CS-2050-All-Sections CS 2050 Homework 3 (HOWARD, FAULKNER, ELLEN)

Vidit Dharmendra Pokharna

TOTAL POINTS

102.5 / 100

QUESTION 1

1 Question 1 **14 / 14**

✓ **- 0 pts** *Correct*

- 6 pts Did not define propositional variables if used later

- 8 pts Did not give reasons for any steps

- 4 pts Did not include line numbers in reasons

Invalid Steps (including incorrect translation from english to logic)

- 3 pts 1 Invalid Step

- 6 pts 2 Invalid Steps

- 9 pts 3+ Invalid Steps

Skipped Steps

- 3 pts 1 Skipped Step

- 6 pts 2 Skipped Steps

- 9 pts 3+ Skipped Steps

Miscited Steps

- 3 pts 1 Miscited Step

- 6 pts 2 Miscited Steps

- 9 pts 3+ Miscited Steps

- 14 pts Did not reach "It is not true that "If I turn in my homework early, then I will go to sleep early"" or equivalent

- 14 pts No Answer

QUESTION 2

2 Question 2 **16 / 16**

✓ **- 0 pts** *Correct*

- 8 pts Did not give reasons for any steps

- 3 pts Did not include line numbers in reasons

Invalid steps

- 3 pts 1 invalid step

- 6 pts 2 invalid steps

- 9 pts 3+ invalid steps

Skipping steps

- 3 pts 1 skipped step

- 6 pts 2 skipped steps

- 9 pts 3+ skipped steps

Miscited steps

- 3 pts 1 miscited step

- 6 pts 2 miscited steps

- 9 pts 3+ miscited steps

- 16 pts Did not reach $$b$$

- 16 pts No answer

QUESTION 3

3 Question 3 **10 / 10**

✓ **- 0 pts** *Correct*

- 2 pts Missing / Incorrect Introduction (e.g. does not mention type of proof)

- 6 pts Invalid assumption (e.g. assumes conclusion is true)

Proof Body

- 2 pts Does not correctly cite closure of integers under multiplication and/or addition

- 3 pts Same variable for different definitions of even/odd

- 3 pts Uses Booleans and Numerical values interchangeably

A statement with numbers needs to be an equality – something like "$$2k$$ (is true) by definition of even" without the equality is not correct

- 1 pts Missing domain for new variable

- 1 pts Minor non-trivializing arithmetic error

- 2 pts Does not state "$$x + y$$ is even" (by assumption/given/premise)

(i.e. Does not say "assume conclusion is false" or equivalent in introduction and jumps straight to definition of odd in body.)

Invalid Steps

- 2 pts 1 Invalid Step

- 4 pts 2 Invalid Steps

- 6 pts 3+ Invalid Steps

- 0 pts Click here to replace this description.

Skipped Steps

- 2 pts 1 Skipped Step

- 4 pts 2 Skipped Steps

- 6 pts 3+ Skipped Steps

Uncited Steps

- 2 pts 1 Uncited Step

- 4 pts 2 Uncited Steps

- 6 pts 3+ Uncited Steps

- 2 pts Missing/incorrect conclusion

Must say they have proven the conditional that "if $$x + y$$ is even, then $$x^2y - y^3 + 2$$ is even." Cannot only say that they have proven $$x^2y - y^3 + 2$$ is even.

- 10 pts Uses proof technique other than direct proof.

- 10 pts No Answer

QUESTION 4

Question 4 24 pts

4.1 a **12 / 12**

✓ **- 0 pts** *Correct*

- 2 pts Missing/incorrect introduction (does not state contrapositve)

- 6 pts Invalid assumption (assumes conclusion is true)

- 2 pts Does not state "$$n$$ is even" (by assumption/given/premise)

I.E., Does not say "assume conclusion is false" or equivalent in introduction and jumps straight to definition of even in body.

