# CS-2050-All-Sections CS 2050 Homework 8 (HOWARD, FAULKNER, ELLEN)

Vidit Dharmendra Pokharna

TOTAL POINTS

**89.5 / 100**

QUESTION 1

*1* Question 1 **23 / 25**

✓ **- 0 pts** *Correct*

Introduction (cap at -6)

✓ **- 2 pts** *Did not define / incorrectly defined $$P(n)$$ before using it*

**- 2 pts** Used predicate as a non-boolean

**- 2 pts** Incorrect/missing domain

Basis step (cap at -6)

**- 2 pts** Minor math error

**- 5 pts** Does not use correct value for the base case. Correct Base Cases are: $$P(7), P(8)$$

**- 6 pts** No/Completely incorrect basis step

IH and Inductive step (cap at -16)

**- 2 pts** Does not explicitly assume $$P(k)$$

**- 2 pts** Incorrect/missing new variable domain definition (e.g. not saying $$k \in \mathbb{Z}^{\geq 0}$$)

**- 2 pts** Using $$n$$ in the inductive step instead of a new variable

**- 2 pts** Switching between booleans and numbers

**- 4 pts** Not citing inductive hypothesis when it is used

**- 2 pts** Minor math error

**- 4 pts** Major math error

**- 3 pts** Minor jump in logic

**- 6 pts** Major jump in logic

**- 5 pts** Did not provide any reasonings

**- 2 pts** Did not provide any reasonings for algebra steps

**- 10 pts** Assumed $$P(k+1)$$ is true

**- 12 pts** Not reaching $$P(k+1)$$

**- 14 pts** Assumed $$P(k)$$ correctly, but did not attempt to reach $$P(k+1)$$

**- 3 pts** Missing or incorrect inductive step conclusion (e.g. only concluded $$P(k+1)$$ instead of $$P(k) \to P(k+1)$$

**- 16 pts** Missing/ completely incorrect inductive step

**- 2 pts** Missing inductive step conclusion (i.e. "this concludes the inductive step")

Conclusion

**- 1 pts** No/Incorrect mention of $$P(n)$$ being true

**- 1 pts** No mention of domain of $$n$$ or incorrect domain for $$n$$ mentioned (domain for $$n$$ should be $$n \in \mathbb{Z}^{\geq 0}$$)

**- 1 pts** No/incorrect mention of principle of mathematical induction

**- 25 pts** No answer

**- 25 pts** Did not use mathematical induction

1 next time, try to start with $$3^k < k!$$ and build up to $$3^{k+1} < (k+1)!$$

2 You need to define $$P(n)$$

QUESTION 2

## 2 Question 2 21 / 25

**- 0 pts** Correct

Introduction (cap at -6)

✓ **- 2 pts** *Did not define / incorrectly defined $$P(n)$$ before using it*

**- 2 pts** Used predicate as a non-boolean

**- 2 pts** Incorrect/missing domain

Basis step (cap at -6)

**- 2 pts** Minor math error

**- 5 pts** Does not use correct value for the base case. Correct Base Cases are: $$P(1)$$

**- 6 pts** No/Completely incorrect basis step

IH and Inductive step (cap at -16)

**- 2 pts** Does not explicitly assume $$P(k)$$

**- 2 pts** Incorrect/missing new variable domain definition (e.g. not saying $$k \in \mathbb{Z}^{\geq 0}$$)

**- 2 pts** Using $$n$$ in the inductive step instead of a new variable

✓ **- 2 pts** *Switching between booleans and numbers*

**- 4 pts** Not citing inductive hypothesis when it is used

**- 2 pts** Minor math error

**- 4 pts** Major math error

**- 3 pts** Minor jump in logic

**- 6 pts** Major jump in logic

**- 5 pts** Did not provide any reasonings

**- 2 pts** Did not provide any reasonings for algebra steps

**- 10 pts** Assumed $$P(k+1)$$ is true

**- 12 pts** Not reaching $$P(k+1)$$

**- 14 pts** Assumed $$P(k)$$ correctly, but did not attempt to reach $$P(k+1)$$

**- 3 pts** Missing or incorrect inductive step conclusion (e.g. only concluded $$P(k+1)$$ instead of $$P(k) \to P(k+1)$$

