A Brief Overview of the Implemented Theory

- Within the assignment we consider phase transformation of spherical inclusion of radius ri, within an ideally elastic-plastic matrix material.

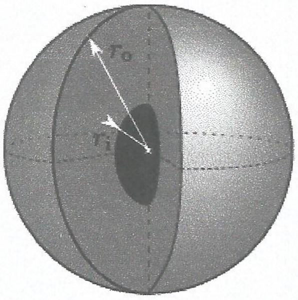


Figure 1: Inclusion in metallic matrix

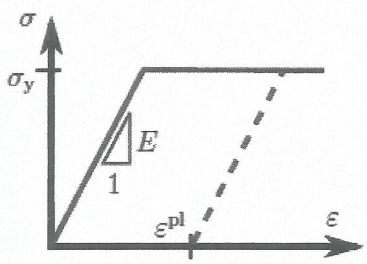


Figure 2: Elastic ideal-plastic material

- Non trivial equilibrium condition in a spherical coordinate system r-phi-theta.

$$0 = \frac{\partial(r^2\sigma_{rr})}{\partial r} - r\,(\sigma_{\phi\phi} + \sigma_{\theta\theta})$$

- With non vanishing stress component sigma_rr sigma_phiphi and sigma_thetatheta.
- Weak form of above equation is given by

$$0 = \delta W = \int_{r_i}^{r_o} \underline{\delta\varepsilon}^{\mathrm{T}} \cdot \underline{\sigma}\, r^2\,\mathrm{d}r - \left[r^2 \sigma_{rr} \delta u_r\right]_{r=r_i}^{r_o}$$

- Boundary condition for the problem are sigma_rr(r = ro) = 0 and ur(r=ri) = 1/3*tau*ev*ri
- Firstly, we discretize the weak form in space r.

$$\delta W = \int_{r_i}^{r_o} \delta\varepsilon^T \cdot \sigma\, r^2\, dr - \left[r^2 \sigma_{rr}\, \delta u_r \right]_{r_i}^{r_o}$$

$$\delta W = \int_{r_i}^{r_o} B^T \partial u^T \cdot \sigma\, (N^T r)^2 \cdot dr - 0$$

$$\delta W = \partial u^T \int_{r_i}^{r_o} B^T \cdot \sigma \cdot (N^T r)^2 \cdot dr - 0$$

- Then we can identify the B matrix and we can obtain strain.

$$\begin{Bmatrix} \varepsilon_{rr} \\ \varepsilon_{\phi\phi} \\ \varepsilon_{\theta\theta} \end{Bmatrix} = \begin{bmatrix} \frac{\partial u_r}{\partial r} \\ \frac{u_r}{r} \\ \frac{u_r}{r} \end{bmatrix} \Rightarrow u_r \begin{bmatrix} \frac{\partial}{\partial r} \\ \frac{1}{r} \\ \frac{1}{r} \end{bmatrix} = N.u. \begin{bmatrix} \frac{\partial}{\partial r} \\ \frac{1}{r} \\ \frac{1}{r} \end{bmatrix} = u. \begin{bmatrix} \frac{\partial N_1}{\partial \xi_e} \cdot \frac{\partial \xi_e}{\partial r_1} & \frac{\partial N_2}{\partial \xi_e} \cdot \frac{\partial \xi_e}{\partial r_2} \\ \frac{N_1}{N_1 r_1 + N_2 r_2} & \frac{N_2}{N_1 r_1 + N_2 r_2} \\ \frac{N_1}{N_1 r_1 + N_2 r_2} & \frac{N_2}{N_1 r_1 + N_2 r_2} \end{bmatrix}$$

$$\Rightarrow \boxed{\varepsilon = B \cdot u_e}$$

- With strain and stiffness matrix we obtain Fe_internal and Kt_e for each element.

$$K_{t\text{-}e} = \text{weight} * B^T * \text{C\_matrix} * B * J * (N^T \cdot r)^2$$

$$F_{int\text{-}e} = \text{weight} * B^T * \text{Stress} * J * (N^T \cdot r)^2$$

- We use Assignment matrix for obtaining global Kt and F_global_internal.

$$\underline{\underline{K_t}}\ \text{global} = \underline{\underline{K_t}}\ \text{global} + \underline{\underline{A}}^{eT} \cdot \underline{\underline{K_{t\text{-}e}}} \cdot \underline{\underline{A}}^{e}$$

$$\underline{F}_{int}\ \text{global} = \underline{F}_{int}\ \text{global} + \underline{\underline{A}}^{eT} \cdot \underline{F}_{int}^{e}$$

$$\underline{G}_K = \underline{F}_{int\text{-}global} - \underline{F}_{ext\text{-}}$$

- With help of G and Kt_global we obtain del_u. And apply Newton method to converge the value of u.

$$\underline{\underline{K_t}}\ \text{global} \cdot \Delta \underline{u} = -\underline{G}_K$$

$$\Delta u = \text{inv}(K_t\ \text{global reduced}) \cdot (-G_K\ \text{reduced})$$

$$u_{k+1} = u_K + \Delta u$$

Convergence Criterial $\quad \|\hat{\underline{G}}\|_\infty < 0.005 \|\hat{F}_{int}\|_\infty \ , \quad \|\Delta \hat{u}_K\|_\infty < 0.005 \|\hat{u}\|_\infty$

Program Structure:

- Parameter used in the assignment is of variant 2 from the Assignment of Parameter table.
  E = 70,000
  Poision\s ratio = 0.25
  sigma y = 70
  r internal = 10
  r external = 40
  volumetric strain = 0.01

- To start the program attach all file in the folder.
- To run the program run the main.m file and change the input from the script for obtaining plots related to the question.
- The file elementrou.m and materialrout.m are excesses in every question solution.

Inputs:

- To change value of other parameters can be done like change in number of elements can done from the main.m file script.

Outputs:

- Graphs are plotted for verification of results in each question and shown in solution part of the report.

Elementrout.m

- In Elementrout.m we calculate value N[N1, N2] for each element at one gauss point.
- For each element calculate Jacobian, B matrix, strain in each element with B*ue.
- To obtain stiffness matrix and stress matrix we call materialrout.m and input the value strain.
- Finally calculate Kt element and Finternal element and return this value to main program.

Materialrout.m

- With the input of strain from elementrout.m for each element, it computes sigma_trail, sigma_trail devitoric and sigma equivalent.
- With sigma equivalent we can obtain phi (phi = sigma_eq – sigma_y).
- Through phi we decide the element is elastic state or in plastic state then we obtain C matrix for element accordingly and update the plastic strain.

Solutions

a) **Verification of FEM solution by means of exact solutions within elastic range.**

i.e.

$$\bar{\varepsilon}_{\mathrm{v}} \; < \; \bar{\varepsilon}_{\mathrm{v}}^{\mathrm{init}}$$

Plasticity initiates at a dilation of the inclusion of

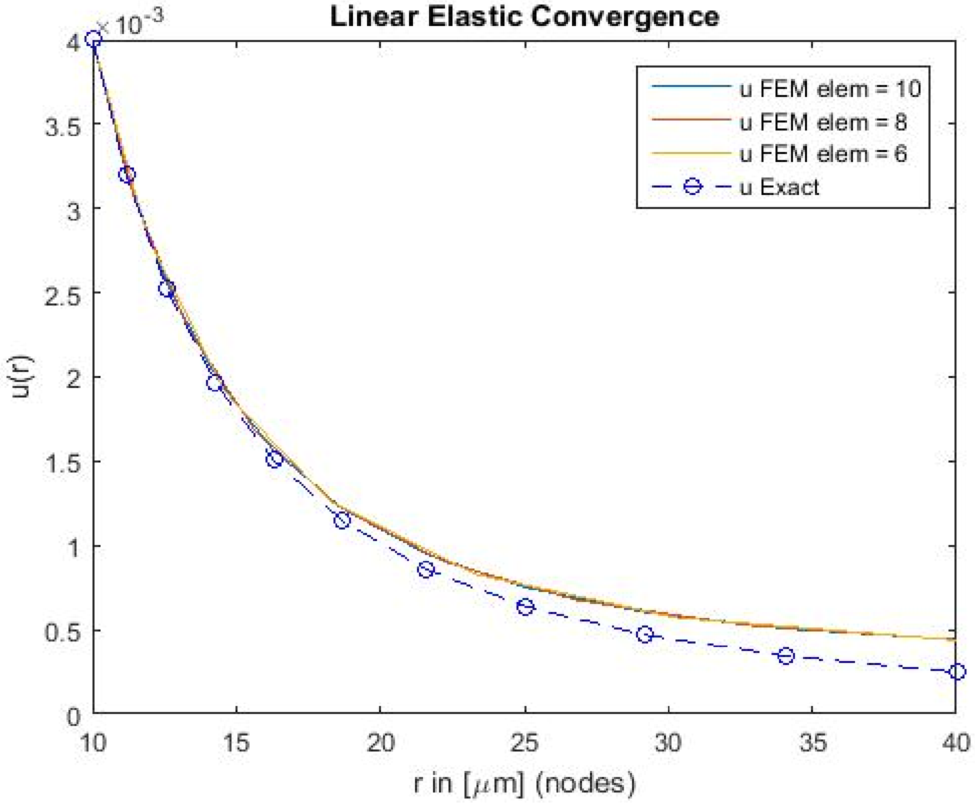
$$\bar{\varepsilon}_{\mathrm{v}}^{\mathrm{init}} = (1+\nu)\frac{\sigma_0}{E} \, ,$$

