MATH-3012-D HW 09

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TOTAL POINTS

40 / 40

QUESTION 1

1 Q1 **13 / 13**

✓ **+ 13 pts** *correct*

QUESTION 2

2 Q2 **13 / 13**

✓ **+ 13 pts** *Correct*

QUESTION 3

3 C **14 / 14**

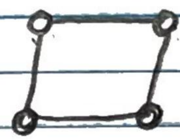
✓ **- 0 pts** *Complete*

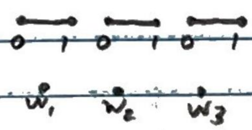
MATH3012 HW09

Q1 11.1

4) $C_e \to$ all binary sequences with even # of 1's
$C_o \to$ all binary sequences with odd # of 1's
It is not possible for a vertex in C_e to be adjacent to a vertex in C_o as that would contradict E as the change in positions would have to be an odd number greater than 0. Therefore, C_e and C_o would be distinct connected components.

$K(G) = 2$

10) 

16) (a) 

010101

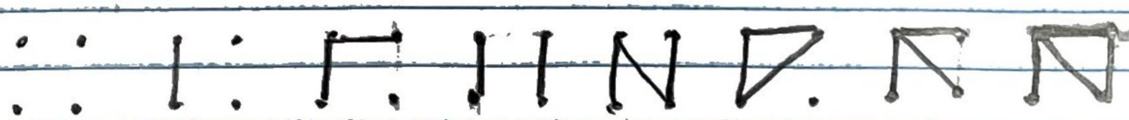
AND 

001011

$\boxed{\begin{matrix} 2 \\ \downarrow \\ 010101 \\ 001011 \end{matrix}}$

(b) $\frac{1}{n+1}\binom{2n}{n} \to \frac{1}{5}\cdot\binom{8}{4} = \frac{8\times 7\times 6}{4\times 3\times 2} = \boxed{14}$

11.2

6) 

(1) (2) (3) (4) (5) (6) (7) (8)

(9) (10) (11)

(5), (7), (8), (9), (10), (11) are connected

↳ $\boxed{\text{6 are connected}}$

8) (a) 

$7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 2520$

$2520/2 = \boxed{1260}$

(b) $(1/2)(n)(n-1)\cdots(n-m)$

9) (a) Graph 1 has four vertices that have degree 3. This is true for graph 2, but they form a cycle. This is not true for graph 1, and therefore they are not isomorphic.

(b) There is a matching of vertices

↳ a b c d e f

v y x w u z

This means they are isomorphic

11) (a) If $\bar{G}_1 = (V_1, \bar{E}_1)$ and $G_2 = (V_2, \bar{E}_2)$ are isomorphic, the function $f: V_1 \to V_2$ preserves the adjacency matrix. Since G_1 and $\bar{G}_1$, as well as G_2 and $\bar{G}_2$, share V_1 and V_2, the function f applies to G_1, G_2 and can be used to prove isomorphism.

(b) They are not isomorphic because there is no way to match vertices. In both graphs, each vertex is adjacent is connected to vertices directly next to itself. However, the connection to the other 5 vertices is directly opposite in graph 1 and 2. Thus, these graphs are not even complements because of their connection to the vertices directly next to each vertex.

1 Q1 **13 / 13**

✓ **+ 13 pts** *correct*

Q2 11.3

1) (a) $2|E| = \Sigma D$

$2(9) = |V|3 \rightarrow \boxed{|V| = 6}$

(b) $2 \cdot |E| = \Sigma D$

$30 = |V| \cdot D$

$|V| = 30/D \rightarrow \boxed{|V| = \{1, 2, 3, 5, 6, 10, 15, 30\}}$

(c) $2|E| = \Sigma D$

$2(10) = 2(4) + x(3)$

$20 = 8 + 3x \rightarrow x = 4 \rightarrow \boxed{|V| = 6}$

5) (a) $|V_1| = 8$, $|E_1| = 14$, $|V_2| = 8$, $|E_2| = 14$

(b)

