Robotics Mini Project: Kinematic Analysis of a Robot Arm

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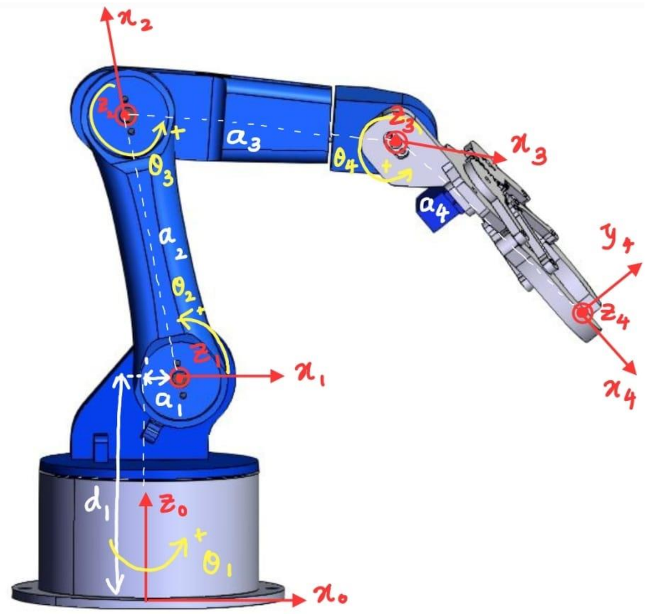
1. Frame Assignment and DH Table



Link	a_i	α_i	d_i	θ_i
1	$1.3cm$	90°	$9.5cm$	θ_1^*
2	$12cm$	0	0	θ_2^*
3	$12cm$	0	0	θ_3^*
4	$12.5cm$	0	0	θ_4^*

2. Forward Kinematics

- Symbols have the following meanings.

$$C_i = \cos(\theta_i)\ , C_{ij} = \cos(\theta_i + \theta_j)\ , \ C_{ijk} = \cos(\theta_i + \theta_j + \theta_k)$$

$$S_i = \sin(\theta_i)\ , S_{ij} = \sin(\theta_i + \theta_j)\ , \ S_{ijk} = \sin(\theta_i + \theta_j + \theta_k)$$

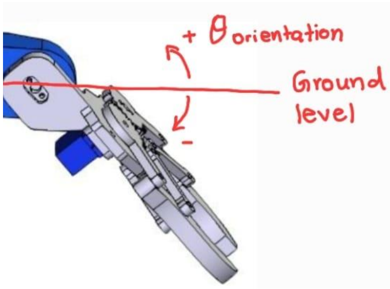
- Transformation Matrices

$$A_1 = \begin{bmatrix} C_1 & 0 & S_1 & 1.3C_1 \\ S_1 & 0 & -C_1 & 1.3S_1 \\ 0 & 1 & 0 & 9.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & 12C_2 \\ S_2 & C_2 & 0 & 12S_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & 12C_3 \\ S_3 & C_3 & 0 & 12S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_4 = \begin{bmatrix} C_4 & -S_4 & 0 & 12.5C_4 \\ S_4 & C_4 & 0 & 12.5S_4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^0 = A_1A_2A_3A_4 = \begin{bmatrix} C_1C_{234} & -C_1S_{234} & S_1 & 12C_1(0.108333 + C_2 + C_{23} + 1.04167C_{234}) \\ S_1C_{234} & -S_1S_{234} & -C_1 & 12S_1(0.108333 + C_2 + C_{23} + 1.04167C_{234}) \\ S_{234} & C_{234} & 0 & 9.5 + 12S_2 + 12S_{23} + 12.5S_{234} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Inverse Kinematics



Take the desired end position as $\boldsymbol{p_4^0} = [\boldsymbol{x} \quad \boldsymbol{y} \quad \boldsymbol{z}]^{\boldsymbol{T}}$.

Take the desired end effector orientation as $\boldsymbol{\theta_{orientation}}$ and we get,

$$\boldsymbol{\theta_2 + \theta_3 + \theta_4 = \theta_{orientation}}$$

Using the geometric relationship, we can take,

$$\boldsymbol{\theta_1 = atan2(y, x) \quad or \quad \theta_1 = atan2(y, x) - \pi}$$

Using the transformation matrix T_4^0 and desired position p_4^0,

$$C_2 + C_{23} = \frac{x}{12C_1} - 0.108333 - 1.04167C_{234} = \boldsymbol{A}$$

or

$$C_2 + C_{23} = \frac{y}{12S_1} - 0.108333 - 1.04167C_{234} = \boldsymbol{A}$$

and

$$S_2 + S_{23} = \frac{z - 12.5S_{234} - 9.5}{12} = \boldsymbol{B}$$

Solving the above two equations we get,

$$C_3 = \frac{A^2 + B^2 - 2}{2} = \boldsymbol{l} \quad \rightarrow \quad S_3 = \pm\sqrt{1 - l^2}$$