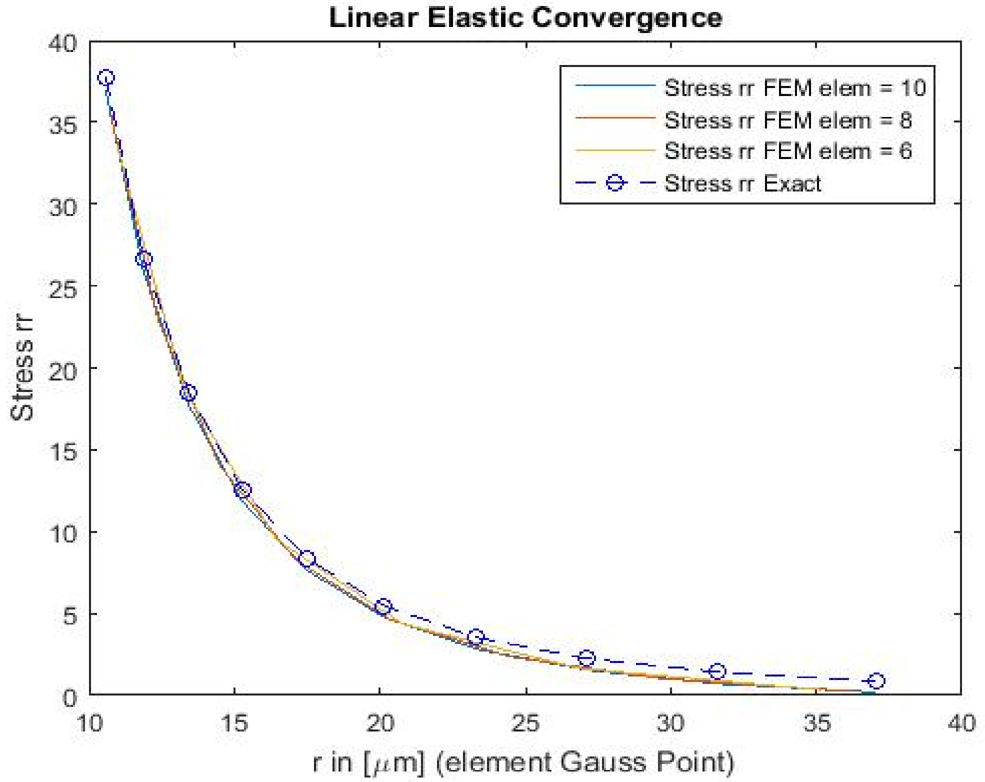
Value for volumetric strain for plotting graph used is 0.01.
& number of element used is 6, 8 and 10.
Exact solution equation used,

$$u_r^{\mathrm{elast}} = \frac{r_{\mathrm{i}}^3 \bar{\varepsilon}_{\mathrm{v}}}{3r^2} \, , \qquad \sigma_{rr}^{\mathrm{elast}} = -\frac{2E\bar{\varepsilon}_{\mathrm{v}}}{3(1+\nu)} \frac{r_{\mathrm{i}}^3}{r^3}$$

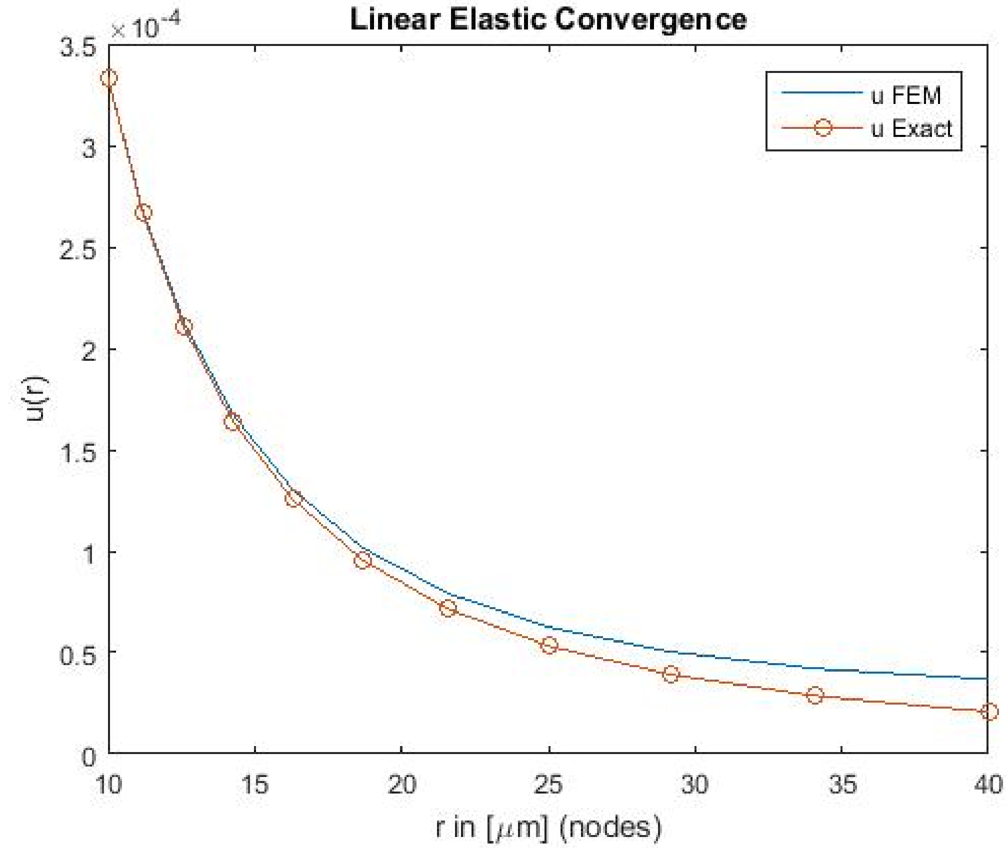
- Incremental value of volumetric strain with time variable tau = 0.01 is used to present the graphs.
- Verification for convergence of solution in 1st iteration for elastic regime can be confirmed from the result of program in command window. (Please select option 0 in main file for required result)
- With in Newton-Raphson Method value of U converges in 1st iteration for all value of volumetric strain in elastic regime.
- From the graph we can observe very slight difference in exact value and FEM obtain value is present.
- Difference between exact value of u and FEM value of u and between exact radial stress and FEM value of radial stress increases as we move away from the point of applied strain.
- This difference is constant irrespective of choice of number of elements.

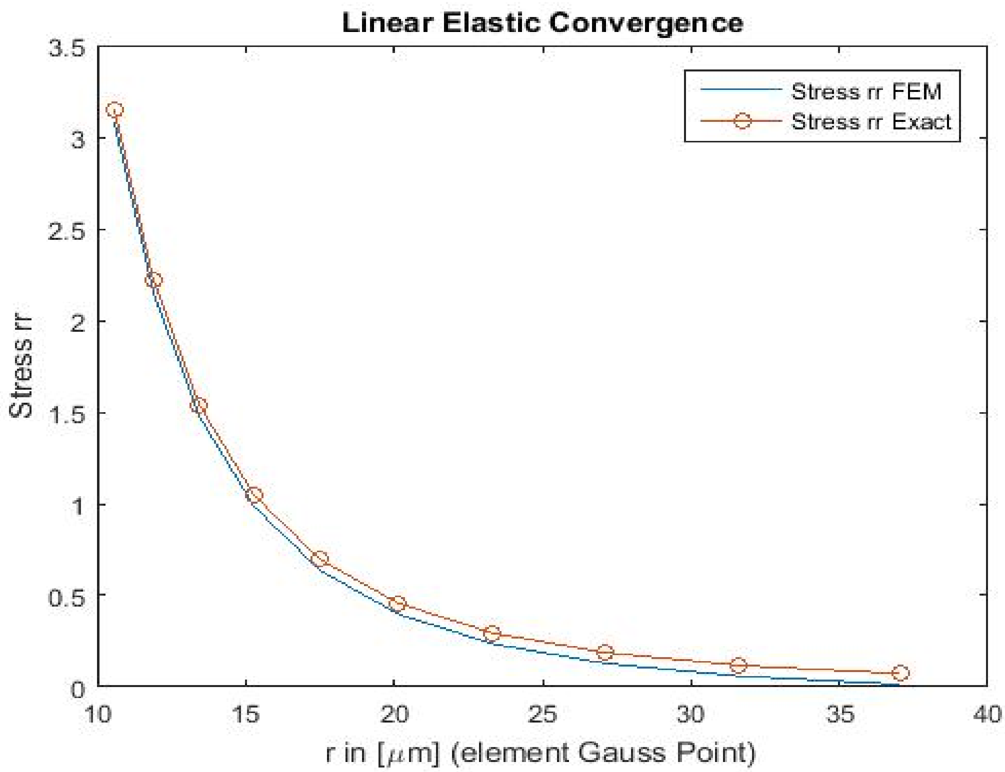
1. For 6, 8 and 10 element (verification in one graph)



