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CST-SPL-1
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Data & Analysis of Algorithm

1. Asymphotic notations are used to find the complexity of an algorithm when input is very large

* **Big O(O):-** $f(n) = O g(n)$
  iff $f(n) \le c g(n)$
  $\forall\ n \ge n_0$
  for some ~~for~~ constant $c > 0$
  $g(n)$ is "tight upper bound" of $f(n)$

* **Big omega(Ω):-**
  $f(n) = \Omega(g(n))$
  iff $f(n) \ge c g(n)$
  $\forall\ n \ge n_0$
  for some constant $c > 0$
  $g(n)$ is "tight lower bound" of $f(n)$

* **Theta(θ):-**
  $f(n) = \theta(g(n))$
  iff $c_1 g(n) \le f(n) \le c_2 g(n)$
  $\forall\ n \ge \max(n_1, n_2)$
  for some constant $c_1 > 0$ & $c_2 > 0$
  $g(n)$ is both "tight upper bound &

lower bound of $f(n)$"

Que 2 for $(i=1$ to $n)$, $\{ P = P \times 2i \}$

~~1+2,4+8~~ $1, 2, 4, 8 \ldots n$

let k^{th} term "

$n = 1 \cdot (2^{k-1})$

Taking log on both sides

$\log n = k-1 \log_2 2$

$k = 1 + \log_2 n$

$O(1 + \log n)$

Ans $O(\log n)$

Ques 3 $T(n) = 3T(n-1)$ — (1)

$n = n-1$ in (1)

$T(n-1) = 3T(n-2)$ — (2)

put (2) in (1)

$T(n) = 9T(n-2)$

$n = n-2$ in (1)

$T(n-2) = 3T(n-3)$ — (3)

$T(n) = 27\,T(n-3)$

$T(n) = 3^k\,T(n-k)$

$n - k = 0$

$n = k$

$T(n) = 3^n\,T(n-n)$

$= 3^n T(0)$

$= 3^n$

Ans $O(3^n)$

Ques 4 $T(n) = 2T(n-1)$ — (1)

$n = n-1$ in eq (1)

$T(n-1) = 2T(n-2)$ — (2)

$T(n) = \cancel{n-2}\ 4T(n-2)$ — (3)

$n = n-2$ in (1)

$T(n-2) = 2T(n-3)$ — (4)

$T(n) = 8T(n-3)$

$T(n) = 2^k T(n-k)$

$n-k=0$

$k=n$

$T(n) = 2^k T(n-n)$

$= 2^n T(0)$

$= 2^n$

Ans $O(2^n)$

Ques 6

```
Void function (int n)
{
  int i, count =0;
  for (i=1; i* i< = n; i++)
     count ++;
}
```

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$O(1+\sqrt{n}+\sqrt{n}+\sqrt{n})$

$O(1+3\sqrt{n})$

$O(3\sqrt{n})$

$O(\sqrt{n})$

Ans $O(n^{1/2})$

Ques7 void function(int n)

```
{
  int i, j, k, count=0;
  for (i=n/2; i<=n; i++)
   for (j=1; j<=n; j=j*2)
    for (k=1; k<=n; k=k*2)
      count++;
}
```

i	j	k
n/2	1	1
		2
		4
$\downarrow$	$\downarrow$	⋮
n/2	log n	n
		log n

$O(n/2 \times \log n \times \log n)$

$O(n(\log n)^2)$

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Ques 8

```
function (int n)
{
  if (n==1)
    return;
  for (i=1 to n)
  {
     for (j=1 to n)
     {
        printf("K");
     }
  }
  function (n-3);
}
```

n	i	j
n	n	n
n-3	n	n
n-6	n	n

$1+4+7+ \ldots n$

$n = 1+3(K-1)$

$= 3K-2$

$K = n+2/3$

no. of terms

$\frac{n+2}{6}\left[2+\left[\frac{n-1}{3}\right]\times 3\right]$

$\left[\frac{n+2}{6}\left[n+1\right]\right]\times n^2$

$O\left[\frac{(n^2+3n+2)}{6}\times n^2\right]$

Ans $O[n^4]$

Ques 9

```
void function (int n)
{
   for (i=1 to n)
   {
      for (j=1 ; j<=n; j=j+1)
         printf("*"); n^2
   }
}
```

$O(n+n^2+n^2+n^2)$

$O(3n^2+n)$

$O(n^2)$ Ans

Ques 10 As given n^k & c^n

relation b/w n^k & c^n is

$n^k = O(c^n)$

$n^k \le a(c^n)$

$\forall\ n \ge n_0$ & constant, $a>0$

for $n_0 = 1$

$c = 2$

$\rightarrow 1^k \le a^{2\cdot 1}$

$\rightarrow n_0 = 1$ & $c = 2$