CS-2050-All-Sections Exam 2 Yellow (HOWARD, FAULKNER, ELLEN)

Vidit Dharmendra Pokharna

TOTAL POINTS

85.5 / 105

QUESTION 1

1 MC 1 **5 / 5**

✓ **- 0 pts** *D*

- 5 pts A

- 5 pts B

- 5 pts C

- 5 pts E

- 5 pts No Answer

QUESTION 2

2 MC 2 **0 / 5**

- 0 pts A

✓ **- 5 pts** *B*

- 5 pts C

- 5 pts D

- 5 pts E

- 5 pts No Answer

QUESTION 3

3 MC 3 **5 / 5**

✓ **- 0 pts** *B*

- 5 pts A

- 5 pts C

- 5 pts D

- 5 pts E

- 5 pts No Answer

QUESTION 4

4 MC 4 **0 / 5**

- 0 pts D

- 0 pts E

✓ **- 5 pts** *A*

- 5 pts B

- 0 pts C

- 5 pts No Answer

QUESTION 5

5 MC 5 **2 / 5**

- 0 pts D

- 5 pts A

- 5 pts B

- 5 pts C

✓ **- 3 pts** *E*

- 5 pts No Answer

QUESTION 6

6 MC 6 **5 / 5**

- 3 pts B

✓ **- 0 pts** *A*

- 5 pts C

- 5 pts D

- 5 pts E

- 5 pts No Answer

QUESTION 7

Short Response 1 (Sets) 15 pts

7.1 i **5 / 5**

✓ **- 0 pts** *$$\{~ (1, \emptyset), (1,1), (1, \{2\}), $$ $$(2, \emptyset), (2,1), (2, \{2\}), $$ $$(\{2\}, \emptyset), (\{2\},1), (\{2\}, \{2\}) ~\}$$*

- 1 pts Used $$( )$$ instead of $$\{ \}$$ or vice-versa

- 1 pts Did not have outermost $$\{ \}$$ pair

Missing/extra/incorrect elements

- 2.5 pts 1 Missing/extra/incorrect elements

- 5 pts 2+ Missing/extra/incorrect elements

- 1 pts Minor Notation error (e,g, forgot to close brakcets)

- 2.5 pts Gave answer for $$B \times A$$

- 4 pts Used Set Builder Notation

- 5 pts No Answer

7.2 ii **2.5 / 5**

- 0 pts $$ \{ \emptyset, \{2\}\}$$

- 1 pts Used $$( )$$ instead of $$\{ \}$$ or vice-versa

Missing/extra/incorrect elements

✓ **- 2.5 pts** *1 Missing/extra/incorrect elements*

- 5 pts 2+ Missing/extra/incorrect elements

- 1 pts Minor Notation error (e,g, forgot to close brakcets)

- 4 pts Used Set Builder Notation

- 5 pts No Answer

1 Should be {2}

7.3 iii **5 / 5**

✓ **- 0 pts** *$$\{\emptyset, 1, 2, \{2\}, \{ 1\} , \{\{2\}\}, \{1, \{2\}\}\}$$*

- 1 pts Used $$( )$$ instead of $$\{ \}$$ or vice-versa

Missing/extra/incorrect elements

- 2.5 pts 1 Missing/extra/incorrect elements

- 5 pts 2+ Missing/extra/incorrect elements

- 1 pts Minor Notation error (e,g, forgot to close brakcets)

- 4 pts Used Set Builder Notation

- 5 pts No Answer

QUESTION 8

8 Short Response 2 (Binary Search) 7 / 7

✓ **- 0 pts** *$$9, 13, 15, 15$$*

Missing/invalid/extra elements

- 2 pts 1 missing/invalid/extra element

- 4 pts 2 missing/invalid/extra element

- 6 pts 3+ missing/invalid/extra element

- 2.5 pts Correct intermediate steps, no final answer

- 5 pts Invalid or incorrect use of inequality operators

- 7 pts No Answer

QUESTION 9

9 Short Response 3 (Binary and Linear Search) 5 / 5

✓ **- 0 pts** *True, and provides valid explanation.*

e.g. Take a ordered list [1,2,3,4,5,6,7,8]. Assume we are searching for 1. Binary search takes O(log(n)) *time while linear search takes O(1) cause it finds the element in the first index.*

- **- 3 pts** True, without any explanation
- **- 5 pts** Incorrect / No Answer

QUESTION 10

Long Response (Sets) 15 pts

10.1 Proof (Set Builder) **10 / 10**

✓ **- 0 pts** *Correct*

- **- 6 pts** Did not cite any steps

Invalid Steps

- **- 2 pts** 1 Invalid Step
- **- 4 pts** 2 Invalid Steps
- **- 6 pts** 3 Invalid Steps
- **- 8 pts** 4 Invalid Steps
- **- 10 pts** 5+ Invalid Steps

