# Regime-Based Factor Allocation

An Empirical Study of Regime-Based Factor Timing Strategies in the US Stock Market

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# Abstract

This thesis investigates whether regime-based factor timing strategies can outperform a static diversified multi-factor model. It combines business cycle regimes with factor investing in the US stock market using data from 1998-2021. The business cycle is split into four regimes, Recovery, Expansion, Slowdown, and Contraction. This paper analyzes factor performances in these regimes to find the optimal factor allocation in each regime. The paper found the Size and Value factors to be the factors with the best performance under the Recovery regime. Momentum was the best performing factor in both the Expansion and Slowdown regimes, whereas the factors, Quality and Minimum Volatility, showed superior returns in the Contraction stage. Two regime-based factor timing strategies have been constructed in this paper, namely the Regime-Based Mean-Variance Model and the Regime-Based Factor Model. Both models showed significant annual outperformance relative to the static Equally Weighted Model by 0.95 and 2.55 percentage points, respectively. Taking transaction costs into account for the Regime-Based Factor Model, the annualized return was reduced by 0.31 percentage points. Even after accounting for transaction costs, this dynamic model still showed significant outperformance relative to the static model.

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# 1. Introduction

Active trading is often mentioned as a zero-sum game, and excessive trading can be directly linked to investor overconfidence. Even rational and not-overconfident portfolio managers could trade excessively to signal their clients and employers that they are worth their fees (Vliet, 2018). This leads us to The Efficient Market Hypothesis (EMH), which states that all publicly available information is immediately priced into securities, implying that active investing would not be profitable (Malkiel, 2003). The EMH is, throughout this thesis, challenged since this paper aims to investigate whether an active investment strategy based on factor indices and macro regimes is superior to a static portfolio.

Several researchers, such as Fama and French, 1992, and Carhart, 1997, have constructed models that could successfully explain some of the cross-section of stock returns. The idea behind the factors is that the factors are the only measures in determining the expected return of an asset. This led to the concept of factor investing, where only the factor risk is considered and not the unsystematic risk that comes from individual firms, which should not be compensated since it can be diversified away (Munk, 2020).

There has been a high interest in factor investing, which has been widely discussed since the global financial crisis in 2008. Here, institutional investors questioned whether their portfolios were adequately diversified (Varsani and Jain, 2018, p.5). Investors use factor indices to seek an excess return, minimize their volatility, and enhance diversification. However, the returns of these single factors typically vary a lot through different periods. Therefore we found it interesting to investigate if it is possible to time the allocations to different factors via a dynamic multi-factor allocation. For example, a value-based investment strategy has performed well in some periods and underperformed in others (Varsani and Jain, 2018, p.5). Also, factors such as the Value and Size factors have tended to be exposed to common macro risks, and they are therefore more cyclical than other factors. So, firms with high accounting value relative to their market value and small-sized firms tend to have high betas and move in conjunction with the overall market.

On the other hand, Low Volatility and Quality have more defensive characteristics (Varsani and Jain, 2018, p.7). As a result, they have lower market betas and are less affected by systematic risk than cyclical factors. Hence, the macro environment needs to be considered when choosing the most optimal allocation to different factors.

The idea of active factor allocation would be to harvest a premium of the optimal factors in good times and then avoid suffering considerable losses in bad times when the optimal factor allocation has changed. With the construction of regime-based dynamic factor models, it would be possible to time the exposure to various factors based on the historically optimal factor allocation.

A motivation for this paper is to understand the link between financial markets and the macroeconomy, and we still think there is a lack of research in this field. Because of this, this paper aims to investigate whether an investment strategy based on macro regimes can outperform a static investment strategy. This thesis differs from other findings in this area with a more nuanced view on dynamic factor allocation by extensively investigating the factor performances under each macro regime. In addition, this paper differs from other papers by analyzing whether outperformance can be made by combining classic portfolio optimization with dynamic factor timing.

## 1.1 Problem Statement

This paper is set to investigate and construct a dynamic factor model which determines the factor loadings in a portfolio based on the state of the economy. The idea is to create a model that chooses among the different factors based on the optimal allocation in each regime. More specifically, the purpose is to investigate whether a dynamic factor model can outperform a static model.

Now, the purpose of this thesis can be formulated as the following overall research question:

**Can regime-based factor timing strategies outperform a static diversified multi-factor model?**

To answer the overall research question, the following sub-questions are essential to analyze:

- How can the business cycle in the US be used to categorize different regimes as the basis for further factor analysis?
- How do the single factors perform across the different regimes?

- How can a multi-factor strategy be constructed to exploit the regime-dependent factor characteristics?
- How robust is the performance of a regime-based factor timing strategy?

First, the business cycle is split into four different macro regimes that will be the foundation for further factor analysis. This will make it possible to explore how the factor performances differ across the regimes. After the single factor analysis, different multi-factor models can be constructed, and a portfolio equally allocated to all the factors is constructed. This model will be defined as the static diversified multi-factor model for comparison purposes. Then the regime-based factor timing models are constructed with an in-depth performance analysis. After the performances of the regime-based strategies have been evaluated, a robustness test will be conducted for the best-performing model. By answering these sub-questions throughout the paper, we are able to answer the overall research question.

## 1.2 Delimitation

Throughout this thesis, some important delimitations have been made.

The data used in this thesis is the factor indices constructed by MSCI. Therefore, we are constrained by using their factor indices instead of eventually adding other relevant factors to include in our analysis. However, the factors used by MSCI are well-known and highly relevant to include. Also, the premade factor indices by MSCI are very reliable, so we are not constructing the indices on our own since it wouldn't add more value than the premade factor indices by MSCI. Additionally, an investment strategy based on the factor indices constructed by MSCI raises the replicability for investors and readers of this paper.

The time horizon is from January 1998 until December 2021. This horizon is chosen because of the data constraints from MSCI, where this was the most extended time horizon with complete data on all five factor indices. This period is considered long enough to do our analysis, including both an estimation period and a test period. Furthermore, we are constrained by the factor definitions and methodologies applied by MSCI. Some researchers might perceive some factors as slightly different from MSCI, eventually leading to minor differences when categorizing stocks into different factor indices.

One assumption is that short selling is not allowed, so we only take long positions when constructing the factor models. In practice, an investor would be able to short sell, but there are some constraints on that, and this thesis does not consider short selling as an option other than in one theoretical subsection.

In practice, factor indices are often tracked by Exchange Traded Funds (ETFs). For example, if an investor pursues a value strategy, she could buy a share of an ETF tracking the MSCI USA Value Index. The relevant cost for holding this ETF would be the Total Expense Ratio, the total annual fee for holding the ETF. However, in this thesis's constructed dynamic factor models, we are not considering the Total Expense Ratio but rather the transaction costs. The reason is that transaction costs are the costs that differ the dynamic factor models from a static factor model since the dynamic model is reallocating its factor exposures through regime changes. The reallocations to different factors result in transaction costs that are considered later in the analysis.

Taxes are not considered in this thesis since they are very investor-dependent. However, taxes do play a significant role in the investor's final return, but it is difficult to take into account. In addition, the taxation varies depending on the investor type and the investor's country of origin.

A final delimitation is that this thesis is restricted by OECD's Composite Leading Indicator (CLI). Their definitions of regimes are applied during our paper since their CLI is well-known and very reliable. We did not find it relevant to create our own indicator because of this. Researchers applying different macroeconomic indicators might categorize the regimes differently, but we found the OECD CLI to be robust and reliable.

## 1.3 Structure of the Thesis

The thesis will start with a review of the most relevant literature. It begins with the traditional relation between risk and return, and after that, it goes through the market beta and the five factors applied in this paper. The section then moves on to a review of other papers investigating Regime-Based Factor Allocation strategies. Afterward, it will further investigate the different macroeconomic regimes. Section 3 describes the data and period applied in this thesis. This section includes how MSCI constructs the factor indices, and it goes through the individual components of the Composite Leading Indicator from OECD. Next, in section 4, the methodologies applied in the analysis are shown, including the economic regimes classification and a subsection with a review of the portfolio performance measures.

Additionally, section 4 explains how the factor portfolio strategies have been constructed and how the robustness test has been made. Section 5 consists of the complete empirical analysis, starting with an analysis of how the factors have performed in the test period. Next, two static factor models are analyzed. The first one is an Equally Weighted Model, which will be used as the primary benchmark in this paper. The second static model is a Mean-Variance optimized model. After that, two regime-based factor allocation strategies are analyzed: a mean-variance optimized model and a model constructed after a more in-depth analysis of the factors across the regimes in the estimation period. Next, the constructed models are compared, where the best model is exposed to different robustness tests. The thesis will then discuss the results of the analysis, the regime-dependent components of the models, and their implications for the future. Finally, a conclusion is made together with a suggestion for further research on this topic.

# 2. Literature Review

We will start this section with an essential mean-variance framework and portfolio theory. Afterward, we will move on to the relevant factors. These factors will be the basis of our further research and analysis, specifically as steppingstones to further model extension. Finally, the factors will be used to construct a dynamic factor model. A factor model allows us to test the risk premium on any individual asset through the asset's exposure to several priced factors that are common for all assets (Munk, 2020).

## 2.1 Risk-Return Relationship

There is a tradeoff between risk and return in financial theory. The investor typically wants to minimize the risk of its portfolio but, on the other hand, maximize its expected return. Generally, the greater the risk is, the higher the return that can be realized. However, the relation is between the *expected* return and risk, not the *actual* return. Expected means what will occur on average; hence the expected return is a weighted average of the possible returns. The expected returns and probabilities can be assessed subjectively or estimated from historical data (Hull, 2018, p.2).

The expected rate of return can, in mathematical terms, be expressed as:

$$E[r] = \sum_{s=1}^{S} p_s r_s \tag{1}$$

This is the weighted average of possible returns as previously described, where $p_s$ are the probabilities of each possible outcome and $r_s$ are the possible returns of each outcome.

A common way to quantify the risk of an investment is to calculate the variance of the return over a period:

$$Var[r] = E(r^2) - (E[r])^2 \tag{2}$$

Here, r is the return over the period, and E symbolizes the expected value, so E(r) means the expected return over a certain period. Hence, the variance is the expectation of the squared return less the squared expected return. So, to get the standard deviation of $r$, we have to take the square root of the variance:

$$Std[r] = \sqrt{Var[r]} \tag{3}$$

The standard deviation is also called the volatility in the case of returns, and this number cannot be negative (Munk, 2020).

Now, we have identified the expected return and standard deviation of the return of individual investments, and now we can consider a portfolio, which is where we combine assets. The rate of return is calculated as:

$$r_p = w \cdot r_1 + (1 - w) \cdot r_2 \tag{4}$$

Where $r_1$ and $r_2$ are the expected return of this portfolio consisting of just two assets. In this equation, $w$ denotes the fraction of the total portfolio invested in asset 1. Then the rest *(1-w)* is invested in asset 2 since the weights have to sum to 100% if we assume everything is invested in this portfolio. However, the weights can also be negative, which would correspond to a short position in that specific asset. As we can see from equation 4, the rate of return on a portfolio is a weighted average of each asset's rate of return.

Equation 4 was for a portfolio consisting of just two assets, but we generalize the way of calculating portfolio returns of portfolios consisting of more than two assets (Munk, 2020):

$$r_p = \sum_{i=1}^{N} w_i \cdot r_i \tag{5}$$

The standard deviation or the volatility of a portfolio depends on each asset's standard deviation of returns, the correlation between each asset, and the weight or fraction that is invested in each asset in the portfolio:

$$\sigma_p = \sqrt{w^2\sigma_1^2 + (1 - w)^2\sigma_2^2 + 2w(1 - w)\rho\sigma_1\sigma_2} \tag{6}$$

Where $w$ denotes the fraction of the total portfolio invested in asset 1, and *(1-w)* is invested in asset 2. $\sigma_1$ and $\sigma_2$ are the standard deviations of the return of the two assets in this portfolio, and the correlation coefficient, $\rho$, is the specific correlation between asset 1 and asset 2.

We know that for a portfolio of no short positions, the portfolio standard deviation is less than the weighted average of the assets' standard deviations because of the asset's correlation coefficient. So, we can reduce the risk of the portfolio by diversifying. The smaller the correlation coefficient is, the more significant the portfolio risk reduction. If we have some assets in the portfolio, the risk will decrease a lot if we add new assets that are negatively correlated with the existing assets. However, this newly added correlation lowers the potential in good periods, but it also reduces the downside risk in bad periods.

The correlation coefficient measures the linear relationship between, for example, two assets, and a correlation of 1 would mean the assets are perfectly correlated. The coefficient can be in the interval from -1 to 1. If the correlation coefficient is positive and the return of asset 1 is high, then the return of asset 2 will also be high. And if the return of asset 1 and low, then the return of asset 2 is also low. Conversely, if the correlation coefficient is negative and the return of asset 1 is high, then the return of asset 2 will be low. A low correlation between assets means that more risk will be diversified away. However, it is also possible to significantly reduce the portfolio risk even with a high and positive correlation if we go short in one of the assets. Mathematically the correlation, *Corr* or $\rho$, between two assets can be calculated as:

$$Corr(r_1, r_2) = \frac{Cov[r_1, r_2]}{Std[r_1] \cdot Std[r_2]} \tag{7}$$

Where the covariance is calculated as

$$Cov[r_1, r_2] = E[r_1, r_2] - E[r_1]E[r_2] \tag{8}$$

(Munk, 2020).

### 2.1.1 Mean-Variance Portfolio Theory

The original mean-variance principles were formulated by Harry Markowitz (1952), where he published his Modern Portfolio Theory. In a mean-variance analysis, the investor only cares about the mean and the variance over a fixed, future period. The Markowitz model describes how to select the portfolio with the highest possible expected return given an acceptable level of risk, where the risk is measured by the standard deviation of the portfolio's return. The model can also explain how to obtain the lowest possible risk by a specific required expected return (Vaclavik and Jablonsky, 2011).

The central idea in his mean-variance portfolio theory is that the market risk of a financial instrument can be reduced by incorporating this instrument into a portfolio of instruments. Hence, the composed portfolio will end up with a lower risk than the sum of the risks of each individual component. The important thing to notice is that each instrument is not selected individually but is instead considered how each financial instrument changes in price relative to how every other instrument in the portfolio changes in price.

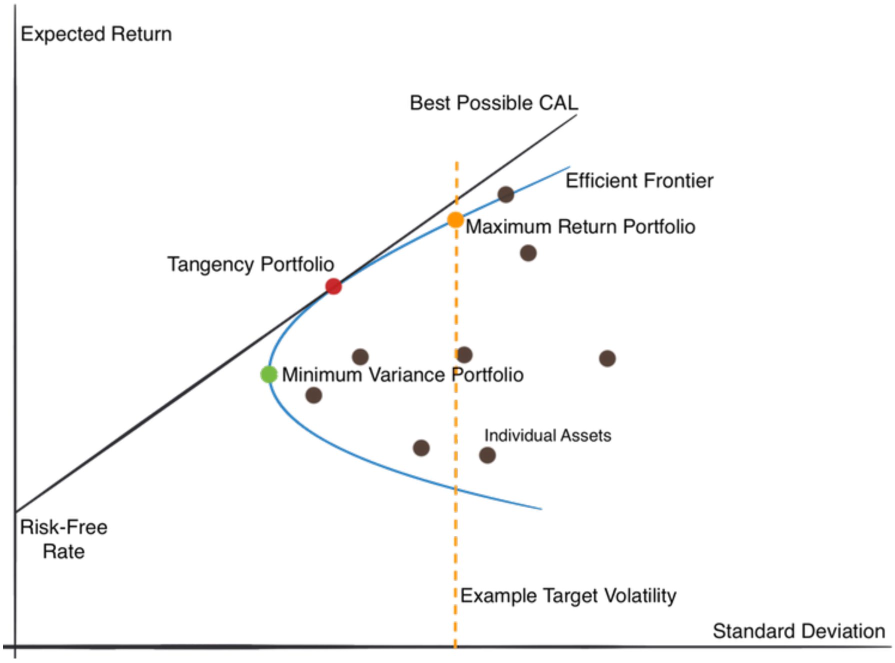


Figure 1: The efficient frontier by Harry Markowitz Source: https://quantpedia.com/markowitz-model/

In figure 1, we can see different individual assets or sub-optimal combinations plotted as the black dots because they do not provide enough expected return for their level of risk. We can obtain and draw the efficient frontier when combing some of these risky assets into a portfolio. The investor has now diversified her portfolio and reduced the asset-specific or idiosyncratic risk. This efficient fronter is a hyperbola that represents different possible portfolios with all the combinations of the individual assets that result in efficient portfolios. Efficient portfolios have the lowest risk given a specific return or the highest return given a specific risk. In order to reach the efficient frontier, a portfolio takes advantage of different correlations between the assets. This could, for example, be assets with negative covariances so that the assets move in the opposite direction of each other, which lowers portfolio volatility because a decrease in one of the assets will be offset by another asset in the portfolio. The red dot in the figure is the optimal portfolio with the highest possible Sharpe Ratio, which is called the tangency portfolio.

The expected return and the standard deviation of the tangency portfolio are calculated as:

$$\mu(\pi) = \pi * \mu \tag{9}$$

$$\sigma(\pi) = \sqrt{\pi * \underline{\underline{\Sigma}}\pi} \tag{10}$$

Where the portfolio weights are calculated by:

$$\pi_{tan} = \frac{\Sigma^{-1}(\mu - r_f 1)}{1 \cdot \Sigma^{-1}(\mu - r_f 1)} \tag{11}$$

(Munk, 2020). Where $\boldsymbol{\mu}$ is a vector of the expected return of the risky assets, and the variance-covariance matrix is represented by $\boldsymbol{\Sigma}$.

The Sharpe Ratio is calculated as:

$$Sharpe\ ratio = \frac{E_r - r_f}{\sigma_{ex}} \tag{12}$$

So, the Sharpe Ratio, sometimes called the excess return-to-risk, tells us how much expected return we can get relative to the risk taken, defined by the standard deviation of the excess return on that asset.

We can draw a tangent from the tangency portfolio point on the efficient frontier to the Y-axis at the rate of return equal to the risk-free rate (often defined as a riskless government bond). This line is called the capital market line, which combines the risk-free asset and the tangency portfolio of risky assets. The intercept of this line is then the risk-free rate, and the slope is the Sharpe Ratio of the tangency portfolio (Munk, 2020). The idiosyncratic risk of the risky assets is the difference in standard deviation between the non-optimal portfolio of risky assets and the capital market line. If an investor is looking for a higher return than the red point at the tangency portfolio, she can borrow at the risk-free rate so that the weight in the tangency portfolio exceeds 100%.

## 2.2 Factor models

The theory of factor models is the basis for this thesis, and it will be applied extensively. A factor model explains the risk premium on any individual asset by the asset's exposure to pricing factors common for all assets. These factors then become the only measures in determining the expected rate of return on an asset. Also, the entire return covariance across assets comes from the exposure to the common factors. Hence, the factor exposures of firms can be determined by the amount of their return variance that can be explained by the covariance between their returns and the factor returns. The variation in returns of different assets can be decomposed into a systematic component and a non-systematic component. The systematic component comes from the factors, and it should be compensated by a higher return. The non-systematic component comes from the asset-specific risk, which should not be compensated since it can be eliminated by forming a well-diversified portfolio (Munk, 2020).

### 2.2.1 Capital Asset Pricing Model - The market beta

The Capital Asset Pricing Model, CAPM, was suggested and analyzed by Jack Treynor (1961), William Forsyth Sharpe (1964), John Lintner (1965), and Jan Mossin (1966). The model is built on Markowitz's mean-variance modern portfolio theory, initially developed in a one-period economy. The CAPM can explain a relationship between systematic risk and expected asset return. The risk in the CAPM is explained by the beta coefficient, which is the only factor that differentiates firms from each other.

$$E[r_i] = r_f + \beta_i(E[r_m - r_f]) \tag{13}$$

Where $E[r_i]$ is the expected rate of return on asset i, and $r_f$ is the risk-free rate, which is typically equal to a 10-year government bond and accounts for the time value of money. $\beta_i$ is the market beta of the asset, and when the beta is equal to one, then the asset or stock has the same amount of systematic risk as the market. If beta is greater than one, the stock is riskier than the market because it has high systematic risk and can be seen as a highly cyclical stock very dependent on the market movements. In contrast, a beta less than one moves less than the market. When a stock's beta is negative, it can be seen as countercyclical because it moves in the opposite direction of the market portfolio. Beta can be calculated as

$$\beta_i = \frac{Cov[r_i, r_m]}{Var[r_m]} \tag{14}$$

$E[r_m]$ is the expected rate of return on the market portfolio, which includes all risky assets in the economy, so the sum of all risky assets is equal to the market portfolio. Hence, a particular asset has the same weight in the market portfolio as its market value divided by the sum of the market value of all the stocks. The market risk premium is calculated as $(E[r_m - r_f])$ and the risk premium of a particular asset is then the market risk premium multiplied with the market beta of the asset.

The benefit of CAPM is a high degree of simplicity because it is relatively straightforward to calculate the required rate of return of any given asset as long as the three components in the model are known, where only the market beta of an asset differs between firms. As a result, the CAPM can be beneficial in comparing different assets to industry peers or the company's historical performance.

There has also been some critique of the CAPM, with one remarkable being Roll (1977). He states that it is impossible to test the CAPM and it is impossible to observe a truly diversified market portfolio (Roll, 1977). His critique is known as "Roll's critique." According to his critique, a true market portfolio would need to include all investments in all markets, which is impossible. Therefore the market portfolio would always be a proxy, even when using the S&P 500 as a proxy for the market return. In addition, the simplicity of using only three inputs means that a slight variation in one of the inputs will significantly impact the output. Hence, the usefulness of the CAPM can be questioned.

### 2.2.2 Arbitrage Pricing Theory

There have been other researchers to expanding the CAPM. Ross (1976) introduced the Arbitrage Pricing Theory (APT) as a more flexible version of the CAPM. APT assumes a linear relationship between various factors and the return of an asset instead of CAPM, which only looks at the overall market as the only factor to explain asset returns. APT suggests that one could use many different macro- or market-based factors to explain the return of an asset, and only little is explained by the CAPM market beta (Ross, 1976). Asset prices and returns are simply a result of exposure to different factors. Hence, an investor could replicate the return of one asset by getting exposure to the same factors differently. APT led the way for many other academics in their search to explain asset returns. Some of the greatest findings stemming from APT are Fama & French (1992) with their original Three-Factor Model.

### 2.2.3 Size

Banz (1981) suggests that the CAPM is misspecified and that small New York Stock Exchange (NYSE) firms have significantly larger risk-adjusted returns than large NYSE firms. His sample includes all common stocks quoted on the NYSE for at least five years between 1926 and 1975. Even though he finds that small firms have significantly larger risk-adjusted returns than large firms, he states no theoretical foundation for the size effect. He says that it is unknown whether the factor is size itself or if the size is just a proxy for one or more factors correlated with size. Hence, the size effect exists, but it is not clear why it exists, and he suggests further research should consider the relationship of size to other factors, and the tests should be expanded to include Over-The-Counter (OTC) stocks as well (Banz, 1981). After Banz investigated the size effect, Fama & French (1992) included the size factor and established their three-factor model. They called the size factor "small-minus-big" (SMB) because they were creating a zero-investment portfolio consisting of a long position in the stocks of small firms and a short position in stocks of large firms measured by the market capitalizations of firms. The market capitalization of a firm is also called the market value of equity, which is calculated by the market price of the stock times the number of outstanding shares. The empirical analysis from 1992 by Fama and French did not precisely explain why their SMB-factor performed well. However, in 1996 they suggested that the reason for the investor demanding a premium for investing in small stocks stems from that small value stocks are more likely to undergo financial distress under recessions (Fama and French, 1996). Investors generally value returns higher under recessions than under booms, so investors should be compensated for investing in small firms with relatively small market capitalizations. They could therefore be less robust under economic recessions.

A Zero Investment Portfolio Strategy is often used to construct factors such as the SMB-factor. This strategy makes the investor's position neutral because the investor does not have a total long or short position because the two positions cancel each other out. Hence, the investor does not need any initial funding when she goes long in a certain portfolio and short in another with the exact same market value. For example, when the investor buys the top 30% of small firms and shorts the top 30% of large firms, the investor has exposed herself to the SMB-factor, where she hopes the long position of her portfolio, which means the small firms, will outperform the large firms.

### 2.2.4 Value

In their famous book, Security Analysis, Graham and Dodd (1934) introduced the investment philosophy of value investing, which has become a widely used and considered way of investing. The idea of value investing is to buy cheap stocks (value stocks) and sell expensive stocks (growth stocks). Whether a stock is considered cheap or expensive depends on the fundamental characteristics of the firm. One widely used method to measure the relative value of a stock is to look at the book value of the equity and divide this number by the market value of the equity, denoted as the book-to-market ratio (B/M). The book value of equity represents the fundamental value of the equity, where the market value of equity is the current paid amount for equity. The first researchers to investigate the value factor was Stattman (1980) and later Rosenberg, Reid, and Lanstein (1985). Stattman found that the stock returns are positively correlated with book-to-market ratios, and his results hold true after adjusting for the exposure to market risk. Also, Rosenberg, Reid, and Lanstein (1985) found significant abnormal performance in the US stock market of a strategy where they buy stocks with a high book/price ratio and sell stocks with low book/price ratios.

Fama & French (1992) established a value factor with a zero-investment portfolio that goes long in high book-to-market ratios and short in low book-to-market ratios, defined as high-minus-low, HML. They argue that the value stocks can be fundamentally riskier than growth stocks, so the higher return of value stocks can simply be compensation for bearing this risk. Hence, the value risk premium could be because value stocks are more cyclical in nature than growth stocks. Growth firms often require new investments, which can be deferred until after a recession, whereas value firms suffer from unused excess capacity under recessions (Munk, 2020). Whether value investing being profitable is because of the stocks being riskier or because they result from naive investors' strategies remains an open question. The behavior of investors can be another explanation that the value premium exists. Lakonishok, Shleifer, and Vishny (1994) take this standpoint in explaining the value premium. They suggest this phenomenon of the value premium could be due to investors being naive, where they assume a trend in stock prices, so they become overly excited about stocks that have performed well in the past. So, the market participants consistently overestimate future growth rates of the stocks that have increased in their stock price in the recent periods. Therefore, the investors buy these stocks that become overpriced “glamour” stocks. Vice versa, they put forward that the investors overreact to stocks that have done very poorly in the past and oversell them, resulting in value stocks becoming underpriced. Hence, investors betting against such irrationality by buying these underpriced stocks and underinvesting in stocks that have become overpriced will outperform the market, resulting in a value premium. Lakonishok, Shleifer, and Vishny (1994) are, therefore, not arguing for value investing being riskier than growth investing, but rather this value premium exists because of the irrationality of investors. This influence of the momentum in stocks affecting the investors could result in some correlation with the momentum factor. This factor will be discussed in a later section. The same researchers also argue that the existence of the value premium can be due to institutional investors that prefer glamour stocks because of their agency problem of institutional money management, where these investors tilt towards the overpriced glamour stocks. Institutional investors view glamour stocks as "prudent" investments because they have performed well in the past, so these investments are easy to justify to sponsors as safe stocks because stocks of these firms seem unlikely to become financially distressed in the near future as opposed to value stocks. Institutional asset managers also need to perform well by achieving abnormal returns even in the short run to prevent their sponsors from withdrawing their funds. The investment managers are restricted from only looking at the long run, which prohibits them from pursuing an otherwise profitable value investment strategy (Lakonishok, Shleifer, and Vishny, 1994).

Therefore, the behavioral perspective that could have created the value factor premium conflicts with the efficient market hypothesis, which states that stock prices reflect all information and that it is impossible to consistently provide positive abnormal returns.

