GTID: Name: Vidit Pokharna

ISYE 6759 Quiz 1

Suppose the "Bet-on-7" game is played at a casino as follows. For each play, a player first pays \$1 to have a dealer roll two fair 6-sided dice behind a screen (namely, the player does NOT see the two numbers on top of the dices after the roll). The dealer announces the sum of the two numbers: if the sum is 7, then the player receives \$6, otherwise receiving \$0.

1) Consider a player who started to play this game with \$10 in hand. After playing this game for 1000 times, how much money does the player most likely to have in hand? Why?
2) After the 1000th play of the game, an announcement rule is enforced:
   - if at least one die shows a 6, then the dealer must announce "I see a 6".
   - If no die shows 6, then the dealer picks a die with equal probability out of the two and truthfully announces its face.

   Following the announcement, the player is offered an option. If the option is accepted, then the current round of the game ends. If the option is rejected by the player, then the sum is revealed and payoff (\$6 or \$0) paid accordingly.

   Now if the player hears the dealer announcing "I see a 6", and the offered option is for the player to receive \$1.05 and stop the current play (namely, if the player decides to accept the option, then he/she receives \$1.05 and stops the current play without hearing the sum and getting the subsequent payoff). Shall the player accept this option? Why or why not?

1) $P(\text{sum}=7) = (3,4),(4,3),(2,5),(5,2),(6,1)(1,6) = 6/36 = 1/6$
$P(\text{sum} \neq 7) = 5/6$

$E[\text{one round}] = \frac{1}{6}(6-1) + \frac{5}{6}(0-1) = \frac{5}{6} - \frac{5}{6} = 0 \implies E[1000 \text{ rounds}] = 10 + 0(1000) = 10$

After playing 1000 rounds, it is expected that the player still has $\boxed{\$10}$ as the expected profit/loss after each round is 0, meaning that over time, it is expected that player gains all the losses back or vice versa.

3/5

2) Given that the player knows one of the dice rolled has a 6, the probability of having a sum of 7 is 1/6 (the other dice rolled is 1).

$E[\text{payoff this round}] = \frac{1}{6}(6) + \frac{5}{6}(0) = 1$

Having the prior knowledge of having a 6 means that we are expected to receive \$1 if we wait to hear the sum, versus \$1.05 without hearing the sum.

Thus it is better to $\boxed{\text{accept the option}}$ and take the advantage of 5 cents through this process.

2/5

5/10