CS-2050-All-Sections CS 2050 Homework 2 (HOWARD, FAULKNER, ELLEN)

Vidit Dharmendra Pokharna

TOTAL POINTS

100.5 / 100

QUESTION 1

Question 1 15 pts

1.1 a **5 / 5**

✓ **- 0 pts** *$$\forall x(C(x)\rightarrow M(x)) \wedge \exists x(M(x) \wedge \neg C(x))$$ or logically equivalent*

- 2.5 pts Only gave half the answer: $$\forall x(C(x)\rightarrow M(x)) $$

- 2 pts Incorrect/Missing quantifier

- 2 pts Incorrect predicate/variable

- 2 pts Incorrect scoping of quantifier

- 3 pts $$\rightarrow$$ instead of $$\land$$ or vice versa

- 5 pts Incorrect / Missing / No Answer

1.2 b **5 / 5**

✓ **- 0 pts** *Question removed*

1.3 c **5 / 5**

✓ **- 0 pts** *$$\forall x(M(x) \rightarrow E(x))$$ or logically equivalent*

- 2 pts Incorrect/Missing quantifier

- 2 pts Incorrect predicate/variable

- 2 pts Incorrect scoping of quantifier

- 3 pts $$\rightarrow$$ instead of $$\land$$ or vice versa

- 5 pts Incorrect / Missing / No Answer

QUESTION 2

Question 2 20 pts

2.1 a **2 / 4**

✓ **- 0 pts** *True*

- 4 pts False

✓ **- 2 pts** *Incorrect Reasoning*

- 4 pts No Answer

1 $y > x^2$ does not imply $y > x$ in general. For example $1/3 > (1/2)^2$ is true but $1/3 > 1/2$ is not. This works if you choose a sufficiently large y, but you have not given a value for y.

2.2 b **4 / 4**

- 4 pts True

✓ **- 0 pts** *False*

- 2 pts Incorrect Reasoning

- 4 pts No Answer

2.3 c **4 / 4**

✓ **- 0 pts** *True*

- 4 pts False

- 2 pts Incorrect Reasoning

- 4 pts No Answer

2.4 d **4 / 4**

- **- 4 pts** True
- ✓ **- 0 pts** *False*
- **- 2 pts** Incorrect Reasoning
- **- 4 pts** No Answer

2.5 e **4 / 4**

- **- 4 pts** True
- ✓ **- 0 pts** *False*
- **- 2 pts** Incorrect Reasoning
- **- 4 pts** No Answer

QUESTION 3

3 Question 3 **10 / 10**

✓ **- 0 pts** *$$\exists x (P(x) \land \forall y (y\neq x \to \lnot P(y)))$$*

OR

$$\exists x \forall y (P(y) \leftrightarrow (y = x))$$

OR

$$\exists x P(x) \wedge \forall y \forall z [P(y) \wedge P(z)) \rightarrow y=z]$$

OR

logically equivalent

- 5 pts Attempted to use the quantifiers in order $$\exists x \forall y$$ but did not reach the correct answer

- 10 pts No Answer/Incorrect Answer

QUESTION 4

4 Question 4 **10 / 10**

✓ **- 0 pts** *Correct*

- 8 pts Did not cite any steps

Invalid steps

- **- 3 pts** 1 invalid step
- **- 5 pts** 2 invalid steps
- **- 7 pts** 3 invalid steps
- **- 10 pts** 4+ invalid steps

Skipped steps

- **- 2 pts** 1 skipped step
- **- 4 pts** 2 skipped steps
- **- 6 pts** 3 skipped steps
- **- 8 pts** 4+ skipped steps

Uncited steps

- **- 1 pts** 1 uncited step
- **- 2 pts** 2 uncited steps
- **- 3 pts** 3 uncited steps
- **- 4 pts** 4+ uncited steps

Miscited steps

- **- 1 pts** 1 miscited step
- **- 2 pts** 2 miscited steps
- **- 3 pts** 3 miscited steps
- **- 4 pts** 4+ miscited steps

- 8 pts Did not reach $$\exists x \forall y (\neg B(y) \rightarrow (\neg A(x) \land \neg C(x,y))) $$

- 10 pts No answer

QUESTION 5

Question 5 10 pts

5.1 a **5 / 5**

✓ **- 0 pts** *$$\forall x \forall y \forall z ((P(x,y) \land S(y, z)) \lor (\lnot P(x, y) \land \lnot S(y, z)))$$ or equivalent*

- 3 pts Equivalent expression but did not push negations such that every negation is immediately to the left of a predicate

- 5 pts Incorrect Expression

- 5 pts No Answer

5.2 b **5 / 5**

✓ **- 0 pts** *$$\exists y \exists x (R(x, y)\to \exists z S(z, y))$$ or equivalent*

- 3 pts Equivalent expression but did not push negations such that every negation is immediately to the left of a predicate

