MOTION IN A PLANE

Chapter Four

Scalar Quantities:

The quantities which has only magnitude along with some unit are classified as scalars quantity

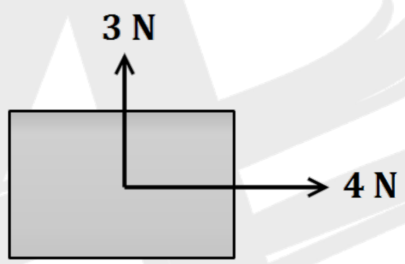
For Example: 1 kg mango, 273 kelvin etc.

The rules for combining scalars are the rules of ordinary algebra. Scalars can be added, subtracted, multiplied and divided just as the ordinary numbers.

Vector Quantitites:

The qunatities which has some magnitude along with the proper unit and direction and follows vector law of addition are classified as vector qunatities.

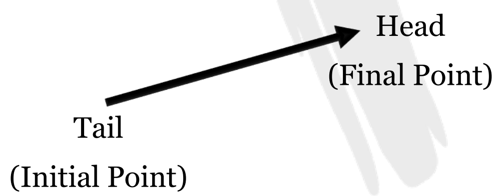
For Example: Force is a vector quantity which has some magnitude as well as direction and also follows vector law of addition.



Net force $\neq$ 7 N

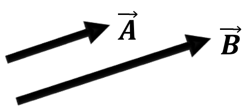
Representation of Vector:

If $\boldsymbol{A}$ is a vector quantity then it is represented by $\vec{A}$ and magnitude of $\vec{A}$ is represented by $|\vec{A}|$ or simply $\boldsymbol{A}$



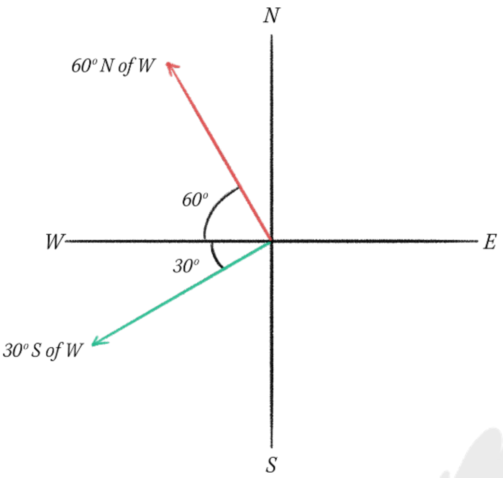
Generally length of a vector represents its magnitude but for convinience we just draw Head and Tail and Write its magnitude numerically.

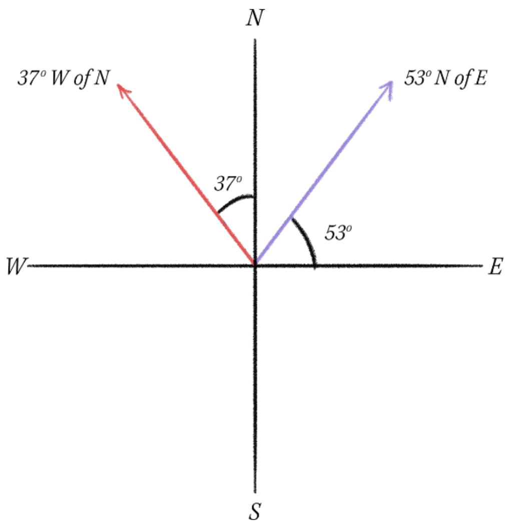
For Example:

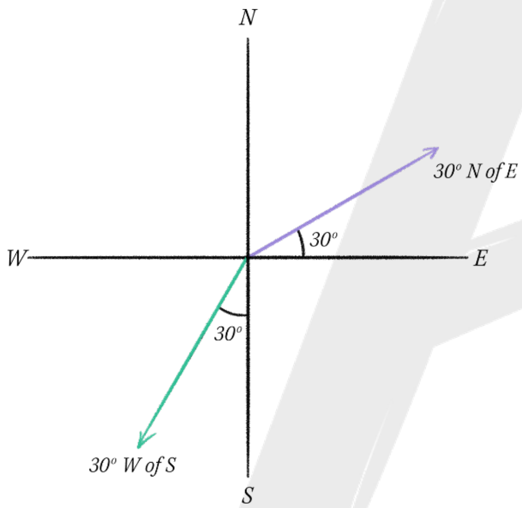


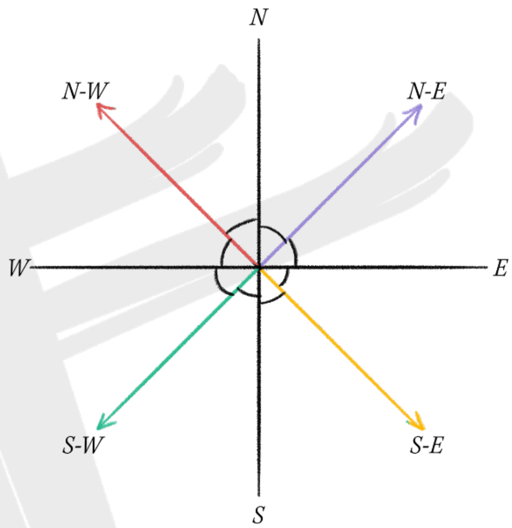
From the above picture we can say magnitude of $\vec{B}$ is greater than the magnitude of $\vec{A}$.

Basic Information about Direction:

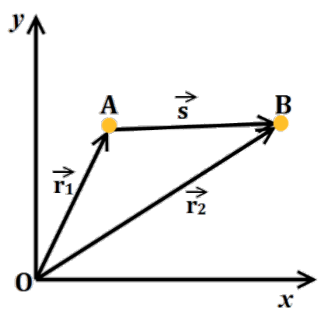








Position Vector:



Position vector is used to specify the position of a object with respect to origin (i.e., head of the position vector is towards the coordinates which needs to be specified and tail at the origins).

$\vec{r}_1$ is the position vector of point A and $\vec{r}_2$ is the position vector of point B.

Displacement Vector:

If the object moves from point A to point B then $\overrightarrow{AB}$ vector or $\vec{s}$ vector is the displacement vector with the initial position A and the final position point B.

