MVA Midterm

```
#installing all the required packages
install.packages("knitr")
## Installing package into 'C:/Users/vidhi/Documents/R/win-library/3.6'
## Error in contrib.url(repos, "source"): trying to use CRAN without setting a mirror
library(knitr)
install.packages("rmarkdown")
## Installing package into 'C:/Users/vidhi/Documents/R/win-library/3.6'
## (as 'lib' is unspecified)
## Error in contrib.url(repos, "source"): trying to use CRAN without setting a mirror
library(rmarkdown)
## Warning: package 'rmarkdown' was built under R version 3.6.3
install.packages("ggplot2")
## Installing package into 'C:/Users/vidhi/Documents/R/win-library/3.6'
## (as 'lib' is unspecified)
## Error in contrib.url(repos, "source"): trying to use CRAN without setting a mirror
library(ggplot2)
## Warning: package 'ggplot2' was built under R version 3.6.3
install.packages("factoextra")
## Installing package into 'C:/Users/vidhi/Documents/R/win-library/3.6'
## (as 'lib' is unspecified)
## Error in contrib.url(repos, "source"): trying to use CRAN without setting a mirror
library(factoextra)
## Warning: package 'factoextra' was built under R version 3.6.3
\textit{## Welcome! Want to learn more? See two factoextra-related books at $https://goo.gl/ve3WBa$ at $ht
install.packages("dplyr")
## Installing package into 'C:/Users/vidhi/Documents/R/win-library/3.6'
## (as 'lib' is unspecified)
## Error in contrib.url(repos, "source"): trying to use CRAN without setting a mirror
library(dplyr)
## Warning: package 'dplyr' was built under R version 3.6.3
## Attaching package: 'dplyr'
## The following objects are masked from 'package:stats':
##
               filter, lag
## The following objects are masked from 'package:base':
                intersect, setdiff, setequal, union
install.packages("GGally")
## Installing package into 'C:/Users/vidhi/Documents/R/win-library/3.6'
## (as 'lib' is unspecified)
```

```
## Error in contrib.url(repos, "source"): trying to use CRAN without setting a mirror
library(GGally)
## Registered S3 method overwritten by 'GGally':
    method from
##
     +.gg ggplot2
## Attaching package: 'GGally'
## The following object is masked from 'package:dplyr':
##
##
       nasa
install.packages("cluster", lib="/Library/Frameworks/R.framework/Versions/3.5/Resources/library")
## Warning in install.packages("cluster", lib = "/Library/Frameworks/R.framework/
## Versions/3.5/Resources/library"): 'lib = "/Library/Frameworks/R.framework/
## Versions/3.5/Resources/library"' is not writable
## Error in install.packages("cluster", lib = "/Library/Frameworks/R.framework/Versions/3.5/Resources/library"): unable to install packages
library(cluster)
## Warning: package 'cluster' was built under R version 3.6.3
install.packages("psych", lib="/Library/Frameworks/R.framework/Versions/3.5/Resources/library")
## Warning in install.packages("psych", lib = "/Library/Frameworks/R.framework/
## Versions/3.5/Resources/library"): 'lib = "/Library/Frameworks/R.framework/
## Versions/3.5/Resources/library"' is not writable
## Error in install.packages("psych", lib = "/Library/Frameworks/R.framework/Versions/3.5/Resources/library"): unable to install packages
library(psych)
## Warning: package 'psych' was built under R version 3.6.3
## Attaching package: 'psych'
## The following objects are masked from 'package:ggplot2':
##
##
       %+%, alpha
data<-read.csv("Protein_Consumption.csv", fill = TRUE)</pre>
#fill = True is added so that if there are rows which have unequal lengths or there is some missing data then it will fill implicitly
attach(data)
##### Check for the dimensions of the dataset attached #########
dim(data)
## [1] 25 11
#Ans- There are 25 observations and 11 variables
head(data)
         i..Country Red.Meat White.Meat Egg Milk Fish Cereals Starchy.Foods
##
## 1
            Albania
                           10
                                        1
                                            1
                                                       0
                                                              42
## 2
                            9
            Austria
                                       14
                                                20
                                                              28
                                                                              4
## 3
            Belgium
                           14
                                        9
                                                              27
                                                18
                                                                              6
## 4
                            8
           Bulgaria
## 5 Czechoslovakia
                           10
                                       11
                                            3
                                                13
                                                              34
## 6
            Denmark
                           11
                                       11
                                            4
                                                25
                                                      10
                                                              22
                                                                              5
    Pulses.Nuts.and.Oilseeds Fruits.and.Vegetables Total
##
## 1
                             6
                                                          72
## 2
                                                          86
## 3
                                                          89
## 4
                             4
                                                     4
                                                          91
## 5
                                                     4
                                                          83
## 6
                             1
                                                     2
                                                          91
tail(data)
```

```
ï..Country Red.Meat White.Meat Egg Milk Fish Cereals Starchy.Foods
## 20
              Sweden
                           10
                                       8
                                              25
## 21
         Switzerland
                           13
                                      10
                                           3
                                               24
                                                            26
                                                                           3
## 22 United Kingdom
                           17
                                       6
                                           5
                                               21
                                                            24
                                                                           5
## 23
                USSR
                            9
                                       5
                                               17
                                                     3
                                                            44
                                                                           6
                                           4
                                                            19
                                                                           5
## 24
        West Germany
                           11
                                      13
                                               19
## 25
          Yugoslavia
                            4
                                       5
                                           1
                                               10
                                                            56
     Pulses.Nuts.and.Oilseeds Fruits.and.Vegetables
##
                                                     Total
## 20
## 21
                             2
                                                   5
                                                        88
## 22
                                                   3
                                                        88
## 23
                                                        92
                             3
                                                   3
## 24
                             2
                                                        80
  25
                             6
                                                        89
# 01) Use principal components analysis to investigate the relationships between the countries on the basis of these variables
cor(data[-1])
                               Red.Meat
                                        White.Meat
##
                                                            Egg
## Red.Meat
                             1.00000000
                                        0.18850977
                                                     0.57532001
                                                                 0.5440251
                                                    0.60095535
                                                                 0.2974816
## White.Meat
                             0.18850977
                                        1.00000000
                                         0.60095535
                                                     1.00000000
                                                                 0.6130310
## Egg
                             0.57532001
## Milk
                             0.54402512
                                        0.29748163
                                                     0.61303102
                                                                 1.0000000
## Fish
                             0.06491072
                                        -0.19719960
                                                     0.04780844
## Cereals
                            -0.50970337
                                        -0.43941908
                                                    -0.70131040 -0.5924925
## Starchy.Foods
                             0.15383673
                                        0.33456770
                                                    0.41266333
                                                                0.2144917
## Pulses.Nuts.and.Oilseeds -0.40988882 -0.67214885 -0.59519381 -0.6238357
## Fruits.and.Vegetables
                            -0.06393465 -0.07329308 -0.16392249 -0.3997753
## Total
                             0.37369919
                                        0.10308602
                                                    0.18970028 0.4603542
                                   Fish
                                            Cereals Starchy.Foods
## Red.Meat
                             0.06491072 -0.50970337
                                                       0.15383673
## White.Meat
                            -0.19719960 -0.43941908
                                                       0.33456770
## Egg
                             0.04780844 -0.70131040
                                                       0.41266333
                             0.16246239 -0.59249246
                                                       0.21449173
## Milk
## Fish
                            1.00000000
                                        -0.51714759
                                                       0.43868411
## Cereals
                            -0.51714759
                                        1.00000000
                                                      -0.57813449
## Starchy.Foods
                             0.43868411
                                        -0.57813449
                                                       1.00000000
## Pulses.Nuts.and.Oilseeds
                            -0.12226043
                                        0.63605948
                                                      -0.49518800
## Fruits.and.Vegetables
                             0.22948842
                                        0.04229293
                                                       0.06835670
## Total
                            -0.09089592
                                        0.18587578
                                                      -0.04418245
                            Pulses.Nuts.and.Oilseeds Fruits.and.Vegetables
##
## Red.Meat
                                          -0.4098888
                                                               -0.06393465
## White.Meat
                                          -0.6721488
                                                               -0.07329308
## Egg
                                          -0.5951938
                                                               -0.16392249
## Milk
                                          -0.6238357
                                                               -0.39977527
## Fish
                                          -0.1222604
                                                                0.22948842
                                                                0.04229293
                                           0.6360595
## Cereals
## Starchy.Foods
                                          -0.4951880
                                                                0.06835670
## Pulses.Nuts.and.Oilseeds
                                           1.0000000
                                                                0.35133227
## Fruits.and.Vegetables
                                           0.3513323
                                                                1.00000000
## Total
                                          -0.0812251
                                                                0.07201466
##
                                 Total
                             0.37369919
## Red.Meat
## White.Meat
                             0.10308602
## Egg
                             0.18970028
## Milk
                             0.46035417
## Fish
                            -0.09089592
## Cereals
                             0.18587578
## Starchy.Foods
                            -0.04418245
## Pulses.Nuts.and.Oilseeds -0.08122510
## Fruits.and.Vegetables
                             0.07201466
## Total
                             1.00000000
# Removing the first variable from the dataset as it is a categorical variable
#while the correlation requires quantitative(numerical values)
data_pca<- prcomp(data[,-1],scale=TRUE)</pre>
# scale=TRUE:- the variable means are set to 0, and variances are set to 1 \,
data_pca #the components for all the variables are displayed here
## Standard deviations (1, .., p=10):
## [1] 2.032257e+00 1.319067e+00 1.144237e+00 1.021544e+00 8.360847e-01
    [6] 6.531975e-01 5.841454e-01 4.366348e-01 3.458098e-01 6.618503e-16
##
## Rotation (n x k) = (10 x 10):
                                   PC1
##
                                               PC2
                                                           PC3
## Red.Meat
                            -0.3180769 -0.17809245 -0.38142753 -0.039766137
                            -0.3140588 -0.11783853
                                                                0.538507972
## White.Meat
                                                   0.36420271
                            -0.4202281 -0.08236350
                                                   0.02047575
## Egg
                                                                0.155623651
## Milk
                            -0.3870300 -0.23356182 -0.19997405
                                                               -0.320360929
                            -0.1271598   0.57388821   -0.33003267   -0.304161366
## Fish
                             0.4177240 -0.31321549 -0.02354236
                                                                0.104798477
## Cereals
## Starchy.Foods
                            -0.2880798
                                        0.41038324
                                                   0.05768490
                                                                0.150709175
## Pulses.Nuts.and.Oilseeds
                            0.4177658
                                       0.04145202 -0.24796403
                                                                0.008042093
## Fruits.and.Vegetables
                             0.1197680
                                       0.34858202 -0.41210384
                                                                0.643455476
                            -0.1062294
                                       -0.41709540 -0.58081103
                                                               0.203145847
## Total
```

