CSE Fall 2024 Project: Traveling Salesman Problem

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1 Introduction

The objective of this project was to explore solving an intractable problem, Traveling Salesman Problem (TSP), using 3 different approaches. The goal was to implement 3 algorithms to find the shortest cycle that connects all N vertices of a given graph. Euclidean distance between the vertices act as Edge costs for the graph.

1.1 Brute Force methodology

Brute force method was implemented to understand the inefficiency of the exact algorithm, as it is an exhaustive approach. This algorithm evaluates all permutations of the given points to find the shortest possible TSP tour. Though brute-force method guarantees optimality, it is computationally infeasible for larger data points, as the complexity is factorial time. Hence a cutoff time was introduced to ensure that algorithm stops if the time limit exceeds. For the algorithm setup, the timer was started when the algorithm started at the current time. Best distance was initialized to infinity and updated the best distance to current distance after each loop. Best path was initialized as an empty set and updated to the current permutation after each loop. Within the loop, for each consecutive pair, the distance was calculated using Euclidean distance. This value was used for the calculation of the current distance. After the cutoff time, the loop breaks and gives the best distance and best path resolved so far. Cutoff times from 5 seconds to 500 seconds were considered.

1.2 2-approximation methodology using Minimum Spanning Tree approach

2-approximation algorithm, based on Minimum Spanning Tree (MST), was implemented to solve the problem in a given time, to also achieve a high-quality solution or low cost solution. As finding an exact solution for TSP is computationally inefficient, a near-optimal approach was considered. Through MST, all vertices are connected with a minimum total edge weight. Preorder traversal approximates the shortest tour and ensures that all nodes are visited, and the cycle is closed by returning to the starting vertex. Though this algorithm does not guarantee an optimal solution but it is efficient. A random seed was considered as an input between the range of 42 to 51. Starting with an empty graph, the Euclidean distance between each pair points was calculated. This distance values became the weight of the edge between two vertices. MST was used to generate a path through the random seed as the starting node. Then preorder traversal of the MST was performed from that random node to a get a sequence of nodes. Total distance was initialized as 0 and the distance between the current node and the next node were added to the total distance. The current node was appended to the tour list.

1.3 Simulated Annealing methodology

Simulated Annealing algorithm was implemented for local search algorithm, without guarantees but provides an effective solution in terms of quality. Simulated Annealing functions by randomly exploring neighboring solutions, generated via 2-opt swaps (reversing sections of the tour). This heuristic approach explores the solution space, allows the algorithm to escape the local optima. A cooling schedule gradually reduces the likelihood of accepting suboptimal moves. 2-opt swap generates a neighboring solution by reversing the segment between two randomly chosen vertices in the tour. This method explored the local solution space efficiently

In combination with Simulated Annealing, MST was used to construct an approximate tour using preoder traversal. This was done as MST minimizes the total edge weight required to connect all the points without forming cycles. A preorder traversal of this tree visits all nodes efficiently, though not optimally. This additional to Simulated Annealing ensures that a valid TSP tour with a guarantee is considered.

This hybrid approach provides a strong initial solution for simulated annealing to start working with. As the solution is already close to the optimal, this reduces the search space for simulated annealing. Without the initial step, simulated annealing might begin with a randomly generated tour, which might be far from the optimal, resulting in poor solution quality. Approximate approach for initial solution accelerates the convergence and also reduces sensitivity to simulated annealing parameters such as initial temperature, number of iterations.

2 Results and Analysis

1. Key points to take way from the results are that for any given data set, brute force method never reaches a full tour within the cut off time (even with higher cutoff times as 500 seconds). However 2-approximation and simulated annealing give a full tour, and a complete path for the vertices. 2-approximation method guarantees a full tour, which is observed in the results. Heuristic algorithms though reduces the cost, but doesn't guarantee a full tour. However with the combination of 2-approximation implementation with simulated annealing, not only the solution quality reduced, but the full tour was observed for all the solutions.

2. The cost or the solution quality is higher for brute force method as it explores every permutation between vertices. This adds up the costs of all the edges between the vertices. Here the solution quality translates to the cost of the tour. Hence brute force method has higher solution quality or the cost of the tour compared to other algorithms. Highest cost is noticed by brute force method, followed by 2-approximation, and then heuristic method. Both 2-approximation and simulated annealing method show a significant drop in the solution quality. As simulated annealing leverages the tour input from 2-approximation method, the solution quality is further dropped down for the last approach.

