

Tutorial No. - 04.

Ques-1. $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

Here, $a=3$, $b=2$

$$f(n) = n^2$$

$$\text{So, } n^{\log_b a} = n^{\log_2 3}$$

$$\text{Since, } n^{\log_2 3} < n^2$$

So, according to Master's Theorem -

$$T(n) = O(n^2).$$

Ques-2. $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

Here, $a=4$, $b=2$ and $f(n) = n^2$

$$\text{So, } n^{\log_b a} = n^{\log_2 4} = n^{\log_2 (2^2)}$$

$$= n^{2 \log_2 2}$$

$$= n^2$$

$$\text{Since, } n^{\log_b a} = f(n)$$

According to Master's Theorem,

$$T(n) = O(n^2 \log n)$$

Ques-3. $T(n) = T\left(\frac{n}{2}\right) + 2^n$

Here, $a=1$, $b=2$ and $f(n) = 2^n$

$$\text{So, } n^{\log_b a} = n^{\log_2 1} = n^{\log_2 2^0}$$

$$\Rightarrow n^0 = 1$$

$$\text{Since, } 1 < f(n)$$

According to Master's Theorem,

$$T(n) = O(2^n)$$

Ques-4.

$$T(n) = 2^n T\left(\frac{n}{2}\right) + n^2$$

Master's Theorem is not applicable since 'a' is a function

Ques-5.

$$T(n) = 16T\left(\frac{n}{4}\right) + n$$

Here, $a=16$, $b=4$ and $f(n)=n$

$$\text{So, } n^{\log_b a} = n^{\log_4 16} = n^{\log_4 4^2} = n^{2 \log_4 4} = n^2$$

Since, $n^2 > n$

Therefore, according to Master's theorem,

$$T(n) = O(n^2)$$

Ques-6.

$$T(n) = 2T\left(\frac{n}{2}\right) + n \log n$$

Here, $a=2$, $b=2$ and $f(n) = n \log n$.

$$\text{So, } n^{\log_b a} = n^{\log_2 2} = n$$

Since, $n^{\log_b a} < f(n)$

According to Master's Theorem,

$$T(n) = O(n \log n).$$

Ques-7.

$$T(n) = 2T\left(\frac{n}{2}\right) + n / \log n$$

Here, $a=2$, $b=2$ and $f(n) = n / \log n$.

$$\text{So, } n^{\log_b a} = n^{\log_2 2} = n$$

Since, $n^{\log_b a} > f(n)$
According to Master's Theorem,
 $T(n) = O(n)$.

Ques-8. $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

Here, $a=2$, $b=4$ and $f(n) = n^{0.51}$

So, $n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

Since, $n^{\log_b a} < f(n)$

According to Master's Theorem,
 $T(n) = O(n^{0.51})$

Ques-9. $T(n) = 0.5T\left(\frac{n}{2}\right) + \frac{1}{n}$

Master's Theorem is not applicable since $a < 1$.

Ques-10. $T(n) = 16T\left(\frac{n}{4}\right) + n!$

Here, $a=16$, $b=4$ and $f(n) = n!$

So, $n^{\log_b a} = n^{\log_4 16} = n^{\log_4 (4^2)} = n^2$

Since, $n^{\log_b a} < n!$

\therefore According to Master's Theorem,

$$T(n) = O(n!)$$

Ques-11. $T(n) = 4T\left(\frac{n}{2}\right) + \log n$

Here, $a=4$, $b=2$ and $f(n) = \log n$.

So, $n^{\log_b a} = n^{\log_2 4} = n^2$

Since, $n^{\log_b a} > f(n)$

According to Master's Theorem,

$T(n) = O(n^2)$

Ques-12. $T(n) = \sqrt{n} T\left(\frac{n}{2}\right) + \log n$

Since, $a \neq \text{constant}$.

\therefore Master's Theorem is not applicable.

Ques-13. $T(n) = 3T\left(\frac{n}{2}\right) + n$

Here, $a=3$, $b=2$ and $f(n) = n$

So, $n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

Since, $n^{\log_b a} > f(n)$

\therefore According to Master's Theorem,

$T(n) = O(n^{1.58})$

Ques-14. $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$

Here, $a=3$, $b=3$ and $f(n) = \sqrt{n}$

So, $n^{\log_b a} = n^{\log_3 3} = n$

Since, $n^{\log_b a} > f(n)$

\therefore According to Master's Theorem,
 $T(n) = O(n)$.

Ques-15. $T(n) = 4T\left(\frac{n}{2}\right) + cn$

Here, $a=4$, $b=2$ and $f(n) = n$.

So, $n^{\log_b a} = n^{\log_2 4}$
 $= n^2$

Since, $n^{\log_b a} > f(n)$

\therefore According to Master's Theorem,
 $T(n) = O(n^2)$

Ques-16. $T(n) = 3T\left(\frac{n}{4}\right) + n \log n$

Here, $a=3$, $b=4$ and $f(n) = n \log n$.

So, $n^{\log_b a} = n^{\log_4 3} = n^{0.187}$

Since, $n^{\log_b a} < f(n)$

\therefore According to Master's Theorem,
 $T(n) = O(n \log n)$

Ques-17. $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

Here, $a=3$, $b=3$ and $f(n) = \frac{n}{2}$

So, $n^{\log_b a} = n^{\log_3 3} = n$

Since, $n^{\log_b a} > f(n)$

\therefore According to Master's Theorem,
 $T(n) = O(n)$

Ques-18. $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$

Here, $a=6$, $b=3$ and $f(n) = n^2 \log n$

So, $n^{\log_b a} = n^{\log_3 6} = n^{1.63}$

Since, $n^{\log_b a} < n^2 \log n$

\therefore According to Master's Theorem,
 $T(n) = O(n^2 \log n)$

Ques-19. $T(n) = 4T\left(\frac{n}{2}\right) + n/\log n$

Here, $a=4$, $b=2$ and $f(n) = n/\log n$

So, $n^{\log_b a} = n^{\log_2 4} = n^2$

Since, $n^{\log_b a} > f(n)$

\therefore According to Master's Theorem,
 $T(n) = O(n^2)$

Ques-20. $T(n) = 64T\left(\frac{n}{8}\right) - n^2 \log n$

Here, $f(n)$ is not an increasing function. Therefore,
Master's Theorem is not applicable.

Ques-21. $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

Here, $a=7$, $b=3$ and $f(n) = n^2$

So, $n^{\log_b a} = n^{\log_3 7} = n^{1.7}$

Since, $n^{\log_b a} < f(n)$

\therefore According to Master's Theorem,
 $T(n) = O(n^2)$

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Ques-22. $T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos n)$

Here, Master's Theorem is not applicable due to violation of regularity condition.