- 5 pts Incorrect Expression

- 5 pts No Answer

QUESTION 6

Question 6 15 pts

6.1 a **5 / 5**

✓ **- 0 pts** *Valid counterexample*

Possible answers:

x=y=2

or anything else > 1

- 4 pts Incorrect counterexample

- 5 pts No counterexample exists

- 5 pts No Answer

6.2 b **5 / 5**

✓ **- 0 pts** *No counterexample exists*

- 5 pts Provided a counter example

- 5 pts No Answer

6.3 c **5 / 5**

✓ **- 0 pts** *Valid counterexample*

Possible answers:

x=y=0

x=y=0.5

x=y=1

x=0.5, y=0.4

- 4 pts Incorrect counterexample

- 5 pts No counterexample exists

- 5 pts No Answer

QUESTION 7

7 Question 7 5 / 5

✓ **- 0 pts** *Neither predicate is bounded by the other quantifier.*

- 5 pts Incorrect

QUESTION 8

Question 8 15 pts

8.1 a **5 / 5**

✓ **- 0 pts** *$$ \forall x(A(x) \to (\lnot D(x) \land \lnot C(x) \land E(x))$$ or logically equivalent*

- 2 pts Incorrect quantifier

- 2 pts Incorrect predicate/variable

- 2 pts Incorrect scoping of quantifier

- 3 pts $$\rightarrow$$ instead of $$\land$$ or vice versa

- 5 pts Incorrect / Missing / No Answer

8.2 b **5 / 5**

✓ **- 0 pts** *$$ \forall x (D(x) \land (\lnot C(x) \lor \lnot H(x))$$ or logically equivalent*

- 2 pts Incorrect quantifier

- 2 pts Incorrect predicate/variable

- 2 pts Incorrect scoping of quantifier

- 3 pts $$\rightarrow$$ instead of $$\land$$ or vice versa

- 5 pts Incorrect / Missing / No Answer

8.3 **C 5 / 5**

✓ **- 0 pts** *$$\exists x (E(x) \land C(x)) \land \exists y (E(y) \land D(y)) \land \exists z (E(z) \land H(z))$$ or logically equivalent*

- 2 pts Incorrect quantifier

- 2 pts Incorrect predicate/variable

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- 3 pts $$\rightarrow$$ instead of $$\land$$ or vice versa

- 5 pts Incorrect / Missing / No Answer

QUESTION 9

9 On Time **2.5 / 0**

✓ **+ 2.5 pts** *On Time (Before Thursday)*

- 0 pts On Time (Friday)

- 10 pts 1 day late

- 25 pts 2 days late

QUESTION 10

10 Matching **0 / 0**

✓ **- 0 pts** *Correct*

- 5 pts Incorrect

Vidit Pokharna

CS 2050 HW 2

1.

a. $[\forall x\ (C(x) \rightarrow M(x))] \wedge [\exists x\ (M(x) \wedge \neg\, C(x))]$

b. $\exists x\ \exists y\ (B(y) \wedge F(x) \wedge E(y, x))$

c. $\forall x\ (M(x) \rightarrow E(x))$

2.

a. **True** because any value of y that is greater than x^2 for any x must also be greater than the value of x and for any y that is less than x^2, the conditional will still hold true.

b. **False** as no positive y can provide a negative value for $y^{\frac{1}{x}}$. A counterexample is $y = 4$, which provides no possible value of x that makes the statement $\sqrt[x]{y} < 0$.

c. **True** because there does exist a value x that satisfies the given conditional for all values of y. This value of x can be $x = 8$, for which all y values either make the conditional $T \rightarrow T$, $F \rightarrow T$, or $F \rightarrow F$, which all evaluate to T.

d. **False** as having an odd positive number for y and a negative x, x^y will provide a negative value which will be greater than the positive value of y. One counterexample to this is $y = 3$ and $x = -5$, where $y > 0 \vee x < 0 \rightarrow x^y \geq y$ makes a $T \rightarrow F \equiv F$.

e. **False** because having an x and y of opposite signs would make this biconditional false. One counterexample for this is $x = -2$ and $y = 2$, where $x^2 = y^2 \leftrightarrow y^3 = x^3$ makes a $T \leftrightarrow F \equiv F$.

3. y is in the domain of x

$$\exists x\ \forall y\ (P(x) \wedge (P(y) \rightarrow (y = x)))$$

4.

Expression	Step
$\exists x\ \forall y\ [(A(x) \rightarrow B(y)) \wedge (C(x, y) \rightarrow B(y))]$	Original
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$\exists x\ \forall y\ [B(y) \vee (\neg\, A(x) \wedge \neg\, C(x, y))]$	Distributivity
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2.

a. **True** because any value of y that is greater than x^2 for any x must also be greater than the value of x and for any y that is less than x^2, the conditional will still hold true.
b. **False** as no positive y can provide a negative value for $y^{\frac{1}{x}}$. A counterexample is $y = 4$, which provides no possible value of x that makes the statement $\sqrt[x]{y} < 0$.
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$$\exists x\, \forall y\,(P(x) \wedge (P(y) \rightarrow (y = x)))$$

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1.2 b **5 / 5**

✓ **- 0 pts** *Question removed*

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1.3 **C** **5 / 5**

- ✓ **- 0 pts** *$$\forall x(M(x) \rightarrow E(x))$$ or logically equivalent*
- **- 2 pts** Incorrect/Missing quantifier
- **- 2 pts** Incorrect predicate/variable
- **- 2 pts** Incorrect scoping of quantifier
- **- 3 pts** $$\rightarrow$$ instead of $$\land$$ or vice versa
- **- 5 pts** Incorrect / Missing / No Answer