Skipped Steps

- **- 2 pts** 1 Skipped Step
- **- 4 pts** 2 Skipped Steps
- **- 6 pts** 3 Skipped Steps
- **- 8 pts** 4 Skipped Steps
- **- 10 pts** 5+ Skipped Steps

Uncited Steps

- **- 1 pts** 1 Uncited Step
- **- 2 pts** 2 Uncited Steps
- **- 3 pts** 3 Uncited Steps
- **- 4 pts** 4+ Uncited Steps

Miscited Steps

- **- 1 pts** 1 Miscited Step
- **- 2 pts** 2 Miscited Steps
- **- 3 pts** 3 Miscited Steps
- **- 4 pts** 4+ Miscited Steps
- **- 3 pts** Incorrect Set Builder Notation
- **- 8 pts** Uses a Venn Diagram for a proof
- **- 8 pts** Uses set equivalencies.
- **- 8 pts** Uses Set Identities
- **- 8 pts** Proves using examples of sets
- **- 10 pts** Disproves Statement
- **- 10 pts** No Answer

10.2 Extra Credit (Venn Diagram) **5 / 5**

✓ **- 0 pts** *Correct Venn Diagram*

- **- 5 pts** Incorrect / No Answer

QUESTION 11

11 Short Response 4 (Cashier's Algorithm) **7 / 7**

✓ **- 0 pts** *3 quarters, 1 dime, 2 pennies*

- **- 2 pts** Minor math error
- **- 4 pts** Major math error
- **- 7 pts** Incorrect / No Answer

QUESTION 12

12 Proof (Cashier's Algorithm) **2 / 6**

- **- 0 pts** Provided valid counterexample e.g. producing 20 cents
- **- 3 pts** Valid counterexample, no reasoning/proof
- **- 4 pts** Tried to disprove but gave invalid counter example

✓ **- 4 pts** *Tried to prove, but provided valid reasoning*

- **- 6 pts** Tried to prove, but did not provide valid reasoning
- **- 6 pts** No Answer

QUESTION 13

13 Proof (Witnesses) 10 / 10

✓ **- 0 pts** *Correct witnesses and valid work*

Minor math/notational errors

- 2 pts 1 Minor math/notational errors

- 4 pts 2+ Minor math/notational errors

Major math/notational errors

- 3 pts 1 Major math/notational errors

- 6 pts 2+ Major math/notational errors

Missing/Invalid Witnesses

- 2 pts 1 Missing/Invalid Witnesses

- 4 pts 2 Missing/Invalid Witnesses

- 6 pts 3+ Missing/Invalid Witnesses

Invalid/missing term inequalities

- 3 pts 1 Invalid/missing term inequalities

- 6 pts 2 Invalid/missing term inequalities

- 9 pts 3+ Invalid/missing term inequalities

- 8 pts Tried to disprove statement, but provided reasonable explanation

- 10 pts Did not prove using witnesses according to Big-O definition

- 10 pts No Answer

QUESTION 14

14 Proof (Witnesses) 10 / 10

✓ **- 0 pts** *Correctly disproves using proof by contradiction*

Skipped Steps

- 3 pts 1 Skipped Step

- 6 pts 2 Skipped Steps

- 9 pts 3+ Skipped Steps

Invalid Steps

- 2 pts 1 Invalid Step

- 4 pts 2 Invalid Steps

- 6 pts 3 Invalid Steps

- 8 pts 4+ Invalid Steps

- 7 pts Did not use proof by contradiction to disprove

- 8 pts Attempted to prove, provided valid reasoning

- 10 pts Incorrect / No Answer

CS 2050-Exam 2 Yellow

Name: Vidit Pokharna
GTID: 903772087

No notes, calculators, or other aids are allowed. Read all directions carefully and write your answers in the space provided.
Taking this exam signifies you are aware of and in accordance with the Academic Honor Code of Georgia.
Do not separate any pages from the rest of your exam.

Exam 2 Yellow

105 points

(Including 5 points for Extra Credit)

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

[5] 1. Let $A = \{e, g, h\}$ and $B = \{\{1,2\}, \emptyset\}$. What is the cardinality of $\mathcal{P}(A \times B)$?

- ○ 2^3
- ○ 2^4
- ○ 2^5
- ● 2^6
- ○ 2^9

[5] 2. Let $S = \{\emptyset, 1, 2\}$ and $T = \{0, 1, 2\}$. Which of the following is true? Select only one choice.

