# CS-2050-All-Sections CS 2050 Homework 4 (HOWARD, FAULKNER, ELLEN)

Vidit Dharmendra Pokharna

TOTAL POINTS

## 96.5 / 101

QUESTION 1

## Question 1 12 pts

*1.1* a **3 / 3**

✓ **- 0 pts** *False*

**- 3 pts** True

**- 3 pts** Incorrect / Missing

*1.2* b **3 / 3**

✓ **- 0 pts** *True*

**- 3 pts** False

**- 3 pts** Incorrect / Missing

*1.3* c **3 / 3**

✓ **- 0 pts** *False*

**- 3 pts** True

**- 3 pts** Incorrect / Missing

*1.4* d **3 / 3**

✓ **- 0 pts** *False*

**- 3 pts** True

**- 3 pts** Incorrect / Missing

QUESTION 2

## Question 2 12 pts

*2.1* a **3 / 3**

✓ **- 0 pts** *0*

**- 3 pts** Incorrect / Missing

*2.2* b **3 / 3**

✓ **- 0 pts** *1*

**- 3 pts** Incorrect / Missing

*2.3* c **3 / 3**

✓ **- 0 pts** *5*

**- 3 pts** Incorrect / Missing

*2.4* d **3 / 3**

✓ **- 0 pts** *3*

**- 3 pts** Incorrect / Missing

QUESTION 3

## Question 3 24 pts

*3.1* a **3 / 3**

✓ **- 0 pts** *$$\{a, b, c, d, e, f, m, n, o\}$$*

**- 3 pts** Incorrect / Missing

*3.2* b **3 / 3**

✓ **- 0 pts** *$$\emptyset$$*

**- 3 pts** Incorrect / Missing

*3.3* c **3 / 3**

✓ **- 0 pts** *$$\{(m, m), (m, n), (m, o), (n, m), (n, n), (n,* *o), (o, m), (o, n), (o, o)\}$$*

- **1.5 pts** Used anything except a set of tuples e.g. used a set of sets
- **3 pts** Incorrect / Missing

### *3.4* d **3 / 3**

✓ **- 0 pts** *$$\{ \emptyset, \{m\}, \{n\}, \{o\}, \{m, n\}, \{n, o\}, \{m, o\}, \{m, n, o\}\}$$*

- **3 pts** Incorrect / Missing

### *3.5* e **3 / 3**

✓ **- 0 pts** *$$\emptyset$$*

- **3 pts** Incorrect / Missing

### *3.6* f **3 / 3**

✓ **- 0 pts** *15*

- **3 pts** Incorrect / Missing

### *3.7* g **3 / 3**

✓ **- 0 pts** *$$2^{2^{2^{|C|}}} = 2^{2^{2^{3}}} = 2^{2^8} = 2^{256}$$*

- **3 pts** Incorrect / Missing

### *3.8* h **3 / 3**

✓ **- 0 pts** *$$\{a, b, c, d, e, m, n, o\}$$*

- **3 pts** Incorrect / Missing

QUESTION 4

## *4* Question 4 **10 / 10**

✓ **- 0 pts** *Correct*

- **6 pts** Uses a Venn Diagram for a proof
- **5 pts** Uses mutual subsets approach but only shows one direction
- **5 pts** Did not cite any steps

Invalid Steps

- **2 pts** 1 Invalid Step
- **4 pts** 2 Invalid Steps
- **6 pts** 3 Invalid Steps
- **8 pts** 4 Invalid Steps
- **10 pts** 5+ Invalid Steps

Skipped Steps

- **2 pts** 1 Skipped Step
- **4 pts** 2 Skipped Steps
- **6 pts** 3 Skipped Steps
- **8 pts** 4 Skipped Steps
- **10 pts** 5+ Skipped Steps

Uncited Steps

- **1 pts** 1 Uncited Step
- **2 pts** 2 Uncited Steps
- **3 pts** 3 Uncited Steps
- **4 pts** 4+ Uncited Steps

Miscited Steps

- **1 pts** 1 Miscited Step
- **2 pts** 2 Miscited Steps
- **3 pts** 3 Miscited Steps
- **4 pts** 4+ Miscited Steps