Unit Vector:

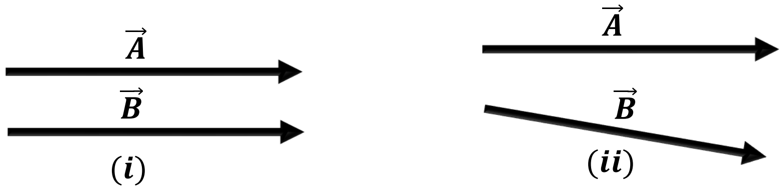
A vector having some direction but magnitude equal to unity is called unit vector. If $\vec{A}$ is a vector then its unit vector is represented by $\hat{A}$.

$$\boxed{\hat{A} = \frac{\vec{A}}{|\vec{A}|}}$$

$\hat{\imath}, \hat{\jmath}, \hat{k}$ are the unit vectors along x, y, z axis respectively.

Equality of Vector:

Two vectors A and B are said to be equal only if they have the same magnitude and the same direction.



In (i) figure vector A and B are equal because they have same magnitude and same direction but in (ii) figure vector A and B are not equal vector because they have same magnitude but different direction.

Multiplication of Vectors by a real number:

Multiplying a vector A with a positive number λ gives a vector whose magnitude is changed by the factor λ but the direction is the same as that of A:

$|\lambda A| = \lambda |A|$ if $\lambda > 0$.

Addition and Subtraction of Vectors (Graphical Method):

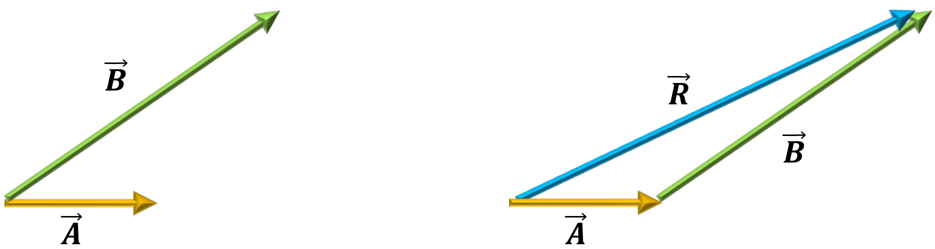
- **Triangle Law of Vector Addition**
- **Parallelogram Law of Vector Addition**

In graphical method magnitude of a vector is represented by drawing vector with proper direction and length but for magnitude scaling is done i.e., if represent 10 N force by length 1cm then 20N force will be represented by a vector of length of 2cm and 35N force will be represented by length of 3.5 cm and so on, this is how the scaling of vector is done.

Triangle Law of Vector Addition:

If two vectors are represented by two sides of a triangle in magnitude and direction taken in same order, then the resultant is given by the third side of the triangle in magnitude and direction taken in opposite order.

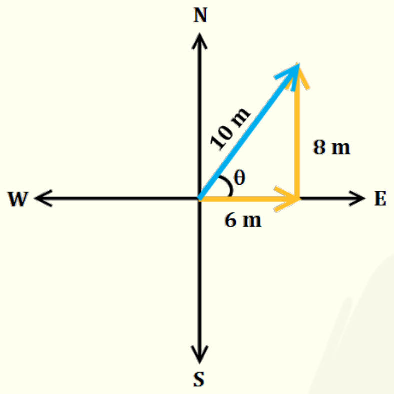
If $\vec{A} + \vec{B} = \vec{R}$



- *Shifting of vector is allowed i.e., we can shift a vector without changing its length and direction.*
- *Angle between two vectors is the angle between them when they are joined head to head or tail to tail.*

Q. A man moves 6 m towards east, then he takes 90° left turn & moves 8 m North. Find displacement of man.

Sol.



$$\tan\theta = \frac{8}{6} = \frac{4}{3}$$

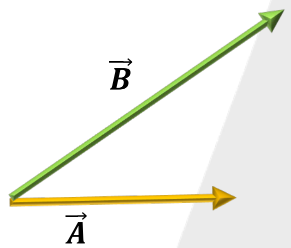
$$\Rightarrow \theta = 53°$$

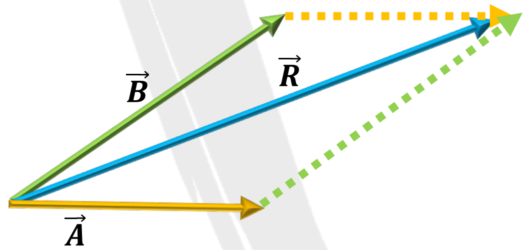
Displacement of man is 10 m, 53° North of East.

Parallelogram Law of Vector Addition:

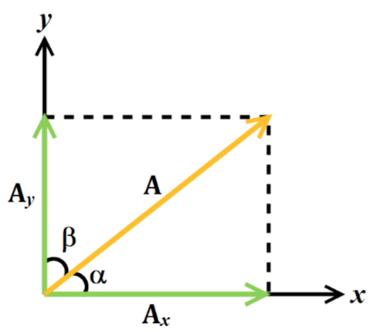
If two vectors are represented by two adjacent sides of a parallelogram in magnitude and direction (Joined tail to tail) then the resultant is given by the diagonal of parallelogram in magnitude and direction starting from common intersection of two vectors.

If $\vec{A} + \vec{B} = \vec{R}$





Resolution of Vector:



$\frac{A_x}{A} = \cos\alpha \Rightarrow A_x = A\cos\alpha$ (component of A in X-direction)

$\frac{A_y}{A} = \cos\beta \Rightarrow A_y = A\cos\beta$ (component of A in Y-direction)

$$\vec{A} = A\cos\alpha\,\hat{\imath} + A\cos\beta\,\hat{\jmath}$$

Also, $\beta = 90 - \alpha$

$$\vec{A} = A\cos\alpha\,\hat{\imath} + A\cos(90-\alpha)\hat{\jmath}$$

$$\boxed{\vec{A} = A\cos\alpha\,\hat{\imath} + A\sin\alpha\,\hat{\jmath}}$$

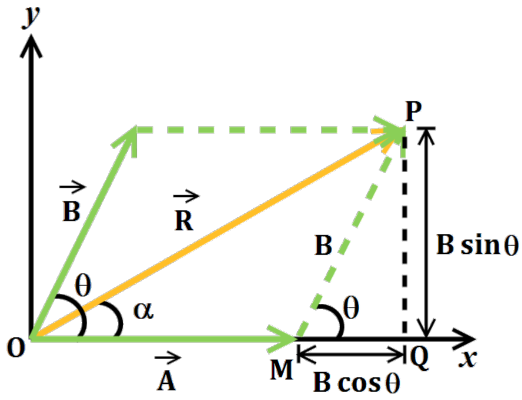
Similarly if a vector $\vec{A}$ is in 3-D making an angle α with x axis, β with y axis and γ with z axis then it can be written as

$$\boxed{\vec{A} = A\cos\alpha\,\hat{\imath} + A\cos\beta\,\hat{\jmath} + A\cos\gamma\,\hat{k}}$$

and magnitude of $\vec{A}$ can be written as

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Vector Addition (Analytical Method):



