

Wave Optics

* Wave Front :-

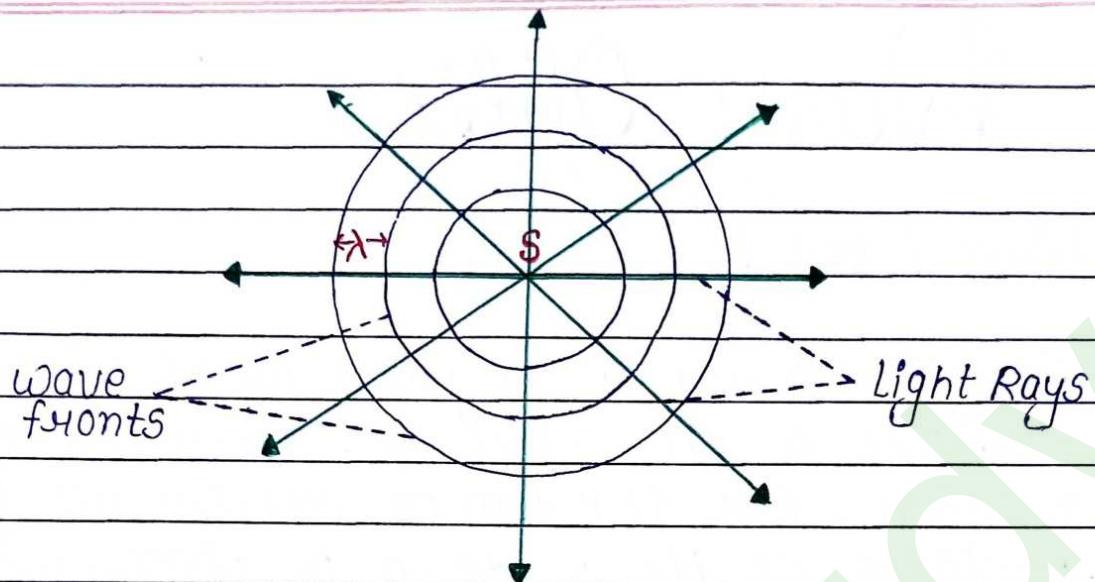
A light source is a point which emits disturbance in all directions. In a homogeneous medium, the disturbance reaches all those particles of the medium in phase, which are located at the same distance from the source of light and hence at all the time, every particle must be vibrating in phase with each other. The locus of all the particles of medium, which at any instant are vibrating in the same phase, is called wave front.

*** Types of Wavefront :-

There are three types of wavefront —

(1) Spherical wavefront

A point source of light produces a spherical wave front. This is because the locus of every points, which are equidistant from the point source, is a sphere.

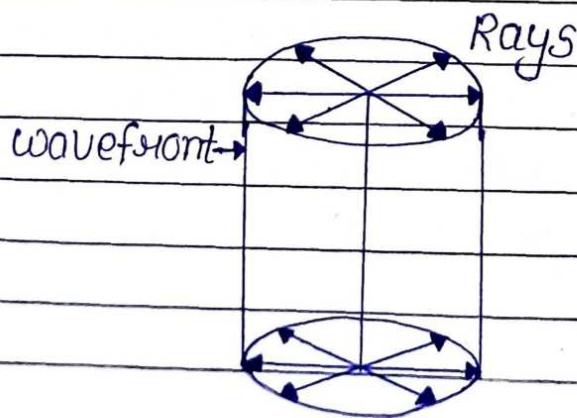


Note: A line perpendicular to wave front is called ray.

A ray represents the path along which light travels.

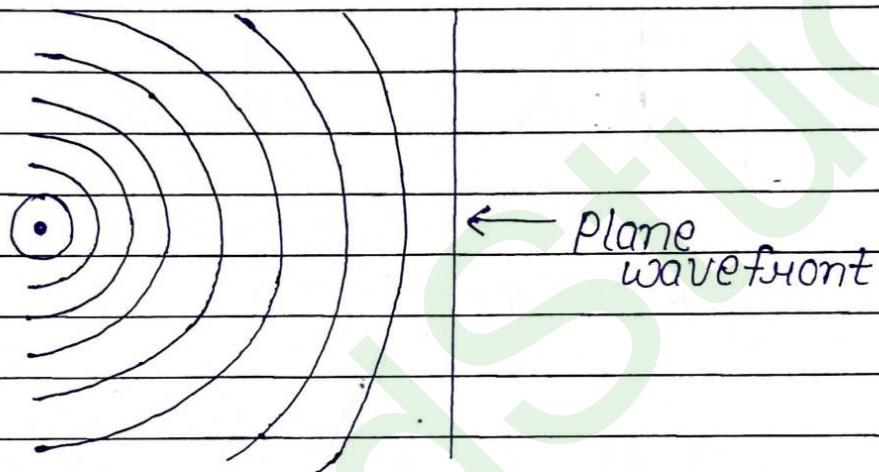
(2) Cylindrical wavefront

If the light source is linear (such as a slit), it produces a cylindrical wave front. Hence, every points, which are equidistant from the linear source, lie on the surface of a cylinder.



(3) Plane wavefront

when the point source or linear source of light is at very large distance a small portion of spherical or cylindrical wavefront appears to be plane. Such wave front is called plane wavelength.



* Ray of Light :-

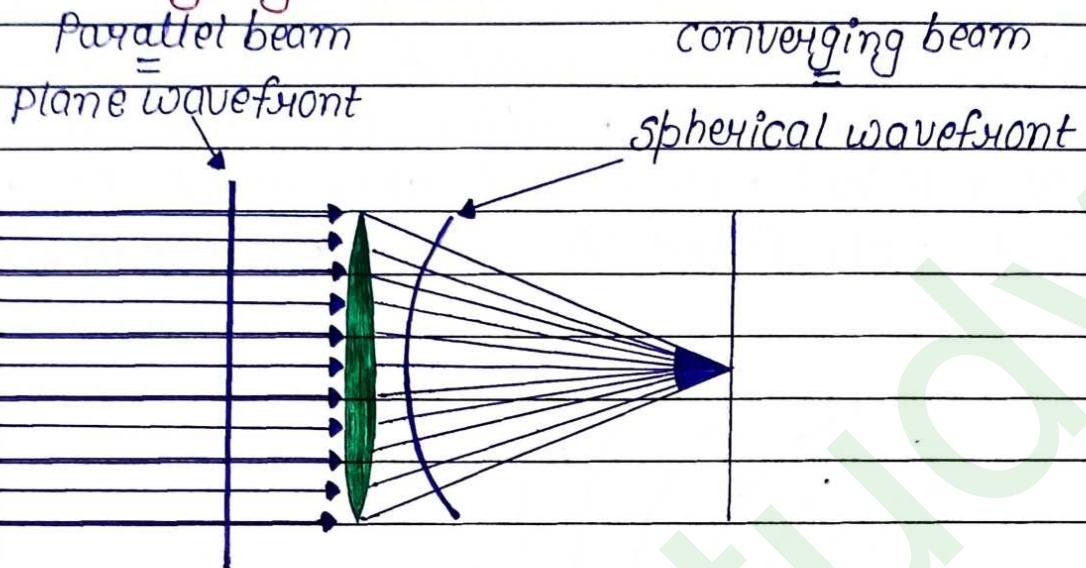
The path along which light travels is known as a ray of light. If we draw an arrow normal to the wave front and which points in the direction of propagation of disturbance represents a ray of light. In a ray diagram, thick arrows represent the ray of light.

It is also called as the wave normal because the ray of light is normal to the wavefront.