Proof body

- 2 pts Does not correctly cite closure of integers under multiplication and/or addition

- 3 pts Same variable for different definitions of even/odd

- 1 pts Missing domain for new variable

- 3 pts Uses Booleans and Numerical values interchangeably

A statement with numbers needs to be an equality – something like "$$2k$$ (is true) by definition of even" without the equality is not correct

- 1 pts Minor non-trivializing arithmetic error

Invalid steps

- 3 pts 1 Invalid step

- 6 pts 2 Invalid steps

- 9 pts 3 Invalid steps

Uncited steps

- 3 pts 1 Uncited step

- 6 pts 2 Uncited steps

- 9 pts 3 Uncited steps

Skipped steps

- 3 pts 1 Skipped step

- 6 pts 2 Skipped steps

- 9 pts 3 Skipped steps

- 3 pts Missing/incorrect conclusion

Must say they have proven the conditional that "If $$n^2 + 2n + 10$$ is odd, then $$n$$ is odd." Cannot only say that they have proven If $$n^2 + 2n + 10$$ is odd.

- 12 pts Uses proof technique other than proof by contrapositive

- 12 pts No answer

1 try splitting this and the thing below into more than 1 step bc it's pretty hard to read

4.2 b **12 / 12**

✓ **- 0 pts** *Correct*

- 2 pts Missing/incorrect introduction

- 6 pts Invalid assumption (assumes conclusion is true)

- 2 pts Does not state "$$n^2+2n+10$$ is odd, and $$n$$ is even" (by assumption/given/premise)

I.E., Does not say "assume hypothesis is true and conclusion is false" or equivalent in introduction and jumps straight to definition of even/odd in body.

Proof body

- 2 pts Does not correctly cite closure of integers under multiplication and/or addition

- 3 pts Same variable for different definitions of even/odd

- 1 pts Missing domain for new variable

- 3 pts Uses Booleans and Numerical values interchangeably

A statement with numbers needs to be an equality – something like "$$2k$$ (is true) by definition of even" without the equality is not correct

- 6 pts Does not reach a valid contradiction

Valid contradictions could be:
$$n$$ is even and $$n$$ is odd
$$3n^2 + 8$$ is both even and odd
$$1 = 0$$
The product/sum of integers is not an integer

(finding a violation of closure) – if a student uses this as their contradiction but incorrectly explains why it is a contradiction, mark off for skipped step

Many more contradictions could work

- **- 1 pts** Minor non-trivializing arithmetic error

Invalid steps

- **- 3 pts** 1 Invalid step
- **- 6 pts** 2 Invalid steps
- **- 9 pts** 3 Invalid steps

Uncited steps

- **- 3 pts** 1 Uncited step
- **- 6 pts** 2 Uncited steps
- **- 9 pts** 3 Uncited steps

Skipped steps

- **- 3 pts** 1 Skipped step
- **- 6 pts** 2 Skipped steps
- **- 9 pts** 3 Skipped steps

- **- 3 pts** Missing/incorrect conclusion

Must say they have proven the conditional that "If $n^2 + 2n + 10$ is odd, then n is odd." Cannot only say that they have proven If $n^2 + 2n + 10$ is odd.

- **- 12 pts** Cites 4a as their answer
- **- 12 pts** Uses proof technique other than proof by contradiction
- **- 12 pts** No answer

QUESTION 5

5 Question 5 **12 / 12**

✓ **- 0 pts** *Correct*

- **- 2 pts** Missing/incorrect introduction
- **- 6 pts** Invalid assumption (assumes conclusion is true)
- **- 2 pts** Does not state appropriate assumption based on type of proof (by assumption/given/premise)

Proof body

- **- 2 pts** Does not correctly cite closure of integers under multiplication and/or addition
- **- 3 pts** Same variable for different definitions of even/odd
- **- 1 pts** Missing domain for new variable
- **- 3 pts** Uses Booleans and Numerical values interchangeably

A statement with numbers needs to be an equality – something like "$2k$ (is true) by definition of even" without the equality is not correct

- **- 6 pts** Does not reach a valid contradiction

Valid contradictions could be:
n is even and n is odd
$3n^2 + 8$ is both even and odd
$1 = 0$
The product/sum of integers is not an integer (finding a violation of closure) – if a student uses this as their contradiction but incorrectly explains why it is a contradiction, mark off for skipped step

Many more contradictions could work

- **- 1 pts** Minor non-trivializing arithmetic error

Invalid steps

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Uncited steps

- **3 pts** 1 Uncited step
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- **9 pts** 3 Uncited steps

