Economics of Climate Change – Problem Set 2

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Question 1: Abatement costs & comparing policy instruments (100%)

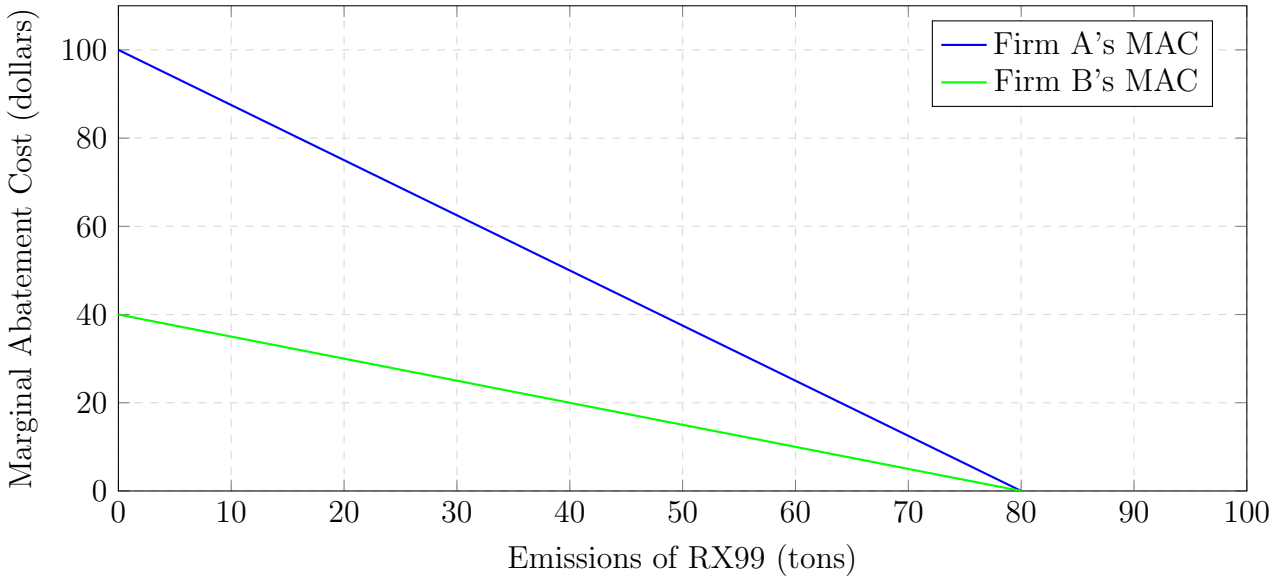

Figure 1: Marginal Abatement Cost Curves for Firms A and B

1.1 In the "no regulation" scenario, each firm would emit up to the point where their marginal abatement cost reaches zero:

- Firm A can emit up to 80 tons of RX99
- Firm B can emit up to 80 tons of RX99

Therefore, the total amount of RX99 emitted by both firms annually in a "no regulation" scenario would be **160 tons**.

1.2

- Firm A's Marginal Abatement Cost (MAC^A):

$$\int_{40}^{80} MAC^A(x) = \int_{40}^{80} 100 - 1.25x = \$1000$$

- Firm B's Marginal Abatement Cost (MAC^B):

$$\int_{40}^{80} MAC^A(x) = \int_{40}^{80} 40 - \frac{x}{2} = \$400$$

The aggregate abatement cost for both firms is **$1400**.