V_1	vertex	a	b	c	d	e	f	g	h
	degree	3	4	4	3	3	4	4	3
V_2	vertex	s	t	u	v	w	x	y	z
	degree	3	4	4	3	4	3	3	4

(c) Not isomorphic; vertices with degree 4 in V_2 form a cycle, but not in V_1

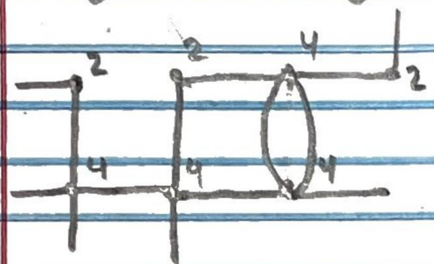
8) (a) $n \cdot 2^{n-1} \rightarrow 8 \cdot 2^7 = \boxed{1024}$

(b) The longest distance would be 00000000 to 11111111 which would be $\boxed{8}$ units.

(c) $2^8 - 1 = 256 - 1 = \boxed{255}$

↑ contains all vertices, but subtract 1 to prevent cycle

20) (a) a → d → h → i → e → f → i → j → k → g → j → f →
b → g → c → b → e → d → b → a
(b) d → a → b → d → h → i → e → f → i → j → k → g →
j → f → b → g → c → b → e

23) 

Each vertex has an even degree, so it is possible to accomplish their goal

33) (a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{C_1 \leftrightarrow C_3} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

These graphs are isomorphic

(b) $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \xrightarrow[R_2 \leftrightarrow R_3]{R_1 \leftrightarrow R_4} \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \xrightarrow[C_2 \leftrightarrow C_3]{C_1 \leftrightarrow C_4} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

These graphs are isomorphic

(c) These graphs are not isomorphic

11.5

3) (a) a → g → k → i → h → b → c → d → j → f → e → a (cycle)
(b) a → c → h → i → j → g → e → b → d → a (cycle)
(c) b → c → d → i → g → f → e → h → a → b (cycle)
(d) a → c → d → b → e → f → g (path)
(e) a → b → c → d → e → j → i → h → g → f → k → l → m → n → o (path)
(f) a → f → k → p → q → l → g → h → m → r → s → t →
o → n → i → j → e → d → c → b → a (path)

7) (a) · order vertices $\rightarrow n!$

· any cycle can be traversed n ways $\rightarrow \frac{1}{n} \cdot n! = (n-1)!$

· direction double counted $\rightarrow \frac{1}{2}(n-1)!$

\# of cycles $\rightarrow \boxed{\frac{(n-1)!}{2}}$

(b) · k edge-disjoint cycles

· # of edges covered by k is kn

· # of edges total is $n(n-1)/2$

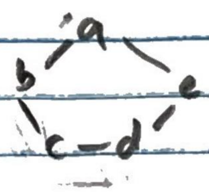
· $kn = n(n-1)/2 \rightarrow k = (n-1)/2$

$(21-1)/2 = \boxed{10}$

(c) $(19-1)/2 = \boxed{9}$

21) Proof by induction

· Base case: $n=5 \rightarrow$ cocycle of C_5 is pentagon, which has a hamiltonian cycle, as shown below.



$a \rightarrow b \rightarrow c \rightarrow d \rightarrow e \rightarrow a$

· Inductive step: Assume cocycle of C_k has Hamiltonian cycle for $k \geq 5$. We will show this is true for C_{k+1}.

Assume x, y are nonadjacent vertices in $\overline{C_{k+1}}$.
$\deg(x)$ and $\deg(y)$ are $k+1-3 = k-2$.
$\deg(x) + \deg(y) = 2k-4$. We also know that
$2k-4 \geq k+1$ for nonadjacent vertices.
This means $2k-k \geq 1+4 \rightarrow k \geq 5$. Thus, for $n \geq 5$, the cocycle of C_n has a Hamiltonian cycle.

2 Q2 **13 / 13**

✓ **+ 13 pts** *Correct*

3 C **14 / 14**

✓ **- 0 pts** *Complete*