3. For brute force method, as the cutoff time increases, the solution quality drops down. This was observed for all datasets. Below one data set graph is given for representation. This occurs as with less cutoff time, the algorithm still has not explored the shortest path. It would still be exploring the permutations between all the vertices. Hence the solution quality obtained at the end of the cutoff time will be higher. As more time is given, algorithm would have computed more number of permutations, found shorter paths between vertices. Hence the edge costs for updated paths will be smaller given more time. Hence as the cutoff times increases, the tour cost or the solution quality decreases.

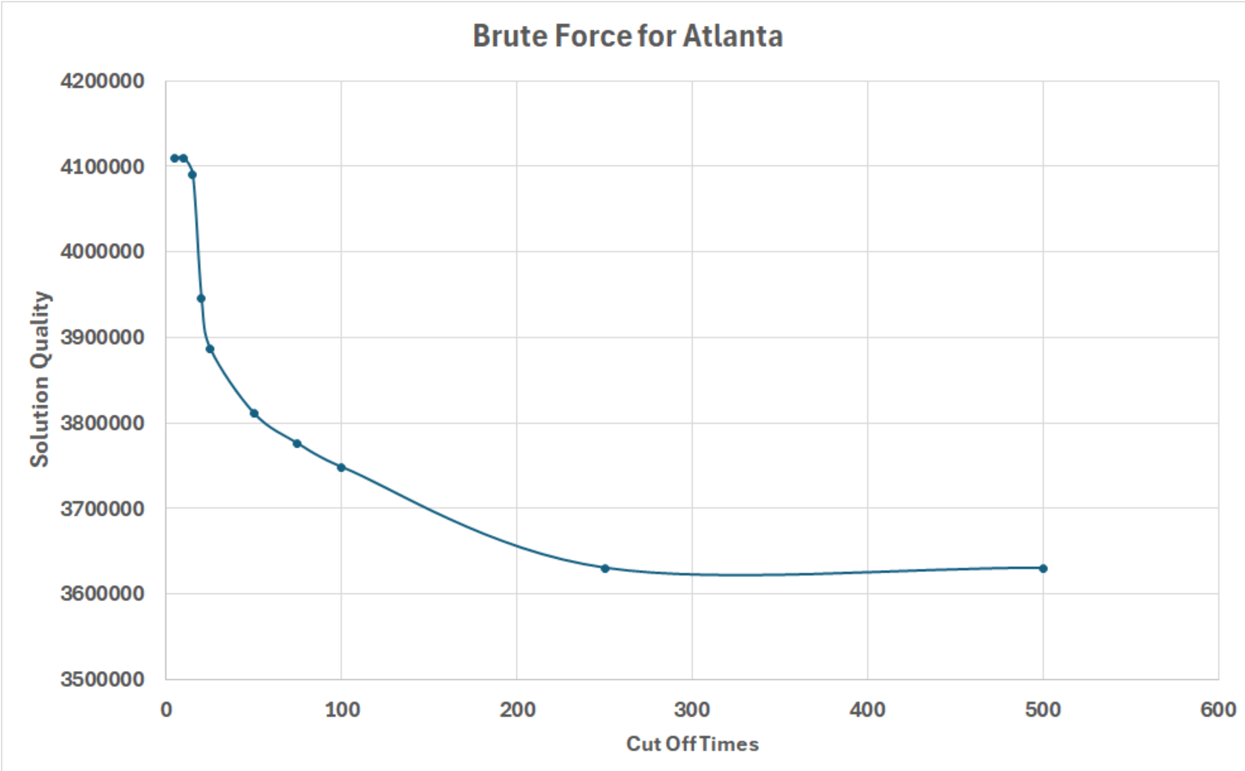

Figure 1: Cutoff times vs. Solution Quality for Atlanta dataset

Dataset	Brute Force					2-Approximation					Simulated Annealing			
	cut off (s)	time (s)	Sol.Qual	Full Tour	Rel.Err	seed	time (s)	Sol.Qual	Full Tour	Rel.Err	time (s)	Sol.Qual	Full Tour	Rel.Err
Atlanta	5	6.14	4109868	No	0.488	42	1.38	2486842	Yes	0.828	1.24	2037607	Yes	0.017
	10	11.11	4109868	No	0.488	43	1.26	2488307	Yes	0.822	1.26	2037607	Yes	0.017
	15	16.11	4090106	No	0.490	44	1.26	2500425	Yes	0.806	1.26	2037607	Yes	0.017
	20	21.15	3945052	No	0.508	45	1.20	2490014	Yes	0.824	1.24	2037607	Yes	0.017
	25	26.16	3886657	No	0.516	46	1.19	2477233	Yes	0.828	1.25	2037607	Yes	0.017
	50	51.19	3811059	No	0.526	47	1.16	2495163	Yes	0.823	1.23	2037607	Yes	0.017
	75	76.18	3775843	No	0.531	48	1.24	2453860	Yes	0.825	1.26	2037607	Yes	0.017
	100	101.21	3748338	No	0.535	49	1.32	2477233	Yes	0.829	1.28	2037607	Yes	0.017
	250	251.18	3630534	No	0.552	50	1.21	2486842	Yes	0.806	1.31	2037607	Yes	0.017
	500	771.50	3630534	No	0.552	51	1.15	2312426	Yes	0.872	1.34	2037607	Yes	0.017
Berlin	5	6.13	20181	No	0.496	42	1.20	10126	Yes	1.060	1.35	10552.8	Yes	0.055
	10	11.28	20181	No	0.496	43	1.18	9976	Yes	1.103	1.50	10552.8	Yes	0.055
	15	16.27	20033	No	0.499	44	1.33	9931	Yes	1.065	1.37	10552.8	Yes	0.055
	20	21.23	19824	No	0.505	45	1.39	10267	Yes	0.974	1.37	10552.8	Yes	0.055
	25	26.19	19786	No	0.506	46	1.15	10194	Yes	1.036	1.30	10552.8	Yes	0.055
	50	51.15	19728	No	0.507	47	1.15	9588	Yes	1.220	1.34	10552.8	Yes	0.055
	75	76.29	19728	No	0.507	48	1.23	10154	Yes	1.002	1.34	10552.8	Yes	0.055
	100	101.43	19728	No	0.507	49	1.17	10194	Yes	1.025	1.34	10552.8	Yes	0.055
