MATH-3012-D HW 05

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TOTAL POINTS

31 / 39

QUESTION 1

1 Q1 **12 / 13**

- ✓ **+ 4 pts** *Credit for (a)*
- ✓ **+ 3 pts** *Partial credit for (b)*
- ✓ **+ 5 pts** *Credit for (c)*

QUESTION 2

2 Q2 **6 / 13**

- ✓ **+ 6 pts** *Partial credit.*

QUESTION 3

3 C **13 / 13**

- ✓ **- 0 pts** *Correct*

Homework 5 Q1

8.1 5. (b) $2 \to 1000$ $\quad 6 \to 333$ $\quad 21 \to 95$ $\quad 70 \to 28$

$3 \to 666$ $\quad 10 \to 200$ $\quad 35 \to 57$ $\quad 105 \to 19$

$5 \to 400$ $\quad 14 \to 142$ $\quad 30 \to 66$ $\quad 210 \to 9$

$7 \to 285$ $\quad 15 \to 133$ $\quad 42 \to 47$

$$2000 - (1000+666+400+285-333-200-142-133-95-57+66+47+28+19-9)$$

$$2000 - 1542 = \boxed{458}$$

(c) $1000+666+400-333-200-133+66 = 1466$

$2000 - 1466 = 534$

$\left\lfloor \frac{534}{7} \right\rfloor = \boxed{76}$

6. (a) $\binom{19+4-1}{19} = \binom{22}{3} = \boxed{1540}$

(b) $\binom{19+3}{3} - \binom{4}{1}\binom{10+3}{3} + \binom{4}{2}\binom{3+3}{3} = \binom{22}{3} - 4\binom{13}{3} + 6\binom{6}{3} = \boxed{516}$

(c) $y_1+y_2+y_3+y_4 = 19-3-3 = 13$

$0 \le y_1 \le 5,\ 0 \le y_2 \le 6,\ 0 \le y_3 \le 4,\ 0 \le y_4 \le 5$

$$\binom{13+3}{3} - \binom{7+3}{3} - \binom{6+3}{3} - \binom{8+3}{3} - \binom{7+3}{3} + \binom{0+3}{3} + \binom{2+3}{3} + \binom{1+3}{3} + \binom{1+3}{3} + \binom{0+3}{3} + \binom{2+3}{3}$$

$$= 560 - 120 - 84 - 165 - 120 + 1 + 10 + 4 + 4 + 1 + 10 = \boxed{101}$$

7. Three groups → IN, ON, IO

$$\frac{11!}{2!2!2!} - 3 \cdot 2! \cdot \frac{9!}{2!2!} + 3! \cdot \frac{7!}{2!} = 4989600 - 544320 + 15120 = \boxed{4460400}$$

17. $$\frac{9!}{3!3!3!} - \frac{7!}{3!3!} \cdot 3 + \frac{5!}{3!} \cdot 3 - 3! = \boxed{1554}$$

22. (c) $5188 = 2^2 \times 1297$

$$5188\left(1 - \tfrac{1}{2}\right)\left(1 - \tfrac{1}{1297}\right) = \boxed{2592}$$

25. (a) $6000\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{5}\right) = \boxed{1600}$

(b) $6000 - 1600 + 1 = \boxed{4399}$

26. Suppose n has prime factors $p_1, \dots, p_a$

n^m would have the same prime factors

Therefore, $\phi(n^m) = n^m \prod_{p_i = p_1}^{p_a} \left(1 - \frac{1}{p_i}\right) = n^{m-1} \cdot n \cdot \prod_{p_i = p_1}^{p_a} \left(1 - \frac{1}{p_i}\right)$