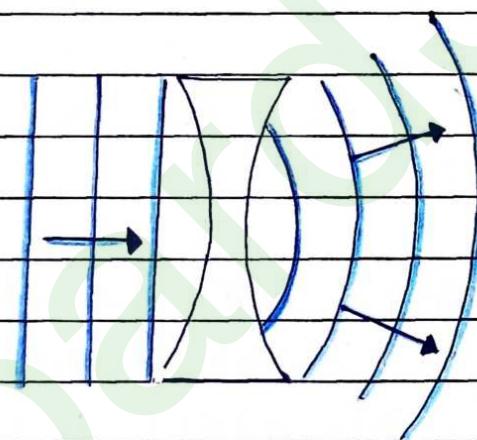
Key points :-

- If we take any two points on a wave front, the phase difference between them will be zero.

* Converging Wavefront :-



* Diverging wavefront :-

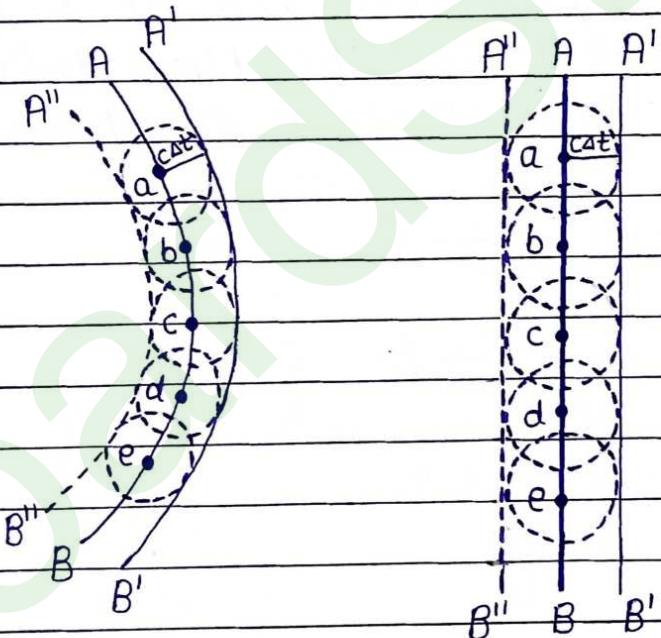


* Huygens Principle :-

Huygens's principle is a geometrical construction, which can be used to obtain new position of a wave front at a later time from its given position at any instant. Or we can quote that this principle gives a method gives an idea about how light spreads out in the medium.

It is developed on the following assumptions:-

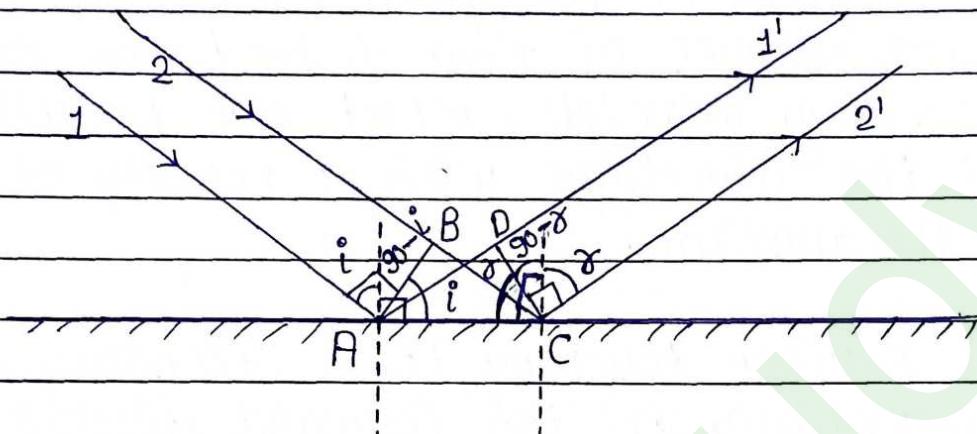
- Every point on the given wavefront acts as fresh source of new disturbance called Secondary wavelets, which are travelling in all the directions with a velocity of light in the medium.
- The surface touching these secondary wavelets tangentially in the forward direction at any instant gives new wavefront at that instant. This is called secondary wavefront.



Key point :-

- Huygens principle is simply a geometrical construction to find the position of wave front at a later time.

* Reflection on the basis of wave theory :-



Let 1, 2 are the incident rays and 1', 2' are the reflected rays. If v is the velocity of light, t is the time taken by light to go from B to C or A to D.

$$t = \frac{BC}{v} \quad \text{--- (1)}$$

$$t = \frac{AD}{v} \quad \text{--- (2)}$$

Equate 1 and 2.

$$\frac{BC}{v} = \frac{AD}{v}$$

$$BC = AD \quad \text{--- (3)}$$

In $\triangle ABC$

$$\sin i = \frac{BC}{AC}$$

$$BC = AC \sin i$$

In $\triangle ADC$

$$\sin \gamma = \frac{AD}{AC}$$

$$AD = AC \sin \gamma$$

From equation ③ —

$$AC \sin i = AC \sin \gamma$$

$$\sin i = \sin \gamma$$

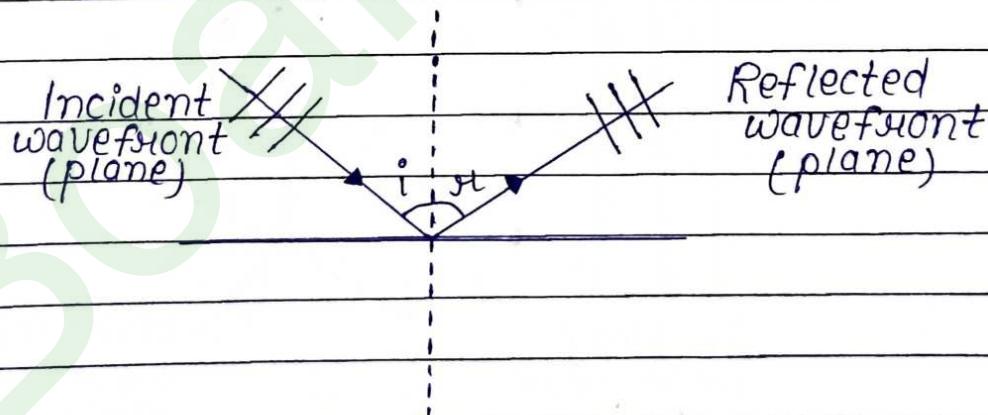
$$i = \gamma$$

law of reflection.

* Wavefronts for Reflection :-

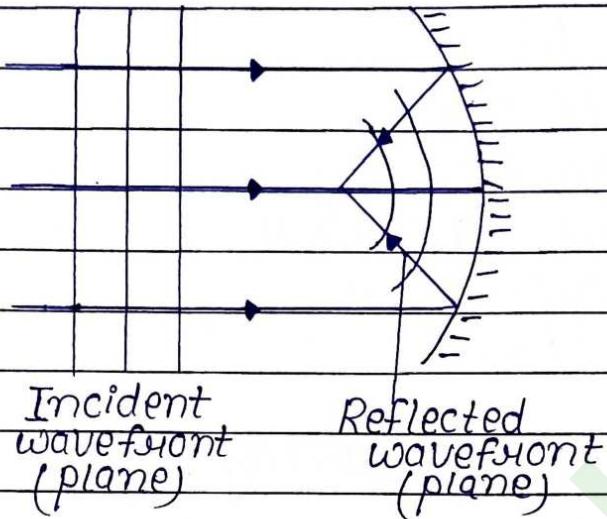
- If light falls on a plane mirror —

If the plane wavefronts are being reflected on the plane mirror, the shape of the wavefront of the reflected light is again planar.