### 2.2.5 Momentum

The momentum strategy of buying stocks that have performed well recently and selling stocks that have performed poorly in the same period was investigated by De Bondt & Thaler (1985). Their sample was monthly return data for NYSE common stocks. They formed an equally weighted arithmetic average rate of return on all Center for Research in Security Prices (CRSP) listed securities that serve as the market index. Their sample period was 1926-1982, and they found prior losers to outperform prior winners by about 25% when doing a 3-year portfolio formation. To the researchers' surprise, this effect was observed as late as five years after portfolio formation, indicating a long-term reversal effect. They argue that most people "overreact" to unexpected and dramatic news events (De Bondt & Thaler, 1985). The momentum effect is because of the under- and overreactions of the investors to a wide extent, a behavioral phenomenon. So this factor is an outcome of fundamentals, which was partly the case for the HML-factor, which is dependent on the book value of equity. Their result conflicts with the idea of the momentum trading strategy, where the belief is that prior winners outperform prior losers. The different result could be that the momentum strategy typically has a 3- to 12-month holding period.

Narasimhan Jegadeesh and Sheridan Titman (1993) did another study. They discovered that stocks that have performed well in the recent months tend to perform well in the next month, and the other way around, with stocks performing poorly in the recent months tend to perform poorly over the next month. Therefore, they argue that the result of De Bondt & Thaler (1985) is unclear since the long-term losers outperform the long-term winners only in the month of January. Also, they have argued that the result of the contrarian portfolios by De Bondt and Thaler can be explained by the systematic risk and the size effect (Jegadeesh and Titman, 1993).

Carhart (1997) observed the result of Jegadeesh and Titman (1993) and identified that it was possible to improve the explanatory power of the multifactor models when incorporating a 12-month momentum factor. Carhart's monthly mutual fund data covers diversified equity funds from 1962 to 1993. The data is free of survivor bias since it includes all known equity funds over this period. His results show that buying last year's top-decile mutual funds and selling last year's bottom-decile funds yields an 8% annual return. He suggests three important rules for investors. The first is to avoid funds with persistently poor performance, and the second is that funds with high returns last year have higher-than-average expected returns next year but not in years after that. The third rule is that the investment costs of expense ratios, transaction costs, and load fees all directly negatively impact performance (Carhart, 1997, p. 81).

The momentum strategy can be viewed as a positive-feedback strategy because investors intend to buy stocks that recently have performed well. An example can now explain the possible correlation between the momentum and the value factor earlier mentioned. When a particular stock price increases, the momentum factor tells the investor to buy that stock because it is now a past winner. However, the rise in stock price lowers the book-to-market ratio, which from a value perspective tells the investor to sell that stock since it now has a lower B/M ratio and has become more like a growth stock. This relationship points towards a negative correlation between the momentum and the value factor, which will later be investigated in our analysis.

Some other researchers have investigated the link between the momentum strategy and macroeconomic variables. They show that the profit related to momentum strategies disappears once stock returns are adjusted for the predictability based on macroeconomic variables. The macroeconomic variables used in the empirical analysis are the value-weighted market dividend yield, default spread, term spread, and the yield on a 3-month treasury bill. They also notice that Berk et al. (1999) find that the momentum strategy's profit represents the compensation for bearing time-varying risk (Chordia and Shivakumar, 2002).

### 2.2.6 Quality

Asness, Frazzini, and Pedersen (2018) define a quality factor with characteristics such as profitability, growth, and safety, and investors should be willing to pay a higher price for these characteristics. These authors show that stocks of high quality based on these measures have shown high risk-adjusted returns. Their quality-minus-junk (QMJ) factor is based on a zero-investment portfolio strategy that goes long in quality stocks and shorts junk stocks. According to their study, this factor has earned significant risk-adjusted returns in the United States and across 24 countries. According to their quality factor, they made a general definition of the factor using stock characteristics that should command a higher price. They rewrote Gordon's growth model for this purpose to express a stock's the price-to-book value (P/B):

$$\frac{P}{B} = \frac{profitability \cdot payout\ ratio}{required\ return - growth} \tag{15}$$

These key variables are:

- Profitability is the profit per unit of book value. The higher the company's profitability, the higher the stock price should be, all else being equal. They measure profits in ways such as using gross profits, margins, earnings, accruals, and cash flows. Here, they focus on the stocks' average rank across these metrics.
- Growth is measured as the prior 5-year growth in each profitability measure. The higher the growth in the profitability measures, the higher the price should be for investors.
- The required rate of return is a measure of the safety in their formula. Investors should pay a higher price for a stock with a lower required return, which is seen as a

safer stock. They mention some required return measures, such as low volatility of profitability, low leverage, and low credit risk (Asness, Frazzini, and Pedersen, 2018).

- The payout ratio is how much profits are paid out to investors, and when this ratio increases, all else equal, the stock price should increase.

Their results suggest that quality stocks as underpriced and junk stocks as overpriced. Furthermore, the paper shows that high-quality stocks have low betas and tend to perform well during periods of extreme market distress. Therefore, these findings challenge the traditional view that higher risk implies a higher return since the junk stock seems to be riskier and yields a lower return than quality stocks. They conclude that the reason for the abnormal return of quality stocks must be due to an anomaly, data mining, or the result of a yet unknown risk factor (Asness, Frazzini, and Pedersen, 2018).

### 2.2.7 Minimum Volatility

Jensen, Black, and Scholes (1972) were the first researchers to reject the standard CAPM and suggest trading to exploit the empirical failure of the CAPM. Their study applied data for all securities listed on the NYSE from 1926 to 1966. They proved that low beta stocks outperformed high beta stocks in their expected returns. This is the opposite view of the CAPM view that assumes a positive relationship between systematic risk and expected return on assets.

Similarly, Vliet (2018) argues that empirical tests show a flat or even negative relation. Early in the 1970s, CAPM tests showed that low-risk stocks have high risk-adjusted returns. However, this low-volatility effect, also mentioned as a low-risk anomaly, did not receive much attention until the 1990s, when some more academic evidence was published. Since the 2000s, low volatility has emerged as a popular investment style (Vliet, 2018).

Walkshäusl (2013) investigated the volatility premium as low volatility stocks outperforming high volatility stocks. This outperformance is economically exceptionally large around the world, amounting on average to 12% per year. Therefore, this low volatility premium is not in line with risk-based mean-variance explanations that fundamentally suggest higher return is followed by higher volatility. This low volatility factor is closely related to the betting-against-beta (BAB) factor that Frazzini and Pedersen (2014) have constructed since low volatility stocks typically have low market betas, whereas high volatility stocks exhibit high market betas (Walkshäusl, 2013). The BAB factor is long low-beta assets and short high-beta assets scaled to the same beta using leverage.

Frazzini and Pedersen (2014) have proved the BAB factor strategy to yield significant positive risk-adjusted returns. The reason for this relationship and that high beta stocks are associated with low alphas is that when investors are constrained, they bid up assets with high betas, making the high-beta stocks overpriced and resulting in negative alphas. The constraints facing some investors are such as leverage constraints and margin requirements. Constrained investor groups are found to hold a portfolio with betas above one on average, so they might be tempted to prioritize risky assets with high betas to obtain a desired rate of return, which contributes to the low-beta stocks outperforming high-beta stocks on a risk-adjusted basis.

On the other hand, buyout funds that have access to leverage buy stocks with betas below one on average. Hence, these investors benefit from the BAB factor by applying leverage to safer assets. So, they are compensated by investors with constraints on borrowing that take the other side. The authors argue that the return on the BAB factor rivals other standard asset pricing factors such as the value, momentum, and size factors in terms of economic magnitude, statistical significance, and robustness across time periods, subsamples of stocks, and global asset classes. They have constructed their BAB factor defined with the following equation:

$$r_{t+1}^{BAB} = \frac{1}{\beta_t^L}(r_{t+1}^L - r^f) - \frac{1}{\beta_t^H}(r_{t+1}^H - r^f) \tag{16}$$

Where $r_{t+1}^L$ is the future return in a low-beta stock, $r^f$ is the risk-free rate, and $\beta_t^L$ is the beta on a low-beta portfolio. The long side has been leveraged to a beta of one, and the short side has been deleveraged to a beta of one. The BAB factor premium is the difference between excess returns on the low-beta and high-beta portfolios. Hence, the factor provides the excess return on a self-financing portfolio similar to HML and SMB (Frazzini and Pedersen, 2014, p.5).

However, Robert Novy-Marx and Mihail Velikov (2021) criticize Frazzini and Pedersen's procedure. They argue that they have used predictable biases and a non-standard beta estimation procedure that have been used as evidence to support the theory of the BAB factor. Further, Novy-Marx and Velikov (2021) argue that for each dollar invested in BAB, the strategy commits $1.05 to stocks in the bottom 1% of total market capitalization on average. The reasons behind the strong performance of the BAB factor are an overweight in the market's smallest, least liquid stocks and ignoring transaction costs and implementation issues. Also, they argue that the significant positive returns of the BAB factor can be, to a large extent, explained by its tilt towards the profitability and investment factors (Novy-Marx and Velikov, 2021).

So overall, the BAB factor and low volatility stocks have gotten much attention. However, the procedure applied to the factor construction has recently been criticized by researchers such as Robert Novy-Marx and Mihail Velikov (2021).

## 2.3 Multi-Factor-Based Asset Allocation

In the previous sections, we have researched the literature for different factors that have a single factor approach. However, it is very often seen that investors combine various factors and create a multi-factor allocation instead of a single-factor allocation. By combing different factors, investors can increase their probability of outperforming a single factor alternative. This is because the factors are not perfectly correlated, so investors can produce a balanced and diversified portfolio in a multi-factor setup, especially when combing factors with low correlations (Hass and Kirk, 2018).

### 2.3.1 Dynamic Factor Investing

Active or dynamic factor allocation is a relatively new and discussed way of investing. Dynamic factor investing involves constantly trading different types of securities over time. Also, dynamic trading involves long or short positions, where we constantly have to adjust portfolio weights. In contrast, the static equity and bond risk premiums can be obtained by only taking long positions (Ang, 2014). By some researchers, "smart beta," "alternative beta," or "exotic beta" are used, but we will stick to the term of dynamic factors in the same way as Ang (2014). In this paper, what we define as dynamic factor investing is allocating dynamically between factors in response to changing market environments. Dynamic factor investing is not the same as forecasting the future, but it allocates to factors with the most predictive power based on historical data. A dynamic factor allocation might be advantageous because historical data shows that years of outperformance in a single factor can be followed by a long period of underperformance. It would be optimal for the portfolio's return to rebalance beforehand but even after markets have moved to a new regime. However, a single factor approach can be optimal for the investor already invested with active managers (Hass and Kirk, 2018). We will do monthly rebalancing or reweighting during our paper to reflect changes in different market conditions.

### 2.3.2 Regime-Based Factor Allocation

Researchers have found a link between excess returns from factor strategies and macroeconomic indicators. Varsani and Jain (2018) investigated some macroeconomic indicators that together become an estimate for economic growth and hence, the state of the economy. The macroeconomic indicators that they have used are:

- Organization for Economic Development's (OECD) Composite Leading Indicator (CLI) is a measure of the overall state of the economy or point in the business cycle.
- US ISM Purchasing Managers Index (PMI) – an indicator of the economic health of the manufacturing sector.
- The Chicago Fed National Activity Index (CFNAI) - a monthly summary of U.S. economic growth, based on a weighted average of 85 indicators of U.S. economic activity weighted to capture the relative importance of historical fluctuations.
- The Federal Reserve Bank of Philadelphia ADS Index – released weekly, is designed to track real business conditions at high frequency. It blends high- and low-frequency information as well as stock and flow data.

MSCI applied these indicators to categorize four different states of the macroeconomy. Each state is defined based on the macro indicator's 3-month vs 12-month moving average.

| Macro State | Macro Indicator |
|---|---|
| **Recovery** | Below long-term trend and has been improving |
| **Expansion** | Above long-term trend and has been accelerating |
| **Slow Down** | Above long-term trend and has been reversing |
| **Contraction** | Below long-term trend and has been deteriorating |

Table 1: Macro States. Source: MSCI

Here, they allocate to three out of six possible factors using a top-down equal weighting, where they apply a monthly rebalancing. They found that in a recovery phase where the economy is below the long-term trend but improving, value, size, and high yield stocks seem to be the best performing factors. In the expansion phase, momentum, size, and value should be preferred, whereas, in the slowdown period, size and value stocks should be replaced by stocks of high quality and low volatility. When moving from the slowdown to the contraction state of the macroeconomy, momentum is now replaced by value. In contrast, quality and low volatility are still two of the three recommended factors to pick out of the six possible factors. The contraction phase is also characterized as beneficial holding so-called defensive factors defined by low volatility, quality, and value stocks. For example, a simulated multi-factor strategy using the PMI as a macro indicator showed a 12.76% annual return from 1986 to 2018 with an annual volatility of 14.14%. This indicates a Sharpe Ratio of 0.84, and the Index Level Turnover was 75% annually using the PMI as an indicator (Varsani and Jain, 2018).

Polk, Haghbin & de Longis (2020) defined the four stages of the business cycle based on the level and change in economic growth. For example, the recovery phase is when growth is below trend and accelerating, similar to the four stages defined by Varsani and Jain (2018) regarding level and change in economic activity. They combine a US leading economic indicator ("US LEI") and a global risk appetite cycle indicator ("GRACI") using the following rules to construct the four macro regimes:

$$Recovery_{t+1}: US\ LEI_t < LT\ LEI\ trend_t\ \&\ GRACI_t \geq MA(GRACI)_t$$

$$Ekspansion_{t+1}: US\ LEI_t \geq LT\ LEI\ trend_t\ \&\ GRACI_t \geq MA(GRACI)_t$$

$$Slowdown_{t+1}: US\ LEI_t \geq LT\ LEI\ trend_t\ \&\ GRACI_t < MA(GRACI)_t$$

$$Contraction_{t+1}: US\ LEI_t < LT\ LEI\ trend_t\ \&\ GRACI_t < MA(GRACI)_t$$

where $LT\ LEI\ trend_t$ stands for the long-term trend in the US LEI at time t, and $MA(GRACI)_t$ stands for short-term moving average in the GRACI at time t (Polk, Haghbin & de Longis, 2020, p. 12). Using this method, the authors are considering the first leg, $US\ LEI_t$ when defining the level and the second leg, $GRACI_t$ regarding the change in the economic growth. Their analysis studied the FTSE Russell Factor Indices. They found size, momentum, and partly value to have sensitivities higher than the Russell 1000 Index and much higher than a static multifactor approach. Hence, they assume these three factors to exhibit systematic risk. On the other hand, the factors quality and especially low volatility have relatively low sensitivities to the Russell 1000.

Shortly after this research, Longis and Haghbin (2020) found the size and value factors beneficial in the recovery phase, just as in the paper by Varsani and Jain (2018). Likewise, momentum, size, and value were the preferred factors in the expansion phase. However, a difference rises from the slowdown and contraction phases. The momentum factor is only chosen for the contraction phase in the paper by Longis and Haghbin (2020), whereas this factor is selected in the slowdown state for the paper by Varsani and Jain (2018). The researchers argue that momentum is a different factor from the others with less persistent fundamental characteristics because of the transitory nature of its price-based definition. They argue that momentum can outperform in the late stages of cyclical up- and downturns, which means in stages of expansion and contraction. Similarly, the momentum factor will be expected to underperform during macroeconomic turning points in the business cycle. Hence, the momentum factor will be expected to underperform under recovery and slowdown stages where price trends and fundamentals are less stable and tend to reverse. They found procyclical performance characteristics for the size and value factors, which means these factors will outperform in the recovery and expansion phase. Defensive or counter-cyclical factors are seen as low volatility and quality, with lower sensitivity to cash-flow news.

Longis and Haghbin (2020) from Invesco further simulated a long-only dynamic factor rotation strategy that seeks to rebalance factor exposures based on the expected stage of the business cycle. They used regional leading economic indicators and a global risk appetite cycle indicator. The approach applied is a bottom-up approach where individual securities are scored and ranked based on their combined factor scores. They compared this dynamic strategy to a static multifactor with equal weight to each factor. Their results showed that the Dynamic Strategy outperformed the performance of the Static Strategy by roughly 2% annualized and Information Ratios by around 0.25 compared to the Static Strategy. Their analysis was economically significant after taking capacity, turnover, and transaction costs into account, and the analysis was also robust across market cap segments and geographies. This outperformance stems from significantly lower downside capture and faster recoveries from drawdowns when the optimal tilts to the different factors change through the macroeconomic business cycles.

## 2.4 The Business Cycle

The financial markets are very much affected by the macro environment. To better understand the link between financial performance and macro regimes, we need to look more specifically at each regime in the business cycle and see what characteristics the recovery, expansion, slowdown, and contraction stages have. Typically, a full business cycle is completed after 7 to 10 years (Longis, 2019). The recurrence in business cycles means that the economy tends to revert to its long-term trend. However, we do not have regimes that can be fully predicted, and some regimes are very often going back and forth, such as expansion and slowdown. Recovery and contractions are observed 15%-20% of the time, whereas expansions and slowdowns take place 30%-35% of the time in a business cycle (Longis, 2019).

Some researchers define their regimes based on trends and GDP growth or industrial production. However, these measures can be criticized because they are released with a lag relative to their reference period. Financial markets tend to lead the real economic activity and not the other way around. Because of this, these measures are based on past information instead of predictive indicators, so criticism could be raised about how relevant these measures are for making investment decisions. Longis (2019) constructed a leading indicator using economic and financial data released in a timely manner to overcome the mentioned criticism of GDP and industrial production as indicators. The leading indicator is based on business and consumer surveys, monetary and financial conditions, and manufacturing activity. Therefore, these indicators are used in their research together with construction and housing data to provide early signals of turning points in the business cycle.

### 2.4.1 Recovery

The recovery stage is characterized as a regime where the economy is relatively weak with a high unemployment rate and relatively low inflation due to low economic activity at the very beginning of the recovery stage. For the recovery phase, economic growth is below the trend but improving. The start of this phase typically has low profit margins because investments have been limited, and the labor force has been reduced due to previously low economic activity (Longis, 2019). Therefore, politicians could be tempted to use either expansive fiscal or monetary policy to get the economic activity back on track by giving companies some incentives to invest. If monetary policy is applied, the central bank is typically buying fixedincome securities that increase the money supply, contributing to lowering the interest rate. This will help improve the economic activity through more lending and investments and increase the inflation rate that could be necessary, especially at the beginning of the recovery stage.

In addition, to help the economy recover from a recession and encourage economic growth, expansionary fiscal policy can be helpful by injecting money through government spending or increased lending to companies. For example, government spending could be by building a new bridge that could employ unemployed people. The new bridge might increase efficiency by lowering transportation time for firms and employers. This could be expensive in the short run but, in some cases, prove to help recover from a recession. An expansionary policy can also be done by lowering taxes to intend people to increase their spending, hence helping the companies' profitability, which should help the economy recover. However, suppose the expansionary policy is not timed well enough and to the right extent. In this case, side effects would rise, such as too high inflation and an overheated economy.

### 2.4.2 Expansion

The expansion phase is when the economy has improved enough to reach a level above the long-term trend, and the economic growth is still accelerating. When the government has imposed an expansionary policy in the recovery phase, many firms benefit from this policy through more investments and increased consumption when getting to the expansion stage. As a result, many companies have now strong consumer confidence with high earnings growth and profit margins. Also, consumption keeps increasing, and the unemployment rate is falling.

Inflation is increasing through an excessive amount of active money circulating in the economy, so the politicians should consider tightening their fiscal policy, and the central bank might consider tightening its monetary policy. A contractionary monetary policy could imply that the central bank is issuing bonds that investors will buy so that the supply of money will decrease, which will increase the interest rate. Hence, this contractionary policy would help in the case of an overheated economy with high inflation. Equities are found to perform well in this stage. An expansion regime can last very long until business begins to respond to an eventually increased contractionary monetary or fiscal policy (Longis, 2019).

### 2.4.3 Slowdown

The slowdown regime is where growth is above the long-term trend but decelerating, which could be a result of lagged contractionary policy. There are no longer increasing profit margins. The unemployment rate has not changed significantly yet, so there is pressure on wages because it is difficult to find new employers. This keeps the inflation yet relatively high in the short run, making it necessary for the policymakers to keep their monetary or fiscal policy tight. Hence, the slowdown continues for a certain period, and it can either result in a new expansion or contraction stage.

Longis (2019) found a slowdown regime from March 2006 until November 2007 using their leading indicator. They found a deceleration in housing activity, manufacturing, and consumer confidence. Still, they found the labor market to stay strong with an even decreasing unemployment rate and rising wages. Here, stocks, in general, were found to outperform, especially because of growth in emerging markets. However, the credit markets seemed to show signs of stress, and the lending conditions tightened.

### 2.4.4 Contraction

Contraction is where the economic growth is below the long-term trend and decelerating. The lending requirements have been tightened, and many inventories are starting to build up because of decreased consumption and investments. In the contraction phase, the unemployment rate increases because companies must fire some employees to adapt to the reduced activity level with lower demand. The inflation is now lowering, so the governments might increase their spending with an expansionary fiscal policy to support economic activity and stop the accelerating unemployment rate. Also, the central banks might use expansionary monetary policy to help the economy stabilize. In this stage, the government bonds are performing relatively well, and risky assets such as stocks are underperforming in the contraction regime (Longis, 2019).

When the expansionary policy starts to stabilize the economy, we are back in the recovery phase, the beginning of a new business cycle. A significant contraction regime took place in 2008 under the recession in the financial crisis with underperforming risky assets. In the second quarter of 2009, the quantitative easing helped the businesses and consumer confidence, which started a recovery regime with economic growth beginning to increase (Longis, 2019).

# 3. Data

This section will describe the data sources used in the analysis in further detail. Data is an essential part of every analysis and choosing the correct data is of the utmost importance for getting valid and reliable results. Therefore, the data used for the analysis is selected to maximize transparency and the robustness of the results.

The time-horizon of the dataset goes back to and includes 1998 and up until December 2021. The data is limited to 1998 to have full data series for all five factor indices.

We have all our data in US Dollars for simplicity and consistency to avoid any exchange rate effect. If we had not kept all our factor indices in the same currency, our data might have been biased by not showing the real changes in index values, but the fluctuations could result from changes in the exchange rates. Also, it was obvious for us to keep the data in USD since we are looking at the US stock market. In addition, all our factor indices are extracted as gross index levels. This allows us to calculate the total returns rather than the changes in the price indices. Furthermore, it captures and includes the dividends that some firms payout to their shareholders. If we had measured our factor indices using only prices, we would not get the complete picture of how the actual returns are from an investor’s point of view who is not only considering any stock price increases or decreases but also the dividends. Some firms do not pay dividends out to shareholders, but in a factor index, there might be firms paying out dividends, and we, therefore, want to include this effect by extracting the gross index levels and hence calculating the total returns.

## 3.1 MSCI USA

One commonly used benchmark in our analysis is the MSCI USA Index (USD). This index measures the performance of the US market's large and mid-cap segments with 629 constituents. Some of the top constituents in the index are Apple, Microsoft, Amazon, Tesla, etc. The index covers around 85% of the free float-adjusted market capitalization in the US (MSCI, 2022). The methodology used for the MSCI USA Index is based on the MSCI Global Investable Market Indices (GIMI) Methodology. This methodology provides exhaustive coverage of the relevant investment opportunity set, focusing on index liquidity, investability, and replicability. The index is reviewed quarterly in February, May, August, and November.

The purpose is to find a balance between reflecting changes in the underlying equity markets while limiting the index turnover. In addition, two times a year, the index is rebalanced in May and November, and the cutoff point of mid and large capitalizations is recalculated (MSCI, 2022). We find the MSCI USA to be a reliable and applicable index to use in our setting because the index is the closest we can get to the US market portfolio regarding the high number of large and mid-cap segments of the US market. MSCI USA is used as the Parent Index for the five following factor indices.

## 3.2 MSCI Factor Indices

To use different factor premiums as the foundation in our analysis, we have applied the already constructed factor indices by MSCI, a leader in factor indices. The indices are used for simple implementation and replicability, and the data is very reliable. For instance, this data will be applied in combination with OECD's Composite Leading Indicator, and it will also be used for other factor portfolios during the analysis.

### 3.2.1 Low Size

The MSCI Low Size index is built on the intuition that smaller companies based on market capitalization yield higher returns than larger firms because they may grow faster. The Size effect can be captured by overweighting small-cap securities through an index or an active strategy. The Size premium has over longer time horizons performed well, but in 1998-2000 and 2014-2018, the MSCI Low Size index underperformed the market. This was because the large and especially technology firms outperformed the market, and the MSCI Low Size index underweighted these stocks (MSCI, 2018).

More specifically, the MSCI Low Size Index is constructed from the MSCI country index, which in our analysis is the USA, so the MSCI USA Index is the parent index. At each rebalancing, which is semi-annually, each security in the index is assigned a weight in proportion to the inverse natural logarithm of the total issuer level market capitalization:

$$w_j = \frac{1/\log_e \textit{Total Issuer Market Capitalization}_j}{\sum_1^k 1/\log_e \textit{Total Issuer Market Capitalization}_j} \tag{17}$$

Where $w_j$ is the weight of the issuer j, and $\log_e \textit{Total Issuer Market Capitalization}_j$ is the natural logarithm of the total market capitalization of the issuer j. k is the total number of issuers in the Parent Index. In addition, a constraint factor is calculated for each constituent at each MSCI Low Size Index rebalancing. The constraint factor is the weight in the MSCI Low Size Index at the trailing rebalancing time divided by the weight in the Parent Index (MSCI, 2018).

### 3.2.2 Quality

Quality is categorized as a defensive factor by MSCI, so the factor tends to be beneficial in economic contraction regimes. Stocks high on the Quality score are classified as having low leverage, stable earnings, and high profitability. Especially, the MSCI Quality Index captures three fundamental variables: Return on Equity (ROE), Debt to equity (D/E), and Earnings Variability (MSCI, 2021). Return on Equity is calculated using the trailing 12-month earnings per share divided by the latest book value per share. Debt to Equity is calculated as the total debt divided by the book value of equity. Lastly, Earnings Variability is defined as the standard deviation of year-on-year earnings per share growth over the most recent five fiscal years (MSCI, 2021).

The final Quality score for each security is calculated by combining z-score and the three previously described fundamental variables. These variables must first be winsorized to minimize the effect from extreme values using the average values. This is done by excluding missing values and ranking the values for the securities in ascending order within each parent index. Afterward, securities below the $5^{th}$ percentile rank or above the $95^{th}$ percentile rank are set equal to the value of these two percentiles, respectively. Then a z-score is computed for each of the three variables, where this score for the ROE is calculated as:

$$z = \frac{x - \mu}{\sigma} \tag{18}$$

Where x is the winsorized variable for each security, $\mu$ is the mean of this variable after being winsorized in the MSCI Parent Index Universe. $\sigma$ is the standard deviation of the same variable in the parent index. For the Debt-to-Equity (D/E) and Earnings Variability measures, the same formula as for the ROE is used, except for putting a minus in front of the expression. The z-scores for these two fundamental variables become negative to ensure that a higher D/E or Earnings Variability gets a lower z-score because we want these variables to be as low as possible from a Quality factor point of view. The composite Z-score for each security is computed by taking the average of the z-scores of all the three variables, and the Quality Score is calculated from the composite Quality Z-scores as follows:

$$Quality\ Score = \begin{cases} 1+Z, & , \quad Z > 0 \\ (1+Z)^{-1} & , \quad Z < 0 \end{cases} \tag{19}$$

The Quality weight is calculated as:

$$Quality\ Weight = Quality\ Score \cdot Market\ Capitalization\ Weight\ in\ the\ Parent\ Index \tag{20}$$

The weights are normalized to 100%. The final security level inclusion factor is determined as the final security level weight ratio and the security level pro forma market capitalization weight in the relevant MSCI Parent Index (MSCI, 2017).

### 3.2.3 Momentum

The Momentum factor is characterized as a persistence factor because it benefits from the continuation of trends. This is because the Momentum factor buys past winners, so the factor benefits from the past winners performing well in the near future. The research done by MSCI has shown that the Momentum factor has been one of the strongest generators of excess returns when looking at historical returns (MSCI, 2021).