	250	251.19	19438	No	0.515	50	1.23	10108	Yes	1.010	1.43	10552.8	Yes	0.055
	500	501.20	19249	No	0.520	51	1.14	9867	Yes	1.025	1.37	10552.8	Yes	0.055
Boston	5	6.18	2261063	No	0.414	42	1.40	1061299	Yes	0.937	1.36	972781.5	Yes	0.040
	10	11.16	2246134	No	0.416	43	1.16	1050695	Yes	0.985	1.37	972781.5	Yes	0.040
	15	16.13	2246134	No	0.416	44	1.14	1045369	Yes	0.896	1.35	972781.5	Yes	0.040
	20	21.16	2246134	No	0.416	45	1.14	1043190	Yes	0.908	1.32	972781.5	Yes	0.040
	25	26.14	2246134	No	0.416	46	1.19	1085113	Yes	0.875	1.31	972781.5	Yes	0.040
	50	51.16	2246134	No	0.416	47	1.14	1125707	Yes	0.884	1.26	972781.5	Yes	0.040
	75	76.15	2244804	No	0.417	48	1.18	1129373	Yes	0.828	1.30	972781.5	Yes	0.040
	100	101.15	2234116	No	0.419	49	1.15	1085113	Yes	0.914	1.30	972781.5	Yes	0.040
	250	251.15	2227323	No	0.420	50	1.18	1085544	Yes	0.878	1.32	972781.5	Yes	0.040
	500	501.19	2226767	No	0.420	51	1.41	1027795	Yes	0.965	1.30	972781.5	Yes	0.040
Champaign	5	6.22	219301	No	0.280	42	1.28	65587	Yes	1.025	1.36	67682.8	Yes	0.101
	10	11.16	219252	No	0.280	43	1.21	68357	Yes	0.910	1.42	67682.8	Yes	0.101
	15	16.14	219252	No	0.280	44	1.20	67546	Yes	1.035	1.40	67682.8	Yes	0.101
	20	21.13	219252	No	0.280	45	1.25	66093	Yes	1.016	1.36	67682.8	Yes	0.101
	25	26.10	219166	No	0.281	46	1.47	67315	Yes	1.020	1.33	67682.8	Yes	0.101
	50	51.17	218823	No	0.281	47	1.30	66516	Yes	1.017	1.37	67682.8	Yes	0.101
	75	76.22	218823	No	0.281	48	1.17	67978	Yes	0.905	1.36	67682.8	Yes	0.101
	100	101.33	218074	No	0.282	49	1.18	67315	Yes	0.965	1.37	67682.8	Yes	0.101
	250	251.19	216391	No	0.284	50	1.19	64759	Yes	1.113	1.36	67682.8	Yes	0.101
	500	501.14	212857	No	0.289	51	1.17	67222	Yes	1.125	1.36	67682.8	Yes	0.101
Cincinnati	5	6.17	277952	No	1.000	42	1.12	320593	Yes	0.867	1.18	278181.1	Yes	0.001
	10	11.25	277952	No	1.000	43	1.09	318227	Yes	0.873	1.17	278181.1	Yes	0.001
	15	16.19	277952	No	1.000	44	1.11	307439	Yes	0.904	1.17	278181.1	Yes	0.001
	20	21.19	277952	No	1.000	45	1.11	307439	Yes	0.912	1.15	278181.1	Yes	0.001
	25	26.13	277952	No	1.000	46	1.11	320593	Yes	0.867	1.17	278181.1	Yes	0.001
	50	37.79	277952	No	1.000	47	1.15	309289	Yes	0.899	1.18	278181.1	Yes	0.001
	75	36.50	277952	No	1.000	48	1.15	317273	Yes	0.876	1.20	278181.1	Yes	0.001
	100	30.61	277952	No	1.000	49	1.12	320593	Yes	0.867	1.22	278181.1	Yes	0.001
	250	34.20	277952	No	1.000	50	1.13	308670	Yes	0.900	1.22	278181.1	Yes	0.001
