CS-2050-All-Sections Exam 4 Blue

Vidit Dharmendra Pokharna

TOTAL POINTS

100 / 100

QUESTION 1

1 MC 1 **5 / 5**

- **- 0 pts** A
- **- 0 pts** B
- **- 0 pts** C
- **- 0 pts** D
- **- 0 pts** E
- ✓ **- 0 pts** *No Answer*

QUESTION 2

2 MC 2 **5 / 5**

- **- 5 pts** A
- **- 5 pts** B
- ✓ **- 0 pts** *C*
- **- 5 pts** D
- **- 5 pts** E
- **- 5 pts** No Answer

QUESTION 3

3 MC 3 **5 / 5**

- **- 5 pts** A
- **- 5 pts** B
- **- 5 pts** C
- **- 5 pts** D
- ✓ **- 0 pts** *E*
- **- 5 pts** No Answer

QUESTION 4

4 MC 4 **5 / 5**

- **- 5 pts** A
- ✓ **- 0 pts** *B*
- **- 5 pts** C
- **- 5 pts** D
- **- 5 pts** E
- **- 5 pts** No Answer

QUESTION 5

5 MC 5 **5 / 5**

- **- 5 pts** A
- **- 5 pts** B
- ✓ **- 0 pts** *C*
- **- 5 pts** D
- **- 5 pts** No Answer

QUESTION 6

6 MC 6 **5 / 5**

- **- 5 pts** A
- **- 5 pts** B
- **- 5 pts** C
- ✓ **- 0 pts** *D*
- **- 5 pts** E
- **- 5 pts** No Answer

QUESTION 7

7 MC 7 **5 / 5**

- **- 5 pts** A

- **5 pts** B
- **5 pts** C

✓ **- 0 pts** *D*

- **5 pts** No Answer

QUESTION 8

8 Short Answer 1 **5 / 5**

✓ **- 0 pts** *C(22, 3) + C(22, 20)*

- **2 pts** Minor error
- **4 pts** Major error
- **5 pts** Incorrect/No Answer

QUESTION 9

Short Answer 2 10 pts

9.1 a **5 / 5**

✓ **- 0 pts** *1*

- **2 pts** Minor error
- **4 pts** Major error
- **5 pts** Incorrect/No Answer

9.2 b **5 / 5**

✓ **- 0 pts** *C(6,2)*

- **2 pts** Minor error
- **4 pts** Major Error
- **5 pts** Incorrect/No Answer

QUESTION 10

10 Short Answer 3 **6 / 10**

- 0 pts 65 is correct since 4 choices for each question implies $$4^3$$ different ways to submit exam.

- **2 pts** Minor error

✓ **- 4 pts** *Major error*

- **5 pts** Did not explain or show work
- **10 pts** Incorrect/No Answer

QUESTION 11

11 Short Answer 4 **5 / 5**

✓ **- 0 pts** *Correct*

- **2 pts** Minor error
- **4 pts** Major error
- **5 pts** Did not explain or show work/Incorrect

QUESTION 12

12 Tree Diagram **10 / 10**

✓ **- 0 pts** *Correct tree diagram and final answer = 12.*

Solutions: (000, 002, 012, 020, 022, 120, 122, 200, 202, 212, 220, 222)

- 2 pts Left in invalid branches but did not include in final sum

- 4 pts Correct tree diagram but incorrect or missing final answer

Incorrect branches

- **3 pts** 1 Incorrect Branch
- **6 pts** 2 Incorrect Branch
- **9 pts** 3 Incorrect Branch

- 8 pts Did not provide tree diagram and had correct final answer

- 10 pts Did not provide tree diagram and had incorrect final answer

- **10 pts** No Answer

QUESTION 13

13 Strong Induction **13 / 15**

- **0 pts** Correct
- **2 pts** Did not define / incorrectly defined

$$P(n)$$ before using it

- **2 pts** Used predicate as a non-boolean

Basis step (cap at -3)

- **1 pts** Minor math error/missing base case

✓ **- 2 pts** *Does not use $$P(12), P(13), P(14)$$ for the base case*

- **3 pts** No/Completely incorrect basis step

IH and Inductive step (cap at -10)

