Economics of Climate Change – Problem Set 3 (5 pts total)

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Question 1: Abatement costs & comparing policy instruments (100%)

Consider two firms that build air-conditioning (AC) systems for cars. These air-conditioning systems require the use of a refrigerant called RX99, which is a potent greenhouse gas. There are only two firms that produce the AC systems for all cars made in the US. One firm is based in Austin (firm A) and the other firm is based in Brooklyn (firm B). The Austin-based firm is older and leaks more refrigerant into the atmosphere. The Brooklyn-based firm is newer and doesn't lose as much refrigerant in its production process. As a result, firm A has larger abatement costs than firm B. Both firms' costs are given below:

$$\text{Firm A's Marginal Abatement Cost} = MAC^A(x) = 100 - 1.25x$$

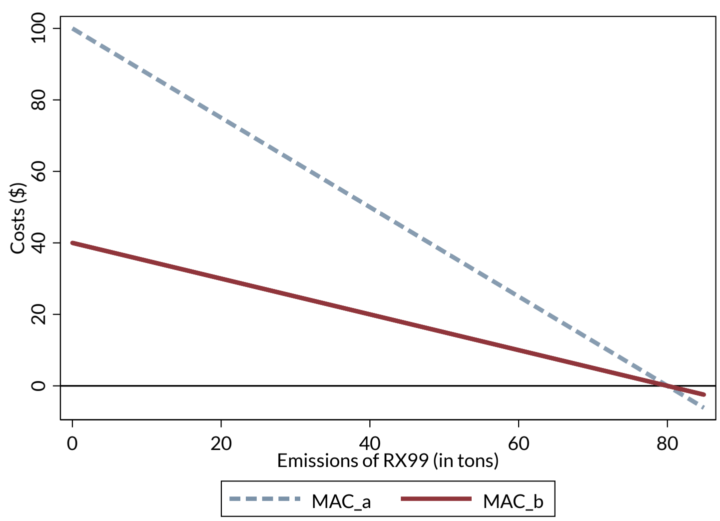
$$\text{Firm B's Marginal Abatement Cost} = MAC^B(x) = 40 - \frac{x}{2}$$

where x is how many tons of RX99 that are emitted by each firm annually.

1.1 (10 pts)

Draw the marginal abatement cost curves for both firms with costs (in dollars) on the vertical axis and emissions of RX99 on the horizontal axis. How many tons of the refrigerant will be emitted in a "no regulation" scenario?

Suggested answer:

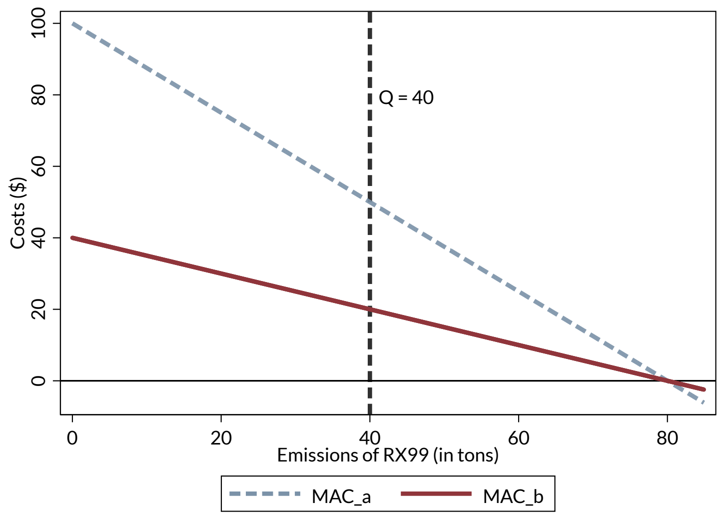


Without any regulation, neither firm will abate any emissions of RX99, so they will both emit until their marginal abatment costs curves intersect with the marginal benefits curve, which in this case is zero because it's nonexistent. So, firm A will emit 80 tons of RX99, firm B will emit 80 tons of RX99, and together 160 tons of RX99 will be emitted.

1.2 (15 pts)

The federal government realizes that RX99 is contributing to climate change and is considering different policy options to reduce its emissions. The government wants to keep total RX99 emissions at no more than 80 tons each year, so it mandates that each firm can only emit 40 tons. What are the abatement costs of limiting RX99 to 40 tons for each firm? What are the aggregate abatement costs arising from both firms?

