CS-2050-All-Sections CS 2050 Homework 5 (HOWARD, FAULKNER, ELLEN)

Vidit Dharmendra Pokharna

TOTAL POINTS

102.5 / 100

QUESTION 1

Question 1 15 pts

1.1 a **3 / 3**

✓ **- 0 pts** *Onto but not one-to-one*

- 3 pts Incorrect / No Answer

1.2 b **3 / 3**

✓ **- 0 pts** *Not a function*

- 3 pts Incorrect / No Answer

1.3 c **3 / 3**

✓ **- 0 pts** *Neither onto nor one-to-one*

- 3 pts Incorrect / No Answer

1.4 d **3 / 3**

✓ **- 0 pts** *Not a function*

- 3 pts Incorrect / No Answer

1.5 e **3 / 3**

✓ **- 0 pts** *Onto but not one-to-one*

- 3 pts Incorrect / No Answer

QUESTION 2

Question 2 9 pts

2.1 a **3 / 3**

✓ **- 0 pts** *1 quarter, 1 dime, 1 nickel, 4 pennies*

- 3 pts Incorrect / No Answer

2.2 b **3 / 3**

✓ **- 0 pts** *2 quarters, 2 dimes, 4 pennies*

- 3 pts Incorrect / No Answer

2.3 c **3 / 3**

✓ **- 0 pts** *3 quarters, 1 dime, 1 nickel, 3 pennies*

- 3 pts Incorrect / No Answer

QUESTION 3

Question 3 8 pts

3.1 a **4 / 4**

✓ **- 0 pts** *Provided valid example*

e.g. producing 14 cents

- 2 pts Valid example, no reasoning or proof

- 3 pts Tried to prove but gave invalid example

- 3 pts Tried to disprove but gave valid reasoning

- 4 pts Tired to disprove gave invalid reasoning

- 4 pts No Answer

3.2 b **4 / 4**

✓ **- 0 pts** *Provided valid counterexample*

e.g. producing 20 cents

- 2 pts Valid counterexample, no reasoning/proof

- 3 pts Tried to disprove but gave invalid counter example

- 3 pts Tried to prove, but provided valid reasoning

- 4 pts Tried to prove, but did not provide valid reasoning

- 4 pts No Answer

QUESTION 4

Question 4 8 pts

4.1 a **4 / 4**

✓ **- 0 pts** *True and provided logical explanation*

- 3 pts True but provided no explanation

Logical error

- 0.5 pts 1 Logical error

- 1 pts 2 Logical errors

- 1.5 pts 3+ logical errors

- 3 pts False, tried to provide explanation

- 4 pts False, no explanation

- 4 pts No Answer

4.2 b **4 / 4**

✓ **- 0 pts** *False, provided valid counter example*

- 3 pts False, but did not provide valid counter example

Logical Errors

- 0.5 pts 1 Logical Error

- 1 pts 2 Logical Errors

- 1.5 pts 3+ Logical Errors

- 3 pts True, tried to provide explanation

- 4 pts True, no explanation

- 4 pts No Answer

QUESTION 5

Question 5 20 pts

5.1 a **5 / 5**

✓ **- 0 pts** *$$f(x)$$ is $$O(g(x))$$ and $$g(x)$$ is $$O(f(x))$$*

- 2.5 pts Only $$f(x)$$ is $$O(g(x))$$

- 2.5 pts Only $$g(x)$$ is $$O(f(x))$$

- 5 pts Incorrect

5.2 b **5 / 5**

✓ **- 0 pts** *Only $$f(x)$$ is $$O(g(x))$$*

- 2.5 pts $$f(x)$$ is $$O(g(x))$$ and $$g(x)$$ is $$O(f(x))$$

- 5 pts Incorrect

5.3 c **5 / 5**

✓ **- 0 pts** *Only $$f(x)$$ is $$O(g(x))$$*

- 2.5 pts $$f(x)$$ is $$O(g(x))$$ and $$g(x)$$ is $$O(f(x))$$

- 5 pts Incorrect

5.4 d **5 / 5**

✓ **- 0 pts** *Only $$f(x)$$ is $$O(g(x))$$*

- 2.5 pts $$f(x)$$ is $$O(g(x))$$ and $$g(x)$$ is $$O(f(x))$$

- 5 pts Incorrect

QUESTION 6

6 Question 6 **10 / 10**

✓ **- 0 pts** *76, 72, 23, 72*

Missing/invalid/extra elements

- **2 pts** 1 missing/invalid/extra element
- **4 pts** 2 missing/invalid/extra element
- **6 pts** 3 missing/invalid/extra element
- **8 pts** 4+ missing/invalid/extra element

- **2.5 pts** Correct intermediate steps, no final answer
- **7 pts** Invalid or incorrect use of inequality operators
- **10 pts** No Answer

QUESTION 7

7 Question 7 10 / 10

✓ **- 0 pts** *Correct*

Invalid/missing term inequalities

- **2 pts** 1 invalid/missing term inequality
- **4 pts** 2 invalid/missing term inequality
- **6 pts** 3 invalid/missing term inequality
- **8 pts** 4 invalid/missing term inequality

