Assignment

Dynamic Programming Assignment

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Problem Analysis

Given n students and n topics, we need to calculate the number of ways to assign topics to students such that each student gets exactly one topic they like.

Dynamic Programming Approach

State Representation

We define a state dp[mask] where mask is a bitmask of length n that represents which topics have already been assigned. Each bit in the mask indicates whether a topic has been assigned or not (1 for assigned, 0 for not assigned).

Transition

For each student i, we iterate through all possible topics. If a topic j has not been assigned yet and the student i likes the topic j, we can assign the topic j to student i and transition to the new state. This can be represented as dp[new_mask] += dp[mask], where new_mask is obtained by setting the j-th bit in mask.

Base Case

The base case is dp[0] = 1, meaning there is one way to assign topics when no topics have been assigned yet.

Recurrence Relation

The recurrence relation can be written as: dp[mask|(1<<j)]+=dp[mask]\text{dp}[mask | (1 << j)] += \text{dp}[mask]dp[mask|(1<<j)]+=dp[mask] where | is the bitwise OR operation, and (1 << j) sets the j-th bit.

Algorithm:

1.Initialize a dp array of size 2n2^n2n with all elements set to 0.

2.Set the base case dp[0] = 1.

3.Iterate through all states represented by mask from 0 to 2n−12^n - 12n−1.

4.For each state mask, count the number of bits set to 1 to determine which student we are considering.

5.For each topic j, check if the j-th bit is not set in mask and if the current student likes topic j. If both conditions are met, update dp[mask | (1 << j)].

Conclusion

The assignment problem of determining the number of ways to assign n topics to n students, where each student gets exactly one topic they like, can be efficiently solved using dynamic programming with bit masking. By defining a state dp[mask] where mask represents the set of already assigned topics, we can systematically explore all possible assignments through a series of state transitions.