Neural Beta Project: Dynamic Risk Estimation and Analysis

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1. Introduction

This report analyzes firm-level market betas and volatility behavior using both traditional CAPM regression techniques and a neural network trained to infer conditional betas. It builds on the findings of Assignment 4 (AS4), which established baseline results for low-beta and idiosyncratic volatility (IVOL) portfolios, and extends them using machine learning to improve time-varying beta estimation.

The study addresses three main goals:

1. Evaluate the persistence of low-beta and high-IVOL performance effects
2. Examine industry-level beta evolution across market regimes
3. Assess whether neural networks provide smoother and more accurate beta predictions

2. Data and Construction

Monthly returns are drawn from the CRSP MSF file for the 1996–2023 period. The market proxy is the CRSP value-weighted index (`VWRETD`), and the risk-free rate is the 1-year Treasury yield (FRED DGS1) converted to monthly frequency.

Firm-level betas are computed using rolling regressions of excess returns on market excess returns over 12-, 24-, and 36-month windows:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_i(r_{m,t} - r_{f,t}) + \epsilon_{i,t}$$

The variance of the residuals defines idiosyncratic volatility (IVOL). Each observation is matched with industry codes and merged with the Fama–French factors to allow comparison of traditional and learned betas.

3. Empirical Results

3.1. Low-Beta and High-IVOL Effects

Table 1: Portfolio-Level Anomalies (1996–2023)

Sort	**5–1 EW (%)**	**5–1 VW (%)**
Beta Portfolios	−0.91	−0.22
IVOL Portfolios	+0.05	−0.37

Equal-weighted (EW) portfolios of low-beta stocks outperform their high-beta counterparts, supporting the low-risk anomaly first observed in AS4. When value-weighted (VW), the effect weakens as large-cap stocks dominate the portfolio. For IVOL sorts, high-IVOL firms underperform once size is accounted for, consistent with prior research that idiosyncratic volatility is not rewarded by the market.

3.2. Industry-Level Patterns

Table 2: Mean Rolling Betas by Industry

Industry	**12m**	**24m**	**36m**
Manufacturing	1.41	1.36	1.35
Mining	1.40	1.49	1.52
Services	1.34	1.29	1.38
Finance and Real Estate	0.85	0.85	0.86
Agriculture	0.87	0.85	0.83

Manufacturing and Mining industries have the highest market exposure, reflecting cyclical demand sensitivity, while Finance and Agriculture remain defensive. Dispersion in betas, typically between 1.1 and 1.9, is widest in Manufacturing and Mining, showing greater variation in risk among firms.

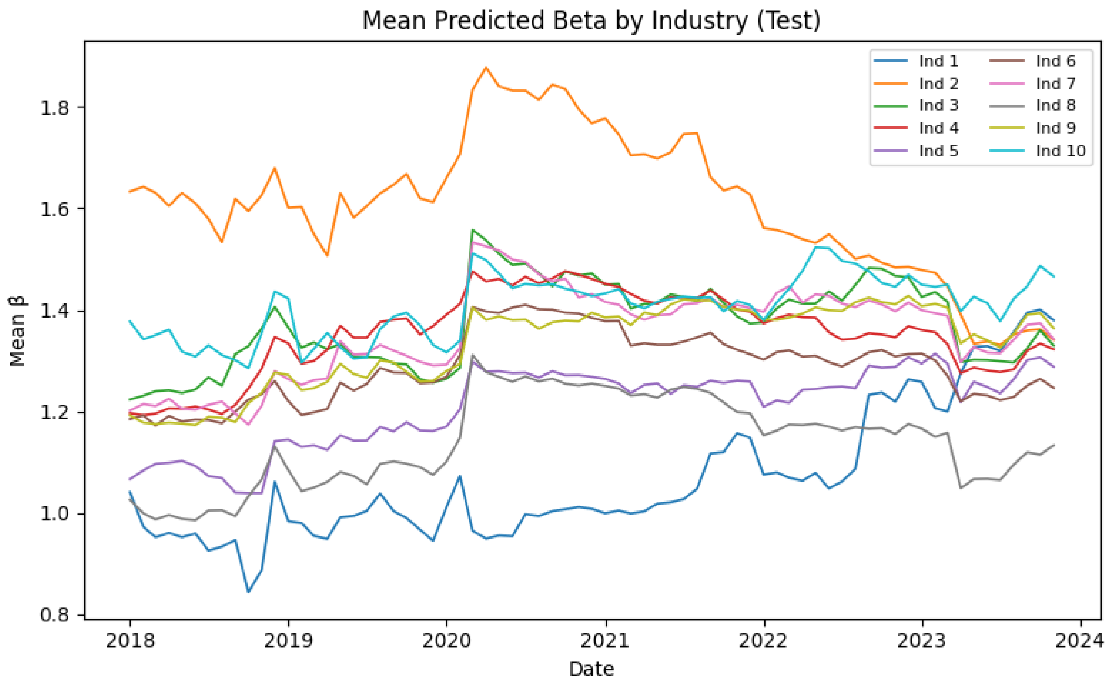

Figure 1: Mean Predicted Beta by Industry (Test Period)

