MATH-3012-D HW 11

Vidit Dharmendra Pokharna

TOTAL POINTS

30 / 30

QUESTION 1

1 Q1 **10 / 10**

- ✓ **+ 1 pts** *graph (a) correct*
- ✓ **+ 1 pts** *graph (b) correct*
- ✓ **+ 1 pts** *graph (c) correct*
- ✓ **+ 2 pts** *P(G_e, k) = k(k-1)(k-2)^3*
- ✓ **+ 2 pts** *P(G_e',k)= k(k-1)(k-2)(k-3)*
- ✓ **+ 1.5 pts** *X(G) =3*
- ✓ **+ 1.5 pts** *The number of 5-coloring = P(G,5)*

QUESTION 2

2 Q2 **10 / 10**

- ✓ **+ 10 pts** *correct*

QUESTION 3

3 C **10 / 10**

- ✓ **- 0 pts** *Complete*

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Q1 11.6

4) This is true when G is a cycle of n vertices where $n \geq 4$ and is odd

6) (a)(i) $a \to \lambda$, $b \to \lambda$, $x, y, z \to (\lambda-1)$

$\lambda(\lambda-1)^3$

(ii) $a \to \lambda$, $b \to \lambda-1$, $x, y, z \to (\lambda-2)$

$\lambda(\lambda-1)(\lambda-2)^3$

(b) $P(K_{2,3}, \lambda) = \lambda(\lambda-1)^3 + \lambda(\lambda-1)(\lambda-2)^3$

$\chi(K_{2,3}) = 2$ (A and B are same color)

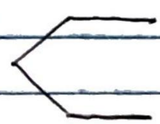
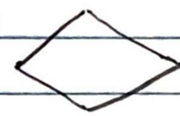
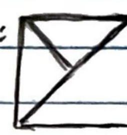
(c) $P(K_{2,n}, \lambda) = \lambda(\lambda-1)^n + \lambda(\lambda-1)(\lambda-2)^n$

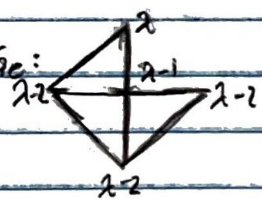
$\chi(K_{2,n}, \lambda) = 2$

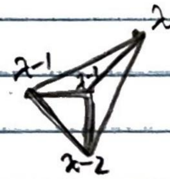
7) (a) $\chi(K_{m,n}) = 2$ (c) 11.59(d): 2 (d) 2

(b) $\begin{cases} 2, & n \text{ is even} \\ 3, & n \text{ is odd} \end{cases}$ 11.62(a): 3

11.85(i): 2

A) (a) G_e:  G_e':  (b) G_e:  G_e': 

(c) G_e: 

G_e': 

$P(G,\lambda) = P(G_e,\lambda) - P(G_e',\lambda) = \lambda(\lambda-1)(\lambda-2)^3 - \lambda(\lambda-1)(\lambda-2)(\lambda-3)$

$= \lambda(\lambda-1)(\lambda-2)(\lambda^2 - 5x + 7)$

$\chi(G) = 3$

$P(G, 5) = 5 \cdot 4 \cdot 3 \cdot 7 = 420$

Q2 11.6

10) (a) Not isomorphic; first graph has two vertices of degree 4 while the second graph has three vertices of degree 4

(b) Graph 1: $P(G,\lambda) = P(G_1,\lambda) + P(G_2,\lambda) = \lambda(\lambda-1)^2(\lambda-2)^2 + \lambda(\lambda-1)(\lambda-2)^2(\lambda-3)^2$

Graph 2: $P(G,\lambda) = P(G_1,\lambda) + P(G_2,\lambda) = \lambda(\lambda-1)^2(\lambda-2)^2 + \lambda(\lambda-1)(\lambda-2)^2(\lambda-3)^2$

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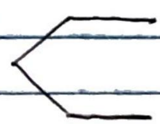
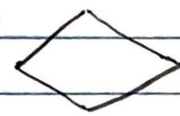
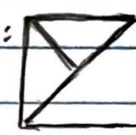
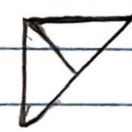
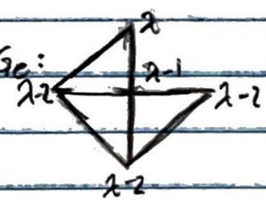
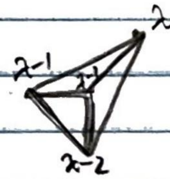
A) (a) G_e:  G_e':  (b) G_e:  G_e': 

(c) G_e: 

G_e': 

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12.1

8) (a) $2|E| = x+4(2)+1(3)+2(4)+1(5)$ $|E| = |V|-1 = x+4+1+2+1-1 = x+7$

$2E = x+8+3+8+5 = x+24$

$2(x+7) = x+24 \to 2x+14 = x+24 \to x=10$

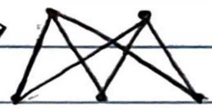
(b) $2|E| = v_1+2v_2+3v_3+\dots+mv_m$ $|E| = |V|-1 = v_1+v_2+v_3+\dots+v_m-1$

$2(v_1+v_2+v_3+\dots+v_m-1) = v_1+2v_2+3v_3+\dots+mv_m \to v_1 = v_3+2v_4+\dots+(m-2)v_m+2$

$|E| = v_2+2v_3+3v_4+\dots+(m-1)v_m+1$

$|V| = (v_3+2v_4+3v_5+\dots+(m-2)v_m+2)+v_2+v_3+\dots+v_m = v_2+2v_3+\dots+(m-1)v_m+2$

13) $K_{2,3}$ ↳

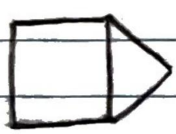


Nonisomorphic spanning trees of $K_{2,3}$ →

*These two trees are the only nonisomorphic spanning trees of $K_{2,3}$

14) When considering graph $G = K_{2,n}$, if another graph T spans G, then T must have $n+1$ edges and the two vertices making up one side of the graph would have a degree sum of $n+1$. The number of isomorphic spanning trees for $K_{2,n}$ would be the division of edges between these two vertices, resulting in $\lfloor \frac{n+1}{2} \rfloor$.

(18)

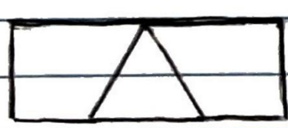


→

$k^{(2)}$

$= \dfrac{k^{(3)} \cdot k(k-1)(k^2-3k+3)}{k(k-1)}$

$= k(k-1)(k-2)(k^2-3k+3)$



→

$k^{(2)}$

$= \dfrac{k(k-1)(k-2)(k^2-3k+3) \cdot k(k-1)(k^2-3k+3)}{k^{(2)}}$

$= k(k-1)(k-2)(k^4-6k^3+15k^2-18k+9)$

2 Q2 **10 / 10**

✓ **+ 10 pts** *correct*

3 C **10 / 10**

✓ **- 0 pts** *Complete*