Invalid/missing witnesses

- **2 pts** 1 Invalid/missing witnesses
- **4 pts** 2 Invalid/missing witnesses
- **6 pts** 3 Invalid/missing witnesses
- **8 pts** 4+ Invalid/missing witnesses

- **8 pts** Tried to disprove statement, but provided reasonable explanation
- **10 pts** Tried to disprove statement and did not provide reasonable explanation
- **10 pts** No Answer

QUESTION 8

Question 8 10 pts

8.1 a 5 / 5

✓ **- 0 pts** *Correct*

Invalid/missing term inequalities

- **0.5 pts** 1 invalid/missing term inequality
- **1 pts** 2 invalid/missing term inequality
- **1.5 pts** 3 invalid/missing term inequality
- **2 pts** 4 invalid/missing term inequality

Invalid/missing witnesses

- **1 pts** 1 Invalid/missing witnesses
- **2 pts** 2 Invalid/missing witnesses
- **3 pts** 3 Invalid/missing witnesses
- **4 pts** 4+ Invalid/missing witnesses

- **4 pts** Tried to disprove statement, but provided reasonable explanation
- **5 pts** Tried to disprove statement and did not provide reasonable explanation
- **5 pts** No Answer

8.2 b 5 / 5

✓ **- 0 pts** *Correct*

- **2.5 pts** Did not use proof by contradiction

Minor/major logic/math error

- **1 pts** 1 minor logical/math error
- **2 pts** 2+ minor logical/math error
- **3 pts** 1 major logical/math error
- **4 pts** 2+ major logical/math error

- **4 pts** Attempted to prove, but provided reasonable explanation
- **5 pts** Attempted to prove and did not provide reasonable explanation
- **5 pts** No Answer

1 you could have stopped here and mentioned

that numerator grows without bound

QUESTION 9

9 Question 9 **5 / 5**

✓ **- 0 pts** *$$O(n\log(n))$$*

- 5 pts Incorrect

QUESTION 10

10 Question 10 **5 / 5**

✓ **- 0 pts** *O(infinity) (though technically this is not completely correct)*

OR

infinite runtime

OR

cannot be analyzed because technically not an algorithm as it is an infinite loop due to def of Big-O

- 5 pts Incorrect

QUESTION 11

11 Matching **0 / 0**

✓ **- 0 pts** *Correct*

- 5 pts Incorrect

QUESTION 12

12 On Time **2.5 / 0**

✓ **+ 2.5 pts** *On Time (Before Thursday)*

- 0 pts On Time (Friday)

- 10 pts 1 day late

- 25 pts 2 days late

Vidit Pokharna

CS 2050 HW 5

1.
   a. Onto but not one-to-one
      i. $f(-1,1) = f(1,1) = 3$, so not one-to-one
   b. Not a function
      i. $y!$ is not defined for negative values of y
   c. Neither onto nor one-to-one
      i. $f(5,1) = f(6,1) = 14$, so not one-to-one
      ii. Not onto because there is no (x, y) for 1
   d. Not a function
      i. $y = 0$ does not map to any value, regardless of the value of x
   e. Onto but not one-to-one
      i. $f(5,5) = f(5,-5) = 155$, so not one-to-one

2.
   a. 1 quarter, 1 dime, 1 nickel, 4 pennies
   b. 2 quarters, 2 dimes, 4 pennies
   c. 3 quarters, 1 dime, 1 nickel, 3 pennies

3.
   a. True; 14 cents
      i. Without 14-cent coin: 1 dime, 4 pennies (5 coins)
      ii. With 14-cent coin: 1 14-cent (1 coin)
   b. False; 46 cents
      i. Without 14-cent coin: 1 quarter, 2 dimes, 1 penny (4 coins)
      ii. With 14-cent coin: 1 quarter, 1 14-cent, 1 nickel, 2 pennies (5 coins)

4.
   a. True; Given that both x and c are positive integers, and that $x + c < 5$, x must have a value less than 5. Therefore, both $\frac{x}{5}$ and $\frac{x+c}{5}$ must have values between 0 and 1. The floor function for a value between 0 and 1 must be 0. Both $\left\lfloor \frac{x}{5} \right\rfloor$ and $\left\lfloor \frac{x+c}{5} \right\rfloor$ must have the value of 0 and are equal to each other.
   b. False; $f(x) = \sqrt{x}, x \in \mathbb{R}^+$ and $g(x) = -|x|, x \in \mathbb{R}$. $f(g(x)) = \sqrt{-|x|}$, which would not be possible for any real numbers x. $g(f(x)) = -|\sqrt{x}|$ which is defined for all non-negative numbers. Therefore, the given statement is inaccurate, where $f(g(x))$ can be undefined.

