MATH1564 K – Linear Algebra with Abstract Vector Spaces
Homework 1

**Due Aug. 29, submit to both Canvas-Assignment and Graadescope**

1. Let $A = \{1, 2, 3, 4\}$, $B = \{1, 3, 5\}$ and $C = \{2, 4, 5\}$.
   i. Find the following sets:

   $A \cup B$, $A \cap B \cap C$, $(A \cap C) \cup B$.
   ii. Assume that $A$, $B$, and $C$ all belong to the universal set $U = \{x \in \mathbb{N} : x \leq 6\}$. Find the following sets:

   $(B \cup C)^c$, $A^c \cap B^c \cap C^c$, $U^c$.
   iii. Find how many elements are in each one of the following sets (these are called the *cardinalities* of the sets):

   $(B \cap C)^c$, $(B \cup C)^c$, $\{X : X \subseteq B\}$,
   $\{X : X \subseteq A$ and $X$ has at most two elements $\}$.
2. Draw a sketch of the following subsets of $\mathbb{R}^2$:
   i. $\{(x, y) : x \leq 2y + 1, x \in \mathbb{R}\}$.
   ii. $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}^c$.
   iii. $\{(-1, 1) + t(1, 2) : t \in \mathbb{R}\}$.
   iv. $\{s(1, 2) + t(-1, 1) : s, t \in \mathbb{R}\}$.
   v. $\{s(1, 2) + t(2, 4) : s, t \in \mathbb{R}\}$.
   vi. $\{t(1, 2) : 0 < t\}$.
3. Express the following sets without the use of 'three dots' nor the use of $\cap, \cup$, or complement.
   i. $\{4, 16, 36, 64, 100, ...\}$
   ii. $\{..., \frac{2}{9}, \frac{2}{3}, 2, 6, 18, ...\}$
   iii. $\{3n : n \in \mathbb{Z}\} \cap \{2n + 1 : n \in \mathbb{Z}\}$.
4. Let $A$,$B$ and $C$ all be sets with a universal set $U$. In each of the following parts, a statement is written about these sets (or some of them). We consider such a statement **true** if it is true for every possible sets $A$,$B$, $C$ and $U$. We consider it **false** if there is at least one example of sets $A$,$B$, $C$ and $U$ for which the statement does not hold. Determine for each one of the following statements if it is **true or false**. If you claim that it is **false** then provide an example for which the statement fails ("counterexample"). If you claim that the statement is **true** then **prove your claim as best you can**.
   i. $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$.
   ii. $(A - B) \cup (B - A) = A \cup B$.
   iii. $(A \cap B) \times C = (A \times C) \cap (B \times C)$.

iv. $(A \cap B)^c = A^c \cap B^c$.

v. $(A \cap B)^c = A^c \cup B^c$.

5. Decide and prove if the following statement true or false.

   i. If $x$ is a multiple of 4, then $x$ is even.

   ii. If $x$ is even, then it is a multiple of 4.

   iii. There exists a real number $a$ such that for $x = a + x$ for all real number $x$.

6. Negate the following statements in quotation.

   i. Suppose $x_1, x_2, x_3$ are given. "If $a_1x_1 + a_2x_2 + a_3x_3 = 0$ for some real numbers $a_1, a_2, a_3$, then $a_1 = a_2 = a_3 = 0$."

   ii. Let $A$ and $B$ are subsets of $X$ and $x \in X$. "$x \in A$ and $x \in B$."

   iii. "There exists a real number $a$ for which $a + x = x$ for every real number x."