$= n^{m-1} \phi(n)$

5.3

7. (a) (i) $2!\, S(7,2) = \boxed{126}$

(ii) $\binom{5}{2} 2!\, S(7,2) = \boxed{1260}$

(iii) $3!\, S(7,3) = \boxed{1806}$

(iv) $\binom{5}{3} 3!\, S(7,3) = \boxed{18060}$

(v) $4!\, S(7,4) = \boxed{8400}$

(vi) $\binom{5}{4} 4!\, S(7,4) = \boxed{42000}$

1 Q1 **12 / 13**

✓ **+ 4 pts** *Credit for (a)*

✓ **+ 3 pts** *Partial credit for (b)*

✓ **+ 5 pts** *Credit for (c)*

Homework 5 Q2

8.3 4. $7!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}+\frac{1}{6!}-\frac{1}{7!}\right) = 1854$

$7! - 1854 = 5040 - 1854 = \boxed{3186}$

6. (a) $\left[4!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}\right)\right]^2 = \boxed{81}$

(b) $(4!)^2 = \boxed{576}$

11. (a) $\left[10!\left(\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\cdots+\frac{1}{10!}\right)\right]^2 = 1334961^2 = 1782120871521$

(b) $= \sum_{k=0}^{n}(-1)^k\binom{n}{k}((n-k)!)^m = \sum_{k=0}^{10}(-1)^k\binom{10}{k}((10-k)!)^2$

$= \boxed{11921584264011}$

14. (a) $N(\bar{c}_1\bar{c}_2\bar{c}_3\cdots\bar{c}_{n-1}) = \sum_{k=0}^{n-1}(-1)^k\binom{n-1}{k}(n-k)!$

9.2 1. (a) $(1+x)^8$

(b) $8(1+x)^7$

(c) $1/(1+x)$

(d) $6x^3/(1+x)$

(e) $1/(1-x^2)$

(f) $x^2/(1-ax)$

2. (c) starts at position 4, skips every 1

$\boxed{0, 0, 0, 1, 0, 1, 0, 1, 0, \ldots}$

(e) $\frac{1}{(3-x)} = \frac{1/3}{(1-1/3x)} \to \boxed{1/3, 1/9, 1/27, \ldots}$

(f) $(\cancel{1}+x+x^2+\ldots)+3x^7-11$

$\boxed{a_0, a_1, a_2, \ldots \mid (a_0=-10,\ a_7=4,\ a_i=1,\ i \neq 0,7)}$

3. (c) $x \to 2a_1x - x$

$x^3 \to 2a_3x^3 - 3x^3 \quad \to \boxed{g(x) = 2f(x) - 2a_1x + x - 2a_3x^3 + 3x^3}$

(d) $x \to 2a_1x + 5x - x$

$x^3 \to 2a_3x^3 + 5x^3 - 3x^3 \quad \to \boxed{g(x) = 2f(x) - \frac{5}{1-x} + x(2a_1+4) + x^3(2a_3+2) + x^7(2a_7-2)}$

$x^7 \to 2a_7x^7 + 5x^7 - 7x^7$

7. $(x^2+x^3+x^4+x^5+x^6)^5 = (x^2)^5(\cancel{1}+x+x^2+x^3+x^4)^5 = x^{10}\left(\frac{1-x^5}{1-x}\right)^5$

$\frac{1}{(1-x)^5} = \sum_{n=0}^{\infty} \binom{n+5-1}{n} x^n \qquad (1-x^5)^5 = 1 - \binom{5}{1}x^5 + \binom{5}{2}x^{10} + \ldots$

$f(x) = \sum \binom{n+5-1}{n} x^{n+10} - \sum \binom{n+5-1}{n}\binom{5}{1} x^{n+15} + \sum \binom{n+5-1}{n}\binom{5}{2} x^{n+20}$

$\text{coef}(x^{20}) = \binom{14}{4}x^{20} + 5\binom{9}{4}x^{20} + 10\binom{4}{0}x^{20}$