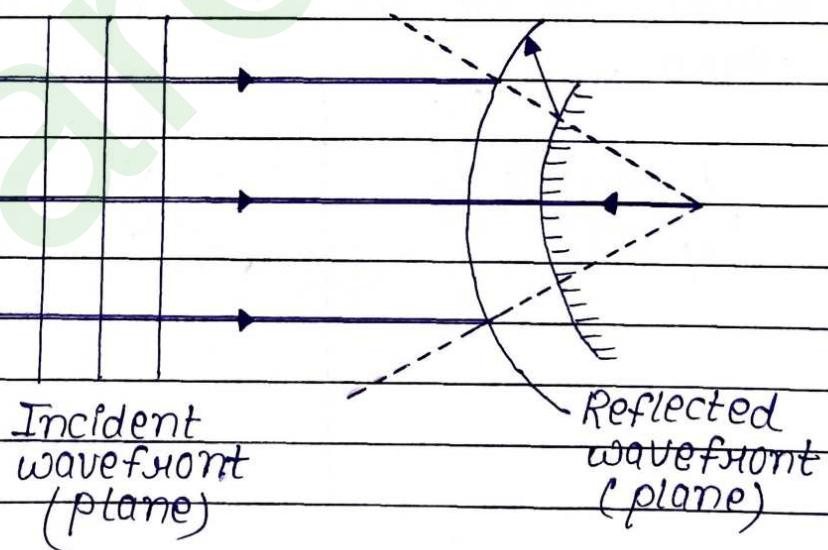
- If light falls on a concave mirror —

If a plane wavefront falls on a concave mirror, the shape of the reflected light is spherical.

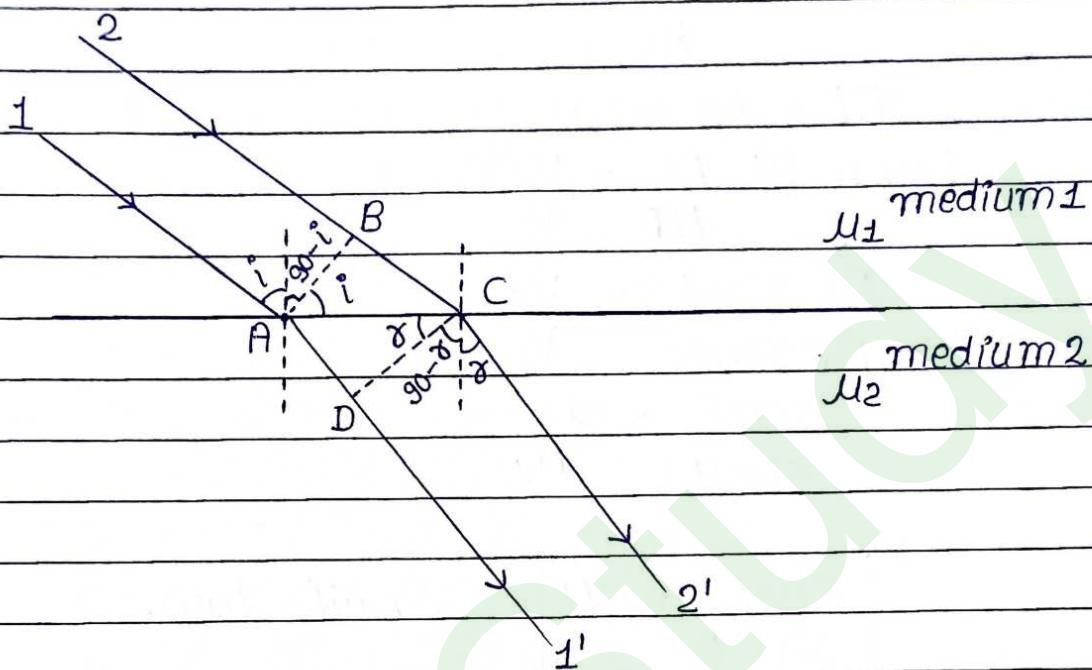


- If light falls on a convex mirror -

If a plane wavefront falls on a convex mirror, the shape of the reflected light is spherical.



* Refraction on the basis of wave theory :



Let 1, 2 are the incident rays and 1', 2' are the refracted rays if v_1 is the velocity in medium 1 (μ_1) and v_2 is the velocity in medium 2 (μ_2), t is the time taken by light to go from B to C or A to D.

$$t = \frac{BC}{v_1} \quad \text{--- (1)}$$

$$t = \frac{AD}{v_2} \quad \text{--- (2)}$$

from (1) and (2) —

$$\frac{BC}{v_1} = \frac{AD}{v_2} \quad \text{--- (3)}$$

In $\triangle ABC$

$$\sin i = \frac{BC}{AC}$$

$$BC = AC \sin i$$

In $\triangle ADC$

$$\sin \mu = \frac{AD}{AC}$$

$$AD = AC \sin \mu$$

$$\text{From (3)} \quad BC = \frac{V_1}{AD} V_2$$

$$\frac{AC \sin i}{AC \sin \mu} = \frac{V_1}{V_2}$$

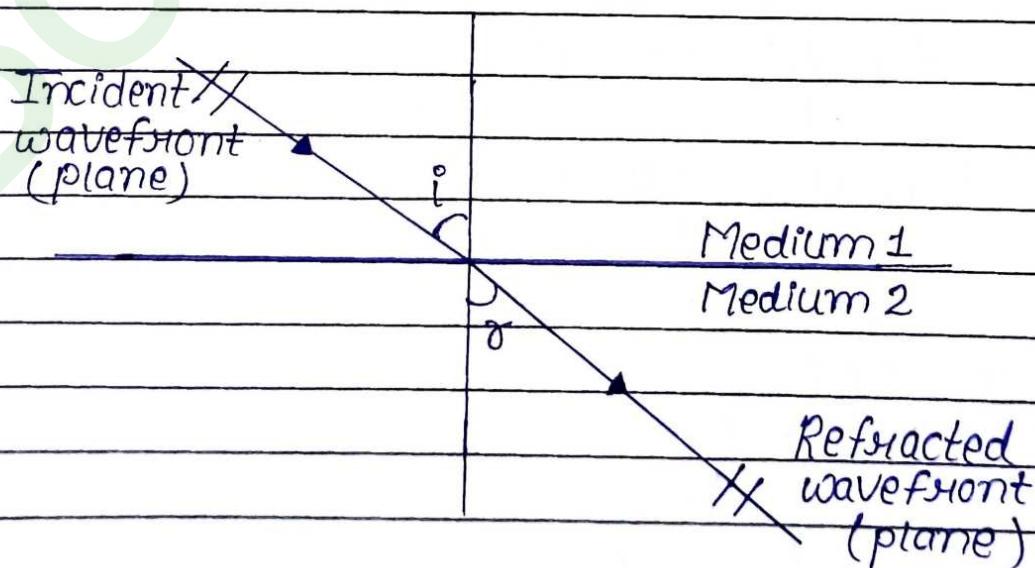
$$\frac{\sin i}{\sin \mu} = \frac{V_2}{V_1}$$

$\sin i$	$= \mu$	Snell's law.
$\sin \mu$		

Wavefronts for Refraction :-

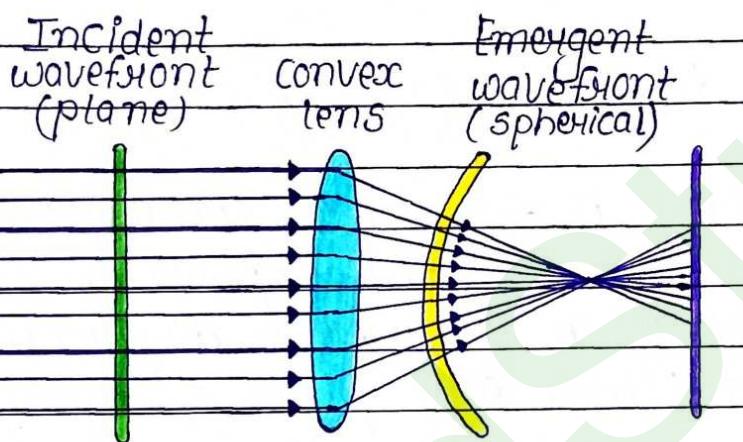
- If light falls on plane surfaces

If a plane wavefront falls on a plane surface, the refracted ray will also have a plane wavefront.