- **8 pts** Uses set equivalencies.
- **10 pts** Disproves Statement
- **10 pts** No Answer

QUESTION 5

## *5* Question 5 **0 / 6**

- **0 pts** $$\{\emptyset, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$$
- **3 pts** Working shown with incorrect order of operators used

✓ **- 6 pts** *Incorrect/No Answer*

QUESTION 6

## Question 6 12 pts

6.1 a **3 / 3**

✓ **- 0 pts** *$$T\cap\overline{S}\cap C$$*

*or*

*$$$T \cap \overline{S} \cap \overline{C}$$$*

**- 3 pts** Incorrect / Missing

6.2 b **3 / 3**

✓ **- 0 pts** *$$(C \cup T) - (C \cap T)$$*

*or*

*$$(C \cup T) \cap \overline{(C \cap T)}$$*

*or*

*$$(T-C) \cup (C-T)$$*

**- 3 pts** Incorrect / Missing

6.3 c **3 / 3**

✓ **- 0 pts** *$$C - (T \cap S)$$ or $$C \cap \overline{T\cap S}$$*

**- 3 pts** Incorrect / Missing

6.4 d **3 / 3**

✓ **- 0 pts** *$$C - T - S$$*

*or*

*$$C - (T \cup S)$$*

*or*

*$$C\cap\overline{T}\cap\overline{S}$$*

*or*

*$$C\cap\overline{T\cup S}$$*

**- 3 pts** Incorrect / Missing

QUESTION 7

7 Question 7 **5 / 5**

✓ **- 0 pts** *A = $$\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$$*

*or*

*A = $$\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$$*

**- 2.5 pts** Correctly satisfied one of the two constraints i.e. either $$|A| = 3$$ or $$A \in P(P(A))$$

**- 5 pts** Does not exist

**- 5 pts** No Answer

QUESTION 8

## Question 8 6 pts

8.1 a **2 / 3**

✓ **- 0 pts** *Correct*

**- 1 pts** Says statement is True but provides an invalid proof

**- 2 pts** Says statement is True but does not provide a proof

**- 3 pts** Says statement is False and provides a counter example

**- 3 pts** No Answer

**- 1** *Point adjustment*

1 You needed to provide a 2 column proof.

8.2 b **3 / 3**

✓ **- 0 pts** *Valid counterexample*

*e.g. $$A = \{1\}$$ and $$B = \{2\}$$*

**- 1 pts** Says statement is False but provides invalid counter example

**- 2 pts** Says statement is False but does not provide a counter example

- **3 pts** Proves the statement
- **3 pts** No Answer

QUESTION 9

## 9 Question 9 **9 / 9**

✓ **- 0 pts** *Correct*

- **5 pts** Only shows one direction
- **5 pts** Did not cite any steps

Invalid Steps

- **2 pts** 1 Invalid Step
- **4 pts** 2 Invalid Steps
- **6 pts** 3 Invalid Steps
- **8 pts** 4 Invalid Steps
- **10 pts** 5+ Invalid Steps

Skipped Steps

- **2 pts** 1 Skipped Step
- **4 pts** 2 Skipped Steps
- **6 pts** 3 Skipped Steps
- **8 pts** 4 Skipped Steps
- **10 pts** 5+ Skipped Steps

Uncited Steps

- **1 pts** 1 Uncited Step
- **2 pts** 2 Uncited Steps
- **3 pts** 3 Uncited Steps
- **4 pts** 4+ Uncited Steps

Miscited Steps

- **1 pts** 1 Miscited Step
- **2 pts** 2 Miscited Steps
- **3 pts** 3 Miscited Steps
- **4 pts** 4+ Miscited Steps

