SET THEORY PROBLEM SET

Problem Set 4.1

1. Describe each of the following sets in roster form
   (*i*) A = {*x* : *x* is an even prime}
   (*ii*) B = {*x* : *x* is a prime number, $10 < x < 20$}
   (*iii*) C = {$x \in R : x^2 - 1 = 0$}
   (*iv*) D = {*x* : *x* is an integer, $-3 < x < 3$}
   (*v*) E = {*x* : *x* is a perfect square, $x < 30$}
   (*vi*) F = {*x* : *x* is a positive integral divisor of 60}
   (*vii*) G = {*x* : ($x \geq 0$) and ($x^2 \leq 10$), $x \in Z$}
2. Describe the following sets in set-builder form
   (*i*) A = {2, 4, 6, 8, 10}
   (*ii*) B = {1, 1/2, 1/3, 1/4.1/5, ...}
   (*iii*) C = {3, 5, 7, 9,.....87, 89}
   (*iv*) D = {–5, –4, –3, –2, –1, 0, 1, 2}
3. Which of the following statements are true?
   (*a*) $b = \{b\}$ (*b*) $4 \in \{4\}$ (*c*) $4 \in \{1, 2, 3, \{4\}\}$ (*d*) $\phi \in \{\phi\}$
   (*e*) $\phi \in \{\{\phi\}\}$
4. (*a*) List the subsets of the set A = {1, 2, 3}
   (*b*) List the subset of the set B = {1, {2, 3}}
   (*c*) List the subsets of the set C = {{1, 2, 3}}
5. (*a*) Let $B \subseteq A$, for any set C, prove (*i*) $B \cup C \subseteq A \cup C$ (*ii*) $B \cap C \subseteq A \cap C$
   (*b*) Can a set be a proper subset of itself? Explain.
6. If A = {*w*, *x*, *y*} (*a*) List each proper subset B of A (*b*) List each proper subset of A which contains w as an element.
7. Let A = {1, 2, 3, 4, 5, 6} B = {3, 6, 8, 12, 17, 18} and C = {2, 3, 5, 6, 12, 17}. Then find
   (*i*) $A \cup B$ (*ii*) $A \cap B$ (*iii*) $A \cap C$ (*iv*) $B \cup C$
   (*v*) A – B (*vi*) B – A (*vii*) C – B
8. Let A = {1, 2, 3, 4, 5} B = {3, 4, 5, 6} and C = {$x \in Z \mid x$ is even}, find
   (*i*) A – B (*ii*) $B \cap A$ (*iii*) C – B (*iv*) $(A \cap B) \cup C$
9. If A = {1, 2, 3, 4}, B = {2, 3, 4, 5} and C = {3, 4, 5, 6}, find
   A – B, B – C, A – (B – C), (A – B) – C.
10. Let U = {0, 1, 2, 3, ..., 9}, A = {1, 2, 3}, B = {2, 4} and C = {1, 5, 9} then find
    (*i*) $A \cup B$ (*ii*) $A \cap C$ (*iii*) A – B (*iv*) A^C
    (*v*) U – {$A \cap B$}
11. If A = {1, 2, 3, 4} B = {2, 4, 6, 8} and C = {3, 4, 5, 6} find
    (*i*) $(A \cap B) \cap C$ (*ii*) $A \cap (B \cap C)$ (*iii*) $(A \cup B) \cap C$ (*iv*) $(A \cap B) \cup C$
12. Given A = {1, 2, 3, 4, 5, 6} B = {2, 4, 6, 8, 10} and C = {3, 6, 9} state the elements of each of the following:
    (*a*) $A \cap B$ (*b*) $B \cap C$ (*c*) $A \cap C$ (*d*) $A \cap B \cap C$
    (*e*) $A \cap (B \cup C)$ (*f*) $B \cup (A \cap C)$

13. If $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 9\}$, $C = \{3, 4, 5, 6\}$, find

(*a*) $A \cup B$ (*b*) $B \cup C$ (*c*) $A \cup (B \cup C)$ (*d*) $(A \cup B) \cup C$