- If light falls on curved Surfaces

If a plane wavefront falls on a converging (or) diverging lens, the emergent light will have a spherical wavefront.



* Coherent Sources :-

Coherent sources are the sources of light which emits continuous light waves with same wavelength, frequency and in phase or having a constant phase difference.

Condition for obtaining coherent source :-

1. Coherent source of light should be obtained from a single source by some device.
2. The two source of light should give monochromatic light i.e., (light of same wave length) The path difference between the light waves from the two sources should be small.

Note: Two independent source can never act as coherent source.

Incoherent source :-

An incoherent source emits a light wave having a different frequency, wavelength and phase.

→ Important terms to know :-

phase :- phase is defined as the argument of sine or cosine in the expression for displacement of a wave. For displacement $y = a \sin \omega t$: term ωt = phase or instantaneous phase.

phase Difference (ϕ) :- phase difference is the difference between the phases of two waves at a point. i.e. if $y_1 = a_1 \sin \omega t$ and $y_2 = a_2 \sin (\omega t + \phi)$ so phase difference = ϕ

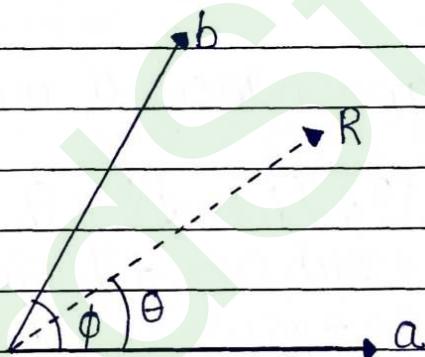
path Difference (Δ) :- path difference between the waves at that point is the difference in path lengths of two waves meeting at a point. Also $\Delta = \frac{\lambda}{2\pi} \times \phi$.

Time Difference (T.D) :- Time difference between the waves meeting at a point is given by $T.D = \frac{T}{2\pi} \times \phi$.

* Principle of Superposition :-

When two or more waves traveling through a medium, superimpose on each other then a new wave is formed in which resultant displacement is equal to vector sum of displacement due to individual waves.

$$\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + \dots + \vec{y}_n$$



Let us consider the light wave from two sources are

$$y_1 = a \sin \omega t$$

$$y_2 = b \sin(\omega t + \phi)$$

when they superimpose on each other, then

$$y = y_1 + y_2$$

$$y = a \sin \omega t + b \sin(\omega t + \phi).$$

$$\therefore \sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$y = a \sin \omega t + b [\sin \omega t \cos \phi + \cos \omega t \sin \phi]$$

$$y = a \sin \omega t + b \sin \omega t \cos \phi + b \cos \omega t \sin \phi$$

$$y = \sin \omega t (a + b \cos \phi) + b \cos \omega t \sin \phi.$$

$$\text{let } a + b \cos \phi = R \cos \theta \quad \text{--- (1)}$$

$$b \sin \phi = R \sin \theta \quad \text{--- (2)}$$

$$y = \sin \omega t R \cos \theta + \cos \omega t (R \sin \theta)$$

$$y = R [\sin \omega t \cos \theta + \cos \omega t \sin \theta]$$

$$y = R [\sin(\omega t + \theta)]$$

$$y = R \sin(\omega t + \theta)$$

where R is Resultant Amplitude.

Resultant Amplitude :-

Squaring and adding (1) and (2) -

$$(a + b \cos \phi)^2 + (b \sin \phi)^2 = (R \cos \theta)^2 + (R \sin \theta)^2$$

$$a^2 + b^2 \cos^2 \phi + 2ab \cos \phi + b^2 \sin^2 \phi = R^2 \cos^2 \theta + R^2 \sin^2 \theta$$

$$a^2 + b^2 (\cos^2 \phi + \sin^2 \phi) + 2ab \cos \phi = R^2 (\cos^2 \theta + \sin^2 \theta)$$

$$a^2 + b^2 + 2ab \cos \phi = R^2$$

$$\therefore \cos^2 \phi + \sin^2 \phi = 1$$

$$R^2 = a^2 + b^2 + 2ab \cos \phi$$

$$R = \sqrt{a^2 + b^2 + 2ab \cos \phi}$$

for R to be maximum, $\cos \phi$ should be max.

$$\phi = 0$$

$$\phi = 0, 2\pi, 4\pi, \dots$$

$$\phi = 2n\pi, \text{ where } n = 0, 1, 2, 3, \dots$$

$$R = \sqrt{a^2 + b^2 + 2ab}$$

$$R = \sqrt{(a+b)^2}$$

$$R = a+b \text{ max.}$$

For R to minimum, $\cos\phi$ should be minimum.

$$\cos\phi = -1$$

$$\phi = \pi, 3\pi, 5\pi \dots$$

$$\phi = (2n-1)\pi \text{ where, } n = 1, 2, 3 \dots$$

$$R = \sqrt{a^2 + b^2 + 2[ab(-1)]}$$

$$R = \sqrt{a^2 + b^2 - 2ab}$$

$$R = |a - b| \text{ min.}$$

Resultant Intensity :-

$$I \propto (\text{Amplitude})^2$$

$$I_1 \propto a^2$$

$$I_2 \propto b^2$$

$$I_R \propto R^2$$

$$I_1 = ka^2$$

$$I_2 = kb^2$$

$$I_R = KR^2$$

putting the value of R

$$I_R = K [a^2 + b^2 + 2ab \cos\phi]$$

$$I_R = K \left[\frac{I_1}{K} + \frac{I_2}{K} + 2 \sqrt{\frac{I_1}{K}} \cdot \sqrt{\frac{I_2}{K}} \cos\phi \right]$$