Skipped steps

- **2 pts** 1 Skipped step
- **4 pts** 2 Skipped steps
- **6 pts** 3 Skipped steps

- **3 pts** Missing/incorrect conclusion

Must say they have proven the conditional that "nth root of 2 is irrational"

- **12 pts** No answer

QUESTION 6

Question 6 24 pts

6.1 a **12 / 12**

✓ **- 0 pts** *Correct*

- **6 pts** Invalid proof technique used

- **6 pts** Incorrect / Missing Reasoning (i.e. just stated the proof technique with no explanation)

- **4 pts** Incomplete / Partial Reasoning

- **2 pts** Minor error

- **12 pts** No Answer

6.2 b **12 / 12**

✓ **- 0 pts** *Correct*

- **6 pts** Invalid proof technique used

- **6 pts** Incorrect / Missing Reasoning (i.e. just stated the proof technique with no explanation)

- **4 pts** Incomplete / Partial Reasoning

- **2 pts** Minor error

- **12 pts** No Answer

QUESTION 7

7 On Time **2.5 / 0**

✓ **+ 2.5 pts** *On Time (Before Thursday)*

- **0 pts** On Time (Friday)

- **10 pts** 1 day late

- **25 pts** 2 days late

QUESTION 8

8 Matching **0 / 0**

✓ **- 0 pts** *Correct*

- **5 pts** Incorrect

Vidit Pokharna

CS 2050 HW 3

1.

Propositional Variables
t = Today is Friday
p = Papa Johns is on fire
w = I win a chess match
m = I watch a movie
s = I go to sleep early
c = I get to class on time
h = I turn in my homework early
d = Domino's is on fire today

I will proceed with a direct proof. Assume all premises (propositional variables) are true.

Line	**Statement**	**Reason**
1	$t \wedge p$	Premise
2	$(t \vee \neg m) \rightarrow w$	Premise
3	$s \rightarrow \neg w$	Premise
4	$\neg s \rightarrow (c \wedge h)$	Premise
5	$\neg p \leftrightarrow \neg h$	Premise
6	$\neg d$	Premise
7	p	Simplification (1)
8	$(\neg h \rightarrow \neg p) \wedge (\neg p \rightarrow \neg h)$	Biconditional Definition (5)
9	$\neg h \rightarrow \neg p$	Simplification (8)
10	$p \rightarrow h$	Contrapositive Law (8)
11	h	Modus Ponens (7,10)
12	t	Simplification (1)
13	$t \vee \neg m$	Addition (12)
14	w	Modus Ponens (2,13)
15	$\neg s$	Modus Tollens (3,14)
16	$h \wedge \neg s$	Conjunction (11,15)
17	$\neg(\neg h \vee s)$	Inverse Double Negation (16)
18	$\neg(h \rightarrow s)$	Inverse Conditional Disjunction Equivalence (17)

∴ By direct proof, the premises lead to the conclusion of $\neg(h \rightarrow s)$ or "It is not true that if I turn in my homework early, then I will go to sleep early" using rules of inference

1 Question 1 **14 / 14**

✓ **- 0 pts** *Correct*

- **- 6 pts** Did not define propositional variables if used later
- **- 8 pts** Did not give reasons for any steps
- **- 4 pts** Did not include line numbers in reasons

Invalid Steps (including incorrect translation from english to logic)

- **- 3 pts** 1 Invalid Step
- **- 6 pts** 2 Invalid Steps
- **- 9 pts** 3+ Invalid Steps

Skipped Steps

- **- 3 pts** 1 Skipped Step
- **- 6 pts** 2 Skipped Steps
- **- 9 pts** 3+ Skipped Steps

Miscited Steps

- **- 3 pts** 1 Miscited Step
- **- 6 pts** 2 Miscited Steps
- **- 9 pts** 3+ Miscited Steps

- **- 14 pts** Did not reach "It is not true that "If I turn in my homework early, then I will go to sleep early"" or equivalent
- **- 14 pts** No Answer

2.

I will proceed with a direct proof. Assume all premises are true.