1.1 a **3 / 3**

✓ **- 0 pts** *Onto but not one-to-one*

- 3 pts Incorrect / No Answer

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1.2 b **3 / 3**

✓ **- 0 pts** *Not a function*

- 3 pts Incorrect / No Answer

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1.3 **C** **3 / 3**

✓ **- 0 pts** *Neither onto nor one-to-one*

- 3 pts Incorrect / No Answer

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1.4 d **3 / 3**

✓ **- 0 pts** *Not a function*

- 3 pts Incorrect / No Answer

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1.5 e **3 / 3**

✓ **- 0 pts** *Onto but not one-to-one*

- 3 pts Incorrect / No Answer

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2.1 a **3 / 3**

✓ **- 0 pts** *1 quarter, 1 dime, 1 nickel, 4 pennies*

- 3 pts Incorrect / No Answer

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2.2 b **3 / 3**

✓ **- 0 pts** *2 quarters, 2 dimes, 4 pennies*

- 3 pts Incorrect / No Answer

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2.3 **C** **3 / 3**

✓ **- 0 pts** *3 quarters, 1 dime, 1 nickel, 3 pennies*

- 3 pts Incorrect / No Answer

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      ii. With 14-cent coin: 1 quarter, 1 14-cent, 1 nickel, 2 pennies (5 coins)

4.
   a. True; Given that both x and c are positive integers, and that $x + c < 5$, x must have a value less than 5. Therefore, both $\frac{x}{5}$ and $\frac{x+c}{5}$ must have values between 0 and 1. The floor function for a value between 0 and 1 must be 0. Both $\left\lfloor \frac{x}{5} \right\rfloor$ and $\left\lfloor \frac{x+c}{5} \right\rfloor$ must have the value of 0 and are equal to each other.
   b. False; $f(x) = \sqrt{x}, x \in \mathbb{R}^+$ and $g(x) = -|x|, x \in \mathbb{R}$. $f\big(g(x)\big) = \sqrt{-|x|}$, which would not be possible for any real numbers x. $g\big(f(x)\big) = -|\sqrt{x}|$ which is defined for all non-negative numbers. Therefore, the given statement is inaccurate, where $f\big(g(x)\big)$ can be undefined.

3.1 a **4 / 4**

✓ **- 0 pts** *Provided valid example*

e.g. producing 14 cents

- 2 pts Valid example, no reasoning or proof

- 3 pts Tried to prove but gave invalid example

- 3 pts Tried to disprove but gave valid reasoning

- 4 pts Tired to disprove gave invalid reasoning

- 4 pts No Answer

Vidit Pokharna

CS 2050 HW 5

1.
   a. Onto but not one-to-one
      i. $f(-1,1) = f(1,1) = 3$, so not one-to-one
   b. Not a function
      i. $y!$ is not defined for negative values of y
   c. Neither onto nor one-to-one
      i. $f(5,1) = f(6,1) = 14$, so not one-to-one
      ii. Not onto because there is no (x, y) for 1
   d. Not a function
      i. $y = 0$ does not map to any value, regardless of the value of x
   e. Onto but not one-to-one
      i. $f(5,5) = f(5,-5) = 155$, so not one-to-one

2.
   a. 1 quarter, 1 dime, 1 nickel, 4 pennies
   b. 2 quarters, 2 dimes, 4 pennies
   c. 3 quarters, 1 dime, 1 nickel, 3 pennies

3.
   a. True; 14 cents
      i. Without 14-cent coin: 1 dime, 4 pennies (5 coins)
      ii. With 14-cent coin: 1 14-cent (1 coin)
   b. False; 46 cents
      i. Without 14-cent coin: 1 quarter, 2 dimes, 1 penny (4 coins)
      ii. With 14-cent coin: 1 quarter, 1 14-cent, 1 nickel, 2 pennies (5 coins)

4.
   a. True; Given that both x and c are positive integers, and that $x + c < 5$, x must have a value less than 5. Therefore, both $\frac{x}{5}$ and $\frac{x+c}{5}$ must have values between 0 and 1. The floor function for a value between 0 and 1 must be 0. Both $\left\lfloor \frac{x}{5} \right\rfloor$ and $\left\lfloor \frac{x+c}{5} \right\rfloor$ must have the value of 0 and are equal to each other.
   b. False; $f(x) = \sqrt{x}, x \in \mathbb{R}^+$ and $g(x) = -|x|, x \in \mathbb{R}$. $f(g(x)) = \sqrt{-|x|}$, which would not be possible for any real numbers x. $g(f(x)) = -|\sqrt{x}|$ which is defined for all non-negative numbers. Therefore, the given statement is inaccurate, where $f(g(x))$ can be undefined.

3.2 b **4 / 4**

✓ **- 0 pts** *Provided valid counterexample*

e.g. producing 20 cents

- 2 pts Valid counterexample, no reasoning/proof

- 3 pts Tried to disprove but gave invalid counter example

- 3 pts Tried to prove, but provided valid reasoning

- 4 pts Tried to prove, but did not provide valid reasoning

- 4 pts No Answer

Vidit Pokharna

CS 2050 HW 5

1.
   a. Onto but not one-to-one
      i. $f(-1,1) = f(1,1) = 3$, so not one-to-one
   b. Not a function
      i. $y!$ is not defined for negative values of y
   c. Neither onto nor one-to-one
      i. $f(5,1) = f(6,1) = 14$, so not one-to-one
      ii. Not onto because there is no (x, y) for 1
   d. Not a function
      i. $y = 0$ does not map to any value, regardless of the value of x
   e. Onto but not one-to-one
      i. $f(5,5) = f(5,-5) = 155$, so not one-to-one