In ΔOPQ,

$|\vec{R}| = R = \sqrt{(A + B\cos\theta)^2 + (B\sin\theta)^2}$

$$R = \sqrt{A^2 + B^2 + 2AB\cos\theta} = \text{Magnitude of resultant vector.}$$

$$\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}$$

Where α is the angle made by the resultant vector with $\vec{A}$

Motion in a Plane:

1. Velocity in a Plane:

$$\vec{v} = \frac{\Delta r}{\Delta t} = \frac{\Delta x\hat{\imath} + \Delta y\hat{\jmath}}{\Delta t} = v_x\hat{\imath} + v_y\hat{\jmath}$$

$$|\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

2. Acceleration in a Plane:

$$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{\Delta(v_c\hat{\imath} + v_y\hat{\jmath})}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{\imath} + \frac{\Delta v_y}{\Delta t}\hat{\jmath}$$

$$\overline{a} = a_x\hat{\imath} + a_y\hat{\jmath}$$

Q. The position of a particle is given is $r = 3.0t\hat{\imath} + 2.0t^2\hat{\jmath} + 5.0\hat{k}$ where t is in seconds and the coefficients have the proper units for r to be in metres. (a) Find $v(t)$ and $a(t)$ of the particle. (b) Find the magnitude and direction of $v(t)$ at $t = 1.0\ s$.

Sol.

a) $v = \frac{dr}{dt} = 3.0\hat{\imath} + 4.0t\hat{\jmath}$

$a = \frac{dv}{dt} = 4.0\hat{\jmath}$ = constant acceleration in y direction

b) velocity at $t = 1s$

$v = 3.0\hat{\imath} + 4.0\hat{\jmath}$

$|\vec{v}| = \sqrt{3^2 + 4^2} = 5.0m/s$

Direction of velocity,

$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 53°$ with x-axis

Equation of Motion for 2-D Motion:

$s_x = u_x t + \frac{1}{2} a_x t^2$ $\qquad$ $s_y = u_y t + \frac{1}{2} a_y t^2$

$v_x = u_x + a_x t$ $\qquad$ $v_y = u_y + a_y t$

$v_x^2 = u_x^2 + 2a_x s_x$ $\qquad$ $v_y^2 = u_y^2 + 2a_y s_y$

Motion in a plane (two-dimensions) can be treated as two separate simultaneous one-dimensional motions with constant acceleration along two perpendicular directions.

Q. A particle starts from origin at $t = 0$ with velocity $5.0\hat{\imath}\ m/s$ and moves in x-y plane under action of a force which produces a constant acceleration of $(3.0\hat{\imath} + 2.0\hat{\imath})\ m/s^2$. (a) What is the y-coordinate of the particle at the instant its x-coordinate is $84\ m$? (b) What is the speed of the particle at this time?

Sol. a) $s_x = 84 = u_x t + \frac{1}{2} a_x t^2 \Rightarrow 5t + \frac{1}{2} \times 3t^2$

$168 = 3t^2 + 10t \Rightarrow t = 6s$

s_y at $t = 6s$

$s_y = u_y t + \frac{1}{2} a_y t^2 = \frac{1}{2} \times 2 \times (6)^2 = 36$

at $t = 0$, $y = 0$ and in $6\ s$ displacement in by direction is 36 that means partical is at $y = 36$ at $t = 6\ s$

b) At $t = 6$

$v_x = u_x + a_x t = 5 + 3 \times 6 = 23$

$v_y = u_y + a_y t = 0 + 2 \times 6 = 12$

Speed = Magnitude of velocity = $\sqrt{v_x^2 + v_y^2} = \sqrt{(23)^2 + (12)^2} = 26$

Relative Velocity in 2-D: (Removed for 2023-2024)

Velocity of A with respect to B is represented by

$$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$$

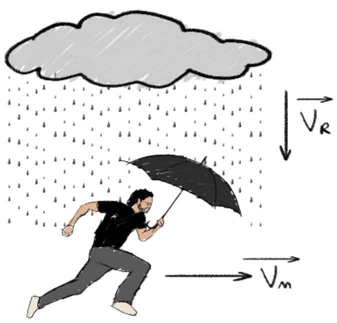
Acceleration of B with respect to A is represented by

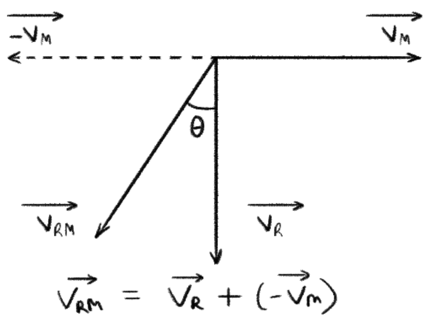
$$\vec{a}_{BA} = \vec{a}_B - \vec{a}_A$$

Relative velocity of Rain: (Removed for 2023-2024)

Case 1: If rain is falling vertically with a velocity $\vec{V}_R$ and an observer is moving horizontally with velocity $\vec{V}_M$ then the velocity of rain with respect to observer will be

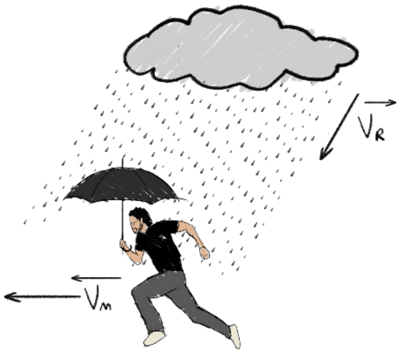
$\boxed{\vec{V}_{RM} = \vec{V}_R - \vec{V}_M}$ $\qquad$ $\boxed{V_{RM} = \sqrt{V_R^2 + V_M^2}}$ $\qquad$ $\boxed{\theta = tan^{-1}\left(\frac{V_M}{V_R}\right)}$

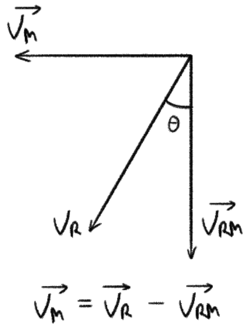




So in order to save himself from wetting person should hold umbrella at angle θ with verticle.