- **9 pts** Disproves Statement
- **9 pts** No Answer

QUESTION 10

## 10 Question 10 **5 / 5**

✓ **- 0 pts** *Correct examples provided*

*e.g. $$A = B = C = D = \emptyset$$*

*OR*

*$$A = \{1,2,3\}$$; $$B  = \{1\}$$; $$C  = \{2\}$$; $$D = \{3\}$$*

- **5 pts** Incorrect / No Answer

QUESTION 11

## 11 On Time **2.5 / 0**

✓ **+ 2.5 pts** *On Time (Before Thursday)*

- **0 pts** On Time (Friday)
- **10 pts** 1 day late
- **25 pts** 2 days late

QUESTION 12

## 12 Matching **0 / 0**

✓ **- 0 pts** *Correct*

- **5 pts** Incorrect

Vidit Pokharna

# CS 2050 HW 4

1.

   a. False; no set is a proper subset of itself
   b. True; the null set is a proper subset of all sets besides itself
   c. False; the right side contains only one element that does not hold the value of the right hand side and therefore is not a subset
   d. False; no set is a proper subset of itself and the right-hand side is simply the null set

2.

   a. 0
   b. 1
   c. 5
   d. 3

3.

   a. $\{a, b, c, d, e, m, n, o, f\}$
   b. $\emptyset$
   c. $\{(m, m), (m, n), (n, o), (n, m), (n, n), (n, o), (o, m), (o, n), (o, o)\}$
   d. $\{\emptyset, \{m\}, \{n\}, \{o\}, \{m, n\}, \{m, o\}, \{n, o\}, \{m, n, o\}\}$
   e. $\emptyset$
   f. 15
   g. $2^{2^{2^3}} = 2^{256}$
   h. $\{a, b, c, d, e, m, n, o\}$

*1.1* a **3 / 3**

✓ **- 0 pts** *False*

**- 3 pts** True

**- 3 pts** Incorrect / Missing

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*1.2* b **3 / 3**

✓ **- 0 pts** *True*

**- 3 pts** False

**- 3 pts** Incorrect / Missing

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   g. $2^{2^{2^3}} = 2^{256}$
   h. $\{a, b, c, d, e, m, n, o\}$

*1.3* **C** **3 / 3**

✓ **- 0 pts** *False*

**- 3 pts** True

**- 3 pts** Incorrect / Missing

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*1.4* d **3 / 3**

✓ **- 0 pts** *False*

**- 3 pts** True

**- 3 pts** Incorrect / Missing

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*2.1* a **3 / 3**

✓ **- 0 pts** *0*

**- 3 pts** Incorrect / Missing

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   h. $\{a, b, c, d, e, m, n, o\}$

*2.2* b **3 / 3**

✓ **- 0 pts** *1*

**- 3 pts** Incorrect / Missing

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*2.3* **C** **3 / 3**

✓ **- 0 pts** *5*

**- 3 pts** Incorrect / Missing

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*2.4* d **3 / 3**

✓ **- 0 pts** *3*

**- 3 pts** Incorrect / Missing

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*3.1* a **3 / 3**

✓ **- 0 pts** *$$\{a, b, c, d, e, f, m, n, o\}$$*

**- 3 pts** Incorrect / Missing

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*3.2* b **3 / 3**

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*3.3* C **3 / 3**

✓ **- 0 pts** *$$\{(m, m), (m, n), (m, o), (n, m), (n, n), (n, o), (o, m), (o, n), (o, o)\}$$*

**- 1.5 pts** Used anything except a set of tuples e.g. used a set of sets

**- 3 pts** Incorrect / Missing

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3.4 d **3 / 3**

✓ **- 0 pts** $$\{ \emptyset, \{m\}, \{n\}, \{o\}, \{m, n\}, \{n, o\}, \{m, o\}, \{m, n, o\}\}$$

**- 3 pts** Incorrect / Missing

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**CS 2050 HW 4**

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*3.5* e **3 / 3**

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*3.6* f **3 / 3**

✓ **- 0 pts** *15*

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*3.7* g **3 / 3**

✓ **- 0 pts** $$2^{2^{2^{|C|}}} = 2^{2^{2^{3}}} = 2^{2^8} = 2^{256}$$

**- 3 pts** Incorrect / Missing

Vidit Pokharna

# CS 2050 HW 4

1.