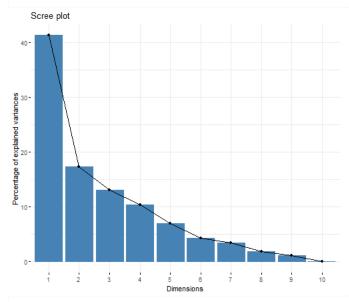
```
## Red.Meat
                               -0.09760147
                                           0.309417061
                                                          0.09254681 -0.2919567
## White.Meat
## Egg
                               0.26932734
                                           -0.059357751 -0.63995627 -0.2652806
## Milk
                              -0.15848975
                                            0.307976584
                                                         -0.17405921 0.5444724
## Fish
                              -0.20323386
                                            0.303075844
                                                          0.06315829 -0.5200308
## Cereals
                              -0.29201244
                                           -0.196460437
                                                          0.06971238 -0.2001491
## Starchy.Foods
                              -0.42198545
                                           -0.680457657
                                                         -0.11769041
                                                                       0.1889672
## Pulses.Nuts.and.Oilseeds 0.22507285
                                           -0.087921207 -0.57816932 -0.0829400
## Fruits.and.Vegetables
                               0.16834367
                                           0.222568384
                                                          0.08684392 0.3701826
## Total
                              -0.47623561
                                           -0.007702046 -0.05178373 -0.1801923
##
                                      PC9
                                                  PC10
## Red.Meat
                              -0.17150487
                                            0.20838019
## White.Meat
                              -0.46186736
                                            0.22903415
## Egg
                               0.48098579
                                            0.06827056
## Milk
                              -0.13218960
                                            0.43456461
## Fish
                               0.01789764
                                            0.21247753
## Cereals
                               0.30436394
                                            0.67412235
## Starchy.Foods
                              -0.14706957
                                            0.10134794
## Pulses.Nuts.and.Oilseeds -0.58938418
                                            0.12362100
## Fruits.and.Vegetables
                               0.20995988
                                           0.11723988
                              -0.04898111 -0.41440004
## Total
summary(data_pca)
## Importance of components:
                              PC1
                                    PC2
                                           PC3
                                                   PC4
                                                           PC5
##
                                                                    PC6
                                                                            PC7
                            2.032 1.319 1.1442 1.0215 0.8361 0.65320 0.58415 0.43663
## Standard deviation
  Proportion of Variance 0.413 0.174 0.1309 0.1044 0.0699 0.04267 0.03412 0.01906
## Cumulative Proportion 0.413 0.587 0.7179 0.8223 0.8922 0.93485 0.96898 0.98804
##
                               PC9
                                         PC10
                           0.34581 6.619e-16
## Standard deviation
## Proportion of Variance 0.01196 0.000e+00
## Cumulative Proportion 1.00000 1.000e+00
# PC1 is able to restore 41% of the total variance, PC2 has 17% of total variance restored, PC3 has 13%, #PC4 has 10%, PC5 has almost 7%, PC6 has 4%, PC7 has 3%, PC8 has almost 2% while PC9 has 1% of the total variance restored
#Conclusion: When I add the variances of 7 Principal Components 95% of the total variance has been restored
#i.e. 95% of the estimated protein consumption comes from these 7 components
#To have a clear idea of choosing the principal components lets' draw the scree plot and then take a final call
# sample scores stored in data_pca$x
# singular values (square roots of eigenvalues) stored in data_pca$sdev
# Loadings (eigenvectors) are stored in data_pca$rotation
# variable means stored in data_pca$center
# variable standard deviations stored in data_pca$scale
# A table containing eigenvalues and %'s accounted, follows
# Eigenvalues are sdev^2
eigen_data<-data_pca$sdev^2
eigen_data
    [1] 4.130067e+00 1.739939e+00 1.309278e+00 1.043551e+00 6.990377e-01
    [6] 4.266669e-01 3.412258e-01 1.906500e-01 1.195844e-01 4.380459e-31
#the eigen values have no names to it so we will now assign the names to it
names(eigen_data) <- paste("PC",1:10,sep="")</pre>
eigen_data
                           PC2
                                        PC3
                                                       PC4
            PC1
                                                                     PC5
## 4.130067e+00 1.739939e+00 1.309278e+00 1.043551e+00 6.990377e-01 4.266669e-01
##
            PC7
                          PC8
                                        PC9
                                                     PC10
## 3.412258e-01 1.906500e-01 1.195844e-01 4.380459e-31
\verb|sumlambdas<-sum(eigen_data)|
sumlambdas
## [1] 10
#Calculating the sum of all the eigen values
propvar <- eigen_data/sumlambdas</pre>
propvar
                           PC2
                                        PC3
                                                       PC4
                                                                     PC5
## 4.130067e-01 1.739939e-01 1.309278e-01 1.043551e-01 6.990377e-02 4.266669e-02
                                        PC9
                                                      PC10
## 3.412258e-02 1.906500e-02 1.195844e-02 4.380459e-32
cumvar_data <- cumsum(propvar)</pre>
cumvar_data
                               PC3
                                          PC4
                                                     PC5
## 0.4130067 0.5870006 0.7179284 0.8222835 0.8921873 0.9348540 0.9689766 0.9880416
##
         PC9
                   PC10
## 1,0000000 1,0000000
```

```
#Calculating the cumulative sum of proportion of the percentage of total variance
matlambdas <- rbind(eigen_data,propvar,cumvar_data)</pre>
                                         PC3
                                                              PC5
                     PC1
                               PC2
                                                   PC4
                                                                          PC6
## eigen_data 4.1300672 1.7399386 1.3092782 1.0435513 0.69903765 0.42666693
               0.4130067 0.1739939 0.1309278 0.1043551 0.06990377 0.04266669
## propvar
## cumvar_data 0.4130067 0.5870006 0.7179284 0.8222835 0.89218729 0.93485398
##
                      PC7
                                PC8
                                           PC9
                                                       PC10
## eigen_data 0.34122581 0.1906500 0.11958440 4.380459e-31
              0.03412258 0.0190650 0.01195844 4.380459e-32
## propvar
## cumvar_data 0.96897656 0.9880416 1.00000000 1.000000e+00
#Putting all these values in a matrix format using the row-wise distribution
# Giving apt names to these variables
rownames(matlambdas)<- c("Eigenvalues", "Prop. variance", "Cum. prop. variance")</pre>
matlambdas
                             PC1
                                       PC2
                                                 PC3
                                                           PC4
                       4.1300672 1.7399386 1.3092782 1.0435513 0.69903765
## Eigenvalues
                       0.4130067 0.1739939 0.1309278 0.1043551 0.06990377
## Prop. variance
## Cum. prop. variance 0.4130067 0.5870006 0.7179284 0.8222835 0.89218729
                              PC6
                                         PC7
                                                   PC8
                                                              PC9
## Eigenvalues
                       0.42666693 0.34122581 0.1906500 0.11958440 4.380459e-31
## Prop. variance
                       0.04266669 0.03412258 0.0190650 0.01195844 4.380459e-32
## Cum. prop. variance 0.93485398 0.96897656 0.9880416 1.00000000 1.000000e+00
#very big values are displayed in each of these components so I am rounding these values till 4 decimal places
round(matlambdas,4)
##
                          PC1
                                 PC2
                                        PC3
                                               PC4
                                                      PC5
                                                             PC6
                                                                     PC7
                       4.1301 1.7399 1.3093 1.0436 0.6990 0.4267 0.3412 0.1906
## Eigenvalues
                       0.4130 0.1740 0.1309 0.1044 0.0699 0.0427 0.0341 0.0191
## Prop. variance
## Cum. prop. variance 0.4130 0.5870 0.7179 0.8223 0.8922 0.9349 0.9690 0.9880
                          PC9 PC10
## Eigenvalues
                       0.1196
## Prop. variance
                       0.0120
                                 0
## Cum. prop. variance 1.0000
                                 1
summary(data_pca)
## Importance of components:
                            PC1
                                  PC2
                                         PC3
                                                PC4
                                                       PC5
                                                               PC6
                                                                        PC7
                                                                                PC8
## Standard deviation
                          2.032 1.319 1.1442 1.0215 0.8361 0.65320 0.58415 0.43663
## Proportion of Variance 0.413 0.174 0.1309 0.1044 0.0699 0.04267 0.03412 0.01906
## Cumulative Proportion 0.413 0.587 0.7179 0.8223 0.8922 0.93485 0.96898 0.98804
                              PC9
                                       PC10
                          0.34581 6.619e-16
## Standard deviation
## Proportion of Variance 0.01196 0.000e+00
## Cumulative Proportion 1.00000 1.000e+00
print(data_pca$rotation)
                                               PC2
## Red.Meat
                            -0.3180769 -0.17809245 -0.38142753 -0.039766137
## White.Meat
                            -0.3140588 -0.11783853
                                                    0.36420271
                                                                 0.538507972
## Egg
                            -0.4202281 -0.08236350
                                                    0.02047575
                                                                0.155623651
                            -0.3870300 -0.23356182 -0.19997405
## Milk
                                                               -0.320360929
## Fish
                             -0.1271598   0.57388821   -0.33003267   -0.304161366
                             0.4177240 -0.31321549 -0.02354236
                                                                 0.104798477
## Cereals
## Starchy.Foods
                             -0.2880798
                                        0.41038324
                                                    0.05768490
                                                                 0.150709175
## Pulses.Nuts.and.Oilseeds 0.4177658
                                        0.04145202 -0.24796403
                                                                 0.008042093
                             0.1197680 0.34858202 -0.41210384
## Fruits.and.Vegetables
                                                                0.643455476
                             -0.1062294 -0.41709540 -0.58081103
## Total
                                                                0.203145847
                                    PC5
                                                 PC6
## Red.Meat
                             0.53138781
                                        -0.393811788
                                                      0.42940825 -0.1592276
## White.Meat
                             -0.09760147
                                         0.309417061
                                                      0.09254681 -0.2919567
## Egg
                             0.26932734
                                        -0.059357751
                                                      -0.63995627 -0.2652806
## Milk
                            -0.15848975
                                         0.307976584 -0.17405921 0.5444724
                            -0.20323386
                                         0.303075844
                                                      0.06315829 -0.5200308
## Fish
## Cereals
                             -0.29201244
                                         -0.196460437
                                                      0.06971238
                                                                  -0.2001491
                                        -0.680457657
## Starchy.Foods
                             -0.42198545
                                                      -0.11769041
## Pulses.Nuts.and.Oilseeds 0.22507285
                                        -0.087921207 -0.57816932 -0.0829400
## Fruits.and.Vegetables
                             0.16834367
                                         0.222568384
                                                      0.08684392
                                                                  0.3701826
                                        -0.007702046 -0.05178373 -0.1801923
## Total
                             -0.47623561
                                    PC9
##
                                               PC10
## Red.Meat
                             -0.17150487
                                         0.20838019
## White.Meat
                             -0.46186736
                                         0.22903415
## Egg
                             0.48098579
                                         0.06827056
## Milk
                             -0.13218960
                                         0.43456461
## Fish
                             0.01789764
                                         0.21247753
                             0.30436394
## Cereals
                                         0.67412235
                             -0.14706957
## Starchy.Foods
                                         0.10134794
                                         0.12362100
## Pulses.Nuts.and.Oilseeds -0.58938418
## Fruits.and.Vegetables
                             0.20995988
                                         0.11723988
## Total
                             -0.04898111 -0.41440004
```