$= 1001 + 630 + 10 = \boxed{1641}$

$\star$ 8. $\left(\frac{(1-x^2)}{(1-x)}\right)^n \to (1-x^2)^n = 1-\binom{n}{1}x^2+\binom{n}{2}x^4-\binom{n}{3}x^6+\binom{n}{4}x^8+\dots$

$$\left(\frac{1}{1-x}\right)^n = \sum_{k=0}^{\infty}\binom{k+n-1}{k}x^k$$

$$f(x) = (1+x+x^2)\left[\sum\binom{k+n-1}{k}x^k - \sum\binom{n}{1}\binom{k+n-1}{k}x^{k+2}+\dots\right]$$

(a) $\binom{5+n-1}{5}+\binom{n}{1}\binom{3+n-1}{3}+\binom{n}{2}\binom{1+n-1}{1}+$

$\binom{6+n-1}{6}+\binom{n}{1}\binom{4+n-1}{4}+\binom{n}{2}\binom{2+n-1}{2}+\binom{n}{3}\binom{0+n-1}{0}+$

$\binom{7+n-1}{7}+\binom{n}{1}\binom{5+n-1}{5}+\binom{n}{2}\binom{3+n-1}{3}+\binom{n}{3}\binom{1+n-1}{1}$

$= \sum_{k=0}^{2}\binom{n}{k}\binom{5-2k+n-1}{5-2k}+\sum_{k=0}^{3}\binom{n}{k}\binom{6-2k+n-1}{6-2k}+\sum_{k=0}^{3}\binom{n}{k}\binom{7-2k+n-1}{7-2k}$

(b) $\sum_{k=0}^{3}\binom{n}{k}\binom{6-2k+n-1}{6-2k}+\sum_{k=0}^{3}\binom{n}{k}\binom{7-2k+n-1}{7-2k}+\sum_{k=0}^{4}\binom{n}{k}\binom{8-2k+n-1}{8-2k}$

(c) $\sum_{k=0}^{\lfloor\frac{r-2}{2}\rfloor}\binom{n}{k}\binom{r-2-2k+n-1}{r-2-2k}+\sum_{k=0}^{\lfloor\frac{r-1}{2}\rfloor}\binom{n}{k}\binom{r-1-2k+n-1}{r-1-2k}+\sum_{k=0}^{\lfloor\frac{r}{2}\rfloor}\binom{n}{k}\binom{r-2k+n-1}{r-2k}$

9. (a) coef(x^{12}) in $(1-2x)^{10} \to \boxed{0}$

(b) coef(x^{14}) in $(x^2-5)/(1-x)^3$

$\frac{1}{(1-x)^3} = \sum_{n=0}^{\infty}\binom{n+3-1}{n}x^n \quad f(x) = \sum\binom{n+3-1}{n}x^{n+2} - \sum 5\binom{n+3-1}{n}x^n$

$= \binom{12+3-1}{12}x^{14} - 5\binom{14+3-1}{14}x^{14}$

$= 91 - 600 = \boxed{-509}$

(c) $(1+x)^4 = 1+\binom{4}{1}x+\binom{4}{2}x^2+\binom{4}{3}x^3+\binom{4}{4}x^4$

$\frac{1}{(1-x)^4} = \sum_{n=0}^{\infty}\binom{n+4-1}{n}x^n$

$f(x) = \sum\binom{n+4-1}{n}x^n + \sum\binom{4}{1}\binom{n+4-1}{n}x^{n+1}+\dots$

$= \binom{15+4-1}{15}x^{15}+\binom{4}{1}\binom{14+4-1}{14}x^{15}+\binom{4}{2}\binom{13+4-1}{13}+\binom{4}{3}\binom{12+4-1}{12}+\binom{4}{4}\binom{11+4-1}{11}$

$= 816+2720+3360+1820+364 = \boxed{9080}$

2 Q2 **6 / 13**

✓ **+ 6 pts** *Partial credit.*

3 C **13 / 13**

✓ **- 0 pts** *Correct*