a. False; no set is a proper subset of itself
b. True; the null set is a proper subset of all sets besides itself
c. False; the right side contains only one element that does not hold the value of the right hand side and therefore is not a subset
d. False; no set is a proper subset of itself and the right-hand side is simply the null set

2.

a. 0
b. 1
c. 5
d. 3

3.

a. $\{a, b, c, d, e, m, n, o, f\}$
b. $\emptyset$
c. $\{(m, m), (m, n), (n, o), (n, m), (n, n), (n, o), (o, m), (o, n), (o, o)\}$
d. $\{\emptyset, \{m\}, \{n\}, \{o\}, \{m, n\}, \{m, o\}, \{n, o\}, \{m, n, o\}\}$
e. $\emptyset$
f. 15
g. $2^{2^{2^3}} = 2^{256}$
h. $\{a, b, c, d, e, m, n, o\}$

*3.8* h **3 / 3**

✓ **- 0 pts** *$$\{a, b, c, d, e, m, n, o\}$$*

**- 3 pts** Incorrect / Missing

4.

I will proceed with a direct proof, showing the equivalence of $(\overline{X \cap Y}) \cap Z$ and $(\bar{X} \cap Z) \cup (\bar{Y} \cap Z)$

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| 5 | $(\overline{X \cap Y}) \cap Z = \{a \mid a \in (\overline{X \cap Y}) \land a \in Z\}$ | Set building notation |
| 6 | $\{a \mid a \notin (X \cap Y) \land a \in Z\}$ | Definition of complement |
| 7 | $\{a \mid \neg(a \in (X \cap Y)) \land (a \in Z)\}$ | Definition of complement |
| 8 | $\{a \mid \neg((a \in X) \land (a \in Y)) \land (a \in Z)\}$ | Definition of intersection |
| 9 | $\{a \mid (\neg(a \in X) \lor \neg(a \in Y)) \land (a \in Z)\}$ | DeMorgan's Law for propositions |
| 10 | $\{a \mid ((a \notin X) \lor \neg(a \in Y)) \land (a \in Z)\}$ | Negation Law |
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$\therefore$ Using proof by logical equivalency, we can conclude that $(\overline{X \cap Y}) \cap Z = (\bar{X} \cap Z) \cup (\bar{Y} \cap Z)$

5. $\{\{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$. The elements of this are $\{\emptyset\}$, $\{\{\emptyset\}\}$, and $\{\emptyset, \{\emptyset\}\}$

6.

a. $T \cap (\bar{S} \cap C)$
b. $C \cup T - (C \cap T)$
c. $C \cap (\overline{S \cap T})$
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## 4 Question 4 10 / 10

✓ **- 0 pts** *Correct*

**- 6 pts** Uses a Venn Diagram for a proof

**- 5 pts** Uses mutual subsets approach but only shows one direction

**- 5 pts** Did not cite any steps

### Invalid Steps

**- 2 pts** 1 Invalid Step

**- 4 pts** 2 Invalid Steps

**- 6 pts** 3 Invalid Steps

**- 8 pts** 4 Invalid Steps

**- 10 pts** 5+ Invalid Steps

### Skipped Steps

**- 2 pts** 1 Skipped Step

**- 4 pts** 2 Skipped Steps

**- 6 pts** 3 Skipped Steps

**- 8 pts** 4 Skipped Steps

**- 10 pts** 5+ Skipped Steps

### Uncited Steps

**- 1 pts** 1 Uncited Step

**- 2 pts** 2 Uncited Steps

**- 3 pts** 3 Uncited Steps

**- 4 pts** 4+ Uncited Steps

### Miscited Steps

**- 1 pts** 1 Miscited Step

**- 2 pts** 2 Miscited Steps

**- 3 pts** 3 Miscited Steps

**- 4 pts** 4+ Miscited Steps

**- 8 pts** Uses set equivalencies.

**- 10 pts** Disproves Statement

**- 10 pts** No Answer

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$\therefore$ Using proof by logical equivalency, we can conclude that $(\overline{X \cap Y}) \cap Z = (\bar{X} \cap Z) \cup (\bar{Y} \cap Z)$