$$I_R = \frac{K}{K} \left[I_1 + I_2 + 2 \sqrt{I_1 \cdot I_2} \cos\phi \right]$$

$$I_R = [I_1 + I_2 + 2 \sqrt{I_1 \cdot I_2} \cos\phi]$$

$$\boxed{I_R = [I_1 + I_2 + 2 \sqrt{I_1 \cdot I_2} \cos\phi]}$$

Resultant Intensity Due to Two Identical waves :-

The resultant intensity for two coherent sources is given by

$$T = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi$$

For identical source $I_1 = I_2 = I_0$

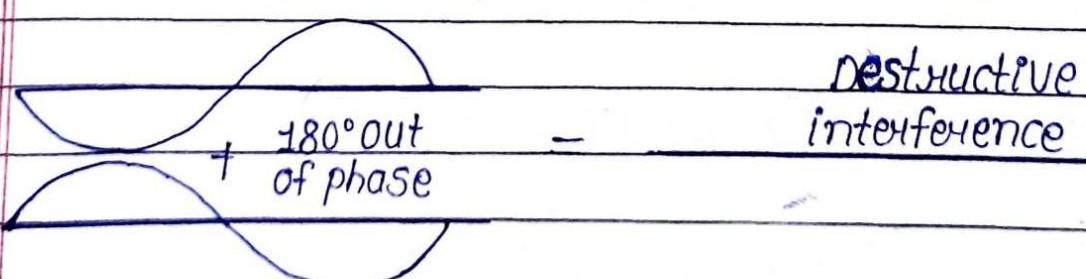
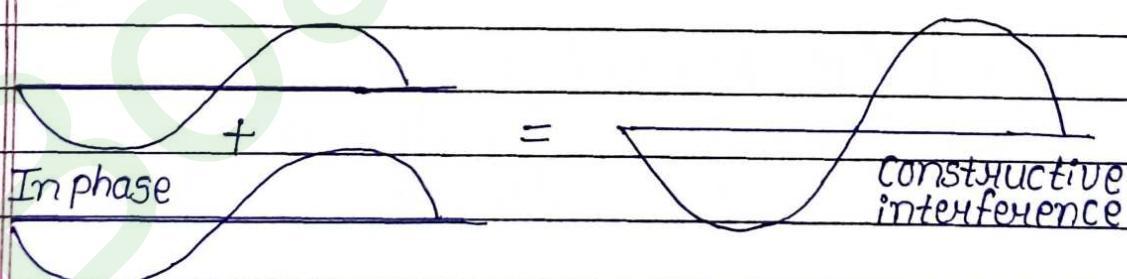
$$\Rightarrow I = I_0 + I_0 + 2\sqrt{I_0 I_0} \cos \phi = 4I_0 \cos^2 \frac{\phi}{2}$$

$$\left[1 + \cos \theta = 2 \cos^2 \frac{\theta}{2} \right]$$

* Interference of light :-

The phenomenon of redistribution of light energy in a medium on account of superposition of light wave from two coherent source.

Types of Interference :-



Constructive Interference

Constructive interference is obtained at a point when the waves meet at that point with same phase (i.e. maximum light)

Destructive Interference

Destructive interference is obtained at that point when the wave meets at that point with opposite phase (i.e. minimum light)

Phase difference between the waves at the point of observation $\phi = 0^\circ \text{ or } 2n\pi$.

$$\phi = 180^\circ \text{ or } (2n-1)\pi; n = 1, 2, \dots$$

$$\text{or } (2n+1)\pi; n = 0, 1, 2, \dots$$

Resultant amplitude at the point of observation will be maximum if $a_1 = a_2 \Rightarrow A_{\max} = 2a_0$

Resultant amplitude at the point of observation will be minimum $A_{\min} = a_1 - a_2$

$$a_1 = a_2 = a_0 \Rightarrow A_{\max} = 2a_0$$

$$\text{If } a_1 = a_2 \Rightarrow A_{\min} = 0$$

Resultant intensity at the point of observation will be maximum,

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

$$I_{\max} = (\sqrt{I_1} + \sqrt{I_2})^2$$

Resultant intensity at the point of observation will be minimum.

$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

$$I_{\min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

$$\text{If } I_1 = I_2 = I_0 \Rightarrow I_{\max} = 2I_0$$

$$\text{If } I_1 = I_2 = I_0 \Rightarrow I_{\min} = 0$$

Young's Double slit Experiments :-

Necessary condition for YDSE -

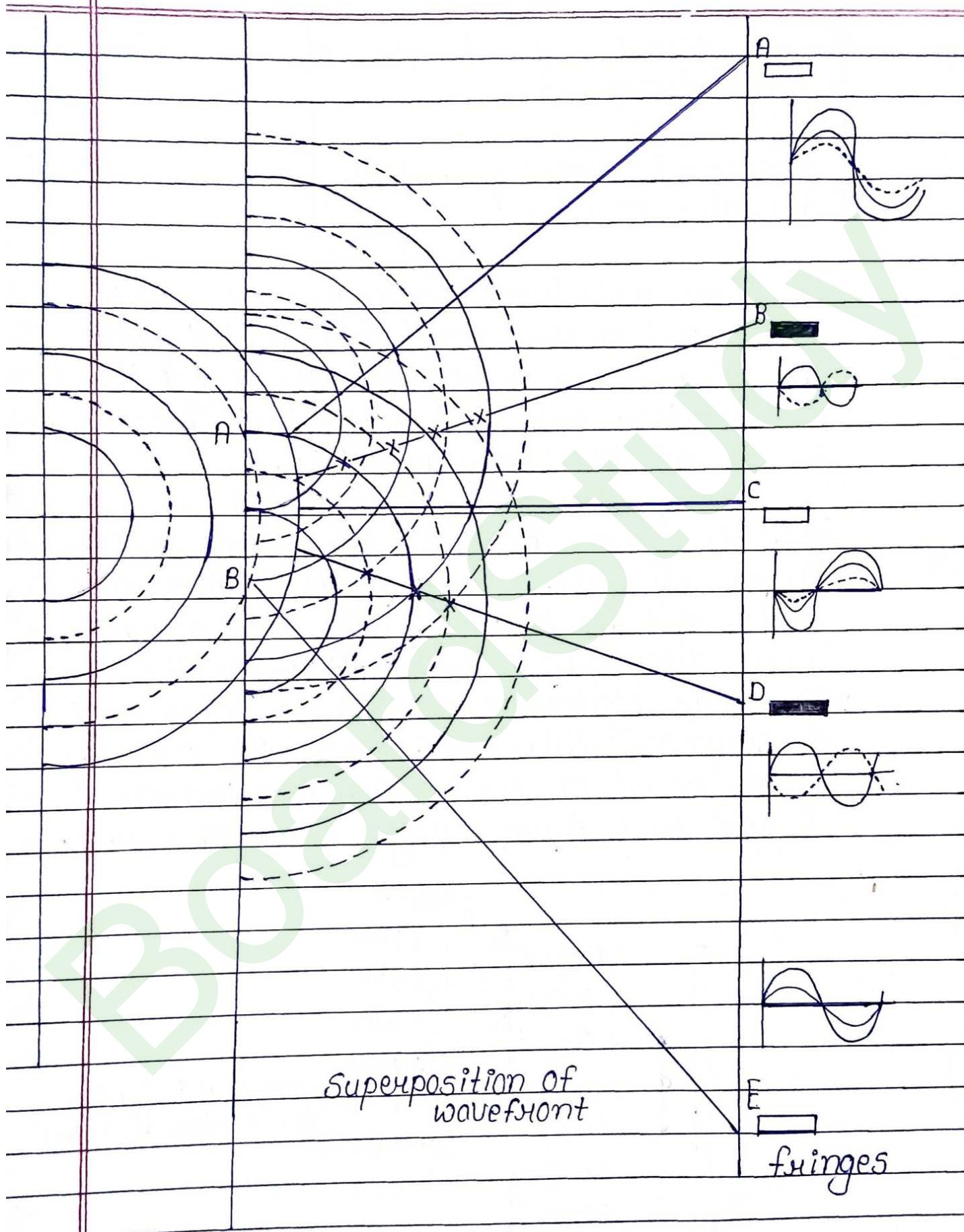
- 1) In interference the source of light should monochromatic.
- 2) Wave should of same frequency.
- 3) direction of wave should be same.
- 4) Amplitude of both wave should also be same.
- 5) The slits should be thin.

S is a narrow slit illuminated by monochromatic source light at a suitable distance (10 cm) from S, there are two fine slits A and B placed symmetrically parallel to S when a screen is placed at a large distance (2m) the slit A and B then alternate bright and dark fringes appears on the screen.

