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Discrete Mathematics Test

Q1. $A \times (B \cup C) = (A \times B) \cup (A \times C)$

Q2. Prove that set of parallel lines in a plane forms an equivalence relation

Q3. Prove that $\sqrt{5}$ is an irrational number

Q4. Solve recurrence relation:

$$a_r - 7a_{r-1} + 12a_{r-2} = 0 \quad ; \quad a_0 = 3 \; ; \; a_1 = 5$$

Q5. Draw the hasse diagram for D_{64} and determine whether it is
i) Complete lattice ii) Modular lattice.

A1. Taking LHS

let $(x, y) \in A \times (B \cup C)$

$\Rightarrow x \in A$ and $y \in (B \cup C)$

$\Rightarrow x \in A$ and $y \in B$ or $y \in C$

$\Rightarrow x \in A$ and $y \in B$ or $x \in A$ and $y \in C$

$\Rightarrow (x,y) \in (A \times B)$ or $(x,y) \in (A \times C)$

$\Rightarrow (x, y) \in (A \times B) \cup (A \times C)$

$\therefore A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ — (1)

Taking RHS

$\Rightarrow (p, q) \in (A \times B) \cup (A \times C)$

$\Rightarrow (p, q) \in (A \times B)$ or $(p, q) \in (A \times C)$

$\Rightarrow p \in A$ and $q \in B$ or $p \in A$ and $q \in C$

$\Rightarrow p \in A$ and $q \in B$ or $q \in C$

$\Rightarrow p \in A$ and $q \in (B \cup C)$

$\Rightarrow (p, q) \in A \times (B \cup C)$

$(A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$ — ②

From eq^n ① and ②,

$A \times (B \cup C) = (A \times B) \cup (A \times C)$ Ag

A2 i) Reflexive :

~~let a line l_1 in a plane~~

let (l_1, l_2) be a set of $\parallel$ lines in a pl

Since, the line is parallel to itself.

$\therefore (l_1, l_1)$ is true.

$\therefore$ It is a reflexive

ii) Symmetric :

let an ordered set (l_1, l_2) be a pair of $\parallel$ lin

Since, $l_1 \parallel l_2$

$\therefore l_2 \parallel l_1$

$\therefore (l_2, l_1)$ exists.

$\therefore$ It is symmetric

iii) Transitive :

let (l_1, l_2) and (l_2, l_3) be a two pairs of $\parallel$ lines in a plane.

$\therefore$ $l_1 \parallel l_2$ and $l_2 \parallel l_3$

$\Rightarrow$ $l_1 \parallel l_2 \parallel l_3$

$\Rightarrow$ $l_1 \parallel l_3$

$\therefore$ (l_1, l_3) is also a pair of $\parallel$ lines.

$\therefore$ It is transitive.

$\because$ The relation is reflexive, symmetric and transitive
Hence, it is an equivalence relation

let $\sqrt{5}$ be a rational number

$\therefore$ $\sqrt{5} = \dfrac{p}{q}$ ($q \neq 0$, and p & q do not have common factor)

On squaring both side

$5 = \dfrac{p^2}{q^2}$

$p^2 = 5q^2$

$\because$ p^2 is a multiple of 5

$\therefore$ p is also a multiple of 5

$\therefore$ $p = 5m$

$\therefore$ $(5m)^2 = 5q^2$

$5m^2 = q^2$

$\because$ q^2 is a multiple of 5.

$\therefore$ q is also a multiple of 5

$\therefore$ $q = 5k$

Since, p and q have a common factor 5. So, it contradicts our assumption.

$\therefore$ $\sqrt{5}$ is an irrational number.

A4.

$$a_r - 7a_{r-1} + 12a_{r-2} = 0$$

Characteristics eqⁿ is given by

$$\alpha^2 - 7\alpha + 12 = 0$$

$$\alpha^2 - 4\alpha - 3\alpha + 12 = 0$$

$$\alpha(\alpha - 4) - 3(\alpha - 4) = 0$$

$$(\alpha - 4)(\alpha - 3) = 0$$

$$\alpha = 4, 3.$$

$$\therefore \quad a_r = A_1 \cdot 4^r + A_2 \cdot 3^r$$

For $r = 0$, $a_0 = 3$

$A_1 + A_2 = 3$

For $r = 1$, $a_1 = 5$

$4A_1 + 3A_2 = 5$

$$A_1 + A_2 = 3 \qquad \times 3$$

$$4A_1 + 3A_2 = 5$$

$$\Rightarrow \quad 3A_1 + \cancel{3A_2} = 9$$

$$4A_1 + \cancel{3A_2} = 5$$

$$- \quad - \quad -$$

$$-A_1 = 4$$

$$\boxed{A_1 = -4}$$

$$-4 + A_2 = 3$$

$$\boxed{A_2 = 7}$$

$$\therefore \quad a_r = -4 \cdot 4^r + 7 \cdot 3^r$$

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