CS-2050-All-Sections CS 2050 Homework 7 (HOWARD, FAULKNER, ELLEN)

Vidit Dharmendra Pokharna

TOTAL POINTS

100.5 / 100

QUESTION 1

1 Question 1 20 / 20

✓ **- 0 pts** *$$x = 52$$ or $$x \equiv 52 (\text{mod} ~280)$$ and showed work using the Chinese Remainder Theorem (refer to answer key)*

- 5 pts Does not check/indicate whether 5, 7, 8 are pairwise relatively prime.

Math errors

- 5 pts 1 Math error

- 10 pts 2 Math errors

- 15 pts 3+ Math errors

- 10 pts Major jump in work / logic

- 20 pts No work using Chinese Remainder theorem is shown

- 20 pts Incorrect / No Answer

- 10 pts Click here to replace this description.

QUESTION 2

2 Question 2 10 / 10

✓ **- 0 pts** *Pineapple pizza is simply superior than normal pizza*

- 8 pts Shift in wrong direction

Incorrect characters

- 2 pts 1 Incorrect characters

- 4 pts 2 Incorrect characters

- 6 pts 3 Incorrect characters

- 8 pts 4+ Incorrect characters

- 10 pts Incorrect / No Answer

QUESTION 3

3 Question 3 10 / 10

✓ **- 0 pts** *IFEEZV DRUV KYZJ HLVJKZFE*

- 8 pts Shift in wrong direction

Incorrect characters

- 2 pts 1 Incorrect characters

- 4 pts 2 Incorrect characters

- 6 pts 3 Incorrect characters

- 8 pts 4+ Incorrect characters

- 10 pts Incorrect / No Answer

QUESTION 4

4 Question 4 10 / 10

✓ **- 0 pts** *SIAOL EMATD OHINS XEXXX*

- 8 pts Shift in wrong direction

Incorrect characters

- 2 pts 1 Incorrect characters

- 4 pts 2 Incorrect characters

- 6 pts 3 Incorrect characters

- 8 pts 4+ Incorrect characters

- 10 pts Incorrect / No Answer

QUESTION 5

5 Question 5 10 / 10

✓ **- 0 pts** *twenty fifty*

- 8 pts Shift in wrong direction

Incorrect characters

- 2 pts 1 Incorrect characters

- 4 pts 2 Incorrect characters

- 6 pts 3 Incorrect characters

- 8 pts 4+ Incorrect characters

- 10 pts Incorrect / No Answer

QUESTION 6

6 Question 6 15 / 15

✓ **- 0 pts** *correct final answer (refer to answer key before using this rubric item)*

- 6 pts Minor Error

- 12 pts Major Error

- 15 pts Incorrect / No Answer

QUESTION 7

7 Question 7 15 / 15

✓ **- 0 pts** *Showed work for calculating $$d$$ and correct final answer (refer to answer key before using this rubric item)*

- 5 pts work shown using incorrect $$d$$

- 7 pts Did not show working for $$d$$

- 6 pts Minor error

- 12 pts Major Error

- 15 pts Incorrect / No Answer

QUESTION 8

8 Question 8 8 / 10

- 0 pts Correct

- 1 pts Did not define / Incorrectly defined $$P(n)$$

- 2 pts Did not clearly label Base case, IH, and IS

Basis Step (cap at -3)

- 2 pts Minor Math Error

- 3 pts Did not use $$P(1)$$ as base case

- 3 pts No basis step

Inductive Step (Cap at -5)

- 1 pts Using $$n$$ in the inductive step instead of a new variable

✓ **- 2 pts** *Minor error in math / logic*

- 4 pts Major error in math / logic

- 2 pts Does not explicitly assume IH that $$P(k)$$ is true for some $$k \geq 1$$

- 2 pts Not citing inductive hypothesis when it is used

- 2.5 pts Did not provide any reasoning

- 5 pts Assumed $$P(k+1)$$ is true

- 5 pts Not reaching $$P(k+1)$$

- 5 pts Assumed IH correctly, but did not attempt to reach $$P(k+1)$$

- 2 pts Missing or incorrect inductive step conclusion (e.g. only concluded $$P(k+1)$$ instead of $$(\forall j P(j)) \to P(k+1)$$ (if doing strong induction)

