

# MATH-3012-D Quiz 1

Vidit Dharmendra Pokharna

TOTAL POINTS

**27 / 30**

QUESTION 1

1 Q1 12 / 15

✓ + 5 pts Correct (a)

✓ + 5 pts Correct (b)

✓ + 2 pts Partial credits (c)

💬 1(c) should be  $(8 \text{ choose } 5) \cdot 13^3$

QUESTION 2

2 Q2 15 / 15

✓ + 15 pts Correct

# D section

Vidit Pokharna

Parts of a question are independent. For example, conditions in 1b do not apply to 1c. Box your answer. **Show work** except for 1ab.

There are two pages of this quiz.

- (1) How many are there strings of length 8 of the 14 letters  $\{A, B, C, D, E, F, G, H, I, J, K, L, M, N\}$  with the following restrictions?

- (a) Distinct letters. (No work needed to show)
- (b) Distinct letters and letters listed alphabetically. (No work needed)
- (c) There are exactly 5 A's, no other restrictions.

$$(a) 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 = \boxed{\frac{14!}{6!}}$$

$$(b) \binom{14}{8} = \boxed{\frac{14!}{8!6!}}$$

$$(c) \binom{14-1}{8-5} = \binom{13}{3} = \boxed{\frac{13!}{3!10!}}$$

1 Q1 12 / 15

✓ + 5 pts Correct (a)

✓ + 5 pts Correct (b)

✓ + 2 pts Partial credits (c)

1(c) should be  $(8 \text{ choose } 5) \cdot 13^3$

(2)

(a) Find  $\# \begin{cases} x_1 + x_2 + x_3 + x_4 = 30 \\ x_1 > 1, x_2 > 1, x_3 > 1, x_4 > 1, \text{ each } x_i \in \mathbb{Z} \end{cases}$

(b) Find  $\# \begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 = 50 \\ x_i \geq 0, x_5 \leq 19, \text{ each } x_i \in \mathbb{Z} \end{cases}$

(Answer should be a sum/difference of no more than 2 terms).

(c) Evaluate the sum

$$1 + (-3) \binom{100}{1} + (-3)^2 \binom{100}{2} + \cdots + (-3)^{100} \binom{100}{100}.$$

(a)  $\begin{aligned} x_1 &= 2 + x_{11} \\ x_2 &= 2 + x_{22} \\ x_3 &= 2 + x_{33} \\ x_4 &= 2 + x_{44} \end{aligned} \quad \begin{cases} 8 + x_{11} + x_{22} + x_{33} + x_{44} = 30 \\ x_{11}, x_{22}, x_{33}, x_{44} \geq 0 \end{cases} \rightarrow \binom{22+4-1}{22} = \binom{25}{22} = \boxed{\frac{25!}{22!3!}}$

(b)  $\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 = 50 \\ x_i \geq 0 \end{cases} - \begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 = 50 \\ x_i \geq 0, x_5 > 19 \end{cases}$

$$\binom{50+5-1}{50} - \binom{30+5-1}{30} = \binom{54}{50} - \binom{34}{30} = \boxed{\frac{54!}{50!4!} - \frac{34!}{30!4!}}$$

(c)  $\sum_{i=0}^n \binom{n}{i} (-3)^i, n=100$

$$(-3+1)^{100} = \sum_{i=0}^{100} \binom{100}{i} (-3)^i (1)^{100-i} = \sum_{i=0}^{100} (-3)^i \binom{100}{i} =$$

$$(-2)^{100} = \boxed{2^{100}}$$

2 Q2 15 / 15

✓ + 15 pts Correct