Sample scores stored in data_pca\$x
data_pca\$x

```
PC1
                                      PC3
                                                  PC4
                                                               PC5
                           PC2
##
         3.5978397 -0.64061101 1.1118946 -1.91119245 1.884437106 -0.37593345
##
         -1.3862854
                   -0.70991905
                                1.1613381 0.93107494 -0.009121937
                                                                   0.75816906
##
    [3,]
        -1.6608482
                    0.10781730 -0.4231894
                                           0.24680766
                                                       0.188016546
                                                                   -0.91001548
                   -1.84361307 -0.0730564
                                           0.30616165 -0.134812297
##
    [4,]
         2.9881523
                                                                   0.29005421
                               1.2155042
                                           0.72202089 -0.062918010
        -0.3686147
                   -0.10141825
                                                                   -0.37091750
##
    [5,]
##
    [6,] -2.4923551
                    0.18474749
                               -0.2075253
                                          -0.93906831 -0.822177041
                                                                   0.65204948
                               1.9302394
                                          0.77259151 -0.139755937
                    1.58140979
##
    [7,]
        -1.2387459
                                                                   -0.58954056
##
    [8,]
        -1.7732789
                   -0.75352175 -0.3644876
                                          -2.28429396 -1.224019848
                                                                   0.17828822
##
    [9,]
        -1.6448018 -0.30606640 -2.4846910
                                          1.25325810
                                                       0.230223125
                                                                   -0.33223855
         2.0943234 -0.61997417 -3.0846378
##
   [10,]
                                           0.31332068
                                                       0.270784604
                                                                   0.64981699
   [11,]
         1.4808993 -0.43978564
                               1.6090270
                                           1.21709297
                                                      -0.143865961
##
                                                                   0.11534733
         -2.6714332 -1.03848419 -0.2833724
                                           0.15763312
                                                       0.181076517
                                                                   -0.86151844
##
   [12,]
                                           0.54266246
                   -0.01064018 -0.5907111
   [13,]
         1.5660043
                                                       1.069631810
                                                                   0.77586008
##
   [14,]
         -1.7006997
                   -0.50438298
                                0.7596605
                                           0.64321026
                                                       0.292062273
                                                                    0.92348043
##
   [15,]
        -0.8828201
                    1.28521025 -0.1832152
                                          -1.71931314 -0.439007528
                                                                    0.41757899
##
   [16,]
        -0.2286613
                    0.19642466 -0.4058046
                                          1,67696384
                                                      -1.334150980
                                                                   0.08818598
                   4.41252506 -0.6718598 -0.03434506 -0.291193444
         2.0912590
                                                                   0.33278906
##
   [17,]
         2.6049767
                   -1.05771521 0.5868844 -0.14252039
##
                                                      -0.533268313
                                                                   -0.20083289
   [18,]
##
   [19,]
         1.5709389
                    2.67472726 -0.2892457
                                           0.23912301
                                                       0.594881631
                                                                   -0.60647031
   [20,]
        -1.8343339
                    0.36443676
                               0.5444138 -1.56417414
                                                                   0.80195706
##
                                                       0.158327086
##
   [21,]
        -0.9293183
                   -0.96269089 -0.3476755 0.27836268
                                                       0.755554148
                                                                   0.70844461
        -1.9728952 -0.55508144 -0.8727628 -0.60997694
##
   [22,]
                                                       1.396218668 -1.20971357
   [23,] 0.7660628 -0.48463412 -0.2720099 -0.40950179 -1.470304012 -1.24044252
##
   [24,] -1.6857673 0.30943116 1.2190705 0.55052071 0.810416131
##
                                                                   0.20076819
         3.7104025 -1.08819138 0.4162119
##
                                          -0.23641829 -1.227034337
                                                                   -0.19516642
   [25,]
                  PC7
                              PC8
                                            PC9
##
                                                        PC10
##
         ##
    [2,]
         0.0005093868 -0.012933034
                                    0.124176638 -9.471590e-16
         0.1534640851 -0.334041295
##
    [3,]
                                    0.023323758
                                                -3.330669e-16
         0.5999541449 -0.762640350
                                                1.665335e-16
    [4,]
                                    0.674235551
##
##
         0.7878924305 -0.039689570
                                    0.241927022
                                                -6.661338e-16
    [5,]
                                    -0.168254146
         -0.0364433564 -0.984127670
                                                -5.551115e-17
##
    [6,]
##
         -0.0632650200 -0.313388346
                                    0.320254182 -8.881784e-16
##
    [8,]
         -0.0506617637
                       0.792618282
                                    0.004268287 -6.661338e-16
##
   [9,]
         1.3629405718 -0.176345585
                                   -0.392094989 -4.440892e-16
                                   -0.185325024 -4.440892e-16
##
        -1.1867279230 -0.252605939
   [10,]
        -0.8173673169 -0.201792286
                                   -0.496946360
                                                -9.714451e-16
##
   [11,]
                                   -0.047542669
   [12,]
         -0.7338089555
                       0.194588527
                                                -5.551115e-16
##
   [13,]
         0.0085984337
                       0.435335074
                                    0.815121519 -8.326673e-16
##
   [14,]
         -0.2530352518
                       0.088559649
                                    -0.434700410 -1.051242e-15
##
   [15,]
         0.0122896156
                       0.009259812
                                    0.182509788 -4.718448e-16
         -0.0295375727
                                    0.341088667 -7.771561e-16
##
                       0.839590880
   [16,]
         0.6466024099 -0.205548666
                                   -0.304794550 -1.110223e-15
##
   [17,]
##
   [18,]
         -0.2135771460 -0.211277632
                                   -0.024663621 -3.677614e-16
   [19,]
         -0.9520057576
                       0.408309790
                                    0.166895175
##
                                                -1.221245e-15
##
   [20,]
         -0.1459371778 -0.241698086
                                    0.340921291 -6.106227e-16
##
   [21,]
         0.6841927749
                      0.678069260
                                   -0.252544924 -1.040834e-15
                                   0.224480622 -2.636780e-16
        -0.4798955917 -0.365578790
##
   [22,]
         ##
   [23,]
         -0.0977006735
                       0.141407912 -0.415483582 -1.110223e-15
   [24,]
  [25,] -0.1558713867 -0.081857747 -0.353665768 -4.718448e-16
```

fviz_eig(data_pca)



```
summary(data_pca)
```

```
## Importance of components:
## Proportion of Variance 0.413 0.174 0.1309 0.1044 0.0699 0.04267 0.03412 0.01906
```

```
## Cumulative Proportion 0.413 0.587 0.7179 0.8223 0.8922 0.93485 0.96898 0.98804

## PC9 PC10

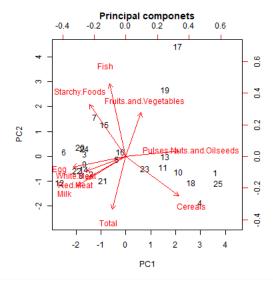
## Standard deviation 0.34581 6.619e-16

## Proportion of Variance 0.01196 0.000e+00

## Cumulative Proportion 1.00000 1.000e+00

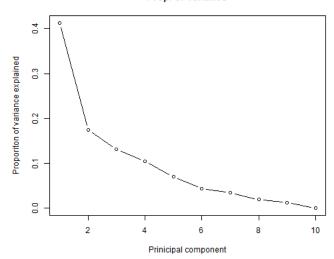
## Plot a biplot to view components on n-dimensional plane

biplot(data_pca, scale = 0, main = 'Principal componets')
```



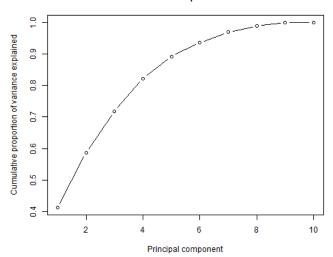
plot(propvar, xlab = 'Prinicipal component',ylab = 'Proporiton of variance explained',type = 'b', main = 'Prop. of Variance')

Prop. of Variance

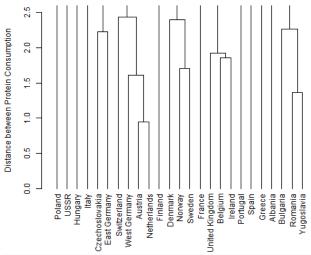


#The optimum number of components are ~ 6 i.e PC1 : PC6
cumulative scree plot
plot(cumvar_data,xlab = 'Principal component',ylab = 'Cumulative proportion of variance explained',type = 'b', main = 'Cumulative Prop.of Variance')

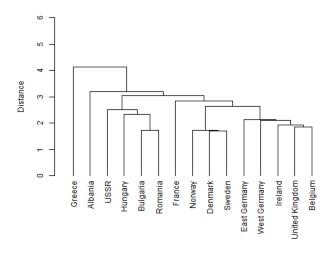
Cumulative Prop. of Variance



Dendrogram of protein consumption for inhabitants in Europe

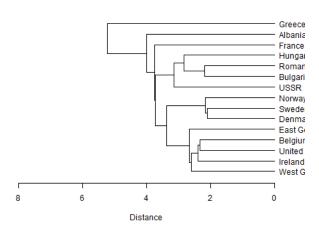


Dendrogram.

