G (since the distribution of $v = r^b \oplus y$ does not depend on b if y is sampled uniformly at random). Thus this gives an efficient adversary for the PRF, which contradicts with the security of the PRF.

Therefore, this scheme is hiding.

- (b) For binding, we can consider computationally unbounded adversary.

Bob can open the commitment in two differently ways if and only if for r chosen by Alice, there exists k_1, k_2 such that $r \oplus F(k_1, 0^n) = F(k_2, 0^n)$. In other words, $r \in R := \{F(k_1, 0^n) \oplus F(k_2, 0^n) : k_1, k_2 \in \{0, 1\}^m\}$. Note that R is a set of size at most 2^{2m} , so for the honest Alice, the event that the randomness r that Alice samples satisfies that $r \in R$ happens with probability at most $\frac{2^{2m}}{2^\ell} = \frac{1}{2^m}$.

Therefore, $p_0 + p_1 \leq 1 + \frac{1}{2^m}$, and thus the scheme is ε -binding for $\varepsilon = \frac{1}{2^m}$.