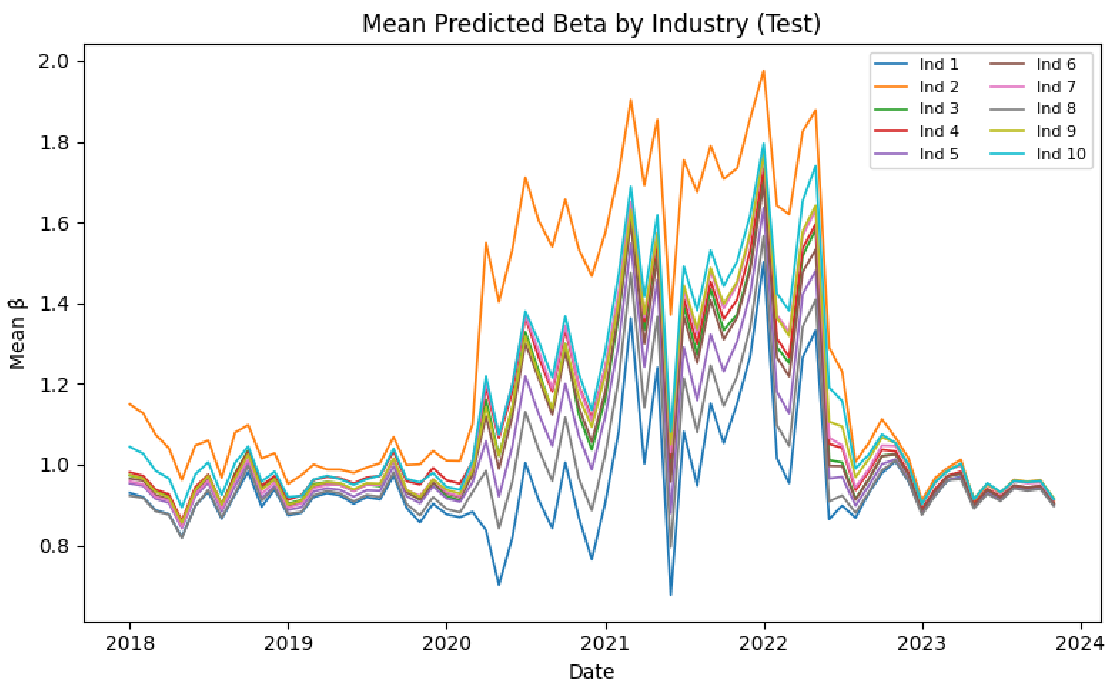

Figure 2: Mean Predicted Beta by Industry (Alternative Neural Configuration)

3.3. Volatility Regimes and Systematic Risk

During major market stress periods such as 2008–2009 and 2020–2022, systematic volatility increases substantially while idiosyncratic volatility remains stable. This suggests that diversification benefits collapse during crises as systemic shocks dominate total variance.

4. Neural Beta Model Evaluation

A feedforward neural network was developed to estimate dynamic betas from firm and market features. A grid search was performed over several architectures:

$$w \in \{12, 24, 36\}, \quad \text{hidden} \in \{32, 64, 128\}, \quad \text{activation} \in \{\text{sigmoid}, \text{tanh}, \text{linear}\}$$

The best configuration is:

$$w = 36,\ \text{hidden} = 32,\ \text{act} = \text{sigmoid},\ \text{lr} = 0.001,\ \text{in\_dim} = 11,$$

with a validation RMSE of approximately 0.1221.