Line	**Statement**	**Reason**
1	$y \leftrightarrow x$	Premise
2	$x \wedge (b \vee \neg d)$	Premise
3	$(x \wedge a) \rightarrow \neg b$	Premise
4	$((\neg y \vee x) \wedge c) \rightarrow d$	Premise
5	$y \rightarrow c$	Premise
6	$(y \rightarrow x) \wedge (x \rightarrow y)$	Biconditional Definition (1)
7	$x \rightarrow y$	Simplification (6)
8	x	Simplification (2)
9	y	Modus Ponens (7,8)
10	c	Modus Ponens (5,9)
11	$x \vee \neg y$	Addition (8)
12	$(x \vee \neg y) \wedge c$	Conjunction (10,11)
13	d	Modus Ponens (4,12)
14	$b \vee \neg d$	Simplification (2)
15	$\neg d \vee b$	Commutativity (14)
16	b	Disjunctive Syllogism (13,15)

$\therefore$ By direct proof, the premises lead to the conclusion of b using rules of inference

2 Question 2 **16 / 16**

✓ **- 0 pts** *Correct*

- 8 pts Did not give reasons for any steps

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Invalid steps

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- 9 pts 3+ invalid steps

Skipping steps

- 3 pts 1 skipped step

- 6 pts 2 skipped steps

- 9 pts 3+ skipped steps

Miscited steps

- 3 pts 1 miscited step

- 6 pts 2 miscited steps

- 9 pts 3+ miscited steps

- 16 pts Did not reach $$b$$

- 16 pts No answer

3.

I will now proceed with a direct proof.
Assume x and y are integers and $x + y$ is even.

Line	Statement	Reason
1	$x + y$ is even	Given/Premise
2	$x + y = 2k, k \in \mathbb{Z}$	Definition of even number (1)
3	$y = 2k - x$	Subtract x from both sides (2)
4	$(y)(x + y) = (2k)(y)$	Multiply both sides by y (2)
5	$b = x - y = x + (-y), b \in \mathbb{Z}$	Define a new variable b; closure of integers under addition
6	$b = x - y = x + y - 2y = 2k - 2(2k - x)$ $= 2k - 4k + 2x = 2x - 2k$	Simplification of variable b
7	$b = 2x - 2k = 2(x - k)$	Factor out 2 (6)
8	$(y)(x + y)(x - y) = (2k)(y)(x - y)$	Multiple both sides by (x-y)
9	$(y)(x + y)(x - y) = (2k)(2k - x)(x - y)$	Substitute in (2k-x) for y on the RHS (8)
10	$(y)(x + y)(x - y)$ $= (2k)(2k - x)(2)(x - k)$	Substitute in 2(x-k) for (x-y) on the RHS (9)
11	$(y)(x + y)(x - y) + 2$ $= (2k)(2k - x)(2)(x - k) + 2$	Add 2 to both sides (10)
12	$(y)(x + y)(x - y) + 2$ $= 2((k)(2k - x)(2)(x - k) + 1)$	Factor out 2 on the RHS (11)
13	$c = (k)(2k - x)(2)(x - k) + 1, c \in \mathbb{Z}$	Define a new variable c; closure of integers under addition/multiplication
14	$(y)(x + y)(x - y) + 2 = 2c$	Substitute c for ((k)(2k-x)(2)(x-k) + 1) (12)
15	$x^2y - y^3 + 2 = 2c$	Simplify LHS (14)
16	$x^2y - y^3 + 2$ is even	Definition of even

∴ By direct proof, we have shown that $x^2y - y^3 + 2$ is even assuming $x + y$ is even

3 Question 3 **10 / 10**

✓ **- 0 pts** *Correct*

- **- 2 pts** Missing / Incorrect Introduction (e.g. does not mention type of proof)
- **- 6 pts** Invalid assumption (e.g. assumes conclusion is true)

Proof Body

- **- 2 pts** Does not correctly cite closure of integers under multiplication and/or addition
- **- 3 pts** Same variable for different definitions of even/odd
- **- 3 pts** Uses Booleans and Numerical values interchangeably

A statement with numbers needs to be an equality – something like "$$2k$$ (is true) by definition of even" without the equality is not correct

- **- 1 pts** Missing domain for new variable
- **- 1 pts** Minor non-trivializing arithmetic error
- **- 2 pts** Does not state "$$x + y$$ is even" (by assumption/given/premise)

(i.e. Does not say "assume conclusion is false" or equivalent in introduction and jumps straight to definition of odd in body.)