Companies with a high Momentum score are characterized as having high price performance over the near team, typically over 6 to 12 months. Therefore, each security is calculated as a combination of the recent 12-month and 6-month local price performance.

$$6\ month\ Price\ Momentum = \left(\left(\frac{P_{t-1}}{P_{t-7}}\right) - 1\right) - (Local\ Risk\ free\ rate) \tag{21}$$

$$12\ month\ Price\ Momentum = \left(\left(\frac{P_{t-1}}{P_{t-13}}\right) - 1\right) - (Local\ Risk\ free\ rate) \tag{22}$$

Where $P_{t-1}$ is the Security Local Price one month before the rebalancing date (T) and vice versa for the seven and thirteenth months. If the 6-month Price Momentum is not available, the Momentum value is not computed. If the 12-month Price Momentum is missing, only the 6-moth Price Momentum is applied to calculate the Momentum value. The Local Risk-free rate for the US is the 3-month T-Bill rate.

A Risk-adjusted Price Momentum is calculated by taking the previously computed Price Momentum and dividing it by the annualized standard deviation of weekly local price returns over three years. These risk-adjusted prices are then standardized into z-scores, which are combined in equal proportion and standardized to get a single Momentum combined score (C):

$$C = 6 \textit{ month Momentum Z score} \cdot 0.5 + 12 \textit{ month Momentum Z score} \cdot 0.5 \tag{23}$$

The combined Momentum score (C) is afterward standardized by calculating the z-scores to compute the standardized Momentum Z-score (Z). Next, the Momentum Z-score is winsorized in a way such that Z-scores above 3 are capped at 3, and Z-scores below -3 are capped at -3.

The Momentum Score is then computed from the Momentum Z-score as follows:

$$\textit{Momentum Score} = \begin{cases} 1 + Z, & , \quad Z > 0 \\ (1 + Z)^{-1} & , \quad Z < 0 \end{cases} \tag{24}$$

When the Momentum Score is known, the Momentum weight can be computed as the Momentum Score multiplied by the Market Capitalization Weight in the Parent Index. These weights are then normalized to 100%. The final security level inclusion factor is determined as the final security level weight ratio and the security level pro forma market capitalization weight in the relevant Parent Index (MSCI, 2017).

### 3.2.4 Value

The MSCI USA Enhanced Value Index (USD) aims to represent the performance of securities that exhibit higher value characteristics relative to their peers within the corresponding GICS sector. The idea behind value investing is that cheap stocks will outperform expensive stocks in the long term. Value is often seen as a pro-cyclical factor, where it will perform well when the economy is improving. The three ratios used to construct the MSCI Enhanced Value Index are:

- Forwards price to earnings (Fwd P/E)
- Enterprise value/operating cash flows (EV/CFO) and
- Price to book value (P/B)

The advantage of using the forward P/E is to avoid the value trap, which means avoiding stock that appears cheap at the moment but is not appreciating. The reason for using the

EV/CFO measure is to reduce the exposure to heavily leveraged companies when considering equity and debt (MSCI, 2021).

More specifically, z-scores for each of the above three variables for each security are calculated. Afterward, a composite z-score is computed by taking a weighted average of the three z-scores for each security and standardizing the composite Value z-score within each sector. The index is then ranked based on its final Value scores, which are computed as:

$$Final\ Value\ Score = \begin{cases} 1 + Z^{i}_{rel_T} & , \quad Z^{i}_{rel_T} \geq 0 \\ \left(1 - Z^{i}_{rel_T}\right)^{-1} & , \quad Z^{i}_{rel_T} < 0 \end{cases} \tag{25}$$

Here, $Z_{rel}$ is the sector relative z-score. The final weights are assigned by multiplying the final Value score and the market capitalization weights. The MSCI Enhanced Value Index consists of a fixed number of securities for each period and is rebalanced semi-annually (MSCI, 2017).

### 3.2.5 Minimum Volatility

The MSCI USA Minimum Volatility Index aims to have the lowest absolute volatility based on some specified constraints and is designed to capture the low volatility effect. The goal is to reflect the performance characteristics of a minimum variance strategy among the large and mid-cap US equity universe. Its parent index is the MSCI USA Index, and the MSCI USA Minimum Volatility Index is calculated by optimizing this index with regard to the lowest absolute risk given some constraints. Therefore, this index is characterized by historically having lower beta, lower volatility, and bias towards stocks with low idiosyncratic risk relative to its parent index. The MSCI USA Minimum Volatility Index is calculated using Barra Optimizer with a specified optimization objective and constraints such as one-way index turnover limits to a maximum of 10%. Also, the maximum weight of an index constituent will be restricted to the lower of 1.5% or 20 times the security weight in the Parent Index. On the other hand, the minimum weight of an index constituent will be 0.05%. Another constraint is that the Minimum Volatility Index sector weights will not deviate more than 5% from the sector weights of the Parent Index. The Minimum Volatility Index is rebalanced semi-annually in May and November (MSCI, 2018).

The minimum volatility strategy reduces portfolio volatility on average by 25%, with a lower drawdown compared to the broad market (MSCI, 2013). This can benefit an investor seeking to de-risk her portfolio by avoiding country, sector, and style bets while still being exposed to equity markets. In addition, the Minimum Volatility index has shown lower beta and volatility characteristics relative to the MSCI USA Index (MSCI, 2022).

## 3.3 OECD's Composite Leading Indicator

This paper has developed regime-based factor allocation strategies using OECD's Composite Leading Indicator (CLI) that will be our foundation for constructing our four different regimes. The CLI was first developed in the 1980s, where the purpose was to provide early signals of turning points in the economic cycles. The CLI is constructed as a composite of different indicators that each provide a leading indication of the evolution of the cycle that the CLI targets. OECD has chosen various specific composite indicators for each country. Regarding the United States, the individual components are:

- Work started for dwellings
- Net new orders - durable goods
- Share prices on NYSE composite
- Consumer - Confidence indicator
- Weekly hours worked in the manufacturing industry
- Manufacturing - Industrial confidence indicator (% balance)
- Spread of interest rates (% p.a.)

(OECD, 2020).

The reason for OECD choosing a composite leading indicator is to improve reliability and accuracy because the CLI provides fewer false signals and missed turning points than any of the individual components. Also, the CLI is resilient to changes that affect only one of the individual components. However, the CLI is still affected when there are changes in different sectors of the economy. Therefore, the CLI will deliver the highest probability of detecting turnings points in the GDP gap. How the CLI has been applied as the basis for categorizing the regimes in the business cycle will be explained in section 4.1.

# 4. Methodology

In the methodology section, we will present the relevant methods used later in the analysis to give an understanding of how the different models are constructed and how the entire analysis is made. In addition, this section will include the methodology of how the business cycle is being modeled, the creation of the different portfolios, performance measurements, and lastly, how we will perform a robustness test on the final model.

## 4.1 Modelling the Business Cycle

We chose OECD's Composite Leading Indicator for defining the regimes due to the well-acknowledged data work by OECD, which has done some extensive research that would have been almost impossible to replicate. It would have required much effort if we wanted to replicate or improve their CLI by using our own data collection. We doubt it would have been possible to provide a better and more reliable leading indicator of the evolution of the business cycles. Therefore, we chose to use their existing CLI as a foundation for categorizing the regimes in this thesis.

The OECD CLI can be used by looking at the level and change in this indicator, creating a 2x2 matrix on these two categories. CLI can be used to predict the development of GDP. There is a GDP gap when there is a difference between actual GDP and its long-term trend. For example, the long-term Trend GDP estimates are set to 100 in the system for all economies and months. Here, a CLI above 100 means that GDP levels will be above trend levels in six to nine months. On the other hand, a CLI below 100 implies that GDP levels will be below their long-term trend in six to nine months (OECD, 2020).

| | | CLI below/above long-term trend | |
|---|---|---|---|
| | | Below 100 | Above 100 |
| Change in month-on-month CLI | Positive | Recovery | Expansion |
| | Negative | Contraction | Slowdown |

Table 2: Categorization of four regimes based on OECD's Composite Leading Indicator (CLI). Source: Own creation.

The four stages of the economy can be categorized as Recovery, Expansion, Slowdown, and Contraction. The upper right corner in table 2 is defined as the Expansion phase, where there is a positive GDP gap that is expected to widen. It is because there is a positive change in the month-on-month CLI. The Slowdown phase is in the bottom right corner. This phase is characterized by a CLI level above the long-term trend, but there is a negative change in the CLI, so the business cycle is going downwards, and the positive GDP gap is expected to narrow. The Contraction phase is in the bottom left corner. This phase is when the CLI level is below the long-term trend, and there is a negative change. Here, we are below the trendline and moving further downwards. The last stage is the Recovery stage in the upper left corner of table 2 and is characterized by a CLI level below 100 but with a positive change, indicating the negative GDP gap is expected to narrow, and the economy is recovering (OECD, 2020).

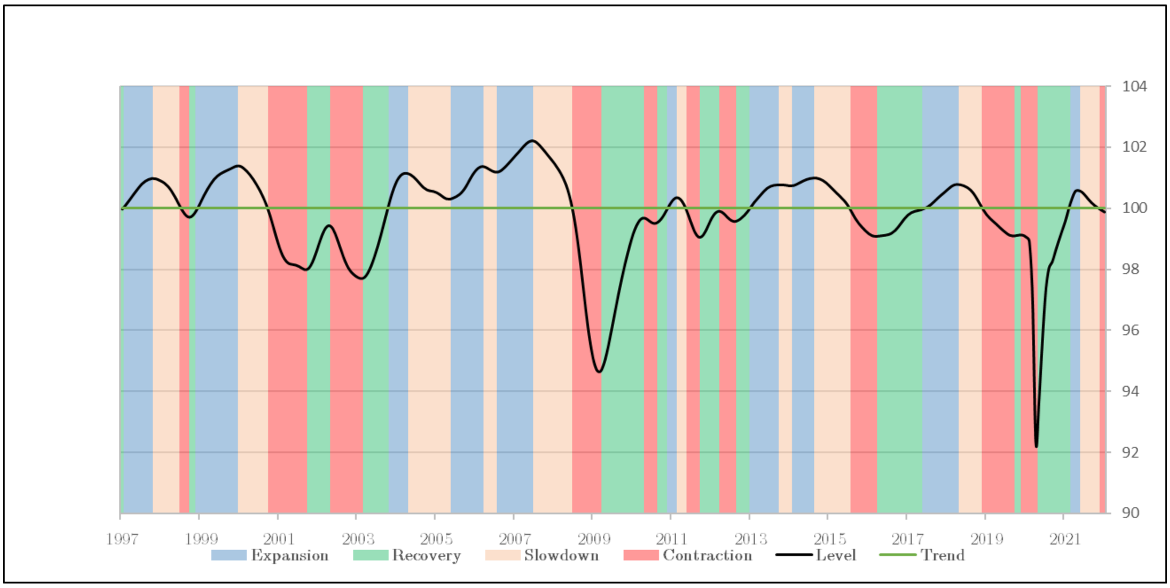


Figure 2: OECD Composite Leading Indicator (CLI) and regimes. Source: Own construction using the data from OECD CLI from 1997 to 2021.

We have illustrated the above four stages in Figure 2. The green horizontal line at level 100 is the long-term trendline. When the black line is above 100, there is a positive GDP gap, and when this gap is expected to widen, the line increases. Hence, the economy is in the Expansion phase, illustrated by the blue area. When the CLI is still above 100 but has a negative month-on-month change, we have moved to the Slowdown stage, and the light red color area illustrates this. If this negative change continues until the CLI is below 100, we have reached the Contraction regime where the CLI still has a negative month-on-month change, and we are below the long-term trend. This Contraction regime is illustrated with a dark red color in Figure 1. Finally, the Recovery is in the green area, characterized as having a positive monthly CLI change but still with a level below the long-term trendline at 100.

## 4.2 Portfolio Creation and Optimization

This thesis will produce several factor investing models, both static and dynamic. In this section, we will describe the model creation procedure and optimization. First, we will describe the estimation period used to construct the models, later tested on separate test data. Then we will show how the four models are created and rebalanced to give a deeper understanding of the coming analysis.

### 4.2.1 Model Estimation Period

To eliminate any possible estimation bias when creating our models, we split the data into two parts: an estimation period and a test period. The estimation period is the first period in which we evaluate factor performance in the different regimes and calculate the input parameters for the models. Input parameters include a return vector and variance-covariance matrix, which are being used in the mean-variance portfolio optimization.

The entire sample data we have available is 24 years, from 1998 until 2021. The estimation period chosen is from 1998 until 2011, and the test period is from 2012 to 2021. These periods are determined such that there are approximately two complete business cycles in each based on the CLI. A complete business cycle is defined as a period where we have experienced all four regimes. The cutoff point in 2012 is chosen since it is a relatively neutral point with a CLI reading close to the average of 100. Both periods are experiencing different bear markets, but the estimation period is exposed to the most severe crises, such as the burst of the dot-com bubble and the financial crisis. In the test period, we have the more recent corona crisis. Based on the CLI readings, the corona crisis seems to be more extreme than the financial crisis but with a much faster rebound, whereas the financial crisis lasted longer.

In the test period, we have experienced what might be characterized as a different financial climate than previously. For the first time and what many thought impossible, we experienced negative interest rates. With such 'low' interest rates, the discount rate for companies lowered, which increased the value of the companies. It has had a significant impact on the returns in the test period, which is one of the reasons we see much higher relative returns in the test period than in the estimation period. At the end of the estimation period, more specifically 2008, FED initialized its QE1 program, their first quantitative easing program that followed the financial crisis. Since then, they have initialized three additional quantitative easing programs, QE2, QE3, and QE4, with QE4 coming because of the corona crisis in 2020 (The American Deposit Management Company, 2022). The QE programs increase the money supply to stimulate the economy in an attempt to create economic growth and inflation. This additional money supply and the low interest rates might change how some factors perform, which will be further analyzed in section 5.1.

### 4.2.2 Equally Weighted Portfolio

An Equally Weighted Portfolio is one of the simplest forms of portfolio construction. As the name suggests, it invests an equal part in all assets in the portfolio. As a result, all the assets in the portfolio have an equal contribution of their returns to the total portfolio return. The rate of return and volatility of the portfolio can be calculated as follows (Munk, 2020):

$$r_p = \sum_{i=1}^{N} w_i r_i \tag{26}$$

$$\sigma_p = \sqrt{\sum_{i=1}^{N} w_i^2 Var[r_i] + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} w_i w_j Cov[r_i, r_j]} \tag{27}$$

where $w_i$ is the weight of each asset and $r_i$ is the asset return. $Var[r_i]$ is the variance on asset $i$, and $Cov$ is the covariance between the two assets.

It needs to be rebalanced to ensure the portfolio has constant equal weights. In this thesis, we do monthly rebalancing, so at the end of each month, we change the weights of the portfolio to $w_i = \frac{1}{N}$.

The Equally Weighted Portfolio serves as a benchmark for the dynamic models to test whether there can be generated alpha from an active rotational factor strategy. The standard MSCI USA cannot be used as the benchmark for the active models since the outperformance then might stem from simple factor investing. The Equally Weighted Portfolio will serve as a static factor portfolio benchmark.

### 4.2.3 Mean-Variance Optimization

Another static benchmark could be a mean-variance optimized portfolio. By using Markowitz's (1952) Mean-Variance approach, one should be able to find the combination of assets, in this case, the five factors which yield either the lowest volatility or the best risk-adjusted returns. This would create an portfolio optimized version of a static factor benchmark.

The returns and volatility of a mean-variance optimized portfolio can be computed as follows (Munk, 2020):

$$\mu(\pi) = \pi * \mu \tag{28}$$

$$\sigma(\pi) = \sqrt{\pi * \underline{\underline{\Sigma}}\pi} \tag{29}$$

where $\boldsymbol{\mu}$ is the return vector including returns of all assets, $\boldsymbol{\pi}$ is a vector of asset weights, and $\underline{\underline{\Sigma}}$ is the variance-covariance matrix.

The weights of the different assets in the model can be computed based on whether the investor wants to minimize the volatility or maximize the risk-adjusted returns. This thesis aims to maximize the risk-adjusted returns to get the most optimal portfolio without worrying about the investor's risk-aversion. The highest risk-adjusted return portfolio is also called the tangency portfolio (Munk, 2020). It is calculated by taking the tangent of the efficient frontier. This gives a line with the steepest slope where the level of the slope is equal to the Sharpe Ratio. Then, the optimal portfolio weights can be calculated as follows (Munk, 2020):

$$\pi_{tan} = \frac{\underline{\underline{\Sigma}}^{-1}(\mu - r_f 1)}{1 * \underline{\underline{\Sigma}}^{-1}(\mu - r_f 1)} \tag{30}$$

where $\boldsymbol{\mu}$ is the return vector including returns of all assets, $\boldsymbol{\pi}$ is a vector of asset weights, $\underline{\underline{\Sigma}}$ is the variance-covariance matrix, and $\mathbf{1}$ is a vector of one's.

The mean-variance optimal portfolio is monthly rebalanced to the weights $\pi$.

Using the mean-variance approach suggested by Markowitz (1952), one needs to be aware of some limitations and possible pitfalls. Estimating the expected return and covariance for the model's inputs can cause problems. The model is highly sensitive to these estimates and thus becomes the optimal weights. An experiment by Frankfurter, Phillips, and Seagle (1971) concluded that the mean-variance optimized portfolio based on the criteria from Markowitz (1952) was not any more efficient than that of the Equally Weighted Portfolio. Another issue coming when optimizing using this approach is extreme portfolio weights. The mean-variance model chooses the optimal portfolio, which may mean extremely high allocation to some assets and none or even very negative to others (Munk, 2020). Using the model without any constraints often yields results stating that one of the assets needs an allocation above 100% and another needs to be heavily shorted. It assumes that the expected returns remain constant, and if these are based on historical returns, they may change a lot in the future. It may result in very risky portfolios even though they attempt to minimize the volatility. This increased risk comes if the estimates turn out to be very wrong. Much of the extreme allocation can be reduced by introducing constraints into the optimization. Constraints such as no shorting are allowed, or some assets cannot have a higher weight than X will help create a more robust model. A more robust model would imply that it is not solely dependent on a single or very few assets and should be better suited for different types of markets.

### 4.2.4 Regime-Based Factor Allocation

The Regime-Based Factor Allocation model is a dynamic factor allocation model which rotates the factor allocations according to the current regime. The factor selection under each regime is based on an assessment of the regime-dependent factor performance of the single factors. Moreover, the factor selection process consists of separating the data into the four regimes and analyzing the performances of the factors under each stage.

The factor performance is based on the factor excess return and excess volatility. The correlation between factors is also essential when constructing the portfolio in each regime since diversification is an important aspect of the portfolio creation. Another aspect considered is the exposure during changes in the regimes and the individual performances in all the individual regimes. The model is more robust when the different sources of outperformance do not stem from a single period. There are significant variations in the factor performance of the various regimes, and we want to minimize the exposure to poor-performing factors.

This model is only exposed to either one or two factors at a time, with equal exposure to each when allocated to more than one factor. It is given the same weight no matter the historical performance but only consists of a maximum of two well-performing factors in each regime. The model is rebalanced each month to $w_i = \frac{1}{n}$ where n is the number of factors, the model is exposed to in that given regime. Each month the model allocates according to the current regime, and an average regime length is 7.4 months before any reallocation.

The idea behind this model is to find an optimal rotation strategy that gives the freedom of taking multiple things into account that may not always be quantifiable. It makes the model more subjective than the more fixed models but also more flexible. It can account for things such as performance during regime changes, consistency in returns, factor correlations, and a more theoretical base of how the factors should perform under different economic conditions.

### 4.2.5 Regime-Based Mean-Variance Optimization

The Regime-Based Mean-Variance Optimized Portfolio is a combination of the two models above. It applies the Mean-Variance framework by Markowitz (1952) described in section

4.2.3 but does so on a regime-dependent basis. It uses formula 30 to compute the optimal weights with the return vector and variance-covariance matrix based on only a single regime. Doing this for all four regimes gives four different optimal allocations. This method utilizes that the factors have different return characteristics depending on the regime, which yields different optimal allocations. The optimal allocation can be any combination of the five factors and is set to give the best risk-adjusted returns in each regime. The model dynamically allocates between the four optimal portfolios depending on the current regime.

The model no longer has equal weights as the previous one, but it rebalances monthly to $\boldsymbol{\pi}$ given by formula 30.

The idea behind this model is to use the Mean-Variance framework to construct a model that optimizes the risk-adjusted returns for each regime. It becomes solely based on the historical data of the factors and gives a more quantifiable set of parameters in the portfolio construction.

## 4.3 Portfolio Performance Measures

**Return and volatility**

This section aims to go through the different performance measures that we will apply throughout our analysis. First, the monthly returns are calculated as:

$$r_{monthly} = \frac{p_t - p_{t-1}}{p_{t-1}} \tag{31}$$

Where $p_t$ is the price or index level in the current period, and $p_{t-1}$ is the price from the previous month. To annualize the monthly returns, the following formula is applied throughout our analysis:

$$r_{annual} = (1 + r_{monthly})^{12} - 1 \tag{32}$$

The volatility is a way of measuring the risk in an investment. It is measured mathematically as the standard deviation, which is the square root of the variance. The variance is calculated as:

$$Var[r] = E(r^2) - (E[r])^2 \tag{33}$$

Where r is the return over the period, and E is the expected value. Then, we can calculate the standard deviation as:

$$Std[r] = \sqrt{Var[r]} \tag{34}$$

(Munk, 2020).

To annualize the standard deviation, the monthly standard deviation is multiplied with the square root of 12:

$$Std[r_{annual}] = Std[r_{monthly}] \cdot \sqrt{12} \tag{35}$$

### Sharpe Ratio

We have calculated the Sharpe Ratio as the difference between the return on the investment and the risk-free rate divided by the volatility of the excess return:

$$Sharpe\ Ratio = \frac{E_r - r_f}{\sigma_{ex}} \tag{36}$$

The Sharpe Ratio is a measure that can be used to measure risk-adjusted returns because it is both taking the return and risk of an investment into account. Note that the volatility of the risk-free rate is often assumed to be zero, but in practice, it is very close to but not exactly zero. Therefore, we chose to include this volatility, and we are therefore dividing by the excess volatility of the investment when calculating the Sharpe Ratio.

### Sortino Ratio

Another risk-reward measure for calculating risk-adjusted returns is the Sortino Ratio. It differentiates harmful volatility from overall volatility. The ratio could be a better risk measure than the Sharpe Ratio because it takes the downside volatility into account instead of the standard deviation of an investment that includes both positive and negative deviations from the expected value (Munk, 2020). Hence, the Sortino Ratio includes the lower partial standard deviation instead of the standard deviation in the denominator:

$$Sortino\ Ratio = \frac{E_r - r_f}{\sigma_{downside\ ex}} \tag{37}$$

Where $E_r$ is the expected portfolio return, $r_f$ is the risk-free rate and $\sigma_{downside\ ex}$ is the downside volatility of excess returns.

### Value-at-Risk (VaR)

The Value-at-Risk can be calculated to measure potential losses with a certain probability, and it is a commonly used risk measure. For example, one way to calculate VaR when the mean, $\mu$, and standard deviation, $\sigma$, is known is by using the following formula:

$$VaR = \mu + \sigma N^{-1}(X) \tag{38}$$

This method assumes that gains and losses are normally distributed. Where X is the confidence level and $N^{-1}$ is the inverse cumulative normal distribution (Hull, 2018). A VaR of $5 million means that with a significance level of 5%, we are not going to lose more than the $5 million with a 95% probability. Another way of calculating VaR is by using the historical method, which looks at the historical actual returns. Then the returns are ranked from the worst to the best outcome. This paper will apply the historical approach taking the 5% worst outcome, used to measure the VaR.

### Expected Shortfall (ES)

The Expected Shortfall measures the expected loss if the VaR threshold is assumed to be crossed. Hence, ES is a way of quantifying the amount of tail risk a portfolio has. When a normal distribution is assumed, ES can be calculated as:

$$ES = \mu + \sigma \cdot \frac{\frac{e^{-Y^2}}{2}}{\sqrt{2\pi}(1-X)} \tag{39}$$

where Y is the Xth percentile of the standard normal distribution, $\mu$ is the mean, and $\sigma$ is the standard deviation (Hull, 2018). Another way of calculating ES is applying the historical approach when the actual return data is known. Here, one simply computes the average of the returns exceeding the VaR threshold. This measures the expected loss when the loss exceeds the VaR.

### Active Return and Active Risk

The Active Return is the difference between the return on an investment minus the return on its benchmark:

$$Active\ Return = r_p - r_{benchmark} \tag{40}$$

This measure is used to analyze the performance of, e.g., active fund managers or hedge funds. For example, this thesis will apply this measure to analyze the performance of our regime-based models by taking their excess returns relative to a static equally weighted model.

The Active Risk is also known as the Tracking Error (TE). It measures how accurately index funds track the index (Hull, 2018). It can be calculated as the standard deviation of the monthly differences in returns. This thesis will apply this measure by comparing the regime-based models to the static Equally Weighted Model.

### Information Ratio

The Information Ratio is a measure that can assess the performance of a portfolio manager because it measures the portfolio manager's ability to achieve high returns in excess of the benchmark while the risk taken is considered simultaneously. It is calculated as the Active Return divided by the TE or Active Risk:

$$Information\ ratio = \frac{Active\ Return}{Active\ Risk} \tag{41}$$

### Maximum Drawdown

An investor might be interested in calculating the Maximum Drawdown (MDD), which is an indicator of the riskiness of an investment. MDD is the historically maximum observed loss from the highest point to the lowest point. The individual drawdowns through the time series are calculated as:

$$DD_t = \frac{HWM_t - P_t}{HWM_t} \tag{42}$$

$DD_t$ are the drawdowns at time $t$. $HWM_t$ is the High-Water Mark, which is the highest peak in value that an investment has reached at the time $t$. $P_t$ is the price of the investment at time $t$. The DD can be calculated for each period. MDD is then the maximum value of all the observed drawdowns:

$$MDD = Max(DD) \tag{43}$$

The MDD does not show how long it took for the investment to recover from the loss, but it is an interesting measure because it focuses on capital preservation that might concern investors.

### T-test

T-tests are relevant to calculate because they can determine if there is a statistically significant difference between the means of two data groups. The null hypothesis states no statistical significance between the two means. If one calculated the t-stat to be higher than the critical value, we would reject the null hypothesis, meaning that there is a statistically significant difference between the two means.

The t-stat is calculated using the formula:

$$t_{stat} = \frac{\mu_1 - \mu_2}{s/\sqrt{n}} \tag{44}$$

The $\mu_1$ and $\mu_2$ are the means of the two sample sets. $s$ is the standard deviation of the difference between $\mu_1$ and $\mu_2$. $n$ is the number of observations in the dataset. In our thesis, a one-sided test is used for comparing two means. It allows us to see if one of the variables is significantly higher than the other. The t-stat can be compared with the critical value. When looking at a 95% confidence level, the critical value is 1.64 for the one-sided Student t-test. The null hypothesis is rejected for a 99% confidence level if the t-stat is higher than the critical value of 2.33 (Stock & Watson, 2015).

## 4.4 Robustness Test

We will apply different analyses and methodologies to test the robustness of the Regime-Based Factor Model. The robustness test is going to look at whether the model is useful in a real-life setting and also to test different cases where the model outperforms and underperforms.

### 4.4.1 Regime Shift Analysis

A Regime Shift Analysis is relevant to analyzing the effect of the regime changes on the performance of the dynamic model. It will help test the model's robustness for different regime types and lengths in the future.

