CS 3510 C: Design & Analysis of Algorithms 09/21/2023

Practice Exam 2

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- This is the CS 3510 practice exam for Exam 2. This does **not** approximate the difficulty or length of the actual exam; it serves as just a big question bank for you to practice!
- Topics include: Graphs, DFS, Topological Sort, Strongly Connected Components, BFS, Dijkstra's, MST, Kruskal's, Max Flow Min Cut
- Note that this assignment does not need to be submitted, but we highly recommend going through it to prepare for the exam!

1.) Determine whether the following statements are true or false. Give a brief explanation if true, or a counterexample if false.

(a) If graph G has more than $|V| - 1$ edges, and there is a unique heaviest edge, then this edge can not be part of the MST.

○ True

○ False

(b) If G has a cycle with a unique heaviest edge e, then e can not be part of any MST.

○ True

○ False

(c) Let e be any edge of minimum weight in G. Then e must be part of some MST.

○ True

○ False

(d) If G has a cycle with a unique lightest edge e, then e must be part of every MST.

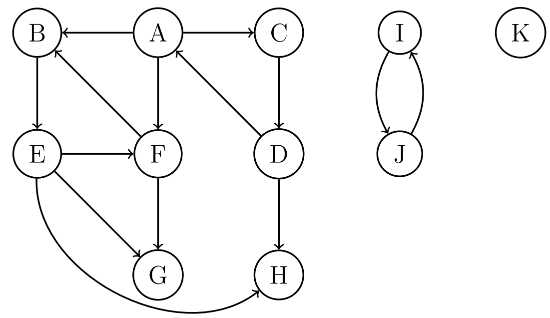
○ True

○ False

(e) The shortest-path tree computed by Djikstra's algorithm is necessarily an MST.

○ True

○ False



2.) Given the above graph, answer the following questions about depth first search algorithms. All exploration should be done in alphabetically order, and start from A unless otherwise stated.

(a) After running explore, in what order would the nodes be found.

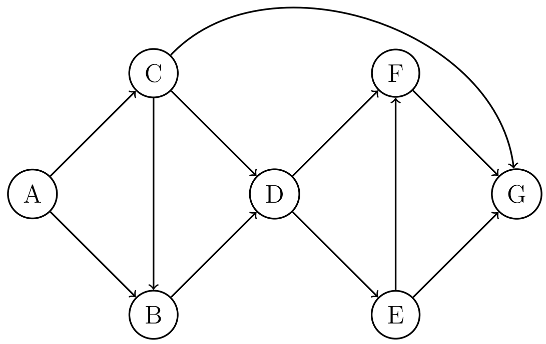
- ◯ A, B, E, F, G
- ◯ A, B, E, F, G, H, C, D, I, J, K
- ◯ A, B, E, F, G, H, C, D
- ◯ A, B, E, F, G, H, C, D, J, I, K

(b) After running DFS, taking note of pre/post labels, which node has the lowest post label?

- ◯ A
- ◯ G
- ◯ K
- ◯ H

(c) After running DFS, and sorting descending by post labels, which ordering would you get?

- ◯ K, I, J, A, B, E, F, G, C, D, H
- ◯ I, J, K, A, B, E, F, G, C, D, H
- ◯ A, B, E, F, G, C, D, H, K, I, J
- ◯ A, B, E, F, G, C, D, H, I, J, K



3.) Given the above graph, answer the following questions.

(a) In the metagraph of the above graph, which node will be in a strongly connected component which leads to the sink.

- ○ A
- ○ B
- ○ F
- ○ G

(b) If the graph was changed such that all the edges were removed, what would the number of edges be that you'd need to add such that the entire graph is strongly connected.

- ○ 6
- ○ 7
- ○ 8
- ○ 9

(c) What is the minimum number of edges which would need to be added to have two strongly connected components.

- ○ 0
- ○ 1
- ○ 2
- ○ 3

4.) Let $G = (V, E)$ be a directed graph in which each vertex $u \in V$ is from the set $\{1, 2, ..., |V|\}$. For each vertex in $u \in V$, let $R(u)$ be all the vertices that are directly reachable from u. $\min(u)$ is defined as the vertex in $R(u)$ which is minimum, i.e., $\min(u)$, for $u \in V$ is the vertex v such that $v = \min\{v \in R(u)\}$. Design an algorithm in $\mathcal{O}(|V| + |E|)$ time that computes $\min(u)$ for all vertices $u \in V$. Justify its correctness and show that its runtime is linear. **Hint: since the vertex labels are integers, think of linear time sorting algorithms.**

5.) Suppose a CS curriculum consists of n courses, all of them mandatory. The prerequisite graph G has a node for each course, and an edge from course v to course w if and only if v is a prerequisite for w. Find an algorithm that works directly with this graph representation, and computes the minimum number of semesters necessary to complete the curriculum (assume that a student can take any number of courses in one semester). The running time of your algorithm should be linear.

6.) Suppose you want to find the shortest path from u to v in a graph G that passes through a specified vertex w. Design an algorithm for each of the following settings.

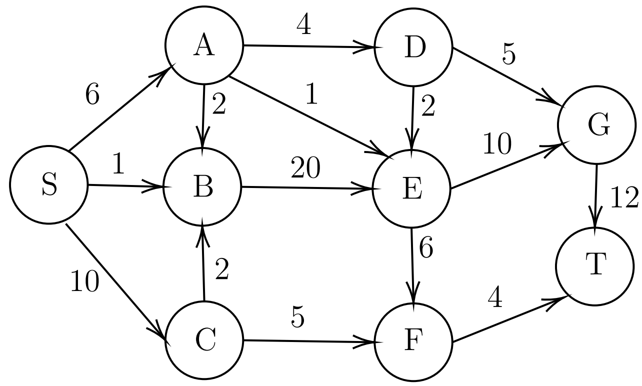
(a) G is an undirected weighted graph.

(b) G is a directed weighted graph.

(c) G is a directed weighted graph, and instead of a single u and v, we have k queries $(u_1, v_1), (u_2, v_2), ..., (u_k, v_k)$ where for each one we want to find only the weight of the shortest path that passes through w (which is fixed).

7.) Suppose you live in a village with n houses and you want to build wells and lay pipes to connect water to all houses with the minimum cost necessary. You're given a list of the costs of building wells for each house $w_1, w_2, ..., w_n$. You can also lay bidirectional pipes between certain houses given as a list of the two houses and the cost to lay the pipe. For example, one such entry in the list would be (h_1, h_2, c_{12}), which tells you that laying a pipe between house 1 and house 2 would cost c_{12}. Give an efficient algorithm to find the minimum total cost to connect water to all houses, and show its runtime.

8.) Consider the network shown below.



(a) What is the maximum flow from S to T? List the flow associated with each edge in the following format: (u, v, f_e) where $u \in G, v \in G$ and f_e is the flow along the edge $u \to v$.

(b) Find a matching minimum cut. Clearly list the edges in the minimum cut.

9.) You've become the head industrial engineer at a company operating on n cities numbered 1 to n along a one-way road. The ith city produces $p_i \in P$ units of goods, and can buy $s_i \in S$ units of goods from other cities. For each pair of cities i and j, such that $1 \leq i \leq j \leq n$, you can transport at most c units of goods from city i to city j. Design an efficient algorithm to determine the maximum number of goods that can be sold in total to all cities after a sequence of transportations given n, c, P, and S. You do not have to prove its runtime.