**- 16 pts** Missing/ completely incorrect inductive step

**- 2 pts** Missing inductive step conclusion (i.e. "this concludes the inductive step")

Conclusion

**- 1 pts** No/Incorrect mention of $$P(n)$$ being true

**- 1 pts** No mention of domain of $$n$$ or incorrect domain for $$n$$ mentioned (domain for $$n$$ should be $$n \in \mathbb{Z}^{\geq 0}$$)

**- 1 pts** No/incorrect mention of principle of mathematical induction

**- 25 pts** No answer

**- 25 pts** Did not use mathematical induction

3 statements need to be equations with an equal sign

4 Need to define $$P(n)$$

QUESTION 3

## 3 Question 3 23 / 25

**- 0 pts** Correct

Minimum $$x$$

- **- 0 pts** $$x=28$$
- **- 5 pts** $$ x \neq 28 $$

✓ **- 2 pts** *Did not define / incorrectly defined $$P(n)$$ before using it*

- **- 1 pts** Did not refer to $$n$$ in the definition of $$P(n)$$.

  E.g. "$$P(n):$$ any amount of postage 28 cents or more can be formed with 5 and 8 cent stamps" is incorrect
- **- 2 pts** Used predicate as a non-boolean

Basis step (cap at -6)

- **- 2 pts** Minor math error
- **- 3 pts** Missing 1 case
- **- 5 pts** Missing 2+ cases
- **- 6 pts** No/completely incorrect basis step

IH and Inductive step (cap at -16)

- **- 0 pts** Completely correct
- **- 2 pts** Does not explicitly assume IH that $$\forall j P(j)$$ or equivalent
- **- 2 pts** Incorrect bounds for $$j$$ (e.g. not saying $$x \leq j \leq k$$)
- **- 2 pts** Incorrect/missing new variable domain definition (e.g. not saying $$j, k \in \mathbb{Z}^{\geq 32}$$)
- **- 2 pts** Incorrect/mismatched bounds for $$k$$ (e.g. if the last base case is $$P(x)$$, need to have $$k \geq x$$)
- **- 2 pts** Using $$n$$ in the inductive step instead of a new variable
- **- 4 pts** Not citing inductive hypothesis when it is used
- **- 2 pts** Minor math error
- **- 4 pts** Major math error
- **- 3 pts** Minor jump in logic
- **- 6 pts** Major jump in logic
- **- 5 pts** Did not provide any reasonings
- **- 2 pts** Did not provide any reasonings for algebra steps
- **- 10 pts** Assumed $$P(k+1)$$ is true
- **- 12 pts** Not reaching $$P(k+1)$$
- **- 14 pts** Assumed IH correctly, but did not attempt to reach $$P(k+1)$$
- **- 3 pts** Missing or incorrect inductive step conclusion (e.g. only concluded $$P(k+1)$$ instead of $$(\forall j P(j)) \to P(k+1)$$ or equivalent)
- **- 16 pts** Missing/ completely incorrect inductive step

Conclusion

- **- 1 pts** No/Incorrect mention of $$P(n)$$ being true
- **- 1 pts** No mention of domain of $$n$$ or incorrect domain for $$n$$ mentioned (domain for $$n$$ should be $$n \in \mathbb{Z}^{\geq 28}$$)
- **- 1 pts** No/incorrect mention of principle of strong induction

- **- 25 pts** No answer
- **- 25 pts** Did not use strong induction

5 need to define P(n) before using it

QUESTION 4

## 4 Question 4 20 / 25

- **- 0 pts** Correct

✓ **- 2 pts** *Did not define / incorrectly defined $$P(n)$$ before using it*

**- 2 pts** Used predicate as a non-boolean

Basis step (cap at -6)

**- 2 pts** Minor math error/missing base case

**- 5 pts** Does not use $$P(2)$$ and $$P(3)$$ for the base case

**- 6 pts** No/Completely incorrect basis step

IH and Inductive step (cap at -16)