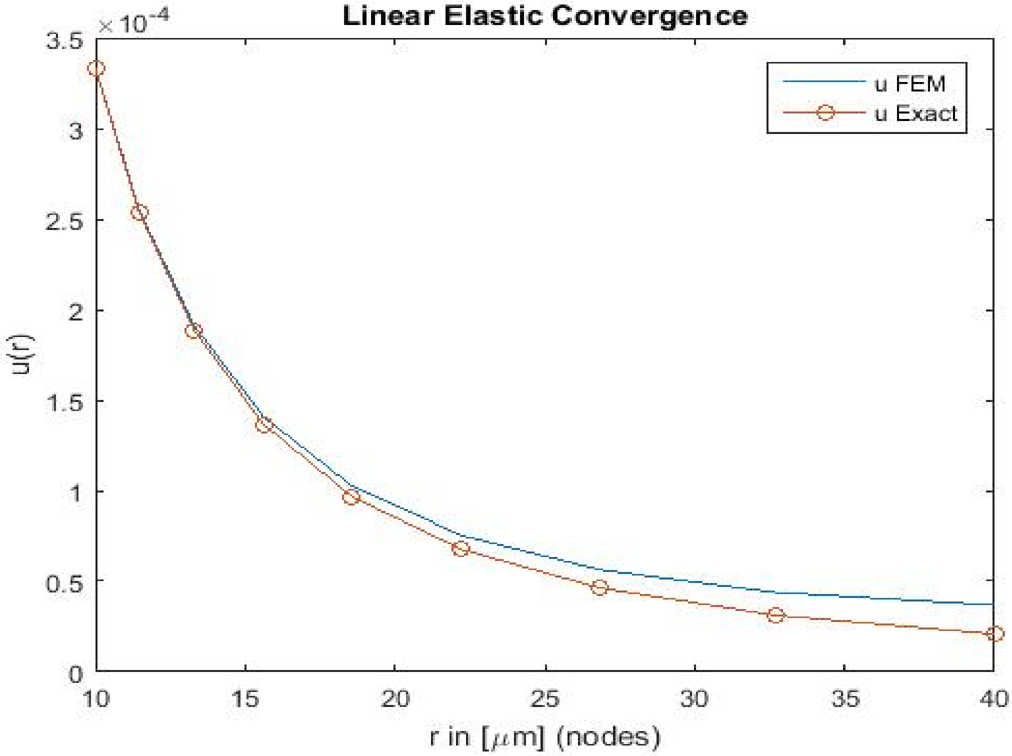


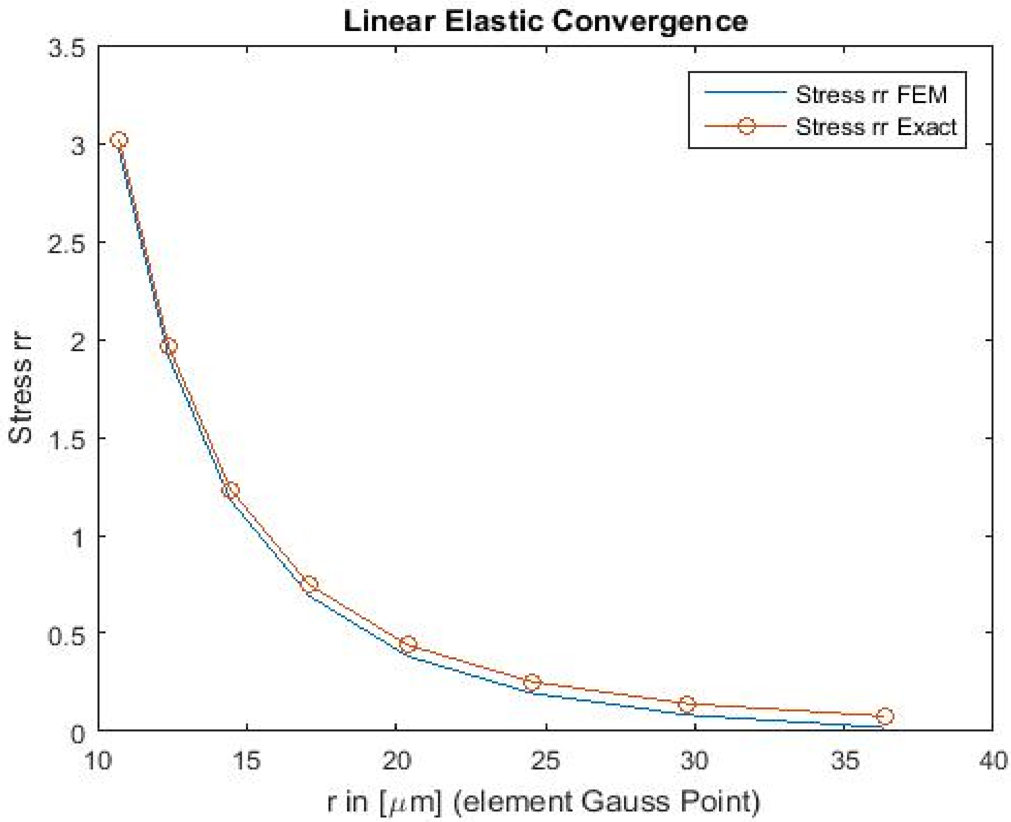
2. For 10 elements (verification in separate graph)



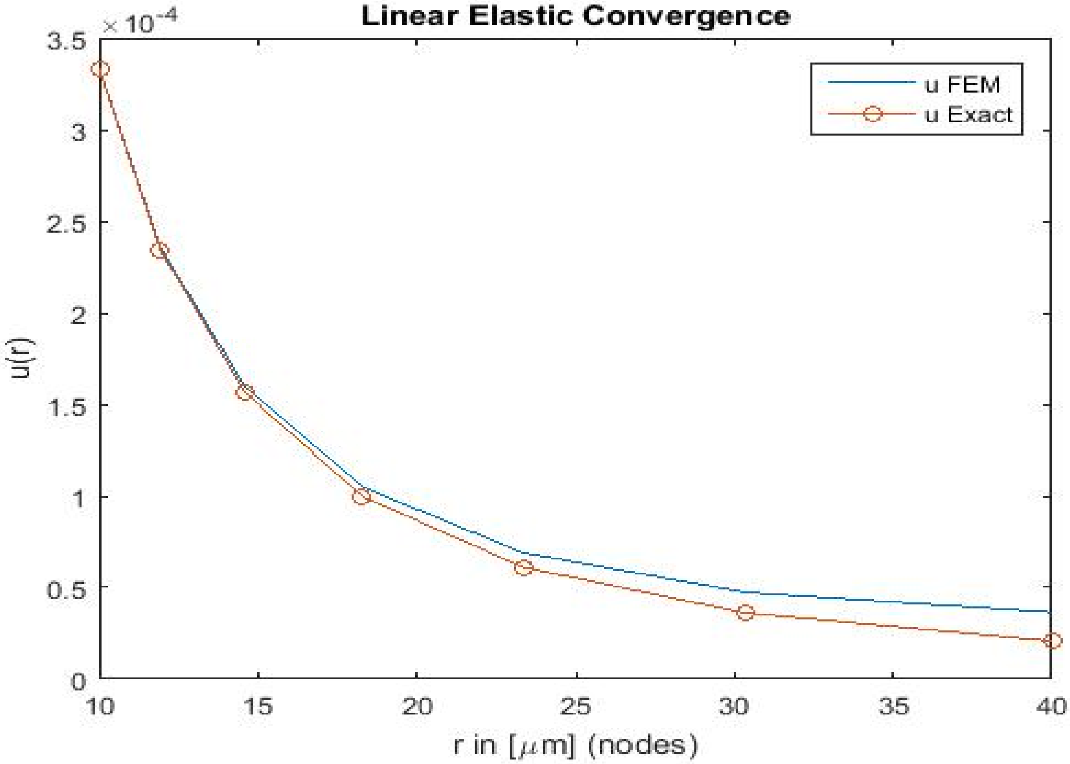


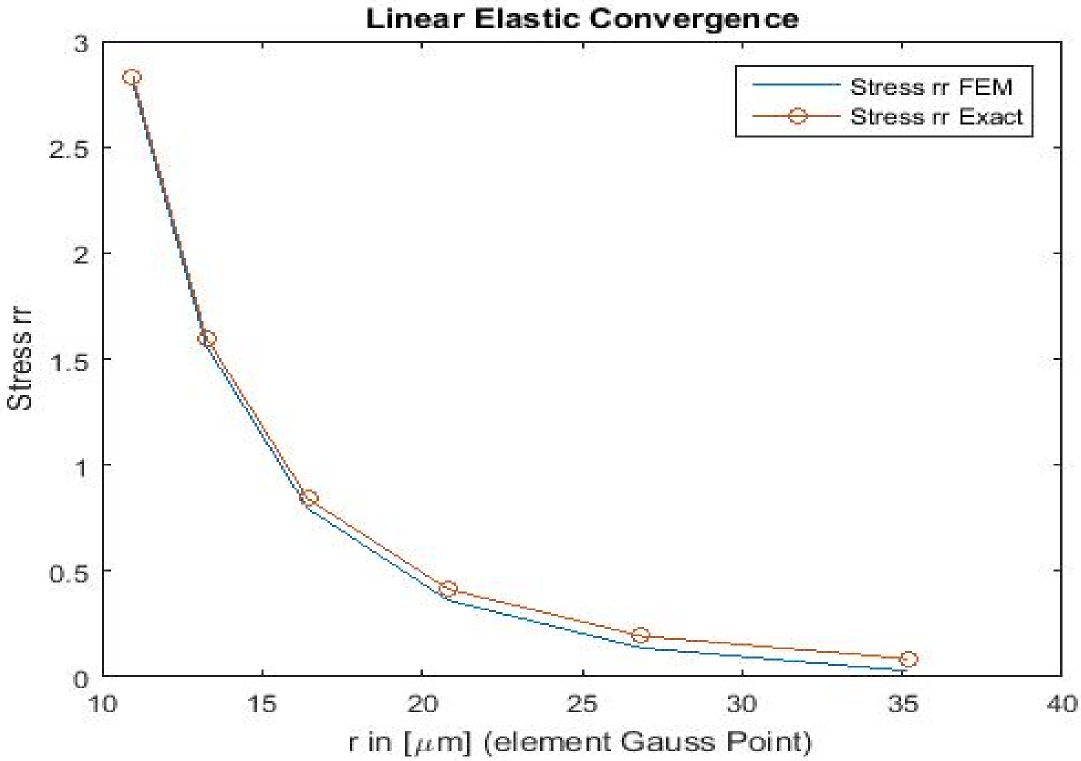
3. For 8 elements (verification in separate graph)





4. For 6 Elements (verification in separate graph)





b) **Convergence study for plastic regime with respect to different number of elements and value of delta tau.**

- In Plastic Regime, convergence study is performed with element no. 10, 20, 30 and delta tau 0.1, 0.01, 0.001.

<u>Expectations</u>

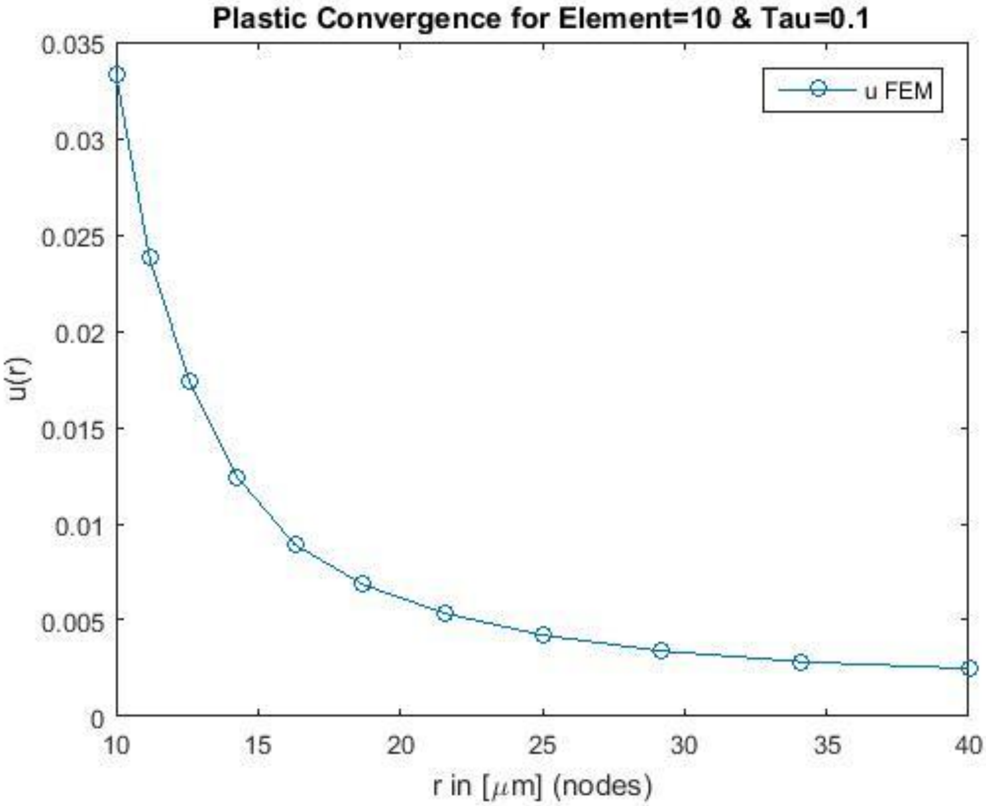
- According to Newton Raphson Method, it is assumed that if the solution for U is very far then it will not converge in 5 to 7 iteration.
- And if it converge, then larger the time step lead to greater number of Newton-Raphson iteration to converge for U.
- Boundary Condition will be satisfied after Newton-Rapshon iterations.