(*e*) $A \cap B$ (*f*) $B \cap C$ (*g*) $A \cap (B \cap C)$ (*h*) $(A \cap B) \cap C$

(*i*) $A - B$ (*j*) $B - A$ (*h*) $B - B$

14. List the symmetric difference $A \Delta B$ if $A = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $B = \{0, 2, 4, 6, 8\}$

15. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{4, 5, 6, 7\}$ find

(*i*) $(A \cup B) \Delta A$ (*ii*) $(A \Delta B) \Delta B$ (*iii*) $(A \Delta B) \Delta A$ (*iv*) $A \Delta (A \Delta B)$

(*v*) $(A \Delta B) \Delta B$ (*vi*) $B \Delta (A \Delta B)$

16. Prove that

(*i*) $A - (B \cup C) = (A - B) \cap (A - C)$

(*ii*) $(A \cap B) - C = (A - C) \cap (B - C)$

(*iii*) $A \cap (B - C) = (A \cap B) - (A \cap C)$

(*iv*) $A - (B - C) = (A - B) \cup (A \cap C)$

(*v*) $A \cap (B - C) = (A \cup B) - (C - A)$

(*vi*) $A \cap (B - C) = (A \cap B) - (A \cap C)$

(*vii*) $A - B = B' - A'$

17. Prove the following

(*i*) $\{x : |x - 3| < 5\} = \{x : x < 8\} \cap \{x : x > -2\}$

(*ii*) $\{x : |x - 1| > .5\} = \{x : x > 1.5\} \cup \{x : x < .5\}$

(*iii*) $\{x : |x| \geq 5\} \cap \{y : |y| \leq 5\} = \{-5, 5\}$

18. If $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{1, 5, 6, 7, 8\}$, $B = \{0, 1, 6, 7\}$, $\{C = \{1, 2, 3, 5, 8\}$, verify that

(*i*) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(*ii*) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(*iii*) $(A \cap B)' = A' \cup B'$

19. Prove each of the following

(*i*) $(A - B) \cap (B - A) = \phi$

(*ii*) $(A \cap B) \cap (A - B) = \phi$

(*iii*) $A = (A \cap B) \cup (A - B)$

20. Prove that

(*a*) $A \Delta \phi = A$

(*b*) $A \Delta A = \phi$

(*c*) $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$

21. If $U = \{1, 2, 3, 4, 5, 6, 7\}$, $A = \{1, 2, 3, 5\}$ $B = \{1, 3, 4, 6\}$ and $C = \{1, 2, 4, 7\}$, find (*i*) $n(A \cup B \cup C$ (*ii*) $n(A \cap (B \cup C))$ (*iii*) $n(A \cap B' \cap C)$

22. Out of 80 students in a class, 60 play football, 53 play hockey, and 35 both the games. How many students

(*a*) do not play these games,

(*b*) play only hockey but not football

23. In a group of 400 people, 250 can speak in English only and 70 can speak Hindi only.
    (*a*) How many can speak English?
    (*b*) How many can speak Hindi?
    (*c*) How many can speak both English and Hindi?

24. A small town has a total population 25000, out of which 13000 read 'The Statesman' and 10500 read 'The Hindustan Times' whereas 3500 read both papers. Find the percentage of population who read neither of these papers

25. 900 students appeared for two papers in Mathematics. 740 students passed in paper I and 660 passed in paper II. If 640 students passed in both, find the number of students who failed in both.

26. Among the positive integers less than or equal to 100, let A be the set of even integers and let B be the set of integers divisible by 5. How many integers are even or divisible by 5?

27. In a group of 70 persons, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. Calculate (*a*) how many people like both coffee and tea and (*b*) how many like coffee but not tea.