Conclusion (Cap at -2)

- 1 pts No / Incorrect mention of $$P(n)$$ or domain of $$n$$

- 1 pts No / incorrect mention of principle of math induction

- 10 pts Did not use Math Induction

- 10 pts No Answer

1 P(k) and P(k+1) are predicates (boolean

statements) so cannot equal a mathematical expression

QUESTION 9

9 Page Matching **0 / 0**

✓ **- 0 pts** *Correct*

- 5 pts Incorrect

QUESTION 10

10 On Time **2.5 / 0**

✓ **+ 2.5 pts** *On Time (Before Thursday)*

- 0 pts On Time (Friday)

- 10 pts 1 day late

- 25 pts 2 days late

Vidit Pokharna

CS 2050 HW 7

1.

$\gcd(5,7) = 1$
$\gcd(7,8) = 1$
$\gcd(5,8) = 1$
Therefore, 5,7,8 are pairwise coprime

$M = 5 \cdot 7 \cdot 8 = 280$

Mod value	Compared value	M / mod value	Inverse expression	Simplified inverse expression	Inverse value	Total product
5	2	280/5 = 56	56a ≡ 1 (mod 5)	1a ≡ 1 (mod 5)	a = 1	$2 \cdot 56 \cdot 1 = 112$
7	3	280/7 = 40	40b ≡ 1 (mod 7)	5b ≡ 1 (mod 7)	b = 3	$3 \cdot 40 \cdot 3 = 360$
8	4	280/8 = 35	35c ≡ 1 (mod 8)	3c ≡ 1 (mod 8)	c = 3	$4 \cdot 35 \cdot 3 = 420$

$x = (112 + 360 + 420)\ mod\ 280$
$= 892\ mod\ 2801$
$= 892 - 280 \cdot \left\lfloor \frac{892}{280} \right\rfloor = 892 - 280 \cdot 3 = 52$

Check answer
$52\ mod\ 5 \equiv 2$
$52\ mod\ 7 \equiv 3$
$52\ mod\ 8 \equiv 4$

2.

C	V	A	R	N	C	C	Y	R
2	21	0	17	13	2	2	24	17
15	34	13	30	26	15	15	37	30
15	8	13	4	0	15	15	11	4
P	I	N	E	A	P	P	L	E

C	V	M	M	N
2	21	12	12	13
15	34	25	25	26
15	8	13	4	0
P	I	Z	Z	A

1 Question 1 20 / 20

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Vidit Pokharna

CS 2050 HW 7

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7	3	280/7 = 40	40b ≡ 1 (mod 7)	5b ≡ 1 (mod 7)	b = 3	$3 \cdot 40 \cdot 3 = 360$
8	4	280/8 = 35	35c ≡ 1 (mod 8)	3c ≡ 1 (mod 8)	c = 3	$4 \cdot 35 \cdot 3 = 420$

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2.

C	V	A	R	N	C	C	Y	R
2	21	0	17	13	2	2	24	17
15	34	13	30	26	15	15	37	30
15	8	13	4	0	15	15	11	4
P	I	N	E	A	P	P	L	E

C	V	M	M	N
2	21	12	12	13
15	34	25	25	26
15	8	13	4	0
P	I	Z	Z	A

V	F
21	5
34	18
15	18
I	S

F	V	Z	C	Y	L
5	21	25	2	24	11
18	34	38	15	37	24
18	15	12	15	11	24
S	I	M	P	L	Y

F	H	C	R	E	V	B	E
5	7	2	17	4	21	1	4
18	20	15	30	17	34	14	17
18	20	15	4	17	15	14	17
S	U	P	E	R	I	O	R

G	U	N	A
6	20	13	0
19	33	26	13
19	7	0	13
T	H	A	N

A	B	E	Z	N	Y
0	1	4	25	13	24
13	14	17	38	26	37
13	14	17	12	0	11
N	O	R	M	A	L

C	V	M	M	N
2	21	12	12	13
15	34	25	25	26
15	8	13	4	0
P	I	Z	Z	A

Decrypted Text: Pineapple pizza is simply superior than normal pizza

2 Question 2 **10 / 10**

✓ **- 0 pts** *Pineapple pizza is simply superior than normal pizza*

- 8 pts Shift in wrong direction

Incorrect characters

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- 6 pts 3 Incorrect characters

