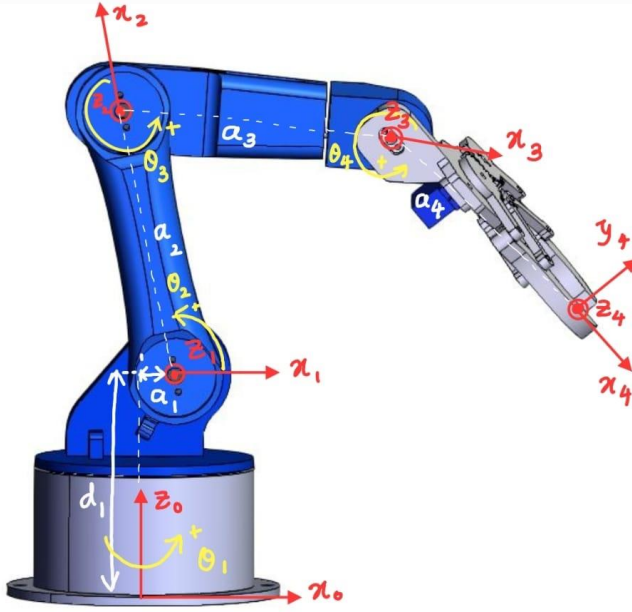


# Robotics Mini Project: Kinematic Analysis of a Robot Arm

Team Members:

W.M.J.C. Kumara	190328V
B.S.V.W. Munasinghe	190397E
R.M.C.D.H. Ranasinghe	190498N

## 1. Frame Assignment and DH Table



Link	$a_i$	$\alpha_i$	$d_i$	$\theta_i$
1	1.3cm	$90^\circ$	9.5cm	$\theta_1^*$
2	12cm	0	0	$\theta_2^*$
3	12cm	0	0	$\theta_3^*$
4	12.5cm	0	0	$\theta_4^*$

## 2. Forward Kinematics

- Symbols have the following meanings.

$$C_i = \cos(\theta_i), C_{ij} = \cos(\theta_i + \theta_j), C_{ijk} = \cos(\theta_i + \theta_j + \theta_k)$$

$$S_i = \sin(\theta_i), S_{ij} = \sin(\theta_i + \theta_j), S_{ijk} = \sin(\theta_i + \theta_j + \theta_k)$$

- Transformation Matrices

$$A_1 = \begin{bmatrix} C_1 & 0 & S_1 & 1.3C_1 \\ S_1 & 0 & -C_1 & 1.3S_1 \\ 0 & 1 & 0 & 9.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & 12C_2 \\ S_2 & C_2 & 0 & 12S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & 12C_3 \\ S_3 & C_3 & 0 & 12S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_4 = \begin{bmatrix} C_4 & -S_4 & 0 & 12.5C_4 \\ S_4 & C_4 & 0 & 12.5S_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^0 = A_1 A_2 A_3 A_4 = \begin{bmatrix} C_1 C_{234} & -C_1 S_{234} & S_1 & 12C_1(0.108333 + C_2 + C_{23} + 1.04167C_{234}) \\ S_1 C_{234} & -S_1 S_{234} & -C_1 & 12S_1(0.108333 + C_2 + C_{23} + 1.04167C_{234}) \\ S_{234} & C_{234} & 0 & 9.5 + 12S_2 + 12S_{23} + 12.5S_{234} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### 3. Inverse Kinematics

Take the desired end position as  $p_4^0 = [x \ y \ z]^T$ .

Take the desired end effector orientation as  $\theta_{orientation}$  and we get,

$$\theta_2 + \theta_3 + \theta_4 = \theta_{orientation}$$

Using the geometric relationship, we can take,

$$\theta_1 = \text{atan2}(y, x) \quad \text{or} \quad \theta_1 = \text{atan2}(y, x) - \pi$$

Using the transformation matrix  $T_4^0$  and desired position  $p_4^0$ ,

$$C_2 + C_{23} = \frac{x}{12C_1} - 0.108333 - 1.04167C_{234} = A$$

or

$$C_2 + C_{23} = \frac{y}{12S_1} - 0.108333 - 1.04167C_{234} = A$$

and

$$S_2 + S_{23} = \frac{y - 12.5S_{234} - 9.5}{12} = B$$

Solving the above two equations we get,

$$C_3 = \frac{A^2 + B^2 - 2}{2} = l \quad \rightarrow \quad S_3 = \pm\sqrt{1 - l^2}$$

When  $S_3 = \sqrt{1 - l^2}$ ,

$$\theta_3 = \text{atan2}(\sqrt{1 - l^2}, l) \quad , \quad \theta_2 = \text{atan2}\left(\frac{B(1 + l) - A\sqrt{1 - l^2}}{(1 + l)^2 + (1 - l^2)}, \frac{A(1 + l) + B\sqrt{1 - l^2}}{(1 + l)^2 + (1 - l^2)}\right)$$

When  $S_3 = -\sqrt{1 - l^2}$ ,

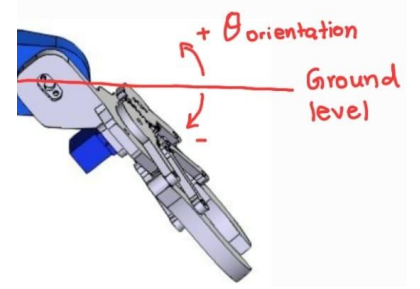
$$\theta_3 = \text{atan2}(-\sqrt{1 - l^2}, l) \quad , \quad \theta_2 = \text{atan2}\left(\frac{B(1 + l) + A\sqrt{1 - l^2}}{(1 + l)^2 + (1 - l^2)}, \frac{A(1 + l) - B\sqrt{1 - l^2}}{(1 + l)^2 + (1 - l^2)}\right)$$

Finally, we solve for  $\theta_4$ , Knowing that  $\theta_{orientation}$  is the addition of all  $\theta_2, \theta_3, \theta_4$ .

$$\theta_4 = \theta_{orientation} - \theta_2 - \theta_3$$

- The solutions are filtered to obtain only one solution by considering the range of the physical joints of the arm. (Each joint is limited to rotate only 180 degrees)
- The limits for the angles defined by our team are as follows. (Convention is the same as used in the frame assignment)

Angle	Minimum	Maximum
$\theta_1^*$	$0^\circ$	$180^\circ$
$\theta_2^*$	$0^\circ$	$180^\circ$
$\theta_3^*$	$-135^\circ$	$0^\circ$
$\theta_4^*$	$-90^\circ$	$90^\circ$



#### 4. Manipulator Jacobian

$$\begin{bmatrix} S_1(-1.3 - 12C_2 - 12C_{23} - 12.5C_{234}) & C_1(-12S_2 - 12S_{23} - 12.5S_{234}) & C_1(-12S_{23} - 12.5S_{234}) & -12.5C_1S_{234}) \\ C_1(1.3 + 12C_2 + 12C_{23} + 12.5C_{234}) & S_1(-12S_2 - 12S_{23} - 12.5S_{234}) & S_1(-12S_{23} - 12.5S_{234}) & -12.5S_1S_{234} \\ 0 & 12C_2 + 12C_{23} + 12.5C_{234} & 12C_{23} + 12.5C_{234} & 12.5C_{234} \\ 0 & S_1 & S_1 & S_1 \\ 0 & -C_1 & -C_1 & -C_1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

#### 5. Controlling the Arm