Dynamic Delta Hedging - Interim Project 1 Report

This project implements a dynamic delta-hedging workflow for European call options. The primary objective is to validate Black–Scholes analytical pricing and daily hedge rebalancing on two fronts: (1) simulated geometric Brownian motion paths under the assignment parameters and (2) historical GOOG data with observed quotes and interest rates from 2011.

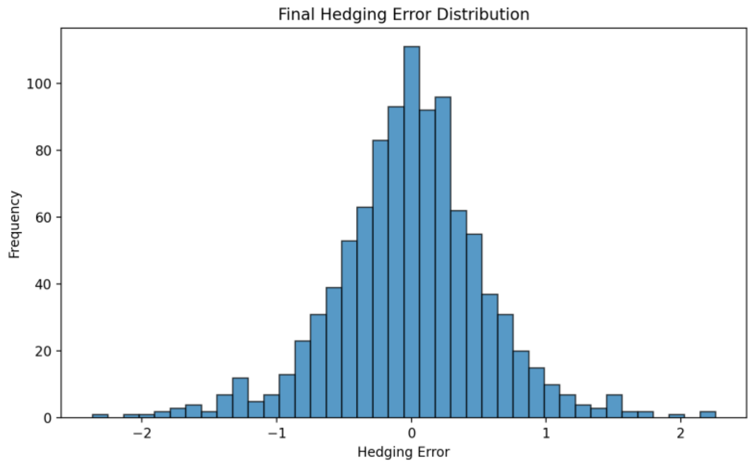
To reproduce the results, compile the project directly with g++. Compile the CLI, compile the unit tests, and run them. Use the commands from the README file. Generate CSV outputs by running ./build/delta_hedging task1 --output outputs for Monte Carlo validation, ./build/delta_hedging task2 --output outputs for the historical test, or ./build/delta_hedging all --output outputs to execute both tasks sequentially. After the CSVs are created, render plots using the helper scripts: python scripts/plot_task1.py --outputs outputs --plots plots --paths 100 for Task 1 and python scripts/plot_task2.py --outputs outputs --plots plots for Task 2.

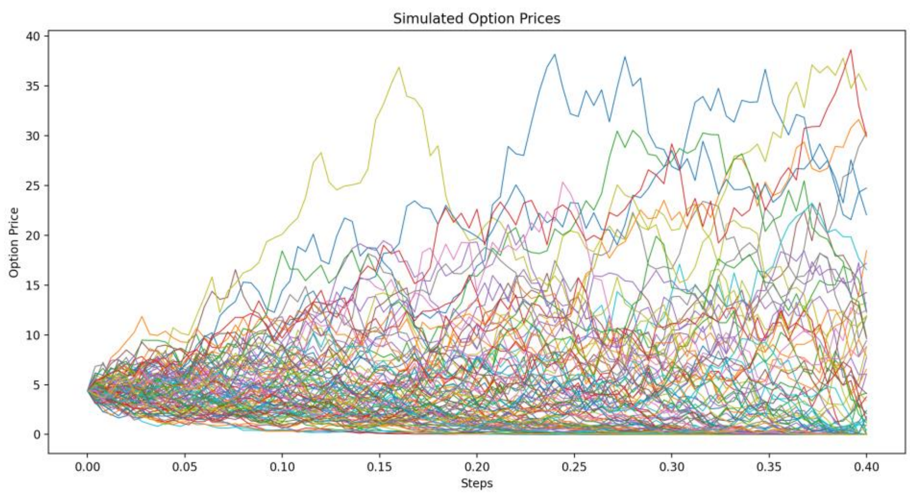
Task 1 focuses on Monte Carlo validation. The simulator creates 1,000 sample paths of a stock initialized at 100, evolving over 100 steps across a 0.4-year horizon with drift 0.05 and volatility 0.24. Analytical Black–Scholes pricing and delta are computed along every node. The hedging engine keeps a cash account and stock holdings aligned with the instantaneous delta, recording cumulative hedging errors and PnL metrics. Outputs include CSV tables for stock paths, option values, deltas, hedging errors, the time grid, and a summary of final hedging-error statistics (mean around –0.009, standard deviation roughly 0.56, and 5th/95th percentiles near –0.93 and 0.88 respectively). These CSVs allow post-processing and plot generation.