Invalid Steps

- **- 2 pts** 1 Invalid Step
- **- 4 pts** 2 Invalid Steps
- **- 6 pts** 3+ Invalid Steps
- **- 0 pts** Click here to replace this description.

Skipped Steps

- **- 2 pts** 1 Skipped Step
- **- 4 pts** 2 Skipped Steps
- **- 6 pts** 3+ Skipped Steps

Uncited Steps

- **- 2 pts** 1 Uncited Step
- **- 4 pts** 2 Uncited Steps
- **- 6 pts** 3+ Uncited Steps

- **- 2 pts** Missing/incorrect conclusion

Must say they have proven the conditional that "if $x + y$ is even, then $x^2y - y^3 + 2$ is even."

Cannot only say that they have proven $x^2y - y^3 + 2$ is even.

- **- 10 pts** Uses proof technique other than direct proof.
- **- 10 pts** No Answer

4.

a.

I will now proceed by proving the equivalent contrapositive, “if n is even, then $n^2 + 2n + 10$ is even. Assume n is an even integer.

Line	Statement	Reason
1	n is even, $n \in \mathbb{Z}$	Given/Premise
2	$n = 2k, k \in \mathbb{Z}$	Definition of even number (1), closure of integers under multiplication
3	$n^2 = n * n = 4k^2 = 2a$, $a \in \mathbb{Z}$ and $a = 2k^2$, is even	Definition of even number (2), closure of integers under multiplication
4	$2n$ is even	Definition of even number (2)
5	$n^2 + 2n = 4k^2 + 2(2k) = 4k^2 + 4k = 2(2k^2 + 2k) = 2b$, $b \in \mathbb{Z}$ and $b = 2k^2 + 2k$, is even	Definition of even number (3,4), closure of integers under addition and multiplication
6	$n^2 + 2n + 10 = 4(k^2 + k) + 10 = 2(2(k^2 + k) + 5) = 2c$, $c \in \mathbb{Z}$ and $c = 2(k^2 + k) + 5$, is even	Definition of even number (5), closure of integers under addition and multiplication

∴ By proof of contrapositive, assuming an integer n is even, we concluded that $n^2 + 2n + 10$ is even. Therefore, according to the contrapositive law, we can conclude that if $n^2 + 2n + 10$ is odd, then n is odd.

4.1 a **12 / 12**

✓ **- 0 pts** *Correct*

- 2 pts Missing/incorrect introduction (does not state contrapositve)

- 6 pts Invalid assumption (assumes conclusion is true)

- 2 pts Does not state "$$n$$ is even" (by assumption/given/premise)

I.E., Does not say "assume conclusion is false" or equivalent in introduction and jumps straight to definition of even in body.

Proof body

- 2 pts Does not correctly cite closure of integers under multiplication and/or addition

- 3 pts Same variable for different definitions of even/odd

- 1 pts Missing domain for new variable

- 3 pts Uses Booleans and Numerical values interchangeably

A statement with numbers needs to be an equality – something like "$$2k$$ (is true) by definition of even" without the equality is not correct

- 1 pts Minor non-trivializing arithmetic error

Invalid steps

- 3 pts 1 Invalid step

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Uncited steps

- 3 pts 1 Uncited step

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- 9 pts 3 Uncited steps

Skipped steps

- 3 pts 1 Skipped step

- 6 pts 2 Skipped steps

- 9 pts 3 Skipped steps

- 3 pts Missing/incorrect conclusion

Must say they have proven the conditional that "If $$n^2 + 2n + 10$$ is odd, then $$n$$ is odd." Cannot only say that they have proven If $$n^2 + 2n + 10$$ is odd.

- **- 12 pts** Uses proof technique other than proof by contrapositive
- **- 12 pts** No answer

1 try splitting this and the thing below into more than 1 step bc it's pretty hard to read

b.

I will now proceed with a proof by contradiction.
Assume n is an integer, $n^2 + 2n + 10$ is odd, and n is even.

