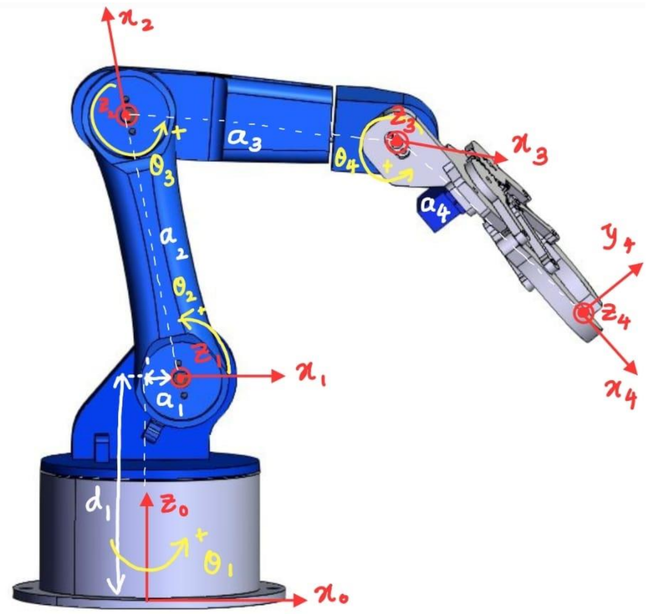
Robotics Mini Project: Kinematic Analysis of a Robot Arm

Team Members:

W.M.J.C. Kumara	190328V
B.S.V.W. Munasinghe	190397E
R.M.C.D.H. Ranasinghe	190498N

1. Frame Assignment and DH Table



Link	a_i	α_i	d_i	θ_i
1	$1.3cm$	90°	$9.5cm$	θ_1^*
2	$12cm$	0	0	θ_2^*
3	$12cm$	0	0	θ_3^*
4	$12.5cm$	0	0	θ_4^*

2. Forward Kinematics

- Symbols have the following meanings.

$$C_i = \cos(\theta_i) \text{ , } C_{ij} = \cos(\theta_i + \theta_j) \text{ , } C_{ijk} = \cos(\theta_i + \theta_j + \theta_k)$$

$$S_i = \sin(\theta_i) \text{ , } S_{ij} = \sin(\theta_i + \theta_j) \text{ , } S_{ijk} = \sin(\theta_i + \theta_j + \theta_k)$$

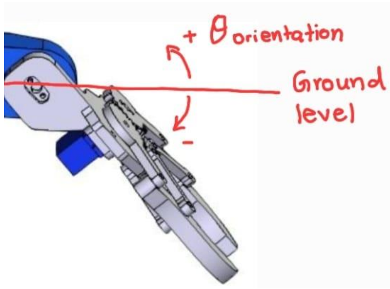
- Transformation Matrices

$$A_1 = \begin{bmatrix} C_1 & 0 & S_1 & 1.3C_1 \\ S_1 & 0 & -C_1 & 1.3S_1 \\ 0 & 1 & 0 & 9.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & 12C_2 \\ S_2 & C_2 & 0 & 12S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & 12C_3 \\ S_3 & C_3 & 0 & 12S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_4 = \begin{bmatrix} C_4 & -S_4 & 0 & 12.5C_4 \\ S_4 & C_4 & 0 & 12.5S_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^0 = A_1A_2A_3A_4 = \begin{bmatrix} C_1C_{234} & -C_1S_{234} & S_1 & 12C_1(0.108333 + C_2 + C_{23} + 1.04167C_{234}) \\ S_1C_{234} & -S_1S_{234} & -C_1 & 12S_1(0.108333 + C_2 + C_{23} + 1.04167C_{234}) \\ S_{234} & C_{234} & 0 & 9.5 + 12S_2 + 12S_{23} + 12.5S_{234} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Inverse Kinematics



Take the desired end position as $\boldsymbol{p_4^0} = [\boldsymbol{x} \quad \boldsymbol{y} \quad \boldsymbol{z}]^{\boldsymbol{T}}$.

Take the desired end effector orientation as $\boldsymbol{\theta_{orientation}}$ and we get,

$$\boldsymbol{\theta_2} + \boldsymbol{\theta_3} + \boldsymbol{\theta_4} = \boldsymbol{\theta_{orientation}}$$

Using the geometric relationship, we can take,

$$\boldsymbol{\theta_1 = atan2(y, x) \quad or \quad \theta_1 = atan2(y, x) - \pi}$$

Using the transformation matrix T_4^0 and desired position p_4^0,

$$C_2 + C_{23} = \frac{x}{12C_1} - 0.108333 - 1.04167C_{234} = \boldsymbol{A}$$

or

$$C_2 + C_{23} = \frac{y}{12S_1} - 0.108333 - 1.04167C_{234} = \boldsymbol{A}$$

and

$$S_2 + S_{23} = \frac{z - 12.5S_{234} - 9.5}{12} = \boldsymbol{B}$$

Solving the above two equations we get,

$$C_3 = \frac{A^2 + B^2 - 2}{2} = \boldsymbol{l} \quad \rightarrow \quad S_3 = \pm\sqrt{1 - l^2}$$

When $S_3 = \sqrt{1 - l^2}$,

$$\boldsymbol{\theta_3 = atan2\left(\sqrt{1 - l^2}, l\right) \; , \; \theta_2 = atan2\left(\frac{B(1 + l) - A\sqrt{1 - l^2}}{(1 + l)^2 + (1 - l^2)}, \frac{A(1 + l) + B\sqrt{1 - l^2}}{(1 + l)^2 + (1 - l^2)}\right)}$$

When $S_3 = -\sqrt{1 - l^2}$,

$$\boldsymbol{\theta_3 = atan2\left(-\sqrt{1 - l^2}, l\right) \; , \; \theta_2 = atan2\left(\frac{B(1 + l) + A\sqrt{1 - l^2}}{(1 + l)^2 + (1 - l^2)}, \frac{A(1 + l) - B\sqrt{1 - l^2}}{(1 + l)^2 + (1 - l^2)}\right)}$$

Finally, we solve for θ_4, Knowing that $\theta_{orientation}$ is the addition of all $\theta_2, \theta_3, \theta_4$.

$$\boldsymbol{\theta_4 = \theta_{orientation} - \theta_2 - \theta_3}$$

- The solutions are filtered to obtain only one solution by considering the range of the physical joints of the arm. (Each joint is limited to rotate only 180 degrees)
- The limits for the angles defined by our team are as follows. (Convention is the same as used in the frame assignment)

Angle	Minimum	Maximum
θ_1^*	0°	180°
θ_2^*	0°	180°
θ_3^*	-135°	0°
θ_4^*	-90°	90°

4. Manipulator Jacobian

$$\begin{bmatrix} S_1(-1.3 - 12C_2 - 12C_{23} - 12.5C_{234}) & C_1(-12S_2 - 12S_{23} - 12.5S_{234}) & C_1(-12S_{23} - 12.5S_{234}) & -12.5C_1S_{234}) \\ C_1(1.3 + 12C_2 + 12C_{23} + 12.5C_{234}) & S_1(-12S_2 - 12S_{23} - 12.5S_{234}) & S_1(-12S_{23} - 12.5S_{234}) & -12.5S_1S_{234} \\ 0 & 12C_2 + 12C_{23} + 12.5C_{234} & 12C_{23} + 12.5C_{234} & 12.5C_{234} \\ 0 & S_1 & S_1 & S_1 \\ 0 & -C_1 & -C_1 & -C_1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

5. Controlling the Arm