**- 2 pts** Does not explicitly assume IH that $$P(j)$$ is true for all $$j$$ in the bounds $$2 \leq j \leq k$$

**- 2 pts** Incorrect/missing new variable domain definition (e.g. not saying $$k \in \mathbb{Z}^{\geq 3}$$ or that $$j \in \mathbb{Z}$$)

**- 2 pts** Using $$n$$ in the inductive step instead of a new variable

**- 2 pts** Switching between booleans and numbers

**- 4 pts** Not citing inductive hypothesis when it is used

**- 2 pts** Minor math error

**- 4 pts** Major math error

**- 3 pts** Minor jump in logic

**- 6 pts** Major jump in logic

**- 5 pts** Did not provide any reasonings

**- 2 pts** Did not provide any reasonings for algebra steps

**- 10 pts** Assumed $$P(k+1)$$ is true

**- 12 pts** Not reaching $$P(k+1)$$

**- 14 pts** Assumed IH correctly, but did not attempt to reach $$P(k+1)$$ (whether by direct proof or contradiction)

✓ **- 3 pts** *Missing or incorrect inductive step conclusion (e.g. only concluded $$P(k+1)$$ instead of $$(\forall j P(j)) \to P(k+1)$$ or equivalent)*

**- 16 pts** Missing/ completely incorrect inductive step

Conclusion

**- 1 pts** No/Incorrect mention of $$P(n)$$ being true

**- 1 pts** No mention of domain of $$n$$ or incorrect domain for $$n$$ mentioned (domain for $$n$$ should be $$n \in \mathbb{Z}^{\geq 2}$$)

**- 1 pts** No/incorrect mention of principle of strong induction

**- 25 pts** No answer

**- 25 pts** Did not use Strong induction

6 need to define P(n)

7 need to conclude the IS

QUESTION 5

## 5 Page Matching **0 / 0**

✓ **- 0 pts** *Correct*

**- 5 pts** Incorrect

QUESTION 6

## 6 On Time **2.5 / 0**

✓ **+ 2.5 pts** *On Time (Before Thursday)*

**- 0 pts** On Time (Friday)

**- 10 pts** 1 day late

**- 25 pts** 2 days late

Vidit Pokharna

**CS 2050 HW 8**

1.

I will use mathematical induction to prove that $3^n < n!$ for all integers $n > 7$.

| Line | Statement | Reason |
|---|---|---|
| 1 | $3^n < n!, n > 7$ | Given Statement |
| 2 | $3^8 < 8!$ <br> $6561 < 40320$ ✓ | Basis Step |
| 3 | $3^k < k!, k > 7$ | Inductive Hypothesis: Assume original statement is true for $n = k$ |
| 4 | $\frac{3^k}{k!} < 1$ | Simplify by dividing $k!$ on both sides |
| 5 | Check if $3^{k+1} < (k+1)!$ | Inductive Step Start |
| 6 | Check if $3 \cdot 3^k < (k+1) \cdot k!$ | Simplify by factoring out |
| 7 | Check if $3^k < \frac{(k+1) \cdot k!}{3}$ | Simplify by dividing 3 on both sides |
| 8 | Check if $\frac{3^k}{k!} < \frac{(k+1)}{3}$ | Simplify by dividing $k!$ on both sides |
| 9 | $\frac{(k+1)}{3} > 1$ | Because $k > 7$, the value of $\frac{(k+1)}{3}$ must be greater than 1 as the numerator will be greater than the denominator |
| 10 | $\frac{3^k}{k!} < 1 < \frac{(k+1)}{3}$ | Inductive Step Proven using (4) and (9) |

We can see that $3^{k+1} < (k+1)!$ is true whenever $3^k < k!$ is true, along with the base case of $3^8 < 8!$ being true as well. This completed the inductive step.