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✓ **- 0 pts** *True*

- 4 pts False

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2.2 b **4 / 4**

- **- 4 pts** True
- ✓ **- 0 pts** *False*
- **- 2 pts** Incorrect Reasoning
- **- 4 pts** No Answer

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✓ **- 0 pts** *True*

- 4 pts False

- 2 pts Incorrect Reasoning

- 4 pts No Answer

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   c. **True** because there does exist a value x that satisfies the given conditional for all values of y. This value of x can be $x = 8$, for which all y values either make the conditional $T \rightarrow T$, $F \rightarrow T$, or $F \rightarrow F$, which all evaluate to T.
   d. **False** as having an odd positive number for y and a negative x, x^y will provide a negative value which will be greater than the positive value of y. One counterexample to this is $y = 3$ and $x = -5$, where $y > 0 \vee x < 0 \rightarrow x^y \geq y$ makes a $T \rightarrow F \equiv F$.
   e. **False** because having an x and y of opposite signs would make this biconditional false. One counterexample for this is $x = -2$ and $y = 2$, where $x^2 = y^2 \leftrightarrow y^3 = x^3$ makes a $T \leftrightarrow F \equiv F$.

3. y is in the domain of x

$$\exists x\, \forall y\, (P(x) \wedge (P(y) \rightarrow (y = x)))$$

4.

Expression	Step
$\exists x\, \forall y\, [(A(x) \rightarrow B(y)) \wedge (C(x,y) \rightarrow B(y))]$	Original
$\exists x\, \forall y\, [(\neg\, A(x) \vee B(y)) \wedge (C(x,y) \rightarrow B(y))]$	Conditional Disjunction Equivalence
$\exists x\, \forall y\, [(\neg\, A(x) \vee B(y)) \wedge (\neg\, C(x,y) \vee B(y))]$	Conditional Disjunction Equivalence
$\exists x\, \forall y\, [(B(y) \vee \neg\, A(x)) \wedge (\neg\, C(x,y) \vee B(y))]$	Commutativity
$\exists x\, \forall y\, [(B(y) \vee \neg\, A(x)) \wedge (B(y) \vee \neg\, C(x,y))]$	Commutativity
$\exists x\, \forall y\, [B(y) \vee (\neg\, A(x) \wedge \neg\, C(x,y))]$	Distributivity
$\exists x\, \forall y\, [\neg B(y) \rightarrow (\neg\, A(x) \wedge \neg\, C(x,y))]$	Conditional Disjunction Equivalence

2.4 d **4 / 4**

- **- 4 pts** True
- ✓ **- 0 pts** *False*
- **- 2 pts** Incorrect Reasoning
- **- 4 pts** No Answer

Vidit Pokharna

CS 2050 HW 2

1.

a. $[\forall x\ (C(x) \to M(x))] \land [\exists x\ (M(x) \land \neg C(x))]$
b. $\exists x\ \exists y\ (B(y) \land F(x) \land E(y, x))$
c. $\forall x\ (M(x) \to E(x))$

2.

a. **True** because any value of y that is greater than x^2 for any x must also be greater than the value of x and for any y that is less than x^2, the conditional will still hold true.
b. **False** as no positive y can provide a negative value for $y^{\frac{1}{x}}$. A counterexample is $y = 4$, which provides no possible value of x that makes the statement $\sqrt[x]{y} < 0$.
c. **True** because there does exist a value x that satisfies the given conditional for all values of y. This value of x can be $x = 8$, for which all y values either make the conditional $T \to T$, $F \to T$, or $F \to F$, which all evaluate to T.
d. **False** as having an odd positive number for y and a negative x, x^y will provide a negative value which will be greater than the positive value of y. One counterexample to this is $y = 3$ and $x = -5$, where $y > 0 \lor x < 0 \to x^y \geq y$ makes a $T \to F \equiv F$.
e. **False** because having an x and y of opposite signs would make this biconditional false. One counterexample for this is $x = -2$ and $y = 2$, where $x^2 = y^2 \leftrightarrow y^3 = x^3$ makes a $T \leftrightarrow F \equiv F$.

3. y is in the domain of x

$$\exists x\ \forall y\ (P(x) \land (P(y) \to (y = x)))$$

4.