5. $\{\{\emptyset\},\{\{\emptyset\}\},\{\emptyset,\{\emptyset\}\}\}$. The elements of this are $\{\emptyset\}$, $\{\{\emptyset\}\}$, and $\{\emptyset,\{\emptyset\}\}$

6.

a. $T \cap (\bar{S} \cap C)$
b. $C \cup T - (C \cap T)$
c. $C \cap (\overline{S \cap T})$
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## 5 Question 5 **0 / 6**

- **- 0 pts** $$\{\emptyset, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\}$$
- **- 3 pts** Working shown with incorrect order of operators used
- ✓ **- 6 pts** *Incorrect/No Answer*

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I will proceed with a direct proof, showing the equivalence of $(\overline{X \cap Y}) \cap Z$ and $(\bar{X} \cap Z) \cup (\bar{Y} \cap Z)$

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$\therefore$ Using proof by logical equivalency, we can conclude that $(\overline{X \cap Y}) \cap Z = (\bar{X} \cap Z) \cup (\bar{Y} \cap Z)$

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a. $T \cap (\bar{S} \cap C)$
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d. $C \cap \bar{S} \cap \bar{T}$

6.1 a **3 / 3**

✓ **- 0 pts** $$T\cap\overline{S}\cap C$$

*or*

$$$T \cap \overline{S} \cap \overline{C}$$$

**- 3 pts** Incorrect / Missing

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I will proceed with a direct proof, showing the equivalence of $(\overline{X \cap Y}) \cap Z$ and $(\bar{X} \cap Z) \cup (\bar{Y} \cap Z)$

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6.2 b **3 / 3**

✓ **- 0 pts** *$$(C \cup T) - (C \cap T)$$*

*or*

*$$(C \cup T) \cap \overline{(C \cap T)}$$*

*or*

*$$(T-C) \cup (C-T)$$*

**- 3 pts** Incorrect / Missing

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∴ Using proof by logical equivalency, we can conclude that $(\overline{X \cap Y}) \cap Z = (\bar{X} \cap Z) \cup (\bar{Y} \cap Z)$

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*6.3* **C** **3 / 3**

- ✓ **- 0 pts** *$$C - (T \cap S)$$ or $$C \cap \overline{T\cap S}$$*
- **- 3 pts** Incorrect / Missing

4.

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∴ Using proof by logical equivalency, we can conclude that $(\overline{X \cap Y}) \cap Z = (\bar{X} \cap Z) \cup (\bar{Y} \cap Z)$

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a. $T \cap (\bar{S} \cap C)$
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6.4 d **3 / 3**

✓ **- 0 pts** $$C - T - S$$

*or*

$$C - (T \cup S)$$

*or*

$$C\cap\overline{T}\cap\overline{S}$$

*or*

$$C\cap\overline{T\cup S}$$

**- 3 pts** Incorrect / Missing

7. $A = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$
We know this meets the condition that $|A| = 3$
Additionally, $P(A)$ contains the null set, and then sets holding the first and second element of $A$. Therefore, each element of $A$ is in $P(A)$. We can use the same logic in $P(P(A))$, where the first three elements of $P\big(P(A)\big)$ would be the same elements as those in $A$. Therefore $A \in P(P(A))$

8.
   a. Suppose $x$ is an element of $A$ and that sets $A$ and $C$ are disjoint. Since we take the union of $A$ and $A \cap C$, we need to check if it is an element of either $A$ or $A \cap C$, or both. However, given the first sentence, this must be true as $x$ is an element of $A$, while it also cannot be in $A \cap C$ because this is the null set as they are disjoint. Now suppose $y$ is an element of $A \cup (A \cap C)$. Then $y$ is an element of either $A$ or $A \cap C$. It cannot be an element of $A \cap C$ as this is the null set (as mentioned before), and therefore, it must be an element of $A$. Then it is clearly an element of $A$. Therefore, every element of $A$ is an element in $A \cup (A \cap C)$ and every element in $A \cup (A \cap C)$ is an element in $A$.
   b. This is not true; given sets $A = \{1,2\}$ and $B = \{2,3\}$, we can determine $P(A) \cup P(B)$ to be $\{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}\}$.
   We also know $A \cup B = \{1,2,3\}$. The power set of this is $P(A \cup B) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$.
   It is noticeable that the set $\{1,2,3\}$ is an element of $P(A \cup B)$, but not an element of $P(A) \cup P(B)$. Therefore $P(A) \cup P(B) \neq P(A \cup B)$.