Explanation :-

The appearance of bright dark fringes can be explained on the basis of interference of light.

According to Huygen's principle, the monochromatic source of light illuminating the slit S sends spherical wavefront.



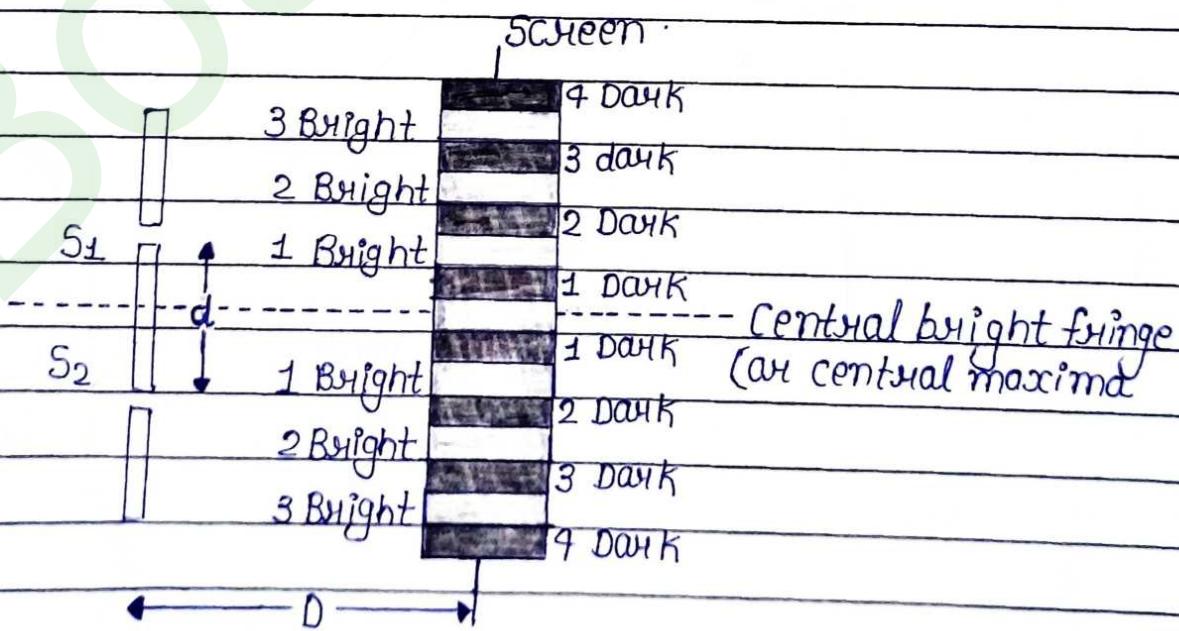
Crust - crust \rightarrow , bright
 through - through \rightarrow

Crust - through \rightarrow , dark
 through - crust \rightarrow

In these wavefront solid are merresents
 crust and dotted are merpresent through.

The wavefronts reach are the slit A and
 B simultaneously which in turn become
 source of secondary wavelets.

Thus two wave of same amplitude and same
 frequency with zero phase difference are
 given out by A and B. The light from
 these two will interfere. If merpresent con-
 structive interference and if merpresent
 destructive interfere. When the pattern
 is observed on the screen; alternate
 bright and dark frings were observed.



d = Distance between slits.

D = Distance between slits and screen.

Calculation of path difference :-

$$\text{Path difference } (x) = \frac{\lambda}{2\pi} \phi$$

* Condition for constructive and Destructive Interference :-

$$\text{For max. } \phi = 2n\pi$$

$$x = \frac{\lambda}{2\pi} \times 2n\pi$$

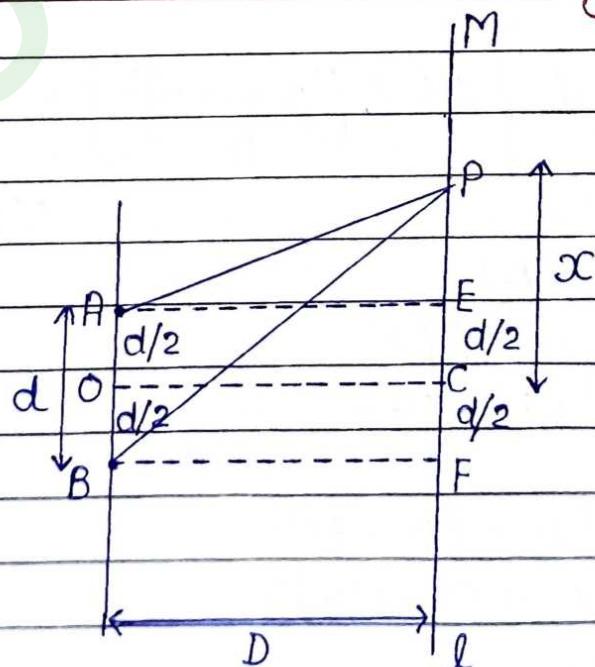
$$x = n\lambda \quad n = 0, 1, 2, 3, \dots$$

$$\text{For min. } \phi = (2n-1)\pi$$

$$x = \frac{\lambda}{2\pi} \times (2n-1)\pi$$

$$x = \frac{\lambda}{2} (2n-1) \quad n = 1, 2, 3, \dots$$

* Calculation of Position of Fringes :-



- A and B are two coherent sources at distance 'd'.
- A screen is placed at distance D.
- O is the centre between the slits A and B.
- Let us choose any one point P at height x.

The path length from two source is AP and BP. Path difference = BP - AP — (1)

In $\triangle BPF$ —

$$BP^2 = BF^2 + PF^2$$

$$BP = \sqrt{BF^2 + PF^2}$$

$$BP = \sqrt{D^2 + (x+d/2)^2}$$

$$BP = [D^2 + (x+d/2)]^{1/2}$$

$$BP = [D^2 (1 + \frac{(x+d/2)^2}{D^2})]^{1/2}$$

$$BP = D [1 + \frac{(x+d/2)^2}{D}]^{1/2}$$

Expanding Binomial

$$BP = D \left[1 + \frac{1}{2} \left(\frac{x+d/2}{D} \right)^2 \right] — (2)$$

$$\because (1+x)^n = 1 + nx \dots$$

Now, in $\triangle APE$

$$AP^2 = AE^2 + PE^2$$

$$AP = \sqrt{AE^2 + PE^2}$$

$$AP = \sqrt{D^2 + (PC - EC)^2}$$

$$AP = \sqrt{D^2 + (x-d/2)^2}$$

$$AP = [D^2 + (x - d/2)^2]^{1/2}$$

$$AP = [D^2 \left(1 + \frac{(x - d/2)^2}{D^2}\right)]^{1/2}$$

$$AP = D \left[1 + \frac{(x - d/2)^2}{D^2}\right]^{1/2}$$

$$AP = D \left[1 + \frac{(x - d/2)^2}{D^2}\right]^{1/2}$$

$$AP = D \left[1 + \frac{1}{2} \left(\frac{x - d/2}{D}\right)^2\right] \quad \text{--- (3)}$$