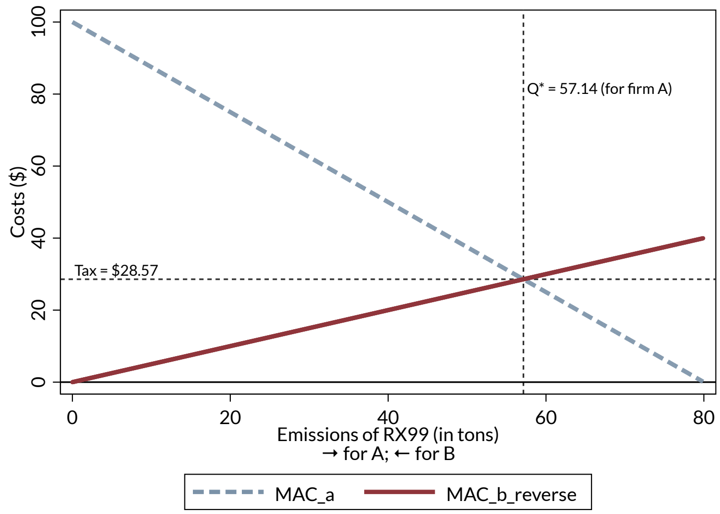
Suggested answer: If each firm is restricted to emitting 40 tons of RX99, the abatement costs that each firm experiences is the area under their marginal abatment curves greater than 40 tons of emissions (e.g., the triangle to the right of 40 tons and below the MAC in the figure below):



- For firm A: $MAC^A(40) = 100 - 1.25(40) = 50$ so, firm A's total abatement costs are $0.5 \times 50 \times (80 - 40) = \$1000$.
- For firm B: $MAC^B(40) = 40 - 0.5(40) = 20$ so, firm A's total abatement costs are $0.5 \times 20 \times (80 - 40) = \$400$.
- So, aggregate abatement costs are \$1400.

1.3 (30 pts – 6 pts each)

You are hired in an advisory role to help the US government design alternative policies for RX99 abatement. One option you and your team are considering is a tax on the emissions of RX99 that attempts to keep total annual emissions below 80 tons.



(a.) What tax rate (per ton of RX99) would you recommend to minimize costs from both firms?

Suggested answer: To minimize costs, you would set the tax rate at the level where both MAC curves intersect. To find this point, just set the curves equal and solve for the optimal level of abatement. The trick is that the MAC for firm B is "reversed", so we can represent the mirror image of $MAC^B(x)$ as $MAC_1^A(x) = 0.5x$. Note that this curve has a y-intercept of 0 on the left axis and intersects the right axis at $x = 80$.

$$MAC^A(x) = MAC_1^B(x) \implies 100 - 1.25x = 0.5x \implies x = 57.1429$$

We can then evaluate either MAC at this level to determine the optimal tax. Using $MAC_1^B(57.14) \approx$ \$28.57/ton.

(b.) How much RX99 would each firm emit under the tax?

Suggested answer: In the previous question, we already calculated that the MAC curves intersect at $x = 57.1429$. So, since moving left to right on the figure above represents emissions for firm A, firm A will emit 57.14 tons under the tax. Total emissions are kept below 80 tons, so firm B will emit the rest, i.e., firm B emits $80 - 57.14 = 22.86$ tons.

(c.) What are each firm's costs of abatement under the RX99 tax? What are the aggregate costs?

Suggested answer: Following a similar process as in Question 1.2, we simply need to find the area under each MAC curve at the optimal point of emissions for each firm.

- For firm A, total costs are given by the triangle with area: $0.5 \times \$28.57 \times (80 - 57.14) = \$326.56$.
- For firm B, total costs are given by the triangle with area: $0.5 \times \$28.57 \times (80 - 22.86) = \$816.25$.

(d.) How much total tax revenue would the tax generate?

Suggested answer: Revenue is always "$P \times Q$" – so, with a tax rate of \$28.57 and a total quantity of 80 tons of emissions. Total tax revenue is $\$28.57 \times 80 = \$2258.60$.

(e.) If you set the tax rate \$10 higher than your tax rate in (a.), how much RX99 would each firm emit?

Suggested answer: If the rate rate was $\$28.57 + \$10 = \$38.57$, we can calculate how much each firm would produce by recognizing that they will set their MAC equal to the tax.

- For firm A, $MAC^A(x) = \$38.57 \implies 100 - 1.25x = \$38.57 \implies x^A = 49.14$

– For firm B, $MAC^B(x) = \$38.57 \implies 40 - 0.5x = \$38.57 \implies x^B = 2.86$

So, in this instance only 52 tons of RX99 would be emitted, which is less than what is "socially optimal".