$\therefore$ By mathematical induction, $3^n < n!$ for all integers $n > 7$ ■

## 1 Question 1 **23 / 25**

✓ **- 0 pts** *Correct*

### Introduction (cap at -6)

✓ **- 2 pts** *Did not define / incorrectly defined $$P(n)$$ before using it*

**- 2 pts** Used predicate as a non-boolean

**- 2 pts** Incorrect/missing domain

### Basis step (cap at -6)

**- 2 pts** Minor math error

**- 5 pts** Does not use correct value for the base case. Correct Base Cases are: $$P(7), P(8)$$

**- 6 pts** No/Completely incorrect basis step

### IH and Inductive step (cap at -16)

**- 2 pts** Does not explicitly assume $$P(k)$$

**- 2 pts** Incorrect/missing new variable domain definition (e.g. not saying $$k \in \mathbb{Z}^{\geq 0}$$)

**- 2 pts** Using $$n$$ in the inductive step instead of a new variable

**- 2 pts** Switching between booleans and numbers

**- 4 pts** Not citing inductive hypothesis when it is used

**- 2 pts** Minor math error

**- 4 pts** Major math error

**- 3 pts** Minor jump in logic

**- 6 pts** Major jump in logic

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**- 2 pts** Did not provide any reasonings for algebra steps

**- 10 pts** Assumed $$P(k+1)$$ is true

**- 12 pts** Not reaching $$P(k+1)$$

**- 14 pts** Assumed $$P(k)$$ correctly, but did not attempt to reach $$P(k+1)$$

**- 3 pts** Missing or incorrect inductive step conclusion (e.g. only concluded $$P(k+1)$$ instead of $$P(k) \to P(k+1)$$

**- 16 pts** Missing/ completely incorrect inductive step

**- 2 pts** Missing inductive step conclusion (i.e. "this concludes the inductive step")

### Conclusion

**- 1 pts** No/Incorrect mention of $$P(n)$$ being true

**- 1 pts** No mention of domain of $n$ or incorrect domain for $n$ mentioned (domain for $n$ should be $n \in \mathbb{Z}^{\geq 0}$)

**- 1 pts** No/incorrect mention of principle of mathematical induction

**- 25 pts** No answer

**- 25 pts** Did not use mathematical induction

1 next time, try to start with $3^k < k!$ and build up to $3^{k+1} < (k+1)!$

2 You need to define $P(n)$

2.

I will use mathematical induction to prove that $3 \mid n^3 + 2n$ for all integers $n > 0$.

| Line | Statement | Reason |
|---|---|---|
| 1 | $3 \mid n^3 + 2n, n > 0$ | Given Statement |
| 2 | $3 \mid 1^3 + 2(1)$<br>$3 \mid 3$<br>$\frac{3}{3} = 1\checkmark$ | Basis Step |
| 3 | $3 \mid k^3 + 2k, k > 0$ | Inductive Hypothesis: Assume original statement is true for $n = k$ |
| 4 | $\frac{k^3 + 2k}{3} = a, a \in \mathbb{Z}$ | Definition of "divides" (3) |
| 5 | Check if $3 \mid (k+1)^3 + 2(k+1)$ | Inductive Step Start |
| 6 | Check if $3 \mid k^3 + 3k^2 + 3k + 1 + 2k + 2$ | Simplify by multiplying out |
| 7 | Check if $3 \mid k^3 + 3k^2 + 5k + 3$ | Simplify by adding like terms |
| 8 | $\frac{k^3 + 3k^2 + 5k + 3}{3}$ | Definition of "divides" (7) |
| 9 | $\frac{k^3 + 2k}{3} + \frac{3k^2 + 3k + 3}{3}$ | Switch around values (8) |
| 10 | $a + k^2 + k + 1$ | Simplify and substitute (4) into (9) |
| 11 | $a + k^2 + k + 1 = b, b \in \mathbb{Z}$ | Closure of integers under addition and multiplication |
| 12 | $\frac{k^3 + 3k^2 + 5k + 3}{3} = b, b \in \mathbb{Z}$ | Substitute (11) into (8) |
| 13 | $3 \mid k^3 + 3k^2 + 5k + 3$ | Definition of "divides" (12) |
| 14 | $3 \mid (k+1)^3 + 2(k+1)$ | Simplify (13) |

We can see that $3 \mid (k+1)^3 + 2(k+1)$ is true whenever $3 \mid k^3 + 2k$ is true, along with the base case of $3 \mid 1^3 + 2(1)$ being true as well. This completed the inductive step.

