5] Order of (1 12 8 10 4) (2 13) (5 11 7) (6 9)

To find the order we note the number of elements in each cycle: 5,2,3,2 & LCM (5,2,3,2) = 30

Hence the order is 30

6] Cycle decomposition of each element of order 4 in S4:

Distinct elements of S4 are {1,2,3,4}

Keeping 1 fixed, { 1 2 3 4}.

{ 1 2 4 3}

{ 1 3 2 4 }

{ 1 3 4 2 }

{ 1 4 2 3 }

{ 1 4 3 2}

7] Cycle decomposition of each element of order 2 in S4:

We note: (1 2), (2 3) ,(3 4) , (1 4) ,( 2 4) 2-cycles

(1 2) (3 4), (1 3) (2 4), ( 1 4)(2 3) -- product of 2-cycles

8] Prove that if λ = { 1 ,2 , 3 …..} then Sλ is an infinite group

Codomain of an injective function must be at least as large as the domain of the function.

So, define the function as: f: N 🡪 S(N)

f(n) = (1 n)

where (1 n) is the cycle decomposition of an element of S(N) {specifically it’s the function given by g(1) = n, g(2) = 2, g(3) = 3,…..). The function f maps every natural number to a distinct one of these functions. Hence f is injective. Hence = |N| <= |S(N)|

10] if S is the m-cycle (a1 a2 ……a(m)), then for all i {1,2,…….m}, S^i(ak) = a(k+i) where k + I is replaced by its least positive residue mod m. Deduce that |S| = m

**Solution:**

We proceed on induction on i

Let i = 1 . Then clearly we have : ) = S() = =

Now assume that ) = for some 1 <= i <= (m-1)

Then, = S . = S(

Thus, by the principle of mathematical induction, the result holds for all 1 <= i <= m

From this proof we see that ) = = . This is very similar to the form e.m = m and hence in this case e is the identity element . Likewise, is the identity. As the smallest positive integer i for which we see that is the smallest positive power of S that’s identity. That is |

11] Let S be the m-cycle (1 2 …..m) . Show that iff its cycle decomposition is a product of commuting 2-cycles.

**Solution** :

First let’s suppose that is an m-cycle and that i is not relatively prime to m and let d = gcd (m, i) > 1. Then there exist numbers j, k such that m = jd and I = kd and also note in particular that 1 <= j < m , consider that : = e

Hence |<= j < m. This contradicts the fact that is an m-cycle. Hence it must be the case that if is an m-cycle , then m and i are relatively prime.

Suppose that m & I are relatively prime for all k, then

Hence

For contradiction’s sake that is not an m-cycle. Then at least one pair I + k1 and I + k2 where k1 =/= k2 are the same modulo m.

That is i + k1 ( mod m) , this implies that k1-k2 and hence that m divides k1 – k2 but by definition k1-k2 < m . Contradiction. Thus it must be the case that if I is relatively prime to m, then is also an m-cycle.