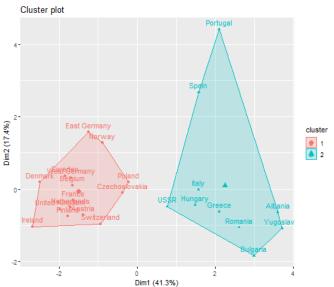


```
# We will use agnes function as it allows us to select option for data standardization, the distance measure and clustering algorithm in one single func
(agn.employ <- agnes(mysample, metric="euclidean", stand=TRUE, method = "single"))</pre>
               agnes(x = mysample, metric = "euclidean", stand = TRUE, method = "single")
## Call:
## Agglomerative coefficient: 0.4640585
## Order of objects:
  [1] West Germany
                     Ireland
                                   United Kingdom Belgium
                                                                East Germany
   [6] Denmark
                     Sweden
                                   Norway
                                                  USSR
                                                                Bulgaria
## [11] Romania
                     Hungary
                                   France
                                                  Albania
                                                                Greece
## Height (summary):
    Min. 1st Qu. Median
2.103 2.352 2.739
                            Mean 3rd Qu.
                                           Max.
##
##
                           3.034
                                 3.640
                                          5.226
##
## Available components:
## [1] "order"
## [7] "method"
                  "height"
                             "ac"
                                        "merge"
                                                    "diss"
                                                               "call"
                  "order.lab" "data"
# Description of cluster merging
agn.employ$merge
        [,1] [,2]
##
##
   [2,]
              -14
   [3,]
[4,]
##
         -10
             -11
##
          -7
              -8
##
   [5,]
          -6
##
   [6,]
##
   [7,]
              -13
##
   [8,]
              -15
##
   [9,]
               8
  [10,]
##
  [11,]
##
               9
          10
##
               -9
  [12,]
          11
##
   [13,]
          12
              -12
##
   [14,]
          13
#Dendogram
```

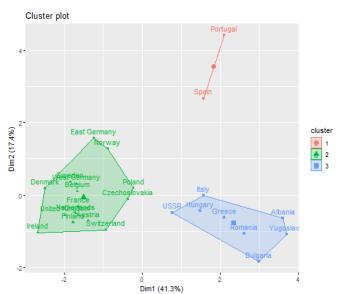
Dendrogram



```
#Interactive Plots
plot(agn.employ,ask=TRUE)
## Error in menu(tmenu, title = "\nMake a plot selection (or 0 to exit):\n"): menu() cannot be used non-interactively
# Standardizing the data with scale()
matstd.employ <- scale(data1)</pre>
# K-means, k=2, 3, 4, 5
# Centers (k's) are numbers thus, 10 random sets are chosen
(kmeans2.employ <- kmeans(matstd.employ,2,nstart = 10))</pre>
## K-means clustering with 2 clusters of sizes 15, 10
##
## Cluster means:
     Red.Meat White.Meat
                                       Milk
                                                 Fish
                                                        Cereals
##
    ## 2 -0.705171 -0.7805887 -0.8788855 -0.8707105 -0.1959456 0.9155065
##
   Starchy.Foods Pulses.Nuts.and.Oilseeds Fruits.and.Vegetables
                                                                Total
## 1
        0.3866381
                              -0.6999903
                                                  -0.2088932 0.1300177
       -0.5799572
                              1.0499854
                                                  0.3133398 -0.1950266
## 2
##
## Clustering vector:
                                                 Bulgaria Czechoslovakia
##
        Albania
                      Austria
                                    Belgium
##
##
         Denmark
                  East Germany
                                    Finland
                                                   France
                                                                Greece
##
                      Ireland
                                      Italy
                                              Netherlands
##
         Hungary
                                                                Norway
##
         Poland
##
                     Portugal
                                    Romania
                                                    Spain
                                                                Sweden
##
##
     Switzerland United Kingdom
                                       USSR
                                             West Germany
                                                             Yugoslavia
##
##
## Within cluster sum of squares by cluster:
## [1] 72.91145 82.27613
##
   (between_SS / total_SS = 35.3 %)
##
## Available components:
##
## [1] "cluster"
                                                "withinss"
                                                             "tot.withinss"
                    "centers"
                                  "totss"
## [6] "betweenss"
                    "size"
                                                "ifault"
                                  "iter'
# Computing the percentage of variation accounted for. Two clusters
perc.var.2 <- round(100*(1 - kmeans2.employ$betweenss/kmeans2.employ$totss),1)</pre>
names(perc.var.2) <- "Perc. 2 clus"
perc.var.2
## Perc. 2 clus
fviz_cluster(kmeans2.employ,data=matstd.employ)
```



```
Dim1 (41.3%)
# Conclusion: Only 2 clusters are formed but the % of variance is 65%
# Computing the percentage of variation accounted for. Three clusters
(kmeans3.employ <- kmeans(matstd.employ,3,nstart = 10))</pre>
## K-means clustering with 3 clusters of sizes 2, 15, 8
##
## Cluster means:
       Red.Meat White.Meat
##
     Red.Meat White.Meat Egg Milk -0.9696102 -1.1815761 -0.9685677 -1.4483663
                                                             Fish
                                                                      Cereals
## 1
                                                       1.7923261 -0.3923599
## 2
      0.4701140 0.5203925 0.5859237 0.5804736
                                                       0.1306304
                                                                   -0.6103377
## 3 -0.6390612 -0.6803419 -0.8564649 -0.7262965 -0.6930136
                                                                   1.2424732
##
     Starchy.Foods Pulses.Nuts.and.Oilseeds Fruits.and.Vegetables
          0.9907602
                                      1.1985682
                                                             1.72336879 -1.4508795
## 1
## 2
          0.3866381
                                     -0.6999903
                                                             -0.20889319 0.1300177
## 3
         -0.9726366
                                      1.0128397
                                                            -0.03916747 0.1189367
##
## Clustering vector:
                                                            Bulgaria Czechoslovakia
##
           Albania
                            Austria
                                            Belgium
##
##
                      East Germany
                                             Finland
           Denmark
                                                              France
                                                                               Greece
##
##
           Hungary
                            Ireland
                                               Italy
                                                         Netherlands
                                                                               Norway
##
##
            Poland
                          Portugal
                                             Romania
                                                                Spain
                                                                               Sweden
##
##
      Switzerland United Kingdom
                                                USSR
                                                        West Germany
                                                                           Yugoslavia
##
## Within cluster sum of squares by cluster:
## [1] 4.167026 72.911454 47.382187
## (between_SS / total_SS = 48.1 %)
##
##
   Available components:
##
## [1] "cluster"
                         "centers"
                                          "totss"
                                                          "withinss"
                                                                           "tot.withinss"
##
   [6]
       "betweenss"
                         "size"
                                          "iter'
                                                          "ifault"
\verb|perc.var.3| <- round(100*(1 - kmeans3.employ\$betweenss/kmeans3.employ\$totss),1)| \\
names(perc.var.3) <- "Perc. 3 clus"</pre>
perc.var.3
## Perc. 3 clus
##
            51.9
fviz_cluster(kmeans3.employ,data=matstd.employ)
```



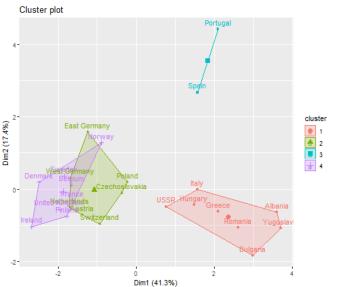
```
#Conclusion: Three clusters with 52% of the variance is restored and there are three separate groups which are visible
# Computing the percentage of variation accounted for. Four clusters
(kmeans4.employ <- kmeans(matstd.employ,4,nstart = 10))</pre>
## K-means clustering with 4 clusters of sizes 8, 7, 2, 8
##
## Cluster means:
##
        Red.Meat
                  White.Meat
                                                Milk
                                                           Fish
                                                                    Cereals
                                     Egg
## 1
     -0.63906125
                  -0.68034187 -0.8564649 -0.7262965 -0.6930136
## 2 -0.02518468
                  1.09068558 0.4407239
                                          0.1618241 -0.4100039 -0.4702091
## 3 -0.96961017 -1.18157605 -0.9685677 -1.4483663
                                                      1.7923261 -0.3923599
## 4 0.90350039 0.02138599 0.7129734 0.9467920 0.6036854 -0.7329502
    Starchy.Foods Pulses.Nuts.and.Oilseeds Fruits.and.Vegetables
##
                                                                          Total
##
        -0.9726366
                                   1.0128397
                                                         -0.03916747
                                                                     0.1189367
## 2
         0.3003350
                                   -0.7471594
                                                         0.19397225 -0.2372401
## 3
         0.9907602
                                   1.1985682
                                                         1.72336879 -1.4508795
## 4
         0.4621534
                                  -0.6587173
                                                         -0.56140044 0.4513683
##
## Clustering vector:
## Albania
                          Austria
                                          Belgium
                                                        Bulgaria Czechoslovakia
##
##
                     East Germany
                                          Finland
##
##
          Hungary
                          Ireland
                                            Italy
                                                     Netherlands
                                                                          Norway
##
##
           Poland
                                                                          Sweden
                         Portugal
                                          Romania
                                                            Spain
##
##
      Switzerland United Kingdom
                                             USSR
                                                    West Germany
                                                                      Yugoslavia
##
##
## Within cluster sum of squares by cluster:
## [1] 47.382187 20.027050 4.167026 34.697369
## (between_SS / total_SS = 55.7 %)
##
## Available components:
##
## [1] "cluster"
## [6] "betweenss"
                                                       "withinss'
                       "centers"
                                                                      "tot.withinss"
                                       "totss'
                       "size"
                                       "iter"
                                                       "ifault"
perc.var.4
## Perc. 4 clus
```

```
file:///C:/Users/vidhi/Desktop/Rutgers/SEM2/Multivariate Analysis/Midterm/MVA-Midterm.html
```

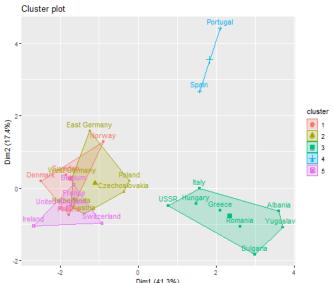
##

44.3

fviz_cluster(kmeans4.employ,data=matstd.employ)