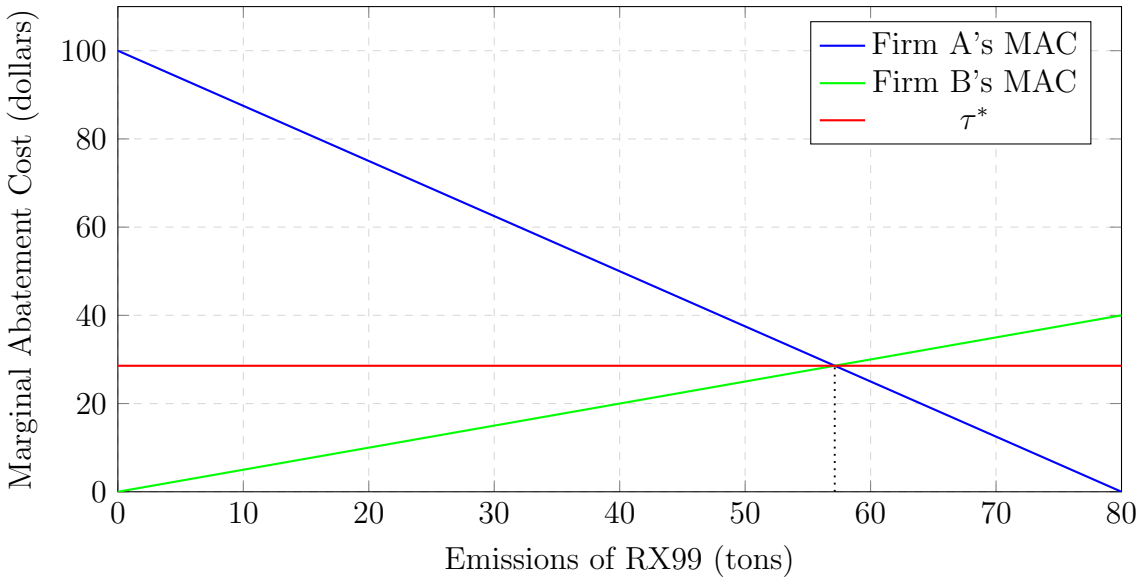

Figure 2: Marginal Abatement Cost Curves

1.3 (a) Setting $MAC^A(x) = \widehat{MAC^B}(x)$, where $\widehat{MAC^B}(x) = \frac{x}{2}$, we get:

$MAC^A(x) = \widehat{MAC^B}(x)$

$100 - 1.25x = 0.5x$

$x \approx 57.14$ tons of RX99

Thus, $\tau^* = 0.5(57.14) =$ **$28.57**

(b) Thus:

- Firm A would emit approximately $\frac{100-28.57}{1.25} \approx$ **57.14** tons
- Firm B would emit approximately $80 - 2(28.57) \approx$ **22.86** tons

(c) Abatement Cost$_A = \int_0^{57.14}(0.5x)\,dx = [0.25x^2]_0^{57.14} = 0.25 \cdot 57.14^2 \approx 816.24$

Abatement Cost$_B = \int_{57.14}^{80}(100 - 1.25x)\,dx = [100x - 0.625x^2]_{57.14}^{80} \approx 326.61$

The abatement costs are:

- Firm A: **$816.24**
- Firm B: **$326.61**
- Aggregate Abatement Cost: **$1142.85**

(d) Total Tax Revenue $= \tau^* \times (x_A + x_B) = 28.57 \times (57.14 + 22.86) = 28.57 \times 80 \approx$ **$2285.71**

(e) $\tau^*_{\text{new}} = 28.57 + 10 = 38.57$ per ton

$MAC^A(x) = \tau^*_{\text{new}} \Rightarrow 100 - 1.25x = 38.57$ so $x_A = \frac{100-38.57}{1.25} = \frac{61.43}{1.25} \approx 49.14$ tons

$MAC^B(x) = \tau^*_{\text{new}} \Rightarrow 40 - \frac{x}{2} = 38.57$ so $x_B = 2 \times (40 - 38.57) = 2 \times 1.43 \approx 2.86$ tons

With the tax rate set \$10 higher than the recommended rate:

- Firm A would emit approximately **49.14** tons
- Firm B would emit approximately **2.86** tons

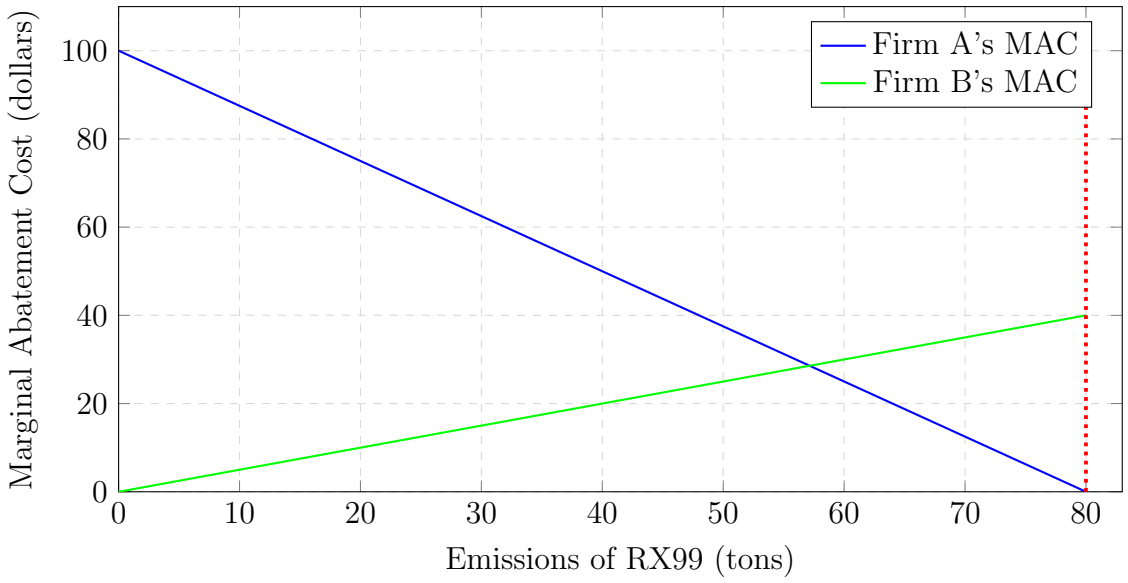

Figure 3: Marginal Abatement Cost Curves

1.4 (a)

