# Discrete Mathematics (BCSC 1010)

## Assignment-2

## Recurrence Relations and Generating Functions

Q1. Consider this problem, which was originally posed by Leonardo Pisano, also known as Fibonacci, in the thirteenth century in his book Liber abaci. A young pair of rabbits (one of each sex) is placed on an island. A pair of rabbits does not breed until they are 2 months old. After they are 2 months old, each pair of rabbits produces another pair each month as shown in Figure 1. Find a recurrence relation for the number of pairs of rabbits on the island after n months, assuming that no rabbits ever die.

| Reproducing pairs (at least two months old) | Young pairs (less than two months old) | Month | Reproducing pairs | Young pairs | Total pairs |
|---|---|---|---|---|---|
| | 🐰🐰 | 1 | 0 | 1 | 1 |
| | 🐰🐰 | 2 | 0 | 1 | 1 |
| 🐰🐰 | 🐰🐰 | 3 | 1 | 1 | 2 |
| 🐰🐰 | 🐰🐰 🐰🐰 | 4 | 1 | 2 | 3 |
| 🐰🐰 🐰🐰 | 🐰🐰 🐰🐰 🐰🐰 | 5 | 2 | 3 | 5 |
| 🐰🐰 🐰🐰 🐰🐰 | 🐰🐰 🐰🐰 🐰🐰 🐰🐰 🐰🐰 | 6 | 3 | 5 | 8 |

**FIGURE 1** Rabbits on an Island.

Q2. A popular puzzle of the late nineteenth century invented by the French mathematician Édouard Lucas, called the Tower of Hanoi, consists of three pegs mounted on a board together with disks of different sizes. Initially these disks are placed on the first peg in order of size, with the largest on the bottom (as shown in Figure 2). The rules of the puzzle allow disks to be moved one at a time from one peg to another as long as a disk is never placed on top of a smaller disk. The goal of the puzzle is to have all the disks on the second peg in order of size, with the largest on the bottom.

Q3. Find a recurrence relation and give initial conditions for the number of bit strings of length n that do not have two consecutive 0s. How many such bit strings are there of length five?

Q4. A computer system considers a string of decimal digits a valid codeword if it contains an even number of 0 digits. For instance, 1230407869 is valid, whereas 120987045608 is not valid. Let $a_n$ be the number of valid n-digit codewords. Find a recurrence relation for $a_n$.

Q5. Suppose that the roots of the characteristic equation of a linear homogeneous recurrence relation are 2, 2, 2, 5, 5, and 9 (that is, there are three roots, the root 2 with multiplicity three, the root 5 with multiplicity two, and the root 9 with multiplicity one). What is the form of the general solution?