	500	33.69	277952	No	1.000	51	1.14	309289	Yes	0.899	1.30	278181.1	Yes	0.001
Denver	5	6.24	564669	No	0.283	42	1.52	128256	Yes	1.402	1.45	175762.3	Yes	0.100
	10	11.49	564669	No	0.283	43	1.37	127937	Yes	1.462	1.46	175762.3	Yes	0.100
	15	16.24	564669	No	0.283	44	1.40	127123	Yes	1.427	1.44	175762.3	Yes	0.100
	20	21.17	563620	No	0.284	45	1.27	128336	Yes	1.425	1.44	175762.3	Yes	0.100
	25	26.22	563620	No	0.284	46	1.17	129206	Yes	1.421	1.46	175762.3	Yes	0.100
	50	51.13	563620	No	0.284	47	1.26	129598	Yes	1.311	1.42	175762.3	Yes	0.100
	75	76.18	563620	No	0.284	48	1.35	129643	Yes	1.233	1.44	175762.3	Yes	0.100
	100	101.20	563620	No	0.284	49	1.33	128223	Yes	1.285	1.58	175762.3	Yes	0.100
	250	251.24	562055	No	0.284	50	1.18	130778	Yes	1.354	1.49	175762.3	Yes	0.100
	500	501.16	546925	No	0.292	51	1.22	128073	Yes	1.338	1.49	175762.3	Yes	0.100
NYC	5	6.18	7244933	No	0.254	42	1.16	1891838	Yes	1.148	1.36	2198950	Yes	0.194
	10	11.23	7244933	No	0.254	43	1.16	1937161	Yes	1.178	1.38	2198950	Yes	0.194
	15	16.22	7244933	No	0.254	44	1.15	1845662	Yes	1.193	1.37	2198950	Yes	0.194
	20	21.21	7244933	No	0.254	45	1.23	1845662	Yes	1.152	1.38	2198950	Yes	0.194
	25	26.20	7244933	No	0.254	46	1.14	1855713	Yes	1.199	1.45	2198950	Yes	0.194
	50	51.23	7244933	No	0.254	47	1.19	1948652	Yes	1.297	1.38	2198950	Yes	0.194
	75	76.20	7244933	No	0.254	48	1.16	1948810	Yes	0.945	1.45	2198950	Yes	0.194
	100	101.22	7239945	No	0.254	49	1.18	1896513	Yes	1.232	1.43	2198950	Yes	0.194
	250	251.18	7218153	No	0.255	50	1.17	1959210	Yes	1.062	1.45	2198950	Yes	0.194
	500	510.32	7210976	No	0.255	51	1.29	1937026	Yes	1.134	1.38	2198950	Yes	0.194
Philadelphia	5	6.53	3765538	No	0.374	42	1.10	1733719	Yes	0.817	1.29	1470186	Yes	0.043
	10	11.26	3765538	No	0.374	43	1.12	1728475	Yes	0.829	1.27	1470186	Yes	0.043
	15	16.23	3724219	No	0.378	44	1.09	1697277	Yes	0.848	1.27	1470186	Yes	0.043
	20	21.18	3710782	No	0.380	45	1.10	1722191	Yes	0.896	1.37	1470186	Yes	0.043
	25	26.28	3710782	No	0.380	46	1.15	1723248	Yes	0.916	1.33	1470186	Yes	0.043
	50	51.18	3710782	No	0.380	47	1.12	1668712	Yes	0.865	1.34	1470186	Yes	0.043
	75	76.19	3710782	No	0.380	48	1.21	1715300	Yes	0.840	1.29	1470186	Yes	0.043
	100	101.28	3710782	No	0.380	49	1.44	1693572	Yes	0.832	1.58	1470186	Yes	0.043
	250	251.28	3624919	No	0.389	50	1.23	1685283	Yes	0.921	1.32	1470186	Yes	0.043
	500	501.49	3624919	No	0.389	51	1.25	1695100	Yes	0.852	1.27	1470186	Yes	0.043