The return for the month where the shift takes place is collected to analyze these cases. We do not know that we have shifted to a new regime before the month has ended. In addition, the OECD Composite Leading Indicator release dates are given between the 7th and 17th day in the month following the regime shift (OECD, 2022). Therefore, we can only rebalance the portfolio at the end of the month following a regime shift since we apply monthly and not daily rebalancing in this thesis. The result is that the model is fitted to the previous regime for two months. Therefore, two months of return data is collected around each regime change.

As an example, if a regime shift happens in April, the model is exposed to the optimal factor exposures from March in this month. Then, when the OECD CLI number from April has its release day in May, the model rebalances to the new optimal factor weights at the end of May. Then the model is only allocated optimally from the beginning of June, so the first monthly return after the rebalancing is observed from June. So, in this case, the return data from April and May is collected in the case where the regime shift took place in April.

The averages are then calculated for each model within each regime. An overall average can be found after the different regime shift types are calculated for the chosen model. This total average across all regimes shows the average monthly return in the period following a regime shift. Therefore, we will end up with average returns from different regime types, which will show how much the different regime shifts affect the models. Furthermore, we will know the total average monthly regime shift return, indicating regime lengths' influence on the models' performances.

### 4.4.2 Transaction Costs

Transaction costs and the total expense are the relevant expenses to consider when considering an ETF. The purchase of an ETF is the cheapest and easiest way to get exposure to an index. So, when an investor wants to be exposed to, e.g., the MSCI USA Momentum Factor, she could buy a share of an ETF tracking the MSCI USA Momentum Index. This index's annual total expense ratio (TER) is 0.20% (Nordnet, 2022). Here, she would hold an ETF tracking a single factor index. If she wanted to pursue a multi-factor index, she would have to hold different ETFs tracking different factor indices. The 0.20% would be spread out to various ETFs, so the total expense would be the same as holding a single factor index. The same concept applies to a dynamic multi-factor model when the investor continuously rebalances her portfolio and still has 0.20% of her investment paid to TER, which is the total management fee for the ETFs. The 0.20% in TER decreases the annual returns with this amount. This amount will be the same for the different investment strategies analyzed during this paper, so TER is not included in our calculations.

On the other hand, transaction costs are relevant when comparing a dynamic and a static investment strategy since there are costs associated with the dynamic approach through the frequency of trading, and hence being an active investor. Before estimating the transaction costs, we first need to calculate the turnover rates for the investment strategies in this paper.

The Turnover Rate is the percentage of the portfolio rebalanced over a certain period. If the turnover for a given month is 100%, the entire portfolio has been replaced. There are two components in determining the monthly turnover. The primary turnover stems from reallocating between factors, but some turnover stems from differences in returns among the factors. A static model with equal weights will not be equally weighted in the next month if not all the factors perform the same. When a fixed amount of money is invested into all factors, a higher return in, for example, Value than Size will make the portfolio more exposed to Value than Size. This is because the higher return in Value has increased its index level compared to Size, whose index level has not increased as much and now has a lower weight than Value. Therefore, all factors need to be rebalanced monthly to the fixed weights depending on the portfolio.

The total turnover is calculated monthly in this paper as:

$$Turnover\ Rate = \frac{\sum_{i=1}^{N} |w_t - w_{t-1}|}{2} \tag{45}$$

Where $w_t$ is the portfolio weight after rebalancing and $w_{t-1}$ is the weight just before rebalancing for one factor, for example, the Size factor. These monthly reallocations are added up and divided by 2. It is because only half of the differences in portfolio weights are being changed. To illustrate this, imagine two factor exposures, where one increases in value by 10 and the other by 20. To rebalance, only 5 from each have to be bought/sold to have equal exposures again, and hence we had a turnover of 5 in this case. The monthly portfolio weights are the average monthly turnovers and annualized by multiplying by 12.

After calculating the Turnover Rates, the transaction costs are calculated using the following formula:

$$Transaction\ costs = 2 \cdot BF \cdot Turnover \tag{46}$$

Here, $BF$ are the brokerage fees for the period. The reason for multiplying with 2 is that we both have a purchase and a sale where a fee is being paid. Hence, when a monthly turnover is 100%, the entire portfolio has been replaced, so 100% is being sold, and 100% is being bought, which equals 200% in transaction costs for this month with a complete rebalancing.

The brokerage fee for a retail investor is 0.10% when purchasing or selling an ETF at Saxo Bank trading on the New York Stock Exchange (Saxo Bank, 2022). For VIP memberships at Saxo Bank, this number is only 0.05%. These numbers are based on the point of view of a retail investor. We assume that institutional investors can negotiate an even lower brokerage fee than this because they might have beneficial deals with banks and other financial institutions. This thesis applies the 0.10% as a conservative estimate when calculating the transaction costs. This is done to make sure all investors will be able to pursue the dynamic investment strategies covered in this thesis. In addition, this conservative estimate harms the performance of the dynamic factor models at a fair rate to make sure these models do not benefit from a lower cost than what applies to all investors.

After the transaction costs have been determined using equation 46, these costs can be subtracted from the gross returns to determine the net returns:

$$r_P^{net} = r_P - Transaction\ costs \tag{47}$$

Where $r_P^{net}$ is the net return on a portfolio, and $r_P$ is the gross return. Hence, the net returns are calculated monthly.

# 5. Empirical Analysis

In this chapter of the paper, the theories and methodologies previously explained will be put into practice. First, the empirical analysis will analyze how the individual factors have performed in the test period to understand their behavior and characteristics. Next, the single-factor portfolios will be expanded to static multi-factor models using non-perfect correlations to combine the factors into a single portfolio. The empirical analysis will then analyze the constructed regime-based multi-factor models and compare them with the previously built models. Finally, the best regime-based model will be exposed to different robustness tests to understand the value drivers behind the model's performance and whether this model seems to provide any significant and durable outperformance.

## 5.1 Single-Factor Analysis

The literature review described different risk factors and previous academic research that proved these factor premiums' existence. However, before using these further in our models, it is essential to look at the performance of the individual factors. How would an investor be compensated for investing in just one of these factor premiums? How does it vary from investing in the market portfolio, here defined as MSCI USA?

The metrics for the single factors in table 3 are based on the test period from 2012 to 2021 because it makes it comparable to the performance of the other models later in the analysis. The performance in the test period has been much better than the performance in the estimation period. It is partly a result of the Quantitative Easing program led by the central bank, resulting in a very low interest rate, which might have contributed to the high market returns in the test period. The low interest rate might have affected the factor performances differently. For example, Value has performed worse in the test period relative to the estimation period compared to the other factors. One reason could be that the Growth firms need to borrow and invest relatively more than Value firms. Additionally, a large portion of the value of the Growth firm comes from cash flows far out in the future. A lower interest rate results in a lower discount rate which benefit the firms with much of their cash flows far out in the future. Hence, Growth firms benefitted from the low interest rate, which has improved the performance of these firms, which, other things being equal, lowered the performance of Value in the test period.

According to Ang (2014), factor investing is an essential part of every investor's portfolio, whether or not they know it. Investors are exposed to various factor risks, including volatility risk, interest rate risk, illiquidity risk, etc. This thesis looks at five of the most documented factors, which are also easily accessible through various ETFs. Whether retail or institutional, investors should at least have their portfolio include market risk and some factor risk (Ang, 2014). The factor portfolios in this thesis all have market risk and a factor exposure.

Looking at the performance metrics in table 3, we see four factors, Size, Value, Momentum, and Quality, yield a higher return than the market portfolio, albeit with higher volatility, except for Quality, which has a higher return and lower volatility. This affirms the risk premium idea regarding Size, Value, and Momentum, where investors get compensated with additional returns by taking on increased risk. Quality and Minimum Volatility seem to have less overall volatility than the market. The risk in these can be underperformance during good times in the market, thus lagging the benchmark. It can be seen as a relative risk since the overall risk is smaller but has the risk of deviating from the benchmark. Quality shows higher returns than MSCI USA even with lower volatility, indicating a possible market anomaly. Minimum Volatility has delivered lower average returns than the market but with lower volatility, resulting in the Sharpe Ratio being higher than for the MSCI USA. Size and Value have slightly lower Sharpe Ratios than the market. However, looking at Momentum, Quality, and Minimum Volatility, the Sharpe Ratios are higher relative to the market. It suggests that investors can get better risk-adjusted returns from investing in these factors. Unconstrained investors should be looking to maximize the risk-adjusted returns since they can use leverage or invest in the risk-free rate in their portfolio to match their level of risk aversion to get the maximum return for a given level of risk.

| Metrics | MSCI USA | Size | Value | Momentum | Quality | Min. Vol |
|---|---|---|---|---|---|---|
| **Return (Ann.)** | 15.36% | 16.52% | 15.75% | 18.94% | 19.03% | 14.84% |
| **Volatility (Ann.)** | 13.25% | 14.72% | 15.70% | 13.33% | 12.94% | 10.87% |
| **Sharpe Ratio** | 1.110 | 1.077 | 0.961 | 1.370 | 1.419 | 1.306 |

Table 3: MSCI USA and single factor performances in the test period 2012-2021. Source: Own creation.

As seen in figure 3, investing in one of the factor indices would, in almost all cases, have yielded a higher return than simply investing in the market portfolio, MSCI USA. Only Minimum Volatility would have given a slightly lower return, albeit with lower volatility. The high returns of Momentum and Quality seem to be consistent throughout the period, and Quality has high performance consistency in the last part of the period. All of the return series are highly correlated due to the high amount of market risk present in all indices.

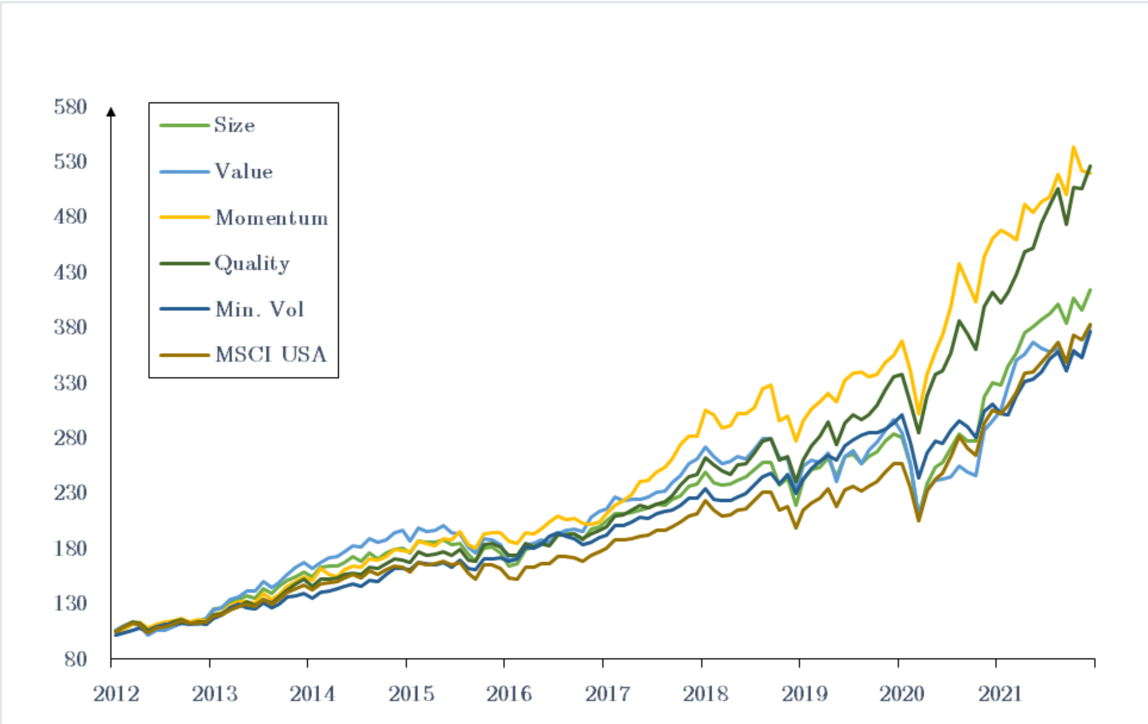


Figure 3: Absolute factor index returns starting from index 100 in the test period (2012). Source: Own construction.

To get a more in-depth look into the actual factor performances and not just the overall market performance, we can look at relative returns. In figure 4, we plot the returns of the five factors relative to MSCI USA, thus removing the market beta of the portfolios. The five factors do not seem correlated except for Size and Value.

The figure shows a predominant Momentum and Quality effect with clear outperformance relative to the rest in the overall returns. The only index which has had a return below the market was Min. Vol. Value seems to have performed decently until 2015 and has since underperformed quite a lot. As shown in figure 4, some factors can experience long times of bad performance. However, they do produce increased returns in the long run due to their risk premiums. The risk premium comes from these long periods of underperformance, and investors need to look at whether they can withhold these bad times to achieve the risk premium (Ang, 2014). Investors get compensated for holding these assets in bad times with an increased return in the long run.

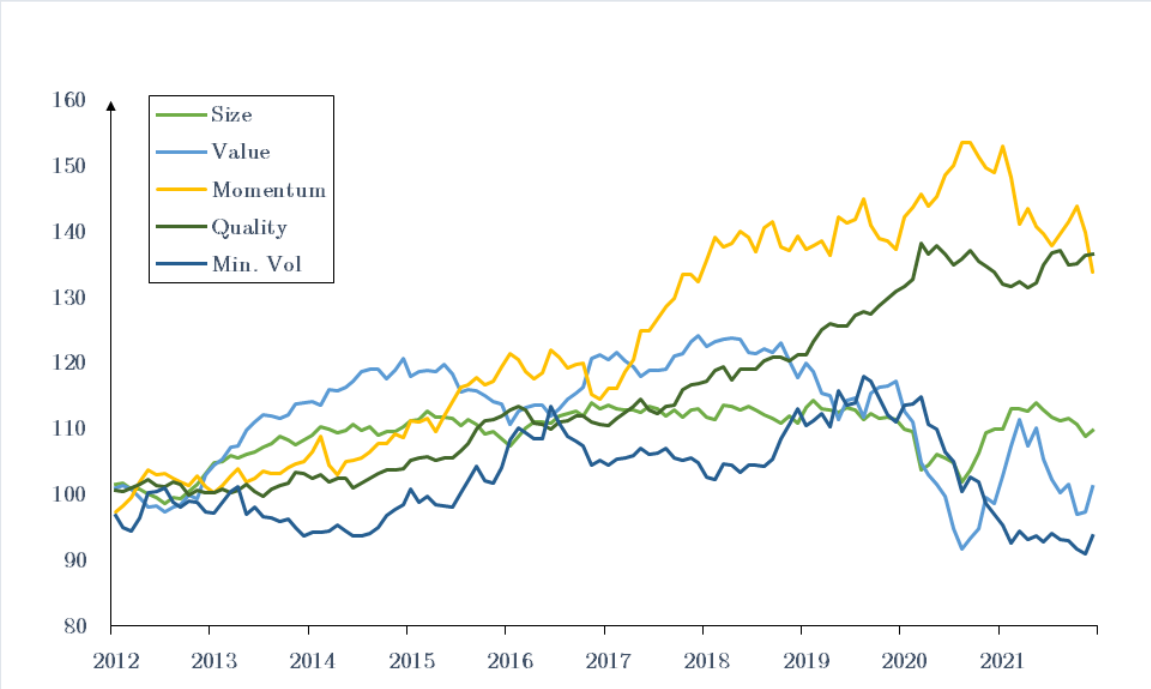


Figure 4: Excess factor returns relative to MSCI USA. Source: Own construction.

Next, a correlation matrix is shown in table 4. Here, a high correlation between Size and Value is observed. We see a negative correlation between Momentum and Size, indicating that the larger firms have a high Momentum score. Also, the large firms might have increased momentum since they have become large in terms of market capitalization. Momentum and Value have a negative correlation. It makes intuitive sense since we expect growth firms to have the highest momentum when we expect these firms to accelerate. The negative relationship was further expected, as discussed in the literature review. In that chapter, we discussed that an increase in a particular stock price tells the Momentum Factor to buy that stock because it is now a past winner. However, from a value perspective, the increased stock price means that we have to sell that stock because its B/M ratio has decreased if we assume the book value of equity to stay unchanged. Quality is also largely negatively correlated to Size and, to some extent, Value, indicating that large and growth firms have high Quality Scores. The negative correlation between Value and Quality could imply that growth firms have higher Quality scores, which might explain why investors demand growth firms, making the book-to-market ratios relatively low. Finally, we observe that the defensive factors, Quality and Min. Vol., have a positive correlation, making it relevant to mention these two factors with some of the same defensive characteristics that they have in common.

Even though most of the factors yield a positive excess return over the market portfolio, in the long run, they have very different periods of over- and underperformance.

| | Size | Value | Momentum | Quality | Min. Vol |
|---|---|---|---|---|---|
| **Size** | 1.0000 | | | | |
| **Value** | 0.6276 | 1.0000 | | | |
| **Momentum** | -0.1762 | -0.1613 | 1.0000 | | |
| **Quality** | -0.5230 | -0.3449 | 0.1330 | 1.0000 | |
| **Min. Vol** | 0.0870 | 0.0970 | 0.0410 | 0.2452 | 1.0000 |

Table 4: Correlation matrix from the test period 2012-2021, where the market beta is excluded. Source: Own construction.

Investing in single factors can significantly impact the returns compared to simply investing in the market portfolio, which otherwise should be the only thing one should invest in according to the CAPM (Sharpe, 1964). Not all the factor returns seem to relate to that factor's risk contribution directly. Factors such as Quality, Momentum, and Minimum Volatility seem to improve the risk-adjusted return of a portfolio by either improving the return, decreasing the risk, or a mix of both.

## 5.2 Static Multi-Factor Models

Multi-Factor models combine multiple single factors into a portfolio of factor exposures. Given that all the factors have periods of relative underperformance, it makes sense to use the low or negative correlations to combine these factors into a single portfolio. It will smooth the equity curve since some factors' underperformance can be balanced by other factors outperformance and thus seek a more consistent harvest of the risk premiums. There are many ways of structuring a multi-factor model depending on the views of the different factor performances in the future. We will start with two simple static models that seek diversification benefits between factors to minimize the bad periods stemming from poor performance in a single factor.

### 5.2.1 Equally Weighted Model

We have constructed an equally weighted static multi-factor model by allocating 20% of the portfolio to each of the five factors. Therefore, we get the average return of the individual five factors by forming this simple portfolio. Mathematically the portfolio return for each month becomes:

$$r_p = 0.2 \cdot r_{size} + 0.2 \cdot r_{value} + 0.2 \cdot r_{momentum} + 0.2 \cdot r_{quality} + 0.2 \cdot r_{\min vol} \tag{48}$$

Where $r_p$ is the Equally Weighted Portfolio return, and $r$ is the return on the individual factor for each month. The return on the Equally Weighted Portfolio of 17.00% is better than the MSCI USA Index with 15.36%. Also, the Equally Weighted Model achieves lower volatility of 12.68% compared to 13.25%. Hence, the risk-adjusted return measured here by the Sharpe Ratio becomes better than the MSCI USA Index.

| Metrics | MSCI USA | Equally Weighted |
|---|---|---|
| **Return (Ann.)** | 15.36% | 17.00% |
| **Volatility (Ann.)** | 13.25% | 12.68% |
| **Sharpe Ratio** | 1.110 | 1.288 |

Table 5: The performance of MSCI USA and the Equally Weighted Portfolio in 2012-2021. Source: Own construction.

Investors would therefore benefit by replicating this Equally Weighted Factor Model when they care about both minimizing the volatility and maximizing their returns. When the investor does not know which factor will outperform in the future, it is safer to diversify by spreading their funds equally between these five factors. It will especially be a desirable strategy for passive investors who do not just want to follow the market and do not want to worry about when to buy and sell their assets.

The outperformance of the static Equally Weighted Model relative to the MSCI USA Index might primarily stem from the risk premiums that the factors imply. For example, for the Size Factor Premium, Fama and French (1996) suggested that investing in small firms might be risky since they could be less robust under economic recessions. Similarly, for the value effect, the value firms should be riskier than growth firms since the value stocks are relatively more cyclical than growth firms. For the Momentum factor premium, the increased return from the factor is compensation for bearing time-varying risk (Berk et al., 1999). This risk stems from the Momentum factor that prioritizes stocks performing well in bull markets. These firms might have higher systematic risk with high market betas, so that a sudden downturn would give the Momentum factor poor performance. Hence, there is a time-varying risk for the Momentum factor that results in a positive risk premium.

Both the Quality factor and the Minimum Volatility factor have lower volatility than the MSCI USA. The other three factor premiums, Size, Value, and Momentum, have higher volatility than MSCI USA, partly explaining their relatively higher returns. On the other hand, Quality has a higher return and lower volatility, so this factor does not increase the risk of the Equally Weighted Portfolio. Hence, the higher return remains unexplained for the Quality factor. Asness, Frazzini, and Pedersen (2018) explain that low-quality firms seem to be riskier and yield a lower return than high-quality stocks. Hence, they argue that the reason for the abnormal Quality returns might result from a market anomaly. Finally, the Min. Vol. factor has a lower return than the MSCI USA Index, but the lower volatility makes the factor more attractive than the MSCI USA in terms of Sharpe Ratio.

The static Equally Weighted Factor Model outperforms the MSCI USA benchmark with lower volatility of 13.25% compared to 12.68% and a better annual return of 17% compared to 15.36%. Because of this, we know that it is beneficial to invest in factors, and this static model with equal weights outperforms the market index, measured by MSCI USA. Because of this, an investor should consider factor investing instead of investing in MSCI USA alone.

### 5.2.2 Mean-Variance Optimized Model

In a Mean-Variance Approach suggested by Markowitz (1952), it is optimal to find the minimum-variance portfolio with the minimum variance of all portfolio combinations that lie on the efficient frontier given a specific expected portfolio return as the target. Another optimal mean-variance solution is finding the highest Sharpe Ratio obtained on the tangency line. We have applied the approach where we want to maximize the Sharpe Ratio since we assume most investors are pursuing the highest possible risk-adjusted returns rather than only minimizing their risk. The tangency portfolio weights are given by formula 30.

*5.2.2.1 Mean-Variance with Short Selling*

In the first case, we conduct the mean-variance optimization with no constraints on short selling. We get portfolio allocations to all five factors when maximizing the Sharpe Ratio in this mean-variance setup. The only constraint is that the weights have to sum to 1. We see a very negative allocation to Size, and we also observe that this factor has a relatively low Sharpe Ratio. Because of this and the correlations with the other assets, the investor of this portfolio is shorting the Size factor to use these funds to invest even more in the other factors with higher Sharpe Ratios and more optimal correlations. The highest allocation is to the Value factor. This factor is especially attractive because of the low correlations to the other factors, except Size, which is shorted in this portfolio choice. Even though the Value factor has the lowest Sharpe Ratio in the test period, this factor still gets the highest allocation. The reason is that the Value factor has performed well in the estimation period from 1998 to 2011, and our model is built on this period. Momentum also gets a very high allocation, above 100%, which is a rather extreme case where we have no short selling constraint. Momentum performed well in both the estimation and test period, with both negative and low correlations to the other factors. Hence, the model gets some diversification benefits by allocating to Momentum. Minimum Volatility has some attractive mean-variance characteristics, especially low volatility, and this factor, therefore, receives a high allocation. Quality is shorted in this model because it has performed relatively poorly in the estimation period.

| Tangency pf | |
|---|---|
| **Size** | -145.50% |
| **Value** | 126.26% |
| **Momentum** | 108.76% |
| **Quality** | -64.53% |
| **Min. Vol** | 75.00% |

Table 6: Optimal portfolio allocation using the Mean-Variance approach, where short selling is allowed. Source: Own construction.

The Sharpe Ratio becomes slightly higher than the MSCI USA Index in table 8. However, the return is somewhat lower than in the Equally Weighted static case. This model's relatively bad performance is because of the differences in factor performances between the estimation and test periods. In addition, we have no short selling constraint and, therefore, some very extreme allocation with -145.50% to Size and 126.26% to Value which magnifies the relatively poor performance resulting in only slightly better performance than MSCI USA. Therefore, it is an extreme case where short selling is assumed to be allowed. In the following sub-chapter, we will do the same mean-variance optimization but with a constraint on short selling where we assume that short selling is not allowed.

#### *5.2.2.2 Mean-Variance without Short Selling*

Now, the same formulas are applied but with a constraint that no short selling is allowed. After maximizing the Sharpe Ratio, the optimal weights become those from table 7, with an optimal allocation of 89% to Momentum and 11% to Value.

| Tangency pf | |
|---|---|
| **Size** | 0.00% |
| **Value** | 11.00% |
| **Momentum** | 89.00% |
| **Quality** | 0.00% |
| **Min. Vol** | 0.00% |

Table 7:Optimal portfolio allocation using the Mean-Variance approach, where short selling is not allowed. Source: Own construction.

Hence, the model behaves almost like a single factor Momentum strategy with some minor diversification benefits by allocating to the Value factor. There is no allocation to Size, Quality, and Min. Vol., which might stem from the three factors having low Sharpe Ratios. The highest allocation is to the Momentum factor, which has an attractive Sharpe Ratio. The Momentum factor had the second-highest exposure before, where it now has the highest factor exposure in the no short selling case. The reason for not allocating the entire portfolio to Momentum, even though it yields a high risk-adjusted return, stems from the correlations between the factors. Value and Momentum have a negative correlation of -0.16 in the estimation period from 1998 to 2011. It is beneficial to combine these two factors when they both have attrActive Risk-return profiles and are negatively correlated. As stated in the delimination part in section 1.2, we assume short selling is not allowed—the model without constraints was conducted for theoretical purposes. Therefore, after this section, we will only refer to the Mean-Variance Model with no short selling.

| Metrics | Mean-Variance with constraint | Mean-Variance without constraint |
|---|---|---|
| Return (Ann.) | 18.58% | 15.29% |
| Volatility (Ann.) | 13.17% | 12.45% |
| Sharpe Ratio | 1.360 | 1.176 |

Table 8: Comparison of the performance of the two Mean-Variance approaches in 2012-2021. Source: Own construction.

After constructing the model, we see that the Mean-Variance Model yields both a higher return and lower volatility than the MSCI USA Index. The return of 18.58% is higher than the Equally Weighted Factor Portfolio with a 17% return, but the volatility is also slightly higher. Moreover, the return is higher than when short selling was allowed.

The primary reason for the increase in return relative to the prior case with short selling is that we now have an exposure of only 11% to Value. In contrast, we had an extreme case with 126.26% factor allocation to Value before. Value performed very well in the estimation period on which our model is based. Value has performed much worse in our test period, where our return and volatility results are calculated. Therefore, this restriction on no short selling makes us less exposed to an extreme case, which has improved our portfolio return, with Value having a less negative effect. Quality performed relatively poorly in our test period but has achieved the highest return in our test period. Hence, the negative exposure to Quality in the short selling case hurts the model's performance because Quality had performed very well in the test period. In the no short selling case, we have zero exposure to Quality. It, therefore, improves the performance of our model, that we now do not have negative exposure to Quality.

Additionally, the now better return and Sharpe Ratio compared to the short selling case are because of the much higher exposure to Momentum with an attractive Sharpe Ratio. The volatility is now higher than in the unconstrained case, and a significant difference is that we have no factor exposure to the Minimum Volatility factor, whereas we had 75% allocation to that factor before. Now, the portfolio optimization is not allowed to short the Size and Quality factors and use these to fund further exposure to Momentum.

To sum up, the Equally Weighted Model yields both better returns and has lower volatility than the MSCI USA benchmark. This investment strategy is beneficial for a passive investor that does not want to choose only one factor but instead wants to diversify the risk equally between the five factors. The Mean-Variance Optimized Model for the case with no short selling constraint allocates to all five factors with some extreme allocations. The Mean-Variance optimization with a short selling constraint only allocates between Momentum and Value. The better performance with the constraint primarily stems from the much lower allocation to Value, which has performed relatively poorly in our test period compared to the estimation period. The Mean-Variance Model with the constraint on short selling yields a better return and lower volatility than the Equally Weighted Model, so an investor seeking to optimize her risk-adjusted return should consider this model. However, an investor replicating this Mean-Variance Model should be aware that the strategy is highly dependent on the future performance of the Momentum factor.