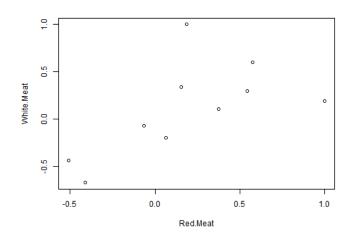


```
Dim1 (41.3%)
# Conclusion: 4 clusters and 44% of the variance is stored in these clusters and there is an overlap
\# Computing the percentage of variation accounted for. Five clusters
(kmeans5.employ <- kmeans(matstd.employ,5,nstart = 10))</pre>
## K-means clustering with 5 clusters of sizes 4, 6, 8, 2, 5
##
## Cluster means:
##
        Red.Meat White.Meat
                                    Egg
                                               Milk
                                                           Fish
                                                                   Cereals
## 1
      0.05876425 -0.1791077
                             0.3766652
                                         1.33424402
                                                     1.2160155 -0.8691863
## 2
     -0.18608680
                  1.1797939
                             0.5261355
                                         0.03099617
                                                    -0.3688388 -0.4529093
## 3 -0.63906125 -0.6803419 -0.8564649 -0.72629651 -0.6930136
                                                                 1.2424732
## 4 -0.96961017 -1.1815761 -0.9685677 -1.44836627
                                                     1.7923261 -0.3923599
     1.58663483 0.2887109 0.8250762 0.63683030 -0.1383146 -0.5921729
## 5
##
    Starchy.Foods Pulses.Nuts.and.Oilseeds Fruits.and.Vegetables
                                                                          Total
## 1
         0.2356076
                                  -0.9063553
                                                        -1.14891253
                                                                     0.06353138
## 2
         0.4873252
                                  -0.7825363
                                                         0.15666989 -0.31814941
## 3
        -0.9726366
                                   1.0128397
                                                        -0.03916747
                                                                     0.11893666
                                   1.1985682
         0.9907602
                                                         1.72336879 -1.45087949
## 4
                                  -0.4358430
                                                         0.10444659 0.72100732
## 5
         0.3866381
##
##
  Clustering vector:
##
          Albania
                         Austria
                                         Belgium
                                                        Bulgaria Czechoslovakia
##
##
          Denmark
                    East Germany
                                         Finland
                                                          France
                                                                         Greece
##
                                               1
##
                                           Italy
          Hungary
                         Ireland
                                                    Netherlands
                                                                         Norway
##
##
           Poland
                                                                         Sweden
                         Portugal
                                         Romania
                                                           Spain
##
##
      Switzerland United Kingdom
                                            USSR
                                                   West Germany
                                                                     Yugoslavia
##
##
## Within cluster sum of squares by cluster:
  [1] 7.849041 15.642697 47.382187 4.167026 15.011394
    (between_SS / total_SS = 62.5 %)
##
##
## Available components:
##
##
   [1] "cluster"
                       "centers"
                                       'totss"
                                                      "withinss"
                                                                      "tot.withinss"
  [6] "betweenss"
                       "size"
                                      "iter"
                                                      "ifault"
\verb|perc.var.5| <- |round| (100*(1 - kmeans5.employ\$betweenss/kmeans5.employ\$totss), 1)|
names(perc.var.5) <- "Perc. 5 clus"</pre>
perc.var.5
## Perc. 5 clus
##
           37.5
fviz_cluster(kmeans5.employ,data=matstd.employ)
```

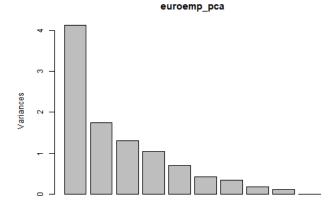


```
Red.Meat
                                          White.Meat
                                                                         Milk
                                                              Egg
                              1.00000000
                                          0.18850977
                                                       0.57532001
                                                                    0.5440251
## Red.Meat
## White.Meat
                              0.18850977
                                          1.00000000
                                                       0.60095535
                                                                    0.2974816
## Egg
                              0.57532001
                                          0.60095535
                                                       1.00000000
                                                                    0.6130310
## Milk
                              0.54402512
                                          0.29748163
                                                       0.61303102
                                                                    1,0000000
                              0.06491072
                                          -0.19719960
                                                       0.04780844
                                                                   0.1624624
## Fish
                             -0.50970337
                                          -0.43941908
                                                      -0.70131040
## Cereals
                                                                   -0.5924925
                              0.15383673
  Starchy.Foods
                                          0.33456770
                                                       0.41266333
                                                                   0.2144917
## Pulses.Nuts.and.Oilseeds
                             -0.40988882
                                         -0.67214885
                                                      -0.59519381
                                                                  -0.6238357
## Fruits.and.Vegetables
                             -0.06393465
                                          -0.07329308
                                                      -0.16392249
                                                                  -0.3997753
## Total
                              0.37369919
                                          0.10308602
                                                      0.18970028 0.4603542
                                                      Starchy.Foods
##
                                    Fish
                                             Cereals
## Red.Meat
                              0.06491072
                                         -0.50970337
                                                         0.15383673
## White.Meat
                             -0.19719960
                                          -0.43941908
                                                         0.33456770
## Egg
                              0.04780844
                                         -0.70131040
                                                         0.41266333
## Milk
                              0.16246239
                                         -0.59249246
                                                         0.21449173
## Fish
                              1,00000000
                                          -0.51714759
                                                         0.43868411
                             -0.51714759
                                          1,00000000
                                                        -0.57813449
## Cereals
                                                         1.00000000
                              0.43868411
                                          -0.57813449
## Starchy.Foods
## Pulses.Nuts.and.Oilseeds
                             -0.12226043
                                          0.63605948
                                                        -0.49518800
## Fruits.and.Vegetables
                              0.22948842
                                          0.04229293
                                                         0.06835670
## Total
                              -0.09089592
                                          0.18587578
                                                        -0.04418245
##
                             Pulses.Nuts.and.Oilseeds Fruits.and.Vegetables
## Red.Meat
                                            -0.4098888
                                                                  -0.06393465
                                            -0.6721488
                                                                  -0.07329308
## White.Meat
## Egg
                                            -0.5951938
                                                                  -0.16392249
                                            -0.6238357
                                                                  -0.39977527
## Milk
## Fish
                                            -0.1222604
                                                                  0.22948842
## Cereals
                                            0.6360595
                                                                  0.04229293
## Starchy.Foods
                                            -0.4951880
                                                                  0.06835670
## Pulses.Nuts.and.Oilseeds
                                            1.0000000
                                                                  0.35133227
## Fruits.and.Vegetables
                                            0.3513323
                                                                  1.00000000
## Total
                                            -0.0812251
                                                                   0.07201466
                                   Total
## Red.Meat
                              0.37369919
## White.Meat
                              0.10308602
                              0.18970028
## Egg
                              0.46035417
## Milk
## Fish
                              -0.09089592
## Cereals
                              0.18587578
## Starchy.Foods
                              -0.04418245
## Pulses.Nuts.and.Oilseeds
                             -0.08122510
## Fruits.and.Vegetables
                              0.07201466
                              1.00000000
## Total
```

plot(corrm.emp)



```
#this is the correlation plot
#calculating the PCA and plotting these variances
euroemp_pca <- prcomp(data1, scale=TRUE)</pre>
summary(euroemp_pca)
\begin{tabular}{ll} \beg
##
                                                                                                                                 PC1
                                                                                                                                                            PC2
                                                                                                                                                                                            PC3
                                                                                                                                                                                                                            PC4
                                                                                                                                                                                                                                                              PC5
                                                                                                                                                                                                                                                                                                    PC6
                                                                                                                                                                                                                                                                                                                                          PC7
                                                                                                                         2.032 1.319 1.1442 1.0215 0.8361 0.65320 0.58415 0.43663
## Standard deviation
## Proportion of Variance 0.413 0.174 0.1309 0.1044 0.0699 0.04267 0.03412 0.01906
## Cumulative Proportion 0.413 0.587 0.7179 0.8223 0.8922 0.93485 0.96898 0.98804
##
                                                                                                                                          PC9
                                                                                                                                                                                  PC10
## Standard deviation 0.34581 6.619e-16
## Proportion of Variance 0.01196 0.000e+00
## Cumulative Proportion 1.00000 1.000e+00
plot(euroemp_pca)
```



```
#Looks like Pc1, pc2, pc3,pc4,pc5 restores maximum of variance

# A table containing eigenvalues and %'s accounted, follows.

# Eigenvalues are the sdev^2
(eigen_euroemp <- round(euroemp_pca$sdev^2,2))

## [1] 4.13 1.74 1.31 1.04 0.70 0.43 0.34 0.19 0.12 0.00

names(eigen_euroemp) <- paste("PC",1:10,sep="")
eigen_euroemp

## PC1 PC2 PC3 PC4 PC5 PC6 PC7 PC8 PC9 PC10
## 4.13 1.74 1.31 1.04 0.70 0.43 0.34 0.19 0.12 0.00
```

```
sumlambdas <- sum(eigen_euroemp)</pre>
sumlambdas
## [1] 10
propvar <- round(eigen euroemp/sumlambdas,2)</pre>
propvar
## PC1 PC2 PC3 PC4 PC5 PC6 PC7 PC8 PC9 PC10
## 0.41 0.17 0.13 0.10 0.07 0.04 0.03 0.02 0.01 0.00
cumvar euroemp <- cumsum(propvar)</pre>
cumvar_euroemp
## PC1 PC2 PC3 PC4 PC5 PC6 PC7 PC8 PC9 PC10
## 0.41 0.58 0.71 0.81 0.88 0.92 0.95 0.97 0.98 0.98
matlambdas <- rbind(eigen euroemp,propvar,cumvar euroemp)</pre>
                   PC1 PC2 PC3 PC4 PC5 PC6 PC7 PC8 PC9 PC10
## eigen_euroemp 4.13 1.74 1.31 1.04 0.70 0.43 0.34 0.19 0.12 0.00
                  0.41 0.17 0.13 0.10 0.07 0.04 0.03 0.02 0.01 0.00
## cumvar_euroemp 0.41 0.58 0.71 0.81 0.88 0.92 0.95 0.97 0.98 0.98
rownames(matlambdas) <- c("Eigenvalues", "Prop. variance", "Cum. prop. variance")</pre>
rownames(matlambdas)
## [1] "Eigenvalues"
                             "Prop. variance"
                                                   "Cum. prop. variance"
eigvec.emp <- euroemp_pca$rotation
print(euroemp_pca)
## Standard deviations (1, .., p=10):
    [1] 2.032257e+00 1.319067e+00 1.144237e+00 1.021544e+00 8.360847e-01
##
    [6] 6.531975e-01 5.841454e-01 4.366348e-01 3.458098e-01 6.618503e-16
##
## Rotation (n x k) = (10 x 10):
                                   PC1
                                               PC2
##
                                                           PC3
                                                                        PC4
## Red.Meat
                            -0.3180769 -0.17809245 -0.38142753 -0.039766137
## White.Meat
                            -0.3140588 -0.11783853
                                                    0.36420271
                                                                0.538507972
## Egg
                            -0.4202281 -0.08236350
                                                    0.02047575
                                                                0.155623651
## Milk
                            -0.3870300 -0.23356182 -0.19997405 -0.320360929
## Fish
                            -0.1271598   0.57388821   -0.33003267   -0.304161366
                            0.4177240 -0.31321549 -0.02354236 0.104798477
## Cereals
## Starchy.Foods
                            -0.2880798 0.41038324 0.05768490
                                                                0.150709175
## Pulses.Nuts.and.Oilseeds 0.4177658
                                        0.04145202 -0.24796403
                                                                0.008042093
## Fruits.and.Vegetables
                             0.1197680 0.34858202 -0.41210384
                                                                0.643455476
## Total
                            -0.1062294 -0.41709540 -0.58081103 0.203145847
##
                                    PC5
                                                 PC6
                                                             PC7
## Red.Meat
                             0.53138781 -0.393811788
                                                      0.42940825 -0.1592276
                                        0.309417061
## White.Meat
                            -0.09760147
                                                      0.09254681 -0.2919567
## Egg
                             0.26932734
                                        -0.059357751 -0.63995627
## Milk
                            -0.15848975
                                         0.307976584 -0.17405921 0.5444724
## Fish
                            -0.20323386
                                         0.303075844
                                                      0.06315829 -0.5200308
## Cereals
                            -0.29201244 -0.196460437
                                                      0.06971238 -0.2001491
## Starchy.Foods
                            -0.42198545 -0.680457657 -0.11769041 0.1889672
## Pulses.Nuts.and.Oilseeds 0.22507285
                                        -0.087921207 -0.57816932 -0.0829400
## Fruits.and.Vegetables
                             0.16834367
                                        0.222568384 0.08684392
                                                                 0.3701826
## Total
                            -0.47623561
                                        -0.007702046 -0.05178373 -0.1801923
##
                                   PC9
                                               PC10
                                         0.20838019
## Red.Meat
                            -0.17150487
                            -0.46186736
                                         0.22903415
## White.Meat
## Egg
                             0.48098579
                                         0.06827056
## Milk
                            -0.13218960
                                         0.43456461
## Fish
                             0.01789764
                                         0.21247753
## Cereals
                             0.30436394
                                         0.67412235
## Starchy.Foods
                            -0.14706957
                                         0.10134794
## Pulses.Nuts.and.Oilseeds -0.58938418
                                         0.12362100
## Fruits.and.Vegetables
                            0.20995988
                                         0.11723988
## Total
                            -0.04898111 -0.41440004
#Taking the first five PCs to generate linear combinations for all the variables
pcafactors.emp <- eigvec.emp[,1:5]</pre>
pcafactors.emp
                                               PC2
                                                           PC3
## Red.Meat
                            -0.3180769 -0.17809245 -0.38142753 -0.039766137
## White.Meat
                            -0.3140588 -0.11783853 0.36420271 0.538507972
## Egg
                            -0.4202281 -0.08236350
                                                    0.02047575
                                                                0.155623651
                            -0.3870300 -0.23356182 -0.19997405 -0.320360929
## Milk
## Fish
                            -0.1271598 0.57388821 -0.33003267
                                                               -0.304161366
## Cereals
                             0.4177240 -0.31321549 -0.02354236
                                                                0.104798477
## Starchy.Foods
                            -0.2880798 0.41038324 0.05768490
                                                                0.150709175
## Pulses.Nuts.and.Oilseeds 0.4177658 0.04145202 -0.24796403
                                                                0.008042093
```