Case 2: If rain is falling at an angle θ





Please note that in rain-man problems, in order to say himself from wetting man should hold the umbrella in the direction opposite to v_{RM}

Swimming into the river: (Removed for 2023-2024)

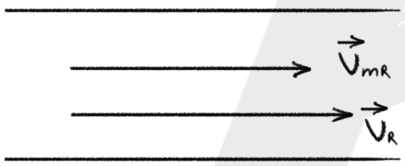
A man can swim with velocity $\vec{V}_{MR}$ i.e. it is the velocity of man w.r.t. still water.

If water is also flowing with velocity $\vec{V}_R$ then the velocity of man relative to ground is

$$\vec{V}_M = \vec{V}_{MR} + \vec{V}_R$$

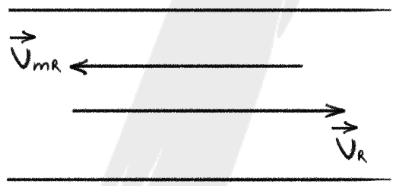
i) If the swimming is in the direction of flow of water then (Downstream)

$V_m = V_{MR} + V_R$



ii) If the swimming is in opposite direction to the flow of water then (Upstream)

$V_M = V_{MR} - V_R$



iii) For shortest Path (Removed for 2023-2024)

If man wants to cross the river such that his "displacement should be minimum" then it means that he wants to reach just opposite point across the river.

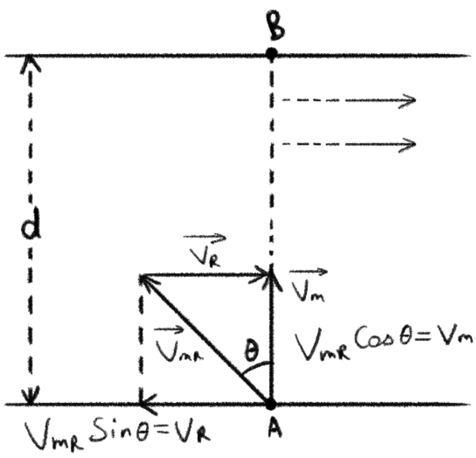
Man should start swimming at an angle θ with the perpendicular to the flow of river.

So that its resultant velocity $\vec{V}_M = \vec{V}_{MR} + \vec{V}_R$ is in the direction of displacement $\overrightarrow{AB}$.

To reach at B

$V_{MR} \sin\theta = V_R$ or $\sin\theta = \dfrac{V_R}{V_{MR}}$

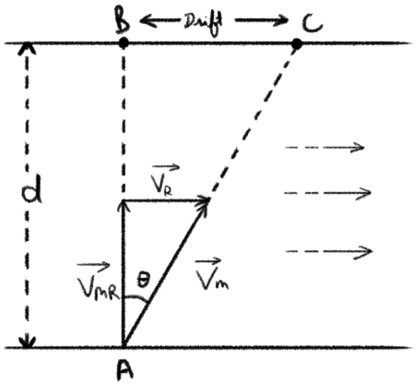
Time taken $T = \dfrac{d}{\sqrt{V_{MR}^2 - V_R^2}}$



(iv) **For Minimum Time** (Removed for 2023-2024)

To cross the river in minimum time the velocity along $AB(V_M \cos\theta)$ should be maximum. It is possible if $\theta = 0$, i.e. Swimming should start perpendicular to water current.

Due to effect of river velocity i.e. his displacement will not be minimum but time taken to cross the river will be minimum.



$$\boxed{t_{min} = \frac{d}{V_{MR}}}$$

In time t_{min} swimmer travels distance BC along the river with speed of river V_R

$$\therefore BC = t_{min} V_R = \frac{d}{V_{MR}} V_R$$

$$\boxed{\text{Distance travelled along river flow} = \text{drift of man} = t_{min}\, V_R}$$

Q. If Velocity of A is given as $3\hat{\imath} + 4\hat{\jmath}$ and velocity of B is given as $5\hat{\imath} - 3\hat{\jmath}$ find out the following

(Removed for 2023-2024)

(a) Velocity of A with respect to B

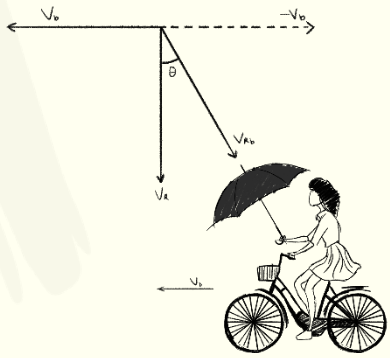
(b) Velocity of B with respect to A

Sol. a) $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = 3\hat{\imath} + 4\hat{\jmath} - (5\hat{\imath} - 3\hat{\jmath}) = -2\hat{\imath} + 7\hat{\jmath}$

b) $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 5\hat{\imath} - 3\hat{\jmath} - (3\hat{\imath} + 4\hat{\jmath}) = 2\hat{\imath} - 7\hat{\jmath}$

Q. Rain is falling vertically with a speed of 35 ms^{-1}. A woman rides a bicycle with a speed of 12 $m\,s^{-1}$ in east to west direction. What is the direction in which she should hold her umbrella.

Sol. Let v_r and v_b are the velocities of rain and bicycle respectively so in order to save herself from wetting she should hold the umbrella in the direction opposite velocity of rain with respect to herself (v_{rb})



$v_{rb} = v_r - v_b$

$\text{Tan}\,\theta = \frac{v_b}{v_r} = \frac{12}{35} = 0.343$

$\Rightarrow \theta = 18.92°$

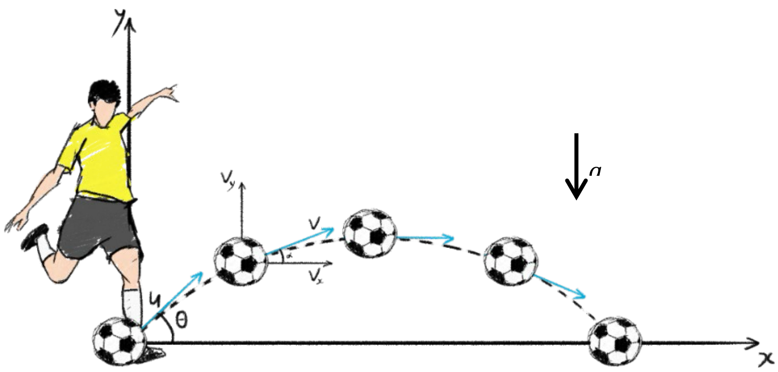
Projectile Motion:

A particle thrown in the space which moves under the effect of gravity alone is called projectile and its motion is called projectile motion.

