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CS 4510 – Automata and Complexity Theory

1. (10 pts) Prove every NFA can be converted into an NFA with at most one accept state.

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Consider NFA n that contains more than 1 accept state. We can say that all accept states are in the set F. Take every element in F and make it not accepting state and add an epsilon transition from each state in F to a new state s. Make s an accepting state. Now, NFA n has only one accepting state.

2. (10 pts) Prove that if L is regular and $x, y \in L$ then $\forall z \in \Sigma^*$ that either xz, yz both in L or xz, yz both not in L. Use Q1.

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Given that L is regular, we know L must have an NFA in which x, y are accepted. Let us convert this NFA to an NFA with at most one accept state, using the method in Q1. Additionally, draw epsilon transitions from each original final state back to the initial state. From here, if $z \in L$, then the path xz and yz will both be in L. However, if $z \notin L$, xz and yz will not be accepted in this newly created NFA. Thus, $z \in L \rightarrow xz, yz \in L$ and $z \notin L \rightarrow xz, yz \notin L$, given L is regular and $xy \in L$.

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3. (15 pts) Give a DFA for $\{w \in \Sigma^* \mid w \text{ contains } aa \text{ or } bb \text{ as a substring }\}$ with $\Sigma = \{a, b\}$

regex: $(\Sigma^* aa \Sigma^*) \cup (\Sigma^* bb \Sigma^*)$

abba x

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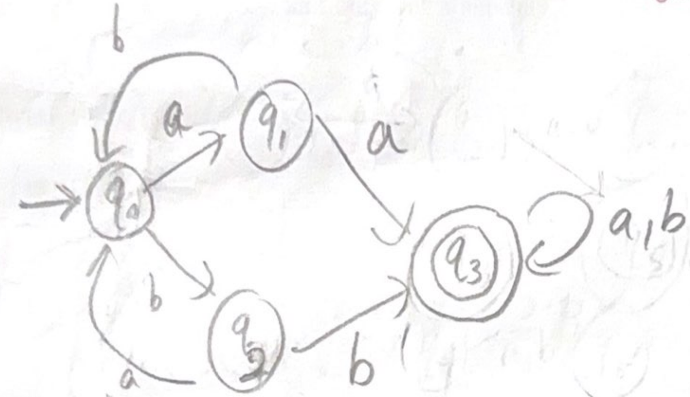
DFA:

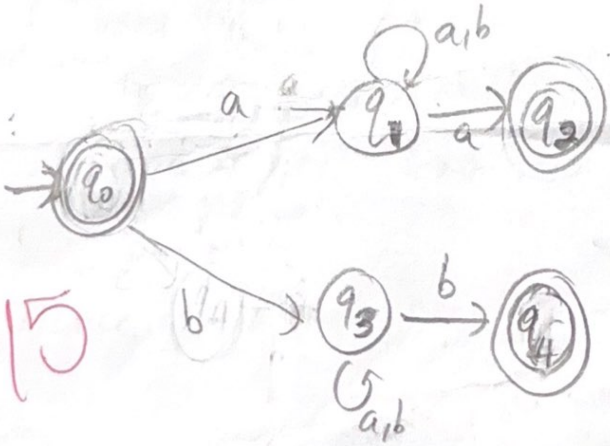


4. (15 pts) Let $L = \{w \in \Sigma^* \mid w$ starts and ends with the same symbol$\}$ with $\Sigma = \{a, b\}$. Give an NFA and regular expression for L.

regex: $(a\Sigma^* a) \cup (b\Sigma^* b) \cup \varepsilon \cup a \cup b$

symbol: only an item in Σ or ε

NFA:



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*the interpretation of same symbol is not fully clear

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5. (20 pts) Prove that

$$L = \{1^{n+m}01^n01^m \mid n, m \in \mathbb{N}\}$$

is not regular by using the pumping lemma for regular languages. I recommend you work out the details on the back of this sheet, and only put a succinct, clean proof on the front.

1. Assume to the contrary that L is regular with pumping length p.

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2. $s = 1^{2p}01^p01^p$, $s \in L$ ✓ $\rightarrow m=p, n=p, m+n=2p$
$|s| = 4p+2 \geq p$ ✓

3. $s = xyz$, $x = 1^a$, $y = 1^b$, $z = 1^{2p-a-b}01^p01^p$
such that $a+b \leq p$ and $b > 0$

4. choose $i = 0$, so $xy^iz = xy^0z = xz = 1^a1^{2p-a-b}01^p01^p$
$= 1^{2p-b}01^p01^p$

using the expression from L, we see $m=p$ and $n=p$ in xz, however $m+n = 2p \neq 2p-b$ because b is nonzero ($b>0$ from step 3). Thus, $xz \notin L$.

5. Hence, L is not regular.

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6. (20 pts) For each of the following, circle only one of T or F. T if the statement is true and F if the statement is false. On the line following the statement, **justify your answer**. You may use more space on the back if necessary.

4 1. T F — The union of two regular languages is always regular

Closure of regular languages holds under intersection and complement $\rightarrow \overline{\overline{R_1} \cap \overline{R_2}} = R_1 \cup R_2$

2. T F — Every subset of a regular language is regular

A nonregular language L_{NR} is a subset of Σ^* which is regular, contradicting the stament

3. T F — If $w \in \Sigma^*$ then $w^2 = w \iff w = \varepsilon$

$(a^*)^2$ is still a^*, so this is not only true for ε

4. T F — The non-regular languages are closed under complement.

if L is regular $\rightarrow \overline{L}$ is regular, thus if $\overline{L}$ is not regular $\rightarrow$ L is not regular

5. T F — $|F| + |\overline{F}| = |Q|$ in a DFA

the only states in Q are either nonaccepting states or accepting states. Therefore, the sum of both must be $|Q|$

You want bonus points? Let's test your decision making skills under uncertainty. Circle the amount of bonus points you want from the selection below. Which ever one gets the **FEWEST NON-ZERO VOTES** will be applied to everyone who answers this question.

-1 0 1 2