- **2 pts** Does not explicitly assume IH that $$P(j)$$ is true for all $$j$$ in the bounds $$12 \leq j \leq k$$

- **2 pts** Incorrect/missing new variable domain definition (e.g. not saying $$k \in \mathbb{Z}^{\geq 14}$$ or that $$j \in \mathbb{Z}$$)

- **2 pts** Using $$n$$ in the inductive step instead of a new variable

- **2 pts** Switching between booleans and numbers

- **4 pts** Not citing inductive hypothesis when it is used

- **2 pts** Minor math error

- **4 pts** Major math error

- **3 pts** Minor jump in logic

- **6 pts** Major jump in logic

- **5 pts** Did not provide any reasonings

- **2 pts** Did not provide any reasonings for algebra steps

- **7 pts** Assumed $$P(k+1)$$ is true

- **8 pts** Not reaching $$P(k+1)$$

- **8 pts** Assumed IH correctly, but did not attempt to reach $$P(k+1)$$ (whether by direct proof or contradiction)

- **3 pts** Missing or incorrect inductive step conclusion (e.g. only concluded $$P(k+1)$$ instead of $$(\forall j P(j)) \to P(k+1)$$ or equivalent)

- **10 pts** Missing/ completely incorrect inductive step

Conclusion (cap at -2)

- **1 pts** No/Incorrect mention of $$P(n)$$ being true

- **1 pts** No mention of domain of $$n$$ or incorrect domain for $$n$$ mentioned (domain for $$n$$ should be $$n \in \mathbb{Z}^{\geq 12}$$)

- **1 pts** No/incorrect mention of principle of strong induction

- **15 pts** No answer

- **15 pts** Did not use Strong induction

1 need P(14) in the basis step

2 should be 14 because you need an additional base case

QUESTION 14

14 Math Induction 10 / 10

✓ **- 0 pts** *Correct*

Introduction (cap at -1)

- **1 pts** Did not define / incorrectly defined $$P(n)$$ before using it / Incorrect/missing domain

- **0.5 pts** Used predicate as a non-boolean

Basis step (cap at -2)

- **1 pts** Minor math error

- **2 pts** Does not use correct value for the base case. Correct Base Cases are: $$P(4)$$

- 2 pts No/Completely incorrect basis step

IH and Inductive step (cap at -6)

- 2 pts Does not explicitly assume $$P(k)$$

- 2 pts Incorrect/missing new variable domain definition (e.g. not saying $$k \in \mathbb{Z}^{\geq 2}$$)

- 1 pts Using $$n$$ in the inductive step instead of a new variable

- 1 pts Switching between booleans and numbers

- 2 pts Not citing inductive hypothesis when it is used

- 2 pts Minor math error

- 4 pts Major math error

- 3 pts Minor jump in logic

- 5 pts Major jump in logic

- 3 pts Did not provide any reasonings

- 2 pts Did not provide any reasonings for algebra steps

- 4 pts Assumed $$P(k+1)$$ is true

- 6 pts Not reaching $$P(k+1)$$

- 5 pts Assumed $$P(k)$$ correctly, but did not attempt to reach $$P(k+1)$$

- 1 pts Missing or incorrect inductive step conclusion (e.g. only concluded $$P(k+1)$$ instead of $$P(k) \to P(k+1)$$

- 6 pts Missing/ completely incorrect inductive step

- 1 pts Missing inductive step conclusion (i.e. "this concludes the inductive step")

Conclusion (cap at -1)

- 0.5 pts No/Incorrect mention of $$P(n)$$ being true

- 0.5 pts No mention of domain of $$n$$ or incorrect domain for $$n$$ mentioned (domain for $$n$$ should be $$n \in \mathbb{Z}^{\geq 2}$$)

- 0.5 pts No/incorrect mention of principle of mathematical induction

- 25 pts No answer

- 25 pts Did not use mathematical induction

QUESTION 15

15 CIOS EC **6 / 0**

✓ **+ 6 pts** *CIOS EC*

CS 2050-Exam 4 Blue

Name: Vidit Pokharna
GTID: 903772087

No notes, calculators, or other aids are allowed. Read all directions carefully and write your answers in the space provided.
Taking this exam signifies you are aware of and in accordance with the Academic Honor Code of Georgia.
Do not separate any pages from the rest of your exam.

