

## Todays Content:

- Recursion?
- How to write a Recursive Code / Tracing
- TC/SC of Recursive Codes → Fridays Session

## Why Recursim?

- Merge Sort / Quick Sort
- Binary Tree / BST / BBST / Segment Tree / Tries
- Dynamic Programming
- Backtracking
- Graphs

Recursion: function calling itself

↳ Solving a Problem, using Smaller Instances of Same Problem  
↳ Sub Problem

$$\text{Sum}(N) = \underbrace{1 + 2 + 3 + \dots}_{\uparrow} N - 1 + N$$

$$\text{Sum}(N) = \underbrace{\text{Sum}(N-1)}_{\uparrow} + N$$

$$\text{Sum}(4) = \text{Sum}(3) + 4 \quad // \text{sum}(3) \text{ is Sub Problem}$$

How to write Recursive Code?

Assumption: Fin what your fun should do

Main logic: Solving assumption using problem

Base condition: Inputs, for which we want stop recursion

↳ at start of function

$N \geq 1$

```
int sum(N) { Ass: Given N, calculate & return, sum of 1^m N
    if(n==1) { return 1 }
    return [ sum(N-1) + N ]
}
1+2+3.. N-1 + N : Sum of N Natural Numbers
```

Natural Numbers  
 $\text{Sum}(3) = 6$   
 $\text{Sum}(4) = 10$

$\text{fact}(3) = 3 * 2 * 1 = 6$ ,  $\text{fact}(4) = 4 * 3 * 2 * 1 = 24$ ,  $\text{fact}(5) = 120$

$N \geq 1$

int fact(N) { Ass: Given N, calculate & return N!

```
if(n==1) return 1;
return fact(N-1) * N
}
↳ Tracing TODO
```

$\text{fact}(N) = \underline{1 * 2 * \dots * N-1 * N}$   
 $\text{fact}(N) = \text{fact}(N-1) * N$

Function Call Tracing:

```
int add(N, M) {
    return (N+M)
}

int mul(n, y) {
    return (n * y)
}

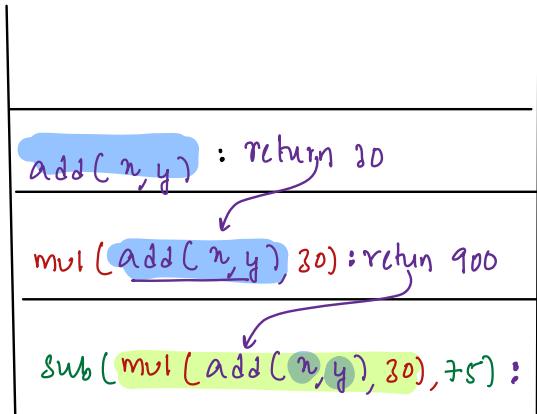
int sub(n, y) {
    return (n - y)
}

main() {
    n = 10, y = 20
    print(sub(mul(add(n, y), 80), 75))
    print(sub(mul(add(n, y), 80), 75)) : 825
    sub(mul(add(n, y), 80), 75) : return 825
    mul(add(n, y), 80) : return 900
    add(n, y) : return 30
```

## = Data Structure :

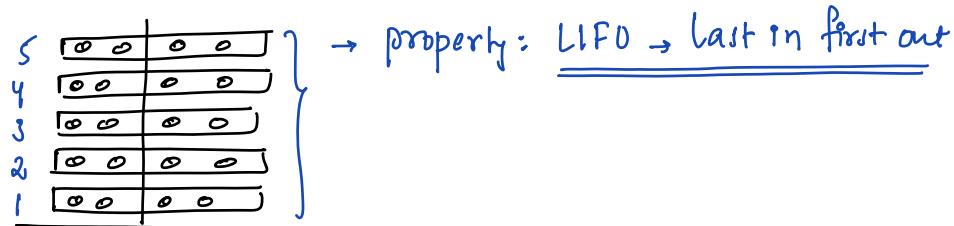
obs1: Whenever a function calls we add the function call at top

