MATH1564 K – Linear Algebra with Abstract Vector Spaces
Homework 8

Due 11/7, submit to both Canvas-Assignment and Gradescope

1. Using a determinant, find the values of a for which the following 3 vectors form a basis for $\mathbb{R}^3$.
$$\begin{pmatrix} a-1 \\ -3 \\ -6 \end{pmatrix}, \begin{pmatrix} 3 \\ a+5 \\ 6 \end{pmatrix}, \begin{pmatrix} -3 \\ -3 \\ a-4 \end{pmatrix}$$

2. Suppose
$$\begin{vmatrix} a & x & l \\ b & y & m \\ c & z & n \end{vmatrix} = 2,$$
Find
$$\begin{vmatrix} 3a-2x & 3b-2y & 3c-2z \\ 2l+x & 2m+y & 2n+z \\ 7l & 7m & 7n \end{vmatrix}.$$

3. Let A and B be two $n \times n$ matrices. Decide each of the following statement is True or False. If it is true, give a proof; if it is false, give a counter example.
 (a) $\det(A+B) = \det(A) + \det(B)$.
 (b) $\det(\lambda A) = \lambda \det(A)$.
 (c) $\det(\lambda A) = \lambda^n \det(A)$.
 (d) $\det(A^T A) = (\det(A))^2$

4. Let $a, b, c \in \mathbb{R}$. Prove that
$$\det \begin{pmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{pmatrix} = (c-a)(c-b)(b-a)$$

5. Compute the determinant of the following $n \times n$ matrix:
$$\begin{pmatrix} 4 & 1 & 1 & \dots & 1 \\ 1 & 4 & 1 & \dots & 1 \\ 1 & 1 & 4 & \dots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 4 \end{pmatrix}$$
(Hint: add row 1 through row (n-1) to row n; then divide row n by (n+3).)