$\therefore$ By mathematical induction, $3 \mid n^3 + 2n$ for all integers $n > 0$ ■

## 2 Question 2 **21 / 25**

- **- 0 pts** Correct

### Introduction (cap at -6)

✓ **- 2 pts** *Did not define / incorrectly defined $$P(n)$$ before using it*

- **- 2 pts** Used predicate as a non-boolean
- **- 2 pts** Incorrect/missing domain

### Basis step (cap at -6)

- **- 2 pts** Minor math error
- **- 5 pts** Does not use correct value for the base case. Correct Base Cases are: $$P(1)$$
- **- 6 pts** No/Completely incorrect basis step

### IH and Inductive step (cap at -16)

- **- 2 pts** Does not explicitly assume $$P(k)$$
- **- 2 pts** Incorrect/missing new variable domain definition (e.g. not saying $$k \in \mathbb{Z}^{\geq 0}$$)
- **- 2 pts** Using $$n$$ in the inductive step instead of a new variable

✓ **- 2 pts** *Switching between booleans and numbers*

- **- 4 pts** Not citing inductive hypothesis when it is used
- **- 2 pts** Minor math error
- **- 4 pts** Major math error
- **- 3 pts** Minor jump in logic
- **- 6 pts** Major jump in logic
- **- 5 pts** Did not provide any reasonings
- **- 2 pts** Did not provide any reasonings for algebra steps
- **- 10 pts** Assumed $$P(k+1)$$ is true
- **- 12 pts** Not reaching $$P(k+1)$$
- **- 14 pts** Assumed $$P(k)$$ correctly, but did not attempt to reach $$P(k+1)$$
- **- 3 pts** Missing or incorrect inductive step conclusion (e.g. only concluded $$P(k+1)$$ instead of $$P(k) \to P(k+1)$$
- **- 16 pts** Missing/ completely incorrect inductive step
- **- 2 pts** Missing inductive step conclusion (i.e. "this concludes the inductive step")

### Conclusion

- **- 1 pts** No/Incorrect mention of $$P(n)$$ being true

**- 1 pts** No mention of domain of $$n$$ or incorrect domain for $$n$$ mentioned (domain for $$n$$ should be $$n \in \mathbb{Z}^{\geq 0}$$)

**- 1 pts** No/incorrect mention of principle of mathematical induction

**- 25 pts** No answer

**- 25 pts** Did not use mathematical induction

3 statements need to be equations with an equal sign

4 Need to define $$P(n)$$

3.

The smallest value of $x$ is 28 cents, such that you can form postage of 28 cents and greater using only 5 cent and 8 cent stamps. I will use strong induction to prove this.

| Line | Statement | Reason |
|---|---|---|
| 1 | $28 = 5 + 5 + 5 + 5 + 8$ ✓<br>$29 = 5 + 8 + 8 + 8$ ✓<br>$30 = 5 + 5 + 5 + 5 + 5 + 5$ ✓<br>$31 = 5 + 5 + 5 + 8 + 8$ ✓<br>$32 = 8 + 8 + 8 + 8$ ✓ | We need 5 basis steps (as a 6th step would just add 5 from the 1st step, so there is no need for more than 5 steps) |
| 2 | Assume that you can form postage of $j$ cents using only 5 cent and 8 cent stamps, where $28 \leq j \leq k$ and $k$ is a fixed arbitrary integer and $k \geq 32$. | Inductive Hypothesis |
| 3 | Consider postage of $k + 1$ cents. Subtract 5 cents from there gives us $k - 4$ cents. Since $k \geq 32$, $k - 4 \geq 28$. Thus, by the Inductive Hypothesis, $k - 4$ cent postage can be made using 5 cent and 8 cent stamps. Using an additional 5 cent stamp, you can make $k + 1$ postage. Thus, you can form postage of $k + 1$ cents using only 5 cent and 8 cent stamps assuming the inductive hypothesis is true. | Inductive Step |

