MATH-3012-D Quiz 2

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TOTAL POINTS

30 / 30

QUESTION 1

1 Q1 **15 / 15**

✓ **+ 15 pts** *Correct*

QUESTION 2

2 Q2 **15 / 15**

✓ **+ 15 pts** *Correct*

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For generating function questions, answers should not have infinite sum.

Q1. For (a)-(c) find the generating functions for the given sequences (all beginning with a_0)

(a) $2, -2, 2, -2, 2, -2, \ldots$

(b) $0, 0, 0, 1, 3, 3, 3^2, 3^3, 3^4, 3^5 \ldots$

(c) the sequence $(a_n)_{n=0}^{\infty}$ defined by $a_n = 2\binom{-5}{n} + 5$. $\rightarrow 2(1+x)^{-5}$
(Remember the binomial formula!)

(d) What is the generating function for the number of partitions of n into summands where each summand cannot occur more than 7 times.

(a) $2 - 2x + 2x^2 - 2x^3 + \ldots \Rightarrow \boxed{f(x) = \frac{2}{1+x}}$

(b) $x^3 + 3x^4 + 9x^5 + 27x^6 + \ldots \Rightarrow \boxed{f(x) = \frac{x^3}{1-3x}}$

(c) $2\sum\binom{-5}{n}x^n + 5\sum 1x^n = 2(1+x)^{-5} + 5(1-x)^{-1} \Rightarrow \boxed{f(x) = \frac{2}{(1+x)^5} + \frac{5}{(1-x)}}$

(d) $x^{0\cdot1} + x^{1\cdot1} + x^{2\cdot1} + x^{3\cdot1} + x^{4\cdot1} + x^{5\cdot1} + x^{6\cdot1} + x^{7\cdot1} = \frac{1-x^8}{1-x}$

$x^{0\cdot2} + \cdots + x^{7\cdot2} = \frac{1-x^{8\cdot2}}{1-x^2}$

$$\boxed{f(x) = \prod_{i=1}^{\infty} \frac{1-x^{8i}}{1-x^i}}$$

1 Q1 **15 / 15**

✓ **+ 15 pts** *Correct*

Q2. (a) Let a_n be the number of ways to give n identical candies to 3 kids A, B, and C such that B gets an odd number of candies, and C gets at most 49 candies. Find the generating function for the sequence (a_n). The answer should be in the form of a quotient of two polynomials, where each is a product of a few simple polynomials. (Here a polynomial is simple if it is a sum of no more than 2 monomials, like $2x^2 - 3x^5$)

(b) Find Coeff(x^9) in $\frac{(1+x^5)^8}{(1-x)^5}$. Answer should be a sum of a few binomial coefficients, like $\binom{7}{3} + 5\binom{8}{4} - 7\binom{9}{6}$. No negative binomial like $\binom{-7}{9}$ is allowed.

(a) $f_A = x^0 + x^1 + x^2 + \cdots = \frac{1}{1-x}$

$f_B = x^1 + x^3 + x^5 + \cdots = \frac{x}{1-x^2}$

$f_C = x^0 + x^1 + x^2 + \cdots + x^{48} + x^{49} = \frac{1-x^{50}}{1-x}$

$$\boxed{f(x) = \frac{x(1-x^{50})}{(1-x)^2(1-x^2)}}$$

(b) $(1+x^5)^8 = 1 + \binom{8}{1}x^5 + \binom{8}{2}x^{10} + \cdots$

$\frac{1}{(1-x)^5} = \sum \binom{n+5-1}{n} x^n$

HOF

$f(x) = \sum \binom{n+5-1}{n} x^n + \sum \binom{n+5-1}{n}\binom{8}{1} x^{5+n} + \cdots$

$= \binom{9+5-1}{9} x^9 + \binom{4+5-1}{4}\binom{8}{1} x^9$

$= \binom{13}{9} + \binom{8}{1}\binom{8}{4} = \boxed{\binom{13}{9} + 8\binom{8}{4}}$

2 Q2 **15 / 15**

✓ **+ 15 pts** *Correct*