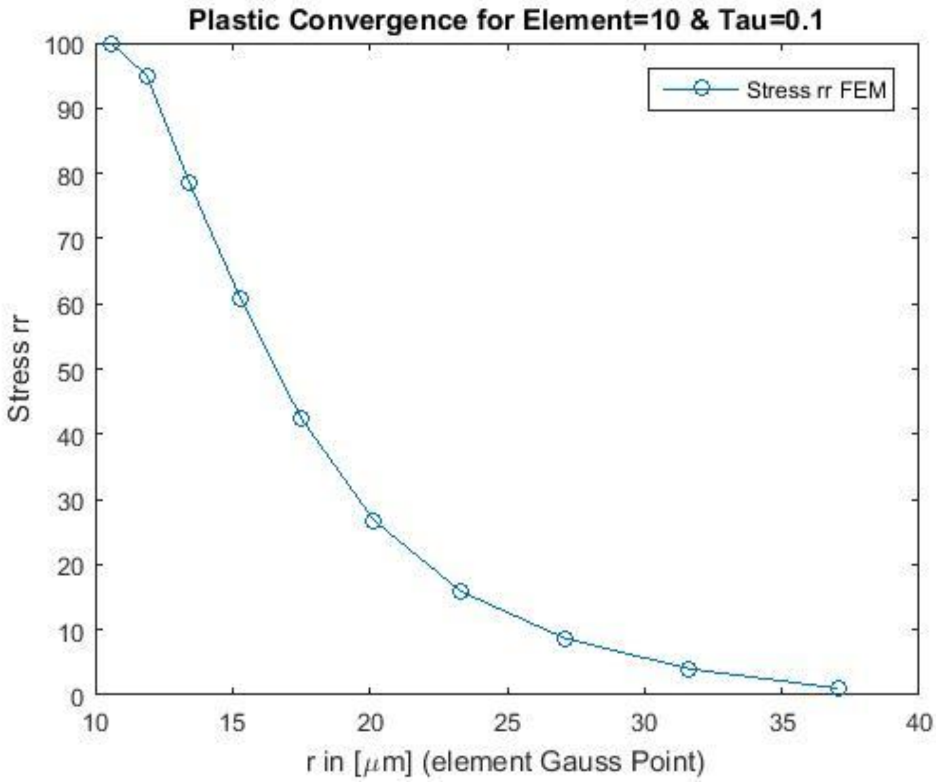
<u>Result after observation</u>

- Table below shows that number of iteration for solution of U reduces as smaller the time increment for volumetric strain is.
- Results can be verified from the output in command window.
  (Please choose 2 option in main program for required results and graph)
- Boundary Condition is satisfied by the solution as the value of stress at ro = 40mm is very small (near to zero).

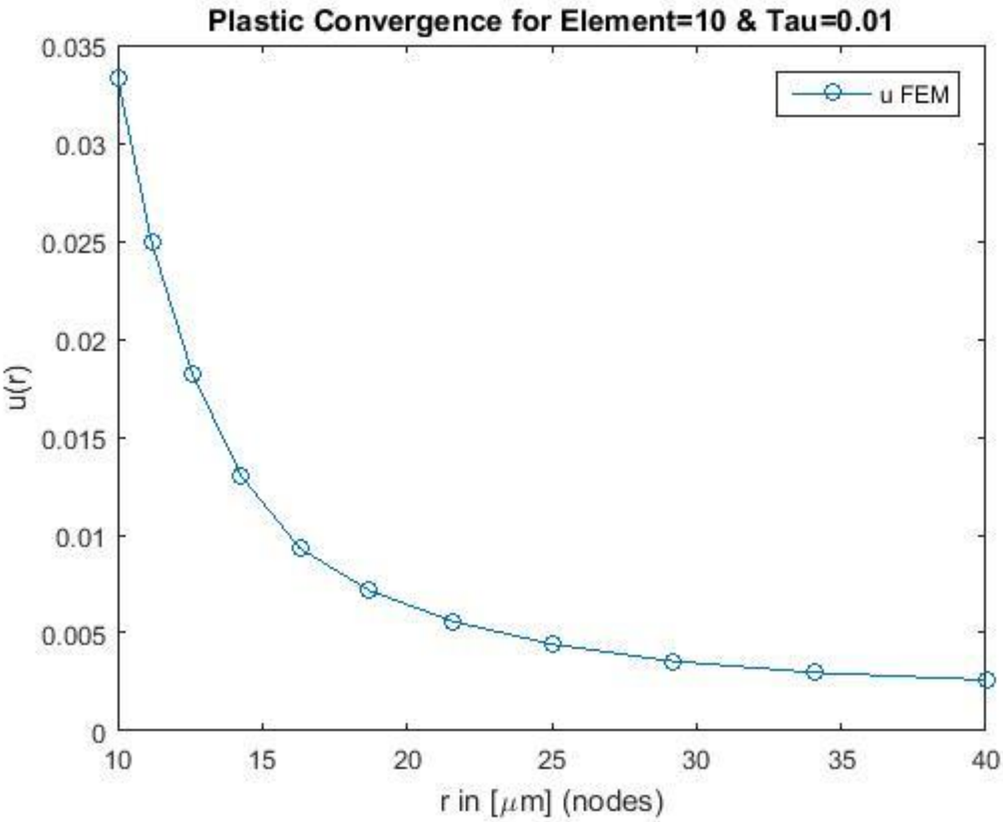
	Tau =0.1	Tau =0.01	Tau =0.001
Element = 10	3 iteration	1 or 2 iteration	0 or 1 iteration
Element = 20	3 iteration	1 or 2 iteration	0 or 1 iteration
Element = 30	3 iteration	2 iteration	0 or 1 iteration

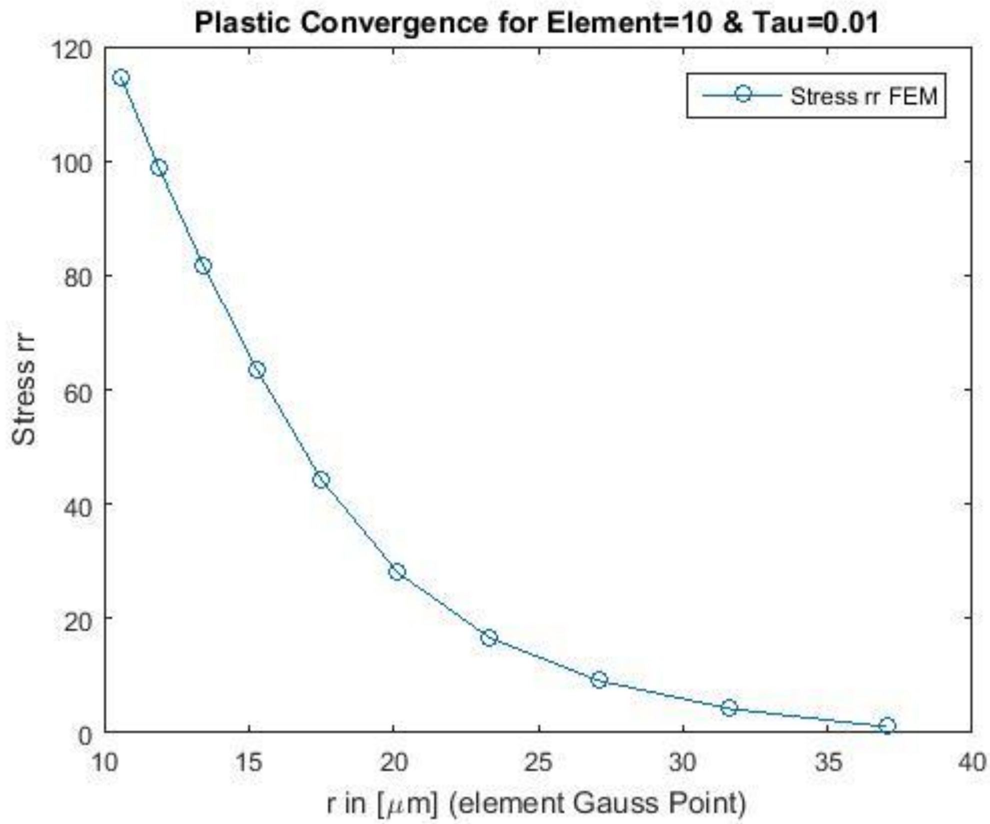
1. Plastic Convergence for Element =10 and tau=0.1