9. We need to prove that every integer is a member of this set. We need to prove that for any integers $a$, $b$ and $n$, $9a + 17b = n$. To do this, we first find that the greatest common divisor of 9 and 17 is 1. Therefore, there must exist some integers $c$ and $d$ such that $9c + 17d = 1$. Multiplying both sides by an integer $n$, we get $9cn + 17dn = n$. This shows that $n$ is a member of the set $\{9a + 17b \mid a, b \in \mathbb{Z}\}$. Given that for $9a + 17b = n$, $n$ is a linear combination of 9 and 17. Being integer coefficients, $n$ must be an integer. Therefore, every integer is a member of the set, and every member of the set is an integer, so we can conclude that $\{9a + 17b \mid a, b \in \mathbb{Z}\} = \mathbb{Z}$.

## 7 Question 7 **5 / 5**

✓ **- 0 pts** *A = $$\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$$*

*or*

*A = $$\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$$*

- **- 2.5 pts** Correctly satisfied one of the two constraints i.e. either $$|A| = 3$$ or $$A \in P(P(A))$$
- **- 5 pts** Does not exist
- **- 5 pts** No Answer

7. $A = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$
   We know this meets the condition that $|A| = 3$
   Additionally, $P(A)$ contains the null set, and then sets holding the first and second element of $A$. Therefore, each element of $A$ is in $P(A)$. We can use the same logic in $P(P(A))$, where the first three elements of $P\big(P(A)\big)$ would be the same elements as those in $A$. Therefore $A \in P(P(A))$

8.
   a. Suppose $x$ is an element of $A$ and that sets $A$ and $C$ are disjoint. Since we take the union of $A$ and $A \cap C$, we need to check if it is an element of either $A$ or $A \cap C$, or both. However, given the first sentence, this must be true as $x$ is an element of $A$, while it also cannot be in $A \cap C$ because this is the null set as they are disjoint. Now suppose $y$ is an element of $A \cup (A \cap C)$. Then $y$ is an element of either $A$ or $A \cap C$. It cannot be an element of $A \cap C$ as this is the null set (as mentioned before), and therefore, it must be an element of $A$. Then it is clearly an element of $A$. Therefore, every element of $A$ is an element in $A \cup (A \cap C)$ and every element in $A \cup (A \cap C)$ is an element in $A$.
   b. This is not true; given sets $A = \{1,2\}$ and $B = \{2,3\}$, we can determine $P(A) \cup P(B)$ to be $\{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}\}$.
      We also know $A \cup B = \{1,2,3\}$. The power set of this is $P(A \cup B) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$.
      It is noticeable that the set $\{1,2,3\}$ is an element of $P(A \cup B)$, but not an element of $P(A) \cup P(B)$. Therefore $P(A) \cup P(B) \neq P(A \cup B)$.

9. We need to prove that every integer is a member of this set. We need to prove that for any integers $a$, $b$ and $n$, $9a + 17b = n$. To do this, we first find that the greatest common divisor of 9 and 17 is 1. Therefore, there must exist some integers $c$ and $d$ such that $9c + 17d = 1$. Multiplying both sides by an integer $n$, we get $9cn + 17dn = n$. This shows that $n$ is a member of the set $\{9a + 17b \mid a, b \in \mathbb{Z}\}$. Given that for $9a + 17b = n$, $n$ is a linear combination of 9 and 17. Being integer coefficients, $n$ must be an integer. Therefore, every integer is a member of the set, and every member of the set is an integer, so we can conclude that $\{9a + 17b \mid a, b \in \mathbb{Z}\} = \mathbb{Z}$.

