Jethwa Vidya. Roll No 529.

M	T	W	T	F	S	S

Page No.: YOUVA

Date:

Operation Research Miniproject.

Q. A company is producing a single product and selling is through five agencies situated in different cities. All of a sudden, there is a demand for the product in five more cities that do not have any agency of company. The company is faced with the problem of deciding on how to assign the existing agencies to dispatch the product to the additional cities in such a way that the travelling distance is minimised. The distance (in km) between the surplus and deficit cities are given in following distance matrix.

Surplus city \ Deficit city	I	II	III	IV	V
A	160	130	175	190	200
B	135	120	130	160	175
C	140	110	155	170	185
D	50	50	80	80	110
E	55	35	70	80	105

Determine the optimum assignment schedule.

Solution :
Subtracting the minimum element of each row from every element of that row, we have.

	I	II	III	IV	V
A	30	0	45	60	70
B	15	0	10	40	55
C	30	0	45	60	75
D	0	0	30	30	60
E	20	0	35	45	70

Subtracting the minimum element of each column from every element of that column, we have

	I	II	III	IV	V
A	30	0	35	30	15
B	15	0	0	10	0
C	30	0	35	30	20
D	0	0	20	0	5
E	20	0	25	15	15

We now assign zeros by drawing rectangles around them as explained.
~~Thus, we get.~~
Now, starting from first row onward, we draw a rectangle around the 0 in each row having a single zero and cross all other zeros in the corresponding column. Here, in first row we find a single zero. So we draw a rectangle & cross all other zeros in corresponding column.

	I	II	III	IV	V
A	30	$\boxed{0}$	35	30	15
B	15	0	$\boxed{0}$	10	0
C	30	0	35	30	20
D	$\boxed{0}$	0	20	0	5
E	20	$\boxed{0}$	25	15	15

Since the number of assignments is less than the number of rows (or columns), we proceed from step 5 onwards of the Hungarian method as follows:

i.) We tick mark (✓) the row in which the assignment has not been made. These are the 3rd and 5th rows.

ii.) We tick mark (✓) the columns which have assignment zeros in the marked rows. This is 2nd column.

iii.) We tick mark the rows which have assignments in marked columns. This is the 1st row.

iv.) Again we tick the column(s) which have zeros in the newly marked row. This is the 2nd column which has already been marked. There is no other such column. So we have.

	I	II	III	IV	V	
A	30	$\boxed{0}$	35	30	15	✓
B	15	$\cancel{0}$	$\boxed{0}$	10	$\cancel{0}$	
C	30	$\cancel{0}$	35	30	20	✓
D	$\boxed{0}$	0	20	$\cancel{0}$	5	
E	20	$\boxed{0}$	25	15	15	✓
		✓				

We draw straight lines through unmarked rows and marked columns as follows:

	I	II	III	IV	V	
A	30	$\boxed{0}$	35	30	15	✓
B	15	0	$\boxed{0}$	10	0	
C	30	0	35	30	20	✓
D	$\boxed{0}$	0	20	0	5	
E	20	0	25	15	15	✓
		✓				

We proceed as follows,

i) We find the smallest element in the matrix not covered by any of the lines. It is 15 in this case.

ii) We ~~off~~ subtract the number '15' from all the uncovered elements and add it to the elements at the intersection of the two lines.

iii) Other elements covered by the lines remain unchanged. Thus, we have

	I	II	III	IV	V
A	15	0	20	15	0
B	15	15	0	10	0
C	15	0	20	15	5
D	0	15	20	0	5
E	5	0	10	0	0

We repeat steps 1 to 4 of the Hungarian method and obtain the following matrix.

	I	II	III	IV	V
A	15	$\cancel{0}$	20	15	$\boxed{0}$
B	15	15	$\boxed{0}$	20	$\cancel{0}$
C	15	$\boxed{0}$	20	15	5
D	$\boxed{0}$	15	20	~~15~~ $\cancel{0}$	5
E	5	$\cancel{0}$	10	$\boxed{0}$	0

Since each row & each column of this matrix has one and only one assigned 0, we obtain the optimum assignment schedule as follows:

A → V, B → III, C → II, D → I, E → IV

Thus, the minimum distance is
200 + 130 + 110 + 50 + 80 ⟹ 570 kms.