# Tutorial-4

Ans1 $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

here, $a = 3$, $b = 2$

$f(n) = n^2$

so, $n^{\log_b a} = n^{\log_2 3}$

since, $n^{\log_2 3} < n^2$

so, according to master's theorum

$T(n) = O(n^2)$

Ans2 $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

here, $a = 4$, $b = 2$ & $f(n) = n^2$

so, $n^{\log_b a} = n^{\log_2 4} = n^{\log (2)^2}$

$= n^{2\log_2 2}$

$= n^2$

since, $n^{\log_b a} = f(n)$

According to master's Theorm,

$T(n) = O(n^2 \log n)$

Ans3 $T(n) = T\left(\frac{n}{2}\right) + 2^n$

here, $a = 1$, $b = 2$ and $f(n) = 2^n$

So, $n^{\log_b a} = n^{\log_2 1} = n^{\log_2 2^0}$

$= n^0 = 1$

since, $1 < f(n)$

Acc. to master's Theorum,

$T(n) = O(2^n)$

Ans4 $T(n) = 2^n T\left(\frac{n}{2}\right) + n^2$

master's theorim is not applicable since 'a' is a function.

Ans5 $T(n) = 16T\left(\frac{n}{4}\right) + n$

here, $a = 16$ $b = 4$ and $f(n) = n$

so,

$n^{\log_b a} = n^{\log_4 16} = n^{\log_4 4^2} = n^{2\log_4 4}$

$= n^2$

since, $n^2 > n$

Therefore, acc to master's theorim

$T(n) = O(n^2)$

Ans6 $T(n) = 2T\left(\frac{n}{2}\right) + n\log n$

here, $a = 2$ $b = 2$ $f(n) = n\log n$

So, $n^{\log_b a} = n^{\log_2 2} = n$

since, $n^{\log_b a} < f(n)$

Acc to master's theorim

$T(n) = O(n\log n)$

Ans7 $T(n) = 2T\left(\frac{n}{2}\right) + n/\log n$

$a = 2$ $b = 2$ $f(n) = n/\log n$

So, $n^{\log_b a} = n^{\log_2 2} = n$

since, $n^{\log_b a} \geq f(n)$

acc. to master's theorim

$T(n) = O(n)$

Ans 8 $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a = 2 \quad b = 4 \quad f(n) = n^{0.51}$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

since $n^{\log_b a} < f(n)$

acc to master's theorem

$T(n) = O(n^{0.51})$

Ans 9 $T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$

not applicable master's theorem since $a<1$

Ans 10

$T(n) = 16T\left(\frac{n}{4}\right) + n!$

here, $a=16$, $b=4$ & $f(n) = n!$

So, $n^{\log_b a} = n^{\log_4 16} = n^{\log_4 (4)^2} = n^2$

since, $n^{\log_b a} < n!$

acc. to master's theorem,

$T(n) = O(n!)$

Ans 12 $T(n) = \text{sqrt}(n)\,T\left(\frac{n}{2}\right) + \log n$

since $a \neq$ constant

~~acc~~. master's theorem not applicable.

Ans 11 $T(n) = 4T\left(\frac{n}{2}\right) + \log n$

here, $a=4$, $b=2$ & $f(n) = \log n$

so, $n^{\log_b a} = n^{\log_2 4} = n^2$

since, $n^{\log_b a} > f(n)$

master's method

$T(n) = O(n^2)$

**Ans 13** $T(n) = 3T\left(\frac{n}{2}\right) + n$

here $a = 3$ $b = 2$ & $f(n) = n$

$n^{\log_b a} = n^{1.58} = n^{\log_2 3}$

$n^{\log_b a} > f(n)$

$T(n) = O(n^{1.58})$

**Ans 14** $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$

$a = 3$ $b = 3$ $f(n) = \sqrt{n}$

$n^{\log_3 3} = n$

$> f n$

$T(n) = O(n)$

**Ans 15** $T(n) = 4T\left(\frac{n}{2}\right) + cn$

$a = 4$ $b = 2$ & $f(n) = n$

$n^{\log_2 4} = n^2$

$> f(n)$

$T(n) = O(n^2)$

**Ans 16** $T(n) = 3T\left(\frac{n}{4}\right) + n \log n$

$a = 3$ $b = 4$ $f(n) = n \log n$

$n^{\log_4 3} = n^{0.187}$

$< f(n)$

$T(n) = O(n \log n)$

**Ans 17** $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

$a = 3$ $b = 3$ $f(n) = n/2$

$n^{\log_3 6} = n^{1.63}$

$< n^2 \log n$

$T(n) = O(n^2 \log n)$

Ans 18 $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$

$a=6 \quad b=3 \quad f(n) = n^2 \log n$

$n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

$< n^2 \log n$

$T(n) = O(n^2 \log n)$

Ans 19

$T(n) = 4T\left(\frac{n}{2}\right) + n/\log n$

$a=4, b=2, f(n) = n/\log n$

$n^{\log_2 4} = n^2$

$> f(n)$

$T(n) = O(n^2)$

Ans 20 $T(n) = 64\,T\left(\frac{n}{8}\right) - n^2 \log n$

here, $f(n)$ is not an increasing function.
Therefore, master's theorem is not applicable.

Ans 21 $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$a=7 \quad b=3 \quad f(n) = n^2$

$n^{\log_3 7} = n^{1.7}$

$n^{\log_b a} < f(n)$

$\therefore$ acc. to master's method $T(n) = O(n^2)$

Ans 22 $T(n) = T(n/2) + n(2 - \cos n)$

here, master's theorem is not applicable due to violation of regularity condition.