Exam 4 Blue
100 points

[5] 1. How many bit strings of length 6 begin with 11 or end with 10?

- ○ 2^6
- ○ 2^4
- ○ $2 * 2^4 - 2 * 2^2$
- ○ $2 * 2^4 \quad 2^2$
- ○ None of the Above

11 _ _ 0 1

0 0 _ _ 1 0

$3 \cdot 2^2 + 3 \cdot 2^2 = 6 \cdot 2^2 = 24$

[5] 2. How many different permutations exist of the string "111122333"

- ○ $\frac{9!}{3!2!3!}$
- ○ $9!$
- ● $\frac{9!}{4!3!2!}$
- ○ $C(9,4) + C(9,2) + C(9,3)$
- ○ $C(9,4) + C(5,2) + C(3,3)$

$\frac{9!}{4! \cdot 2! \cdot 3!}$

[5] 3. How many ways can you rearrange the string "HELLOWORLD", if the string "LORD" must exist in the rearranged string?

- ○ $\frac{10!}{3! \cdot 3!}$
- ○ $\frac{10!}{3!}$
- ○ $\frac{7!}{3! \cdot 2!}$
- ○ $\frac{7!}{3!}$
- ● $\frac{7!}{2!}$

$\boxed{\text{LORD}}$ HE~~L~~WO~~L~~

$\frac{7!}{2!}$

[5] 4. Juan and Carmen are in a discrete mathematics course of 80 students which has asked for volunteers to form a committee of 7 students. How many ways are there to form this committee if both Juan and Carmen must be on the committee?

- ○ $C(80,5)$
- ● $C(78,5)$
- ○ $C(80,2) \cdot C(78,5)$
- ○ $C(80,2) + C(78,5)$
- ○ $C(80,7) - C(80,2)$

$C(78,5)$

[5] 5. How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 \leq 39$, where each x_i is an integer satisfying $x_i \geq 2$?

- ○ $C(39, 5)$
- ○ $C(29, 24)$
- ● $C(34, 29)$
- ○ $C(39, 4)$

$X_1 + X_2 + X_3 + X_4 + X_5 + t = 29$

$\binom{29+5}{5} = \binom{34}{5}$

$\binom{29+4}{4} = \binom{33}{4} + \binom{32}{4} + \cdots + \binom{?}{4}$

$= \binom{34}{5}$

[5] 6. A pirate robs the queen and makes off with 21 pieces of treasure. He then hides it in 4 treasure chests. What can we say about the quantity of treasure in each chest?

- ○ There is exactly one chest containing 6 pieces of treasure
- ○ All four chests could contain 6 pieces of treasure X
- ○ Each chest contains 4.25 pieces of treasure X
- ● It is possible that two chests both contain 8 pieces of treasure ✓
- ○ It is impossible to make any conclusions about the amount of treasure in each chest

[5] 7. Suppose you are proving P(n) is true for all positive integers using mathematical induction. Which of the following is true?

- ○ The basis step is $P(0)$
- ○ In the inductive step, you prove $P(k + 1)$ where $k \geq 1$ is a fixed arbitrary integer
- ○ The inductive hypothesis assumes $P(k)$ to be true for some fixed arbitrary real number $k \geq 1$. ✓
- ● In the inductive step, you prove $P(k) \to P(k + 1)$ where $k \geq 1$ is a fixed arbitrary integer ✓

[5] 8. Let A be a set with 22 elements. How many sets in the powerset of A contain exactly 3 or 20 elements? Show your work. You need not simplify to an integer.