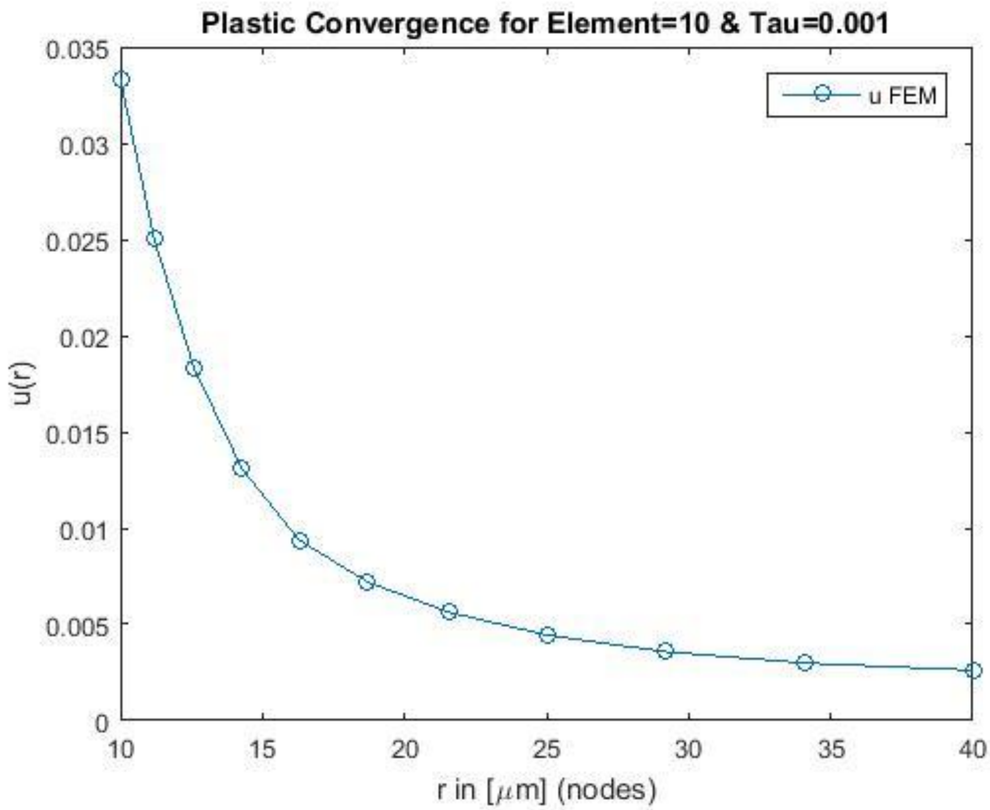


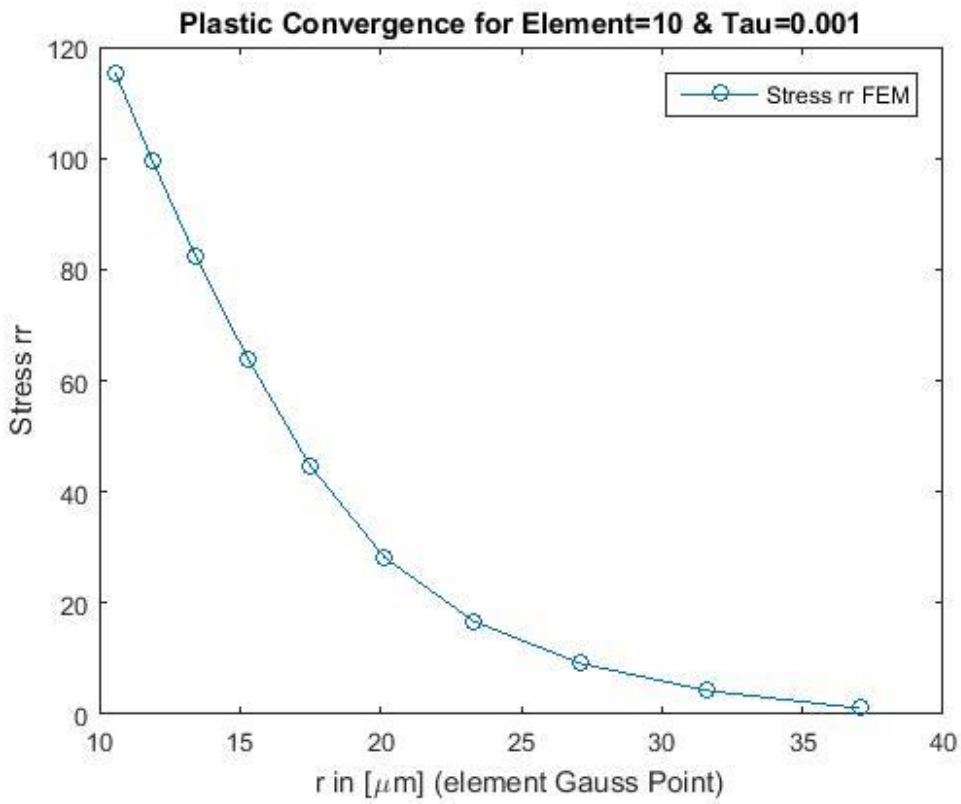
2. Plastic Convergence for Element =10 and tau=0.01



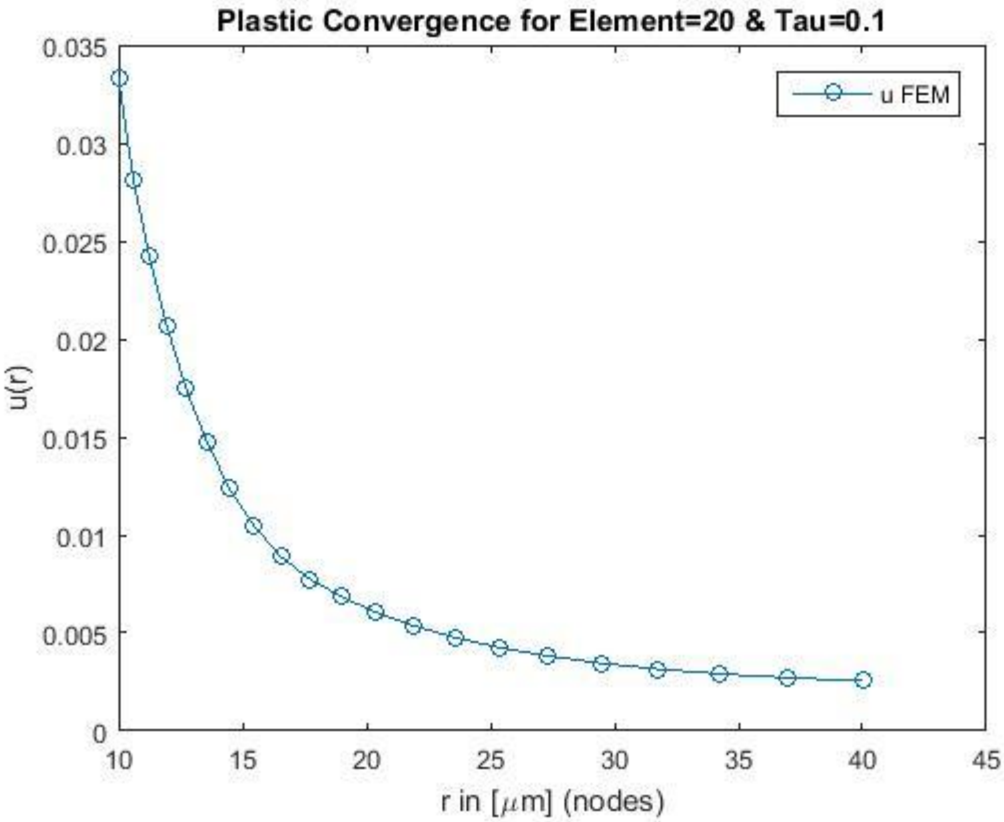


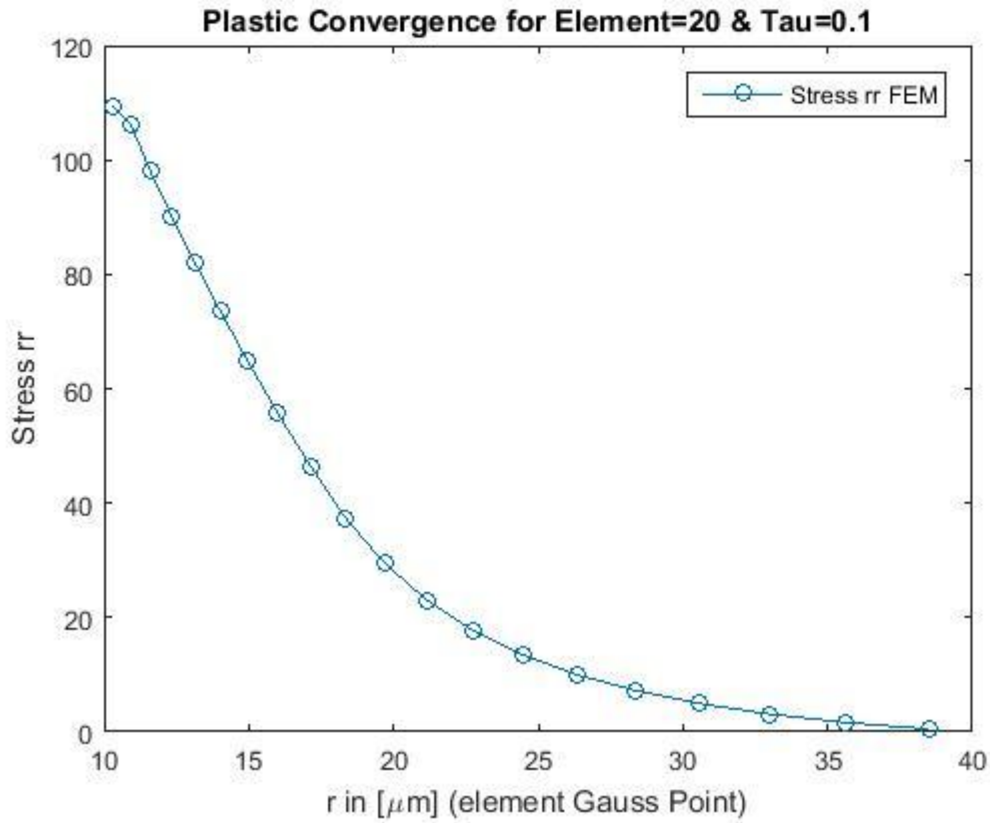
3. Plastic Convergence for Element =10 and tau=0.001