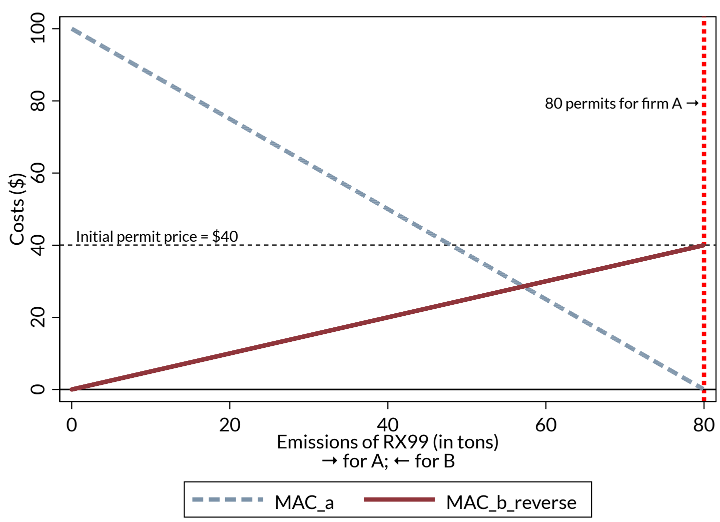
Hint: Draw firm B's MAC curve as a "mirror image" on a secondary vertical axis as we've done in class. The width of the graph should represent the total amount of emissions allowed from both firms. So, the horizontal axis for firm B should move from right (at 0 tons) to left (at 80 tons). Think about how this "mirror image" would change the MAC equation: the magnitude of the slope stays the same (but becomes negative).

1.4 (30 pts – 6 pts each)

Now, you are considering an alternative policy—a cap–and-trade program—that sets the total emissions of RX99 to 80 tons annually, but allows firms to trade permits for emitting RX99. Because the Austin-based firm (firm A) has been around longer and the CEO is golf buddies with one of the key regulators at the EPA, firm A receives all 80 permits to emit RX99.

(a.) Demonstrate graphically what this initial allocation of permits looks like (i.e., before trading occurs) using a similar format from the "Hint" in the previous question.

Suggested answer:



In the figure above, firm A receives all 80 permits, so the initial allocation is: firm A has 80 permits, firm B has 0 permits.

(b.) What are total costs of abatement under this inital allocation to each firm (before trading occurs)?

Suggested answer: Remember, there is an 80 ton cap in the market. So, if firm A has 80 permits, they can emit 80 tons without recourse (and that's what they'll do initially), so they will not abate any tons of RX99. Firm B, on the other hand, doesn't have any permits so they can't emit any RX99. This means that they have to abate all 80 units of RX99, which is effectively "closing down" their operation. The total area under their MAC curve from 0 to 80 tons are firm B's costs, which can be calculated as: $0.5 \times 80 \times \$40 = \$1600$.

(c.) What is the initial permit price (i.e., what will firm B be willing to pay firm A for the *first* permit)?

Suggested answer: The initial permit price will be \$40. Because firm B's MAC curve equals \$40 at 0 tons of emissions, they would be willing to pay \$40 for the first permit. Firm A's MAC at 80 tons of emissions equals \$0. (Note: 0 tons for firm A is 80 tons for firm B.) So, firm A should be willing to pay firm B up to \$40 for the first permit, and firm B would happily accept anything greater than \$0.

(d.) Once trading occurs, describe (in words) what will happen in the market? How will permit prices and emissions from each firm evolve?

Suggested answer: Once trading occurs, firm B will continue to buy permits from firm A (moving right to left in the figure) so long as $MAC^B > MAC^A$. The permit price will get incrementally smaller with each additional permit bought/sold because, moving right to left, firm B's willingness-to-pay for permits becomes smaller and smaller. The firms will stop trading once they hit the point where $MAC^A = MAC^B$. Beyond that point, it would cost firm B more to buy an additional permit than it would to abate an additional unit of RX99.

(e.) What is the equilibrium price of permits? What are the aggregate costs of abatement at the equilibrium permit price?

Suggested answer: Once trading occurs and firms reach an equilibrium, it should be obvious that we've ended up in the same price-quantity equilibrium that we observed with the tax (i.e., in question 1.3.a and 1.3.b). The equilibrium price of the permits is equal to \$28.57, which is the location at which both MAC curves intersect. This is also the tax rate that we saw previously. Each firms' costs of abatement (and hence aggregate costs) are identical to those in question 1.3.c. So, aggregate abatement costs are $\$326.56 + \$816.25 = \$1142.81$. Comparing these costs with the aggregate costs with the initial allocation (i.e., \$1600 from 1.4.a), we can see that buying/selling permits decreases total costs by about \$450.