Expression	Step
$\exists x\ \forall y\ [(A(x) \to B(y)) \land (C(x,y) \to B(y))]$	Original
$\exists x\ \forall y\ [(\neg A(x) \lor B(y)) \land (C(x,y) \to B(y))]$	Conditional Disjunction Equivalence
$\exists x\ \forall y\ [(\neg A(x) \lor B(y)) \land (\neg C(x,y) \lor B(y))]$	Conditional Disjunction Equivalence
$\exists x\ \forall y\ [(B(y) \lor \neg A(x)) \land (\neg C(x,y) \lor B(y))]$	Commutativity
$\exists x\ \forall y\ [(B(y) \lor \neg A(x)) \land (B(y) \lor \neg C(x,y))]$	Commutativity
$\exists x\ \forall y\ [B(y) \lor (\neg A(x) \land \neg C(x,y))]$	Distributivity
$\exists x\ \forall y\ [\neg B(y) \to (\neg A(x) \land \neg C(x,y))]$	Conditional Disjunction Equivalence

2.5 e **4 / 4**

- **- 4 pts** True
- ✓ **- 0 pts** *False*
- **- 2 pts** Incorrect Reasoning
- **- 4 pts** No Answer

Vidit Pokharna

CS 2050 HW 2

1.

   a. $\left[\forall x\,(C(x) \rightarrow M(x))\right] \wedge [\exists x\,(M(x) \wedge \neg\, C(x))]$
   b. $\exists x\, \exists y\, (B(y) \wedge F(x) \wedge E(y,x))$
   c. $\forall x\, (M(x) \rightarrow E(x))$

2.

   a. **True** because any value of y that is greater than x^2 for any x must also be greater than the value of x and for any y that is less than x^2, the conditional will still hold true.
   b. **False** as no positive y can provide a negative value for $y^{\frac{1}{x}}$. A counterexample is $y = 4$, which provides no possible value of x that makes the statement $\sqrt[x]{y} < 0$.
   c. **True** because there does exist a value x that satisfies the given conditional for all values of y. This value of x can be $x = 8$, for which all y values either make the conditional $T \rightarrow T$, $F \rightarrow T$, or $F \rightarrow F$, which all evaluate to T.
   d. **False** as having an odd positive number for y and a negative x, x^y will provide a negative value which will be greater than the positive value of y. One counterexample to this is $y = 3$ and $x = -5$, where $y > 0 \vee x < 0 \rightarrow x^y \geq y$ makes a $T \rightarrow F \equiv F$.
   e. **False** because having an x and y of opposite signs would make this biconditional false. One counterexample for this is $x = -2$ and $y = 2$, where $x^2 = y^2 \leftrightarrow y^3 = x^3$ makes a $T \leftrightarrow F \equiv F$.

3. y is in the domain of x

$$\exists x\, \forall y\, (P(x) \wedge (P(y) \rightarrow (y = x)))$$

4.

Expression	Step
$\exists x\, \forall y\, [(A(x) \rightarrow B(y)) \wedge (C(x,y) \rightarrow B(y))]$	Original
$\exists x\, \forall y\, [(\neg\, A(x) \vee B(y)) \wedge (C(x,y) \rightarrow B(y))]$	Conditional Disjunction Equivalence
$\exists x\, \forall y\, [(\neg\, A(x) \vee B(y)) \wedge (\neg\, C(x,y) \vee B(y))]$	Conditional Disjunction Equivalence
$\exists x\, \forall y\, [(B(y) \vee \neg\, A(x)) \wedge (\neg\, C(x,y) \vee B(y))]$	Commutativity
$\exists x\, \forall y\, [(B(y) \vee \neg\, A(x)) \wedge (B(y) \vee \neg\, C(x,y))]$	Commutativity
$\exists x\, \forall y\, [B(y) \vee (\neg\, A(x) \wedge \neg\, C(x,y))]$	Distributivity
$\exists x\, \forall y\, [\neg B(y) \rightarrow (\neg\, A(x) \wedge \neg\, C(x,y))]$	Conditional Disjunction Equivalence

3 Question 3 **10 / 10**

✓ **- 0 pts** *$$\exists x (P(x) \land \forall y (y\neq x \to \lnot P(y)))$$*

OR

$$\exists x \forall y (P(y) \leftrightarrow (y = x))$$

OR

$$\exists x P(x) \wedge \forall y \forall z [P(y) \wedge P(z)) \rightarrow y=z]$$

OR

logically equivalent

- 5 pts Attempted to use the quantifiers in order $$\exists x \forall y$$ but did not reach the correct answer

- 10 pts No Answer/Incorrect Answer

Vidit Pokharna

CS 2050 HW 2

1.
   a. $[\forall x\ (C(x) \rightarrow M(x))] \wedge [\exists x\ (M(x) \wedge \neg\, C(x))]$
   b. $\exists x\ \exists y\ (B(y) \wedge F(x) \wedge E(y,x))$
   c. $\forall x\ (M(x) \rightarrow E(x))$

2.
   a. **True** because any value of y that is greater than x^2 for any x must also be greater than the value of x and for any y that is less than x^2, the conditional will still hold true.
   b. **False** as no positive y can provide a negative value for $y^{\frac{1}{x}}$. A counterexample is $y = 4$, which provides no possible value of x that makes the statement $\sqrt[x]{y} < 0$.
   c. **True** because there does exist a value x that satisfies the given conditional for all values of y. This value of x can be $x = 8$, for which all y values either make the conditional $T \rightarrow T$, $F \rightarrow T$, or $F \rightarrow F$, which all evaluate to T.
   d. **False** as having an odd positive number for y and a negative x, x^y will provide a negative value which will be greater than the positive value of y. One counterexample to this is $y = 3$ and $x = -5$, where $y > 0 \vee x < 0 \rightarrow x^y \geq y$ makes a $T \rightarrow F \equiv F$.
   e. **False** because having an x and y of opposite signs would make this biconditional false. One counterexample for this is $x = -2$ and $y = 2$, where $x^2 = y^2 \leftrightarrow y^3 = x^3$ makes a $T \leftrightarrow F \equiv F$.