Dataset	Brute Force					2-Approximation					Simulated Annealing			
	cut off (s)	time (s)	Sol.Qual	Full Tour	Rel.Err	seed	time (s)	Sol.Qual	Full Tour	Rel.Err	time (s)	Sol.Qual	Full Tour	Rel.Err
Roanoke	5	6.11	6861679	No	0.407	42	1.32	802562	Yes	4.471	1.87	3195472	Yes	0.145
	10	11.19	6861679	No	0.407	43	1.30	802080	Yes	3.735	1.89	3195472	Yes	0.145
	15	16.24	6861679	No	0.407	44	1.27	794281	Yes	4.235	1.84	3195472	Yes	0.145
	20	21.19	6861679	No	0.407	45	1.29	801675	Yes	3.481	1.91	3195472	Yes	0.145
	25	26.26	6861679	No	0.407	46	1.34	803892	Yes	4.084	1.93	3195472	Yes	0.145
	50	51.26	6861290	No	0.407	47	1.35	806892	Yes	4.003	1.91	3195472	Yes	0.145
	75	76.21	6857992	No	0.407	48	1.40	795380	Yes	4.106	1.91	3195472	Yes	0.145
	100	101.15	6857992	No	0.407	49	1.40	795431	Yes	4.081	1.92	3195472	Yes	0.145
	250	251.22	6849948	No	0.407	50	1.24	795468	Yes	3.854	1.95	3195472	Yes	0.145
	500	501.26	6849948	No	0.407	51	1.52	795524	Yes	3.928	2.02	3195472	Yes	0.145
San Francisco	5	6.19	5712903	No	0.280	42	1.22	1078204	Yes	1.505	1.42	1675613	Yes	0.047
	10	11.22	5697031	No	0.281	43	1.25	1045326	Yes	1.621	1.42	1675613	Yes	0.047
	15	16.34	5697031	No	0.281	44	1.23	1076544	Yes	1.569	1.51	1675613	Yes	0.047
	20	21.22	5683031	No	0.282	45	1.14	1069058	Yes	1.557	1.56	1675613	Yes	0.047
	25	26.25	5683031	No	0.282	46	1.46	1078212	Yes	1.536	1.70	1675613	Yes	0.047
	50	51.28	5604793	No	0.286	47	1.29	1068233	Yes	1.526	1.70	1675613	Yes	0.047
	75	76.18	5604793	No	0.286	48	1.24	1068021	Yes	1.586	1.62	1675613	Yes	0.047
	100	101.18	5604793	No	0.286	49	1.23	1090167	Yes	1.635	1.58	1675613	Yes	0.047
	250	251.30	5600573	No	0.286	50	1.12	1045326	Yes	1.531	1.56	1675613	Yes	0.047
	500	501.16	5600573	No	0.286	51	1.14	1093751	Yes	1.573	1.59	1675613	Yes	0.047
Toronto	5	6.24	9223694	No	0.275	42	1.41	1652074	Yes	1.720	1.49	2870172	Yes	0.132
	10	11.19	9223694	No	0.275	43	1.58	1651938	Yes	1.990	1.48	2870172	Yes	0.132
	15	16.17	9219828	No	0.275	44	1.62	1651069	Yes	1.695	1.62	2870172	Yes	0.132
