Solving the weighted bipartite matching problem (with contextualizations)

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Min-cost bipartite matching: given N jobs and M people and a $n\,x\,m$ matrix of costs of performing these jobs. Each job has to be assigned to *one* of m people, while one person can perform maximum one job. Assume $m \leqslant n$ (or matrix can be rotated). Another interpretation can be assigning each of n keywords a tag from m tags available (and we have a likelihood matrix). This can be efficiently solved by the well-known Hungarian/Munkres[1] algorithm in $\Theta(n^2m)$.

Contextualization adds additional interdependencies between solution costs, e.g. the person A would agree to perform the job X cheaper, if person be is assigned job Y (e.g. as he wants to be nearby). In the context of keyword search: the tag_j of a keyword kw_i is more likely if it's nearby a related tag_y.

Known approaches and their combinations

Notation: n keywords; m is total number of possible tags; $\widetilde{m}$ is the average # of possible tags; k - # of top-k results to return.

Method	**Advantages**	**Disadvantages**
Exhaustive search	▷ easy pruning and contextualization ▷ optimal answers	**slow**, $O(\widetilde{m}^n)$
Munkres[3] gives one best solution in $\Theta(n^2m)$.	▷ quite fast	▷ no contextualization ▷ only one best result
Keymantic[2] - Munkres modified for contextualizations, $\Theta(n^3m^2)$. recursively evaluate[2] all mappings pruning on the cost. do contextualization inside Munkres	▷ quite fast ▷ some contextualization ▷ some of top-k answers	▷ **approximate - not global top-k** ▷ **correctness unproven** ▷ no guarantee of all contextualizations
Murty[6] + *Munkres* - top-k matchings in $\Theta(n^3m)$. to get each additional result, call Munkres to solve $n-1$ smaller assignments of sizes $2..n-1$. Heuristics can greatly improve expected run time[4]	▷ *top-k optimal* solutions ▷ quite fast	▷ no early pruning (partial matching may change a lot) ▷ **no contextualization** [3]
HMM[1] + *List Viterbi*[7] can start with estimated HMM params: transition probs from contextualizations, output probs from cost matrix	▷ *top-k optimal* solutions ▷ contextualization of *limited length* ▷ quite fast	▷ no pruning ▷ **a tag may get selected many times**
Murty + Dynamic Munkres on contextualized cost-matrix 1) enumerate over all contextualization possibilities[4] 2) use Murty's to get top-k results over contextualized cost-matrix reusing older sub-solutions[5] costing only $\Theta(nm)$ per modified "line".	▷ **top-k optimal results** ▷ **fast if # dependencies is small**	▷ exponential for complex contextualizations

Table 1: comparison of different methods

[1] *Munkres* splits the assignment problem into easier ones: 1) maintaining a set of constraints that restrict the currently allowed matches (edges) to be cheap enough, and 2) solving N unweighted bipartite assignments: starting with an empty matching, find an augmenting path to increase the size of matching - new edges are selected or existing deselected; if no augmenting path exist, loosen the constraints on weights.

[2] recursively solve multiple weighted matchings with Munkres (modifying the matrices to force or prevent specific matches)

[3] could do same unproven contextualization within Munkres as in Keymantic; would at least guarantee top-k with limited contextualization

[4] exploring contextualizaions in depth-first order cost-matrix modifications can be reused

References

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