For Example:

i) Motion of a ball after it strikes with bat.

ii) Motion of football after kick



$u_x = x$ component of initial velocity $= u\cos\theta$

$u_y = y$ component of initial velocity $= u\sin\theta$

$a_y = -g\ m/s^2$

x component of velocity at any time t

$v_x = u_x + a_x t = u\cos\theta$

y component of velocity at any time t

$v_y = u_y + a_y t = u\sin\theta - gt$

Displacement in x in time t

$s_x = u_x t + \frac{1}{2}a_x t^2 = (u\cos\theta)t$

Displacement in y in time t

$s_y = u_y t + \frac{1}{2}a_y t^2 = (u\sin\theta)t - \frac{1}{2}gt^2$

Angle of velocity with x-axis at any time t

$$\boxed{tan\,\alpha = \frac{v_y}{v_x} = \frac{u\,sin\,\theta - gt}{u\,cos\,\theta}}$$

A) Maximum Height:

Maximum height achieved during the whole motion.

We can observe that during upward motion velocity is decreasing because of acceleration due to gravity and at some point velocity in y direction became zero. This is the point where maximum height is achieved.

We know that,

$v_y^2 = u_y^2 + 2a_y s_y$

When s_y is equal to maximum height $v_y = 0$

$0 = (u\sin\theta)^2 - 2gH_{max}$

$$\boxed{H_{max} = \frac{u^2\sin^2\theta}{2g} = \frac{u_y^2}{2g}}$$

B) Time of Flight:

Total time T during which the projectile is in flight.

When ball is in projectile motion thrown from the ground reaches to the ground again after time T we can observe that displacement in y direction is zero.

$$s_y = u_y t + \frac{1}{2}a_y t^2$$

$$0 = (u\sin\theta)T - \frac{1}{2}gT^2$$

$$\boxed{T = \frac{2u\sin\theta}{g} = \frac{2u_y}{g}}$$

C) Horizontal Range:

It is the displacement in x direction during time T (time of light)

$$s_x = u_x t + \frac{1}{2} a_x t^2$$

$$R = (u\cos\theta)T$$

$$\boxed{R = (u\cos\theta)\left(\frac{2u\sin\theta}{g}\right) = \frac{2u_x u_y}{g} = \frac{u^2\sin 2\theta}{g}}$$

D) Equation of Trajectory:

We know that $x = (u\cos\theta)t$ and $y = (u\sin\theta)t - \frac{1}{2}gt^2$ on eliminating t from these two equations, we get

$$y = (u\sin\theta)\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2}g\left(\frac{x}{u\cos\theta}\right)^2$$

Or

$$\boxed{y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}}$$

This is the equation of parabola. [$y = ax - bx^2$]

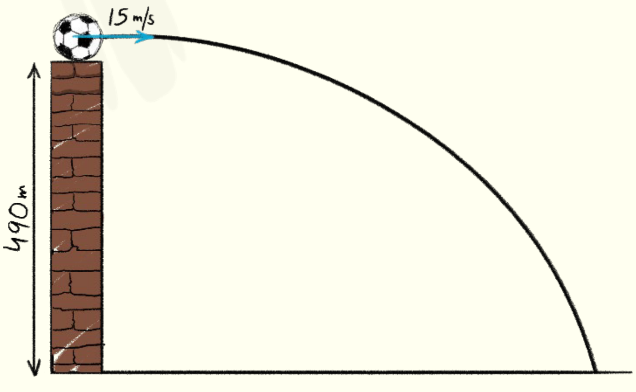
Q. A hiker stands on the edge of a cliff 490 m above the ground and throws a stone horizontally with an initial speed of 15 ms^{-1}. Neglecting air resistance, find the time taken by the stone to reach the ground, and the speed with which it hits the ground. (Take $g = 9.8\ m\ s^{-2}$).

Sol. When ball reaches to ground its displacement in y will be $-490\ m$.

$$s_y = u_y t + \frac{1}{2}a_y t^2$$

$$-490 = \frac{1}{2}(-9.8) \times t^2$$

$$t = \sqrt{\frac{2 \times 490}{9.8}} = 10\ s$$



Let, x-component of velocity when it reaches to ground is v_x and y-component of velocity when it is reaches to ground is v_y

$$v_x = u_x + a_x t = 15$$

$$v_y = u_y + a_y t = 0 + (-9.8)10 = -9.8$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15)^2 + (98)^2} = 99\ m/s$$

Uniform Circular Motion:

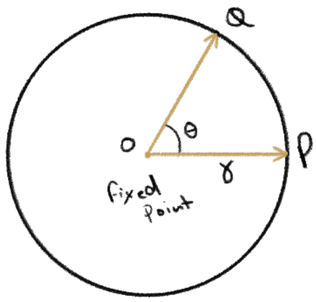
When an object follows a circular path at a constant speed, the motion of the object is called uniform cicular motion.

In uniform circular motion speed is constant but direction is changing continuously so there is some acceleration present in circular motion

Angular Displacement (θ):

Angle traced by position vector of a particle moving w.r.t. some fixed point is called angular displacement.

$$\text{Angular displacement } \theta = \frac{Arc\ PQ}{r}$$



- Its direction is perpendicular to plane of rotation and given by right hand screw rule.
- It is dimensionless and has SI unit 'Radian'

  2π radian $= 360° = 1$ revolution.

Angular Velocity (ω):

It is defined as the rate of change of angular displacement of moving object with time.

$$\text{Angular Velocity} = \frac{\text{Angle traced}}{\text{Time taken}}$$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

- Its SI unit is radian/sec.
- It is an axial vector quantity. Its direction is perpendicular to the plane of rotation and along the axis according to right hand screw rule.

Frequency (n):

Number of revolutions describes by particle per second is its frequency.

Its unit is revolution per sec. (r.p.s.)