3. y is in the domain of x

$$\exists x\ \forall y\ (P(x) \wedge (P(y) \rightarrow (y = x)))$$

4.

Expression	Step
$\exists x\ \forall y\ [(A(x) \rightarrow B(y)) \wedge (C(x,y) \rightarrow B(y))]$	Original
$\exists x\ \forall y\ [(\neg\, A(x) \vee B(y)) \wedge (C(x,y) \rightarrow B(y))]$	Conditional Disjunction Equivalence
$\exists x\ \forall y\ [(\neg\, A(x) \vee B(y)) \wedge (\neg\, C(x,y) \vee B(y))]$	Conditional Disjunction Equivalence
$\exists x\ \forall y\ [(B(y) \vee \neg\, A(x)) \wedge (\neg\, C(x,y) \vee B(y))]$	Commutativity
$\exists x\ \forall y\ [(B(y) \vee \neg\, A(x)) \wedge (B(y) \vee \neg\, C(x,y))]$	Commutativity
$\exists x\ \forall y\ [B(y) \vee (\neg\, A(x) \wedge \neg\, C(x,y))]$	Distributivity
$\exists x\ \forall y\ [\neg B(y) \rightarrow (\neg\, A(x) \wedge \neg\, C(x,y))]$	Conditional Disjunction Equivalence

4 Question 4 **10 / 10**

✓ **- 0 pts** *Correct*

- **- 8 pts** Did not cite any steps

Invalid steps

- **- 3 pts** 1 invalid step
- **- 5 pts** 2 invalid steps
- **- 7 pts** 3 invalid steps
- **- 10 pts** 4+ invalid steps

Skipped steps

- **- 2 pts** 1 skipped step
- **- 4 pts** 2 skipped steps
- **- 6 pts** 3 skipped steps
- **- 8 pts** 4+ skipped steps

Uncited steps

- **- 1 pts** 1 uncited step
- **- 2 pts** 2 uncited steps
- **- 3 pts** 3 uncited steps
- **- 4 pts** 4+ uncited steps

Miscited steps

- **- 1 pts** 1 miscited step
- **- 2 pts** 2 miscited steps
- **- 3 pts** 3 miscited steps
- **- 4 pts** 4+ miscited steps

- **- 8 pts** Did not reach $$\exists x \forall y (\neg B(y) \rightarrow (\neg A(x) \land \neg C(x,y))) $$
- **- 10 pts** No answer

5.

a.

Expression	Step
$\forall x\, \neg\exists y\, \exists z\, (P(x,y) \leftrightarrow \neg\, S(y,z))$	Original
$\forall x\, \forall y\, \neg\exists z\, (P(x,y) \leftrightarrow \neg\, S(y,z))$	DeMorgan's Law for Quantifiers
$\forall x\, \forall y\, \forall z\, \neg\, (P(x,y) \leftrightarrow \neg\, S(y,z))$	DeMorgan's Law for Quantifiers
$\forall x\, \forall y\, \forall z\, \neg\, ((P(x,y) \rightarrow \neg\, S(y,z)) \wedge (\neg\, S(y,z) \rightarrow P(x,y)))$	Expansion of Biconditional
$\forall x\, \forall y\, \forall z\, (\neg\, (P(x,y) \rightarrow \neg\, S(y,z)) \vee \neg\, (\neg\, S(y,z) \rightarrow P(x,y)))$	DeMorgan's Law
$\forall x\, \forall y\, \forall z\, (\neg\, (\neg\, P(x,y) \vee \neg\, S(y,z)) \vee \neg\, (\neg\, S(y,z) \rightarrow P(x,y)))$	Conditional Disjunction Equivalence
$\forall x\, \forall y\, \forall z\, (\neg\, (\neg\, P(x,y) \vee \neg\, S(y,z)) \vee \neg(\neg\, (\neg\, S(y,z)) \vee P(x,y)))$	Conditional Disjunction Equivalence
$\forall x\, \forall y\, \forall z\, (\neg\, (\neg\, P(x,y) \vee \neg\, S(y,z)) \vee \neg\, \big(S(y,z) \vee P(x,y)\big))$	Double Negation
$\forall x\, \forall y\, \forall z\, (\neg\, (\neg\, P(x,y)) \wedge \neg\, (\neg\, S(y,z))) \vee \neg\, \big(S(y,z) \vee P(x,y)\big)))$	DeMorgan's Law
$\forall x\, \forall y\, \forall z\, (\neg\, (\neg\, P(x,y)) \wedge \neg\, (\neg\, S(y,z))) \vee \big(\neg\, S(y,z) \wedge \neg\, P(x,y)\big))$	DeMorgan's Law
$\forall x\, \forall y\, \forall z\, (P(x,y) \wedge \neg\, (\neg\, S(y,z))) \vee \big(\neg\, S(y,z) \wedge \neg\, P(x,y)\big))$	Double Negation
$\forall x\, \forall y\, \forall z\, ((P(x,y) \wedge\, S(y,z)) \vee \big(\neg\, S(y,z) \wedge \neg\, P(x,y)\big))$	Double Negation

b.