## 5.3 Regime-Based Factor Timing

This thesis investigates whether it is possible to time the exposures to the factors in order to maximize the factor risk premium received by being exposed to factor risks. Factor premiums are the payments of being exposed to factor risks. The factors tend to have some periods of bad performance, and it is by being exposed to these bad times, that one should be entitled to the excess returns stemming from factor premiums. However, we would like to investigate whether one can time the exposure to the factors to avoid the bad times in the different factors and thus seek only to be exposed to them in good times.

The idea of timing market exposure is if one were to believe the "Efficient Market Hypothesis" (EMH) impossible. The EMH states in its semi-strong form that markets reflect all past information and that stock prices follow a random walk (Fama, 1970). It implies that any attempt to time the market would be pointless as this would be purely a random bet. Our endeavor with this study is to test whether any timing attempts would be futile or whether there are any inefficiencies in the markets that investors can utilize, thus falsifying the Efficient Market Hypothesis.

This section will take a timing mechanism, macro regimes, and use these signals in our tactical asset allocation. The purpose of the timing indicator is to signal which factors have the best conditions going into the next month. Our model should then allocate funds to the factors with the best prospects and avoid those likely to have poor performance.

### 5.3.1 Performance of MSCI USA in each regime

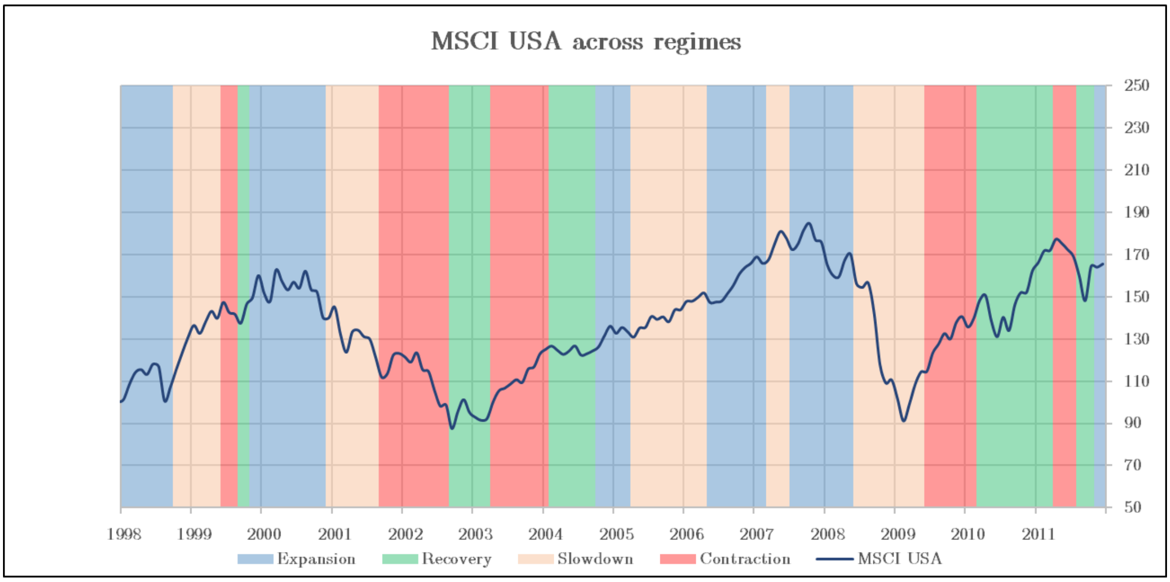


Figure 5: The performance of MSCI USA plotted across the regimes from 1998-2011. Source: Own construction.

The performance of the MSCI USA Index is compared to the four different regimes to illustrate the business cycles. As the MSCI USA Index is not expected to follow the regimes precisely, we still see a very close connection between the regimes and the change in the index. For example, the CLI had some wrong predictions, such as in late 2002, when the market increased when the CLI predicted a Contraction. Another example of the non-perfect prediction is in late 2007 to 2008, when CLI predicted an Expansion regime, but we observed a decreasing market index. However, in most of the regimes, we see a prediction from OECD that has been very close to the actual market performance.

| Metrics | Recovery | Expansion | Slowdown | Contraction |
|---|---|---|---|---|
| Return (monthly) | 3.07% | 1.74% | 0.27% | -3.05% |
| Return (ann.) | 43.82% | 23.02% | 3.30% | -31.07% |
| Volatility (ann.) | 13.88% | 9.64% | 12.52% | 21.27% |

Table 9: The performance of MSCI USA in the four regimes in 1998-2011. Source: Own construction.

After calculating returns and volatilities for the four different regimes, we see some substantial differences - indicating how different the performance of the MSCI USA Index is in the various stages of the economy. The Recovery stage is where the highest average returns are observed, followed by the Expansion phase. These two stages have in common that the economy is booming through a positive change in the month-on-month CLI, with one above and one below the long-term average. The returns are more stable in the Expansion phase than in Recovery. It might be because most firms are doing well in the stage where there is a positive change in the CLI, and the CLI is above the long-term trend, so many industries have excellent conditions to perform well in their businesses. The Slowdown stage has a relatively low expected return, which could result from the negative change in the CLI, and many sectors are beginning to have more difficulties expanding their businesses. Also, it is difficult for firms to accelerate in the Slowdown stage because many of them have reached a high stock price coming from a previous Expansion phase. The Contraction regime has a large negative average return and the highest volatility among the four regimes. This regime is where many businesses are struggling due to the negative change and level in the CLI.

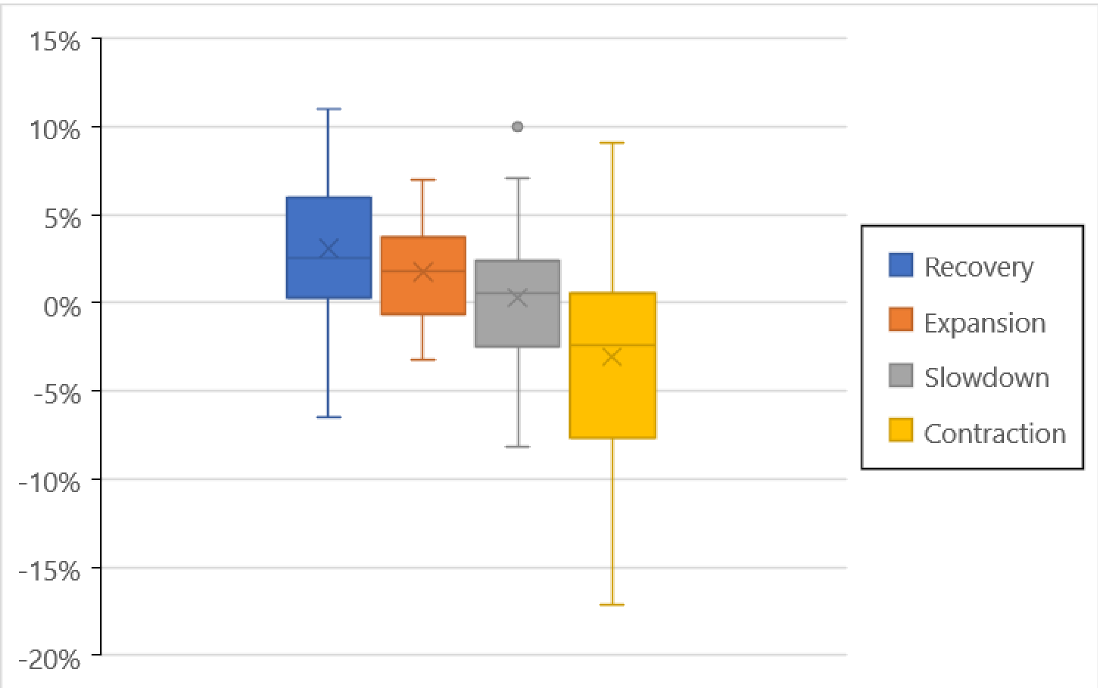


Figure 6: The boxplot illustrates the distribution of monthly returns in the four regimes. Source: Own construction.

When illustrating the dispersions of the factors in a box plot, it becomes clear how different the returns in the Contraction regime have been. We see a negative skewness for Contraction because the difference between the third and second quartile is less than the difference between the second and first quartile - indicating that there have been many returns above the median level but also some very negative returns that contribute to the large dispersion in returns regarding the Contraction regime. The Expansion stage is characterized by having the most consistent returns with the lowest dispersion in returns, which is in connection with the relatively low volatility. Recovery has shown the most positive returns but with relatively high dispersion. An outlier is observed from the Slowdown stage, whereas the other regimes have shown no outliers relative to the other returns in the period.

### 5.3.2 Factor Testing Across Regimes

This section will analyze all periods in the four different macroeconomic regimes, where we will end up choosing one or two factors in each of the four regimes. These two factors of each regime are added to our overall dynamic Regime-Based Factor Model, which we will test the performance of in a test period. The test period is being seen as yet unknown data, and we will see the actual effect of our model constructed in the estimation period. The estimation period is known data in our model that we will use as the basis for our dynamic multi-factor construction. The optimal factors in each regime will be picked after an overall assessment. The analysis of average returns, volatility, and correlations of factors will be the basis for our comprehensive assessment and final recommendation. Furthermore, the five factors are being indexed to start at level 100 to track the performance development in all periods in the Recovery, Expansion, Slowdown, and Contraction regimes, respectively.

The development in factor performance within each regime is analyzed by subtracting the MSCI USA return, shown in table 9, from the individual factor returns. This is to exclude the market risk, and hence, we are analyzing the excess returns of the factors relative to their benchmark. Because of this, the relative differences in factor performances are becoming clearer and easier to analyze when excluding the market risk that would have made the factors highly correlated and difficult to separate in their performances. The performance statistics are first illustrated in a table of monthly returns, which we also have annualized. Annualizing makes it easier to compare to other performances in our analysis, even though many periods are only a few months in a row in the same regime and not necessarily an entire year. The volatility is also stated as annualized volatility. It is important to notice that the volatility in this section is based on the standard deviation of the excess returns of the factors and not based on the factor index returns themselves. Therefore, the purpose is not to compare with all other models in this thesis but rather to compare the five factors to get an intuition about how the factors differ in the regimes. As a basis for further correlation analysis within each regime, the full sample correlations from the estimation period excluding the market risk component are shown in table 10.

| | Size | Value | Momentum | Quality | Min. Vol |
|---|---|---|---|---|---|
| Size | 1.0000 | | | | |
| Value | 0.6276 | 1.0000 | | | |
| Momentum | -0.1762 | -0.1613 | 1.0000 | | |
| Quality | -0.5230 | -0.3449 | 0.1330 | 1.0000 | |
| Min. Vol | 0.0870 | 0.0970 | 0.0410 | 0.2452 | 1.0000 |

Table 10: Factor correlations in 1998-2011. Source: Own construction.

The highest positive correlation is between Size and Value, whereas the most negative correlation is between Size and Quality. This negative correlation might indicate that the larger firms have the highest Quality scores. There are relatively low correlations between Momentum and the remaining four factors. In the next subsections, the factor performances across the four regimes will be investigated.

*5.3.2.1 Recovery*

| Factors | Size | Value | Momentum | Quality | Min. Vol |
|---|---|---|---|---|---|
| Excess return (monthly | 0.88% | 0.79% | 0.23% | -0.25% | -0.58% |
| Excess return (ann.) | 11.15% | 9.87% | 2.74% | -2.93% | -6.69% |
| Volatility (ann.) | 5.84% | 6.18% | 6.76% | 4.10% | 5.36% |

Table 11: Factor performances in Recovery in 1998-2011. Source: Own construction.

The Recovery stage is characterized by anticipating GDP levels below the long-term level and with a positive change, which indicates the economy is recovering. The average excess returns relative to the US market are highest for the Size and Value factors, which are much higher than the other factor returns. Size and Value are seen as cyclical factors, and when the CLI has a positive one-on-month change, these factors will intuitively perform well when the market performs well. The Quality and Minimum Volatility factors are seen as defensive factors. They perform relatively better than the other factors under macroeconomic downturns because they react relatively less to cash-flow news than the other factors. Hence, these defensive factors are the two worst-performing ones with negative excess returns, meaning that they have underperformed MSCI USA. Minimum Volatility is related to the

BAB factor, and it is the worst-performing factor in this stage. The Recovery regime favors cyclical factors with high market betas such as Value and Size.

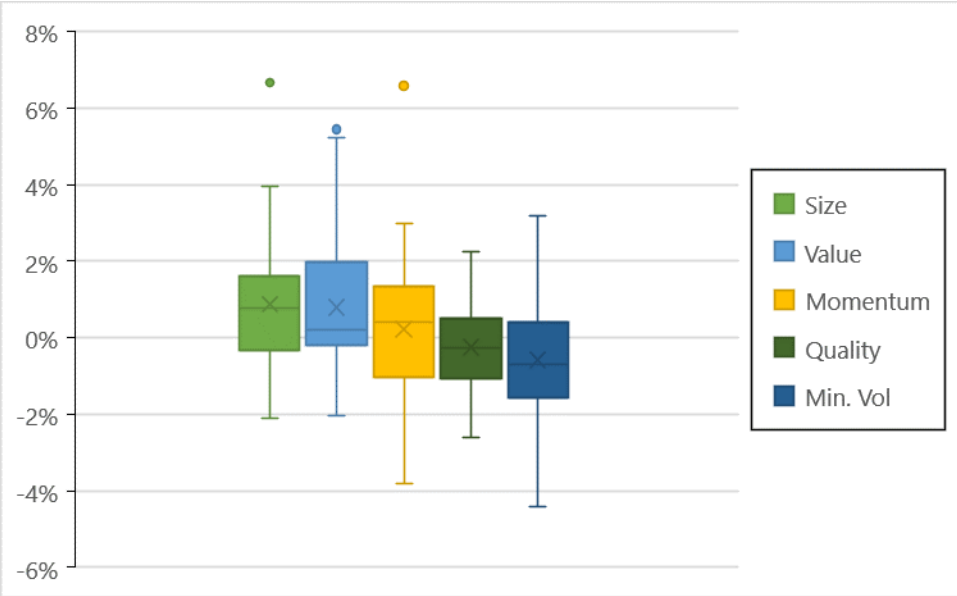


Figure 7: The boxplot illustrates the distribution of monthly factor returns in Recovery in 1998-2011. Source: Own construction.

When plotting the factor excess returns in a boxplot, we see the variability of returns relative to each other. The averages illustrated with an X are highest for Size and then Value. Momentum has had the second lowest observed monthly return, which is much lower than Quality, even though the mean and median are higher for Momentum. These bad monthly performances by the Momentum factor could be the results of holding stocks performing well in Contraction stages, which then perform poorly when there is a macroeconomic turning point. Even though the average return for Value has been higher than Momentum, the median is higher for Momentum than Value. This indicates that Value has many months of underperformance, but the returns have been less negative than the returns of Momentum. On the other hand, Value has some relatively high returns that have increased the average returns relative to Momentum. The dispersions are highest for Value and Minimum Volatility and smallest for the Quality factor, so the Quality factor has been much more consistent in its returns. However, the returns for Quality have been negative on average. Furthermore, we observe how Size and Value have had some outliers with very high returns relative to most other monthly returns.

| | *Size* | *Value* | *Momentum* | *Quality* | *Min. Vol* |
|---|---|---|---|---|---|
| **Size** | 1.0000 | | | | |
| **Value** | 0.5504 | 1.0000 | | | |
| **Momentum** | -0.2152 | -0.0243 | 1.0000 | | |
| **Quality** | -0.5660 | -0.2974 | 0.4402 | 1.0000 | |
| **Min. Vol** | -0.1861 | 0.0286 | 0.5554 | 0.3182 | 1.0000 |

Table 12: Correlation matrix in Recovery in 1998-2011. Source: Own construction.

The correlations in the Recovery stage are shown above, and we get a very high but slightly lower correlation between Value and Size than in the entire sample case with all regimes. Value and Size are highly cyclical, and the high correlation is important to notice when considering pooling these two factors in a portfolio since, from a diversification point of view, they might not be beneficial to combine. Momentum and Value now have a less negative correlation relative to the total sample. The very low and negative correlation is positively related when the portfolio's goal is to diversify. Some of the most significant changes in correlations relative to the total sample are the now relatively high correlations that Momentum has to both the Minimum Volatility and Quality factors. A possible explanation might be that many of the Momentum firms at the beginning of the Recovery regime are defensive ones, similar to the stocks in the Min. Vol. and Quality factors. This would be the case because a Contraction regime always occurs before moving on to a Recovery regime. Therefore, the Momentum factor has not had time to get rid of the defensive stocks at the beginning of the Recovery regime.

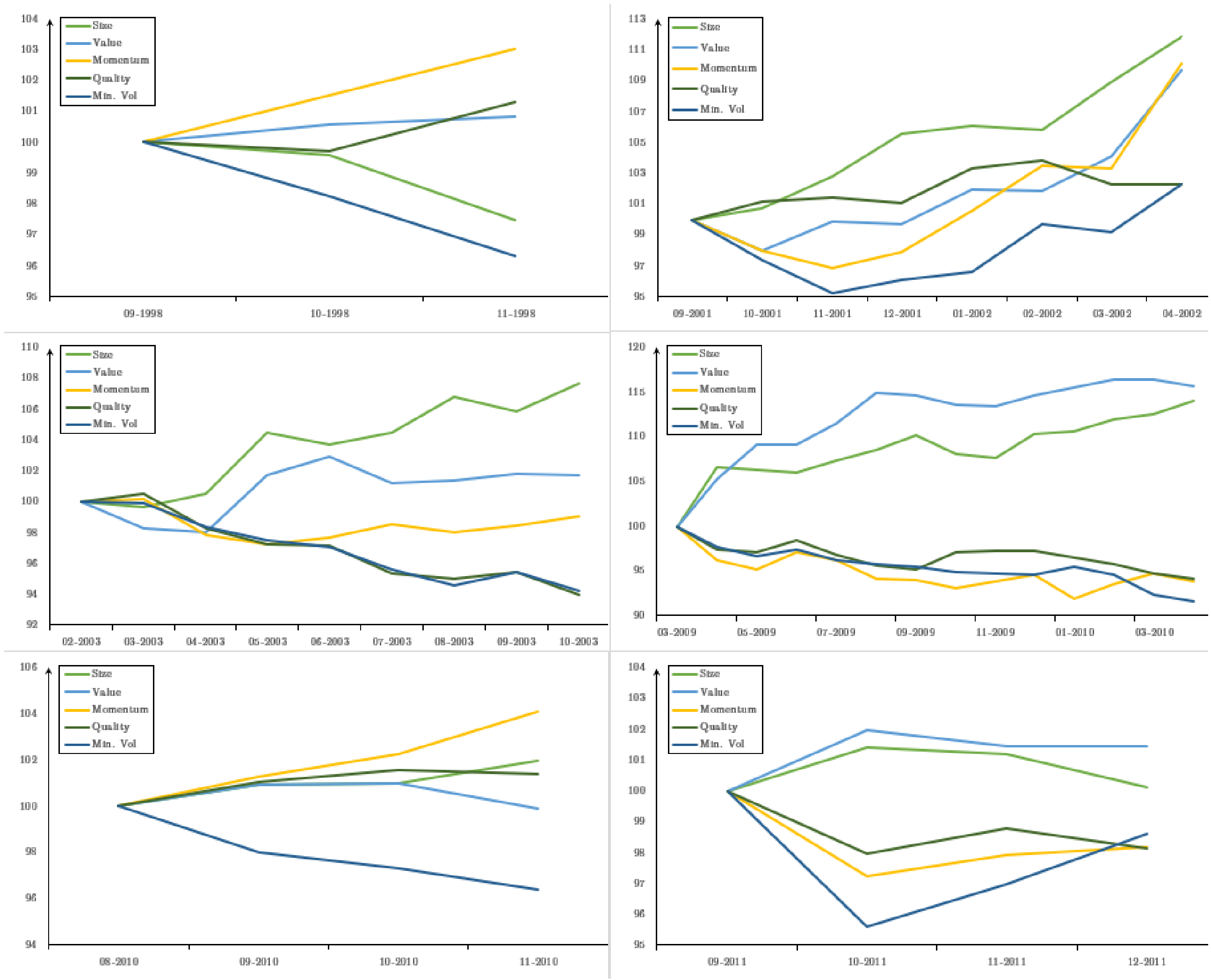


Figure 8: Factor performances across all Recovery regimes in the estimation period. Source: Own creation.

When comparing all the periods in the Recovery stage, we see how Value and Size are the best performers in most periods with a few exceptions, such as in the periods starting in September 1998 and August 2010. However, these two periods have only three and four months in the Recovery phase, so the other periods with more sample months are more robust when analyzing the factor performances. Min. Vol. is the worst-performing factor in Recovery. Momentum has shown many poor performances at the beginning of the Recovery stages, where it starts to increase after a few months. This is because a Recovery regime is always after Contraction, so the Momentum factor consists in these first months of stocks performing well recently, which have been the defensive stocks such as Minimum Volatility and Quality. We know that the two defensive factors are the worst-performing ones in Recovery, so that is one of the key drivers for the bad performance of the Momentum factor in the first months.

After a few months in Recovery, Momentum has changed its allocations to other better performing firms, such as the cyclical stocks, which contributes to the fact that Momentum is the third-best factor in Recovery when looking at the average returns. In March 2009, when the economy started to recover after the brutal financial crisis, the cyclical stocks such as Value and Size performed very well when the market began to recover since the cyclical firms typically have high market betas. On the other hand, the three other factors were underperforming relative to the market.

To sum up, Value and Size have been the best performing factors, with excess returns much higher than the other factors and outperformance in most of the Recovery periods. The correlation between the two factors is relatively high, so they do not provide high diversification, but their returns have been superior to the other factors in Recovery.

*5.3.2.2 Expansion*

| Factors | Size | Value | Momentum | Quality | Min. Vol |
|---|---|---|---|---|---|
| Excess return (monthly) | 0.03% | -0.01% | 0.51% | -0.24% | -0.59% |
| Excess return (ann.) | 0.30% | -0.13% | 6.31% | -2.84% | -6.82% |
| Volatility (ann.) | 5.73% | 6.53% | 7.90% | 3.42% | 4.12% |

Table 13: Factor performances in Expansion in 1998-2011. Source: Own construction.

Size and Value have excess returns close to zero, indicating returns very close to the market. However, they sometimes deviate from the market because of their relatively high tracking error, shown as the volatility of the excess returns. Momentum is the best performing factor in this stage, with an excess annual return of 6.31%. The outperformance might stem from the fact that stocks performing well in the Recovery phase are bought in the Momentum factor, and these firms keep performing well in the Expansion phase, where the change in CLI keeps on improving. The Expansion phase can also come right after a Slowdown stage, where the Momentum factor has an even higher excess return relative to the market. Therefore, the Momentum is well set in the Expansion phase, where the previous stage could have been both a Recovery or a Slowdown regime. On the other hand, Momentum has shown the highest excess volatility relative to the other factors, which might stem from holding expensive stocks since the Momentum factor buys the recently best-performing stocks. As a result, those stocks could struggle to keep up with their high monthly returns since they have performed well recently and achieved a relatively high stock price. Quality and Min. Vol. have shown lower returns than the market but also lower volatility of excess returns relative to the other factors. The negative excess returns by the Quality factor might be because Quality stocks have lower debt to equity ratios contributing to being less risky than stocks in other factors. The Min. Vol. factor has relatively low risk, and this factor does not benefit as much from the increased risk appetite in the Expansion phase. So, the defensive factors, Quality and Min. Vol., are not performing as well as the market in bull markets.

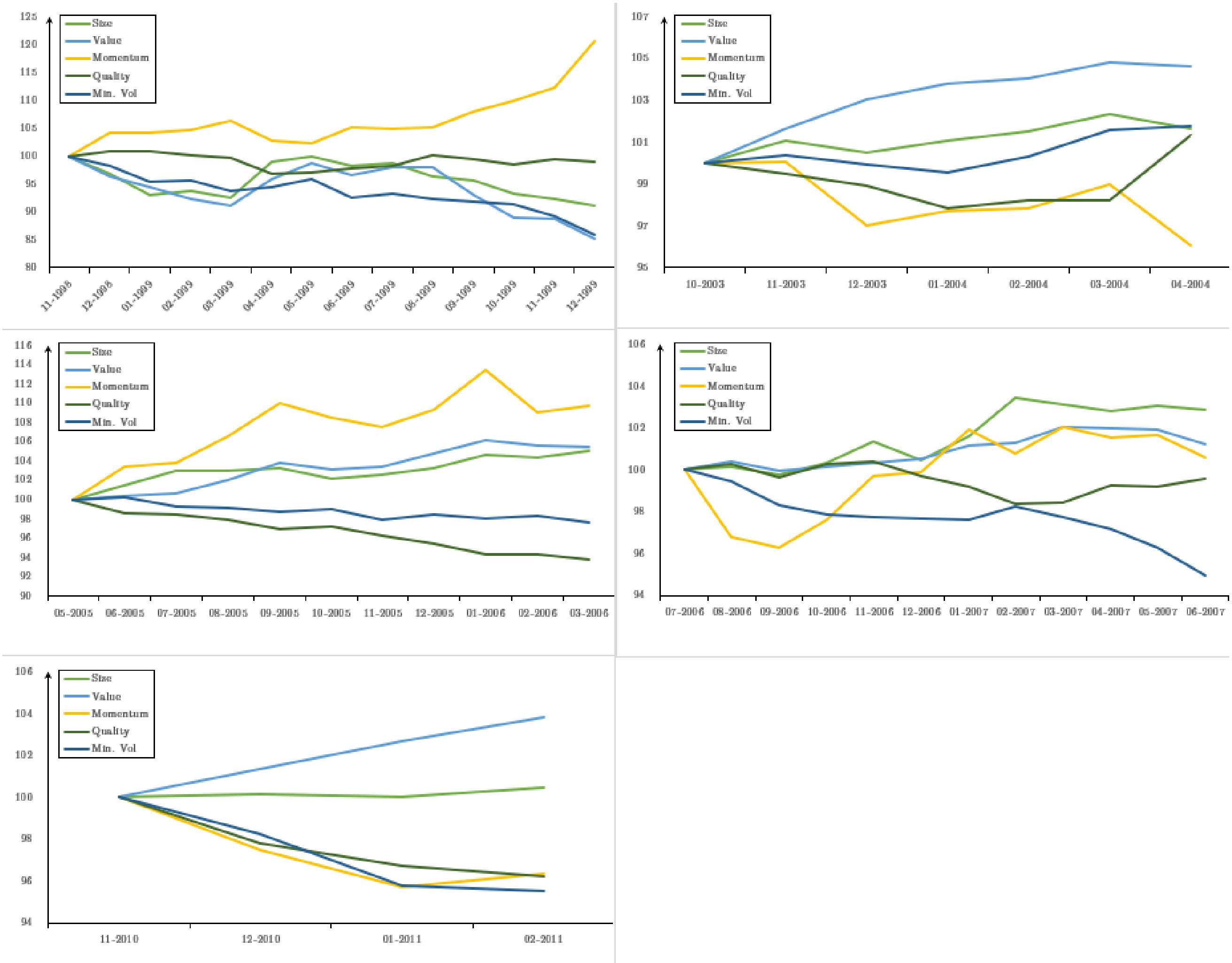

Figure 9: Factor performance across all Expansion regimes in the estimation period. Source: Own creation.