0.1197680 0.34858202 -0.41210384 0.643455476

Fruits.and.Vegetables

```
-0.1062294 -0.41709540 -0.58081103 0.203145847
## Total
##
                                      PC5
## Red.Meat
                              0.53138781
## White.Meat
                              -0.09760147
## Egg
                              0.26932734
## Milk
                              -0.15848975
## Fish
                              -0.20323386
## Cereals
                              -0.29201244
## Starchy.Foods
                              0.42198545
## Pulses.Nuts.and.Oilseeds
                              0.22507285
## Fruits.and.Vegetables
                              0.16834367
## Total
                              -0.47623561
# Multiplying each column of the eigenvector's matrix by the square-root of the corresponding eigenvalue in order to get the factor loadings unrot.fact.emp <- sweep(pcafactors.emp,MARGIN=2,euroemp_pca$sdev[1:5],`*`)
unrot.fact.emp
                                    PC1
                                                PC2
##
                                                             PC3
                                                                           PC4
                              -0.6464140 -0.2349159 -0.43644348 -0.040622842
## Red.Meat
## White.Meat
                              -0.6382482 -0.1554370
                                                     0.41673420
                                                                  0.550109362
## Egg
                              -0.8540114 -0.1086430
                                                     0.02342911
## Milk
                              -0.7865443 -0.3080838
                                                    -0.22881770
                                                                  -0.327262651
## Fish
                              -0.2584213 0.7569972 -0.37763557 -0.310714091
## Cereals
                              0.8489223 -0.4131523 -0.02693804
                                                                  0.107056211
## Starchy.Foods
                              -0.5854521 0.5413231 0.06600519
                                                                  0.153955990
## Pulses.Nuts.and.Oilseeds
                             0.8490074
                                        0.0546780 -0.28372960
                                                                  0.008215348
                              0.2433992 0.4598032 -0.47154445
## Fruits.and.Vegetables
                                                                  0.657317811
## Total
                              -0.2158855 -0.5501769 -0.66458545
                                                                  0.207522336
##
                                     PC5
                              0.44428523
## Red Meat
                              -0.08160309
## White.Meat
                              0.22518048
## Egg
## Milk
                              -0.13251086
## Fish
                              -0.16992073
## Cereals
                              -0.24414713
## Starchy.Foods
                              -0.35281559
## Pulses.Nuts.and.Oilseeds
                              0.18817997
## Fruits.and.Vegetables
                              0.14074957
## Total
                              -0.39817332
# Computing communalities
communalities.emp <- rowSums(unrot.fact.emp^2)</pre>
communalities.emp
##
                    Red.Meat
                                            White.Meat
                                                                              Egg
                                                                        0.8176674
##
                   0.8625590
                                             0.9144681
##
                        Milk
                                                  Fish
                                                                          Cereals
##
                   0.8905850
                                             0.9078512
                                                                        0.9631584
##
              Starchy.Foods Pulses.Nuts.and.Oilseeds
                                                           Fruits.and.Vegetables
                  0.7883228
                                                                        0.9448934
##
                                             0.8397850
##
                       Total
##
                   0.9925825
# Performing the varimax rotation. The default in the varimax function is norm=TRUE thus, Kaiser normalization is carried out
rot.fact.emp <- varimax(unrot.fact.emp)</pre>
View(unrot.fact.emp)
rot.fact.emp
## $loadings
##
## Loadings:
                             PC1
                                    PC2
                                            PC3
                                                           PC5
##
                                                   PC4
## Red.Meat
                                            -0.228
                                                            0.897
                              -0.936
## White.Meat
                                                            0.150
## Egg
                              -0.588
                                                    -0.103
                                                            0.671
## Milk
                              -0.233
                                      0.243 -0.427 -0.521
                                                            0.568
## Fish
                              0.180
                                     0.923
                                                            0.112
                              0.419 -0.559
                                            -0.252
## Cereals
                                                           -0.634
                              -0.550
## Starchy.Foods
                                     0.695
## Pulses.Nuts.and.Oilseeds 0.709
                                     -0.259
                                                     0.412 -0.309
## Fruits.and.Vegetables
## Total
                                            -0.977
                                                            0.168
##
##
                     PC1
                           PC2
                                 PC3
                                       PC4
                  2.299 1.830 1.268 1.386 2.139
## SS loadings
  Proportion Var 0.230 0.183 0.127 0.139 0.214
## Cumulative Var 0.230 0.413 0.540 0.678 0.892
##
## $rotmat
##
              [,1]
                          [,2]
                                      [,3]
                                                  [,4]
                                0.1399026
         0.6255823 -0.3589826
                                           0.2682405 -0.6230992
##
   [1,]
        0.0252904 0.7722244
                                0.4833311
                                            0.3851547 -0.1451784
##
   [2,]
   [3,]
        -0.4909586 -0.2604259
                                0.6394138
                                           -0.3857373 -0.3653693
##
        -0.5631966 -0.2793097 -0.1587512
                                            0.7523791 -0.1162730
        0.2231063 -0.3591174 0.5592552 0.2546280 0.6660754
#The print method of varimax omits loadings less than abs(0.1). In order to display all the loadings, it is necessary to ask explicitly the contents of
```