Expression	Step
$\neg\, \forall y\, \neg\, \exists x\, (R(x,y) \rightarrow \exists z S(z,y))$	Original
$\exists y\, \neg\, (\neg\, \exists x)\, (R(x,y) \rightarrow \exists z S(z,y))$	DeMorgan's Law for Quantifiers
$\exists y\, \exists x\, (R(x,y) \rightarrow \exists z\, S(z,y))$	Double Negation

5.1 a **5 / 5**

✓ **- 0 pts** $$\forall x \forall y \forall z ((P(x,y) \land S(y, z)) \lor (\lnot P(x, y) \land \lnot S(y, z)))$$ *or equivalent*

- 3 pts Equivalent expression but did not push negations such that every negation is immediately to the left of a predicate

- 5 pts Incorrect Expression

- 5 pts No Answer

5.

a.

Expression	Step
$\forall x \neg\exists y\, \exists z\, (P(x,y) \leftrightarrow \neg S(y,z))$	Original
$\forall x\, \forall y\, \neg\exists z\, (P(x,y) \leftrightarrow \neg S(y,z))$	DeMorgan's Law for Quantifiers
$\forall x\, \forall y\, \forall z\, \neg (P(x,y) \leftrightarrow \neg S(y,z))$	DeMorgan's Law for Quantifiers
$\forall x\, \forall y\, \forall z\, \neg ((P(x,y) \rightarrow \neg S(y,z)) \wedge (\neg S(y,z) \rightarrow P(x,y)))$	Expansion of Biconditional
$\forall x\, \forall y\, \forall z\, (\neg (P(x,y) \rightarrow \neg S(y,z)) \vee \neg (\neg S(y,z) \rightarrow P(x,y)))$	DeMorgan's Law
$\forall x\, \forall y\, \forall z\, (\neg (\neg P(x,y) \vee \neg S(y,z)) \vee \neg (\neg S(y,z) \rightarrow P(x,y)))$	Conditional Disjunction Equivalence
$\forall x\, \forall y\, \forall z\, (\neg (\neg P(x,y) \vee \neg S(y,z)) \vee \neg(\neg (\neg S(y,z)) \vee P(x,y)))$	Conditional Disjunction Equivalence
$\forall x\, \forall y\, \forall z\, (\neg (\neg P(x,y) \vee \neg S(y,z)) \vee \neg \big(S(y,z) \vee P(x,y)\big))$	Double Negation
$\forall x\, \forall y\, \forall z\, (\neg (\neg P(x,y)) \wedge \neg (\neg S(y,z))) \vee \neg \big(S(y,z) \vee P(x,y)\big)))$	DeMorgan's Law
$\forall x\, \forall y\, \forall z\, (\neg (\neg P(x,y)) \wedge \neg (\neg S(y,z))) \vee \big(\neg S(y,z) \wedge \neg P(x,y)\big))$	DeMorgan's Law
$\forall x\, \forall y\, \forall z\, (P(x,y) \wedge \neg (\neg S(y,z))) \vee \big(\neg S(y,z) \wedge \neg P(x,y)\big))$	Double Negation
$\forall x\, \forall y\, \forall z\, ((P(x,y) \wedge\ S(y,z)) \vee \big(\neg S(y,z) \wedge \neg P(x,y)\big))$	Double Negation

b.

Expression	Step
$\neg\, \forall y \neg\, \exists x\, (R(x,y) \rightarrow \exists z S(z,y))$	Original
$\exists y \neg\, (\neg\, \exists x)\, (R(x,y) \rightarrow \exists z S(z,y))$	DeMorgan's Law for Quantifiers
$\exists y\, \exists x\, (R(x,y) \rightarrow \exists z\, S(z,y))$	Double Negation

5.2 b **5 / 5**

✓ **- 0 pts** *$$\exists y \exists x (R(x, y)\to \exists z S(z, y))$$ or equivalent*

- 3 pts Equivalent expression but did not push negations such that every negation is immediately to the left of a predicate

- 5 pts Incorrect Expression

- 5 pts No Answer

6.

   a. $x = 2$ and $y = 2$
   b. No counterexample ($x = 1$ works)
   c. $x = 0.5$ and $y = 0.25$

7. Suppose statement 1 = $\forall x\, P(x) \vee \exists x\, Q(x)$ and statement 2 = $\forall x\, \exists y\, (P(x) \vee Q(y))$. The reason why statement 1 and statement 2 are logically equivalent is because in statement 2, $P(x)$ does not rely on the quantifier $\exists y$. This can be illustrated in statement 1, where they $P(x)$ and $Q(y)$ are separated by an OR, both having the same variable x, but the quantifier separation makes each value for the variable for each quantifier allowed to have different variables. This plays a similar role as having two different variables, like we saw with statement 2. Therefore, these two statements serve the same purpose and can be considered logically equivalent.