Time Period (T):

It is time taken by particle to complete one revolution.

$$\mathrm{T} = \frac{1}{\mathrm{n}}$$

Relation between Linear and Angular Velocity:

$\text{Angle} = \dfrac{Arc}{Radius}$ or $\Delta\theta = \dfrac{\Delta S}{r}$

or $\Delta S = r\,\Delta\theta$

$\dfrac{\Delta S}{\Delta t} = \dfrac{r\Delta\theta}{\Delta t}$ if $\Delta t \to 0$ then $\dfrac{ds}{dt} = \dfrac{rd\theta}{dt}$

or $\quad V = r\omega$

In vector form $\vec{V} = \vec{\omega} \times \vec{r}$

Direction of $\vec{V}$ is according to right hand thumb rule.

Average Angular Velocity (ω_{av}):

$$\omega_{av} = \frac{total\ angle\ of\ rotation}{total\ time\ taken}$$

$$\omega_{av} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta\theta}{\Delta t}$$

Where θ_1 and θ_2 are angular position of the particle at instant t_1 and t_2.

Instantaneous Angular Velocity (ω):

It is the angular velocity at a particular instant.

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt} \qquad \text{or} \qquad \vec{\omega} = \frac{d\vec{\theta}}{dt}$$

Angular Acceleration (α):

The rate of change of angular velocity is called angular acceleration.

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$$

or

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

- It is an axial vector quantity. Its direction is along the axis according to the 'Right hand Rule'.
- Its SI unit is radian/sec^2.

Relation between Angular and Linear Acceleration:

We know that $\vec{v} = \vec{\omega} \times \vec{r}$

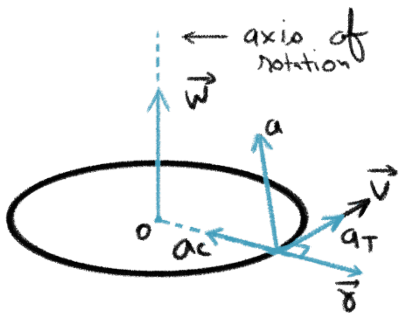
Here $\vec{v}$ is a tangential vector, $\vec{\omega}$ is a axial vector and $\vec{r}$ is a radial vector. These three vectors are mutually perpendicular.

but $\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow \vec{a} = \frac{d}{dt}(\vec{\omega} \times \vec{r})$

or $\vec{a} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$

$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$$

$$\vec{a} = \vec{a}_T + \vec{a}_C$$



Here $\vec{a}_T$ is tangential acceleration and $\vec{a}_C$ is centripetal acceleration.

Hence $\vec{a}_T$ and $\vec{a}_C$ are two components of net linear acceleration.

Tangential Acceleration ($\vec{a}_r$):

$\vec{a}_T = \vec{\alpha} \times \vec{r}$, its direction is parallel to velocity.

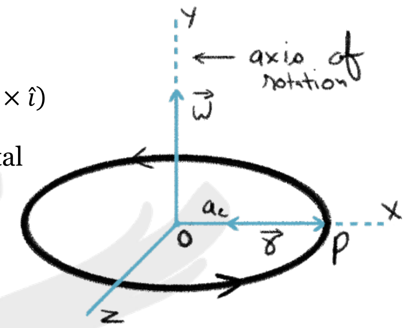
As $\vec{\omega}$ and $\vec{\alpha}$ both are parallel and along the axis so that $\vec{v}$ and $\vec{a}_T$ are also parallel and along the tengential direction.

Magnitude of tangential acceleration is

$$a_T = \alpha\, r \sin 90° \Rightarrow a_T = \alpha\, r$$

As $\vec{a}_T$ is along the direction of motion (along $\vec{v}$) so that $\vec{a}_T$ is responsible for change in speed of particle. Its magnitude is rate of change of speed of the particle. On circular path with constant speed tangential acceleration is zero.

Centripetal Acceleration (a_c):



$\vec{a}_c = \vec{\omega} \times \vec{v} \Rightarrow \vec{a}_c = \vec{\omega} \times (\vec{\omega} \times \vec{r})$

Let $\vec{r}$ is in $\hat{\imath}$ direction and $\vec{\omega}$ is in $\hat{\jmath}$ direction, then direction of $\vec{a}_c$ is along $\hat{\jmath} \times (\hat{\jmath} \times \hat{\imath})$

$= \hat{\jmath} \times (-\hat{k}) = -\hat{\imath}$ opposite direction of $\vec{r}$ i.e. from P to O and it is centripetal direction.

Magnitude of centripetal acceleration is

$$a_c = \omega v = \frac{v^2}{r} = \omega^2 r \Rightarrow \vec{a}_c = \frac{v^2}{r}(-\hat{r})$$

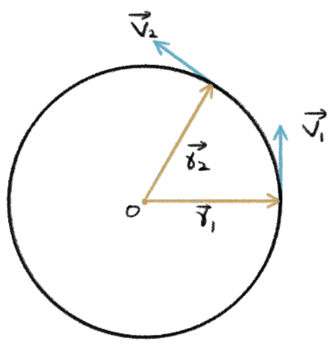
Net Linear Acceleration:

As $a_T \perp a_c$ so that

By $\vec{a} = \vec{a}_T + \vec{a}_c \Rightarrow |\vec{a}| = \sqrt{a_T^2 + a_c^2}$

Uniform Circular Motion:

When a particle moves in a circle at a constant speed then the motion is said to be a uniform circular motion.



In this motion, position vector Keep changing continuously.

Speed is constant, so that

$\vec{a}_T = 0$ $\qquad\qquad \alpha = 0$

Acceleration of particle $\vec{a} = \vec{a}_c = \vec{\omega} \times \vec{v}$

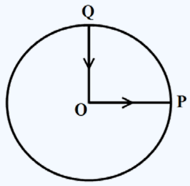
Or $a = \omega v$

$$\therefore a = \frac{v^2}{r} = \omega^2 r = \text{Centripetal acceleration}$$

Due to centripetal acceleration the velocity of the particle keeps on changing the direction i.e. the particle is accelerated.