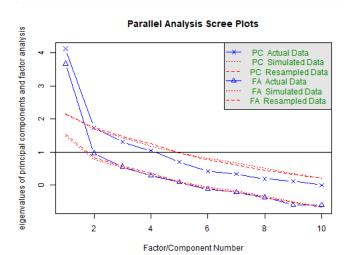
#The print method of varimax omits loadings less than abs(0.1). In order to display all the loadings, it is necessary to ask explicitly the contents of fact.load.emp <- rot.fact.emp\$loadings[1:5,1:5] fact.load.emp

```
PC1
                                   PC2
                                                PC3
                                                            PC4
## Red.Meat
              -0.07404907
                           0.01610055 -0.22812738 -0.01295720 0.8969986
## White.Meat -0.93583294
                           -0.12378641 -0.03092196
                                                     0.00129115 0.1496794
## Egg
              -0.58780050
                           0.09130706 -0.05631238
                                                    -0.10301458 0.6708487
## Milk
               -0.23275046
                            0.24303057 -0.42740930 -0.52134596 0.5682148
                           0.92349352 0.04255822
## Fish
               0.17996718
                                                     0.09086870 0.1120465
#Computing the rotated factor scores for the 25 European Countries.
scale.emp <- scale(data1)</pre>
scale.emp
##
                      Red.Meat White.Meat
                                                               Milk
                                                                            Fish
                                                    Egg
## Albania
                   0.05876425
                               -1.84988830
                                            -1.86538958
                                                        -1.16658295 -1.23330478
## Austria
                   -0.23505701
                                1.62533538
                                            0.82507616
                                                         0.38322532
                                                                     -0.65699414
                   1.23404931
                                0.28871089
                                            0.82507616
                                                         0.10144200
## Belgium
                                                                     0.20747183
## Bulgaria
                   -0.52887828
                                -0.51326380
                                            -0.96856767
                                                         -1.30747461
                                                                     -0.94514946
## Czechoslovakia
                   0.05876425
                                0.82336069
                                            -0.07174575
                                                         -0.60301630
                                                                     -0.65699414
                                            0.82507616
                                0.82336069
                                                                     1.64824845
                   0.35258552
                                                         1.08768362
## Denmark
## East Germany
                   -0.52887828
                                1.09068558
                                            0.82507616
                                                         -0.88479963
                                                                     0.20747183
## Finland
                    0.05876425
                               -0.78058870
                                            -0.07174575
                                                         2.35570856
                                                                     0.49562716
## France
                    2.40933437
                                0.55603579
                                            -0.07174575
                                                         0.38322532
                                                                     0.49562716
## Greece
                    0.05876425
                                -1.31523850
                                            -0.07174575
                                                         0.10144200
                                                                     0.49562716
                   -1.41034207
## Hungary
                                1.09068558
                                            -0.07174575
                                                         -1.02569129
                                                                     -1.23330478
                                0.55603579
                   1,23404931
                                            1.72189807
                                                         1.22857528
                                                                     -0.65699414
## Ireland
                   -0.23505701
                               -0.78058870
                                            -0.07174575
                                                         -0.46212464 -0.36883881
## Italy
## Netherlands
                   0.05876425
                                1.62533538
                                            0.82507616
                                                         0.80590030
                                                                     -0.36883881
                   -0.23505701
                               -0.78058870
                                            -0.07174575
                                                         0.80590030
## Norway
                                                                     1.64824845
## Poland
                   -0.82269954
                                0.55603579
                                            -0.07174575
                                                         0.24233366
                                                                     -0.36883881
## Portugal
                   -1.11652080
                                -1.04791360
                                            -1.86538958
                                                         -1.73014959
                                                                     2.80086974
                   -1.11652080 -0.51326380
                                                        -0.88479963
## Romania
                                            -0.96856767
                                                                     -0.94514946
                   -0.82269954
                                            -0.07174575
## Spain
                               -1.31523850
                                                         -1.16658295
                                                                     0.78378248
                    0.05876425
                                0.02138599
                                            0.82507616
                                                         1.08768362
                                                                     1.07193780
##
  Sweden
                                0.55603579
  Switzerland
                    0.94022805
                                            -0.07174575
                                                         0.94679196
                                                                     -0.65699414
## United Kingdom
                   2.11551310
                                -0.51326380
                                            1.72189807
                                                         0.52411698 -0.08068349
## USSR
                   -0.23505701
                               -0.78058870
                                            -0.96856767
                                                         -0.03944966 -0.36883881
## West Germany
                   0.35258552
                                1.35801048
                                            0.82507616
                                                         0.24233366 -0.36883881
                   -1.70416333 -0.78058870 -1.86538958
                                                        -1.02569129 -0.94514946
## Yugoslavia
                      Cereals Starchy.Foods Pulses.Nuts.and.Oilseeds
##
## Albania
                    0.8791769
                                 -2.0298502
                                                           1.44620630
## Austria
                   -0.3923599
                                 -0.2174840
                                                          -1.03017435
## Belgium
                   -0.4831840
                                  0.9907602
                                                          -0.53489822
## Bulgaria
                    2.2415378
                                 -2.0298502
                                                           0.45565404
                                                          -1.03017435
                                  0.3866381
## Czechoslovakia
                   0.1525844
                   -0.9373043
                                  0.3866381
                                                          -1.03017435
## Denmark
## East Germany
                   -0.6648321
                                  1.5948823
                                                          -1.03017435
                   -0.5740081
                                  0.3866381
                                                          -1.03017435
## Finland
## France
                   -0.3923599
                                  0.3866381
                                                          -0.53489822
## Greece
                   0.8791769
                                 -1.4257281
                                                           2.43675857
                   0.6975288
                                 -0.2174840
                                                           0.95093017
## Hungary
                   -0.7556562
                                  0.9907602
                                                           -0.53489822
## Ireland
                   0.4250566
                                 -1.4257281
                                                           0.45565404
##
  Italy
## Netherlands
                   -0.9373043
                                 -0.2174840
                                                           -0.53489822
## Norway
                   -0.8464803
                                  0.3866381
                                                          -0.53489822
## Poland
                   0.3342325
                                  0.9907602
                                                          -0.53489822
                                  0.9907602
                                                           0.95093017
## Portugal
                   -0.4831840
                   1.6057694
                                 -0.8216060
## Romania
                                                           0.95093017
## Spain
                   -0.3015359
                                  0.9907602
                                                           1.44620630
                                 -0.2174840
                                                          -1.03017435
   Sweden
                   -1.1189524
## Switzerland
                   -0.5740081
                                 -0.8216060
                                                          -0.53489822
## United Kingdom
                   -0.7556562
                                  0.3866381
                                                          -0.03962209
## USSR
                   1.0608250
                                  0.9907602
                                                          -0.03962209
                                  0.3866381
                                                          -0.53489822
## West Germany
                   -1.2097765
## Yugoslavia
                                 -0.8216060
                                                           1.44620630
                   2.1507138
##
                   Fruits.and.Vegetables
## Albania
                              -1.1489125
                                          -2.11574280
## Austria
                              -0.1044466
                                          -0.04727917
## Belgium
                              -0.1044466
                                          0.39596304
                              -0.1044466
                                          0.69145784
## Bulgaria
                              -0.1044466
                                          -0.49052138
## Czechoslovakia
  Denmark
                              -1.1489125
                                          0.69145784
##
## East Germany
                              -0.1044466
                                          1.37700579
## Finland
                              -1.6711455
                                          0.69145784
## France
                               1.4622523
                                          1.87343706
## Greece
                               1,4622523
                                          1.87343706
                              -0.1044466
                                          -0.49052138
## Hungary
## Ireland
                              -0.6266796
                                          0.83920525
## Italy
                               1.4622523
                                          -0.34277397
## Netherlands
                              -0.1044466
                                          -0.04727917
## Norway
                              -0.6266796
                                          -0.49052138
                                          0.98695265
## Poland
                               1.4622523
                               1.9844853
                                          1.52475319
## Portugal
## Romania
                              -0.6266796
                                          0.10046823
## Spain
                               1.4622523
                                          1.37700579
                               -1.1489125
                                          -0.63826878
## Sweden
## Switzerland
                               0.4177864
                                          0.24821564
## United Kingdom
                              -0.6266796
                                          0.24821564
## USSR
                              -0.6266796
                                          0.83920525
                              -0.1044466
## West Germany
                                          -0.93376358
   Yugoslavia
                              -0.6266796
                                          0.39596304
## attr(,"scaled:center")
##
                    Red.Meat
                                            White.Meat
                                                                             Egg
##
                       9.80
                                                  7.92
                                                                            3.08
                       Milk
                                                  Fish
##
                                                                         Cereals
##
                       17.28
                                                  4.28
                                                                           32.32
              Starchy.Foods Pulses.Nuts.and.Oilseeds
                                                          Fruits.and.Vegetables
```

```
4.20
##
                        4.36
                                                   3.08
##
                       Total
                       86.32
   attr(,"scaled:scale")
##
                    Red.Meat
                                             White.Meat
                                                                              Egg
##
                    3.403430
                                               3.740766
                                                                         1.115049
##
                        Milk
                                                  Fish
                                                                          Cereals
                    7.097652
                                               3.470351
##
                                                                        11.010298
##
               Starchy.Foods Pulses.Nuts.and.Oilseeds
                                                           Fruits.and.Vegetables
##
                    1.655295
                                               2.019076
                                                                         1.914854
##
                       Total
##
                    6.768309
#as.matrix(scale.emp)%*%fact.load.emp%*%solve(t(fact.load.emp))%*%fact.load.emp)
library(psych)
install.packages("psych", lib="/Library/Frameworks/R.framework/Versions/3.5/Resources/library")
## Warning in install.packages("psych", lib = "/Library/Frameworks/R.framework/
## Versions/3.5/Resources/library"): 'lib = "/Library/Frameworks/R.framework/
## Versions/3.5/Resources/library"' is not writable
## Error in install.packages("psych", lib = "/Library/Frameworks/R.framework/Versions/3.5/Resources/library"): unable to install packages
fit.pc <- principal(data1, nfactors=5, rotate="varimax")</pre>
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was done
## In factor.stats, I could not find the RMSEA upper bound . Sorry about that
## Warning in principal(data1, nfactors = 5, rotate = "varimax"): The matrix is not
## positive semi-definite, scores found from Structure loadings
fit.pc
## Principal Components Analysis
## Call: principal(r = data1, nfactors = 5, rotate = "varimax")
\hbox{\tt \#\# Standardized loadings (pattern matrix) based upon correlation matrix}
##
                               RC1
                                     RC5
                                            RC2
                                                  RC4
                                                         RC3 h2
                                                                       u2 com
                                                        0.23 0.86 0.1374 1.1
## Red.Meat
                              0.07
                                     0.90
                                           0.02 -0.01
                              0.94
## White.Meat
                                     0.15
                                                        0.03 0.91 0.0855 1.1
                                          -0.12 0.00
## Egg
                              0.59
                                     0.67
                                           0.09
                                                 -0.10
                                                        0.06 0.82 0.1823 2.1
## Milk
                              0.23
                                     0.57
                                           0.24 -0.52
                                                        0.43 0.89 0.1094 3.6
## Fish
                              -0.18
                                    0.11
                                           0.92
                                                  0.09
                                                       -0.04 0.91 0.0921 1.1
## Cereals
                              -0.42 -0.63
                                          -0.56
                                                  0.10
                                                        0.25 0.96 0.0368 3.2
## Starchy.Foods
                              0.55 0.01
                                           0.69
                                                  0.05
                                                        0.00 0.79 0.2117 1.9
## Pulses.Nuts.and.Oilseeds -0.71 -0.31
                                                       -0.07 0.84 0.1602 2.4
                                          -0.26
                                                  0.41
## Fruits.and.Vegetables
                              -0.06 -0.03
                                           0.16
                                                  0.95
                                                        0.07 0.94 0.0551 1.1
                              0.03 0.17
## Total
                                          -0.09
                                                  0.04
                                                        0.98 0.99 0.0074 1.1
##
##
                           RC1 RC5 RC2 RC4 RC3
## SS loadings
                          2.30 2.14 1.83 1.39 1.27
## Proportion Var
                          0.23 0.21 0.18 0.14 0.13
## Cumulative Var
                          0.23 0.44 0.63 0.77 0.89
## Proportion Explained 0.26 0.24 0.21 0.16 0.14
## Cumulative Proportion 0.26 0.50 0.70 0.86 1.00
## Mean item complexity = 1.9
## Test of the hypothesis that 5 components are sufficient.
## The root mean square of the residuals (RMSR) is 0.05
    with the empirical chi square 4.86 with prob < 0.43
## Fit based upon off diagonal values = 0.99
round(fit.pc$values, 3)
## [1] 4.130 1.740 1.309 1.044 0.699 0.427 0.341 0.191 0.120 0.000
fit.pc$loadings
## Loadings:
                                     RC5
                              RC1
                                            RC2
                                                           RC3
## Red.Meat
                                      0.897
                                                             0.228
## White.Meat
                              0.936
                                      0.150 -0.124
                                                    -0.103
## Egg
                              0.588
                                      0.671
                                             0.243 -0.521
## Milk
                              0.233
                                                           0.427
                                      0.568
## Fish
                                      0.112
                                             0.923
                              -0.180
                              -0.419
## Cereals
                                      -0.634
                                             -0.559
                                                             0.252
## Starchy.Foods
                              0.550
                                             0.695
## Pulses.Nuts.and.Oilseeds -0.709
                                     -0.309
                                             -0.259
                                                     a 412
## Fruits.and.Vegetables
                                             0.156
                                                     0.955
                                      0.168
                                                             0.977
## Total
##
                     RC1 RC5 RC2 RC4 RC3
```