∴ By strong induction, you can form postage of $n$ cents using only 5 cent and 8 cent stamps for $n \geq 28$ ■

## 3 Question 3 **23 / 25**

- **- 0 pts** Correct

### Minimum $$x$$

- **- 0 pts** $$x=28$$
- **- 5 pts** $$ x \neq 28 $$

✓ **- 2 pts** *Did not define / incorrectly defined $$P(n)$$ before using it*

- **- 1 pts** Did not refer to $$n$$ in the definition of $$P(n)$$.

E.g. "$$P(n):$$ any amount of postage 28 cents or more can be formed with 5 and 8 cent stamps" is incorrect

- **- 2 pts** Used predicate as a non-boolean

### Basis step (cap at -6)

- **- 2 pts** Minor math error
- **- 3 pts** Missing 1 case
- **- 5 pts** Missing 2+ cases
- **- 6 pts** No/completely incorrect basis step

### IH and Inductive step (cap at -16)

- **- 0 pts** Completely correct
- **- 2 pts** Does not explicitly assume IH that $$\forall j P(j)$$ or equivalent
- **- 2 pts** Incorrect bounds for $$j$$ (e.g. not saying $$x \leq j \leq k$$)
- **- 2 pts** Incorrect/missing new variable domain definition (e.g. not saying $$j, k \in \mathbb{Z}^{\geq 32}$$)
- **- 2 pts** Incorrect/mismatched bounds for $$k$$ (e.g. if the last base case is $$P(x)$$, need to have $$k \geq x$$)
- **- 2 pts** Using $$n$$ in the inductive step instead of a new variable
- **- 4 pts** Not citing inductive hypothesis when it is used
- **- 2 pts** Minor math error
- **- 4 pts** Major math error
- **- 3 pts** Minor jump in logic
- **- 6 pts** Major jump in logic
- **- 5 pts** Did not provide any reasonings
- **- 2 pts** Did not provide any reasonings for algebra steps

**- 10 pts** Assumed $$P(k+1)$$ is true

**- 12 pts** Not reaching $$P(k+1)$$

**- 14 pts** Assumed IH correctly, but did not attempt to reach $$P(k+1)$$

**- 3 pts** Missing or incorrect inductive step conclusion (e.g. only concluded $$P(k+1)$$ instead of $$(\forall j P(j)) \to P(k+1)$$ or equivalent)

**- 16 pts** Missing/ completely incorrect inductive step

Conclusion

**- 1 pts** No/Incorrect mention of $$P(n)$$ being true

**- 1 pts** No mention of domain of $$n$$ or incorrect domain for $$n$$ mentioned (domain for $$n$$ should be $$n \in \mathbb{Z}^{\geq 28}$$)

**- 1 pts** No/incorrect mention of principle of strong induction

**- 25 pts** No answer

**- 25 pts** Did not use strong induction

5 need to define P(n) before using it

4.

The value of $a_n$ is defined as follows: $a_0 = 1$, $a_1 = 3$, and $a_t = a_{t-1} + a_{t-2}, t \in \mathbb{Z}^{\geq 2}$. I will use strong induction to prove that $a_n \leq 2^n$ for all $n \geq 2$.

