

**ECON 3161 Fall 2023**

## Homework 1

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Due: September 10th 2023

- **Instruction:** There are 85 points in total. You can either type or handwrite your answer. Then upload your answer file on canvas. Please clearly number your answers. Incomplete or non-legible answers will only receive partial or no credits. For the first two questions, you can refer to my math review lecture notes or the math refreshers A – C of our reference book.

1.) (20 points) (24 pts) Let  $X$  be a random variable taking on the values  $-1$  and  $2$ , each with probability  $0.5$ . Answer the following two questions:

(i) Find  $E(X)$  and  $E(X^2)$ . (12 points)

**Solution:**

$$E(X) = (-1) \cdot (0.5) + (2) \cdot (0.5) = 0.5$$

$$E(X^2) = (-1)^2 \cdot (0.5) + (2)^2 \cdot (0.5) = (1) \cdot (0.5) + (4) \cdot (0.5) = 2.5$$

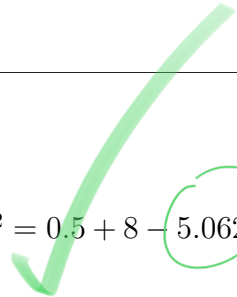


(ii) Find  $\text{var}(X)$  and  $\text{var}(X^2)$ . (12 points)

**Solution:**

$$\text{var}(X) = E(X^2) - (E(X))^2 = 2.5 - (0.5)^2 = 2.5 - 0.25 = 2.25$$

$$\text{var}(X^2) = E(X^4) - (E(X^2))^2 = (-1)^4 \cdot (0.5) + (2)^4 \cdot (0.5) - (2.5)^2 = 0.5 + 8 - 5.0625 = 3.4375$$



$$2.5^2 = 6.25$$

$$\text{var}(X^2) = 8.5 - 6.25 = 2.25$$

2.) (25 pts) Let  $Y_1, Y_2, Y_3$  be independent, identically distributed random variables from a population with mean  $\mu$  and variance  $\sigma^2$ . Let  $\bar{Y} = \frac{1}{3} \sum_{i=1}^3 Y_i$  denote the average of these three random variables. Answer the following questions:

(i.) What are the expected value and variance of  $\bar{Y}$ ? (10 points)

<p><b>Solution:</b></p> $E(\bar{Y}) = \frac{1}{3} \cdot 3\mu = \mu$ $\text{var}(\bar{Y}) = \left(\frac{1}{3}\right)^2 \cdot 3\sigma^2 = \sigma^2 = \frac{\sigma^2}{3}$	$\begin{aligned} \text{var}(aX) &= E[(aX - aE(X))^2] \\ &= E[a^2(X - E(X))^2] \\ &= a^2 E[(X - E(X))^2] = a^2 \text{Var}(X) \end{aligned}$ <p><i>"a" is a constant</i></p>
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(ii.) Now consider a different estimator of  $\mu$  :  $W = \frac{1}{2}(Y_1 + Y_2)$ . Show that  $W$  is an unbiased estimator of  $\mu$ . Find the variance of  $W$ . (10 points)

<p><b>Solution:</b></p> $E(W) = \frac{1}{2} \cdot 2\mu = \mu = E(\mu)$ $\text{var}(W) = \left(\frac{1}{2}\right)^2 \cdot 2\sigma^2 = \sigma^2 = \frac{\sigma^2}{2}$	$\text{var}(aX) = a^2 \text{Var}(X)$ <p><i>"a" is a constant</i></p>
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(iii.) Based on your answers to parts (i) and (ii), which estimator do you prefer? (5 points)

<p><b>Solution:</b></p> <p><math>\bar{Y}</math> because it incorporates all three random variables, while <math>W</math> only incorporates two out of the three.</p>
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$$\text{var}(\bar{Y}) = \frac{\sigma^2}{3} \quad \text{var}(W) = \frac{\sigma^2}{2}$$

$$\frac{\sigma^2}{3} < \frac{\sigma^2}{2}$$



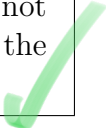
lower variance → more efficient

3.) (18 points) Read section 1-4 on pages 10-14 and section 2-7a on pages 53-54 of our reference book, and watch the video (by clicking the link YouTube), answer the following questions:

- (i.) What does a counterfactual outcome mean? Can you observe a counterfactual outcome in practice? (8 points)

**Solution:**

A counterfactual outcome is a hypothetical or alternative outcome that could have occurred in a situation but did not actually happen. In practice, it usually is not observable. With clones, it is possible to have perfect ceteris paribus and witness the counterfactual outcome playing out.




- (ii.) What does ceteris paribus mean? If we collect earnings data from two groups of people (group 1 attended public colleges, group 2 attended private colleges), and compare the earnings between those two groups, will this be a ceteris paribus comparison? Why? (10 points)

**Solution:**

It means other relevant factors being equal, and thus a clearer picture of the cause and effect.

Comparing earnings between these two groups of people would not be a ceteris paribus comparison by default. The reason is that attending a public college versus a private college is just one of many factors that can influence earnings. To make it a ceteris paribus comparison, you would need to ensure that all other relevant factors that could affect earnings are held constant.



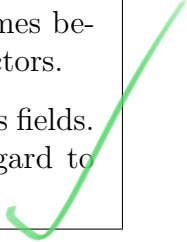
4.) (18 pts) Read section 2-7a on pages 53-54 of our reference book, and watch the video (by clicking the link YouTube), answer the following questions:

- (i.) What does RCT stand for? What are the advantages and disadvantages of RCTs? (9 points)

**Solution:**

In an RCT, participants are randomly assigned to either an experimental group that receives the intervention or treatment being tested or a control group that does not. This random assignment helps ensure that any observed differences in outcomes between the groups are likely due to the intervention itself rather than other factors.

Advantages of RCTs are that they determine causality and are useful in various fields. Disadvantages are that they are very expensive and can be unethical in regard to randomizing subjects.



- (ii.) In the West Point study, what is the treatment? What is the treatment group? What is the control group? (9 points)

**Solution:**

In the West Point study, the treatment is "permission to use electronics in class." The treatment group is the group of students who were granted this permission, and the control group is the group of students who were in the technology-free class.