2.
   a. 1 quarter, 1 dime, 1 nickel, 4 pennies
   b. 2 quarters, 2 dimes, 4 pennies
   c. 3 quarters, 1 dime, 1 nickel, 3 pennies

3.
   a. True; 14 cents
      i. Without 14-cent coin: 1 dime, 4 pennies (5 coins)
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   b. False; 46 cents
      i. Without 14-cent coin: 1 quarter, 2 dimes, 1 penny (4 coins)
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4.
   a. True; Given that both x and c are positive integers, and that $x + c < 5$, x must have a value less than 5. Therefore, both $\frac{x}{5}$ and $\frac{x+c}{5}$ must have values between 0 and 1. The floor function for a value between 0 and 1 must be 0. Both $\left\lfloor \frac{x}{5} \right\rfloor$ and $\left\lfloor \frac{x+c}{5} \right\rfloor$ must have the value of 0 and are equal to each other.
   b. False; $f(x) = \sqrt{x}, x \in \mathbb{R}^+$ and $g(x) = -|x|, x \in \mathbb{R}$. $f\big(g(x)\big) = \sqrt{-|x|}$, which would not be possible for any real numbers x. $g\big(f(x)\big) = -|\sqrt{x}|$ which is defined for all non-negative numbers. Therefore, the given statement is inaccurate, where $f\big(g(x)\big)$ can be undefined.

4.1 a **4 / 4**

✓ **- 0 pts** *True and provided logical explanation*

- 3 pts True but provided no explanation

Logical error

- 0.5 pts 1 Logical error

- 1 pts 2 Logical errors

- 1.5 pts 3+ logical errors

- 3 pts False, tried to provide explanation

- 4 pts False, no explanation

- 4 pts No Answer

Vidit Pokharna

CS 2050 HW 5

1.
   a. Onto but not one-to-one
      i. $f(-1,1) = f(1,1) = 3$, so not one-to-one
   b. Not a function
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   d. Not a function
      i. $y = 0$ does not map to any value, regardless of the value of x
   e. Onto but not one-to-one
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      ii. With 14-cent coin: 1 14-cent (1 coin)
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      i. Without 14-cent coin: 1 quarter, 2 dimes, 1 penny (4 coins)
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   b. False; $f(x) = \sqrt{x}, x \in \mathbb{R}^+$ and $g(x) = -|x|, x \in \mathbb{R}$. $f(g(x)) = \sqrt{-|x|}$, which would not be possible for any real numbers x. $g(f(x)) = -|\sqrt{x}|$ which is defined for all non-negative numbers. Therefore, the given statement is inaccurate, where $f(g(x))$ can be undefined.

4.2 b **4 / 4**

✓ **- 0 pts** *False, provided valid counter example*

- 3 pts False, but did not provide valid counter example

Logical Errors

- 0.5 pts 1 Logical Error

- 1 pts 2 Logical Errors

- 1.5 pts 3+ Logical Errors

- 3 pts True, tried to provide explanation

- 4 pts True, no explanation

- 4 pts No Answer

5.

a. $f(x)$ is $O(g(x))$ and $g(x)$ is $O(f(x))$
   i. $f(x) \leq 4g(x), \forall x > 1$
   ii. $g(x) \leq f(x), \forall x > 1$
b. $f(x)$ is $O(g(x))$
   i. $f(x) \leq g(x), \forall x > 1$
   ii. There are no groups of witnesses c, k making the following true: $g(x) \leq Cf(x), \forall x > k$
c. $f(x)$ is $O(g(x))$
   i. $f(x) \leq g(x), \forall x > 3$
   ii. There are no groups of witnesses c, k making the following true: $g(x) \leq Cf(x), \forall x > k$
d. $f(x)$ is $O(g(x))$
   i. There are no groups of witnesses c, k making the following true: $g(x) \leq Cf(x), \forall x > k$
   ii. $f(x) \leq g(x), \forall x > 2$

6.

a. Low = 1, High = 10, Mid = 5
   i. $30 < 76$
b. Low = 1, High = 5, Mid = 3
   i. $30 < 72$
c. Low = 1, High = 3, Mid = 2
   i. $30 > 23$
d. Low = 3, High = 3, Mid = 3
   i. $30 \neq 72$
e. Return 0 since value is not found

All values compared: $\{76,72,23,72\}$

5.1 a **5 / 5**

✓ **- 0 pts** *$f(x)$ is $O(g(x))$ and $g(x)$ is $O(f(x))$*

- 2.5 pts Only $f(x)$ is $O(g(x))$

- 2.5 pts Only $g(x)$ is $O(f(x))$

- 5 pts Incorrect

5.

a. $f(x)$ is $O(g(x))$ and $g(x)$ is $O(f(x))$
   i. $f(x) \leq 4g(x), \forall x > 1$
   ii. $g(x) \leq f(x), \forall x > 1$
b. $f(x)$ is $O(g(x))$
   i. $f(x) \leq g(x), \forall x > 1$
   ii. There are no groups of witnesses c, k making the following true: $g(x) \leq Cf(x), \forall x > k$
c. $f(x)$ is $O(g(x))$
   i. $f(x) \leq g(x), \forall x > 3$
   ii. There are no groups of witnesses c, k making the following true: $g(x) \leq Cf(x), \forall x > k$
d. $f(x)$ is $O(g(x))$
   i. There are no groups of witnesses c, k making the following true: $g(x) \leq Cf(x), \forall x > k$
   ii. $f(x) \leq g(x), \forall x > 2$