After plotting the factor performances in all Expansion periods, it becomes clear how well the Momentum has performed in the first Expansion phase, which started at the end of 1998 and ended end of 1999. As the technology sector performed very well in the dot-com boom, the Momentum factor greatly enjoyed the long Expansion period. In this period, internet-related companies have become a part of the Momentum factor since they have performed well recently. The internet companies performed well when the high increase in internet adoption occurred. The Momentum factor has performed well in the Expansion phases in 2005 and 2006 but has underperformed in the Expansion phases starting end of 2003 and end of 2010. Therefore, the Momentum factor has been the most volatile, but it has, on average, a high excess return relative to the market. Size and Value have performed well in some periods, but overall, they are not providing any significant excess returns. Min. Vol. and Quality have primarily performed poorly in all the Expansion periods, except the Quality factor reaching the second-highest return in the dot-com boom. However, the Quality factor has in this period still an index level slightly below 100, which has been better than all the other factors, except for the Momentum factor.

| | **Size** | **Value** | **Momentum** | **Quality** | **Min. Vol** |
|---|---|---|---|---|---|
| **Size** | 1.0000 | | | | |
| **Value** | 0.6881 | 1.0000 | | | |
| **Momentum** | -0.2488 | -0.4437 | 1.0000 | | |
| **Quality** | -0.5132 | -0.2545 | -0.0317 | 1.0000 | |
| **Min. Vol** | 0.5704 | 0.5021 | -0.3272 | -0.0725 | 1.0000 |

Table 14: Correlation matrix in Expansion in 1998-2011. Source: Own construction.

From the correlation matrix, we observe how the correlation between Size and Value has increased relative to the correlation in the case of the full sample across all four regimes. Momentum is now negatively correlated to the two defensive factors, Quality and Min. Vol. Another significant difference relative to the full sample case is the increased correlation between Size and Min. Vol. going from 0.0870 to 0.5704. These two factors move in the same direction to a large extent in the Expansion phase. Also, Value and Min. Vol. now have a very high correlation, contrary to the full sample case. Additionally, the Momentum factor is even more negatively correlated to the cyclical stocks than in the full sample, so Momentum moves very differently from the other factors during Expansions and has shown the highest returns.

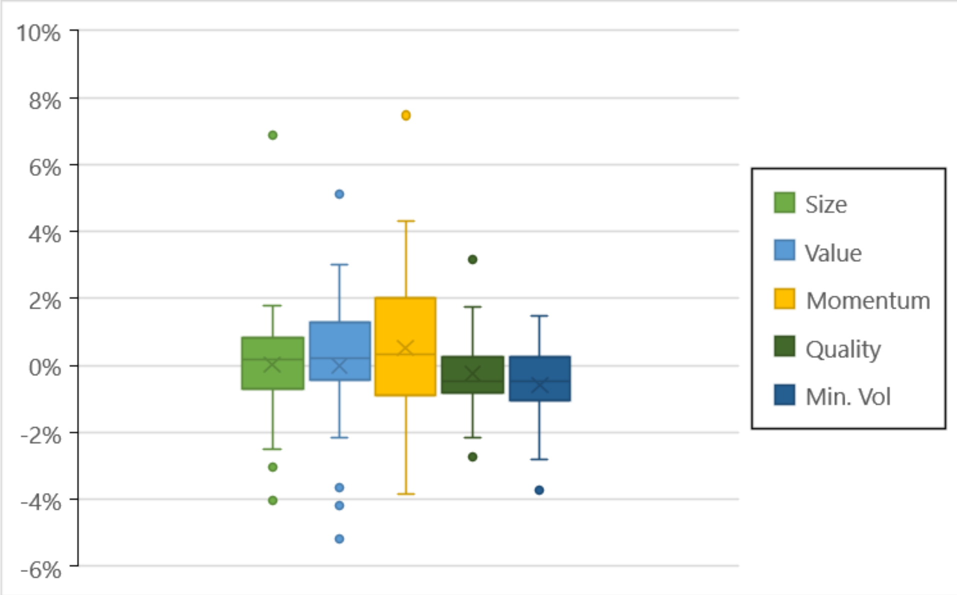


Figure 10: The boxplot illustrates the distribution of monthly returns in Expansion in 1998-2011. Source: Own construction.

There have been many outliers in the Expansion phase, where Value has shown three very low returns as outliers, whereas Momentum has had a very positive outlier. Also, Size has had some outliers, with two very low returns and one high. The biggest dispersion is observed from the Momentum factor with a mean higher than the median, so there have been some very high returns observed here, which have increased the average Momentum return. Quality has been very consistent, with a minimal variation in returns. Similarly, Min. Vol. has had a relatively low dispersion, and both defensive factors have had negative excess returns in the Expansion phase.

Overall, the Momentum factor has shown much better returns than the other factors, especially in the dot-com boom but also in other Expansion periods when looking at the individual periods. Momentum negatively correlates with all other factors in Expansions, so it behaves very differently from the remaining factors during this regime. The cyclical factors have almost just tracked the market but with positive excess volatility. The defensive factors have been very consistent with only minor variations in returns.

#### *5.3.2.3 Slowdown*

| Factors | Size | Value | Momentum | Quality | Min. Vol |
|---|---|---|---|---|---|
| Excess return (monthly) | -0.05% | 0.08% | 0.82% | 0.25% | 0.34% |
| Excess return (ann.) | -0.59% | 0.98% | 10.29% | 3.08% | 4.12% |
| Volatility (ann.) | 5.61% | 6.40% | 8.82% | 4.12% | 4.90% |

Table 15: Factor performances in Slowdown in 1998-2011. Source: Own construction.

The Slowdown regime is characterized by having a measure of the CLI above the mean (100), but the rate of change is declining. This indicates a solid economy, previously trending upwards but is starting to show weakness. When the economy is beginning to show weakness, the more defensive stocks tend to outperform the market as a whole. In general, the defensive stocks are less interest rate and cash flow sensitive, thus not suffering from increases in interest rates or decreases in cash flows. In addition, some of the main characteristics of both Quality and Min. Vol. are that the firms generally have a lower debt to equity ratio and are thereby not as affected when the overall economy starts to weaken.

Momentum is the top-performing factor in the Slowdown phase by having an annualized outperformance relative to MSCI USA of 10.29%. This is done while having a volatility of the excess returns of 8.82%, showing a large dispersion in the excess returns. One possible explanation of the performance of Momentum might be that Momentum invests in the best-performing stocks. Hence it should be exposed to the best-performing sectors and stock which has driven the past bull market. The recent top performers might not be the first stocks to turn around and react to the economy's weakening but instead, continue to perform well in the near future. Momentum is said not to be good in turning points, as seen in the Recovery phase. However, it doesn’t seem to be the case in the turning point from Expansion to Slowdown. This might be due to the two regimes not being as different in characteristics as Recovery and Contraction. When going from Expansion to Slowdown, we are still in a solid economy, however not with as good prospects for the future.

The more cyclical factors such as Size and Value do not seem to yield different returns than MSCI USA on average. They do, however, have large dispersions from the overall market. In this regime, it seems to act more like idiosyncratic risk, thus not being compensated. Being more cyclical in nature, it would also be logical that these stocks do not have the greatest prospects when the economy is weakening. Cyclical stocks tend to perform well in expanding economies and worse in declining economies.

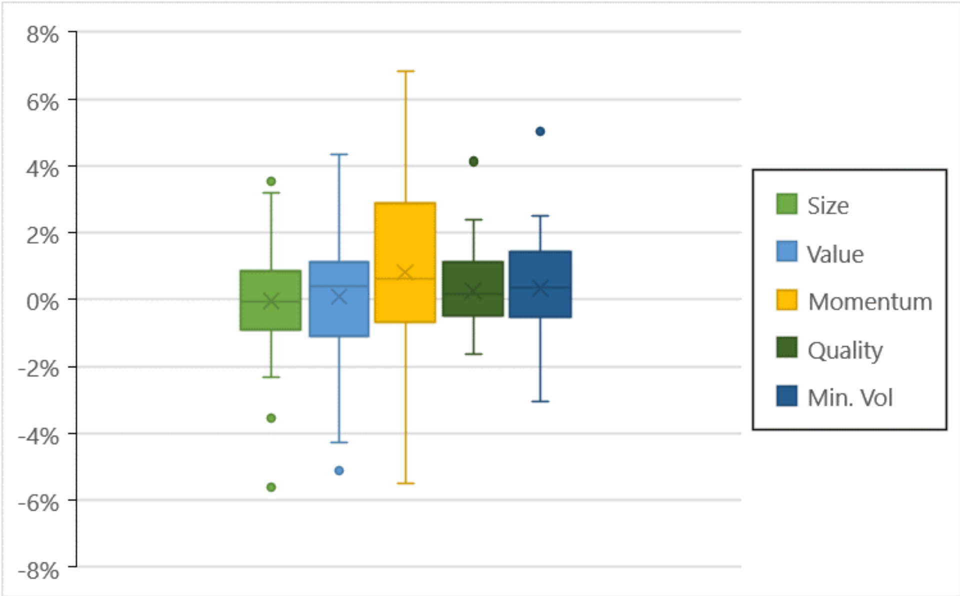


Figure 11: The boxplot illustrates the distribution of monthly returns in Slowdown in 1998-2011. Source: Own construction.

The medians of the box plots lie close to 0%, with Momentum having the highest median of monthly excess returns. However, the median of Momentum is slightly less than the average, indicating some relatively high monthly returns dragging the average return upwards.

Momentum has larger whiskers than the other factors showing a higher dispersion of excess returns in the Momentum factor. These more extreme returns are both on the positive and negative sides. The interquartile range for Momentum is larger than the four other factors indicating that not only are the tails more extreme, but there is also a higher variation in excess returns in the middle 50% returns, which is between the first and third quartile. The average return of Value didn't show any significant deviations from MSCI USA, but the dispersion of excess returns relative to MSCI USA ranged from +/- 4%, indicating a relatively large tracking error. Quality again has a pretty narrow range of excess returns, which is also why we see the lowest standard deviation of excess returns in Quality. Quality is a quite stable factor without having large deviations from the market. The Quality factor invests in stocks with a large Return on Equity (RoE), low Debt to Equity (D/E), and low Earnings Variability, which often leads to investing in companies that do not move as much, leading to more stable returns.

All factors but Momentum has experienced outliers, either positive or negative. This is because the interquartile ranges are relatively narrow, making large positive or negative excess returns look extreme. However, when looking at Momentum, there are not any outliers. This is not because Momentum doesn't have extreme values. Instead, the extreme values observed for Momentum are simply more common and don't look as extreme compared to the remaining excess returns of that factor.

Momentum is positively skewed, showing many excess returns centered around 0% with some relatively high returns. The opposite seems to be the case when looking at Value. Value is negatively skewed again, with many excess returns centered around 0% and some being relatively more negative, giving a negative skew. When looking at Minimum Volatility, the interquartile range looks normally distributed with the median around the center of the range. The lower quartile is larger than the upper quartile indicating larger dispersion among the negative returns than the positive. Min. Vol. doesn't have large positive excess returns except for one outlier but has more consistent positive excess returns.

| | Size | Value | Momentum | Quality | Min. Vol |
|---|---|---|---|---|---|
| **Size** | 1.0000 | | | | |
| **Value** | 0.5993 | 1.0000 | | | |
| **Momentum** | -0.2354 | -0.1633 | 1.0000 | | |
| **Quality** | -0.5112 | -0.5375 | 0.2567 | 1.0000 | |
| **Min. Vol** | 0.4386 | 0.1945 | -0.5632 | -0.1656 | 1.0000 |

Table 16: Correlation matrix in Slowdown in 1998-2011. Source: Own construction.

Looking at the correlations in the Slowdown regime, there seem to be some changes compared to the entire period. Size and Min. Vol. increase their correlation in the Slowdown regime. This could indicate that Min. Vol. would increase its exposure to small companies. It went from being nearly uncorrelated to a correlation of 0.44. Quality and Min. Vol. went from having a slightly positive correlation of 0.25 to a slight negative correlation of -0.17 in this regime. Hence, when in a Slowdown phase, the two more defensive factors become negatively correlated even though they both have positive excess returns.

The most significant correlation change is between Momentum and Min. Vol. There seems to be almost zero correlation between the two when looking across all regimes. However, when looking only at a Slowdown regime, the correlation becomes significantly negative with -0.56.

This suggests that the two factors move in the opposite direction most of the time. This negative correlation could provide significant diversification benefits when constructing a multi-factor portfolio.

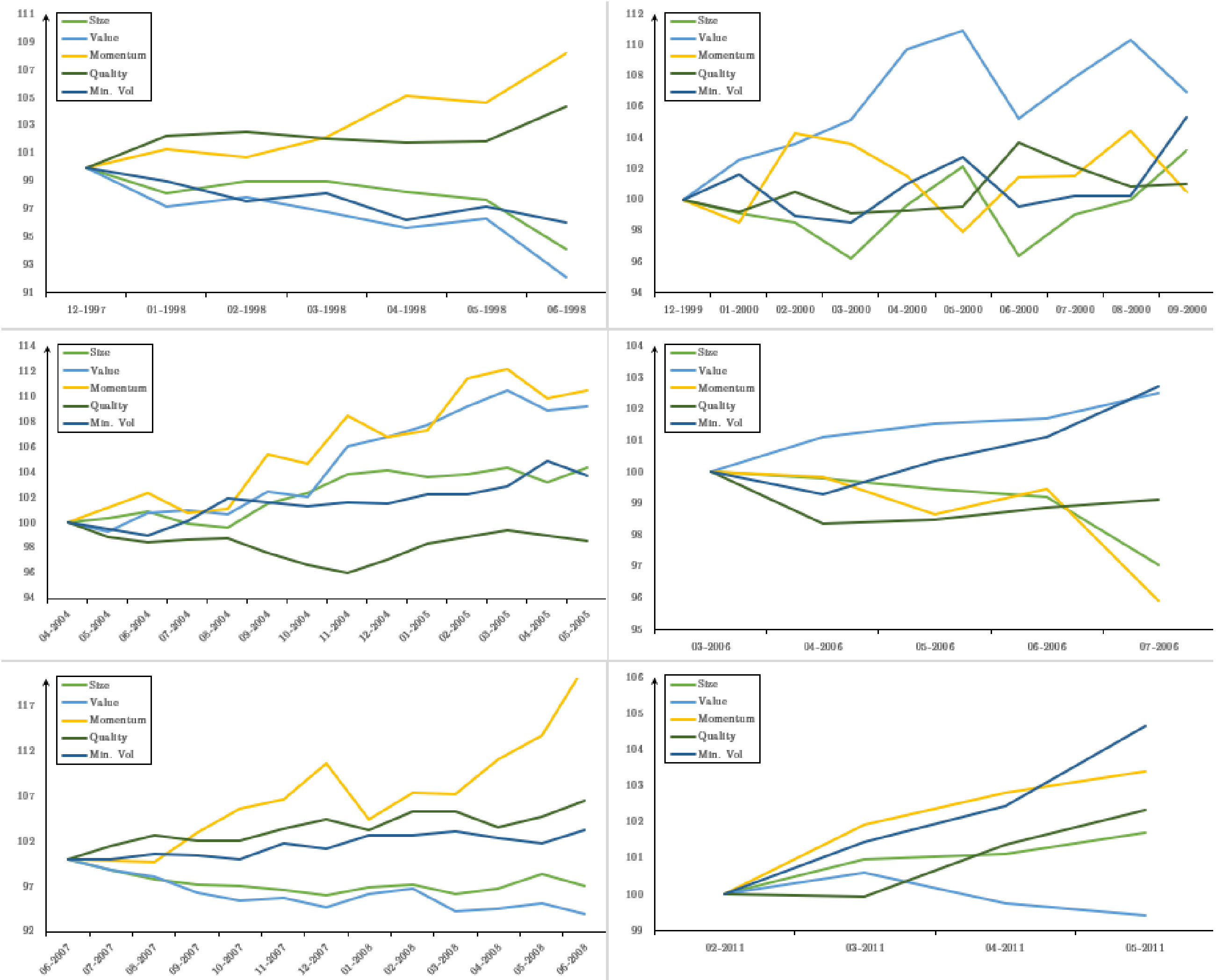


Figure 12: Factor performances in Slowdown in 1998-2011. Source: Own construction.

From 1998 to 2011, there were six Slowdown regimes, with the shortest one being only three months and the longest being an entire year. Looking at the performance of the factors in each individual regime, we see that Momentum does reasonably consistently, only having one bad performing regime and one being mediocre. Momentum is one of the best performing factors in the remaining four Slowdown regimes.

As the boxplot indicated, Value seems to vary a lot in the outperformance, with some periods of great outperformance and, conversely, significant underperformance. Value and Size seem to do very poorly in the first regime from 01-1998 to 06-1998. Hence, stocks with a more cyclical nature suffered from lower returns during the dot-com boom. Looking at the lower-left corner of figure 12, we see the period leading up to and at the beginning of the financial crisis where cyclical stocks were punished again.

The negative correlation between Momentum and Min. Vol. can also be seen in most regimes. In some regimes, both do reasonably well. However, for the most part, when one of them is doing poorly, the other is doing quite well. Nevertheless, in all regimes, either Momentum or Min. Vol. is one of the top two performing factors.

Overall, Momentum seems to be the best performing single factor during the Slowdown phase, both in terms of excess returns alone and risk-adjusted excess returns. Momentum has large dispersions in the excess returns relative to the market, indicating systematic factor risk being compensated. The more defensive factors, Quality and Minimum Volatility, also perform well during this regime. The two factors benefit relative to the other factors from the economy weakening. Momentum and Min. Vol. have a negative correlation of –0.56, providing good diversification benefits for a multi-factor investor.

#### *5.3.1.4 Contraction*

| Factors | Size | Value | Momentum | Quality | Min. Vol |
|---|---|---|---|---|---|
| **Excess return (monthly)** | 0.00% | 0.24% | -0.33% | 0.77% | 1.12% |
| **Excess return (ann.)** | -0.04% | 2.88% | -3.94% | 9.62% | 14.32% |
| **Volatility (ann.)** | 6.97% | 7.74% | 11.14% | 4.61% | 9.59% |

Table 17: Factor performances in Contraction in 1998-2011. Source: Own construction.

The Contraction phase is characterized by the CLI anticipating GDP levels below the long-term average and continuing to decline. In the Contraction regime, the general market has large drawdowns with average annualized returns of -31.07% from 1998 until 2011. When the market is experiencing low levels of GDP and declines in the general economy as well as the stock market, the more defensive stocks tend to do better than the overall market. The defensive stocks benefit from not being overly leveraged, having strong earnings, and not being as cash flow sensitive. As a result, we observe a strong outperformance relative to MSCI USA in both Quality and Min. Vol. However, the outperformance comes from the more defensive stocks still declining, but not as much as the broader market. Declines in this phase can be challenging to avoid. However, one can aim to be positioned to minimize the drawdowns.

The worst performing factor in a Contraction phase is the Momentum factor. Momentum has annualized excess returns of -3.94% resulting in larger losses in this regime than the benchmark. It might be due to Momentum investing in stocks that have increased the most in price in the past. These stocks may be overpriced and suffer from large declines in price when a Contraction comes. In addition, the excess returns have large volatility, indicating large variations from MSCI USA. These large volatilities may result in some periods having extreme negative or positive returns by being invested in Momentum.

The more cyclical stocks don't seem to diverge much from the benchmark during this stage. For example, Size doesn't have any significant difference in the returns than MSCI USA. Being invested in Size during this phase only makes the investor subject to idiosyncratic risk that doesn't seem to get compensated by a risk premium. Value, however, has an annualized excess return of 2.88%, suggesting that there is some risk premium to be gained by being invested in Value during a Contraction phase. One possible explanation for this could be that the Value factor invests in relatively underpriced stocks. These stocks may already have low prices, so they don't fall as much as stocks with higher relative prices.

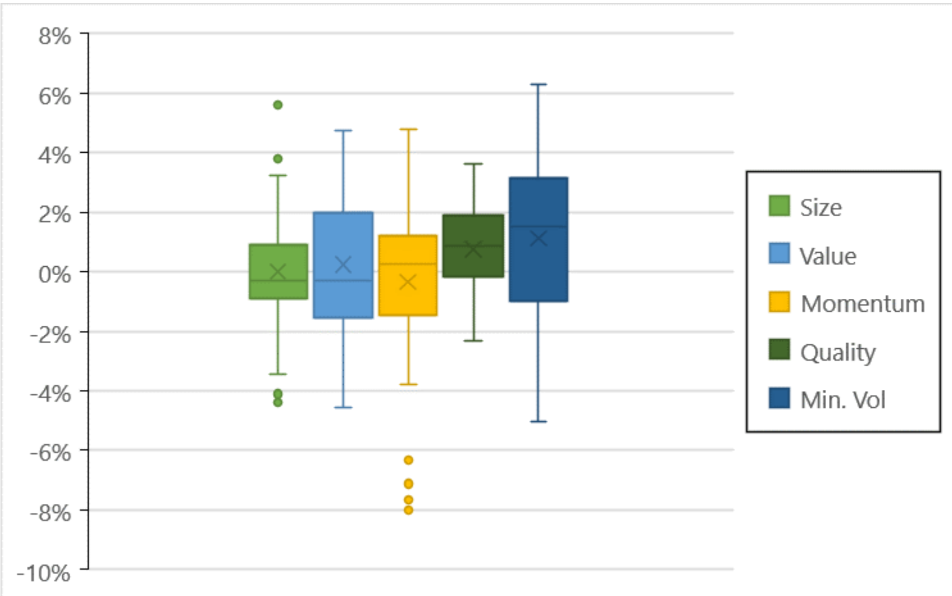


Figure 13: The boxplot illustrates the distribution of monthly returns in Contraction in 1998-2011. Source: Own construction.

Contrary to what one would think, Min. Vol. in the Contraction phase has the largest volatility of excess returns of all the five factors. The median of returns for Min. Vol. is relatively high, with close to 2% excess return every month. Most of the excess returns lie above 0% but with a tail down to -5%, showing that there still is a significant underperformance of Min. Vol. in some months. Quality has more stable returns and is the most stable of all factors, with mean and median excess returns close to 1% a month. The excess return distribution of Quality is centered above 0%, ranging from approximately -2% to 4%, showing a tendency of above 0% excess returns.

Momentum has average excess returns below 0%, however, the median of excess returns lies above 0%. This suggests that Momentum, more than half the time, actually has higher monthly returns than the MSCI USA. The average return is being dragged down by some significant outliers between -6% and -8% in excess of MSCI USA. Besides these outliers, Momentum looks to be relatively centered around 0%, indicating no significant risk premium.

Size and Value are centered around zero, indicating no significant risk premium. Value has an average excess return slightly above 0%, however, with a median just below 0% showing that more than half the excess returns are negative. This suggests that the risk premium of Value comes from some large positive gains, which can also be seen by the distribution of excess returns having a positive skewness. Size has a small body which is closely centered around zero showing that in a Contraction phase, it, for the most part, follows the market relatively close. It does, however, have some significant outliers showing the presence of some idiosyncratic volatility.

| | **Size** | **Value** | **Momentum** | **Quality** | **Min. Vol** |
|---|---|---|---|---|---|
| **Size** | 1.0000 | | | | |
| **Value** | 0.6210 | 1.0000 | | | |
| **Momentum** | -0.0826 | -0.0275 | 1.0000 | | |
| **Quality** | -0.5573 | -0.3224 | 0.1092 | 1.0000 | |
| **Min. Vol** | -0.0235 | -0.0180 | 0.3611 | 0.2961 | 1.0000 |

Table 18: Correlation matrix in Contraction in 1998-2011. Source: Own construction.

During the Contraction stage, some of the factors increase their correlations. Looking at Momentum and Min. Vol., we see that they change from being close to uncorrelated to having a positive correlation of 0.36. This might, to some extent, be due to Momentum allocating more to the defensive stocks during the Slowdown phase, which in many cases precedes the Contraction phase. Min. Vol. and Quality remain slightly correlated with a correlation of 0.30, which only slightly increases the correlation of the two factors in this regime.

Momentum goes from being somewhat negatively correlated to the cyclical factors such as Size and Value to being close to uncorrelated to the two factors during a Contraction.


Figure 14: Factor performances in Contraction in 1998-2011. Source: Own construction.

From 1998 to 2011, the CLI predicted the economy to be in a Contraction phase six times. The two longest being after the burst of the dot-com bubble and following the financial crisis in 2008. When looking at the relative factor performances, it is important to remember that it is the return in excess of MSCI USA. On average, the MSCI USA yielded an annualized return of -31.07%, indicating that the market, in general, is declining at this stage.

The individual factor performances for the defensive factors, Quality and Min. Vol. are consistent over almost all six Contraction regimes. Only following the dot-com crash in the early 2000's did the more cyclical factors do best, however still with very good performance of Min. Vol.

Overall, both Momentum and the cyclical factors seem to perform poorly in this regime. The returns of Size and Value come solely from the one period of good performance, which may be considered an outlier. During the financial crisis of 2008, Momentum has severe underperformance and is lagging MSCI USA by around 15% by the end of the regime.

When the market reaches the Contraction stage, defensive factors show clear outperformance over the other factors and the market in general. Quality and Min. Vol. have significantly higher excess returns than the remaining three factors. Momentum becomes the worst performing factor with negative excess returns. During the Contraction regime, the market tends to fall a lot. However, being exposed to defensive factors tends to minimize this regime's otherwise large drawdowns.

*5.3.1.5 Sub Conclusion to Factor Testing Across Regimes*

Testing the five factors in different macro regimes from 1998 to 2011 showed significant differences in how the factors performed in different regimes. No factor was the top-performing factor across all regimes, suggesting that a more dynamic approach could be preferable. By dynamically switching between the factors according to the current regime, one might be able to generate returns in excess of a static model.

In the Recovery phase, Size and Value were the best performing factors, with annualized excess returns of 11.15% and 9.87%, respectively. They were consistent and outperformed the other factors in almost all the Recovery periods. The two factors do, however, have a relatively large correlation which suggests that the diversification benefits from combining these two factors are not very high.

One factor performs much better in the Expansion phase than the rest, namely Momentum. Momentum has an annualized excess return of 6.31%, being the only factor with significantly positive excess returns. On the other hand, Size has a minimal excess return, and the remaining factors show negative returns in excess of MSCI USA during the Expansion stage.

Momentum is again the best performing single factor during the Slowdown regime with annualized excess returns of 10.29%. Momentum has large dispersions in the excess returns suggesting a systematic risk, since higher returns compensate the high risk. The defensive factors also perform well during this regime. This is due to the previous bull market that is now showing signs of weakness. This weakness is good for defensive factors relative to the cyclical ones. Min. Vol. has an annualized outperformance of 4.12%, which is negatively correlated with the high excess returns of Momentum.

Not surprisingly, the defensive factors show clear outperformance in the Contraction phase relative to the market. Quality has annualized excess returns of 9.62%, where the excess returns of Min. Vol. is 14.32% being the highest excess average return among all factors and across all regimes. Even though the factors are defensive in nature and show high levels of outperformance in this regime, the correlation between the two is only 0.30 in a Contraction stage.

### 5.3.3 Regime-Based Factor Model

We have constructed a Regime-Based Factor Model based on the optimal factors analyzed for each regime in the previous section. Therefore, this model is based on an overall assessment of our in-depth analysis based on our estimation period from 1998 to 2011. We will test the model from 2012 through 2021 to avoid any forward-looking bias.

We found Value and Size to be the best performing factors in the Recovery phase, primarily because of their high outperformance in returns. The outperformance of these two cyclical stocks might stem from these factors having procyclical performance characteristics, so they are performing well when the market is recovering and improving. Even though their correlation with each other is relatively high, we found their superior outperformance to be strong enough to solely choose these two factors in the Recovery phase for our Regime-Based Factor Model.

Momentum is the only factor chosen in our model for the Expansion phase, and this is based on superior outperformance with an annual excess return of 6.31%. The Momentum factor performs well when price trends and fundamentals are relatively stable, so the firms recently performing well have high probabilities of continuing to perform well. The Momentum excess return is much higher than the second-best performing factor, Size, with an annual average return of 0.30%. The other three factors have delivered negative excess returns relative to the market. Therefore, we have chosen not to include any other factors than the Momentum factor in the Expansion regime.

