Solving weighted bipartite matching problem (with contextualizations)

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Min-cost bipartite matching: given N jobs and M people and a NxM matrix of costs of performing these jobs. Each job has to be assigned to *one* of M people, while one person can perform maximum one job. Assume $M \leqslant N$ (or matrix can be rotated). Another interpretation can be assigning each of N keywords a tag from M tags available (and we have a likelihood matrix). This can be efficiently solved by a well-known Munkres[1] algorithm in $\Theta(N^2M)$.

Contextualization adds additional interdependencies between solution costs, e.g. the person A would agree to perform the job X cheaper, if person be is assigned job Y (e.g. as he wants to be nearby). In the context of keyword search: the tag_j of a keyword kw_i is more likely if it's nearby a related tag_y.

1 Comparison of existing approaches

Notation: N keywords; M is total number of possible tags; $\widetilde{M}$ is the average # of possible tags

Method	Advantages	Disadvantages
Exhaustive search	easy pruning and contextualization optimal answers	**slow**, $O(\widetilde{M}^N)$
Munkres[3] gives one best solution in $\Theta(N^2M)$.	quite fast	no contextualization only one best result
Keymantic[2] - a modified Munkres recursively evaluate[2] all mappings pruning on the cost. do contextualization inside Munkres	quite fast some contextualization some of top-k answers	approximate - **no real top-k** no guarantee of all contextualizations seems hard to prove 100% correctness
Murty[6] + *Munkres* to get each additional result, call Munkres to solve $n-1$ smaller assignments of sizes $2..n-1$. expected runtime improved through heuristics[4]	*top-k optimal* solutions quite fast	• no early pruning (an augmenting path may change the matching a lot...) • **no contextualization** (or hard to prove similar method as in Keymantic, but provides guaranteed top-k with limited contextualization)
HMM[1] + *List Viterbi*[7] can start with estimated HMM params: transition probs from contextualizations, output probs from cost matrix	*top-k optimal* solutions allows some contextualization of *limited length* quite fast	no pruning **same tag may get selected multiple times**
Murty + Dynamic Munkres (proposed by us in cases of few dependencies) 1) enumerate over all contextualization possibilities[3] 2) use Murty's to get top-k results over contextualized cost-matrix reusing older sub-solutions[5] costing only $\Theta(NM)$ per modified "line".	**top-k optimal results** **fast** if # contextualizations is small	intractable/exponential for complex contextualizations

Table 1: comparison of different methods

[1] Munkres algorithm transforms the problem into N smaller ones: maintaining the "dual" problem to restrict the currently allowed matches (edges) to be cheap enough, it solves multiple unweighted bipartite assignments: starting with an empty matching, find an augmenting path to increase the size of matching - new edges are selected or existing deselected; if no augmenting path exist, loosen the constraint on weights.

References

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[3] Francois Bourgeois and Jean-Claude Lassalle. An extension of the munkres algorithm for the assignment problem to rectangular matrices. *Communications of the ACM*, 14(12):802–804, 1971.

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[6] Katta G Murty. Letter to the editor—an algorithm for ranking all the assignments in order of increasing cost. *Operations Research*, 16(3):682–687, 1968.

[7] N. Seshadri and C.E.W. Sundberg. List viterbi decoding algorithms with applications. *Communications, IEEE Transactions on*, 42(234):313–323, 1994.

[2] by solving multiple weighted matchings with Munkres (and modifying the matrices to (not) to choose specific matches)

[3] exploring contextualizaions in depth-first order cost-matrix modifications can be reused