6.

a. Low = 1, High = 10, Mid = 5
   i. $30 < 76$
b. Low = 1, High = 5, Mid = 3
   i. $30 < 72$
c. Low = 1, High = 3, Mid = 2
   i. $30 > 23$
d. Low = 3, High = 3, Mid = 3
   i. $30 \neq 72$
e. Return 0 since value is not found

All values compared: $\{76,72,23,72\}$

5.2 b **5 / 5**

✓ **- 0 pts** *Only $$f(x)$$ is $$O(g(x))$$*

- 2.5 pts $$f(x)$$ is $$O(g(x))$$ and $$g(x)$$ is $$O(f(x))$$

- 5 pts Incorrect

5.

a. $f(x)$ is $O(g(x))$ and $g(x)$ is $O(f(x))$
   i. $f(x) \leq 4g(x), \forall x > 1$
   ii. $g(x) \leq f(x), \forall x > 1$
b. $f(x)$ is $O(g(x))$
   i. $f(x) \leq g(x), \forall x > 1$
   ii. There are no groups of witnesses c, k making the following true: $g(x) \leq Cf(x), \forall x > k$
c. $f(x)$ is $O(g(x))$
   i. $f(x) \leq g(x), \forall x > 3$
   ii. There are no groups of witnesses c, k making the following true: $g(x) \leq Cf(x), \forall x > k$
d. $f(x)$ is $O(g(x))$
   i. There are no groups of witnesses c, k making the following true: $g(x) \leq Cf(x), \forall x > k$
   ii. $f(x) \leq g(x), \forall x > 2$

6.

a. Low = 1, High = 10, Mid = 5
   i. $30 < 76$
b. Low = 1, High = 5, Mid = 3
   i. $30 < 72$
c. Low = 1, High = 3, Mid = 2
   i. $30 > 23$
d. Low = 3, High = 3, Mid = 3
   i. $30 \neq 72$
e. Return 0 since value is not found

All values compared: $\{76,72,23,72\}$

5.3 **C** **5 / 5**

✓ **- 0 pts** *Only $$f(x)$$ is $$O(g(x))$$*

- 2.5 pts $$f(x)$$ is $$O(g(x))$$ and $$g(x)$$ is $$O(f(x))$$

- 5 pts Incorrect

5.

a. $f(x)$ is $O(g(x))$ and $g(x)$ is $O(f(x))$
   i. $f(x) \leq 4g(x), \forall x > 1$
   ii. $g(x) \leq f(x), \forall x > 1$
b. $f(x)$ is $O(g(x))$
   i. $f(x) \leq g(x), \forall x > 1$
   ii. There are no groups of witnesses c, k making the following true: $g(x) \leq Cf(x), \forall x > k$
c. $f(x)$ is $O(g(x))$
   i. $f(x) \leq g(x), \forall x > 3$
   ii. There are no groups of witnesses c, k making the following true: $g(x) \leq Cf(x), \forall x > k$
d. $f(x)$ is $O(g(x))$
   i. There are no groups of witnesses c, k making the following true: $g(x) \leq Cf(x), \forall x > k$
   ii. $f(x) \leq g(x), \forall x > 2$

6.

a. Low = 1, High = 10, Mid = 5
   i. $30 < 76$
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   i. $30 < 72$
c. Low = 1, High = 3, Mid = 2
   i. $30 > 23$
d. Low = 3, High = 3, Mid = 3
   i. $30 \neq 72$
e. Return 0 since value is not found

All values compared: $\{76,72,23,72\}$

5.4 d **5 / 5**

✓ **- 0 pts** *Only $f(x)$ is $O(g(x))$*

- 2.5 pts $f(x)$ is $O(g(x))$ and $g(x)$ is $O(f(x))$

- 5 pts Incorrect

5.

a. $f(x)$ is $O(g(x))$ and $g(x)$ is $O(f(x))$
    i. $f(x) \leq 4g(x), \forall x > 1$
    ii. $g(x) \leq f(x), \forall x > 1$
b. $f(x)$ is $O(g(x))$
    i. $f(x) \leq g(x), \forall x > 1$
    ii. There are no groups of witnesses c, k making the following true: $g(x) \leq Cf(x), \forall x > k$
c. $f(x)$ is $O(g(x))$
    i. $f(x) \leq g(x), \forall x > 3$
    ii. There are no groups of witnesses c, k making the following true: $g(x) \leq Cf(x), \forall x > k$
d. $f(x)$ is $O(g(x))$
    i. There are no groups of witnesses c, k making the following true: $g(x) \leq Cf(x), \forall x > k$
    ii. $f(x) \leq g(x), \forall x > 2$

6.