obs2: When function return we remove it from top

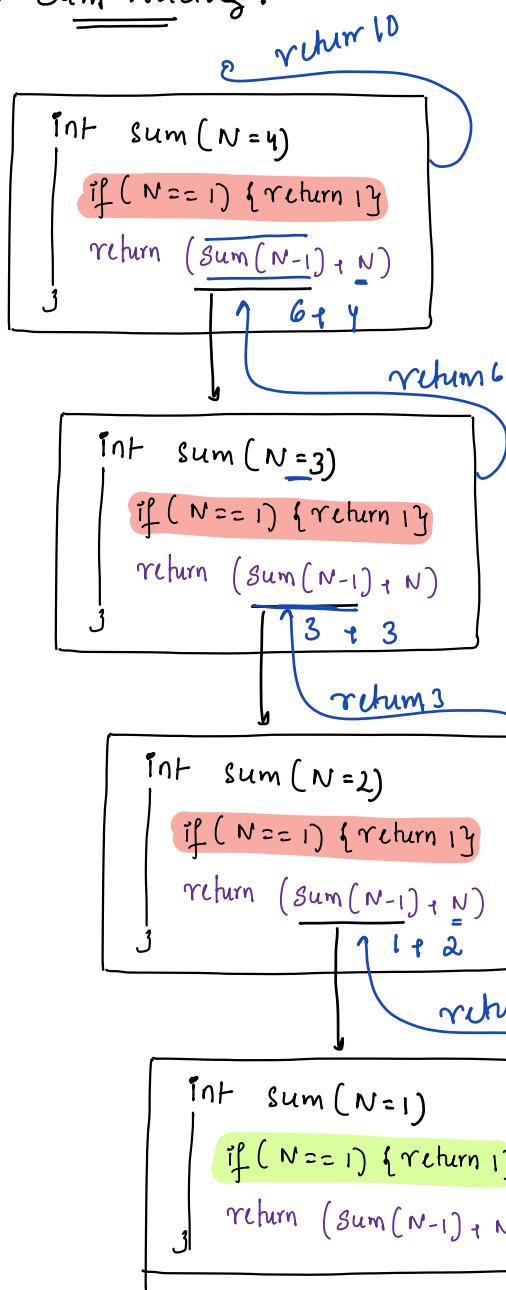


Note: All function calls are stored in Stack

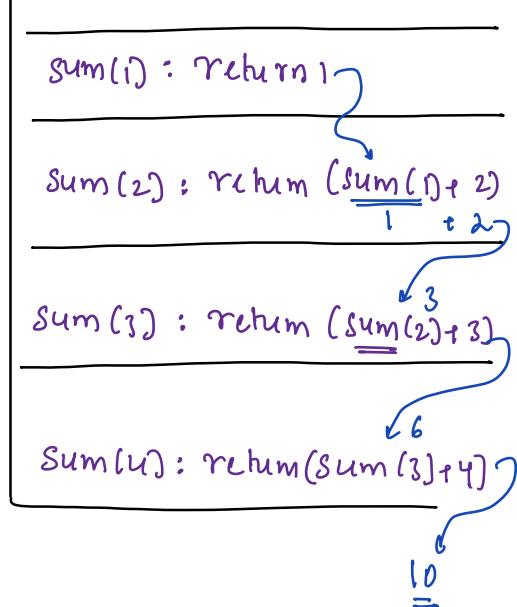
?:



## // Sum Tracing:



## Stack Trac.:



Without Base Condition: Recursion won't stop, memory limit occurs first

a) TLE

b) Memory limit Exceeded / Stack Overflow

→ Stack limit can be changed, in your language of choice

Note: In Recursion, if your code gives Memory limit exceed, that means, code is not properly stopped, verify base conditions.