	20	21.18	9219492	No	0.275	45	1.51	1650915	Yes	1.718	1.45	2870172	Yes	0.132
	25	26.14	9219353	No	0.275	46	1.46	1607855	Yes	1.745	1.47	2870172	Yes	0.132
	50	51.17	9219353	No	0.275	47	1.42	1623952	Yes	1.863	1.51	2870172	Yes	0.132
	75	76.19	9219353	No	0.275	48	1.58	1676544	Yes	1.677	1.47	2870172	Yes	0.132
	100	101.20	9219353	No	0.275	49	1.53	1597242	Yes	1.587	1.52	2870172	Yes	0.132
	250	251.22	9192020	No	0.276	50	1.29	1628988	Yes	1.935	1.58	2870172	Yes	0.132
	500	501.28	9179267	No	0.276	51	1.19	1662499	Yes	1.569	1.51	2870172	Yes	0.132
Ukansas	5	6.20	62962	No	1.000	42	1.13	70143	Yes	0.898	1.19	62962	Yes	0.000
	10	11.24	62962	No	1.000	43	1.11	70318	Yes	0.895	1.16	62962	Yes	0.000
	15	16.17	62962	No	1.000	44	1.11	69987	Yes	0.900	1.17	62962	Yes	0.000
	20	21.19	62962	No	1.000	45	1.16	69987	Yes	0.900	1.20	62962	Yes	0.000
	25	26.20	62962	No	1.000	46	1.26	70143	Yes	0.898	1.20	62962	Yes	0.000
	50	35.08	62962	No	1.000	47	1.13	71542	Yes	0.880	1.17	62962	Yes	0.000
	75	36.82	62962	No	1.000	48	1.10	67636	Yes	0.931	1.18	62962	Yes	0.000
	100	35.82	62962	No	1.000	49	1.14	70143	Yes	0.898	1.20	62962	Yes	0.000
	250	34.51	62962	No	1.000	50	1.13	70944	Yes	0.887	1.22	62962	Yes	0.000
	500	36.27	62962	No	1.000	51	1.14	70318	Yes	0.895	1.22	62962	Yes	0.000
Umissouri	5	6.17	670811	No	0.353	42	7.93	171306	Yes	1.532	1.45	266850.2	Yes	0.126
	10	11.25	670811	No	0.353	43	1.19	165181	Yes	1.496	1.49	266850.2	Yes	0.126
	15	16.17	670811	No	0.353	44	1.16	171054	Yes	1.696	1.44	266850.2	Yes	0.126
	20	21.21	670811	No	0.353	45	1.30	170078	Yes	1.642	1.51	266850.2	Yes	0.126
	25	26.20	670811	No	0.353	46	1.20	166855	Yes	1.593	1.49	266850.2	Yes	0.126
	50	51.29	668983	No	0.354	47	1.24	169548	Yes	1.504	1.49	266850.2	Yes	0.126
	75	76.33	668983	No	0.354	48	1.20	170112	Yes	1.659	1.54	266850.2	Yes	0.126
	100	101.22	668983	No	0.354	49	1.12	171483	Yes	1.382	1.51	266850.2	Yes	0.126
	250	251.21	662953	No	0.357	50	1.17	170432	Yes	1.639	1.49	266850.2	Yes	0.126
	500	501.21	662935	No	0.357	51	1.15	170155	Yes	1.588	1.51	266850.2	Yes	0.126