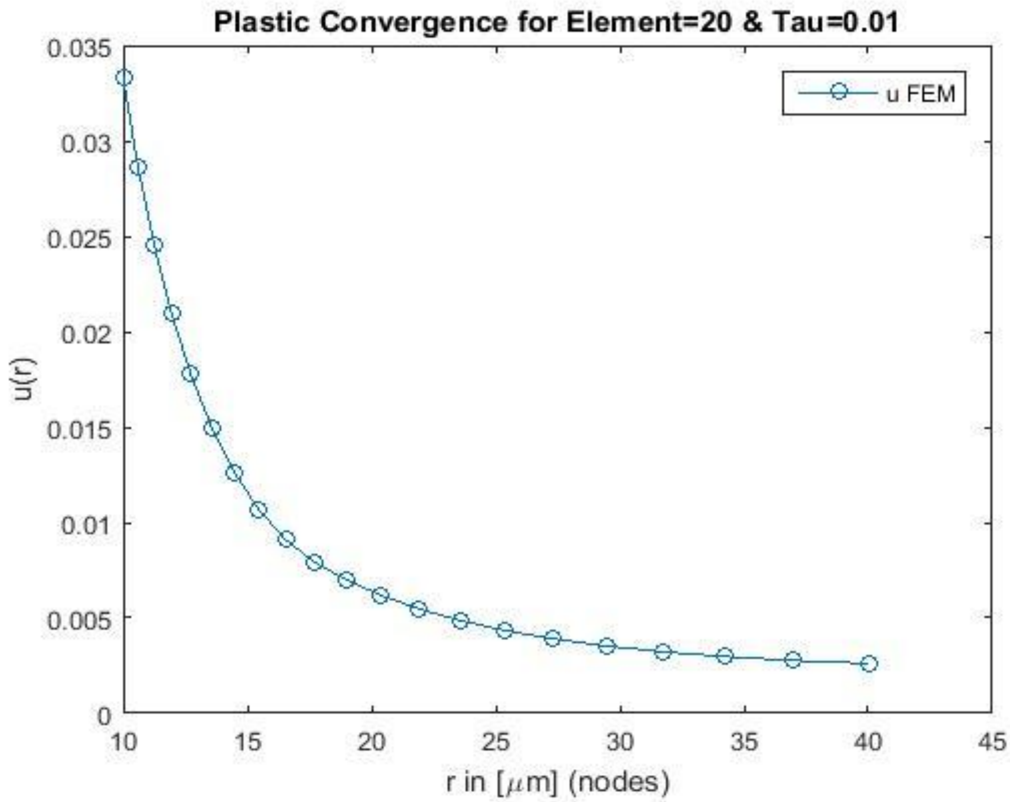


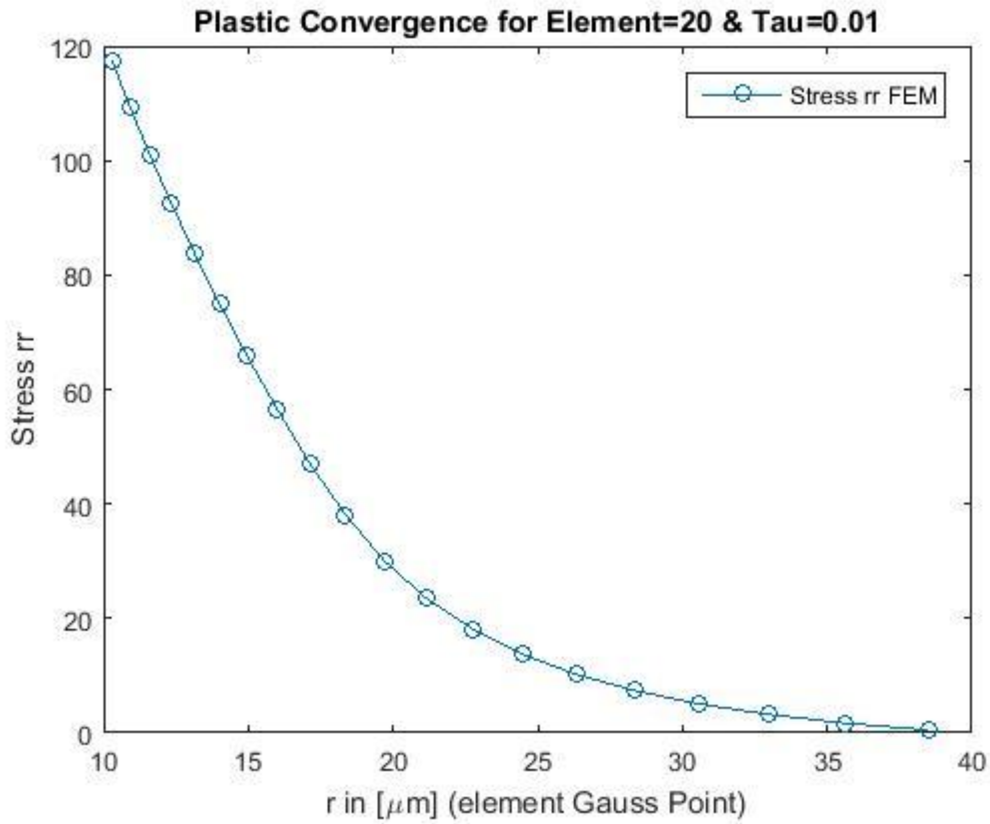
4. Plastic Convergence for Element =20 and tau=0.1



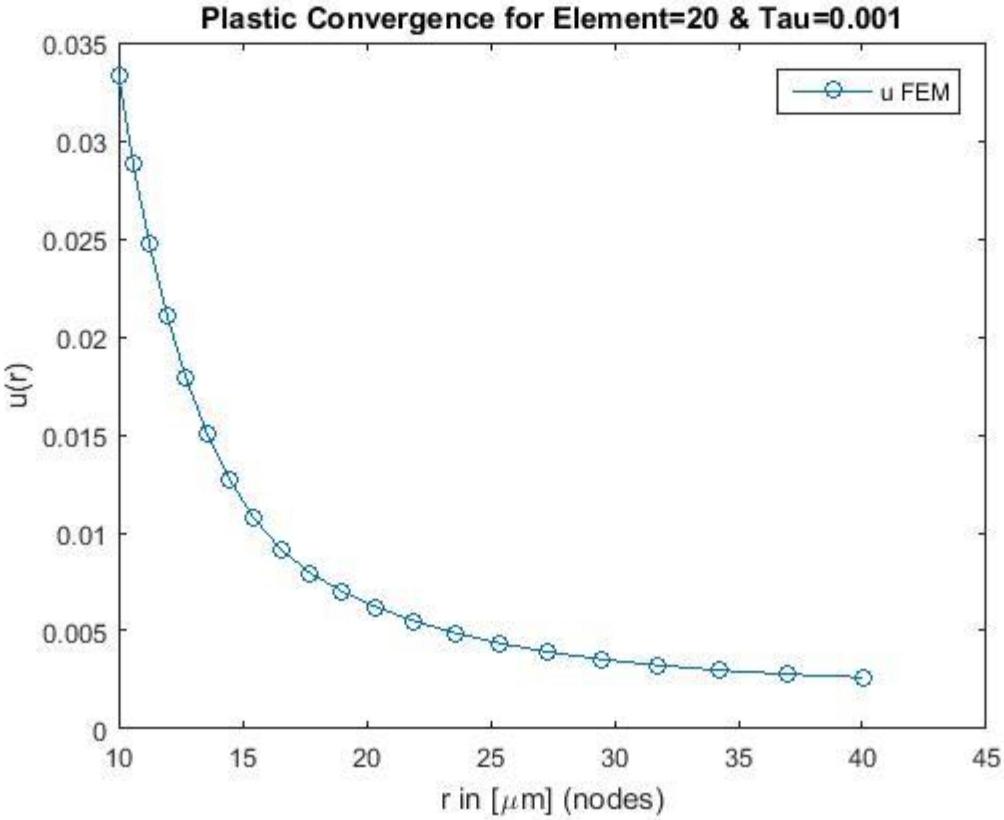


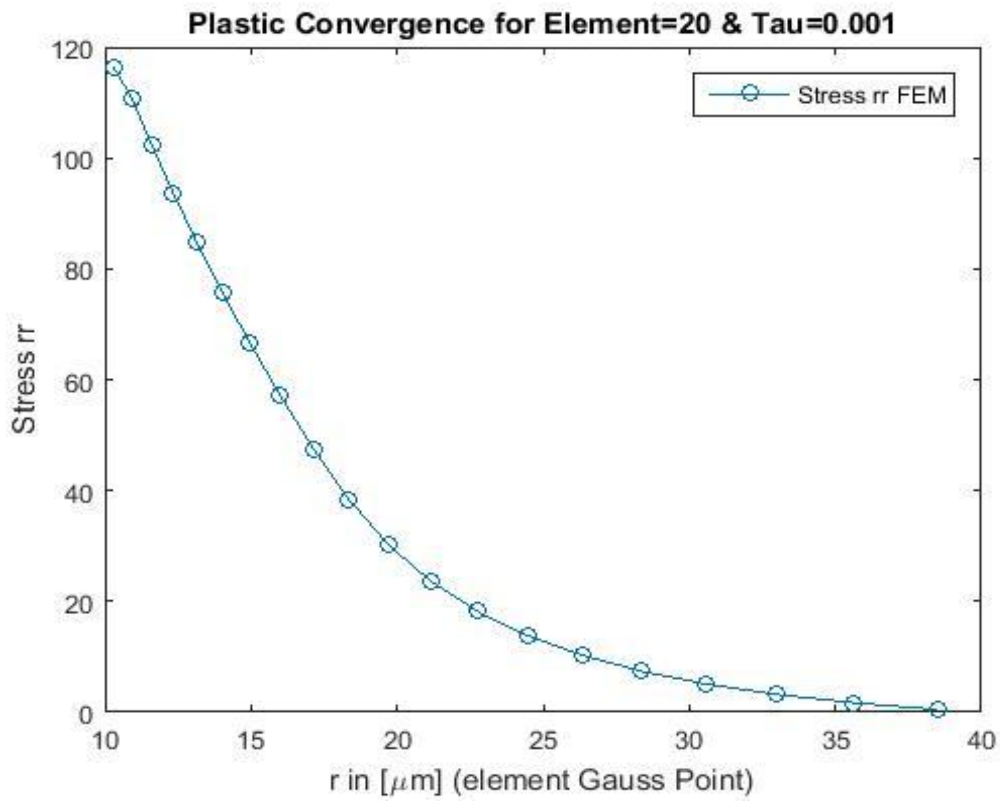
5. Plastic Convergence for Element =20 and tau=0.01



