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Spring 2024

CS 4510 – Automata and Complexity Theory

Exam 2 Don't forget that $0 \in \mathbb{N}$

5 1. (5 pts) Give a context-free grammar for $\emptyset$.

$S \rightarrow S$

5 2. (5 pts) Give a context-free grammar for Σ^* with $\Sigma = \{a, b\}$.

$S \rightarrow aS \mid bS \mid \varepsilon$

5 3. (5 pts) Give a context-free grammar for $\{a^n b^m \mid n, m \in \mathbb{N} \text{ and } n \leq m\}$ with $\Sigma = \{a, b\}$.

$S \rightarrow aSb \mid R$

$R \rightarrow Rb \mid \varepsilon$

n=0, m=0 $\rightarrow \varepsilon$
n=1, m=2 $\rightarrow abb$
$aSb \rightarrow aRb \rightarrow aRbb \rightarrow abb$

5 4. (5 pts) Give a context-free grammar for $\{bxayb \mid x, y \in \Sigma^* \text{ and } |x| = |y|\}$ with $\Sigma = \{a, b\}$.

$S \rightarrow bXb$

$X \rightarrow aXa \mid bXb \mid aXb \mid bXa \mid a$

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$bXb \rightarrow baXab \rightarrow baaab$

$\downarrow$

$babXaab \rightarrow babaaab$

$\downarrow$

$babaXbaab \rightarrow baabaaab$

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5. (5 pts) Give a context-free grammar for $\{w \in \Sigma^* \mid \#a(w) = 2\}$ with $\Sigma = \{a, b\}$.

$S \to XaXaX$
$X \to bX \mid \varepsilon$

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6. (5 pts) Give a context-free grammar for $\{w \in \Sigma^* \mid \#a(w) = \#b(w)\}$ with $\Sigma = \{a, b\}$.

$S \to SaSbS \mid SbSaS \mid \varepsilon$

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7. (5 pts) Give a context-free grammar for $\{w\#w^R \mid w \in (a \cup b)^*\}$ with $\Sigma = \{a, b, \#\}$ but w, w^R do not contain any $\#$, only a's and b's.

$S \to aSa \mid bSb \mid \#$

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8. (5 pts) Give a context-free grammar for the Dyck language. The language of valid, or balanced parenthesis over the alphabet $\Sigma = \{(,)\}$.

$S = (S)S \mid \varepsilon$

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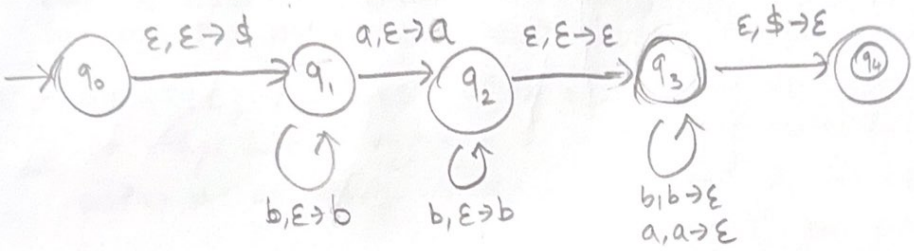
9. (10 pts) Give the state diagram of a PDA for $\{ww^R \mid \#a(w) = 1\}$ with $\Sigma = \{a, b\}$. Do not make a CFG and then convert it to a PDA.



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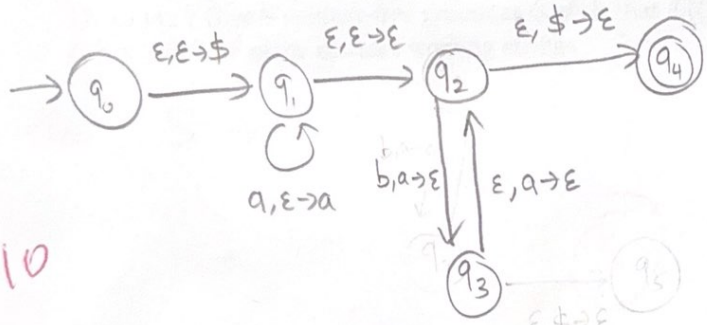
10. (10 pts) Give the state diagram of a PDA for $L = \{a^{2n}b^n \mid n \in \mathbb{N}\}$. Do not make a CFG and convert it to a PDA.



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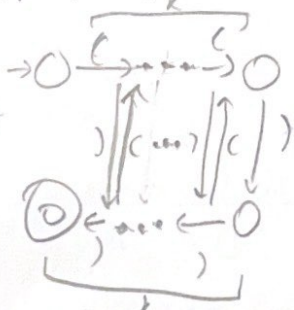
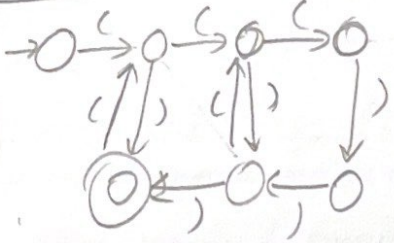
11. (10 pts) Consider the subset L of the Dyck language such that PDA computation on the strings in this language do not exceed depth k. This subset is the balanced matching parenthesis where each matching open and close are not arbitrarily far away from each other. It may contain strings like (())()()(())() but not ((((((()))))) for example. Prove that this subset of the Dyck Language is regular.

Subset L, the subset of Dyck such that the PDA on strings in this language do not exceed depth k, can be written as an NFA. Given a constant k, we can have a set of transitions of k "(" and k ")" that reach a final state, as well as transition from the i "("th node to the 2k+2-i "("th node, and vice versa, as shown below. Since it can be drawn as an NFA, it must be regular.



This will implicitly reject any unbalanced strings, as well as any depth > k.

ex: k=3 shown here



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12. (5 pts) Give a context-free grammar G such that $L(G)$ is finite, $L(G) \neq \emptyset$, and G has infinitely many distinct working strings.

$S \rightarrow SS \mid \varepsilon$

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13. (5 pts) Whats the point of Chomsky Normal Form?

Chomsky Normal Form (CNF) is a normalized version of Context Free Grammars (CFG). It is useful in comparing multiple CGF's. However, it is mainly used for visualizing productions. Given a string of length $n \geq 1$, you can get that string from the CNF in $2n-1$ productions.

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14. (20 pts) For each of the following, circle only one of T or F. T if the statement is true and F if the statement is false. On the line following the statement, **justify your answer**. You may use more space on the back if necessary.

1. (T) F english contains some regular substructure.

The concept of plural words shows regular substructure:
Seat + s = seats

2. (T) F english contains some non-regular substructure.

There are some substructures like hello + s = hellos,
which is non-regular in English

3. (T) F english is probably more context-free than it is regular.

the complex recursive nature of English (sensitive
grammar) is not plausibly displayed through NFA/DFAs

4. T (F) Language is only spoken and not written.

communication through language via text, email, letters
are all written. Therefore, this is not correct.

5. T (F) All syntactically correct sentences have semantic value.

As stated in class "colorless green ideas sleep
furiously" is a syntactically correct sentence, but does not
make sense (therefore does not hold semantic value)

2 15. (2 pts bonus) Give me all palindromes over the alphabet $\Sigma = \{(,)\}$ of exactly length four. The symbols are the open and close parenthesis.

((((
(())
))))
)(()

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