a. Low = 1, High = 10, Mid = 5
    i. $30 < 76$
b. Low = 1, High = 5, Mid = 3
    i. $30 < 72$
c. Low = 1, High = 3, Mid = 2
    i. $30 > 23$
d. Low = 3, High = 3, Mid = 3
    i. $30 \neq 72$
e. Return 0 since value is not found

All values compared: $\{76,72,23,72\}$

6 Question 6 **10 / 10**

✓ **- 0 pts** *76, 72, 23, 72*

Missing/invalid/extra elements

- **- 2 pts** 1 missing/invalid/extra element
- **- 4 pts** 2 missing/invalid/extra element
- **- 6 pts** 3 missing/invalid/extra element
- **- 8 pts** 4+ missing/invalid/extra element

- **- 2.5 pts** Correct intermediate steps, no final answer
- **- 7 pts** Invalid or incorrect use of inequality operators
- **- 10 pts** No Answer

7.

$f(\mathrm{x}) = \log(x)\left(2x^4 + x^3 + 3x + \frac{9}{x}\right)$, where $f: R \to R$
$f_1(\mathrm{x}) = \log(x)$, where $f: R \to R$
$f_2(\mathrm{x}) = \left(2x^4 + x^3 + 3x + \frac{9}{x}\right)$, where $f: R \to R$
$f(x) = f_1(\mathrm{x}) \cdot f_2(\mathrm{x})$

I will proceed with direct proof. I will find values for witnesses for all terms of $f(\mathrm{x}) = \log(x)\left(2x^4 + x^3 + 3x + \frac{9}{x}\right)$ are $O(x^5)$.

$f_1(\mathrm{x})$ is $O(x)$ because

$\log(x) \leq x$	$\forall x > 1$

Then $\log(x) \leq x, \forall x > 1$ with witnesses $c = 1, k = 1$ so $f_1(\mathrm{x})$ is $O(x)$

$f_2(\mathrm{x})$ is $O(x^4)$ because

$2x^4 \leq 2x^4$	$\forall x > 1$
$x^3 \leq x^4$	$\forall x > 1$
$3\mathrm{x} \leq x^4$	$\forall x > 1$
$\frac{9}{x} \leq x^4$	$\forall x > 1$

Then $2x^4 + x^3 + 3x + \frac{9}{x} \leq 5x^4, \forall x > 1$ with witnesses $c = 5, k = 1$ so $f_2(\mathrm{x})$ is $O(x^4)$

Since $f_1(\mathrm{x})$ is $O(x)$ and $f_2(\mathrm{x})$ is $O(x^4)$ and $f(x) = f_1(\mathrm{x}) \cdot f_2(\mathrm{x})$, $f(x)$ is $O(x \cdot x^4) = O(x^5)$ by theorem 3.2.3.

8.

a.

$f(x) = 2x^2 \log(x^3)$, where $f: R \to R$
$g(x) = 3x^3$, where $g: R \to R$

I will proceed with direct proof. I will find values for witnesses c, k to show that $2x^2 \log(x^3)$ is $O(3x^3)$.

1	$2x^2 \log(x^3) \leq 3x^3$	$\forall x > 1$

$2x^2 \log(x^3) \leq 3x^3, \forall x > 1$
Thus, when $x = 2$ and $c = 1$ as witnesses, $2x^2 \log(x^3)$ is $O(3x^3)$ by the definition of big-O.

7 Question 7 10 / 10

✓ **- 0 pts** *Correct*

Invalid/missing term inequalities

- **- 2 pts** 1 invalid/missing term inequality
- **- 4 pts** 2 invalid/missing term inequality
- **- 6 pts** 3 invalid/missing term inequality
- **- 8 pts** 4 invalid/missing term inequality

Invalid/missing witnesses

- **- 2 pts** 1 Invalid/missing witnesses
- **- 4 pts** 2 Invalid/missing witnesses
- **- 6 pts** 3 Invalid/missing witnesses
- **- 8 pts** 4+ Invalid/missing witnesses

- **- 8 pts** Tried to disprove statement, but provided reasonable explanation
- **- 10 pts** Tried to disprove statement and did not provide reasonable explanation
- **- 10 pts** No Answer

7.

$f(\mathrm{x}) = \log(x)\left(2x^4 + x^3 + 3x + \frac{9}{x}\right)$, where $f: R \to R$
$f_1(\mathrm{x}) = \log(x)$, where $f: R \to R$
$f_2(\mathrm{x}) = \left(2x^4 + x^3 + 3x + \frac{9}{x}\right)$, where $f: R \to R$
$f(x) = f_1(\mathrm{x}) \cdot f_2(\mathrm{x})$

I will proceed with direct proof. I will find values for witnesses for all terms of $f(\mathrm{x}) = \log(x)\left(2x^4 + x^3 + 3x + \frac{9}{x}\right)$ are $O(x^5)$.