Put (2) and (3) in eqn - (1) :-

$$\text{Path difference} = BP - AP$$

$$= D \left[1 + \frac{1}{2} \left(\frac{x + d/2}{D}\right)^2\right] - D \left[1 + \frac{1}{2} \left(\frac{x - d/2}{D}\right)^2\right]$$

$$= D \left[\frac{1}{2} \left(\frac{x + d/2}{D}\right)^2 - \frac{1}{2} \left(\frac{x - d/2}{D}\right)^2\right]$$

$$= D \left[\frac{1}{2} \left(\frac{(x + d/2)^2}{D^2}\right) - \frac{1}{2} \left(\frac{(x - d/2)^2}{D^2}\right)\right]$$

$$\Rightarrow \frac{1}{2} \frac{D}{D^2} \left[(x + d/2)^2 - (x - d/2)^2\right]$$

$$\Rightarrow \frac{1}{2D} \left[x^2 + \left(\frac{d}{2}\right)^2 + 2x \frac{d}{2} - \left(x^2 + \left(\frac{d}{2}\right)^2 - 2x \frac{d}{2}\right)\right]$$

$$\Rightarrow \frac{1}{2D} \left[x^2 + \frac{d^2}{4} + dx - x^2 - \frac{d^2}{4} + dx\right]$$

$$\Rightarrow \frac{1}{2D} \times 2dx$$

$$= \frac{dx}{D}$$

$$\therefore \text{Path difference} = \frac{dx}{D}$$

→ Far Bright Fringes :

For maxima -

Path difference = $n\lambda$, $n=0, 1, 2, \dots$

$$\frac{x_{cd}}{D} = n\lambda$$

$$x_c = \frac{n\lambda D}{d}$$

for $n=0$, $x_0=0$ i.e. at C central Bright Fringe.

$n=1$, $x_1 = \frac{\lambda D}{d}$. First Bright fringe after centre.

$n=n$, $x_n = \frac{n\lambda D}{d}$ n^{th} Bright fringe.

→ For Dark Fringes :

for minima,

Path difference = $\frac{(2n-1)\lambda}{2}$ for $n=1, 2, 3, \dots$

$$\frac{x_{cd}}{D} = \frac{(2n-1)\lambda}{2}$$

$$x_n = \frac{(2n-1)\lambda}{2} \frac{D}{d}$$

for $n=1$, $x_1 = \frac{\lambda D}{2}$ 1st dark fringe.

$n=2$, $x_2 = \frac{3\lambda D}{2}$ 2nd dark fringe.

$n=n$, $x_n = \frac{(2n-1)\lambda D}{2}$ n^{th} dark fringe.

* Fringe width (B) :-

Fringe width is the distance between two consecutive bright or dark fringes is called fringe width.

For Bright fringe (constructive or maxima) :-

$$B = x_n - x_{n-1}$$

$$\text{Put } n=2 \quad B = x_2 - x_{2-1}$$

$$B = x_2 - x_1$$

$$B = \frac{3\lambda D}{2d} - \frac{\lambda D}{2d}$$

$$B = \frac{2\lambda D}{2d}$$

$B = \frac{\lambda D}{d}$

For dark fringe (destructive or minima) :-

$$B = x_n - x_{n-1}$$

$$\text{Put } n=2 \quad B = x_2 - x_1$$

$$B = \frac{2\lambda D}{d} - \frac{\lambda D}{d}$$

$B = \frac{\lambda D}{d}$

* Intensity of Fringes :-

$$I_R = [I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi]$$

For Bright fringes -

$$\cos \phi = 1$$

$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

In young's double slit experiment,

$$\text{As } I_1 = I_2 = I$$

$$I_{\max} = I + I + 2\sqrt{I^2}$$

$$I_{\max} = 2I + 2I$$

$$I_{\max} = 4I$$

For Dark fringes -

$$\cos \phi = -1$$

$$I_{\min} = I_1 + I_2 + 2\sqrt{I_1 I_2} \times -1$$

$$I_{\min} = I + I + 2\sqrt{I^2} \times -1$$

$$I_{\min} = 2I - 2I$$

$$I_{\min} = 0$$

- Q) Find the ratio of intensities at the two point X and Y on a screen in young's double slit experiment where wave from the two sources S_1 and S_2 having path difference of zero and $\frac{\lambda}{4}$ respectively.

Soln: $I_R = K [a^2 + b^2 + 2ab \cos \phi]$

$$I_X = K [a^2 + b^2 + 2ab \cos 0^\circ]$$

$$I_X = K [a^2 + b^2 + 2ab]$$

$$I_X = K [a+b]^2$$

Similarly,

$$I_y = k \left[a^2 + b^2 + 2ab \cos \frac{\pi}{2} \right]$$

$$I_y = k [a^2 + b^2]$$

$$\frac{I_x}{I_y} = \frac{(a+b)^2}{(a-b)^2}$$

$$x = \frac{\lambda}{2\pi} \phi$$

$$\therefore \phi = \frac{2\pi x}{\lambda}$$

$$\phi = \frac{2\pi x \lambda}{\lambda \times 4}$$

$$\phi = \frac{\pi}{2}$$

* Ratio of Intensity of maxima and minima in Interference :-

$$I_{\max} = k (a+b)^2$$

$$I_{\min} = k (a-b)^2$$

$$\frac{I_{\max}}{I_{\min}} = \frac{k(a+b)^2}{k(a-b)^2}$$

$I_{\max} = (a+b)^2$
$I_{\min} = (a-b)^2$

if,

$$\frac{a}{b} = \mu$$

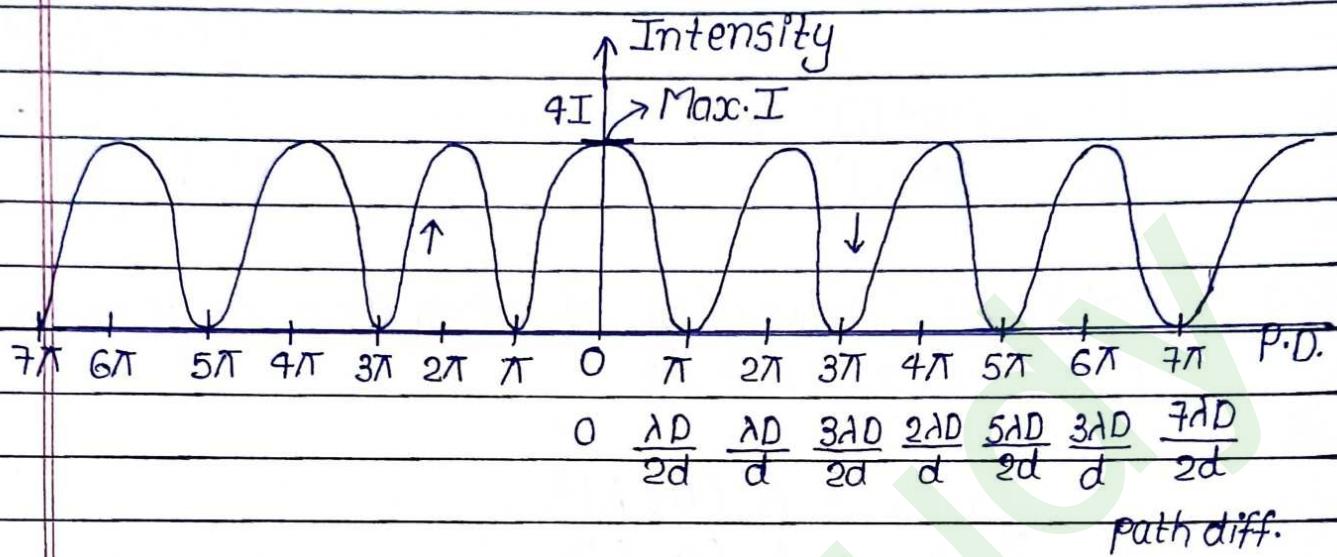
$$\text{then } \frac{a^2}{b^2} = \mu^2$$

$$\frac{I_{\max}}{I_{\min}} = \left(\frac{\mu+1}{\mu-1} \right)^2$$

Note: If w_1 and w_2 are width of two slits from which the intensities of light I_1 and I_2 produce -

$\frac{w_1}{w_2} = \frac{I_1}{I_2} = \frac{a^2}{b^2}$

* Graph between Intensity and phase difference:



* Condition for Sustained Interference of light :-

- The two source providing interference must be coherent.
- The wave must be having same amplitude.
- The two sources must be very close to each and pattern observed at large distance. So that sufficient width of fringes are formed ($\frac{\lambda D}{d}$).
- The source must be monochromatic.
- The two source should be point source.

