Thursday, 11 March 2021

Cantor Diagonalisation Argument

Analysis

- 1] To prove: that for any list of real numbers between 0 and 1, there exists some real number that is between 0 and 1, but is not in the list
- 2] Obviously we can have lists that include at least some real numbers. In such lists, the first real number in the list is the number that is matched to the number one, the second real number in the list is matched to the number two, and so on.For any such list, we call the list a function and we give it the name r(x). So r(1) means the real number matched up to the number 1, while r(2) means the real number matched up to the number 2 and r(17) means the real number match dup to the number 17.And so on. There can be many such lists, and we know we can have some lists that have some finite quantity of real numbers. We will later address the question of whether there can be such a list that includes every real number.
- 3] Now we suppose that the beginnings of the binary expansions of some list of real numbers are as follows (of course we cannot actually write down infinitely long binary expansions) :
- r(1) = 0.101011110101….
- r(2) = 0.00010100011….
- r(3) = 0.0010111011110…
- r(4) = 0.111101010111…
- r(5) = 0.1011110111….
- r(6) = 0.11101011111001…
- 4] For any list of real numbers, there exists a number which we call d which is defined by the following rule. We start off with a zero followed by a point viz '0'; then we take the first digit of the first number in the list and if the digit is 0 we change it to 1 and write it down; if it is 1 we change it to 0 and write it down. This is called the complement, and so the complement of 0 is 1 and the complement of 1 is 0. We then take the second digit of the second number in the list and do the same, writing the changed digit after the previous one. And so on and so forth. For the first few numbers in the list above, this would work out like this
- r(1) = 0.**1**01011110101
- r(2) = 0.0**0**010100011….

- r(3) = 0.00**1**0111011110….
- r(4) = 0.111**1**01010111….
- r(5) = 0.1011**1**101111….
- r(6) = 0.11101**0**11111001…
- 5] From this list, we obtain the following number : d = 0.010001. This is called the 'diagonal' number. This real number d differs from every other number in the list since it is different from every other number in the list by at least one digit. For any finite list, the number d is a rational number since the sequence of digits is finite. But if the list is limitless, then d is an endless expansion that is a real number. In this case, we cannot follow the instruction to write down the digits and the number d is given only by definition- it is defined as the number where its nth digit is the complement of the *nth* digit of the *nth* number in the list.
- 6] So, given any list of real numbers we can always define another real number that is not there in the list - the Diagonal number
- 7] We now assume that there can be a list that includes every real number
- 8] And now we have a contradiction - because the Diagonal number would be at the same time defined as a number that is in the list and also cannot be in the list - because it differs from every number in the list, since it is always different at the *nth* digit.
- 9] That means that the assumption that there can be a list that includes every real number (Step 7 above) is incorrect.
- 10] Therefore there cannot be a list that includes every real number
- This concludes the diagonal argument or proof.