$f_1(\mathrm{x})$ is $O(x)$ because

$\log(x) \le x$	$\forall x > 1$

Then $\log(x) \le x, \forall x > 1$ with witnesses $c = 1, k = 1$ so $f_1(\mathrm{x})$ is $O(x)$

$f_2(\mathrm{x})$ is $O(x^4)$ because

$2x^4 \le 2x^4$	$\forall x > 1$
$x^3 \le x^4$	$\forall x > 1$
$3\mathrm{x} \le x^4$	$\forall x > 1$
$\frac{9}{x} \le x^4$	$\forall x > 1$

Then $2x^4 + x^3 + 3x + \frac{9}{x} \le 5x^4, \forall x > 1$ with witnesses $c = 5, k = 1$ so $f_2(\mathrm{x})$ is $O(x^4)$

Since $f_1(\mathrm{x})$ is $O(x)$ and $f_2(\mathrm{x})$ is $O(x^4)$ and $f(x) = f_1(\mathrm{x}) \cdot f_2(\mathrm{x})$, $f(x)$ is $O(x \cdot x^4) = O(x^5)$ by theorem 3.2.3.

8.

a.

$f(x) = 2x^2 \log(x^3)$, where $f: R \to R$
$g(x) = 3x^3$, where $g: R \to R$

I will proceed with direct proof. I will find values for witnesses c, k to show that $2x^2 \log(x^3)$ is $O(3x^3)$.

1	$2x^2 \log(x^3) \le 3x^3$	$\forall x > 1$

$2x^2 \log(x^3) \le 3x^3, \forall x > 1$
Thus, when $x = 2$ and $c = 1$ as witnesses, $2x^2 \log(x^3)$ is $O(3x^3)$ by the definition of big-O.

8.1 a **5 / 5**

✓ **- 0 pts** *Correct*

Invalid/missing term inequalities

- **- 0.5 pts** 1 invalid/missing term inequality
- **- 1 pts** 2 invalid/missing term inequality
- **- 1.5 pts** 3 invalid/missing term inequality
- **- 2 pts** 4 invalid/missing term inequality

Invalid/missing witnesses

- **- 1 pts** 1 Invalid/missing witnesses
- **- 2 pts** 2 Invalid/missing witnesses
- **- 3 pts** 3 Invalid/missing witnesses
- **- 4 pts** 4+ Invalid/missing witnesses

- **- 4 pts** Tried to disprove statement, but provided reasonable explanation
- **- 5 pts** Tried to disprove statement and did not provide reasonable explanation
- **- 5 pts** No Answer

b.

$f(x) = 3x^3$, where $f: R \to R$
$g(x) = (2x^2 \log(x^3))$, where $g: R \to R$

I proceed by using a proof by contradiction and assume $3x^3$ is $O(2x^2 \log(x^3))$. By definition of big-O, there exists constant real numbers c, k assuming that the following statement is true: $3x^3 \le C[2x^2 \log(x^3)], \forall x > k$.

1	$3x^3 \le C[2x^2 \log(x^3)], \forall x > k$	Given
2	$\frac{3}{2}x \le C[\log(x^3)]$	Simplify (1) by dividing by $2x^2$ on both sides
3	$\frac{\frac{3}{2}x}{C[\log(x^3)]} \le 1$	Simplify (2) by dividing by $C[\log(x^3)]$ on both sides
4	$\lim_{x\to\infty} \frac{\frac{3}{2}x}{C[\log(x^3)]} \le \lim_{x\to\infty} 1$	Take limit of both sides of (4)
5	$\lim_{x\to\infty} \frac{\frac{3}{2}x}{C[\log(x^3)]} \le \lim_{x\to\infty} 1 = 1$	Limit of 1 for any x is 1
6	$\lim_{x\to\infty} \frac{3}{2}x = \infty$	Limit of numerator in (5)
7	$\lim_{x\to\infty} C[\log(x^3)] = \infty$	Limit of denominator in (5)
8	$\lim_{x\to\infty} \frac{\frac{3}{2}x}{C[\log(x^3)]} = \lim_{x\to\infty} \frac{\infty}{\infty} \le 1$	Substituting (6) and (7) into (5)
9	$\lim_{x\to\infty} \frac{\frac{3}{2}x}{C[\log(x^3)]} = \lim_{x\to\infty} \frac{\infty}{\infty} \le 1$	Use of L'Hospitals rule due to (8) being undefined
10	$\frac{d(\frac{3}{2}x)}{dx} = \frac{3}{2}$	Derivative of numerator in left hand side in (9)
11	$\frac{d(C[\log(x^3)])}{dx} = \frac{C \cdot 3x^2}{x^3} = \frac{3C}{x}$	Derivative of denominator in left hand side in (9)
12	$\lim_{x\to\infty} \frac{\frac{3}{2}x}{C[\log(x^3)]} \to \lim_{x\to\infty} \frac{\frac{3}{2}}{\frac{3C}{x}} \le 1$	Substituting (10) and (11) into (9)
13	$\lim_{x\to\infty} \frac{x}{2c} \le 1$	Simplifying (12)

14	$\lim_{x\to\infty} \frac{x}{2c} = \infty \leq 1$	Taking limit of left hand side of (12)
15	$\infty \leq 1$	Contradiction

As shown by line 15, we are led to $\infty \leq 1$, which is a contradictory statement. Since we got to this conclusion by assuming that $3x^3$ is $\mathrm{O}(2x^2 \log(x^3))$, our assumption must be incorrect. Therefore, $3x^3$ is not $\mathrm{O}(2x^2 \log(x^3))$ and there exists no c, k as witnesses that make the following statement true: $3x^3 \leq \mathrm{C}(2x^2 \log(x^3)), \forall x > k$.