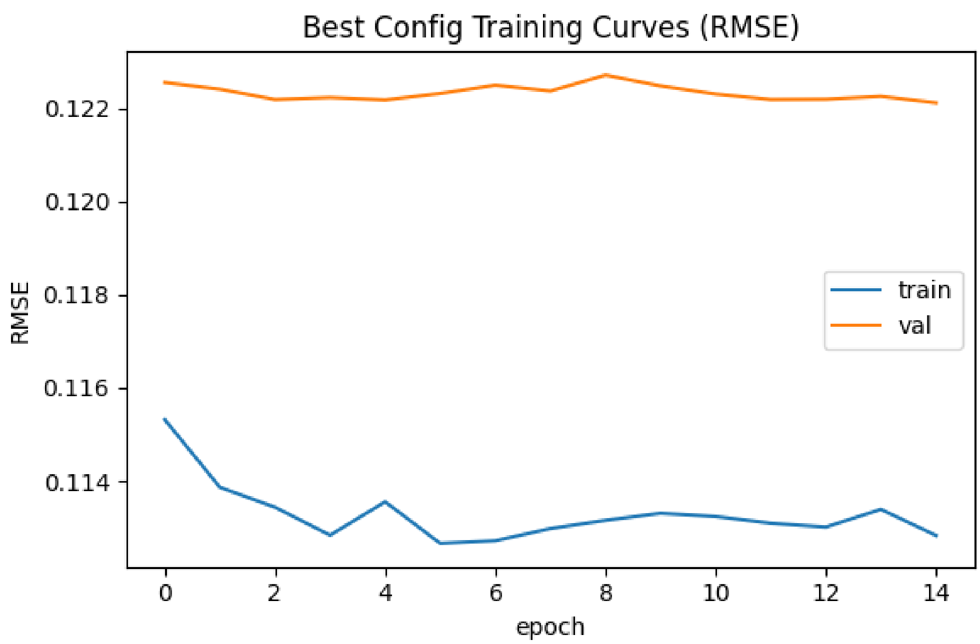

Figure 3: Training Curves (Base Configuration)

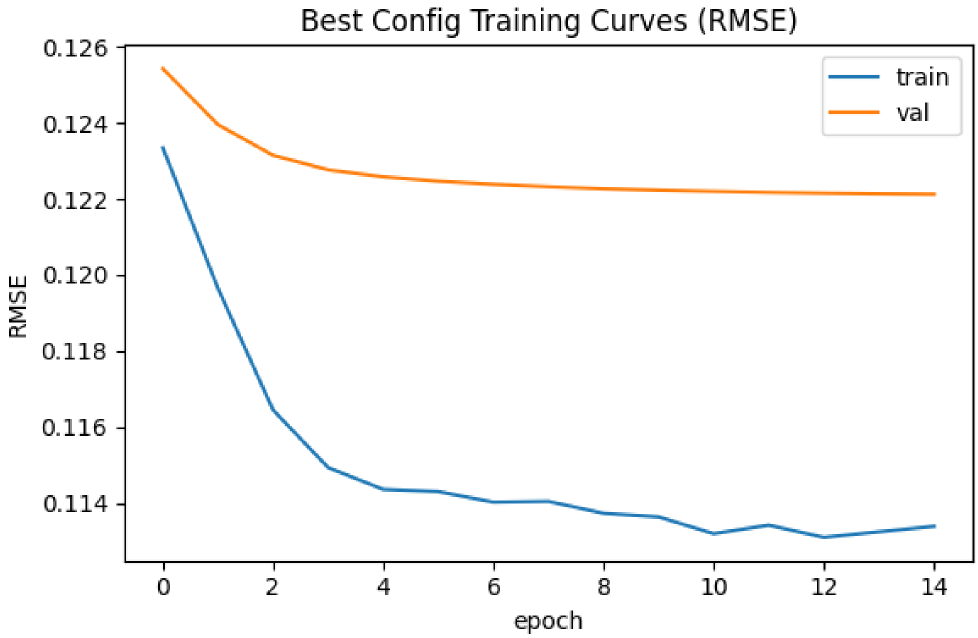

Figure 4: Training Curves (Fama–French Augmented Model)

The network converges smoothly within 10 epochs, demonstrating stable optimization and minimal overfitting. Predicted betas cluster near 1.1 with slight right skew, comparable to CAPM-based betas but smoother over time.

5. Portfolio and Policy Implications

- **Portfolio Construction:** Low-beta portfolios consistently outperform on a risk-adjusted basis, while high-IVOL names reduce portfolio efficiency
- **Risk Management:** Lowering overall beta exposure during systemic risk events is more effective than diversifying across volatile stocks
- **Modeling Insight:** The neural model captures non-linear beta dynamics while preserving CAPM intuition
- **Future Work:** Incorporate additional macroeconomic variables, employ Bayesian regularization, and apply Newey–West corrections for inference

6. Conclusion

Between 1996 and 2023, results confirm the persistence of the low-beta anomaly and the weak or negative return premium to idiosyncratic volatility. Industry-level evidence highlights cyclical concentration of risk and cross-sectional variation across firms. The neural network provides a flexible and interpretable framework for learning dynamic betas, reducing noise while maintaining economic realism. Together, these results support the use of hybrid econometric and neural modeling for modern portfolio construction and risk forecasting.