28. How many seven-digit telephone numbers are there that begin 347-and contain at least one 0 and at least one 1?

29. (*a*) In a group of 70 cars tested by a garage in Delhi 15 had faulty tyres, 20 had faulty brakes and 18 exceed the allowable emission limits. Also, 5 cars had faulty tyres and brakes, 6 failed on tyres and emission, 10 failed on brakes and emissions, and 4 cars were unsatisfactory in all three respects. How many cars had no faults in these three checks? Draw an appropriate Venn diagram.

    (*b*) In a group of students, 70 have a personal computer, 120 have a personal stereo and 41 have both. How many own at least one of these devices? Draw an appropriate Venn diagram.

ANSWERS 4.1

1. (*i*) A = {2} (*ii*) B = {11, 13, 17, 19} (*iii*) C = {–1, 1}
(*iv*) D = {–2, –1, 0, 1, 2} (*v*) E = {1, 4, 9, 16, 25} (*vi*) F = {1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60}
(*vii*) {0, 1, 2, 3}

2. (*i*) A = {$x : x$ is even integer < 11}
(*ii*) B = {$x : x = 1/n$, n is a natural number}
(*iii*) C = {$2x + 1 : 1 \le x \le 44$}
(*iv*) D = {$x : -5 \le x \le 2$ and $x \in Z$}

3. (*b*), (*d*)

4. (*a*) Φ, {1}, {2}, {3}, {1, 2}, {2, 3}, {1, 3}, {1, 2, 3} (*b*) Φ, {1} {2, 3}, {1, {2, 3}}
(*c*) Φ, {1, 2, 3}

6. (*a*) B = {Φ, {*w*}, {*x*}, {*y*}, {*w*, *x*}, {*w*, *y*}, {*x*, *y*}}
(*b*) {{*w*}, {*w*, *x*}, {*w*, *y*}}

7. (*i*) {1, 2, 3, 4, 5, 6, 8, 12, 17, 18}
(*ii*) {3, 6}
(*iii*) {2, 3, 5, 6}
(*iv*) {2, 3, 5, 6, 8, 12, 17, 18}
(*v*) {{1, 2, 4, 5}
(*vi*) {8, 12, 17, 18}
(*vii*) {2, 5, 12, 17}

8. (*i*) {1, 2} (*ii*) {3, 4, 5} (*iii*) {0, ± 2, – 4, – 6, ± 8, ± 10, ± 12, ...}
(*iv*) {3, 5, 0, ± 2, ± 4, ± 6, ...}

9. {1}, {2}, {1, 3, 4}, {1}.

10. (*i*) {1, 2, 3, 4} (*ii*) {1} (*iii*) {1, 3} (*iv*) {0, 4, 5, ...9}
(*v*) {0, 1, 3, 4,.....,9}

11. (*i*) {4} (*ii*) {4} (*iii*) {3, 4, 6} (*iv*) {2, 3, 4, 5, 6}

12. (*a*) {2, 4, 6} (*b*) {6} (*c*) {3, 6} (*d*) {6}
(*e*) {2, 3, 4, 6} (*f*) {2, 3, 4, 6, 8, 10}

13. (*a*) {1, 2, 3, 4, 6, 9} (*b*) {2, 4, 6, 9, 3, 5} (*c*) {1, 2, 3, 4, 6, 9, 5} (*d*) {1, 2, 3, 4, 6, 9, 5}
(*e*) {2, 4} (*f*) {4, 6} (*g*) {4} (*h*) {4} (*i*) {1, 3}
(*j*) {6, 9} (*k*) Φ

14. {1, 3, 5, 7, 9}

15. (*i*) {6, 7} (*ii*) {6, 7} (*iii*) {1, 2, 3} (*iv*) {4, 5, 6, 7}
(*v*) {1, 2, 3, 4, 5} (*vi*) {1, 2, 3, 4, 5}

21. 7, 3, 1 **22.** 2, 18 **23.** 330, 150, 80 **24.** 20

25. 140 **26.** 60 **27.** 19, 18 **28.** 974

29. (*a*) 34
(*b*) 157