$$\boxed{\binom{22}{3} + \binom{22}{20}}$$

$$\binom{22}{3} + \binom{22}{2}$$

$$\frac{22 \times 21 \times 20}{3 \times 2} + \frac{22 \times 21}{2}$$

$$\boxed{\frac{22!}{19!\,3!} + \frac{22!}{20!\,2!}}$$

$$\frac{22!}{19!}\left(\frac{1}{3!} + \frac{1}{20 \cdot 2}\right)$$

$$\frac{1}{6} + \frac{1}{40} \qquad \frac{46}{240}$$

Answer: 1771

$$\frac{22 \times \cancel{21} \times \cancel{20}}{\cancel{3} \times \cancel{2}} \qquad \frac{11\ \cancel{22} \times 21}{\cancel{2}}$$

$$\begin{array}{r} 231 \\ +\ 1540 \\ \hline 1771 \end{array} \qquad \begin{array}{r} 77 \\ \hline 161 \\ 1610 \\ \hline 1771 \end{array}$$

$$\frac{\cancel{22} \times \cancel{21} \times \cancel{20} \times \cancel{46}}{\cancel{240}\ \cancel{12}\cancel{4}\cancel{2}}$$

$\{\emptyset, 1, 2\}$

$\{\emptyset, 1, 2, 3\}$

$\binom{4}{2}$

[10] 9. Assume a fair coin is flipped 6 times. Write your **answer as an integer** in the provide blank for each part.

a) How many possible outcomes are there where the coins never land on heads?

Answer: 1

b) How many ways are there to toss exactly 2 heads?

Answer: 15

HHTTTT

$$\frac{6!}{4! \cdot 2!} = \binom{6}{2} = \frac{6 \times 5}{2} = 15$$

[10] 10. A professor gives a multiple-choice quiz that has 3 questions, each with 4 possible responses, a, b, c, d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least 2 answer sheets must be identical, assuming that no answers are left blank. Please write your final answer as an integer in the designated space below. **You must show your work and provided justification for your answer.**

Answer: 13

$3q \times 4r = 12$ combinations of answers

$12+1 = 13$ answer sheets, where if 12 are different, the last one must match any of those 12.

$\left\lceil \frac{13}{12} \right\rceil = 2$ ✓

[5] 11. A chef sits a family of 6 around a round table. The grandma requires that her husband sit on her right hand side. Given this constraint, how many possible seatings are there if two seatings are considered the same when each person has the same left neighbor and right neighbor? **Show your work to provide a justification for your solution.**

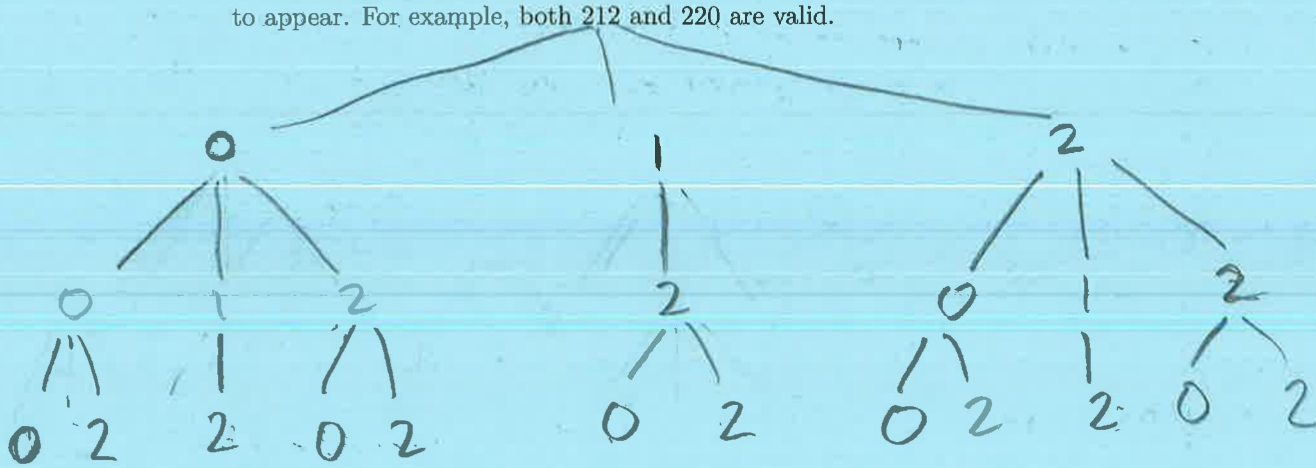
family of 5, where one member is grandma on left and husband on right.