(b) Since Firm A has all 80 permits, it incurs no abatement costs. Firm B, however, needs to abate all of its emissions. Abatement $\text{Cost}_B = \int_0^{80} \left(40 - \frac{x}{2}\right) dx = \left[40x - \frac{x^2}{4}\right]_0^{80} = 1600$.

Therefore, the total abatement cost for Firm B under the initial allocation (before trading) is **$1600**.

(c) $MAC^B(0) = 40 - \frac{1}{2} = 39.5$. So, the initial permit price that Firm B would be willing to pay Firm A for the first permit is **$39.5**.

(d) Once trading begins in the cap-and-trade market, Firm B, which has no permits, will have a strong incentive to purchase permits from Firm A to reduce its abatement costs. Firm B's marginal abatement cost is initially high, so it is willing to pay a substantial price for the first few permits. Conversely, Firm A, holding all 80 permits initially, faces zero abatement cost and will be willing to sell permits to Firm B if the price is greater than its marginal cost of abatement. As permits are traded, Firm A will begin to abate some emissions as it sells permits, while Firm B will reduce its abatement as it buys permits and gains the right to emit.

This trading process will continue until the permit price reaches an equilibrium where both firms' marginal abatement costs are equal. At this point, neither firm has an incentive to buy or sell more permits, as the cost of further abatement

would exceed the permit price for Firm A and be lower than the permit price for Firm B. In this equilibrium, each firm's marginal abatement cost is balanced with the permit price, leading to a cost-effective allocation of emissions. Firm A will end up emitting slightly more than Firm B, but the total emissions will remain at the capped level of 80 tons, distributed in a way that minimizes total abatement costs across both firms.

(e) As calculated in 1.3, the equilibrium permit price is \$28.57 per ton. Firm A emits approximately 57.14 tons. Firm B emits approximately 22.86 tons. Firm A's Abatement Cost would be \$816.24. Firm B's Abatement Cost would be \$326.61. The Total Aggregate Abatement Cost would be \$1142.85.

1.5 (a) For abatement costs between \$0 and \$40:

- In this range, only Firm B will abate, as Firm A's MAC starts higher (at \$100) and only decreases below \$40 when abatement is substantial
- Therefore, the aggregate MAC curve for abatement costs between \$0 and \$40 is simply Firm B's MAC
- The aggregate MAC is the same as Firm B's MAC in this range:

$$MAC(x) = 40 - \frac{x}{2}$$

$$x = 2(40 - MAC) = 80 - 2 \cdot MAC$$

For abatement costs between \$40 and \$100:

- In this range, both Firm A and Firm B will abate
- To find the total abatement at each cost level, we sum the quantities abated by each firm at a given MAC level within this range
- For Firm B $x_B = 2(40 - MAC)$
- For Firm A, rearrange $MAC = 100 - 1.25x$ to solve for x_A:

$$x_A = \frac{100 - MAC}{1.25}$$

- The total abatement X at each MAC level in this range is the sum of x_A and x_B:

$$X = x_A + x_B = \frac{100 - MAC}{1.25} + 2(40 - MAC) = \frac{200 - 3.5MAC}{1.25} = 160 - 2.8 \cdot MAC$$

$$Agg(x) = \begin{cases} 80 - 2 \cdot MAC & \text{for } 0 \leq MAC \leq 40 \\ 160 - 2.8 \cdot MAC & \text{for } 40 < MAC \leq 100 \end{cases}$$

(b) This curve is upward sloping with respect to emissions x, meaning that the marginal damage increases as more RX99 is emitted.

- If there is uncertainty about abatement costs, a cap-and-trade system might be preferable because it provides certainty about the quantity of emissions (capped at a specific level) but allows the price (permit price) to adjust.
- In contrast, an emissions tax provides certainty about the cost of abatement but allows the total quantity of emissions to vary depending on firms' responses to the tax.
- Given the upward-sloping marginal damage curve, if the EPA prioritizes controlling the quantity of emissions (to limit the environmental damage from RX99), a cap-and-trade system would be more effective.