- 8 pts 4+ Incorrect characters

- 10 pts Incorrect / No Answer

3.

A	B	C	D	E	F	G	H	I	J
17	18	19	20	21	22	23	24	25	0
R	S	T	U	V	W	X	Y	Z	A
K	L	M	N	O	P	Q	R	S	T
1	2	3	4	5	6	7	8	9	10
B	C	D	E	F	G	H	I	J	K
U	V	W	X	Y	Z				
11	12	13	14	15	16				
L	M	N	O	P	Q				

R	O	N	N	I	E		M	A	D	E		T	H	I	S		Q	U	E	S	T	I	O	N
I	F	E	E	Z	V		D	R	U	V		K	Y	Z	J		H	L	V	J	K	Z	F	E

Encrypted Text: IFEEZV DRUV KYZJ HLVJKZFE

4.

I	A	L	S	O		M	A	D	E	T		H	I	S	O	N
S	I	A	O	L		E	M	A	T	D		O	H	I	N	S

E	X	X	X	X
X	E	X	X	X

Encrypted Text: SIAOL EMATD OHINS XEXXX

5.

W	E	T		T	Y	N		I	F	F		Y	X	T
T	W	E		N	T	Y		F	I	F		T	Y	X

Decrypted Text: TWENTYFIFTY

3 Question 3 **10 / 10**

✓ **- 0 pts** *IFEEZV DRUV KYZJ HLVJKZFE*

- 8 pts Shift in wrong direction

Incorrect characters

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- 4 pts 2 Incorrect characters

- 6 pts 3 Incorrect characters

- 8 pts 4+ Incorrect characters

- 10 pts Incorrect / No Answer

3.

A	B	C	D	E	F	G	H	I	J
17	18	19	20	21	22	23	24	25	0
R	S	T	U	V	W	X	Y	Z	A
K	L	M	N	O	P	Q	R	S	T
1	2	3	4	5	6	7	8	9	10
B	C	D	E	F	G	H	I	J	K
U	V	W	X	Y	Z				
11	12	13	14	15	16				
L	M	N	O	P	Q				

R	O	N	N	I	E		M	A	D	E		T	H	I	S		Q	U	E	S	T	I	O	N
I	F	E	E	Z	V		D	R	U	V		K	Y	Z	J		H	L	V	J	K	Z	F	E

Encrypted Text: IFEEZV DRUV KYZJ HLVJKZFE

4.

I	A	L	S	O		M	A	D	E	T		H	I	S	O	N
S	I	A	O	L		E	M	A	T	D		O	H	I	N	S

E	X	X	X	X
X	E	X	X	X

Encrypted Text: SIAOL EMATD OHINS XEXXX

5.

W	E	T		T	Y	N		I	F	F		Y	X	T
T	W	E		N	T	Y		F	I	F		T	Y	X

Decrypted Text: TWENTYFIFTY

4 Question 4 **10 / 10**

✓ **- 0 pts** *SIAOL EMATD OHINS XEXXX*

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Incorrect characters

- 2 pts 1 Incorrect characters

- 4 pts 2 Incorrect characters

- 6 pts 3 Incorrect characters

- 8 pts 4+ Incorrect characters

- 10 pts Incorrect / No Answer

3.

