MATH 3215 Assignment 1

1. Read the syllabus carefully. In particular, you should know how to contact the instructor, what to do if you plan to submit a late assignment, what happens when you submit a wrong PDF file for an assignment, etc.

2. (Warn-up question, not to be graded) The table below shows the number of successes over the number of total treatments for treatments involving either small or large kidney stones, where Treatment A includes open surgical procedures and Treatment B includes closed surgical procedures. Compute and compare the six success rates in percentage. What have you discovered? This is called Simpson's paradox.

	Treatment A	Treatment B
Small stones	81/87	234/270
Large stones	192/263	55/80
Both	273/350	289/350

Even though the success rates for Treatment A are higher in the groups of small stones and large stones separately, the overall success rate for Treatment A is in fact lower. This is known as Simpson's paradox. Because there are such counterintuitive situations, we have to be careful when doing data analysis.

3. A box contains three marbles—one red, one green, and one blue. Consider an experiment that consists of taking one marble from the box, then replacing it in the box and drawing a second marble from the box. Let r denote red, g denote green, and b denote blue. Use (b, r) to denote the outcome where the first draw is blue and the second draw is red, for example.

(a) Describe the sample space.
$\{(r,r),(r,g),(r,b),(g,r),(g,g),(g,b),(b,r),(b,g),(b,b)\}$

(b) Describe the sample space for the case in which the second marble is drawn without first replacing the first marble.
$\{(r,g),(r,b),(g,r),(g,b),(b,r),(b,g)\}$

4. Let $E, F, G \subset S$ be three events in the sample space S. Find expressions for the following events described in terms of E, F, G. For example, the event that exactly two of them occur is $(E \cap F \cap G^c) \cup (E \cap F^c \cap G) \cup (E^c \cap F \cap G)$ (think about why this makes sense).

Note: The same set may be described in different forms, so the answers below may appear to be different from yours even if they are equivalent mathematically.

(a) only E occurs;
$E \cap F^c \cap G^c$

(b) both E and G but not F occur;
$E \cap F^c \cap G$

(c) at least one of the events occurs;
$E \cup F \cup G$

(d) at least two of the events occur;
$(E \cap F) \cup (E \cap G) \cup (F \cap G)$

(e) all three occur;
$E \cap F \cap G$

(f) none of the events occurs;
$E^c \cap F^c \cap G^c$

(g) at most one of them occurs;
$(E^c \cap F^c) \cup (E^c \cap G^c) \cup (F^c \cap G^c)$

(h) at most two of them occur.
$(E \cap F \cap G)^c$

5. Let X be chosen uniformly at random from the set $\{1, 2, \ldots, m\}$ for an integer m.

(a) What is the PMF of X?
$f(x) = 1/m$ for all $x \in \{1, 2, \ldots, m\}$

(b) What is the CDF of X? (Define $F(x)$ case by case depending on the value of x.)
$F(x) = 0$ if $x < 1$, $F(x) = k/m$ if $k \leq x < k + 1$, and $F(x) = 1$ if $x \geq m$

6. Roll a typical six-sided die twice. Let X be the sum of the two numbers we see. What is the PMF of X?

$f(2) = f(12) = 1/36$, $f(3) = f(11) = 1/18$, $f(4) = f(10) = 1/12$, $f(5) = f(9) = 1/9$, $f(6) = f(8) = 5/36$, and $f(7) = 1/6$

7. Roll a typical six-sided die three times. Let E denote the event that we see 1 or 2 on the first roll, let F denote the event that we see 3 or 4 on the second roll, and let G denote the event that we see 5 or 6 on the third roll.

(a) How to describe the outcome (which is a triplet) as a random variable? (It suffices to specify the sample space and the PMF.)
We can denote the outcome by a random variable (X, Y, Z). The sample space is $S = \big\{(x, y, z) : x, y, z \in \{1, 2, 3, 4, 5, 6\}\big\} = \{1, 2, 3, 4, 5, 6\}^3$. The random variable can be defined by $f(x, y, z) = \mathbb{P}\{(X, Y, Z) = (x, y, z)\} = 1/6^3$ for any $(x, y, z) \in S$.

(b) What is the mathematical expression for the event $E \cap F$ (i.e., what is it as a set)? What is $\mathcal{P}(E \cap F)$?
$E \cap F = \{(x, y, z) : x = 1, 2,\ y = 3, 4,\ z = 1, 2, 3, 4, 5, 6\}$ and $\mathcal{P}\{E \cap F\} = 24/6^3 = 1/9$

(c) What is $\mathcal{P}(E \cup F \cup G)$?
$E \cup F \cup G = (E^c \cap F^c \cap G^c)^c$, so $\mathcal{P}\{E \cup F \cup G\} = 1 - \mathcal{P}\{E^c \cap F^c \cap G^c\} = 1 - 2^3/3^3 = 19/27$