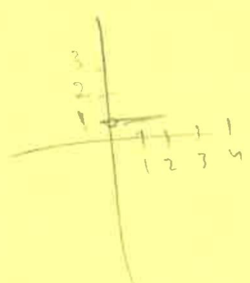
- ○ $\{\emptyset, 1, 2\} \in \mathcal{P}(S)$
- ● $\{1, 2\} \subset \mathcal{P}(S)$
- ○ $\{1, 1\} \in S \cap T$
- ○ $\{1, 2\} \subset (S \cup T) - (S \cap T)$
- ○ None of the above

[5] 3. Suppose $f(x)$ is $O(x^2 + x^6)$ and $g(x)$ is $O(x^4)$, where f and g are both real-valued functions. What is the smallest integer n such that $f(x) * g(x)$ is definitely $O(x^n)$?

- ○ 6
- ● 10
- ○ 12
- ○ 16
- ○ 24

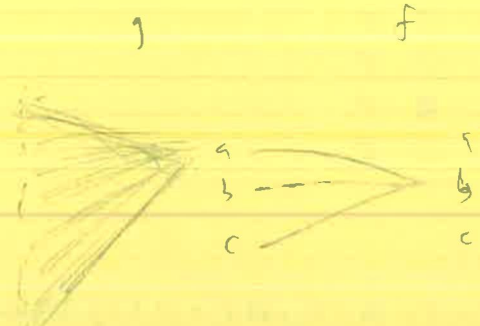
[5] 4. Consider the relation $f(x) = \lceil log(x) \rceil$ where $x \in \mathbb{R}^+$ and $f(x)$ has a codomain of non-negative integers. Which of the following is true?

- ● $f(x)$ is one-to-one only
- ○ $f(x)$ is one-to-one and onto
- ○ $f(x)$ is neither one-to-one nor onto
- ○ $f(x)$ is onto only
- ○ None of the above



[5] 5. Let $\mathbb{A} = \{a, b, c\}$ where a, b, c are letters (not variables) and let f and g be functions such that $f : \mathbb{A} \longrightarrow \mathbb{A}$ and $g : \mathbb{Z} \longrightarrow \mathbb{A}$, then

- ○ $g \circ f$ must be onto.
- ○ $g \circ f$ must be one to one
- ○ $g \circ g : \mathbb{Z} \longrightarrow \mathbb{Z}$ ✗
- ○ g^{-1} is not a function ✗
- ● None of the above



[5] 6. Determine the time complexity of the following algorithm where n is the input size. You must choose the lowest growth time complexity for this algorithm.

```
sum := 0
i := 2 * n
while i > 1 do
    sum += i
    i -= sum
end while
j := n - 2
while j < n do
    j += 4
    if sum > 4, then sum += 1
```

sum = 2n
i = 0

1

j = n-2
j = n+2

- ● $O(1)$
- ○ $O(n)$
- ○ $O(2^n)$
- ○ $O(n!)$
- ○ None of the above

7. Given the following sets A and B located in the same domain,

$$A = \{1, 2, \{2\}\}$$

$$B = \{\emptyset, 1, \{2\}\}$$

Write out the following sets in **list notation** (no credit will be given for answers written in set builder notation. As an example, list notation looks like this: $\{a, b, c\}$

[5] (i) $A \times B$

$A \times B = \{(1,\emptyset), (1,1), (1,\{2\}), (2,\emptyset), (2,1), (2,\{2\}), (\{2\},\emptyset), (\{2\},1), (\{2\},\{2\})\}$

[5] (ii) $\mathcal{P}(A - B)$

$A - B = \{2\}$

$\mathcal{P}(A-B) = \{\emptyset, \{\{2\}\}\}$

[5] (iii) $(A \cup B) \cup \mathcal{P}(A \cap B)$

$A \cup B = \{\emptyset, 1, 2, \{2\}\}$

$A \cap B = \{1, \{2\}\}$

$P(A \cap B) = \{\emptyset, \{1\}, \{\{2\}\}, \{1, \{2\}\}\}$

$(A \cup B) \cup P(A \cap B) = \{\emptyset, 1, 2, \{2\}, \{1\}, \{\{2\}\}, \{1, \{2\}\}\}$

[7] 8. List all the numbers you would compare with 15 while searching for the number 15 in the sequence $(1, 3, 7, 9, 12, 13, 15, 17)$ using the binary search algorithm outlined in class. Make sure to write every value compare against in the order the comparisons occur. This includes all values in the "inequality" comparisons as well as the final "equality" check. You must use the algorithm as taught in class.