**N >= 0**      #Input      0      1      2      3      4      5      6      7      8      9      10

$$\underline{\underline{\text{fib}}}(\underline{\underline{}}) : 0 \quad 1 \quad \underbrace{1}_{\text{sum}} \quad \underbrace{2 \quad 3}_{\text{sum}} \quad 5 \quad 8 \quad 13 \quad 21 \quad 34 \quad 55$$

int fib(N) { Ass: Calculate & return  $N^{\text{th}}$  fib number }

if ( $N <= 1$ ) return  $N$   
 return  $\{ \text{fib}(N-1) + \text{fib}(N-2) \}$

$\text{fib}(5) = 5$   
 $\text{fib}(7) = 13$

$\text{fib}(N) = \text{sum of previous 2 fib numbers}$

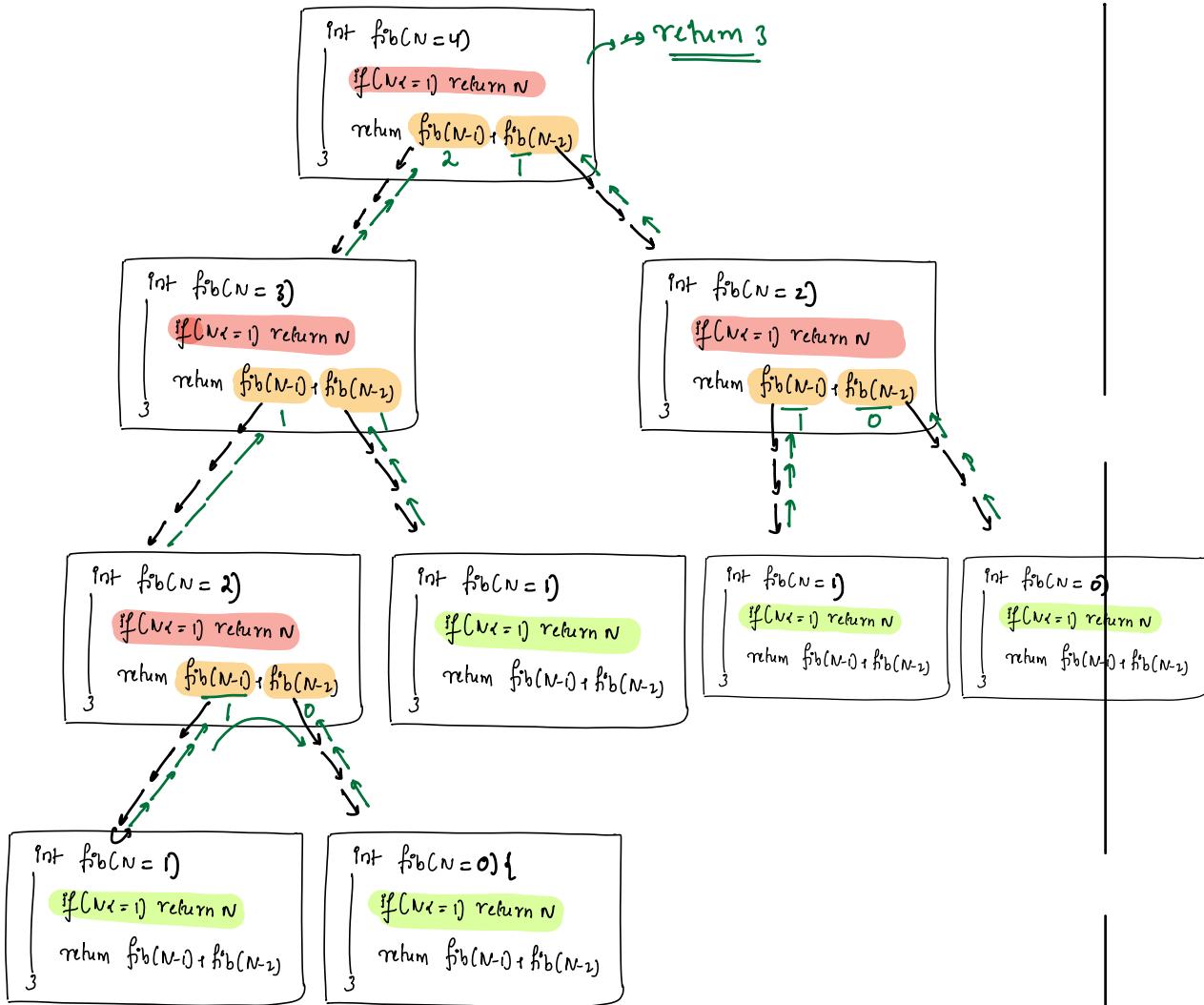
// Note: how to figure out conditions:

For which valid, inputs expression is invalid

$$\int \text{fib}(o) = \text{fib}(-1) + \text{fib}(-2)$$

$$f_i^{\circ} b(i) = f_i^{\circ} b(0) + f_i^{\circ} b(-i)$$

$$f_{\text{ib}}(2) = f_{\text{ib}}(1) + f_{\text{ib}}(0)$$



→ Try tracing with stack → TODO

// given N, print all numbers from  $1 \rightarrow N$  in increasing order

void Inc(N){ Ass: Given N, print all numbers from [1, N] in inc

if ( $N == 1$ ) { print(1), return }

Inc( $N-1$ ) //  $1, 2, 3 \dots N-1$ ,  $N$   
print( $N$ ) //

Stack Frame  $\rightarrow$  TODO

Inc(3)  $\rightarrow$  1 2 3

Inc(4)  $\rightarrow$  1 2 3 4

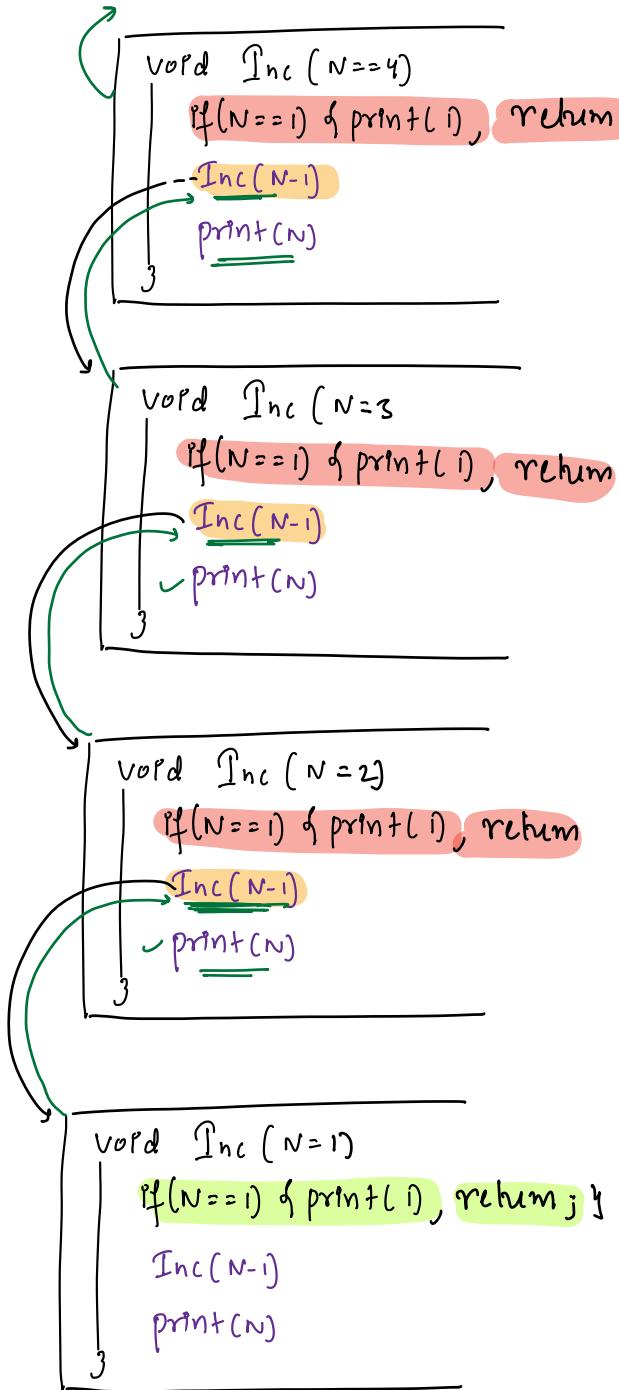
Inc( $N$ ):  $1 2 3 \dots N-1$ ,  $N$   
[ Inc( $N-1$ )  
print( $N$ ) ]  $\leftarrow$