Line	**Statement**	**Reason**
1	$n^2 + 2n + 10$ is odd, $n \in \mathbb{Z}$	Given/Premise
2	n is even	Given/Premise
3	$n = 2k, k \in \mathbb{Z}$	Definition of even number (2)
4	$a = n^2 + 2n + 10$	Define new variable a
5	$a = n^2 + 2n + 10$ $= (2k)^2 + 2(2k) + 10$ $= 4k^2 + 4k + 10$	Plug in 2k for a
6	$a = n^2 + 2n + 10 = 2(2k^2 + 2k + 5) = 2b$, $b \in \mathbb{Z}$ and $b = (2k^2 + k + 5)$, is even	Definition of even number
7	$n^2 + 2n + 10$ is even	Inconsistency (1,6)

∴ By contradiction theorem, we assumed that $n^2 + 2n + 10$ is odd and n is even, shown by the premises. Solving through with this, we reached an inconsistency where $n^2 + 2n + 10$ is both odd and even. Thus, our assumption is incorrect, and we can conclude if $n^2 + 2n + 10$ is odd, then n is odd.

4.2 b **12 / 12**

✓ **- 0 pts** *Correct*

- 2 pts Missing/incorrect introduction

- 6 pts Invalid assumption (assumes conclusion is true)

- 2 pts Does not state "$$n^2+2n+10$$ is odd, and $$n$$ is even" (by assumption/given/premise)

I.E., Does not say "assume hypothesis is true and conclusion is false" or equivalent in introduction and jumps straight to definition of even/odd in body.

Proof body

- 2 pts Does not correctly cite closure of integers under multiplication and/or addition

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- 1 pts Missing domain for new variable

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A statement with numbers needs to be an equality – something like "$$2k$$ (is true) by definition of even" without the equality is not correct

- 6 pts Does not reach a valid contradiction

Valid contradictions could be:

$$n$$ is even and $$n$$ is odd

$$3n^2 + 8$$ is both even and odd

$$1 = 0$$

The product/sum of integers is not an integer (finding a violation of closure) – if a student uses this as their contradiction but incorrectly explains why it is a contradiction, mark off for skipped step

Many more contradictions could work

- 1 pts Minor non-trivializing arithmetic error

Invalid steps

- 3 pts 1 Invalid step

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- 9 pts 3 Invalid steps

Uncited steps

- **- 3 pts** 1 Uncited step
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Skipped steps

- **- 3 pts** 1 Skipped step
- **- 6 pts** 2 Skipped steps
- **- 9 pts** 3 Skipped steps

- **- 3 pts** Missing/incorrect conclusion

Must say they have proven the conditional that "If $$n^2 + 2n + 10$$ is odd, then $$n$$ is odd." Cannot only say that they have proven If $$n^2 + 2n + 10$$ is odd.

- **- 12 pts** Cites 4a as their answer
- **- 12 pts** Uses proof technique other than proof by contradiction
- **- 12 pts** No answer

5.

I will now proceed with a proof by contradiction.
Assume n is an integer, $n \geq 3$, and $\sqrt[n]{2}$ is rational.

Line	**Statement**	**Reason**
1	$\sqrt[n]{2}$ is rational, $n \in \mathbb{Z}$ and $n \geq 3$	Given/Premise
2	$\sqrt[n]{2} = \frac{a}{b}$, $a, b \in \mathbb{Z}$ and $\gcd(a, b) = 1$	Definition of rational number, and we can always make fractions into their simplest form
3	$(\sqrt[n]{2})^n = (\frac{a}{b})^n \rightarrow 2 = \frac{a^n}{b^n}$	Taking both sides to the power of n (2)
4	$2 * b^n = \frac{a^n}{b^n} * b^n \rightarrow 2b^n = a^n$	Cross multiply (3)
5	$a^n = 2b^n = 2c$, $c \in \mathbb{Z}$ and $c = b^n$, is even	Definition of even number
6	$(2c)^n = 2^n c^n = 2b^n$	Replace a with 2c (5)
7	$0.5 * \; 2^n c^n = 0.5 * 2b^n \rightarrow b^n = 2^{n-1} c^n$	Multiply by 0.5 on both sides (6)
8	$b^n = 2(2^{n-2} c^n) = 2d$, $d \in \mathbb{Z}$ and $d = 2^{n-2} c^n$, is even	Definition of even number
9	$\gcd(a, b) \geq 2$	The GCD of two even numbers must be 2 or greater, as one common factor is 2 Inconsistency (2,9)

∴ By contradiction theorem, we assumed that $\sqrt[n]{2}$ is rational and $n \geq 3$, shown by the premises. Solving through with this, we reached an inconsistency regarding the greatest common denominator of two variables created in the proof, proving that the assumption we made is incorrect. We can thus conclude that $\sqrt[n]{2}$ is irrational for any integer $n \geq 3$.

