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1. (a) $t=0 \to 50$

$t=1 \to \underset{D}{40}, \underset{U}{60}$

$t=2 \to \underset{DD}{32}, \underset{UD}{48}, \underset{UU}{72}$

$t=3 \to \underset{DDD}{25.6}, \underset{UDD}{38.4}, \underset{UUD}{57.6}, \underset{UUU}{86.4}$

Risk free zero coupon:

$B_t = \frac{100}{(1.05)^{3-t}}$

$B_0 = 86.3838$

$B_1 = 90.7029$

$B_2 = 95.2381$

$B_3 = 100$

(b) risk neutral probability is:

$q = \frac{1.05 - 0.8}{1.2 - 0.8} = 0.625$ and $d < 1+r < u$

$0.8 < 1.05 < 1.2$ ✓

between 0 and 1 ✓

thus the market is arbitrage free

(c) At $t=3$, terminal payoffs are: $S_3 \in \{25.6, 38.4, 57.6, 86.4\}$

$\Rightarrow V_3 \in \{0, 0, \cancel{12.6}, \cancel{41.4}\}$

6.6 -1 35.4

$t=2 \Rightarrow S_2 = 32 : \max(0, \frac{1}{1.05}(0.625 \cdot 0 + 0.375 \cdot 0))$

exercise=3 $S_2 = 48 : \max(0, \frac{1}{1.05}(0.625 \cdot 12.6 + 0.375 \cdot 0)) = 7.5$ = continue

exercise: 27 $S_2 = 72 : \max(0, \frac{1}{1.05}(0.625 \cdot 41.4 + 0.375 \cdot 12.6)) = 29.1429$ = continue

-2

$t=1 \Rightarrow S_1 = 40 : \max(0, \frac{1}{1.05}(0.625 \cdot 7.5 + 0.375 \cdot 0)) = 4.4643$

exercise:15 $S_1 = 60 : \max(0, \frac{1}{1.05}(0.625 \cdot 29.1429 + 0.375 \cdot 7.5)) = 20.0255$ = continue

$t=0 \Rightarrow \frac{1}{1.05}(0.625 \cdot 20.0255 + 0.375 \cdot 4.4643) = 13.5143$

exercise: 5

$\boxed{C_0 = \$13.51}$ → never exercise early, not optimal

(d) No. As pricing the call option and found early exercise is not optimal, therefore the American call value equals European call value and has no dividends before maturity. Selling at 13.51 and delta hedging leaves no riskless profit. If up probability becomes 0.9 if $S_n > 55$, it does not create arbitrage because pricing and hedging use the risk neutral measure.

-10

2. (a) $P(TT) = P(T|A)^2 \cdot P(A) + P(T|B)^2 \cdot P(B)$

$= 0.4 \cdot 0.4 \cdot 0.5 + 0.65 \cdot 0.5 \cdot 0.65 = 0.29125$

$P(HH) = 0.24125$

$P(HT) = 0.4675$

$E[\$2 \to T] = P(TT) \cdot (2) + P(TH) \cdot (0) + P(HH)(-2)$

$= 0.1$

So $E[\$2 \to T] = 50 \cdot 0.1 = \boxed{\$5}$

10/10.

(b) After seeing tails:

$P(A|T) = \dfrac{P(T|A) \cdot P(A)}{P(T)} = \dfrac{0.4 \cdot 0.5}{0.525} = 0.3810$

$P(B|T) = \dfrac{P(T|B) \cdot P(B)}{P(T)} = \dfrac{0.65 \cdot 0.5}{0.525} = 0.6190$

$P(A|H) = \dfrac{P(H|A) \cdot P(A)}{P(H)} = \dfrac{0.6 \cdot 0.5}{0.475} = 0.6316$

$P(B|H) = \dfrac{P(H|B) \cdot P(B)}{P(H)} = \dfrac{0.35 \cdot 0.5}{0.475} = 0.3684$

$P(NT)$ = probability of tails on next toss

$P(NT)(+1) + (1 - P(NT))(-1) = 2P(NT) - 1 > 0$

$P(NT) > 1/2$

strategy: compute $P(NT)$ before every play

bet \$1 on tails if $P(NT) > 0.5$, otherwise skip that round for betting

4/10

3. (a) $\boxed{\$2}$ $\rightarrow$ \$4/2 10

1.17

2.14

3.16

47

(b) 24.3827 23.7807 23.1935

1 cent 1 cent 1 cent

$2500 \cdot 0.01 \cdot e^{-rt}$

3 months

2500 gpu hours

sum = 71.3571

+4.2500 =

$10071.35707 \approx \boxed{\$10071.36}$

6