8.

   a. $\forall x\, (A(x) \rightarrow (\neg D(x) \wedge \neg C(x) \wedge E(x)))$
   b. $\forall x\, (D(x) \wedge \neg (H(x) \wedge C(x)))$
   c. $(\exists x\, (E(x) \wedge D(x))) \wedge (\exists x\, \big(E(x) \wedge C(x)\big)) \wedge (\exists x\, (E(x) \wedge H(x)))$

6.1 a **5 / 5**

✓ **- 0 pts** *Valid counterexample*

Possible answers:

x=y=2

or anything else > 1

- 4 pts Incorrect counterexample

- 5 pts No counterexample exists

- 5 pts No Answer

6.

   a. $x = 2$ and $y = 2$
   b. No counterexample ($x = 1$ works)
   c. $x = 0.5$ and $y = 0.25$

7. Suppose statement 1 = $\forall x\ P(x) \vee \exists x\ Q(x)$ and statement 2 = $\forall x\ \exists y\ (P(x) \vee Q(y))$. The reason why statement 1 and statement 2 are logically equivalent is because in statement 2, $P(x)$ does not rely on the quantifier $\exists y$. This can be illustrated in statement 1, where they $P(x)$ and $Q(y)$ are separated by an OR, both having the same variable x, but the quantifier separation makes each value for the variable for each quantifier allowed to have different variables. This plays a similar role as having two different variables, like we saw with statement 2. Therefore, these two statements serve the same purpose and can be considered logically equivalent.

8.

   a. $\forall x\ (A(x) \rightarrow (\neg D(x) \wedge \neg C(x) \wedge E(x)))$
   b. $\forall x\ (D(x) \wedge \neg (H(x) \wedge C(x)))$
   c. $(\exists x\ (E(x) \wedge D(x))) \wedge (\exists x\ \big(E(x) \wedge C(x)\big)) \wedge (\exists x\ (E(x) \wedge H(x)))$

6.2 b **5 / 5**

✓ **- 0 pts** *No counterexample exists*

- 5 pts Provided a counter example

- 5 pts No Answer

6.

   a. $x = 2$ and $y = 2$
   b. No counterexample ($x = 1$ works)
   c. $x = 0.5$ and $y = 0.25$

7. Suppose statement 1 = $\forall x\, P(x) \vee \exists x\, Q(x)$ and statement 2 = $\forall x\, \exists y\, (P(x) \vee Q(y))$. The reason why statement 1 and statement 2 are logically equivalent is because in statement 2, $P(x)$ does not rely on the quantifier $\exists y$. This can be illustrated in statement 1, where they $P(x)$ and $Q(y)$ are separated by an OR, both having the same variable x, but the quantifier separation makes each value for the variable for each quantifier allowed to have different variables. This plays a similar role as having two different variables, like we saw with statement 2. Therefore, these two statements serve the same purpose and can be considered logically equivalent.

8.

   a. $\forall x\, (A(x) \rightarrow (\neg D(x) \wedge \neg C(x) \wedge E(x)))$
   b. $\forall x\, (D(x) \wedge \neg (H(x) \wedge C(x)))$
   c. $(\exists x\, (E(x) \wedge D(x))) \wedge (\exists x\, \big(E(x) \wedge C(x)\big)) \wedge (\exists x\, (E(x) \wedge H(x)))$

6.3 **C** **5 / 5**

✓ **- 0 pts** *Valid counterexample*

Possible answers:

x=y=0

x=y=0.5

x=y=1

x=0.5, y=0.4

- 4 pts Incorrect counterexample

- 5 pts No counterexample exists

- 5 pts No Answer

6.

   a. $x = 2$ and $y = 2$
   b. No counterexample ($x = 1$ works)
   c. $x = 0.5$ and $y = 0.25$

7. Suppose statement 1 = $\forall x\, P(x) \vee \exists x\, Q(x)$ and statement 2 = $\forall x\, \exists y\, (P(x) \vee Q(y))$. The reason why statement 1 and statement 2 are logically equivalent is because in statement 2, $P(x)$ does not rely on the quantifier $\exists y$. This can be illustrated in statement 1, where they $P(x)$ and $Q(y)$ are separated by an OR, both having the same variable x, but the quantifier separation makes each value for the variable for each quantifier allowed to have different variables. This plays a similar role as having two different variables, like we saw with statement 2. Therefore, these two statements serve the same purpose and can be considered logically equivalent.

8.

   a. $\forall x\, (A(x) \rightarrow (\neg D(x) \wedge \neg C(x) \wedge E(x)))$
   b. $\forall x\, (D(x) \wedge \neg (H(x) \wedge C(x)))$
   c. $(\exists x\, (E(x) \wedge D(x))) \wedge (\exists x\, (E(x) \wedge C(x))) \wedge (\exists x\, (E(x) \wedge H(x)))$

7 Question 7 **5 / 5**

✓ **- 0 pts** *Neither predicate is bounded by the other quantifier.*

- 5 pts Incorrect

6.

   a. $x = 2$ and $y = 2$
   b. No counterexample ($x = 1$ works)
   c. $x = 0.5$ and $y = 0.25$

7. Suppose statement 1 = $\forall x\, P(x) \vee \exists x\, Q(x)$ and statement 2 = $\forall x\, \exists y\, (P(x) \vee Q(y))$. The reason why statement 1 and statement 2 are logically equivalent is because in statement 2, $P(x)$ does not rely on the quantifier $\exists y$. This can be illustrated in statement 1, where they $P(x)$ and $Q(y)$ are separated by an OR, both having the same variable x, but the quantifier separation makes each value for the variable for each quantifier allowed to have different variables. This plays a similar role as having two different variables, like we saw with statement 2. Therefore, these two statements serve the same purpose and can be considered logically equivalent.

