MGT 6078 – Finance and Investments
Assignment 2

Vidit Pokharna, Anjalika Arora, Krishaang Gupta

October 9, 2025

1. Trading Securities

$P_0 = \$20$, $N = 1000$, own equity $E_0 = \$15000$, loan $L_0 = \$5000$, loan rate $i = 8\%$

(a) Immediate (no interest):

$$E_1 = NP_1 - L_0, \quad R = \frac{E_1 - E_0}{E_0} = \frac{N(P_1 - P_0)}{E_0} = \frac{4}{3} \cdot \frac{P_1 - P_0}{P_0}$$

$$\boxed{P_1 = 22: \;\; R = +13.33\%} \quad \boxed{P_1 = 20: \;\; R = 0.00\%} \quad \boxed{P_1 = 18: \;\; R = -13.33\%}$$

(b) Maintenance margin $m = 25\%$:

$$\frac{NP - L_0}{NP} = m \;\Rightarrow\; P_{\text{call}} = \frac{L_0}{N(1-m)} = \boxed{\$6.67}$$

(c) If only \$10000 of own money ($L_0 = \$10000$):

$$\boxed{P_{\text{call}} = \frac{10000}{1000(0.75)} = \$13.33}$$

(d) One-year (with interest):

$$L_1 = L_0(1+i) = \$5400, \quad E_1 = NP_1 - L_1, \quad R = \frac{E_1 - E_0}{E_0}$$

$$\boxed{P_1 = 22: \;\; R = +10.67\%} \quad \boxed{P_1 = 20: \;\; R = -2.67\%} \quad \boxed{P_1 = 18: \;\; R = -16.00\%}$$

Relationship: approximately $R \approx \frac{4}{3} \cdot \%\Delta P \; - \; \underbrace{\tfrac{iL_0}{E_0}}_{2.67\%}$

(e) One-year margin call (interest accrued):

$$P_{\text{call}} = \frac{L_1}{N(1-m)} = \frac{5400}{1000(0.75)} = \boxed{\$7.20}$$

The portfolio's return equals 1.33 times the percentage change in Xtel's price, reduced by 2.67% to reflect the marginal borrowing cost.

2. Risk and Return

(a) Annual real yield on the 10Y TIPS zero:

$$84.49 = \frac{100}{(1+y)^{10}} \;\Rightarrow\; y = \left(\frac{100}{84.49}\right)^{1/10} - 1 = \boxed{1.6997\%\text{ p.a.}}$$

(Continuously compounded $r_f = \ln(1+y) = \boxed{1.6854\%\text{ p.a.}}$)

(b) CC annual risk premium on real estate:

$$\mu_c = 4\ln(1+0.02) = \boxed{7.9208\%\text{ p.a.}}, \qquad \pi = \mu_c - r_f = \boxed{6.2354\%\text{ p.a.}}$$

3. Preferences

Mean–variance with risk aversion $A = 4$ and a risky market with mean *excess* return μ_M and standard deviation σ_M:

$$y^* = \frac{\mu_M}{A\sigma_M^2} \qquad \Rightarrow \qquad \text{Equity weight } = y^*, \quad \text{T-bills weight } = 1 - y^*$$

(a) **1927–2021:** $\mu_M = 0.0887,\ \sigma_M = 0.2025$

$$\sigma_M^2 = (0.2025)^2 = 0.04100625, \qquad A\sigma_M^2 = 4 \times 0.04100625 = 0.164025$$

$$y^* = \frac{0.0887}{0.164025} = 0.5408 \text{ (to 4 d.p.)}$$

$$\boxed{\text{Equity } = 54.08\%, \quad \text{T-bills } = 45.92\%}$$

(Sharpe check: $S = \mu_M/\sigma_M = 0.0887/0.2025 = 0.4388 \approx 0.44$*)*

(b) **1975–1998:** $\mu_M = 0.11,\ \sigma_M = 0.144$

$$\sigma_M^2 = (0.144)^2 = 0.020736, \qquad A\sigma_M^2 = 4 \times 0.020736 = 0.082944$$

$$y^* = \frac{0.11}{0.082944} = 1.3262 \text{ (to 4 d.p.)}$$

$$\boxed{\text{Equity } = 132.62\%, \quad \text{T-bills } = -32.62\% \text{ (borrow at } R_f)}$$

(Sharpe check: $S = \mu_M/\sigma_M = 0.11/0.144 = 0.7639 \approx 0.76$*)*

(c) Because the later period has a much higher Sharpe ratio (about 0.76 vs. 0.44), the optimal risky share $y^* = \mu_M/(A\sigma_M^2)$ is much larger: with $A = 4$ the investor would lever the market in 1975–1998, but hold a roughly balanced stock/T-bill mix over 1927–2021.

4. Mean–Variance Investing

Tangency (optimal) portfolio

$$E[r_p] \approx \mathbf{4.19}\%, \quad \sigma_p \approx \mathbf{5.63}\%, \quad \text{Sharpe} \approx \mathbf{0.56}$$

Given the imposed Min/Max bounds, the attainable frontier exists only for targets of approximately **4%–7%**. Targets at **1%–3% are infeasible**: with positive minimum allocations to several high–return sleeves and caps on low–return bond sleeves, the constraint set forces a floor on $w'\mu$ near 4%. Solver therefore cannot satisfy $E[r_p] = 1\%, 2\%, 3\%$ simultaneously with the bounds and $1'w = 1$.