5! ways to organize around the table.

However, due to rotations, you must divide by 5.

$\frac{5!}{5} = 4! = \boxed{24}$

[10] 12. Use a tree diagram to count the number of ternary strings (strings containing 0,1,2) of length 3 such that any occurrence of the digit 1 always appears directly before a 2. Not all digits have to appear. For example, both 212 and 220 are valid.



12 ternary strings

* continue onto page 9

[15] 13. Suppose you can buy chicken nuggets in two forms: a pack of 3 nuggets or a pack of 7 nuggets. Prove using strong induction that by using combinations of the two forms mentioned above that you can buy any quantity of nuggets greater than or equal to 12.

Let p(n) be the statement that a quantity of n nuggets can be made using packs of 3 nuggets or packs of 7 nuggets. I will use strong induction to prove that p(n) is true for all positive integers greater than or equal to 12.

Basis Step

- P(12) is true because $12 = 4(3)$, so 12 nuggets can be made using 4 packs of 3 nuggets.
- P(13) is true because $13 = 2(3) + 1(7)$, so 13 nuggets can be made using 2 packs of 3 nuggets and 1 pack of 7 nuggets.

Inductive Hypothesis

Let us assume p(j) is true for $12 \leq j \leq k$, where $k \geq 13$ is an arbitrary fixed integer. Also assume j is an integer.

Inductive Step

Line	Statement	Reasoning
1	$k = a(3) + b(7)$, $a, b \in \mathbb{Z}^{0,+}$	P(k) is true from inductive hypothesis
2	$a \geq 4$ or $b \geq 2$	k is a positive integer greater than 13

3	if $a \geq 4$:
	$k+1 = a(3) + b(7) + 1$
	$= a(3) + b(7) + 7 - 6$
	$= (a-2)(3) + (b+1)(7)$
	and thus a, b remain non-negative integers
	
	if $b \geq 2$:
	$k+1 = a(3) + b(7) + 1$
	$= a(3) + b(7) + 15 - 14$
	$= (a+5)(3) + (b-2)(7)$
	and thus a, b remain non-negative integers
		Add 1 to LHS of (1), testing if P(k+1) is true

[10] 14. Use mathematical induction, prove that $3n + 3 \leq 3^n$ for all positive integers $n \geq 2$.

Let p(n) be the statement $3n+3 \leq 3^n$ for some integer n. I will use mathematical induction to prove p(n) is true for all positive integers $n \geq 2$.

Basis Step

P(2) is true because $3(2)+3 = 6+3 = 9 \leq 3^2 = 9$

Inductive Hypothesis

Let us assume P(K) is true for some fixed arbitrary integer $k \geq 2$.

Inductive Step

Line	Statement	Reasoning
1	$3k+3 \leq 3^k$	known from inductive hypothesis
2	$3k+6 \leq 3^k+3$	add 3 to both sides of (1)
3	$3^k+3 \leq 3^{k+1}$	given $k \geq 2$, a power of 3 with exponent greater than or equal to 2 will always be less than the next power of 3 by atleast 18 $(3^3-3^2=18)$
4	$3k+6 \leq 3^k+3 \leq 3^{k+1}$	Substitute (3) into (2)
5	$3k+6 \leq 3^{k+1}$	Definition of "$\leq$"
6	$3k+3+3 \leq 3^{k+1}$	Split 6 into 3+3
7	$3(k+1)+3 \leq 3^{k+1}$	Factor out k+1 from 3k+3

Therefore, P(k+1) is true given the assumption of P(k).

Thus, we can conclude that P(n) is true for all positive integers $n \geq 2$ using mathematical induction.

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From this table, we can see that in both cases ($a \geq 4$ or $b \geq 2$) when testing if $k+1$ can be created with multiples of 3 and 7, the number of packs of 3 and 7 are still non-negative integers. Therefore $p(k+1)$ is true given the inductive hypothesis.

Thus, $p(n)$ is true for all integers $n \geq 12$ using strong induction. ■

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