Base:

Inc(1) =  $\begin{bmatrix} \text{Inc}(0) \\ \text{print}(1) \end{bmatrix}$  \*

TODO:

void dec(N): Ass: Given N print all number from  $N \rightarrow 1$



### Output

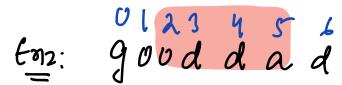
print(1)  
 print(2)  
 print(3)  
 print(4)

Note1: Even for void return type we can return inside function.

Note2: Once a function is completely created it will go back to its parent function.

Q8) Given a Substring Check if it's palindrome or Not?

Ex1:   $s=4, e=6 \Rightarrow$  return True

Ex2:   $s=2, e=5 \Rightarrow$  return False

Ass: Return, If substring from  $[s \dots e]$  is palindrome or Not?

bool ispal(char ch[], int s, int e) {  $\overbrace{s \dots e}^{\underline{s_1=e}}$

if ( $s > e$ ) { return True }

if ( $ch[s] == ch[e]$  &&

ispal(ch, s+1, e-1))

} return True

return False

ch[]: 

$\rightarrow$  if ( $ch[s] == ch[e]$ )

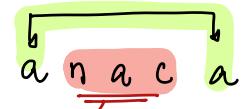
$\rightarrow$  Substring from  $[s+1, e-1]$   
should be a problem

ispal(ch, s+1, e-1)

Ex:







not palindrome

0 1 2 3 4 5

Input: m a d d a m , s=0, e=5      return True

```
bool ispal(char[], int s=0, int e=5){  
    if(s > e) { return true; }  
    if(ch[s] == ch[e] && ispal(ch, s+1, e-1)) {  
        return true;  
    } else {  
        return false;  
    }  
}
```

```
bool ispal(char[], int s=1, int e=4){  
    if(s > e) { return true; }  
    if(ch[s] == ch[e] && ispal(ch, s+1, e-1)) {  
        return true;  
    } else {  
        return false;  
    }  
}
```

```
bool ispal(char[], int s=2, int e=3){  
    if(s > e) { return true; }  
    if(ch[s] == ch[e] && ispal(ch, s+1, e-1)) {  
        return true;  
    } else {  
        return false;  
    }  
}
```

```
bool ispal(char[], int s=3, int e=2){  
    if(s > e) { return true; }  
    if(ch[s] == ch[e] && ispal(ch, s+1, e-1)) {  
        return true;  
    } else {  
        return false;  
    }  
}
```

Input: a n m e t h a     $s=0, e=6$

```
bool ispal(char ch[], int s=0, int e=6){  
    if(s>e) { return true }  
    if(ch[s] == ch[e] && ispal(ch, s+1, e-1)){  
        return true  
    } else {  
        return false  
    }  
}
```

```
bool ispal(char ch[], int s=1, int e=5){  
    if(s>e) { return true }  
    if(ch[s] == ch[e] && ispal(ch, s+1, e-1)){  
        return true  
    } else {  
        return false  
    }  
}
```

```
bool ispal(char ch[], int s=2, int e=4){  
    if(s>e) { return true }  
    if(ch[s] == ch[e] && ispal(ch, s+1, e-1)){  
        return true  
    } else {  
        return false  
    }  
}
```

```
bool ispal2(char ch[], int s, int e){  
    if(s>e) { return true }  
    if(ch[s] == ch[e]) {  
        return ispal((ch, s+1, e-1))  
    } else {  
        return false  
    }  
}
```

This function will return, if Substring from [s+1, e-1] is palindrom or not

Note: Don't use post increment  
operators, while writing  
recursion, Use pre increment