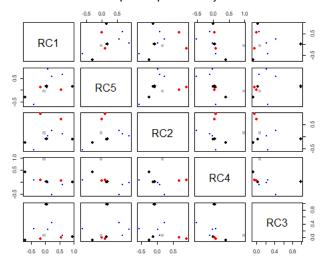
```
2.299 2.139 1.830 1.386 1.268
## SS loadings
## Proportion Var 0.230 0.214 0.183 0.139 0.127
## Cumulative Var 0.230 0.444 0.627 0.765 0.892
# Loadinas with more digits
for (i in c(5,1)) { print(fit.pc$loadings[[1,i]])}
## [1] 0.2281274
## [1] 0.07404907
# Communalities
fit.pc$communality
                   Red.Meat
                                          White.Meat
##
                                                                          Egg
##
                  0.8625590
                                           0.9144681
                                                                    0.8176674
                                                                      Cereals
##
                  0.8905850
                                           0.9078512
                                                                    0.9631584
##
              Starchy.Foods Pulses.Nuts.and.Oilseeds
                                                        Fruits.and.Vegetables
##
                  0.7883228
                                           0.8397850
                                                                    0.9448934
##
                      Total
                  0.9925825
##
#Cereals is able to restore 96% of the total variance
# Rotated factor scores
fit.pc$scores
## Albania
                  -5.37916900 -3.6216649 -3.6291464
                                                    0.07734096 -2.6190935
## Austria
                   2.97591312
                               1,2902085
                                         -0.3207655
                                                    -0.91529580
                                                                 0.1523285
                   1,97709986
                               2.3347936
## Belgium
                                         1.3207201 -0.43410258
                                                                 0.6652047
                                                                 0.4919409
## Bulgaria
                  -3.57721608
                              -3.5117365
                                         -4.0830333
                                                    0.93116104
## Czechoslovakia
                  1.58199341 -0.1427662 -0.3837129
                                                    -0.24742031
                                                                -0.5733394
## Denmark
                   2.65898620
                               2.8611177
                                          2.5831142 -2.07000038
                                                                 0.9748179
## East Germany
                   3.07703727
                               0.2892108
                                          1.7609008
                                                    -0.15976200
                                                                 -1.8776263
                                                    -3.20488435
## Finland
                   0.98892592
                               2.1318518
                                          1.6540365
                                                                 1.4531404
                                                     1.05679941
## France
                   1.38357845
                               3.1595434
                                         1.2018894
                                                                 2,5762733
                  -4.24503719 -1.1301364 -1.4109331
                                                     2.48665475
                                                                 1.9808523
## Greece
## Hungary
                  -0.23708684 -2.6884066
                                         -2.3119008
                                                     0.77848839
                                                                 1.0532456
                   3.32867621
## Ireland
                               3.7822855
                                          0.8762098
                                                    -1.69910669
## Italy
                  -2.20973495 -1.2028148
                                         -1.4532792
                                                    1.75379920
                                                                -0.4184105
## Netherlands
                   2.92133471
                               2.0188394
                                          0.2293759
                                                    -0.96211172
                                                                 0.2170547
## Norway
                   0.06928655
                               0.9081268
                                          2.6301684 -1.16234778 -0.5078814
                   1.26985863 -0.5205768
## Poland
                                          0.4099243
                                                     1.15911974
                                                                 1.1324699
                  -3.14878421 -3.3703562
                                          3.2664394
                                                     3.58992895
## Portugal
                                                                -2.7856233
## Romania
                  -2.92936249 -3.6215274
                                          2.9519579
                                                     0.40059015
                                                                -0.2694526
                  -2.22231411 -2.0776776
                                          1.4177987
                                                     2.65335072
## Spain
                                                                -2.1796632
## Sweden
                   1.69629854
                               2.2993811
                                          1.9459837
                                                    -2.22359646
                                                                -0.4368219
## Switzerland
                   1.03749592
                               1.8942025
                                         -0.5053111 -0.46744107
                                                                 0.8242122
## United Kingdom 1.42506807
                               3.8194452
                                          0.8887990 -1.14386138
                                                                 0.8015974
                  -1.07190408 -1.5337029
## USSR
                                         -0.4131728 -0.33630301
                                                                 0.9112671
## West Germany
                   2.98326743 1.9516984
                                         0.7813568 -0.70417689 -0.8995641
## Yugoslavia
                  -4.35421133 -5.3193385 -3.5035040 0.84317713 -0.1302773
# Play with FA utilities
fa.parallel(data1) # See factor recommendation
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc, smcs < 0 were set to .0
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc. smcs < 0 were set to .0
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc, smcs < 0 were set to .0
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was done
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
## In factor.scores, the correlation matrix is singular, an approximation is used
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was done
\#\# Warning in cor.smooth(r): The estimated weights for the factor scores are
## probably incorrect. Try a different factor score estimation method.
```

```
## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected. Examine the results carefully
## Warning in fa.stats(r=r, f=f, phi = phi, n.obs = n.obs, np.obs = np.obs, : ## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected. Examine the results carefully
## Warning in fa.stats(r=r, f=f, phi = phi, n.obs = n.obs, np.obs = np.obs, : ## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected. Examine the results carefully
## Warning in fa.stats(r=r, f=f, phi = phi, n.obs = n.obs, np.obs = np.obs, : ## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
## Warning in fa.stats(r=r, f=f, phi = phi, n.obs = n.obs, np.obs = np.obs, : ## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected. Examine the results carefully
## Warning in fa.stats(r=r, f=f, phi = phi, n.obs = n.obs, np.obs = np.obs, : ## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected. Examine the results carefully
```



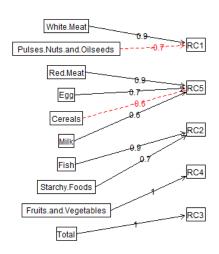
```
## Parallel analysis suggests that the number of factors = 2 and the number of components = 1
fa.plot(fit.pc) # See Correlations within Factors
```

Principal Component Analysis



fa.diagram(fit.pc) # Visualize the relationship

Components Analysis



```
vss(data1) # See Factor recommendations for a simple structure

## Warning in sqrt(e$values): NaNs produced

## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done

## In smc, smcs < 0 were set to .0

## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done

## Un smc, smcs < 0 were set to .0

## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done

## Un smc, smcs < 0 were set to .0

## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.

## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done

## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done

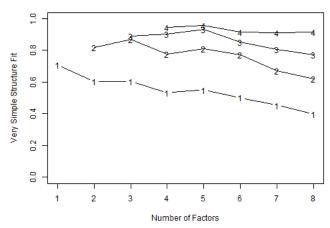
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done</pre>
```

```
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc, smcs < 0 were set to .0
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc. smcs < 0 were set to .0
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was done
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected. Examine the results carefully
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc, smcs < 0 were set to .0
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc, smcs < 0 were set to .0
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc. smcs < 0 were set to .0
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was done
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc, smcs < 0 were set to .0
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc, smcs < 0 were set to .0
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc, smcs < 0 were set to .0
## Warning in cor.smooth(r): Matrix was not positive definite. smoothing was done
\#\# In factor.stats, I could not find the RMSEA upper bound . Sorry about that
## Warning in fa.stats(r=r, f=f, phi = phi, n.obs = n.obs, np.obs = np.obs, : ## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected. Examine the results carefully
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc, smcs < 0 were set to .0
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc, smcs < 0 were set to .0
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc, smcs < 0 were set to .0
```

```
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was done
## In factor.stats, I could not find the RMSEA upper bound . Sorry about that
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected. Examine the results carefully
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc, smcs < 0 were set to .0
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc, smcs < 0 were set to .0
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc. smcs < 0 were set to .0
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was done
## Warning in fa.stats(r=r, f=f, phi = phi, n.obs = n.obs, np.obs = np.obs, : ## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected. Examine the results carefully
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc, smcs < 0 were set to .0
## Warning in cor.smooth(R): Matrix was not positive definite. smoothing was done
## In smc, smcs < 0 were set to .0
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc, smcs < 0 were set to .0
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was done
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
## Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
## ultra-Heywood case was detected. Examine the results carefully
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc, smcs < 0 were set to .0
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc, smcs < 0 were set to .0
## Warning in cor.smooth(R): Matrix was not positive definite, smoothing was done
## In smc, smcs < 0 were set to .0
## Warning in cor.smooth(r): Matrix was not positive definite, smoothing was done
## Warning in fa.stats(r = r, f = f, phi = phi, n.obs = n.obs, np.obs = np.obs, :
## The estimated weights for the factor scores are probably incorrect. Try a
## different factor score estimation method.
```

Warning in fac(r = r, nfactors = nfactors, n.obs = n.obs, rotate = rotate, : An
ultra-Heywood case was detected. Examine the results carefully

Very Simple Structure



```
## Very Simple Structure
## Call: vss(x = data1)
## VSS complexity 1 achieves a maximimum of 0.71 with 1 ## VSS complexity 2 achieves a maximimum of 0.87 with 3
                                                                factors
                                                                 factors
## The Velicer MAP achieves a minimum of 0.09 with 1 factors
## BIC achieves a minimum of NA with 5 factors
## Sample Size adjusted BIC achieves a minimum of NA with 5 factors
##
## Statistics by number of factors
##
    vss1 vss2
                 map dof chisq
                                     prob sqresid fit RMSEA BIC SABIC complex
## 1 0.71 0.00 0.087
                        35
                             453
                                 3.6e-74
                                              6.94 0.71
                                                         0.69 341
                                                                               1.0
## 2 0.60 0.82 0.106
                        26
                             419 2.0e-72
                                              4.25 0.82
                                                          0.78 335
                                                                       416
                                                                               1.3
## 3 0.60 0.87 0.145
                        18
                              390 1.0e-71
                                              2.60 0.89
                                                          0.91 332
                                                                       388
                                                                               1.6
## 4 0.53 0.78 0.173
                        11
                              356 1.1e-69
                                              1.28 0.95
                                                          1.12 321
                                                                       355
                                                                               1.7
## 5 0.55 0.81 0.209
                             323 1.4e-67
                                              0.70 0.97
                                                          1.59 306
                                                                       322
                                                                               1.8
                         5
## 6 0.50 0.77 0.314
                         0
                             294
                                              0.61 0.97
                                                            NA
                                                                 NA
                                                                       NA
                                       NA
                                                                               2.1
## 7 0.45 0.67 0.477
                                              0.40 0.98
                              277
                                                            NA
                                                                               2.2
## 8 0.40 0.62 1.000
                             250
                                        NΑ
                                              0.12 0.99
                                                            NA
                                                                 NA
                                                                        NA
                                                                               2.3
##
      eChisq
                SRMR eCRMS eBIC
## 1 56.2527 0.1581 0.179
                             -56
## 2 27.4318 0.1104 0.145
                              -56
## 3 12.8827 0.0757 0.120
                              -45
      4.6981 0.0457 0.092
                              -31
## 5
      0.9579 0.0206 0.062
                              -15
## 6
      0.3033 0.0116
                         NΑ
                              NA
## 7
      0.0199 0.0030
                         NΑ
                               NΑ
## 8
      0.0031 0.0012
                         NA
                               NA
```

Conclusion: The goal of FCA is to identify groups items when considered together and explains as much of the observed co-variance as possible.
I can see that out of 5 factors taken into consideration we have reduced the factors to two which contains most of the information in the dataset.
When I consider 5 factors then I will be able to