* Diffraction :-

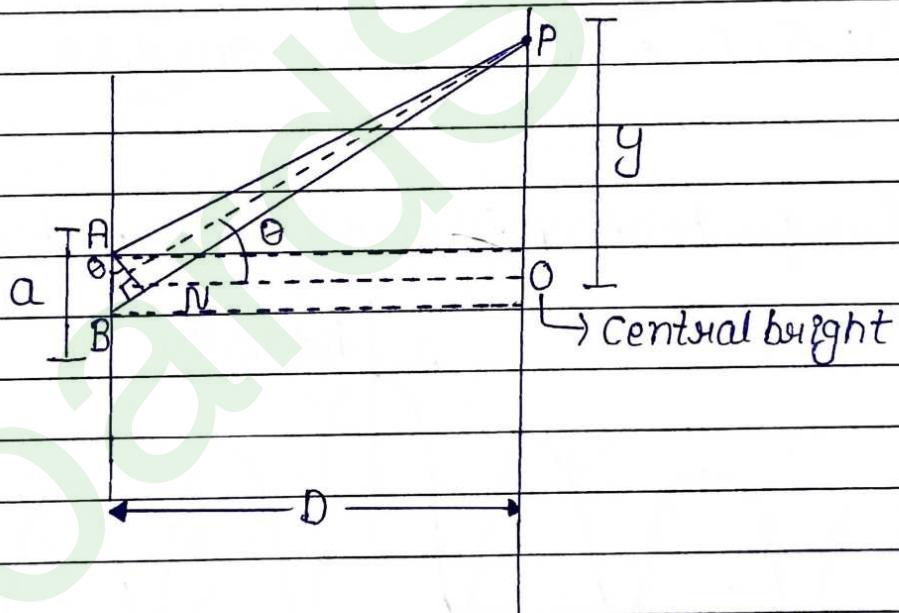
The phenomenon of bending of light around the sharp corners and the spreading of light within the geometrical shadow of opaque obstacles. Obstacles is called diffraction of light.

→ Diffraction is more when slit width ' a ' is smaller than wavelength ' λ ' of light.

$$a < \lambda : \text{Diffraction} \uparrow$$

Diffraction of light at single slit :

AB is an opening 'a' wide. O is a point on a screen at distance 'D'. such that the waves from A and B. reach at same distance, same phase i.e; O is a central bright. Let P is a point at elevation θ .



path difference between wave from A and B.

$$\text{Path difference} = BP - AP$$

$$= BP - NP$$

$$= BN$$

$$\because AP = NP$$

In $\triangle ABN$ -

$$\sin\theta = \frac{BN}{a}$$

$$BN = a \sin\theta$$

Condition for secondary maxima (Bright) :-

We observe that P is a Bright point when path difference is $\frac{3d}{2}, \frac{5d}{2}, \frac{7d}{2} \dots$

$$a \sin \theta_n = (2n+1) \frac{\lambda}{2}$$

$$n = 1, 2, 3 \dots$$

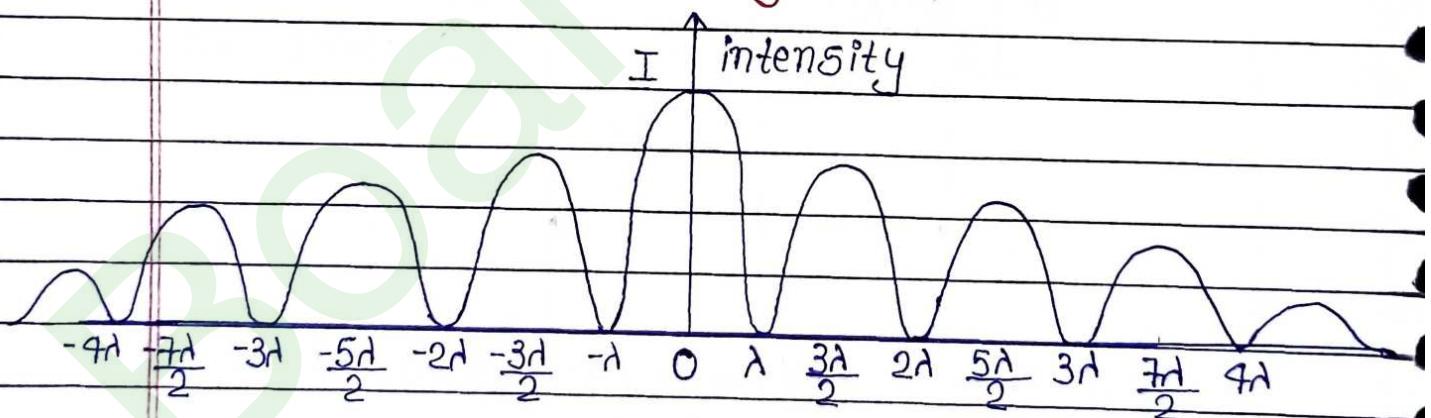
Condition for secondary minima (Dark) :-

We observed that P is dark point, if path difference is $\lambda, 2\lambda, 3\lambda \dots$

$$a \sin \theta_n = n\lambda$$

$$n = 1, 2, 3 \dots$$

Graph between Intensity and path difference :-



point a corresponds to position of central maxima and positions $-3\lambda, -2\lambda, -\lambda, \lambda, 2\lambda, 3\lambda$ are secondary minima (dark) and position $\frac{3\lambda}{2}, \frac{5\lambda}{2}, \frac{7\lambda}{2}$ are secondary maxima (bright).

Width of central maximum :

The width of central maximum is the distance between first secondary minimum, on either side of central bright fringe O.

For first secondary minimum

$$a \sin \theta = n\lambda$$

$$\text{for } n=1, a \sin \theta = \lambda$$

$$\sin \theta = \frac{\lambda}{a} \quad \text{--- (1)}$$

From $\triangle OPC$

$$\text{for small angle, } \sin \theta = \frac{y}{D} \quad \text{--- (2)}$$

From (1) and (2) —

$$\frac{y}{D} = \frac{\lambda}{a}$$

$y = \frac{\lambda D}{a}$

$$\text{width of central maximum} = 2y = \frac{2\lambda D}{a}$$

$W = \frac{2\lambda D}{a}$

As the slit width 'a' increases, then width of central maximum decreases.

Angular width of central maximum —

$$= 2\theta = \frac{2d}{a}$$

$\text{Angular width} = \frac{2\lambda}{a}$

$\sin \theta = \frac{\lambda}{a}$
for small angle
 $\theta = \frac{\lambda}{a}$

* Difference between Interference and Diffraction:

Interference	Diffraction
Produced by the superposition of waves from two coherent sources.	Produced by the superposition of wavelets from different parts of same wave front (single coherent source)
In interference intensity of maxima's are same.	In diffraction intensity of maxima's goes on decreasing.
For maxima, path difference is nd . For minima, path difference is $(2n-1)\frac{\lambda}{2}$	For maxima, path difference is $(2n+1)\frac{\lambda}{2}$. For minima, path difference is nd .
Intensity of all minimum may be zero.	Intensity of minima is not zero.
Coherent source are required.	Not necessary.