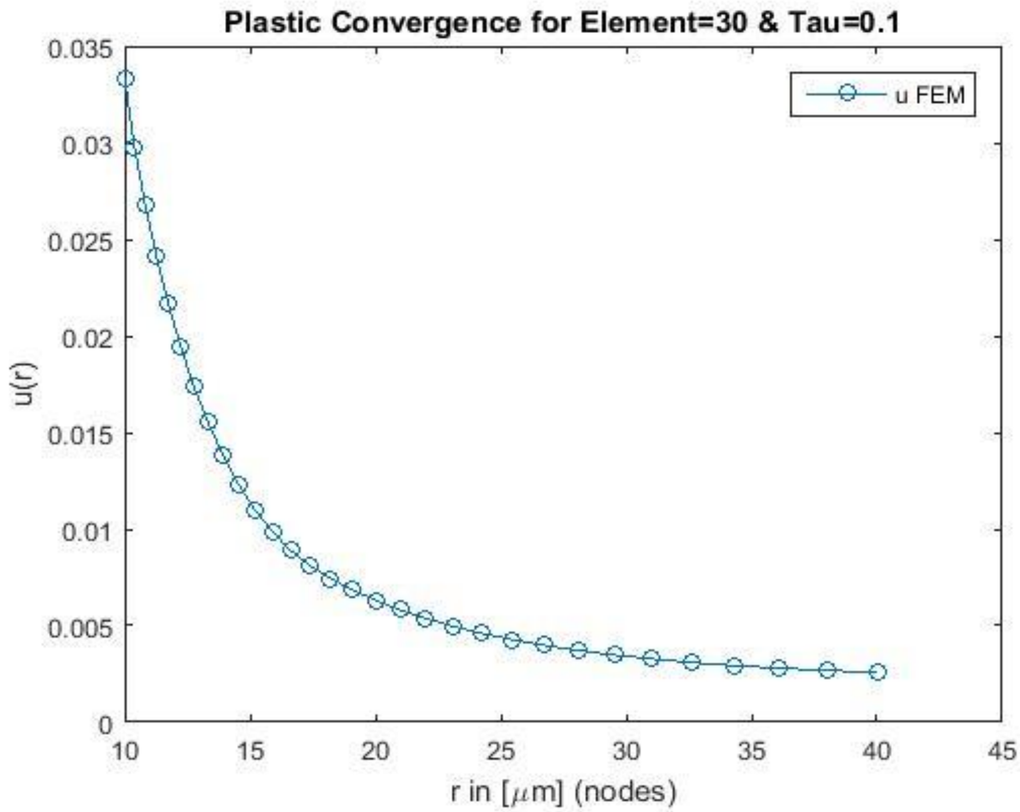


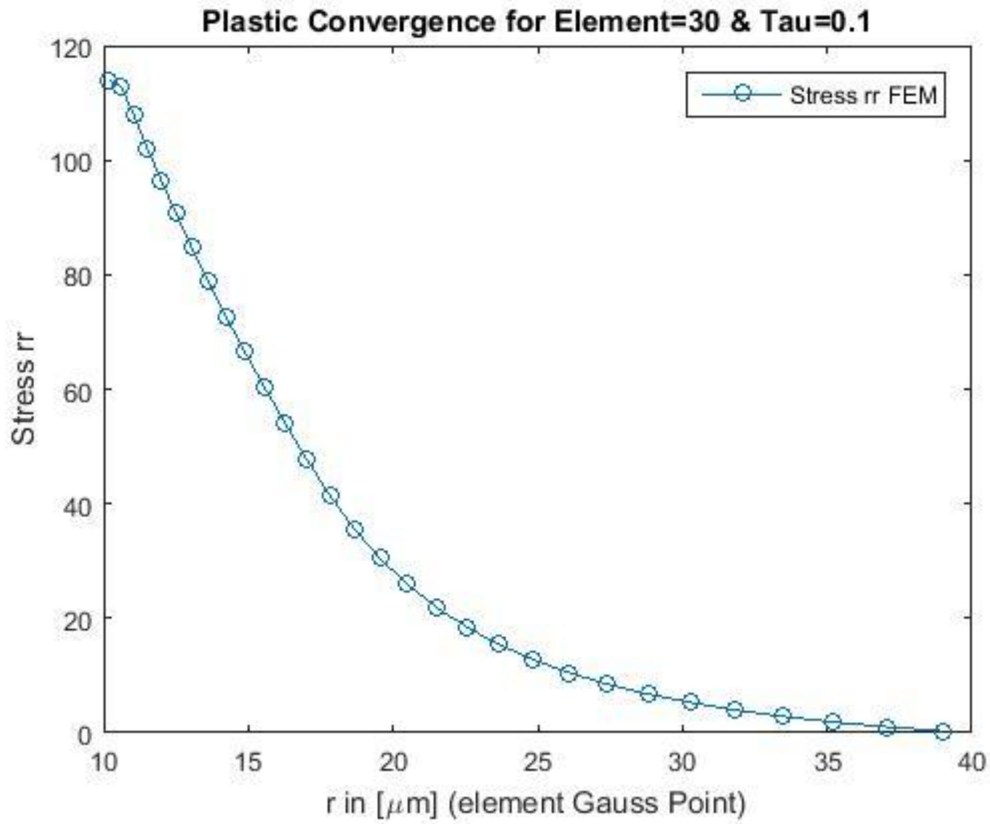
6. Plastic Convergence for Element =20 and tau=0.001



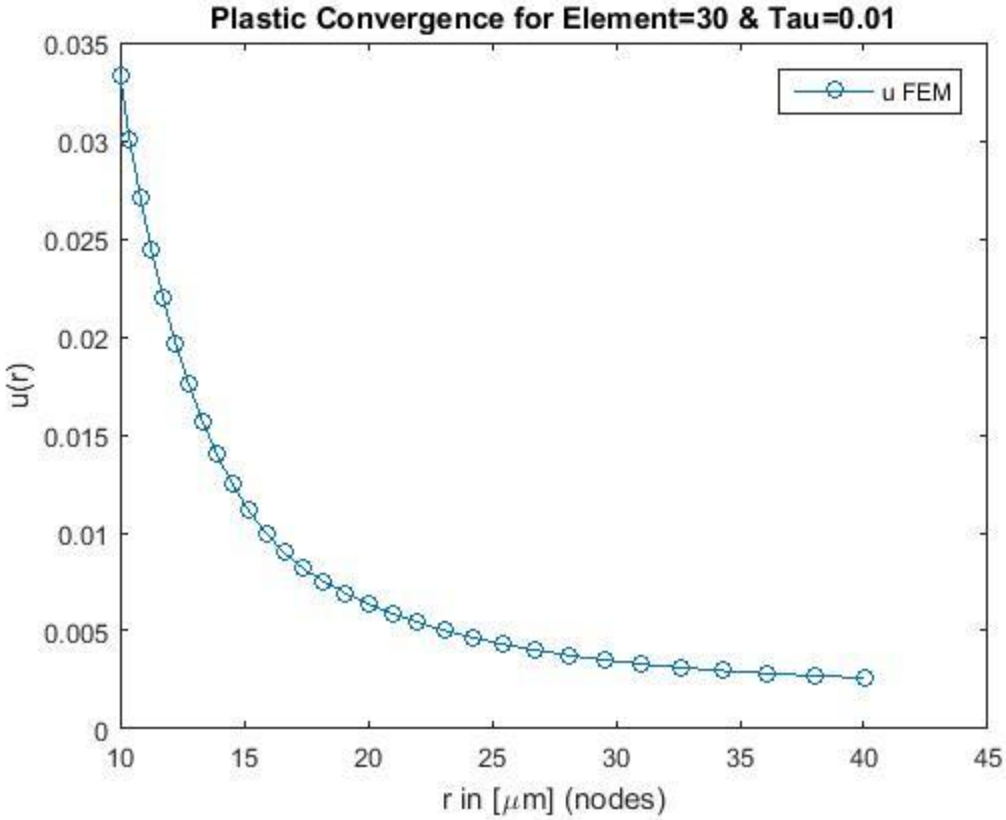


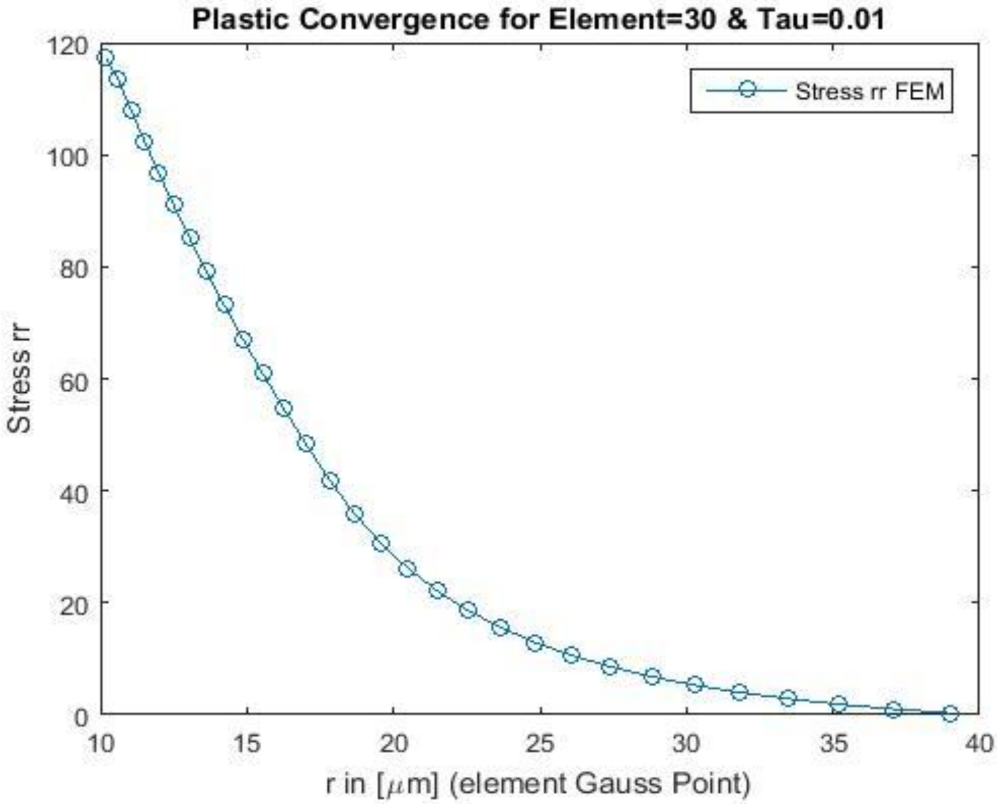
7. Plastic Convergence for Element =30 and tau=0.1



