Problems 3 to 7

3] Show that G is non abelian

Proof Idea:

Take any 2 elements from this group and name them a and b , then prove that a.b b.a

This should suffice for the proof:

So a = , b = ab = and ba = so in this case they do commute

But suppose c = and d = and cd = , we see that dc = and cd dc and hence we say that the group is non abelian

4] Show that if n is not prime, then Z/n Z is not a field

Solution :

Suppose n is a composite, then n = ab (say and > 1)

If Z/n Z were a field, then a Z would have a multiplicative inverse say c

And in this case we have ac = 1 and if we multiply by b both sides, we have

bac = b

0 = b

And so b would be 0 in Z/n Z, that is b would be divisible by n. Since n = ab

This would imply that b = n and a = 1 contrary to the assumption that a > 1

5] Show that G is a finite group only iff F has a finite number of elements

Proof Idea:

We look at some theoretical arguments and then try and build on that.

First, if F is finite, then the group that is built from this field should also be finite and in this case GLn since there would be only finitely many n X n matrices with entries from F. On the other hand, if F is not finite then for every with , the matrix has a nonzero determinant, therefore qualitatively GLnF is infinite

6] If |F| = q is finite, prove that |GLn(F)| <

Proof:

Since F has q elements, there are only possible n X n matrices over F that can be formed. Since at least one of these matrices has zero determinant (the 0 matrix for instance), it follows that |GLn(F)| <

Alternatively consider that an arbitrary matrix A = [] with elements in a field F with q elements. Each element has q possibilities, thus the number of all matrices over F is equal to since we can choose q numbers times, but it is obvious that not all of them can be invertible matrices, namely the zero matrix. The statement of the problem follows.

7] Let p be a prime. Prove that the order of () is

Proof:

Let A be a 2 X 2 matrix over that is not in (). Let A = , and note

a d – b c = 0

We have two cases: a = 0 or a

First if a = 0 then d can take any of p possible values while b c = 0, Again there are two

Cases: if b = 0 then are p possible values that c could possibly take.

If b (this can happen in p – 1 ways) then c =

Thus in this scenario the total number of cases = p + (p – 1) = 2p – 1 possibilities for b and c which give a total of 2

Next, if a . Now b and c can take any value and then d is determined by the other variables, so there are a total of possibilities for this case. So taken together we have ( ) + (2 = possible matrices that can be. Since there are a total of matrices which are 2 X 2 over Fp , it follows that | () | =