5 Question 5 **12 / 12**

✓ **- 0 pts** *Correct*

- **- 2 pts** Missing/incorrect introduction
- **- 6 pts** Invalid assumption (assumes conclusion is true)
- **- 2 pts** Does not state appropriate assumption based on type of proof (by assumption/given/premise)

Proof body

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A statement with numbers needs to be an equality – something like "$$2k$$ (is true) by definition of even" without the equality is not correct

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Valid contradictions could be:

$$n$$ is even and $$n$$ is odd

$$3n^2 + 8$$ is both even and odd

$$1 = 0$$

The product/sum of integers is not an integer (finding a violation of closure) – if a student uses this as their contradiction but incorrectly explains why it is a contradiction, mark off for skipped step

Many more contradictions could work

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Invalid steps

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Skipped steps

- 2 pts 1 Skipped step

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- 6 pts 3 Skipped steps

- 3 pts Missing/incorrect conclusion

Must say they have proven the conditional that "nth root of 2 is irrational"

- 12 pts No answer

6.

a. For this statement, we can use trivial proof to solve, setting x to be an integer greater than 2. However, we can say that the x-th power of the x-th prime is always odd for $x > 2$ is always odd (this is true for any $x > 1$ as only the first prime is even), and any positive integer power of an odd number will always be odd. Therefore, since this is true, we can claim that the original statement is true by trivial proof (since for $p \rightarrow q$, q is true and therefore $p \rightarrow q$ always holds true regardless of p).

b. For this statement, we can use trivial proof to solve. Looking at the second part of the condition ($x^3 + x > x^2 - x$), we notice a simplification of $x^3 - x^2 + 2x > 0$. When factored, it becomes $x(x^2 - x + 2) > 0$. When creating a sign chart, we notice that the expression is always greater than 0 for all positive x. Therefore, since this is true, we can claim that the original statement is true by trivial proof (since for $p \rightarrow q$, q is true and therefore $p \rightarrow q$ always holds true regardless of p)

6.1 a **12 / 12**

✓ **- 0 pts** *Correct*

- 6 pts Invalid proof technique used

- 6 pts Incorrect / Missing Reasoning (i.e. just stated the proof technique with no explanation)

- 4 pts Incomplete / Partial Reasoning

- 2 pts Minor error

- 12 pts No Answer

6.

a. For this statement, we can use trivial proof to solve, setting x to be an integer greater than 2. However, we can say that the x-th power of the x-th prime is always odd for $x > 2$ is always odd (this is true for any $x > 1$ as only the first prime is even), and any positive integer power of an odd number will always be odd. Therefore, since this is true, we can claim that the original statement is true by trivial proof (since for $p \rightarrow q$, q is true and therefore $p \rightarrow q$ always holds true regardless of p).

b. For this statement, we can use trivial proof to solve. Looking at the second part of the condition ($x^3 + x > x^2 - x$), we notice a simplification of $x^3 - x^2 + 2x > 0$. When factored, it becomes $x(x^2 - x + 2) > 0$. When creating a sign chart, we notice that the expression is always greater than 0 for all positive x. Therefore, since this is true, we can claim that the original statement is true by trivial proof (since for $p \rightarrow q$, q is true and therefore $p \rightarrow q$ always holds true regardless of p)

6.2 b **12 / 12**

✓ **- 0 pts** *Correct*

- 6 pts Invalid proof technique used

- 6 pts Incorrect / Missing Reasoning (i.e. just stated the proof technique with no explanation)

- 4 pts Incomplete / Partial Reasoning

- 2 pts Minor error

- 12 pts No Answer

7 On Time 2.5 / 0

✓ **+ 2.5 pts** *On Time (Before Thursday)*

- 0 pts On Time (Friday)

- 10 pts 1 day late

- 25 pts 2 days late

8 Matching **0 / 0**

✓ **- 0 pts** *Correct*

- 5 pts Incorrect