When $S_3 = \sqrt{1 - l^2}$,

$$\boldsymbol{\theta_3 = atan2\left(\sqrt{1 - l^2}, l\right) \ , \ \theta_2 = atan2\left(\frac{B(1 + l) - A\sqrt{1 - l^2}}{(1 + l)^2 + (1 - l^2)}, \frac{A(1 + l) + B\sqrt{1 - l^2}}{(1 + l)^2 + (1 - l^2)}\right)}$$

When $S_3 = -\sqrt{1 - l^2}$,

$$\boldsymbol{\theta_3 = atan2\left(-\sqrt{1 - l^2}, l\right) \ , \ \theta_2 = atan2\left(\frac{B(1 + l) + A\sqrt{1 - l^2}}{(1 + l)^2 + (1 - l^2)}, \frac{A(1 + l) - B\sqrt{1 - l^2}}{(1 + l)^2 + (1 - l^2)}\right)}$$

Finally, we solve for θ_4, Knowing that $\theta_{orientation}$ is the addition of all $\theta_2, \theta_3, \theta_4$.

$$\boldsymbol{\theta_4 = \theta_{orientation} - \theta_2 - \theta_3}$$

- The solutions are filtered to obtain only one solution by considering the range of the physical joints of the arm. (Each joint is limited to rotate only 180 degrees)
- The limits for the angles defined by our team are as follows. (Convention is the same as used in the frame assignment)

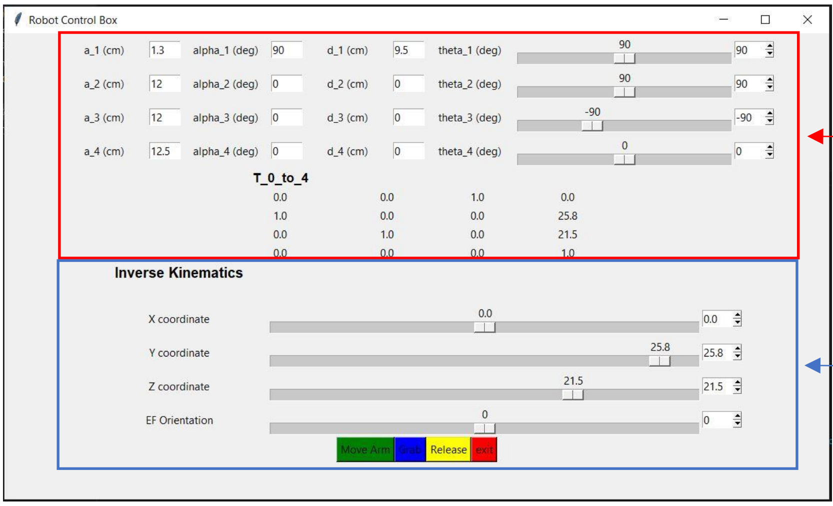
Angle	Minimum	Maximum
θ_1^*	0°	180°
θ_2^*	0°	180°
θ_3^*	-135°	0°
θ_4^*	-90°	90°

4. Manipulator Jacobian

$$
\begin{bmatrix}
S_1(-1.3 - 12C_2 - 12C_{23} - 12.5C_{234}) & C_1(-12S_2 - 12S_{23} - 12.5S_{234}) & C_1(-12S_{23} - 12.5S_{234}) & -12.5C_1S_{234}) \\
C_1(1.3 + 12C_2 + 12C_{23} + 12.5C_{234}) & S_1(-12S_2 - 12S_{23} - 12.5S_{234}) & S_1(-12S_{23} - 12.5S_{234}) & -12.5S_1S_{234} \\
0 & 12C_2 + 12C_{23} + 12.5C_{234} & 12C_{23} + 12.5C_{234} & 12.5C_{234} \\
0 & S_1 & S_1 & S_1 \\
0 & -C_1 & -C_1 & -C_1 \\
1 & 0 & 0 & 0
\end{bmatrix}
$$

5. Controlling the Arm

- We have designed an interface to control the robot arm's joints using python. We can use either forward kinematics or inverse kinematics.
- The set angles are sent to the Arduino board to move the servo motors.



Set the DH parameters of the arm and control each angle independently

Set the desired position and orientation of the end effector to move the arm

- For the angle movement, we obtained a generalized trajectory equation in joint space. The parameters for the equation are change of angle $\boldsymbol{\theta}$ and desired time frame **T.**

$$q(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3$$

Time	angle	velocity
t=0	0	0
t=T	θ	0

- Solving the equation for boundary conditions, we get,

$$q(t) = \left(\frac{3\theta}{T^2}\right) t^2 + \left(-\frac{2\theta}{T^3}\right) t^3$$

- When an angle change is present, the trajectory points for each joint is calculate within a time loop which lasts for a time frame of **T**. Then the calculated angles are written to motors at each time step.