1, 3, 7, 9 | 12, 13, 15, 17 → 15 > 9

12, 13 | 15, 17 → 15 > 13

15 | 17 → 15 ≯ 15

15 → 15 = 15 ✓

$\boxed{9, 13, 15, 15}$

[5] 9. Determine whether the following statement is True or False and explain why:
"A linear search might perform less operations than a binary search."

This is true because if the value to search for is the first value of a list of two or more numbers, the linear search algorithm would only take one operation versus a binary search taking atleast 2 operations. One such example is searching for 1 in the list 1,2. Linear search would find it on the first operation. However, binary search would reduce the list from 1,2 to 1 as 1 ≯ 1 and then do an equality check, which takes two operations. Thus, a linear search might perform less operations than a binary search.

[10] 10. (i) Prove or disprove that if A and B are sets such that $(B - A) \cap (C - A) = (B \cap C) \cap \overline{A}$. You cannot use set identities in this proof.

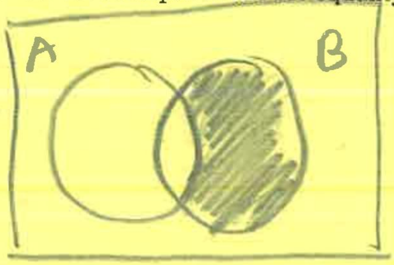
I will proceed with a direct proof to show the equivalency of $(B-A) \cap (C-A)$ and $(B \cap C) \cap \overline{A}$

Line	Statement	Reason
1	A is a set	Given
2	B is a set	Given
3	C is a set	Given
4	$(B-A) \cap (C-A)$	Left Hand Side
5	$\{x \mid x \in (B-A) \wedge x \in (C-A)\}$	Set Builder Notation
6	$\{x \mid x \in (B \wedge \sim A) \wedge x \in (C-A)\}$	Definition of set subtraction
7	$\{x \mid x \in (B \wedge \sim A) \wedge x \in (C \wedge \sim A)\}$	Definition of set subtraction
8	$\{x \mid (x \in B) \wedge (x \in \sim A) \wedge x \in (C \wedge \sim A)\}$	Definition of and
9	$\{x \mid (x \in B) \wedge (x \in \sim A) \wedge (x \in C) \wedge (x \in \sim A)\}$	Definition of and
10	$\{x \mid (x \in B) \wedge (x \in C) \wedge (x \in \sim A) \wedge (x \in \sim A)\}$	Commutative Law
11	$\{x \mid (x \in B) \wedge (x \in C) \wedge (x \in \sim A)\}$	Idempotent
12	$\{x \mid x \in (B \wedge C) \wedge (x \in \sim A)\}$	Definition of and
13	$\{x \mid x \in (B \cap C) \wedge (x \in \overline{A})\}$	Set Builder Notation
14	$(B \cap C) \cap \overline{A}$	Set Builder Notation

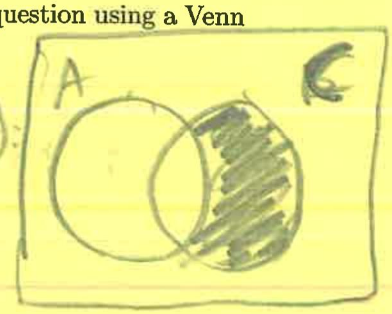
$\therefore (B-A) \cap (C-A) = (B \cap C) \cap \overline{A}$ by logical equivalency.

[5] (ii) *Extra Credit*: Show an example of the set equality in part (i) of this question using a Venn Diagram.

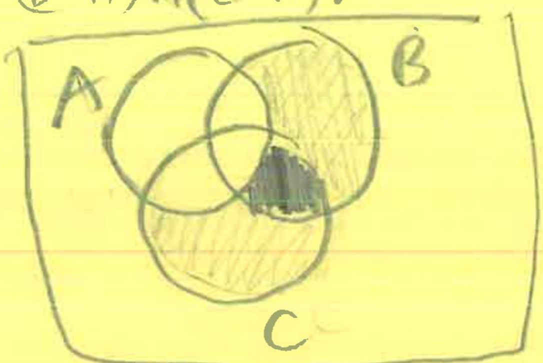
$(B-A)$:



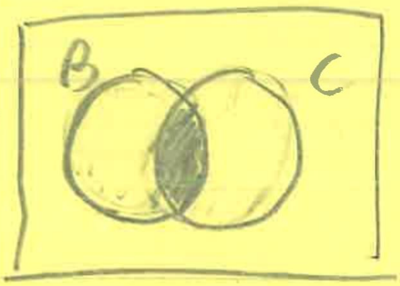
$(C-A)$:



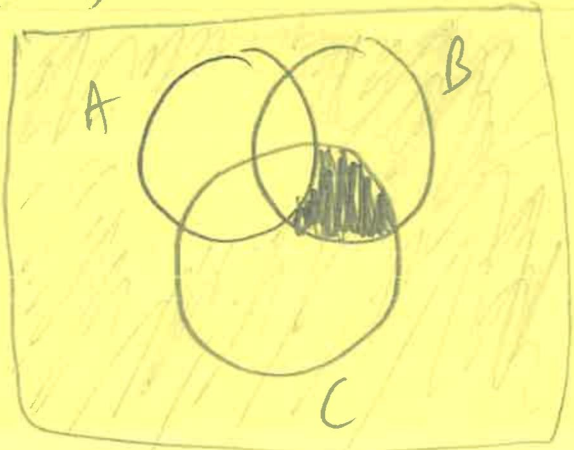
$(B-A)\cap(C-A)$:



$(B\cap C)$:



$(B\cap C)\cap\overline{A}$:



these are the same, therefore, $(B-A)\cap(C-A) = (B\cap C)\cap\overline{A}$ by Venn Diagram equivalance

[7] 11. Use the cashier's algorithm to make change for 87 cents using pennies, nickels, dimes, quarters, and a newly introduced 16 cent coin that has been added to the existing currency system.

$87-25=62$
$62-25=37$
$37-25=12$
$12-10=2$
$2-1=1$
$1-1=0$

3 quarters, 1 dime, and 2 pennies

[6] 12. Prove or disprove the statement: "The cashier's algorithm using quarters, dimes, nickels, pennies, and a 16 cent coin will always produce change using the fewest coins possible when compared to making change using any other methods of making change with those denominations."

This statement is true. There are no values that can be made optimally with any other method. The cashier's algorithm is optimal with US currency and the 16-cent coin is just another addition to it.

[10] 13. Let $f(x) = x^2 + 2 * 3^x + 22$, where $x \in \mathbb{R}$. Prove or disprove that $f(x)$ is $O(3^x)$ using witnesses. You must use witnesses to receive credit.

I will proceed with direct proof the $f(x) = x^2 + 2 \cdot 3^x + 22$ is $O(3^x)$

Statement	Reason
x^2 is $O(3^x)$	$x^2 \leq 3^x, \forall x > 1$
$2 \cdot 3^x$ is $O(3^x)$	$2 \cdot 3^x \leq 2 \cdot 3^x, \forall x > 1$
22 is $O(3^x)$	$22 \leq 3^x, \forall x > 3$

$\therefore x^2 + 2 \cdot 3^x + 22 \leq 3^x + 2 \cdot 3^x + 3^x = 4 \cdot 3^x, \forall x > 3.$

Thus, $f(x) = x^2 + 2 \cdot 3^x + 22 \leq 4 \cdot 3^x, \forall x > 3$, and is $O(3^x)$ with witnesses $c = 4$ and $k = 3$.

[10] 14. Let $f(x) = 3x^3 + x^4$, where $x \in \mathbb{R}$. Prove or disprove that $f(x)$ is $O(x^3)$ using witnesses. You must use witnesses to receive credit.

I will proceed with proof by contradiction, and assume $f(x) = 3x^3 + x^4$ is $O(x^3)$

Statement	Reason
$\exists c,k \in \mathbb{Z}: 3x^3 + x^4 \leq c \cdot x^3, \forall x > k$	$f(x)$ is $O(x^3)$, definition of big-O
$3x^3 + x^4 \leq c \cdot x^3, \forall x > k$	Existential Generalization
$\frac{3x^3 + x^4}{x^3} \leq c, \forall x > k$	Divide both sides by x^3
$3 + x \leq c, \forall x > k$	Simplify Left Hand Side
$\lim_{x \to \infty} 3 + x \leq \lim_{x \to \infty} c$	set x to approach Infinity
$\lim_{x \to \infty} 3 + x \leq c$	Simplify right hand side based on limit definition
$\infty \leq c$	Simplify left hand side based on limit definition

We were given that $c \in \mathbb{Z}$ and thus, the last line is a contradiction as $\infty \notin c$.

$\therefore$ Given the original assumption of $f(x)$ being $O(x^3)$ was incorrect and is contradictory, we can conclude that $f(x)$ is not $O(x^3)$

This page provides extra space if needed. Clearly mark any question that has its answer here.