8.1 a 2 / 3

✓ **- 0 pts** *Correct*

**- 1 pts** Says statement is True but provides an invalid proof

**- 2 pts** Says statement is True but does not provide a proof

**- 3 pts** Says statement is False and provides a counter example

**- 3 pts** No Answer

**- 1** *Point adjustment*

1 You needed to provide a 2 column proof.

7. $A = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$
   We know this meets the condition that $|A| = 3$
   Additionally, $P(A)$ contains the null set, and then sets holding the first and second element of $A$. Therefore, each element of $A$ is in $P(A)$. We can use the same logic in $P(P(A))$, where the first three elements of $P\big(P(A)\big)$ would be the same elements as those in $A$. Therefore $A \in P(P(A))$

8.
   a. Suppose $x$ is an element of $A$ and that sets $A$ and $C$ are disjoint. Since we take the union of $A$ and $A \cap C$, we need to check if it is an element of either $A$ or $A \cap C$, or both. However, given the first sentence, this must be true as $x$ is an element of $A$, while it also cannot be in $A \cap C$ because this is the null set as they are disjoint. Now suppose $y$ is an element of $A \cup (A \cap C)$. Then $y$ is an element of either $A$ or $A \cap C$. It cannot be an element of $A \cap C$ as this is the null set (as mentioned before), and therefore, it must be an element of $A$. Then it is clearly an element of $A$. Therefore, every element of $A$ is an element in $A \cup (A \cap C)$ and every element in $A \cup (A \cap C)$ is an element in $A$.
   b. This is not true; given sets $A = \{1,2\}$ and $B = \{2,3\}$, we can determine $P(A) \cup P(B)$ to be $\{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}\}$.
      We also know $A \cup B = \{1,2,3\}$. The power set of this is $P(A \cup B) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$.
      It is noticeable that the set $\{1,2,3\}$ is an element of $P(A \cup B)$, but not an element of $P(A) \cup P(B)$. Therefore $P(A) \cup P(B) \neq P(A \cup B)$.

9. We need to prove that every integer is a member of this set. We need to prove that for any integers $a$, $b$ and $n$, $9a + 17b = n$. To do this, we first find that the greatest common divisor of 9 and 17 is 1. Therefore, there must exist some integers $c$ and $d$ such that $9c + 17d = 1$. Multiplying both sides by an integer $n$, we get $9cn + 17dn = n$. This shows that $n$ is a member of the set $\{9a + 17b \mid a, b \in \mathbb{Z}\}$. Given that for $9a + 17b = n$, $n$ is a linear combination of 9 and 17. Being integer coefficients, $n$ must be an integer. Therefore, every integer is a member of the set, and every member of the set is an integer, so we can conclude that $\{9a + 17b \mid a, b \in \mathbb{Z}\} = \mathbb{Z}$.

8.2 b **3 / 3**

✓ **- 0 pts** *Valid counterexample*

*e.g. $$A = \{1\}$$ and $$B = \{2\}$$*

**- 1 pts** Says statement is False but provides invalid counter example

**- 2 pts** Says statement is False but does not provide a counter example

**- 3 pts** Proves the statement

**- 3 pts** No Answer

7. $A = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}\}$
   We know this meets the condition that $|A| = 3$
   Additionally, $P(A)$ contains the null set, and then sets holding the first and second element of $A$. Therefore, each element of $A$ is in $P(A)$. We can use the same logic in $P(P(A))$, where the first three elements of $P\big(P(A)\big)$ would be the same elements as those in $A$. Therefore $A \in P(P(A))$