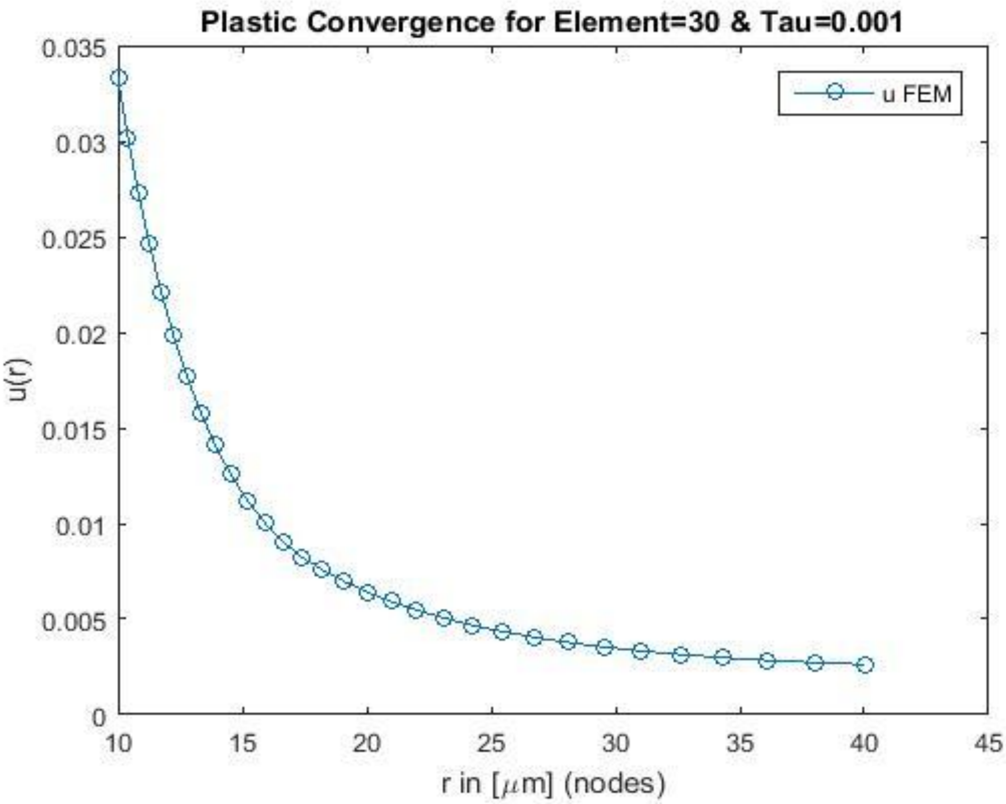


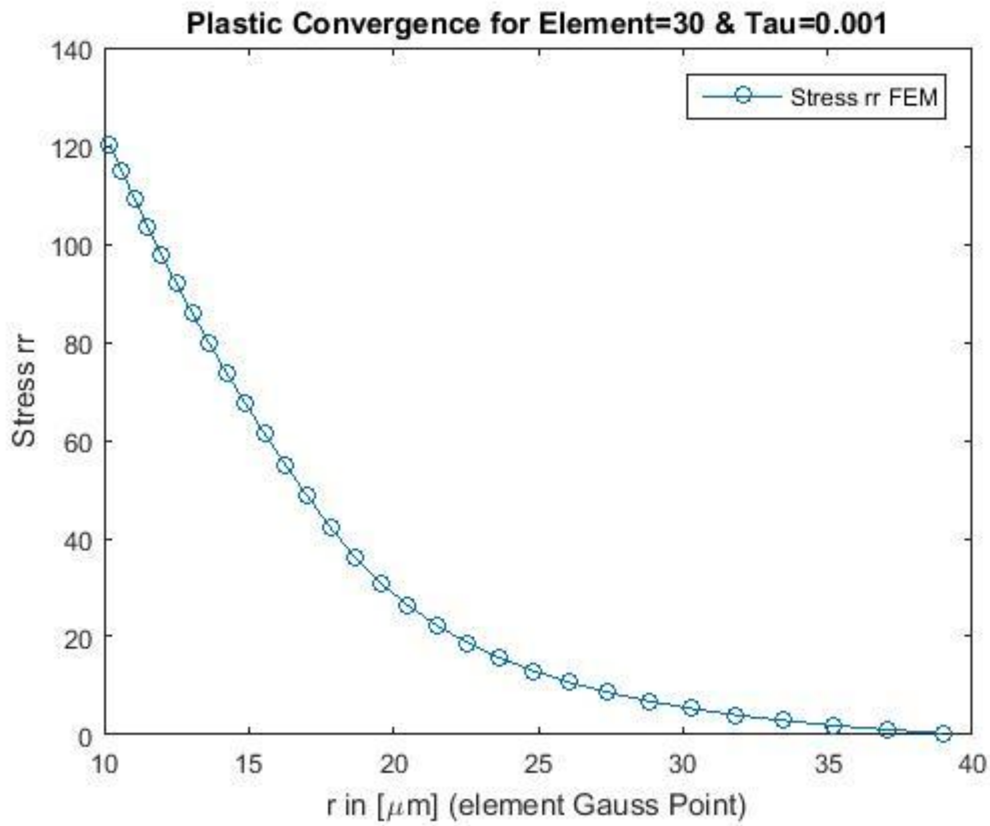
8. Plastic Convergence for Element =30 and tau=0.01.





9. Plastic Convergence for Element =30 and tau=0.001.

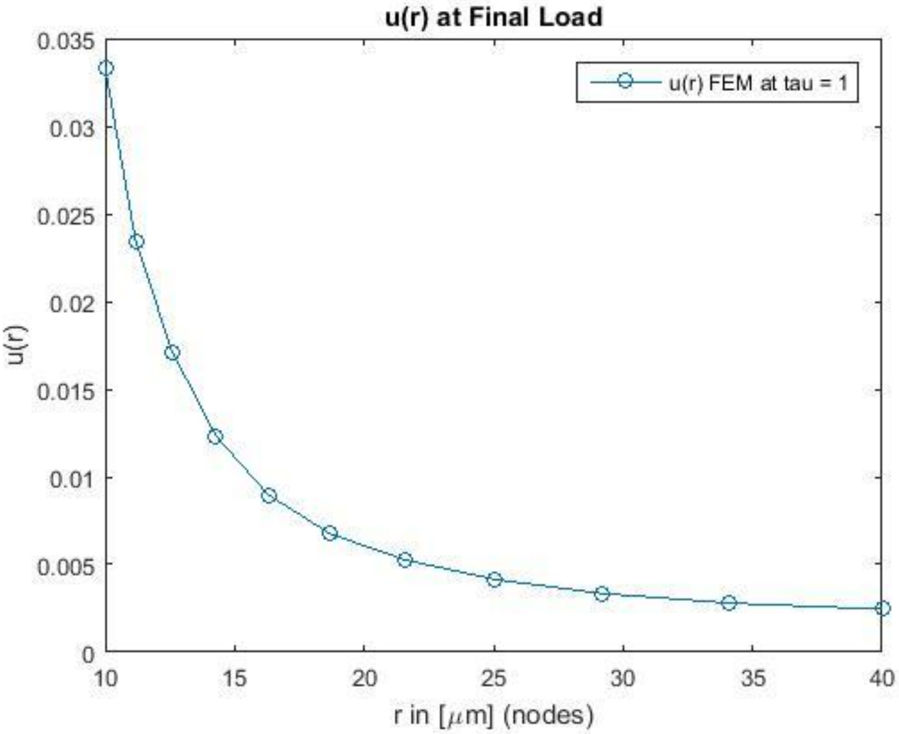


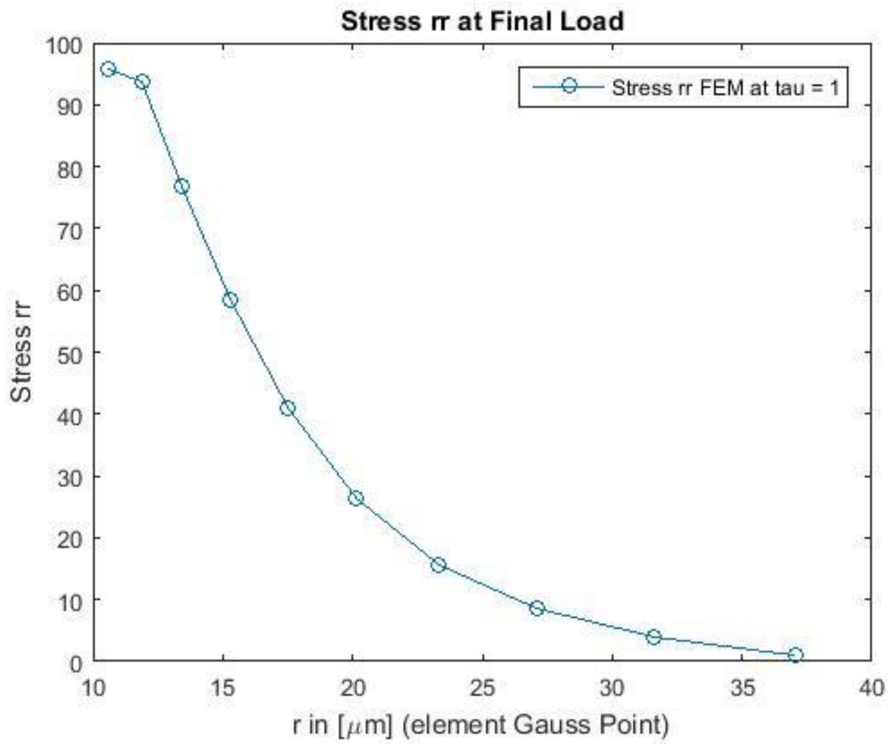


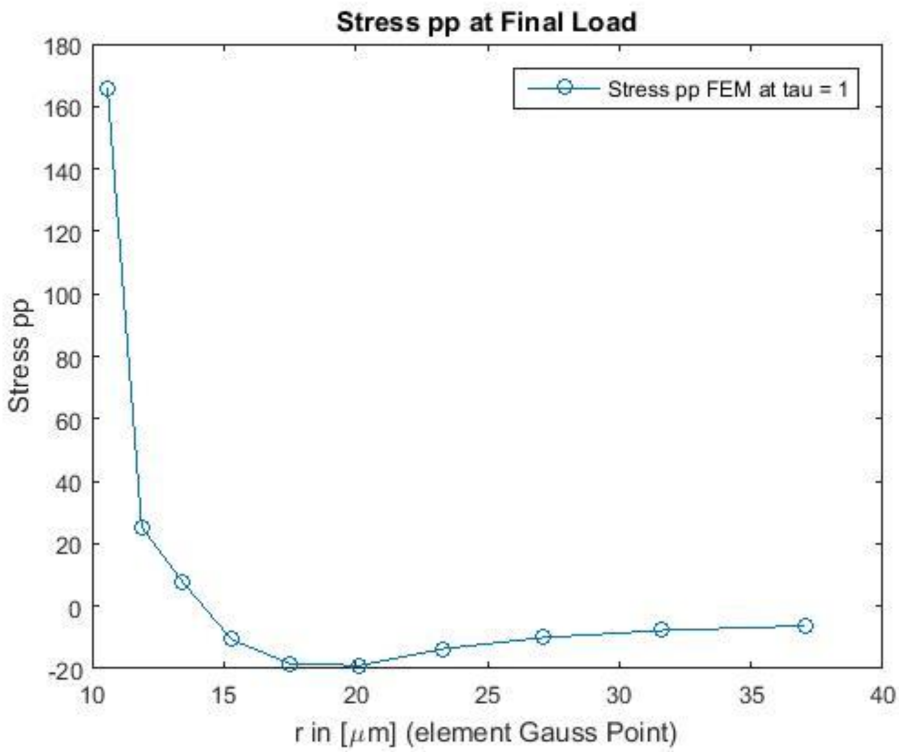
c) **Extract distribution of following at tau = 1**

$$u_r(r),\ \sigma_{rr}(r) \text{ and } \sigma_{\phi\phi}(r)$$

For graphs number elements used are 10.







d) **Demonstrating the nonlinearity of the problem by extracting the time history of the radial stress at point ri. for tau[0,1].**