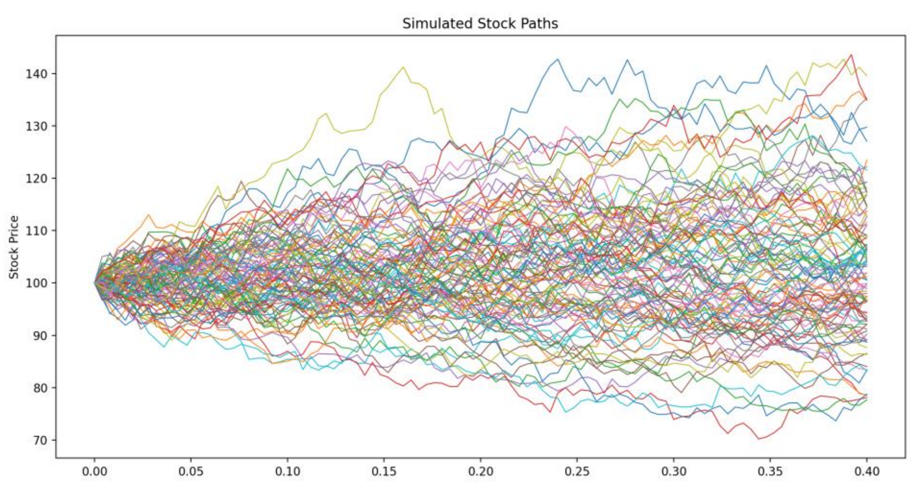
Task 2 tests the hedge on real data. Historical closing prices, interest rates, and option quotes for GOOG during July 2011 are ingested. Each row is enhanced with implied volatility via numerical inversion of Black–Scholes, the resulting delta, and annualized business-day time-to-maturity. Using the same hedging routine (with time increments derived from business-day differences), the program produces a result.csv that mirrors the grading rubric: date, stock price, option price, implied volatility, delta, hedging error, PnL without hedge, PnL with hedge, time-to-expiry, and rate. Over the selected window the hedging errors average roughly –6.83 with standard deviation near 4.98, reflecting a sizable residual mismatch when the market deviates from model assumptions. The CSV makes it straightforward to examine the hedge distribution or compare against buy-and-hold strategies in downstream analytics packages.

Several extensions are suggested for future work. Incorporating transaction costs, discrete financing spreads, or dividend-adjusted drift could add realism to the hedging simulation. Additional unit tests might cover hedging-error invariants and numerical stability of the implied-volatility solver under extreme option prices. Finally, adding R or MATLAB plotting scripts would satisfy alternative visualization preferences without modifying the core C++ codebase.

Task 1 Plots


Final Hedging Error Distribution
Frequency
Hedging Error


Simulated Option Prices
Option Price
Steps


Simulated Stock Paths
Stock Price
Steps

Task 2 Plots

	date	stock	option	implied_vol	delta	hedging_error	pnl_call	pnl_with_hedge	tau	rate
1	2011-07-05	532.44	44.2	0.259686	0.722654	0	0	0	0.210317	0.0019
2	2011-07-06	535.36	46.9	0.269337	0.733318	-0.592419	-2.7	-0.592419	0.206349	0.0019
3	2011-07-07	546.6	55.3	0.269861	0.787576	-0.752541	-11.1	-0.752541	0.202381	0.002
4	2011-07-08	531.99	43.95	0.268563	0.719396	-0.912005	0.25	-0.912005	0.198413	0.0017
5	2011-07-11	527.28	41	0.27595	0.691509	-1.35265	3.2	-1.35265	0.194444	0.0017
6	2011-07-12	534.01	46.4	0.286788	0.722767	-2.10098	-2.2	-2.10098	0.190476	0.0018
7	2011-07-13	538.26	49.3	0.287137	0.745056	-1.93166	-5.1	-1.93166	0.186508	0.0016
8	2011-07-14	528.94	41.15	0.272178	0.706903	-0.727831	3.05	-0.727831	0.18254	0.0015
9	2011-07-15	597.62	99.65	0.28149	0.940744	-10.6797	-55.45	-10.6797	0.178571	0.0015
10	2011-07-18	594.94	97.65	0.300413	0.926444	-11.2037	-53.45	-11.2037	0.174603	0.0015
11	2011-07-19	602.55	103.8	0.266228	0.96029	-10.3063	-59.6	-10.3063	0.170635	0.0017
12	2011-07-20	595.35	97.8	0.299707	0.931925	-11.2236	-53.6	-11.2236	0.166667	0.0019
13	2011-07-21	606.99	108.15	0.275666	0.964257	-10.7295	-63.95	-10.7295	0.162698	0.002
14	2011-07-22	618.23	118.7	0.24842	0.985999	-10.4452	-74.5	-10.4452	0.15873	0.002
15	2011-07-25	618.98	119.95	0.294243	0.971589	-10.9596	-75.75	-10.9596	0.154762	0.002
16	2011-07-26	622.52	123.25	0.28697	0.978582	-10.8241	-79.05	-10.8241	0.150794	0.0021
17	2011-07-27	607.22	108.65	0.304805	0.957691	-11.2006	-64.45	-11.2006	0.146825	0.0021
18	2011-07-28	610.94	112.1	0.302652	0.964977	-11.092	-67.9	-11.092	0.142857	0.0021
19	2011-07-29	603.69	106.8	0.370045	0.924709	-12.7922	-62.6	-12.7922	0.138889	0.002