Practice Questions

Q1. A cyclist starts from the centre O of a circular park of radius 1 km, reaches the edge P of the park, then cycles along the circumference, and returns to the centre along QO as shown in Fig. If the round trip takes 10 min, what is the (a) net displacement, (b) average velocity, and (c) average speed of the cyclist? **[NCERT Exercise]**



Sol. a) As the initial and the final pointscoincide at the centre of the circle therefore, the net displacement of the cyclist is zero.

b) The average velocity is the ratio of net displacement to the total time taken during the motion. Since, the net displacement of the cyclist is zero therfore, the average velocity is also zero.

c) Average speed of the cyclist is given by the relation:

$$\text{Average speed} = \frac{Total\ path\ length}{Total\ time}$$

$$\text{Total path length } = OP + PQ + QO = 1 + \frac{1}{4}(2\pi \times 1) + 1$$

$$= 2 + \frac{1}{2}\pi = 3.570\ km$$

$$\text{Time taken } = 10 \text{ min} = \frac{10}{60} = \frac{1}{6}\ h$$

$$\therefore \text{ Average speed} = \frac{3.570}{\frac{1}{6}} = 21.42\ km/h$$

Q2. A passenger arriving in a new town wishes to go from the station to a hotel located 10 km away on a straight road from the station. A dishonest cabman takes him along a circuitous path 23 km long and reaches the hotel in 28 min. What is (a) the average speed of the taxi, (b) the magnitude of average velocity? Are the two equal? **[NCERT Exercise]**

Sol. Given, the actual path length travelled is 23 km, displacement is 10 km and the time taken is 28 min.

a) Average speed of the taxi is given as,

$$v_{avg} = \frac{\text{Total Path length}}{Time}$$

Substitute the values in above equation.

$$v_{avg} = \frac{\text{Total Path length}}{Time} = \frac{23km}{28\ min} = \frac{23km}{\frac{28}{60}hr} = 49.3\ km/h$$

Hence, the average speed of the taxi is 49.3 km/h.

b) Magnitude of the average velocity is given as,

v_{avg} = Displacement/Time

Substitute the values in the above equation.

v_{avg} = Displacement/Time = 10 km/(28 min) = $\frac{10\ km}{\frac{28}{60}hr}$ = 21.4 km/h

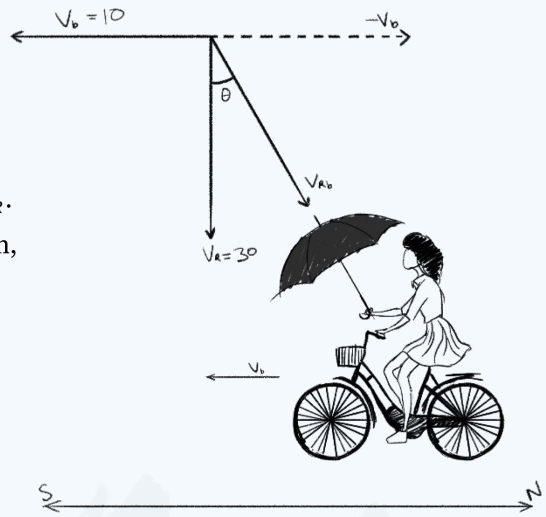
c) From the calculation in (a) and (b), it is obtained that the average speed is not equal to the magnitude of average velocity.

Q3. Rain is falling vertically with a speed of 30 $m\ s^{-1}$. A woman rides a bicycle with a speed of 10 $m\ s^{-1}$ in the north to south direction. What is the direction in which she should hold her umbrella?

(Removed for 2023-2024) **[NCERT Exercise]**

Sol. Given: The velocity of rain falling vertically is 30 ms^{-1} and velocity of bicycle is 10 ms^{-1} in north to south direction.

The situation can be described as shown in the figure below.



The direction in which the woman holds the umbrella is given by,

$$\tan\theta = \frac{|v_b|}{|v_R|}$$

Where, velocity of bicycle is v_b and velocity of rain is v_R.

By substituting the given values in the above expression, we get

$$\tan\theta = 10/30 = \theta = \tan^{-1}\left(\frac{1}{3}\right) = 18°26'$$

Thus, the woman must hold the umbrella toward the south, at an angle of $18°26'$ with the vertical.

Q4. A man can swim with a speed of 4.0 km/h in still water. How long does he take to cross a river 1.0 km wide if the river flows steadily at 3.0 km/h and he makes his strokes normal to the river current? How far down the river does he go when he reaches the other bank? **[NCERT Exercise]**

(Removed for 2023-2024)

Sol. Here vector $v_{MR} = 4\ kmh^{-1}$ = velocity of man in still water.

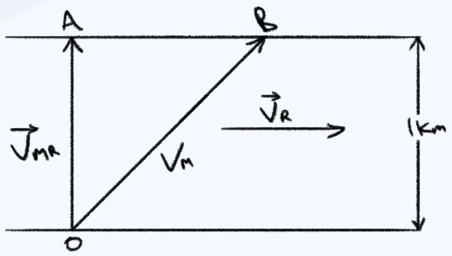
Vector $v_r = 3\ kmh^{-1}$

OA = 1 km

Let t = time taken by man to reach the other bank

Then, $t = OA/v_{MR} = \frac{1}{4} = 0.25\ h = 15\ min.$

Distance, $AB = v_R \times t = 3 \times 0.25 = 0.75\ km = 750\ m.$



Q5. A cricketer can throw a ball to a maximum horzontal distance of 100 m. How much high above the ground can the cricketer throw the same ball? **[NCERT Exercise]**

Sol. Maximum horizontal distance, $R = 100\ m$

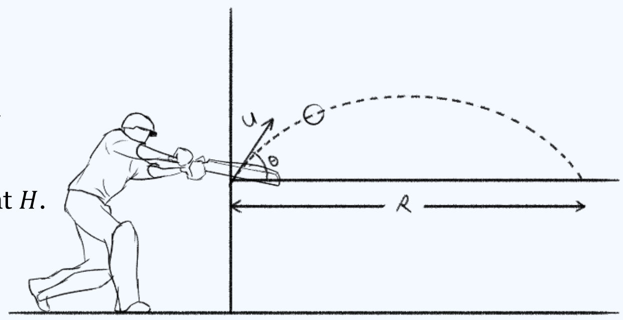
The cricketer will only be able to throw the ball to the maximum horizontal distance when the angle of projection is 45°, i.e., $\theta = 45°$.

The horizontal range for a projection velocity v, is given by the relation:

$R = (u^2 \sin 2\theta)/g$

$100 = (u^2 \sin 90°)/g$

$u^2/g = 100$ ...(i)



The balll will achieve the maximum height when it is thrown vertcally upward. For such motion, the final velocity v is zero at the maximum height H.