9.

- The first while loop has a time complexity of $\log(n)$
- The first for loop has a time complexity of n
- The second while loop has a time complexity of $\log(n)$
- Since the first for loop is embedded in the first while loop, their time complexity is $n\log(n)$
- Adding the first for and while loop to the second while loop gives $n\log(n) + \log(n)$
- The total time complexity would be $O(n\log(n))$ as $n\log(n) \leq n\log(n), \forall x > 1$ and $\log(n) \leq n\log(n), \forall x > 1$. Therefore, $n\log(n) + \log(n) \leq 2n\log(n), \forall x > 1$. Having witnesses $c = 2$ and $k = 1$, we can conclude $n\log(n) + \log(n)$ gives a time complexity of $O(n\log(n))$.

10. This would run infinitely as the condition for the outer while loop has a condition that will never terminate as x will always be less than $x + 10$, and therefore it is not an algorithm as it contains an infinite loop.

8.2 b **5 / 5**

✓ **- 0 pts** *Correct*

- 2.5 pts Did not use proof by contradiction

Minor/major logic/math error

- 1 pts 1 minor logical/math error

- 2 pts 2+ minor logical/math error

- 3 pts 1 major logical/math error

- 4 pts 2+ major logical/math error

- 4 pts Attempted to prove, but provided reasonable explanation

- 5 pts Attempted to prove and did not provide reasonable explanation

- 5 pts No Answer

1 you could have stopped here and mentioned that numerator grows without bound

14	$\lim_{x\to\infty} \frac{x}{2c} = \infty \leq 1$	Taking limit of left hand side of (12)
15	$\infty \leq 1$	Contradiction

As shown by line 15, we are led to $\infty \leq 1$, which is a contradictory statement. Since we got to this conclusion by assuming that $3x^3$ is $\mathrm{O}(2x^2 \log(x^3))$, our assumption must be incorrect. Therefore, $3x^3$ is not $\mathrm{O}(2x^2 \log(x^3))$ and there exists no c, k as witnesses that make the following statement true: $3x^3 \leq \mathrm{C}(2x^2 \log(x^3)), \forall x > k$.

9.

- The first while loop has a time complexity of $\log(n)$
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- The total time complexity would be $O(n\log(n))$ as $n\log(n) \leq n\log(n), \forall x > 1$ and $\log(n) \leq n\log(n), \forall x > 1$. Therefore, $n\log(n) + \log(n) \leq 2n\log(n), \forall x > 1$. Having witnesses $c = 2$ and $k = 1$, we can conclude $n\log(n) + \log(n)$ gives a time complexity of $O(n\log(n))$.

10. This would run infinitely as the condition for the outer while loop has a condition that will never terminate as x will always be less than $x + 10$, and therefore it is not an algorithm as it contains an infinite loop.

9 Question 9 **5 / 5**

✓ **- 0 pts** *$$O(n\log(n))$$*

- 5 pts Incorrect

14	$\lim_{x\to\infty} \frac{x}{2c} = \infty \leq 1$	Taking limit of left hand side of (12)
15	$\infty \leq 1$	Contradiction

As shown by line 15, we are led to $\infty \leq 1$, which is a contradictory statement. Since we got to this conclusion by assuming that $3x^3$ is $O(2x^2 \log(x^3))$, our assumption must be incorrect. Therefore, $3x^3$ is not $O(2x^2 \log(x^3))$ and there exists no c, k as witnesses that make the following statement true: $3x^3 \leq C(2x^2 \log(x^3)), \forall x > k$.

9.

- The first while loop has a time complexity of $\log(n)$
- The first for loop has a time complexity of n
- The second while loop has a time complexity of $\log(n)$
- Since the first for loop is embedded in the first while loop, their time complexity is $n\log(n)$
- Adding the first for and while loop to the second while loop gives $n\log(n) + \log(n)$
- The total time complexity would be $O(n\log(n))$ as $n\log(n) \leq n\log(n), \forall x > 1$ and $\log(n) \leq n\log(n), \forall x > 1$. Therefore, $n\log(n) + \log(n) \leq 2n\log(n), \forall x > 1$. Having witnesses $c = 2$ and $k = 1$, we can conclude $n\log(n) + \log(n)$ gives a time complexity of $O(n\log(n))$.

10. This would run infinitely as the condition for the outer while loop has a condition that will never terminate as x will always be less than $x + 10$, and therefore it is not an algorithm as it contains an infinite loop.

10 Question 10 **5 / 5**

✓ **- 0 pts** *O(infinity) (though technically this is not completely correct)*

OR

infinite runtime

OR

cannot be analyzed because technically not an algorithm as it is an infinite loop due to def of Big-O

- 5 pts Incorrect

11 Matching **0 / 0**

✓ **- 0 pts** *Correct*

- 5 pts Incorrect

12 On Time **2.5 / 0**

✓ **+ 2.5 pts** *On Time (Before Thursday)*

- 0 pts On Time (Friday)

- 10 pts 1 day late

- 25 pts 2 days late