A	B	C	D	E	F	G	H	I	J
17	18	19	20	21	22	23	24	25	0
R	S	T	U	V	W	X	Y	Z	A
K	L	M	N	O	P	Q	R	S	T
1	2	3	4	5	6	7	8	9	10
B	C	D	E	F	G	H	I	J	K
U	V	W	X	Y	Z				
11	12	13	14	15	16				
L	M	N	O	P	Q				

R	O	N	N	I	E		M	A	D	E		T	H	I	S		Q	U	E	S	T	I	O	N
I	F	E	E	Z	V		D	R	U	V		K	Y	Z	J		H	L	V	J	K	Z	F	E

Encrypted Text: IFEEZV DRUV KYZJ HLVJKZFE

4.

I	A	L	S	O		M	A	D	E	T		H	I	S	O	N
S	I	A	O	L		E	M	A	T	D		O	H	I	N	S

E	X	X	X	X
X	E	X	X	X

Encrypted Text: SIAOL EMATD OHINS XEXXX

5.

W	E	T		T	Y	N		I	F	F		Y	X	T
T	W	E		N	T	Y		F	I	F		T	Y	X

Decrypted Text: TWENTYFIFTY

5 Question 5 10 / 10

✓ **- 0 pts** *twenty fifty*

- 8 pts Shift in wrong direction

Incorrect characters

- 2 pts 1 Incorrect characters

- 4 pts 2 Incorrect characters

- 6 pts 3 Incorrect characters

- 8 pts 4+ Incorrect characters

- 10 pts Incorrect / No Answer

6.

1. $\emptyset(77) = (11-1)(7-1) = (10)(6) = 60$
   $\gcd(17,60) = 1$ ✓
2.

M	A	T	H	I	S	F	U	N
12	00	19	07	08	18	05	20	13

3. $25 < 77$
   $N = 1$, so we encrypt in blocks of size $2N = 2$
4. $12^{17}\ mod\ 77 = 45$
   $0^{17}\ mod\ 77 = 0$
   $19^{17}\ mod\ 77 = 24$
   $7^{17}\ mod\ 77 = 28$
   $8^{17}\ mod\ 77 = 57$
   $18^{17}\ mod\ 77 = 72$
   $5^{17}\ mod\ 77 = 3$
   $20^{17}\ mod\ 77 = 48$
   $13^{17}\ mod\ 77 = 62$

   Encrypted Text: 450024285772034862

6 Question 6 **15 / 15**

✓ **- 0 pts** *correct final answer (refer to answer key before using this rubric item)*

- 6 pts Minor Error

- 12 pts Major Error

- 15 pts Incorrect / No Answer

7.

1. $\emptyset(55) = (11-1)(5-1) = (10)(4) = 40$
   $23d \equiv 1\ (mod\ 40)$
   $d = 7$
2. $25 < 55$
   $N = 1$, so we encrypt in blocks of size $2N = 2$
3. $9^7\ mod\ 55 = 04 \rightarrow$ E
   $11^7\ mod\ 55 = 11 \rightarrow L$
   $49^7\ mod\ 55 = 14 \rightarrow O$
   $52^7\ mod\ 55 = 13 \rightarrow$ N
   $49^7\ mod\ 55 = 14 \rightarrow O$
   $33^7\ mod\ 55 = 22 \rightarrow W$
   $52^7\ mod\ 55 = 13 \rightarrow$ N
   $2^7\ mod\ 55 = 18 \rightarrow$ S
   $0^7\ mod\ 55 = 00 \rightarrow A$
   $10^7\ mod\ 55 = 10 \rightarrow$ K
   $17^7\ mod\ 55 = 08 \rightarrow I$
   $39^7\ mod\ 55 = 19 \rightarrow T$
   $8^7\ mod\ 55 = 02 \rightarrow C$
   $13^7\ mod\ 55 = 07 \rightarrow H$
   $9^7\ mod\ 55 = 04 \rightarrow E$
   $52^7\ mod\ 55 = 13 \rightarrow$ N
   $2^7\ mod\ 55 = 18 \rightarrow$ S
   $17^7\ mod\ 55 = 08 \rightarrow I$
   $52^7\ mod\ 55 = 13 \rightarrow$ N
   $10^7\ mod\ 55 = 10 \rightarrow K$

   Encrypted Text: $ELON\ OWNS\ A\ KITCHEN\ SINK$

7 Question 7 15 / 15

✓ **- 0 pts** *Showed work for calculating $$d$$ and correct final answer (refer to answer key before using this rubric item)*

- 5 pts work shown using incorrect $$d$$

- 7 pts Did not show working for $$d$$

- 6 pts Minor error

- 12 pts Major Error

- 15 pts Incorrect / No Answer

8.