Acceleration , $a = -g$

Using the third equation of motion:

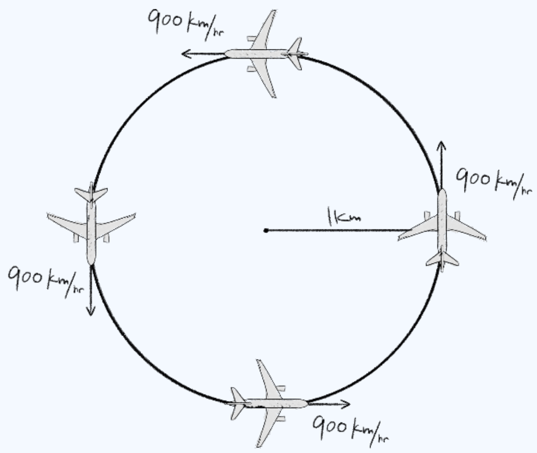
$v^2 - u^2 = -2gH$

$$H = \frac{u^2}{2g} = \frac{100}{2} = 50\ m$$

Q6. An aircraft executes a horizontal loop of radius 1.00 km with a steady speed of 900 kmh^{-1}. compare its centripetal acceleration with the acceleration due to gravity. **[NCERT Exercise]**

Sol. The correct option is A 62.5 m/s^2

Centripetal acceleration



$$a_c = \frac{V^2}{R}$$

$$V = 900\ kmp = 900 \times \frac{5}{18} = 250\ ms^{-1}$$

$$R = 1\ km = 10^3\ m$$

$$\Rightarrow a_c = \frac{250 \times 250}{1000} = 62.5\ ms^{-2}$$

Acceleartion due to gravity, $g = 9.8\ m/s^2$

$$\frac{a_c}{g} = \frac{62.5}{9.8} = 6.38$$

$$a_c = 6.38\ g$$

Q7. An aircraft is flying at a height of 3400 m above the ground. If the angle subtended at a ground observation point by the aircraft positions 10.0 s apart is 30°, what is the speed of the aircraft? **[NCERT Exercise]**

Sol. Given: The height at which aircraft is flying is 3400 m, the angle subtended at a ground observation point by the aircraft positions 10.0 s apart is 30°.

The situation is shown in figure below.

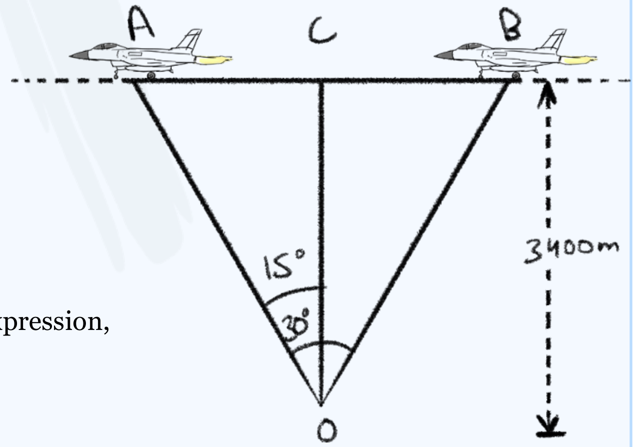
OC is the height of the aircraft. In 10 s the aircraft covers a distance AB.

In triangle AOC,

$\tan\theta = AC/OC \Rightarrow\ AC = OC \times \tan\theta$

By substituting the given values in the above expression,

We get,



$AC = 3400 \times \tan(15°) = 911\ m$

The distance covered by aircraft in 10 s is,

$AB = 2AC = 2 \times 911 = 1822\ m$

The speed of the aircraft is given as,

$v = \frac{Distance}{Time} = \frac{AB}{t}$

By substituting the given values in the above expression,

We get

$v = 1822/10 = 182.2\ ms^{-1}$

Thus, the speed of the aircraft is 182.2 ms^{-1}.

Q8. The horizontal range of a projectile fired at an angle of 15° is 50 m. If it is fired with the same speed at an angle of 45°, its range will be **[NCERT Exemplar]**

A) 60 m B) 71 m C) 100 m D) 141 m

Sol. When a particle is thrown obliquely near the earth's surface, it moves along a curved path under constant acceleration that is directed towards the centre of the earth. The path of such a particle is called a projectile and the motion is called projectile motion.

Let us consider a ball projected at an angle θ with respect to the horizontal x-axis with the initial velocity u as shown below:

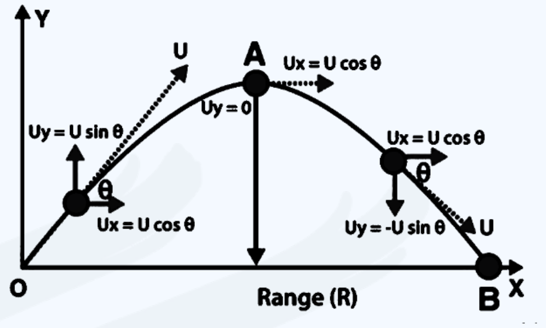
The point O is called the point of projection; θ is the angle of projection and OB = Horizontal Range or Simply Range. The total time taken by the particle from reaching O to B is called the time of flight.

In a projectile motion the Horizontal Range is given by (R)

$$Horizontal\ Range\ (R) = \frac{u^2 \sin 2\theta}{g}$$

Given

- The horizontal range of a projectile fixed at an angle = 15°
- Range = 50 m



In first case range

$50 = u^2(\sin 30°)/g$

$u^2/g = 100$

In 2nd case range

$u^2(\sin 2 \times 45°)/g$

Since $\sin 90° = 1$

$= 100\ m$

Q9. It is found that $|A + B| = |A|$. This necessarily implies, **[NCERT Exemplar]**

A) $B = 0$
B) A, B are antiparallel
C) A, B are perpendicular
D) $A.B \leq 0$

Sol. $B = 0$

Q10. A boy throws a ball in air at 60° to the horizontal along a road with a speed of 10 m/s (36km/h). Another boy sitting in a passing by car observes the ball. Sketch the motion of the ball as observed by the boy in the car, if car has a speed of (18km/h). Give explanation to support your diagram. **[NCERT Exemplar]**

Sol. Horizontal component of velocity $u_x = 10 \cos\theta$ Vertical component of velocity $u_y = 10 \sin\theta$

$$u_x = 10 \cos 60° = 10 \times \frac{1}{2} = 5\ m/s$$

Since the speed of car matches with the horizontal speed of the projectile, boy sitting in the car will see only vertical component of motion as shown in Figure.

As, Horizontal Component of velocity

$u_x = 10 \cos\theta$

$u_x = 10 \cos 60° = 10 \times \frac{1}{2} = 5\ m/s$

Vertical component of velocity,

$u_y = 10 \sin\theta$

$u_y = 10 \sin 60° = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}\ m/s$