1.5 (15 pts – 5 pts for (a.), 10 pts for (b.))

The government now wants to decide whether to move forward with an emissions tax on RX99 or a cap-and-trade program. They ask you for some additional analysis to make this comparison.

(a.) What is the aggregate abatement cost curve for RX99 industry?

Suggested answer: To do this correctly is somewhat tricky. The aggregate abatement cost curve can be found by adding the two MACs horizontally. It's easier to do it graphically by just "shifting" MAC^B rightward until the x-axis intercepts with $x = 160$.

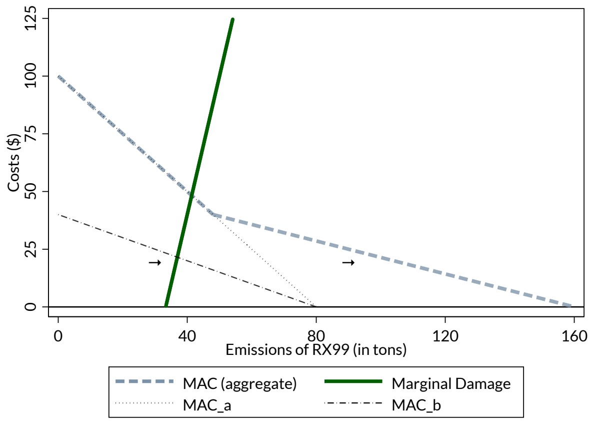
To do this quantitatively: first, solve each MAC curve for x:

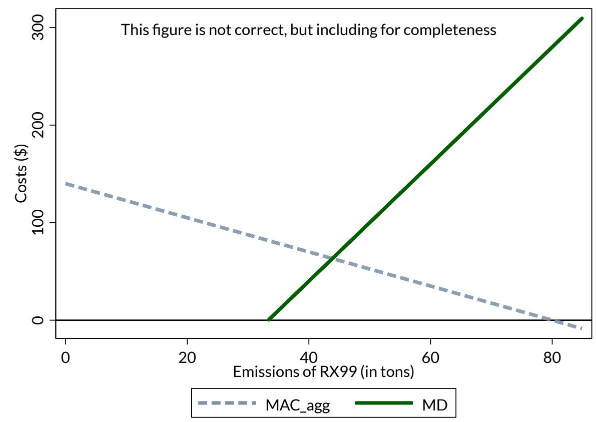
- $MAC^A = 100 - 1.25x^A \implies x^A = 80 - \frac{4/5}{M}AC^A$
- $MAC^b = 40 - \frac{x^B}{2} \implies x^B = 80 - 2MAC^B$
- Then, $x^{Agg} = x^A + x^B = 160 - 2.8MAC^{Agg}$
- Rearranging gives: $\frac{400}{7} - \frac{5}{14}x^{Agg.}$ – this function applies for values of $X \geq 48$ and ≤ 160.
- For values ≤ 48, $MAC^{Agg} = MAC^A = 100 - 1.25x^{Agg.}$.

Note: I've also given full credit if you added the two functions (since we didn't go over this in class in detail), although this approach is not fully accurate. This would look like: $MAC^A(x) + MAC^B(x) = 100 - 1.25x + 40 - 0.5x \implies MAC^{Agg}(x) = 140 - 1.75x$.

(b.) The EPA suspects that the marginal damages of RX99 emissions are given by the following equation: $MD(x) = 6x - 200$. This equation is upward sloping because x is units of *emissions* rather than units of *abatement*. (Note: You can think of marginal damages of emissions as the same thing as the marginal benefits of abatement.) Assuming that there is uncertainty (from the EPA's perspective) about the abatement cost curves for each firm, should the EPA prefer a tax or a cap-and-trade policy to reduce RX99 emissions?

Suggested answer:





In the figures above (doesn't matter which), the marginal damage function is plotted alongside the aggregate MAC function. As shown, the MD function is relatively steep compared to the MAC function. The slope of the MD function is 6. The (absolute value) of the MAC function is 1.25 at where it intersects with the MD function (or 1.75 on the graph on the right). So, because the MD (or marginal benefits) function is much steeper than the MAC function, we should prefer a quantity policy based on the Weitzman rule. You could demonstrate this graphically by showing that the welfare loss from a quantity policy is smaller than that of a price policy like we did in class, but the key is to recognize that with uncertainty about costs, we can determine which policy is better simply by comparing the steepness (or slopes) of the MD and MAC curves.