Let $p(n)$ be $3 + 7 + 11 + \cdots + (4n - 1) = n(2n + 1)$. I will prove by induction that $p(n)$ is true $\forall n \in \mathbb{Z}^+$.

Line	Statement	Reason
1	$p(n) = 3 + 7 + 11 + \cdots + (4n - 1) = n(2n + 1), n \in \mathbb{Z}^+$	Given Statement
2	$p(1) = 1(2(1) + 1) = 1(3) = 3$ $3 = 3$ ✓	Basis Step
3	$p(k) = 3 + 7 + 11 + \cdots + (4k - 1) = k(2k + 1), k \in \mathbb{Z}^+$	Inductive Hypothesis: Assume $p(k)$ is true
4	$p(k + 1) = 3 + 7 + 11 + \cdots + (4k - 1) + (4(k + 1) - 1)$	Inductive Step Start
5	$p(k + 1) = 3 + 7 + 11 + \cdots + (4k - 1) + (4k + 3)$	Simplify (4)
6	$p(k + 1) = k(2k + 1) + (4k + 3)$	Substitute (3) into (5)
7	$p(k + 1) = 2k^2 + k + 4k + 3$	Simplify (6)
8	$p(k + 1) = 2k^2 + 5k + 3$	Simplify (7)
9	$p(k + 1) = (k + 1)(2k + 3)$	Factor Out (8)
10	$p(k + 1) = (k + 1)(2(k + 1) + 1)$	Rewrite (9)

We can see that $p(k + 1) = 3 + 7 + 11 + \cdots + (4k - 1) + (4k + 3) = (k + 1)(2(k + 1) + 1)$ is true whenever $p(k)$ is true. This completed the inductive step.

By mathematical induction, $p(n)$ is true $\forall n \in \mathbb{Z}^+$ ■

8 Question 8 8 / 10

- **0 pts** Correct
- **1 pts** Did not define / Incorrectly defined $$P(n)$$
- **2 pts** Did not clearly label Base case, IH, and IS

Basis Step (cap at -3)

- **2 pts** Minor Math Error
- **3 pts** Did not use $$P(1)$$ as base case
- **3 pts** No basis step

Inductive Step (Cap at -5)

- **1 pts** Using $$n$$ in the inductive step instead of a new variable

✓ **- 2 pts** *Minor error in math / logic*

- **4 pts** Major error in math / logic
- **2 pts** Does not explicitly assume IH that $$P(k)$$ is true for some $$k \geq 1$$
- **2 pts** Not citing inductive hypothesis when it is used
- **2.5 pts** Did not provide any reasoning
- **5 pts** Assumed $$P(k+1)$$ is true
- **5 pts** Not reaching $$P(k+1)$$
- **5 pts** Assumed IH correctly, but did not attempt to reach $$P(k+1)$$
- **2 pts** Missing or incorrect inductive step conclusion (e.g. only concluded $$P(k+1)$$ instead of $$(\forall j P(j)) \to P(k+1)$$ (if doing strong induction)

Conclusion (Cap at -2)

- **1 pts** No / Incorrect mention of $$P(n)$$ or domain of $$n$$
- **1 pts** No / incorrect mention of principle of math induction
- **10 pts** Did not use Math Induction
- **10 pts** No Answer

1 P(k) and P(k+1) are predicates (boolean statements) so cannot equal a mathematical expression

9 Page Matching **0 / 0**

✓ **- 0 pts** *Correct*

- 5 pts Incorrect

10 On Time **2.5 / 0**

✓ **+ 2.5 pts** *On Time (Before Thursday)*

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