MATH-3012-D Quiz 3

Vidit Dharmendra Pokharna

TOTAL POINTS

28 / 45

QUESTION 1

1 Q1 **1 / 15**

   ✓ **+ 1 pts** *(b) partial credit*

QUESTION 2

2 Q2 **15 / 15**

   ✓ **+ 15 pts** *Correct*

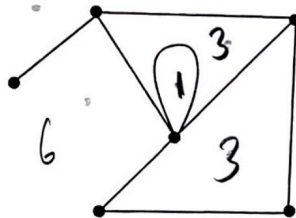QUESTION 3

3 Q3 **12 / 15**

   (c)

   ✓ **- 3 pts** *partial credit*

   1 Why?

NAME: Vidit Pokharna   903772087

1. $(7+4+4=15$ points. Always justify your answers. Answers alone have no credit.)

   (a) A convex polyhedron has in total 2 pentagon faces, 5 quadrilateral faces. and 20 triangle faces. How many vertices does it have?

   (b) In each face of the plane multigraph below, write down the degree of the face.

   (c) A plane multigraph G has 6 connected components. For G, what is the value of $v - e + f$?

(a).

$$\ldots - 39 + 28 = 28 \times 2$$
$$v = 59 + 11 = \boxed{70}$$

(b) in drawing

(c) 12

✓ **+ 1 pts** *(b) partial credit*

2. $(5 + 8 + 2 = 15$ points. Always justify your answers. Answers alone have no credit.)

  (a) Let H be a loop-free graph with 12 vertices. If H has 30 edges then how many edges does the complement graph $\bar{H}$ have?

  (b) Let G be a loop-free connected graph with 12 vertices. Prove that either G or its complement $\bar{G}$ must be non-planar.

  (c) Does the hypercube Q_5 have a cycle of length 5?

(a) $\binom{12}{2} - 30 = \frac{\overset{6}{\cancel{12}} \times 11}{\cancel{2}} - 30 = 66 - 30 = 36$

(b) If both G and $\bar{G}$ are planar, then the following would be true: $E(G) \leq 3|v| - 6 = 3 \cdot 12 - 6 = 30$

$E(\bar{G}) \leq 3|v| - 6 = 3 \cdot 12 - 6 = 30$

However, $E(G) + E(\bar{G})$ must equal $\binom{12}{2} = 66$. This is not possible when $E(G), E(\bar{G}) \leq 30$. Therefore, either G or $\bar{G}$ must be non-planar.
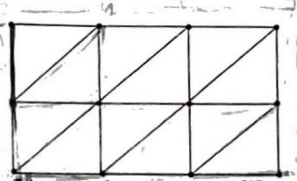
(c) $\boxed{N}$ because a cycle would require an even number of steps

2

✓ **+ 15 pts** *Correct*

NAME: Vidit Palchaira   9037720087

3. (7 + 2 + 6 = 15 points)

(a) Does the graph

have an Euler circuit? If yes, draw one, if no, explain why?
Does it have an Euler trail? If yes, where should the Euler trail begin? If no, explain why.
(Answers alone have 0 credit.)
(b) Does there exist a multigraph with degree sequence 5, 5, 3, 2, 2? If yes, explain why. If no, draw an example of such a multigraph.
(c) Is the following graph bipartite? Does it have a Hamiltonian cycle? (Answers alone have no credit).
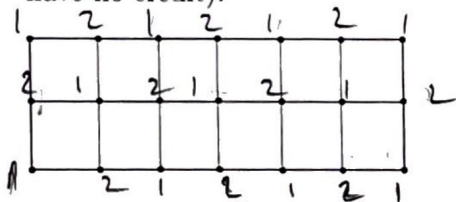
(a) It does not have Euler cycle because there are odd degrees

(b) It does have a Euler Trail, beginning at either the top right vertex or bottom left vertex

(b) No because the sum of the degrees in a multigraph must be even and have an even number of odd degrees.

(c) Yes, it is bipartite (as shown in the diagram.

It does not have a hamiltonian cycle [1]

3

(c)

✓ **- 3 pts** *partial credit*

1 Why?