8.

   a. $\forall x\, (A(x) \rightarrow (\neg D(x) \wedge \neg C(x) \wedge E(x)))$
   b. $\forall x\, (D(x) \wedge \neg (H(x) \wedge C(x)))$
   c. $(\exists x\, (E(x) \wedge D(x))) \wedge (\exists x\, \left(E(x) \wedge C(x)\right)) \wedge (\exists x\, (E(x) \wedge H(x)))$

8.1 a **5 / 5**

✓ **- 0 pts** *$$ \forall x(A(x) \to (\lnot D(x) \land \lnot C(x) \land E(x))$$ or logically equivalent*

- 2 pts Incorrect quantifier

- 2 pts Incorrect predicate/variable

- 2 pts Incorrect scoping of quantifier

- 3 pts $$\rightarrow$$ instead of $$\land$$ or vice versa

- 5 pts Incorrect / Missing / No Answer

6.

   a. $x = 2$ and $y = 2$
   b. No counterexample ($x = 1$ works)
   c. $x = 0.5$ and $y = 0.25$

7. Suppose statement 1 = $\forall x\, P(x) \vee \exists x\, Q(x)$ and statement 2 = $\forall x\, \exists y\, (P(x) \vee Q(y))$. The reason why statement 1 and statement 2 are logically equivalent is because in statement 2, $P(x)$ does not rely on the quantifier $\exists y$. This can be illustrated in statement 1, where they $P(x)$ and $Q(y)$ are separated by an OR, both having the same variable x, but the quantifier separation makes each value for the variable for each quantifier allowed to have different variables. This plays a similar role as having two different variables, like we saw with statement 2. Therefore, these two statements serve the same purpose and can be considered logically equivalent.

8.

   a. $\forall x\, (A(x) \rightarrow (\neg D(x) \wedge \neg C(x) \wedge E(x)))$
   b. $\forall x\, (D(x) \wedge \neg (H(x) \wedge C(x)))$
   c. $(\exists x\, (E(x) \wedge D(x))) \wedge (\exists x\, (E(x) \wedge C(x))) \wedge (\exists x\, (E(x) \wedge H(x)))$

8.2 b **5 / 5**

✓ **- 0 pts** *$$ \forall x (D(x) \land (\lnot C(x) \lor \lnot H(x))$$ or logically equivalent*

- 2 pts Incorrect quantifier

- 2 pts Incorrect predicate/variable

- 2 pts Incorrect scoping of quantifier

- 3 pts $$\rightarrow$$ instead of $$\land$$ or vice versa

- 5 pts Incorrect / Missing / No Answer

6.

   a. $x = 2$ and $y = 2$
   b. No counterexample ($x = 1$ works)
   c. $x = 0.5$ and $y = 0.25$

7. Suppose statement 1 = $\forall x\, P(x) \lor \exists x\, Q(x)$ and statement 2 = $\forall x\, \exists y\, (P(x) \lor Q(y))$. The reason why statement 1 and statement 2 are logically equivalent is because in statement 2, $P(x)$ does not rely on the quantifier $\exists y$. This can be illustrated in statement 1, where they $P(x)$ and $Q(y)$ are separated by an OR, both having the same variable x, but the quantifier separation makes each value for the variable for each quantifier allowed to have different variables. This plays a similar role as having two different variables, like we saw with statement 2. Therefore, these two statements serve the same purpose and can be considered logically equivalent.

8.

   a. $\forall x\, (A(x) \to (\neg D(x) \land \neg C(x) \land E(x)))$
   b. $\forall x\, (D(x) \land \neg (H(x) \land C(x)))$
   c. $(\exists x\, (E(x) \land D(x))) \land (\exists x\, (E(x) \land C(x))) \land (\exists x\, (E(x) \land H(x)))$

8.3 **C** **5 / 5**

✓ **- 0 pts** *$$\exists x (E(x) \land C(x)) \land \exists y (E(y) \land D(y)) \land \exists z (E(z) \land H(z))$$ or logically equivalent*

- 2 pts Incorrect quantifier

- 2 pts Incorrect predicate/variable

- 2 pts Incorrect scoping of quantifier

- 3 pts $$\rightarrow$$ instead of $$\land$$ or vice versa

- 5 pts Incorrect / Missing / No Answer

9 On Time **2.5 / 0**

✓ **+ 2.5 pts** *On Time (Before Thursday)*

- 0 pts On Time (Friday)

- 10 pts 1 day late

- 25 pts 2 days late

10 Matching **0 / 0**

✓ **- 0 pts** *Correct*

- 5 pts Incorrect