| Line | Statement | Reason |
|---|---|---|
| 1 | $a_2 = a_1 + a_0 = 3 + 1 = 4 \leq 2^2 = 4$ ✓<br>$a_3 = a_2 + a_1 = 4 + 3 = 7 \leq 2^3 = 8$ ✓ | We need 2 basis steps (since the recursive term uses two terms from before) |
| 2 | Assume $a_j \leq 2^j$ such that $2 \leq j \leq k$ and $k$ is a fixed arbitrary integer and $k \geq 3$. | Inductive Hypothesis |
| 3 | $a_{k+1} = a_k + a_{k-1}$ | Inductive Step (Definition of $a_{k+1}$) |
| 4 | $a_k \leq 2^k$<br>$a_{k-1} \leq 2^{k-1}$ | Inductive Step (known from Inductive Hypothesis) |
| 5 | $a_k + a_{k-1} \leq 2^k + 2^{k-1}$ | Inductive Step (Add the inequalities from (4)) |
| 6 | $2^k \leq 2^k$<br>$2^{k-1} \leq 2^k$ | Inductive Step (known since $k \geq 3$) |
| 7 | $2^k + 2^{k-1} \leq 2^k + 2^k$ | Inductive Step (Add the inequalities from (6)) |
| 8 | $2^k + 2^{k-1} \leq 2(2^k)$ | Inductive Step (Combine like terms in (7)) |
| 9 | $2^k + 2^{k-1} \leq 2^{k+1}$ | Inductive Step (Addition property of exponents from (8)) |
| 10 | $a_k + a_{k-1} \leq 2^k + 2^{k-1} \leq 2^{k+1}$ | Inductive Step (Combine (5) and (9)) |
| 11 | $a_k + a_{k-1} \leq 2^{k+1}$ | Inductive Step (Definition of $\leq$) |
| 12 | $a_k + a_{k-1} = a_{k+1} \leq 2^{k+1}$ | Inductive Step (Substitute (3) into (11)) |

$\therefore$ By strong induction, for the value $a_n$, $a_n \leq 2^n$ for all $n \geq 2$ ■

## *4* Question 4 **20 / 25**

- **- 0 pts** Correct
- ✓ **- 2 pts** *Did not define / incorrectly defined $$P(n)$$ before using it*
- **- 2 pts** Used predicate as a non-boolean

### Basis step (cap at -6)

- **- 2 pts** Minor math error/missing base case
- **- 5 pts** Does not use $$P(2)$$ and $$P(3)$$ for the base case
- **- 6 pts** No/Completely incorrect basis step

### IH and Inductive step (cap at -16)

- **- 2 pts** Does not explicitly assume IH that $$P(j)$$ is true for all $$j$$ in the bounds $$2 \leq j \leq k$$
- **- 2 pts** Incorrect/missing new variable domain definition (e.g. not saying $$k \in \mathbb{Z}^{\geq 3}$$ or that $$j \in \mathbb{Z}$$)
- **- 2 pts** Using $$n$$ in the inductive step instead of a new variable
- **- 2 pts** Switching between booleans and numbers
- **- 4 pts** Not citing inductive hypothesis when it is used
- **- 2 pts** Minor math error
- **- 4 pts** Major math error
- **- 3 pts** Minor jump in logic
- **- 6 pts** Major jump in logic
- **- 5 pts** Did not provide any reasonings
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- **- 10 pts** Assumed $$P(k+1)$$ is true
- **- 12 pts** Not reaching $$P(k+1)$$
- **- 14 pts** Assumed IH correctly, but did not attempt to reach $$P(k+1)$$ (whether by direct proof or contradiction)
- ✓ **- 3 pts** *Missing or incorrect inductive step conclusion (e.g. only concluded $$P(k+1)$$ instead of $$(\forall j P(j)) \to P(k+1)$$ or equivalent)*
- **- 16 pts** Missing/ completely incorrect inductive step

### Conclusion

- **- 1 pts** No/Incorrect mention of $$P(n)$$ being true
- **- 1 pts** No mention of domain of $$n$$ or incorrect domain for $$n$$ mentioned (domain for $$n$$ should be $$n \in \mathbb{Z}^{\geq 2}$$)

- **- 1 pts** No/incorrect mention of principle of strong induction
- **- 25 pts** No answer
- **- 25 pts** Did not use Strong induction

6 need to define P(n)

7 need to conclude the IS

*5* Page Matching **0 / 0**

✓ **- 0 pts** *Correct*

**- 5 pts** Incorrect

## *6* On Time **2.5 / 0**

- ✓ **+ 2.5 pts** *On Time (Before Thursday)*
- **- 0 pts** On Time (Friday)
- **- 10 pts** 1 day late
- **- 25 pts** 2 days late