We found a combination of Momentum and Min. Vol. for the Slowdown phase to be most beneficial. These two factors have been the best performing factors in terms of excess returns. They have a negative correlation, so this combination is beneficial for diversification purposes and when seeking a high excess return. The Quality factor would also be a candidate because of the excess return, which is only slightly below the Min. Vol. factor, and the volatility is below Min. Vol. However, the correlation between Quality and Momentum is positive, so Min. Vol. is better to combine with Momentum from a diversification point of view. The two defensive factors are among the top 3 best performers because the economy shows signs of weakness. The countercyclical firms begin to outperform the cyclical firms. The strong performance in the Momentum factor might be firms performing well in the Expansion phase that continue to perform well in some months following the regime change, where the economic activity level is still above the long-term trend.

When looking at the Contraction phase, the obvious choice is to pick the defensive factors, Quality and Min. Vol., that have shown clear outperformance with excess returns of 9.62% and 14.32%, respectively. These excess returns of the two defensive factors relative to the market return might stem from the two factors having counter-cyclical performance characteristics with relatively low sensitivity to cash-flow news. Their correlation is positive but still relatively low, so we found these two factors relevant to combine in the Contraction regime. The Regime-Based Factor Model's chosen factor allocation weights can be seen in Table 19.

| | Size | Value | Momentum | Quality | Min. Vol |
|---|---|---|---|---|---|
| **Recovery** | 50.00% | 50.00% | 0.00% | 0.00% | 0.00% |
| **Expansion** | 0.00% | 0.00% | 100.00% | 0.00% | 0.00% |
| **Slowdown** | 0.00% | 0.00% | 50.00% | 0.00% | 50.00% |
| **Contraction** | 0.00% | 0.00% | 0.00% | 50.00% | 50.00% |

Table 19: Factor allocations across the four regimes for the Regime-Based Factor Model. Source: Own construction.

In addition to our previous analysis, it is further relevant to look at different possible shifts in regimes to see how robust and well prepared our model is when moving on to a different regime, where the optimal factor allocation choice changes. This is especially useful to look at because there is a lag from when there is a regime change and to when OECD is publishing their CLI number from that month. Therefore, our model will have a lag with some non-optimal allocations until the data from OECD is published so that we can rebalance the portfolio. Because of this, the robustness of our model in regime shifts affects the performance of our Regime-Based Factor Model.

When looking at the Recovery regime, we are exposed to the cyclical factors, Size and Value, and we can move into Expansion or back to Contraction. The model will not perform very well in a shift towards the Contraction phase since Quality and Min. Vol. are the best performing factors in that regime. However, Value is the third-best performing factor with a positive excess return. Size only has a slightly negative excess return, so our model would still yield a positive expected excess return relative to MSCI USA in this stage. Our model is in the case of a regime change from Recovery to Contraction, benefitting from not having any exposure to the Momentum factor. If we move from Recovery to Expansion, the model is well prepared since both Size and Value are in the top 3 among the highest excess returns in the Expansion phase. Value and Size have excess returns close to zero, so the model would almost follow the MSCI USA Index but with excess volatility. Our model benefits when moving from Recovery to Expansion by not being exposed to Quality and Min. Vol., that are the worst-performing factors in the Expansion regime.

When the model is solely exposed to Momentum in the Expansion phase, the only stage that can follow this period is Slowdown, and Momentum is the best performing factor here. Hence, our model is robust for a regime shift after the Expansion phase.

In the Slowdown stage, the possible scenarios are going into the Contraction phase or back to an Expansion. Here, our model is well equipped since a change into the Expansion phase makes it optimal for allocating to especially Momentum, and here we are already exposed to that factor in the Slowdown stage. When moving on to a Contraction regime, we found Quality and Min. Vol. to be the best factors, and we are already exposed to Min. Vol. from Slowdown. Therefore, we do not have to rebalance the entire portfolio, whether moving to the Contraction or Expansion phase, since we already have some optimal factor allocations in the Slowdown phase. This minimizes the turnover rate for our model and the non-optimal exposures that follow from the lag in rebalancing to the new optimal factor allocation in a regime change.

However, our model is not well prepared for regime changes after the Contraction stage, where the Recovery phase is the only possible regime to follow. Quality and Min. Vol. are the worst-performing factors in Recovery, so our model is negatively affected during the beginning of the shifts towards Recovery, where we have some non-optimal factor exposures.

To sum up the regime changes, our model is well prepared for some regime changes and less for others. The model is well equipped for a new regime in the Expansion and Slowdown regimes. In the Recovery regime, the model has some decent exposures when moving on to the Expansion phase since we are exposed to the second and third best-performing factors in excess returns. However, our model is less prepared to shift from Recovery toward the Contraction stage because of the defensive factors, Quality and Min. Vol., are the preferred factors there. Still, the model has exposure to Value in this regime shift, which has a positive excess return. Our model becomes vulnerable when standing in the Contraction stage since we are exposed to the defensive factors. We found these factors to provide negative excess returns when moving on to Recovery, the only possible regime to follow after a Contraction phase.

After constructing our model based on the estimation period from 1998 to 2011, we tested our model's performance from 2012 to 2021 to avoid any biases that we would have if we had tested our model on the same data we used to construct the model. The test period is in our data, therefore, assumed to be yet unknown, which is similar to a real-life implication for an investor.

| Metrics | Equally Weighted | Regime-Based Factor Model |
|---|---|---|
| Return (Ann.) | 17.00% | 19.55% |
| Volatility (Ann.) | 12.68% | 12.89% |
| Sharpe Ratio | 1.288 | 1.463 |
| Sortino Ratio | 1.542 | 2.137 |
| VaR | 5.49% | 5.23% |
| Expected Shortfall | 8.67% | 7.99% |
| Active Return | | 2.55% |
| Active Risk | | 4.12% |
| Information Ratio | | 0.620 |
| Turnover | 7.06% | 128.14% |
| T-test | | 6.765 |
| Max Drawdown | 21.09% | 19.34% |

Table 20: Performance measures of the Equally Weighted Model and the Regime-Based Factor Model in 2012-2021. Source: Own Construction.

After calculating the performance statistics shown in table 20, we get an annual average return of 19.55% produced by our Regime-Based Factor Model compared to 17.00% of the static Equally Weighted Portfolio of all the five factors. The excess return produced by our model relative to the Equally Weighted Model is 2.55 percentage points higher but with a slightly higher annual volatility of 0.21 percentage points. The higher volatility of our dynamic Regime-Based Factor Model might stem from some more concentrated exposures with less diversification because some factors get more exposure than others compared to the Equally Weighted Model. Nevertheless, the outperformance in return outweighs the downside of slightly higher volatility. Hence, the Sharpe Ratio has increased to 1.463 in Sharpe Ratio instead of 1.288, which is equivalent to an improvement of 13.63%. Our model has outperformed the Equally Weighted Model significantly when looking at the t-stat of 6.77, which is above the critical level of 1.64 when looking at a 5% significance level. The outperformance is even significant when applying a 1% significance level with a critical value of 2.33.

Since our model seeks to time the factor exposures by allocating to the most optimal factors in each regime, it is relevant to look further into other risk measures other than the volatility. The Sortino Ratio differs from the Sharpe Ratio by looking only at downside volatility instead of total volatility. We get a Sortino Ratio of 2.137 instead of 1.542, meaning a 38.61%

improvement. This improvement is larger than the increase in Sharpe Ratio, which means that our model successfully minimizes the 'bad' times because we now have lower downside volatility relative to the excess return. Hence, the higher volatility must stem from the Regime-Based Factor Model having higher upside volatility. The lower downside risk can also be seen after calculating 95% Value-at-Risk (VaR) and Expected Shortfall (ES). The VaR has decreased in our model, so the maximum monthly loss at 95% probability is now 5.23% instead of 5.49%, so the difference is not very large but still relevant. The Expected Shortfall is the weighted average of the "extreme" or tail losses in the distribution of return outcomes. This measure has decreased from 8.67% to 7.99% when comparing our constructed Regime-Based Factor Model to the static Equally Weighted Model.

The Active Return is the average return difference between our Regime-Based Factor Model and the static Equally Weighted Model. We get a positive number of 2.55%, indicating outperformance stemming from the rotation between the factors. The model dynamically changes its factor exposures when the regimes change, but with a lag at the beginning of a new regime, as earlier mentioned. The Active Return is obtained with an Active Risk of 4.12%, which is the annualized standard deviation of the monthly differences in returns between the two models. The Information Ratio becomes 0.62 after dividing the Active Return by the Active Risk. The Regime-Based Factor Model could sometimes underperform due to the Active Risk but, on average, delivers higher risk-adjusted returns.

Furthermore, the turnover rate is interesting for an investor to consider. The static Equally Weighted Model only has an annual turnover of 7.06% to maintain equal weights to each factor for every month. The Regime-Based Factor Model has an average annual turnover rate of 128.1%, with some stemming from holding constant weights through all the months within the same regime. However, the turnover rate for our dynamic model is primarily a function of having to change the allocations when reallocating according to a new regime. The turnover rate is further investigated in the robustness test in section 5.5.

The Maximum Drawdown (MDD) is another way to look at downside risk, where this measure looks at the maximum loss from a peak. For example, MDD is 19.34% for the Regime-Based Factor Model and 21.09% for the Equally Weighted Model, showing that the Regime-Based Factor Model has a lower risk than the Equally Weighted Model.

MDD is shown graphically for our dynamic model below. The grey dots show the drawdowns in percentage terms, measured on the vertical axis on the right side. The blue line is the index value for our model, and the orange line is High-Watermark (HWM). HWM shows the all-time highest point in the index value up until this point. The gap between the HWM and blue index line illustrates the same as the Drawdown. We see the biggest drawdown in March 2020, when the Corona Crisis started. This is where the Maximum Drawdown is found at 19.34%, which is the maximum loss from the peak.

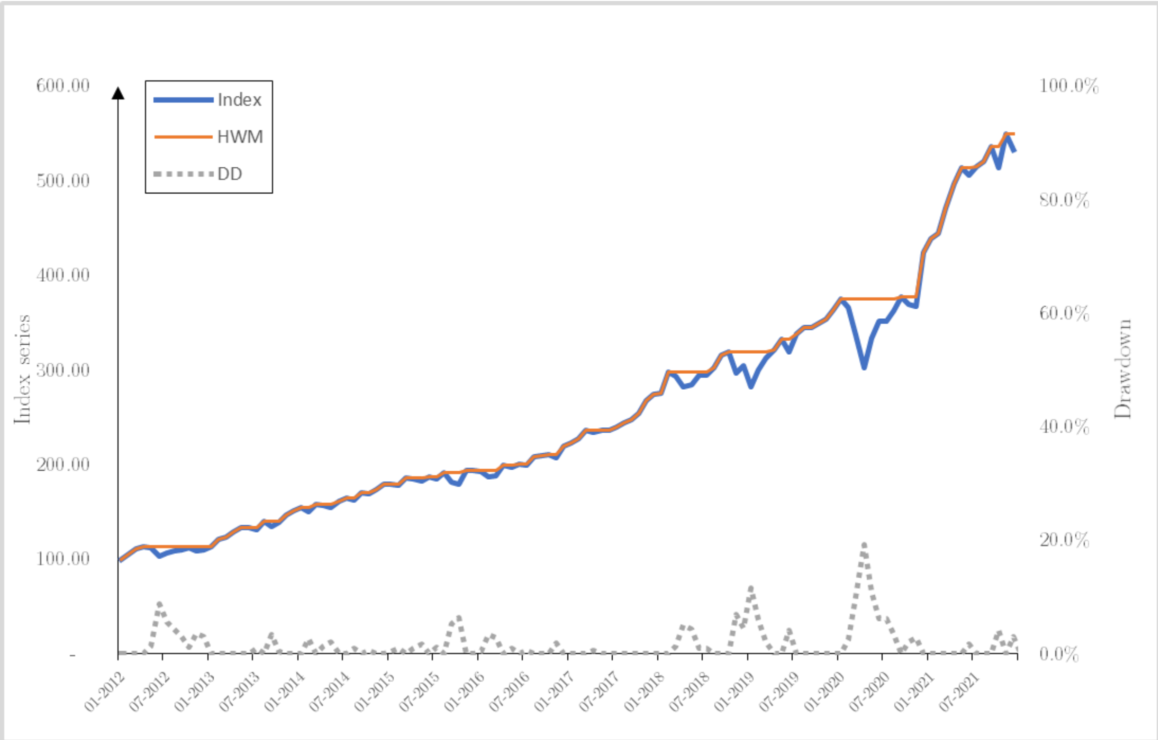


Figure 15: Drawdowns for the Regime-Based Factor Model in 2012-2021. Source: Own construction.

### 5.3.4 Regime-Based Mean-Variance Model

The mean-variance model is optimized based on the data's return vector and covariance matrix from 1998 to 2011. It is then tested on the timeframe 2012 to 2021 to avoid any estimation biases in the model. The static mean-variance model stated an optimal allocation of 89% to Momentum and 11% to Value which makes the model behave almost like a single factor Momentum strategy. However, in this section, we will construct a regime-dependent mean-variance optimized portfolio. Since the factors behave differently during different regimes, it may provide better portfolios by having regime-dependent estimates. In the regime-based model, we see large variations in how the optimal portfolio should be according to the current regime.

In the Recovery phase, the optimal allocation should be 24.50% to Size and 75.50% to Momentum. This allocation is different from the one suggested by the previous model, which allocated to Size and Value in this stage. The Mean-Variance Model chooses Momentum and not Value due to the negative correlation between Momentum and Size relative to the large correlation between Size and Value. Surprisingly, the primary allocation is to Momentum, which has the lower return of the two and higher excess volatility.

The Expansion phase adds one more factor to the portfolio, namely the Value factor. The model is now exposed to Size, Value, and Momentum with weights of 34.41%, 25.10%, and 40.49%, respectively. Momentum is still the factor with the highest exposure, in line with the previous model, which allocated only to Momentum. Size and Value don't have any significant excess returns in the Expansion phase but might be included solely based on their negative correlation to Momentum. Value goes from not being correlated in the Recovery phase to having a negative correlation of -0.44, providing better diversification benefits to the portfolio. Value and Size are still highly correlated but are both included in this regime.

The model is again only exposed to two factors in the Slowdown phase. Momentum has a very high exposure of 77.92% because it is the best performing single factor in this regime. The model also allocates 22.08% to Minimum Volatility. Min. Vol. also has decent excess returns and low volatility of excess returns. Momentum and Min. Vol. in the Slowdown stage become negatively correlated by -0.56, giving high diversification benefits even with the two highest excess returns. The two factors chosen in the previous model are the same due to their high returns and negative correlation.

In the last regime, namely, the Contraction phase, the model only allocates to one factor. The only factor in the portfolio in this regime is Min. Vol. This factor has the largest single factor excess return in this regime. Quality also performs well during this regime, however, it is not in the portfolio in this model due to the correlation with Min. Vol.

Compared to the static Mean-Variance Model taking only the full sample return data into account, we still see a relatively high exposure to Momentum, however not as significant as in the static model with 89% Momentum. We see higher allocations to both Size and Min. Vol. and still nothing in Quality.

| Metrics | Size | Value | Momentum | Quality | Min. Vol |
|---|---|---|---|---|---|
| Full sample | 0.00% | 11.00% | 89.00% | 0.00% | 0.00% |
| Recovery | 24.50% | 0.00% | 75.50% | 0.00% | 0.00% |
| Expansion | 34.41% | 25.10% | 40.49% | 0.00% | 0.00% |
| Slowdown | 0.00% | 0.00% | 77.92% | 0.00% | 22.08% |
| Contraction | 0.00% | 0.00% | 0.00% | 0.00% | 100.00% |

Table 21: Factor allocations across the four regimes for the Regime-Based Mean-Variance Model together with the Mean-Variance full sample allocations from section 5.2.2. Source: Own construction.

The way the portfolio is positioned going into a regime change is important for the model. Since there is a lag between when you observe a regime change and when one can rebalance the portfolio to have the new regime-dependent weights, some time has passed in which the model was wrongly exposed according to the ‘old’ regime. Therefore, it can be important to investigate how the model is exposed going into these changes in regimes.

Standing in the Recovery regime, we can either go into an Expansion or back to a Contraction. The model is in Recovery exposed to Size and Momentum, which will still be expected to perform well entering an Expansion. However, if we go back to a Contraction, the model is poorly allocated since Size and Momentum are the two worst-performing factors in this regime.

The model is allocated to Size, Value, and Momentum in an Expansion. In the Expansion phase, the only way we can move is into a Slowdown. Here the model would be decently exposed since Momentum is the best performing factor, and we have a 40.49% allocation to that factor. Size and Value are not the best factors in Slowdown but don’t seem to have any excess return significantly different from zero.

We can move either back to further Expansion or into a Contraction from the Slowdown regime. Since the model suggests allocations to Momentum and Min. Vol., it would be decently exposed regardless of the next regime. Min. Vol. has significant outperformance in a Contraction, but Momentum does poorly. The skewed allocation to Momentum in this regime can affect the performance going into a Contraction. Going back to an Expansion, Momentum is still the best performing factor and has high exposure in both regimes. Hence the model is well equipped for this regime change.

The only way the economy can go from a Contraction is into Recovery. Unfortunately, the model does a poor job during the change from the Contraction regime to a Recovery. Min. Vol., which the model allocates 100% to in a Contraction, is the worst-performing factor in the Recovery phase.

Overall, it is quite mixed when looking at the changes in regimes and how the model is exposed during these changes. The model is well equipped for any regime changes in the Expansion and Slowdown phase. However, it is only positioned to a shift to Expansion in the Recovery phase and does a poor job when going back to a Contraction. In a Contraction, it is not allocated well to a change in the regime.

The average annual returns produced by the Regime-Based Mean-Variance optimized portfolio seem to outperform the returns of an Equally Weighted Portfolio, which allocates to all five factors. The model has an annual outperformance of 0.95 percentage points (pp.), giving an annualized average return of 17.95% from 2012 to 2021. In addition, the model is superior when looking at both the return and volatility. The model generates a higher return and does so with lower volatility, resulting in a 9.6% improvement in the Sharpe Ratio. The outperformance relative to the Equally Weighted Portfolio is significant at the 5% level with a t-stat of 2.36.

| Metrics | Equally Weighted | Regime-Based Mean-Variance |
|---|---|---|
| Return (Ann.) | 17.00% | 17.95% |
| Volatility (Ann.) | 12.68% | 12.24% |
| Sharpe Ratio | 1.288 | 1.412 |
| Sortino Ratio | 1.542 | 1.810 |
| VaR | 5.49% | 5.27% |
| Expected Shortfall | 8.67% | 8.34% |
| Active Return | | 0.95% |
| Active Risk | | 4.41% |
| Information Ratio | | 0.215 |
| Turnover | 7.06% | 120.93% |
| T-test | | 2.356 |
| Max Drawdown | 21.09% | 19.04% |

Table 22: Performance measures of the Equally Weighted and the Regime-Based Mean-Variance Model in 2012-2021. Source: Own Construction.

A dynamic allocation between regimes aims to avoid the ‘bad’ times in the different factors. Factors have periods of significant underperformance, and these are the periods the model seeks to avoid. Therefore, it is relevant not only to look at the risk-adjusted returns in the Sharpe Ratio but also to consider downside volatility. The Sortino Ratio of the model is 1.810 compared to 1.542 in an equal-weighted portfolio, which is an increase of 17.4%. The increase in the Sortino Ratio is relatively larger than in the Sharpe Ratio, implying that the model minimizes the ‘bad’ times. It can also be seen when looking at the Value-at-Risk (VaR) and Expected Shortfall (ES). Both VaR and ES are lower for the Regime-Based Factor Model than the static Equally Weighted, showing a lower tail for losses in the return distribution. The Maximum Drawdown of the model is also lower than for the Equally Weighted Portfolio. The Regime-Based Mean-Variance Model shows a lower risk than the Equally Weighted Model in all risk parameters.

The Active Return of the model in relation to the Equally Weighted Portfolio is 0.95%. The Active Return being a positive number is also an indication of outperformance. The model does generate a higher return by actively selecting and rotating between factors rather than having a static allocation to each. The Active Return is achieved with Active Risk of 4.41% resulting in an Information Ratio of 0.215. This suggests that though the model outperforms the static model, it does so with a relatively high return variation. It could mean that the model could go through periods of relative underperformance in order to outperform on average.

The portfolio turnover is also of great importance when looking into an actively managed portfolio. The Equally Weighted Portfolio has a yearly turnover of 7.06% on average to maintain a 20% allocation to each factor. The Regime-Based Mean-Variance portfolio has, on average, an annual turnover of 120.9% as a function of having to change allocations according to each regime. Only a tiny portion of this turnover comes from holding constant weights within the regime. Turnover becomes an essential factor when considering the impact of transaction costs, which will be considered later in the robustness test in section 5.5.

Maximum Drawdown is another way to investigate the downside risk of the portfolio. The MDD for the Regime-Based Mean-Variance model is 19.04% which is lower than for the Equally Weighted Model and is even the lowest of both the two models constructed. The Regime-Based Mean-Variance Model only lost 19.04% during the period, whereas the EW model lost 21.09%, which is done even though there was generated an excess return. As shown in figure 16, we see only two or three significant drawdowns during the entire period. For most of the period, the portfolio index keeps making new highs and thus improving the High-Water-Mark. The two major drawdowns happened in late 2018 and in March 2020, when the COVID-19 crisis occurred. Both of the drawdowns had relatively fast rebounds and made new highs.

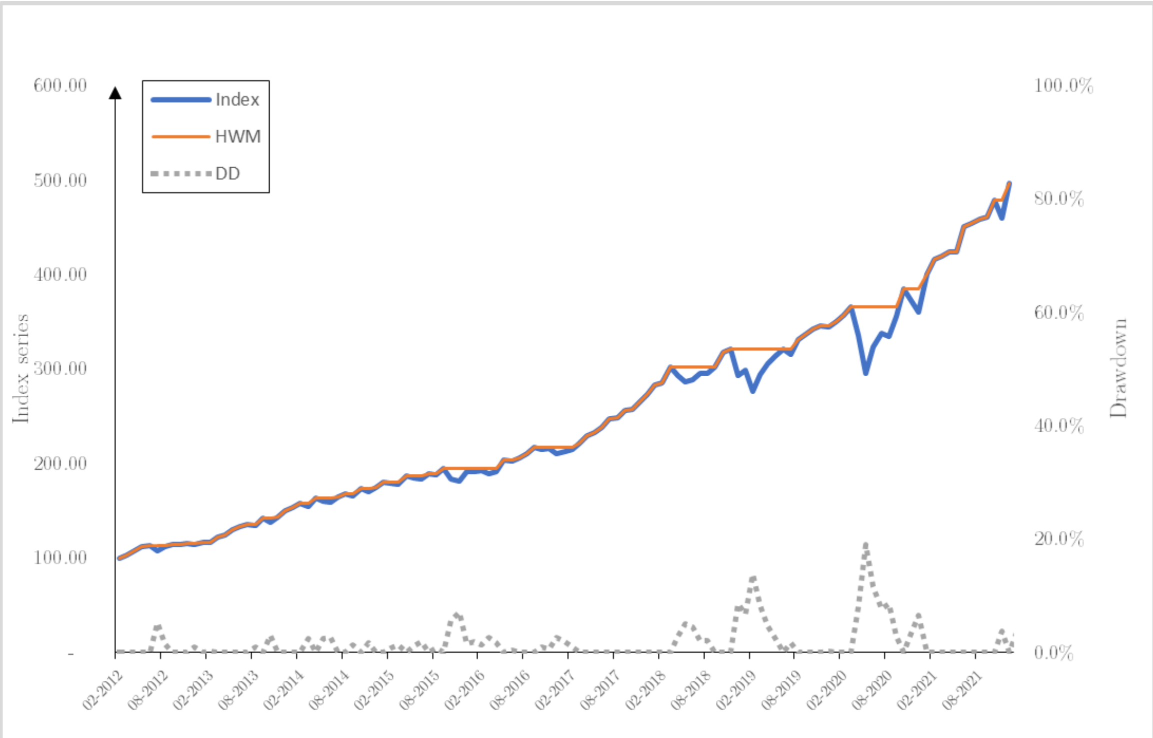


Figure 16: Maximum Drawdown for the Regime-Based Mean-Variance Model in 2012-2021. Source: Own construction.

## 5.4 Model Comparison

In order to summarize and compare the models constructed during the analysis, an overview in terms of returns and volatilities is illustrated in the figure below. The best position in the scatter plot is being as far as possible in the top left corner with low volatility and high return. The MSCI USA Index is on the opposite end of the graph, so this has the worst risk-return combination. Going from bottom right to top left, the two static models, Mean-Variance and Equally Weighted, are observed in the second layer. Next, the last layer is the two regime-based models that seem to provide the investor with the most attractive risk-return combination. If drawing a line between the two regime-based models, we would see something looking like an efficient frontier, illustrating the most optimal mean-variance combinations.

Even though the standard Mean-Variance Model yields a higher return than the Regime-Based Mean-Variance Model, it comes with relatively higher volatility and a less attractive Mean-Variance profile, which is further observed from their Sharpe Ratios. The best model seems to be the Regime-Based Factor Model, with a return that appears to be high enough to make up for its higher volatility than the Regime-Based Mean-Variance Model.

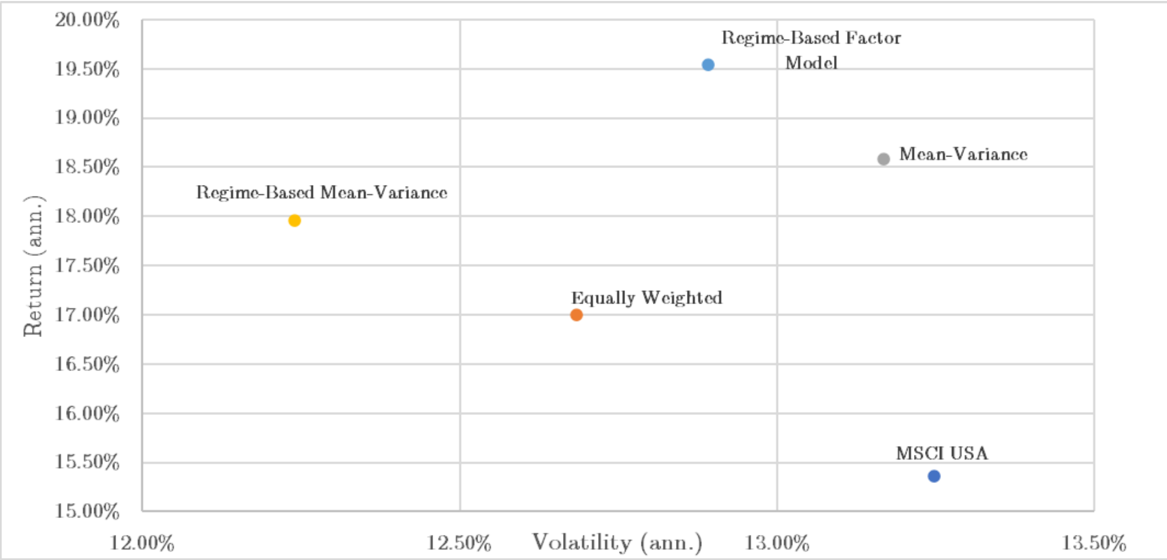


Figure 17: The scatter plot compares the models' performances in this paper in the period 2012-2021. The annualized volatilities are measured on the horizontal axis, and the annualized average returns are measured on the vertical axis. Source: Own construction.