8.
   a. Suppose $x$ is an element of $A$ and that sets $A$ and $C$ are disjoint. Since we take the union of $A$ and $A \cap C$, we need to check if it is an element of either $A$ or $A \cap C$, or both. However, given the first sentence, this must be true as $x$ is an element of $A$, while it also cannot be in $A \cap C$ because this is the null set as they are disjoint. Now suppose $y$ is an element of $A \cup (A \cap C)$. Then $y$ is an element of either $A$ or $A \cap C$. It cannot be an element of $A \cap C$ as this is the null set (as mentioned before), and therefore, it must be an element of $A$. Then it is clearly an element of $A$. Therefore, every element of $A$ is an element in $A \cup (A \cap C)$ and every element in $A \cup (A \cap C)$ is an element in $A$.
   b. This is not true; given sets $A = \{1,2\}$ and $B = \{2,3\}$, we can determine $P(A) \cup P(B)$ to be $\{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}\}$.
      We also know $A \cup B = \{1,2,3\}$. The power set of this is $P(A \cup B) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$.
      It is noticeable that the set $\{1,2,3\}$ is an element of $P(A \cup B)$, but not an element of $P(A) \cup P(B)$. Therefore $P(A) \cup P(B) \neq P(A \cup B)$.

9. We need to prove that every integer is a member of this set. We need to prove that for any integers $a$, $b$ and $n$, $9a + 17b = n$. To do this, we first find that the greatest common divisor of 9 and 17 is 1. Therefore, there must exist some integers $c$ and $d$ such that $9c + 17d = 1$. Multiplying both sides by an integer $n$, we get $9cn + 17dn = n$. This shows that $n$ is a member of the set $\{9a + 17b \mid a, b \in \mathbb{Z}\}$. Given that for $9a + 17b = n$, $n$ is a linear combination of 9 and 17. Being integer coefficients, $n$ must be an integer. Therefore, every integer is a member of the set, and every member of the set is an integer, so we can conclude that $\{9a + 17b \mid a, b \in \mathbb{Z}\} = \mathbb{Z}$.

## 9 Question 9 **9 / 9**

✓ **- 0 pts** *Correct*

- **- 5 pts** Only shows one direction
- **- 5 pts** Did not cite any steps

Invalid Steps

- **- 2 pts** 1 Invalid Step
- **- 4 pts** 2 Invalid Steps
- **- 6 pts** 3 Invalid Steps
- **- 8 pts** 4 Invalid Steps
- **- 10 pts** 5+ Invalid Steps

Skipped Steps

- **- 2 pts** 1 Skipped Step
- **- 4 pts** 2 Skipped Steps
- **- 6 pts** 3 Skipped Steps
- **- 8 pts** 4 Skipped Steps
- **- 10 pts** 5+ Skipped Steps

Uncited Steps

- **- 1 pts** 1 Uncited Step
- **- 2 pts** 2 Uncited Steps
- **- 3 pts** 3 Uncited Steps
- **- 4 pts** 4+ Uncited Steps

Miscited Steps

- **- 1 pts** 1 Miscited Step
- **- 2 pts** 2 Miscited Steps
- **- 3 pts** 3 Miscited Steps
- **- 4 pts** 4+ Miscited Steps

- **- 9 pts** Disproves Statement
- **- 9 pts** No Answer

10. There exists sets $A, B, C, D$ such that

1. $|A|, |B|, |C|, |D| \geq 0$,
2. $B, C, D \in A$
3. $\mathrm{B} \cap \mathrm{C} = \emptyset$
4. $\mathrm{B} \cap \mathrm{D} = \emptyset$
5. $\mathrm{C} \cap \mathrm{D} = \emptyset$

An example of this would be:
$A = \{1,2,3,4\}$
$B = \{1\}$
$C = \{2\}$
$D = \{3\}$

$|A - B - C - D| = |\{4\}| = 1$
$|A| - |B| - |C| - |D| = 4 - 1 - 1 - 1 = 1$

## 10 Question 10 **5 / 5**

✓ **- 0 pts** *Correct examples provided*

*e.g. $$A = B = C = D = \emptyset$$*

*OR*

*$$A = \{1,2,3\}$$; $$B  = \{1\}$$; $$C  = \{2\}$$; $$D  = \{3\}$$*

**- 5 pts** Incorrect / No Answer

## 11 On Time 2.5 / 0

✓ **+ 2.5 pts** *On Time (Before Thursday)*

**- 0 pts** On Time (Friday)

**- 10 pts** 1 day late

**- 25 pts** 2 days late

## 12 Matching 0 / 0

✓ **- 0 pts** *Correct*

**- 5 pts** Incorrect