$\sqrt{3}$ is Irrational Proof

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Proposition. $\sqrt{3}$ *is irrational.*

Proof. Suppose $\sqrt{3}$ is rational, such that

$$\sqrt{3} = \frac{a}{b} \quad \forall a, b \in \mathbb{Z}.$$

If we square the fraction, we get $3 = \frac{a^2}{b^2}$. Without loss of generality, we can suppose that a^2 is even and b^2 odd. We do this because 3 would result from a fraction with integers of the same type. We set $a^2 = 2k$ and $b^2 = 2j + 1$ $\forall k, j \in \mathbb{Z}$.

$$\begin{aligned} 3 &= \frac{2k}{2j+1} \\ 6j + 3 &= 2k \\ 3j + \frac{3}{2} &= k \implies k \notin \mathbb{Z}. \end{aligned} \tag{1}$$

Then $3 = \frac{a^2}{b^2}$ and $3 \neq \frac{a^2}{b^2}$ for $\forall a, b \in \mathbb{Z}$. From this we can conclude that $\sqrt{3}$ is irrational. ■