After illustrating the risk-return profiles of the different models, a more precise comparison of the four models and the MSCI USA Index can be made by observing table 23 with their respective performance measures.

| Metrics | MSCI USA | Equally Weighted | Mean-Variance | Regime-Based Factor Model | Regime-Based Mean-Variance |
|---|---|---|---|---|---|
| Return (Ann.) | 15.36% | 17.00% | 18.58% | 19.55% | 17.95% |
| Volatility (Ann.) | 13.25% | 12.68% | 13.17% | 12.89% | 12.24% |
| Sharpe Ratio | 1.110 | 1.288 | 1.360 | 1.463 | 1.412 |
| Sortino Ratio | 1.417 | 1.542 | 1.856 | 2.137 | 1.810 |
| VaR | 6.39% | 5.49% | 5.02% | 5.23% | 5.27% |
| Expected Shortfall | 8.78% | 8.67% | 8.64% | 7.99% | 8.34% |
| Active Return | | | | 2.55% | 0.95% |
| Active Risk | | | | 4.12% | 4.41% |
| Information Ratio | | | | 0.620 | 0.215 |
| Turnover | | 7.06% | 3.18% | 128.14% | 120.93% |
| T-test | | | | 6.765 | 2.356 |
| Max Drawdown | 20.10% | 21.09% | 18.86% | 19.34% | 19.04% |

Table 23: The model compares the models' performances in this paper with the relevant performance measures. The period is 2012-2021. Source: Own construction.

Regarding comparing the regime-based models that seemed to provide the most attractive mean-variance profiles from the figure above, we see that the Sharpe Ratio is higher for the Regime-Based Factor Model than for the Regime-Based Factor Mean-Variance Model. Furthermore, the Sortino Ratio difference is relatively higher than the difference in Sharpe Ratios between the two models. This indicates that the higher volatility from the Regime-Based Factor Model stems from higher upside volatility rather than downside volatility. Hence, the Regime-Based Factor Model is a better model to avoid 'bad' times through its different factor exposures across regimes. As a result, VaR and ES are lower for this model, but it has a slightly higher turnover rate of 128.14% compared to 120.93% for the Regime-Based Mean-Variance Model.

The reason for the better performance of the Regime-Based Factor Model relative to the Regime-Based Mean-Variance Model stems from the different factor exposures that the two models have across the regimes in the business cycle. For the Regime-Based Factor Model, the Value factor does not get any weight in the Recovery stage, even though it is highly correlated with Size, which performs well during this stage. Regarding the Expansion phase, Momentum only gets 40.49% weight in the Regime-Based Mean-Variance Model, whereas it gets 100% allocation in the Regime-Based Factor Model. Hence, the Regime-Based Factor Model benefits, especially in the Expansion stage, because Momentum is superior to the other factors here. Similarly, the Regime-Based Factor Model is benefitting in the Contraction stage. The model has a 50% allocation to the Quality factor, which has performed very well in the test period. On the other hand, the Regime-Based Mean-Variance Model does not allocate to Quality in Contractions, which might stem from the much poorer performance of Quality in the estimation period. Instead, the Regime-Based Mean-Variance Model solely allocates to the Min. Vol. factor in the Contraction regime, whereas an allocation to Quality might have improved the model's performance.

After calculating the t-stat of 3.043 between the two regime-based models with the most attractive risk-return profiles, we know that the return of the Regime-Based Factor Model is significantly higher than the Regime-Based Mean-Variance Model when looking at a significance level of 5%. In addition, the t-stat of 3.043 is also higher than 2.33, indicating a significant outperformance of the Regime-Based Factor Model relative to the Regime-Based Mean-Variance Model also on a 1% significance level.

The static Mean-Variance optimized model is, as previously mentioned, almost like a single factor model since 89% of its exposure is towards the Momentum factor. The Regime-Based Mean-Variance Model has an annual return of 17.95%, which is below the return of the standard Mean-Variance Model of 18.58%. Still, the Sharpe Ratio is better for the Regime-Based Mean-Variance Model, and it has a lower Expected Shortfall. On the other hand, the Regime-Based Mean-Variance Model has a lower Sortino Ratio and higher Max Drawdown and VaR, indicating some more downside volatility than the static Mean-Variance Model for the period 2012-2021.

Another way to compare the models is by looking at their index values plotted on the entire test period from 2012 to 2021 in figure 18. The performances of MSCI USA and the Equally Weighted Model are mainly below the other three models that seem to be very close until the middle of 2020 when the static Mean-Variance Model suddenly outperforms the others with 89% allocation to Momentum. The significant outperformance of the two regime-based models relative to the Equally Weighted Model can be observed from the figure where these two models have a relatively far distance to the benchmark. However, the Regime-Based Factor Model has a very steep performance acceleration from the end of 2020 until the end of the test period. This boost in performance from the Regime-Based Factor Model is primarily a result of the high excess returns in the Recovery period following the corona crisis.

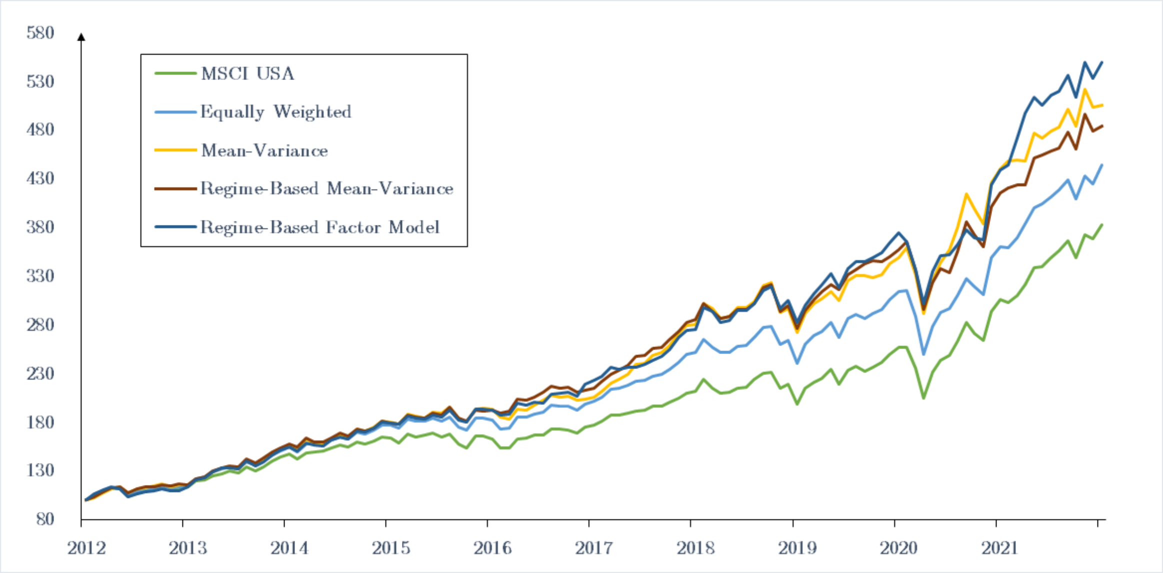


Figure 18: Absolute returns of the models in 2012-2021. Source: Own construction.

To sum up, the Regime-Based Factor Model significantly outperforms the Regime-Based Mean-Variance Model and the Equally Weighted Model. The Regime-Based Factor Model is exposed to the best-performing factors in each regime. It avoids the worst performance by dynamically rotating its factor exposures to avoid 'bad' times and harvesting factor outperformance across regimes. As a result, the Regime-Based Factor Model is superior to all the others in our analysis based on our 10 years of the test period. However, the outperformance stemming from the Regime-Based Factor Model relative to the two mean-variance models comes with some uncertainty. The performance of these three models has been very similar until the end of 2020, whereafter the Regime-Based Factor Model has been superior.

## 5.5 Robustness Test

In the analysis, we found that the Regime-Based Factor Model was the superior model showing the highest returns and the best Sharpe Ratio, among other things. The following section will put the model through a robustness test to check whether the model's performance is robust, considering different parameters. The robustness test aims to prove whether the model is applicable in a real-life setting, what causes the model performance and which conditions might harm the performance. The model will be tested against a new benchmark with the exact same average factor weights. In addition, the model performance will be decomposed into a year-by-year performance. Then, we will test the performance around a change in regimes and, lastly, the effect of transaction costs of this active investment strategy.

### 5.5.1 Comparison with Average Factor Allocation Model

We have constructed another static factor model as a benchmark to analyze the effect stemming from factor timing by our dynamic model. The static model is built by taking the average weights of the five factors from the dynamic model. Here, the static model becomes mostly exposed to Momentum and Min. Vol., which are the most commonly used factors in the dynamic model. However, the three other factors are also used in the dynamic model but have smaller weights. The weights are shown in table 24 below:

| Size | Value | Momentum | Quality | Min. Vol |
|---|---|---|---|---|
| 14.58% | 14.58% | 36.25% | 11.67% | 22.92% |

Table 24: The average factor allocations for the Regime-Based Factor Model. Source: Own construction.

On average, these weights are the same as in the dynamic Regime-Based Factor Model but here with a static approach. Hence, the only difference between the static and dynamic models stems from the active rebalancing based on the regimes.

After calculating the static model's performance statistics below, we get an average annual return of 17.18%, above the static Equally Weighted Model and with lower volatility. We still see the dynamic Regime-Based Factor Model outperforming the newly constructed static model. This outperformance is both in terms of a higher return and lower volatility, as shown in table 25. The T-stat of 5.92 is above 1.64, so the outperformance by our dynamic model relative to the static model with the same average weights is statistically significant at a 5% significance level. Hence, we can conclude that most of the superior performance of the dynamic Regime-Based Factor Model is a result of the dynamic factor timing in each regime.

| Metrics | Regime-Based Factor Model | Average Allocation |
|---|---|---|
| Return (Ann.) | 19.55% | 17.18% |
| Volatility (Ann.) | 12.89% | 12.40% |
| Sharpe Ratio | 1.463 | 1.331 |
| T-stat | 5.924 | |

Table 25: The performance of the Regime-Based Factor Model relative to the Average Allocation Model. Source: Own construction.

### 5.5.2 Year-by-Year Performance Analysis

A decomposition of the model performance on a year-by-year basis can show whether the model is consistent in its outperformance or if all the excess returns are coming from a single year of high outperformance. The model might not be as robust if this was the case since it may have been an outlier period. Therefore, in testing the robustness of the model, it is important to look at the performance on a year-by-year basis to get an indication of the consistency of outperformance.

The model performance is decomposed into yearly returns, as shown in figure 19 below. In the graphs, we see the model performance minus the performance of MSCI USA. At the beginning of each year, the models start at index 100 to give them the same starting point, which allows for looking at the yearly performances. It is plotted this way to eliminate the high correlation stemming from the market risk component present in all portfolios and to get a better view of the relative model outperformance.


Figure 19: Year-by-Year performance comparison of the models in 2012-2021. Source: Own construction.

The overall picture from the ten years of returns shows a relatively consistent outperformance of the Regime-Based Factor Model. In seven out of the ten years, the model shows the best performance and lags some of the other models in only three years. It suggests that the outperformance of the model is not coming from an outlier event and tends to show a consistent outperformance year by year. There were years of underperformance during the ten years in 2012, 2017, and 2020. In 2012 we had a change in regime from Recovery to

Contraction. As shown, we are not very well positioned for this type of regime change since we would be allocated to the worst-performing factors at the beginning of the regime before being able to rebalance. However, a performance improvement at the end of 2012 was observed after changing factor allocations. In 2017 we experienced a regime change from Recovery to Expansion, however, with most of the time being in a Recovery phase. In the Recovery regime, the model is exposed to Size and Value, which performed very well in the estimation period but have had poorer performance in the test period. This poorer performance leads the Recovery phase to often show a lag of performance relative to some of the other models. In 2020 the Corona Crisis came, and the market had a significant drawdown but with a rapid recovery. All models except the standard Mean-Variance model showed an underperformance that year. The Mean-Variance is 89% exposed to Momentum, which happened to be very good during the Corona Crisis. The Regime-Based Factor Model was exposed to the defensive factors at the beginning and the cyclical factors at the end of the year, which were not the best factors for coping with the crisis.

In the years following the years of underperformance, we see strong performance of the Regime-Based Factor Model. For example, looking at 2013, 2018, and 2021 we see a strong rally at the beginning of the years. This suggests that following a period of underperformance, the model quickly recovers most of the returns it was lagging in the ‘bad’ years.

Overall, the model proves to be relatively robust when looking at the performance on a year-by-year basis outperforming the remaining models in seven out of ten years. Following the three years of underperformance, the model had a strong performance in the first half of the following year, quickly trying to recover the lost returns.

### 5.5.3 Performance During Changes in Regimes

Since the model is dynamic and reallocates every time the regime changes, it is important to look closer into the model's performance during the regime shifts. When a regime shift occurs, the model isn’t rebalanced instantly, and the model is wrongly allocated at the beginning of every regime until it is rebalanced. Therefore, in this section, we take a closer look at the performance of the model in the following two months after a regime shift has occurred.

| | Equally Weighted | Regime-Based Factor Model |
|---|---|---|
| Recovery | 1.15% | 0.73% |
| Expansion | 2.48% | 2.57% |
| Slowdown | 1.95% | 2.23% |
| Contraction | -1.34% | -2.02% |
| All regimes | 0.88% | 0.66% |

Table 26: Monthly returns during regime changes for the Equally Weighted Model and the Regime-Based Factor Model in 2012-2021. Source: Own construction.

When the current regime changes into a Recovery, the dynamic model suffers from relatively lower average monthly returns. The monthly return of the Regime-Based Factor Model is 37% lower relative to the Equally Weighted. This suggests that the dynamic model performs poorly when going into a Recovery. When the regime changes to a Recovery, it implies that the previous regime must have been a Contraction. In a Contraction, the model is exposed to the defensive factors, which are the poorest performing factors in a Recovery phase. So, being exposed to the defensive factors at the beginning of a Recovery yields a lower average monthly return.

Going into an Expansion, the model does similar to the Equally Weighted Portfolio. The model has the highest returns going into this stage. When the regime changes to an Expansion, there are two possible previous regimes, either Recovery or Slowdown. Both regime shifts don't give significantly different monthly returns than a more diversified portfolio. This suggests that the model is well suited for a regime shift into the Expansion phase.

When the regime changes to Slowdown, the model also does a good job with higher monthly returns two months after the change compared to the Equally Weighted Model. This is because a regime shift to Slowdown implies that the previous regime must have been Expansion. In an Expansion, the model is 100% allocated to Momentum, which is also the best performing factor in the Slowdown phase. Hence the model yields high returns in these types of regime changes.

The model does not perform very well at the beginning of a Contraction. Going into a Contraction, the previous regime can be either Slowdown or Recovery. When coming from a Slowdown, the model is allocated to Momentum and Min. Vol. Minimum Volatility is the best performing in this regime, while Momentum is the worst-performing factor in a Contraction. When coming from a Recovery, the model is exposed to Size and Value, which does not yield significant excess returns over MSCI USA. In both cases, the model is not optimally allocated towards a regime shift resulting in lower average monthly returns compared to the Equally Weighted Portfolio.

Across all regime shifts, the model has a lower average monthly return than the Equally Weighted Model, suggesting that the model has worse performance than the benchmark overall. The underperformance under a regime shift lasts until the model has been rebalanced to reflect the optimal factor allocation in the new regime. Looking closer into the different regime shifts, we see a more nuanced picture that varies depending on the regime to which it changes. Overall, the model does not perform well during regime shifts, which would be expected due to the allocation to fit the previous regime. Regime changes hurt the active investment strategy, so the more the regime changes, the more the strategy's returns are affected.

### 5.5.4 Effect of Transaction Costs

One of the highly relevant differences to discuss when comparing a dynamic strategy to a static strategy is the effect of transaction costs. This makes an important distinction between the static Equally Weighted Factor Model and the dynamic Regime-Based Factor Model because the dynamic model pays fees for each transaction more frequently. In addition to transaction costs, the total expense ratio is also a fee that must be subtracted from the gross returns to arrive at an actual net return. However, this ratio is the same for a dynamic and a static strategy, so this cost does not make any significant difference in comparing a dynamic and a static model. As previously mentioned, the total expense ratio is typically around 0.20%, and this percentage would be subtracted from the returns of both models. Again, because it is the same for both a static and a dynamic model, only the transaction costs are considered in our calculation.

The transaction cost disadvantage stemming from higher portfolio turnover for the dynamic model relative to the static model influences the net returns. The static model has simply decreased from 17% in gross return to 16.98% in net return. This relatively small decrease stems from rebalancing to keep the weights constant and equal. On the other hand, the dynamic model has an annual gross return of 19.55%, and the net return is 19.24%. This relatively larger decrease results from the dynamic feature of the Regime-Based Factor Model, which reallocates when there is a regime shift.

After calculating the t-statistic, we still get a statistically significant outperformance of the dynamic Regime-Based Factor Model relative to the static Equally Weighted Model. This is because the t-stat of 5.970 is higher than the critical value of 1.64 at a 5% significance level. The t-statistic is now lower for the net returns than the gross return because we see a relatively smaller excess return between the two models for the net returns compared to the gross returns. Hence, we see the negative effect of transaction costs on the net return for the dynamic model. However, the outperformance is still highly statistically significant, even when looking at a 1% significance level, as the t-stat is higher than the 2.33 on a 1% significance level.

Because of this, we can conclude that the Regime-Based Factor Model relative to the static Equally Weighted Model provides significant outperformance even after subtracting transaction costs.

| Metrics | Equally Weighted | Regime-Based Factor Model |
|---|---|---|
| Return (Ann.) Gross | 17.00% | 19.55% |
| Return (Ann.) Net | 16.98% | 19.24% |
| T stat | | 5.970 |

Table 27: The gross and net returns for the Equally Weighted Model and the Regime-Based Factor Model in 2012-2021. Source: Own creation.

# 6. Discussion

In this section, we will address some relevant implications that the results from the analysis have led up to. These implications are more specifically related to the dynamic Regime-Based Factor Model, which this section discusses.

## 6.1 Discussion of the Efficient Market Hypothesis

The Efficient Market Hypothesis (EMH) states that all publicly available information is immediately priced into securities. The EMH is associated with the idea of a "random walk," which states that all price changes represent random departures from previous prices (Malkiel, 2003). Hence, if the information is immediately reflected in the stock prices, returns are independent since the news from yesterday already incorporated this information into the security pricing. Therefore, today's returns only reflect today's news and are independent of yesterday's returns. In this way, returns are unpredictable and follow a "random walk." Because of this, prices reflect all known information. It means that uninformed investors who buy a diversified portfolio at the market price will obtain a return equal to that achieved by the experts (Malkiel, 2003).

The results of our analysis seem to question the EMH. For example, how is it possible for our active model to achieve higher returns than the static Equally Weighted benchmark by timing factor exposures across regimes after the regime changes have been made public?

If the EMH held true, the prices of the factors would immediately adapt to a new equilibrium according to the new regime. Hence it would not be profitable to time the factor allocations.

The fact that we see this significant outperformance of the dynamic model indicates that the market mechanisms misprice the factors. It means that an investor can benefit by pursuing the dynamic strategy investigated in this paper relative to a static strategy. An example of mispricing would be in the Recovery regime, where the Value and Size factor indices are too cheap because they outperform the market and the other factors. Similarly, for the Expansion phase, where the Momentum factor is the outperforming factor, investors should look for an ETF tracking the Momentum factor in this regime. The same concept applies to the optimal factor allocations in the Slowdown and Contraction regimes.

If all investors were informed about the outperformance of the factors in each regime, the demand for these factors under these regimes would increase, which would lower the expected factor returns. Hence, the expected return on the Regime-Based Factor Model would decrease when the demand for its factor components increases. This is because of the inverse relation between the expected return and the prices of securities. As a result, there is an underpricing in terms of the higher returns for some factors under specific regimes. If we assume all investors are becoming aware of this phenomenon, the relative underpricing of the factors in each regime will be minimized. This process would eventually continue until we see an equilibrium where the dynamic strategy no longer outperforms a static model when looking at a net return level.

This mispricing of the optimal factors in the different regimes would not be the first inefficiency for the capital markets. For example, Daniel and Titman (2000) found evidence that rejects the notion of efficient markets in favor of an alternative theory that suggests that investor overconfidence influences asset prices. They argue that some market participants have behavioral biases, are risk-averse, and have limited capital. Hence, these investors cannot eliminate pricing biases that less rational counterparts have created. However, if a significant number of investors are rational and not biased, the profits from market inefficiencies would quickly disappear.

Furthermore, even though we see a significant outperformance of the dynamic model relative to the static model, the excess return might not be large enough for some investors to care about timing their factor exposures by rebalancing after regime shifts. For example, 19.24% relative to 16.98% could, for some investors, not be large enough a gap to put in the effort that an active investment strategy requires. However, the average length of a regime is 7.4 months, so a reallocation does not take up much time. Moreover, many financial institutions and some individuals might be able to set up an algorithm that executes this dynamic strategy without any effort after setting up the algorithm. Another reason investors do not pursue this strategy might be that they are underconfident about the factor performances investigated in this thesis. Or they are not informed about this dynamic outperformance. Hence, if all investors become aware of this beneficial investment strategy, we might see higher pricing of the optimal factors in the different regimes, which would lower the outperformance of the dynamic model.

## 6.2 Future Performance of the Dynamic Model

In section 5.5.3, the model performance during changes in regimes was analyzed. This robustness check found the dynamic model to show a lower average monthly return than the static model. This means that the dynamic model has worse performance than the static model in the period following a regime shift. This remains until the model is rebalanced to reflect the optimal allocation in the new regime. On average, a regime length is 7.4 months. We now know that if the regime lengths become very short, the dynamic model will likely perform worse because the model will experience relatively bad return periods more frequently due to regime shifts. On the other hand, if we experience much longer regime lengths in the future, it will be easier for static models to match the performance of the Regime-Based Factor Model. The reason is that it will lose some of its dynamic character, which is the driver for outperformance. A static model with the same average allocations as the dynamic model from section 5.5.1 will then start tracking the performance of the Regime-Based Factor Model. However, it is not very likely that either of the extreme cases will occur in the future because the economy measured by the monthly CLI is most likely to go either up or down, which will result in a regime shift at some point.

Another and probably the most critical implication for the future performance of the dynamic model is how the factors are going to perform in the future. For example, when comparing the performance of the factors in the estimation relative to the test period, the Value factor had the worst Sharpe Ratio in the test period, but the Size factor had the worst Sharpe Ratio in the estimation period. Another example is that the Quality factor only had the third-best Sharpe Ratio in the estimation period but has shown the highest Sharpe Ratio in the test period among the five factors. This could imply that the dynamic model constructed in this thesis would have to be re-estimated in the future to update the model with the new factor performances. On the other hand, suppose the model is not updated with the new factor performances. In that case, the model might be outdated at some point because the factors could change enough to no longer be optimal in the same regimes as previously. However, it is quite unlikely that the factor patterns will significantly change since the factors have some fundamental characteristics which make them suitable for certain economic conditions.

A third implication that should be considered for an investor or a financial institution that aims to implement the dynamic model constructed in this thesis would be to track the development in the transaction costs. The analysis shows that the transaction costs do not make any significant difference when choosing the optimal model between a dynamic and a static factor model. However, if market conditions change, where the transaction costs are being raised for some reason, this should be considered before choosing this dynamic investment strategy that needs to rebalance after regime shifts. Still, the transaction costs would have to increase significantly to invalidate the significantly higher net returns for the dynamic model relative to the static model.

# 7. Conclusion

The thesis aimed to construct regime-based factor timing strategies and investigate whether they can outperform a static diversified multi-factor model. To investigate this, four regimes have been established as the foundation where the business cycle has been separated into repeatable regimes. The regimes in the business cycle are categorized as Recovery, Expansion, Slowdown, and Contraction. An entire business cycle consists of all four regimes, and the average regime length has been 7.4 months. Each regime has certain economic conditions that make the individual factors perform differently.

In the Recovery regime, the Size and Value factors were the best performing factors due to the cyclical characteristics of these factors that benefit from the improving economic conditions. Momentum shows clear outperformance relative to the other factors moving to the Expansion phase. During an Expansion, the best-performing stocks tend to continue their high relative performance, leading Momentum to become the best performing factor. In the Slowdown phase, Momentum again shows the highest relative performance among the five factors, while the more defensive factors also begin to show outperformance relative to the market. When the business cycle goes into a Contraction, the defensive factors are superior. In this regime, the market is suffering from very negative returns. Still, Minimum Volatility and Quality show the least negative returns, providing good protection against the bad times in this regime.

In order to utilize the different performance characteristics of the four regimes, we have constructed two dynamic models that are changing their factor exposures depending on the optimal allocations across the regimes. The model creation process applied data from 1998 to 2011, and we tested the models on data from 2012 to 2021.

The Regime-Based Mean-Variance Model has been constructed using regime-dependent return vectors and variance-covariance matrices and then maximizing the risk-adjusted returns measured by the Sharpe Ratio. This resulted in four different optimal factor allocation choices. As a result, the model outperformed the static Equally Weighted benchmark with a higher Sharpe Ratio and Sortino Ratio and showed lower risk in all risk parameters.

The Regime-Based Factor Model has been constructed after analyzing the performance of the five factors across the four regimes. In addition, the yearly performance in the estimation period and the correlations across the regimes were considered in the creation of the model. This model significantly outperforms both the Regime-Based Mean-Variance Model and the Equally Weighted Model with the highest return, Sharpe Ratio, and Sortino Ratio.

Because we found the Regime-Based Factor Model to be a superior model, we have conducted a robustness test. This was to check whether the model's performance is robust, considering different things and seeing if the model is applicable after considering real-life implications. One of the relevant robustness tests was to include transaction costs in the return calculations because the model is an active investment strategy. As a result, the yearly net return was only 0.31 percentage points lower than the gross returns. The net return is still significantly outperforming the net return of the static diversified multi-factor model.

The findings of this paper contribute to the existing literature with a more nuanced view of dynamic factor allocation. Furthermore, this paper adds a deeper perspective on the factor performances within the different regimes. The possible dangers of latency in the rebalancing of the model have also been investigated in this thesis. This paper also includes the effect of transaction costs for a dynamic relative to a static model. In addition, this paper combines portfolio optimization with dynamic factor timing by constructing a Regime-Based Mean-Variance Model.

# 8. Further Research

This thesis contributes to a practical field of research dominated primarily by practitioners rather than academics. While it makes some valuable contributions to the field of dynamic investing, several aspects remain unexplored and would thus be ideal for further research and improvement of the model.

First, it could be interesting to analyze the strategies in different markets. In this thesis, the analysis has been based on only US data. Therefore, it could be interesting to see if the results could be replicated in other markets. Replication in different markets would also improve the robustness of the model. Analyzing a different time period could also be interesting. One does, however, run into problems with data on factor indices back in time before 1998.

Another interesting study could be to create the four regimes based on other things than the CLI. It could be interesting to see if other parameters than the CLI would produce similar or better results. Since the CLI is an external variable computed by OECD, it could be interesting to construct an internal indicator that could minimize the lag needed before the rebalancing. This lag significantly impacts the strategy's returns, and improving this could yield considerably even better performance.

This thesis looked at the most common factors for the factor allocation. One could make an analysis using a different set of factors and maybe choose some more and other types of factors in order to find the absolute best-performing factors in each regime. One could take a more general factor approach like the one in this thesis or look at abstract factors, which are purely mathematically factors found in the dataset.

The combination of factors was in this thesis done by using the mixed approach in which you allocate to the factors separately, say you want to buy Momentum and Value, then you invest in Momentum by itself and Value by itself. Another way to do this could be by looking at an integrated approach. In the integrated approach, you look at the combined factor score, say you again want Momentum and Value and buy the companies with the best combined Value and Momentum scores. Using the integrated approach, you maximize the exposure to the factors you want. Also, this approach makes sure you don’t have any low Momentum stocks in the portfolio if you are invested in Momentum as well as other factors. The integrated approach could, at this moment at least, not be made on an index level, and the strategy needs to be adapted to a single stock level.

Lastly, it could be interesting to analyze the regime-dependent factor performance using rolling window estimates. In this thesis, the estimation was done in a single estimation- and a single test period. Since it has become evident that the factor does not continue to perform as they have done in the past, it could be interesting to look at only the more recent past when analyzing and choosing the optimal factors. Using rolling estimates would ensure that only the more recent data is considered and would capture any changes in performance patterns.

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