Date / /
Page

Electrostatic Potential & Capacitance

Electric potential :-

The work done to carry unit positive charge from electric field to infinity is called electric potential.

→ It is Scalar quantity.
→ It is denoted by V.

$\therefore$ Electric potential = $\frac{\text{work}}{\text{charge}}$

$$\boxed{V = \frac{W}{q}}$$

→ SI unit $= \frac{J}{C}$
$= JC^{-1}$
$=$ Volt (v)

→ Dimension $= \frac{ML^2T^{-2}}{AT}$
$= ML^2T^{-3}A^{-1}$

Potential difference :-

The difference in the electric potential between two point or electric field is called potential difference.

Date / /
Page

→ It is denoted by V.
→ Its SI unit is volt.
→ Its dimension is $ML^2T^{-3}A^{-1}$.

i.e., $\boxed{V_{AB} = V_A - V_B}$

Here,
V_{AB} = Potential difference between A & B.
V_A = Electric potential of A.
V_B = Electric potential of B.

Electric potential due to point charge :-

We know,

$$V = \frac{W}{q}$$

$$V = \frac{F \cdot r}{q} \quad [\because W = F \cdot r \quad \text{Force} \times \text{displacement}] \text{ using}$$

$$V = \frac{1/4\pi\varepsilon_0 \cdot q_1 q_2 / r^2 \times r}{q} \quad \left[\because F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}\right] \text{ using}$$

$$V \Rightarrow \frac{\frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r}}{q}$$

For point charge
$q_1 = q_2 = q$

$$\therefore V = \frac{\frac{1}{4\pi\varepsilon_0} \cdot \frac{q \cdot q}{r}}{q}$$

$$\therefore V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

Date __/__/__
Page ____

Q.1) Find electric potential at 2 cm distance from a charge of $5\mu C$.

Solution :- A/q

$r = 2cm = 2 \times 10^{-2}$
$q = 5\mu C = 5 \times 10^{-6} C$

$$\therefore V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r}$$

$$= 9 \times 10^9 \times \frac{5 \times 10^{-6}}{2 \times 10^{-2}}$$

$$= \frac{45 \times 10^3}{2 \times 10^{-2}}$$

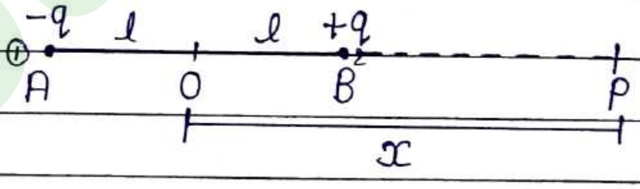
$$= 22.5 \times 10^3 \times 10^2$$

$$= 22.5 \times 10^5$$

$$= 2.25 \times 10^6 \, V. \text{ ans}$$

Electric potential due to a dipole :-

case. i :- Electric potential at an axial point of dipole



Consider an electric dipole consting of two point charge $-q$ and $+q$, seperated by a small distance $2l$. Let P be a point on axis of dipole at distance x from its centre O.

Electric potential at point P due to dipole is —

Date / /
Page

$$V = V_1 + V_2$$

$$V = \frac{-kq}{(l+x)} + \frac{kq}{(x-l)}$$

$$V = kq\left[\frac{1}{(l+x)} + \frac{-1}{(x-l)}\right]$$

$$V = kq\left[\frac{(x-l) - (x+l)}{(x+l)\,(x-l)}\right]$$

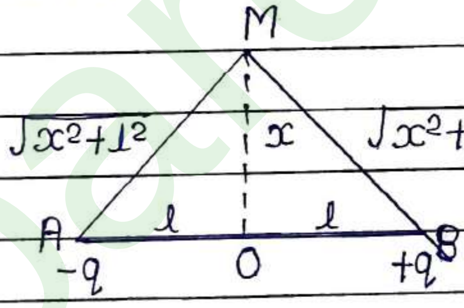
$$V = kq\left[\frac{2l}{x^2-l^2}\right]$$

$$V = \frac{kP}{x^2-l^2}$$

$\therefore$ generally $l^2 <<< x$
$l^2 \to 0$

$$\boxed{V = \frac{kP}{x^2}}$$

case ii) Electric potential at an equatorial point of dipole :
broadside·on



Consider an electric dipole consisting of charges $-q$ and $+q$ and separated by distance $2l$. Let M be a point at x distance from Centre O of dipole.

Electric potential at point P due to dipole is –

$$V = V_1 + V_2$$

$$V = \frac{-Kq}{r} + \frac{Kq}{r}$$

$$V = \frac{-Kq}{\sqrt{x^2+l^2}} + \frac{Kq}{\sqrt{x^2+l^2}}$$

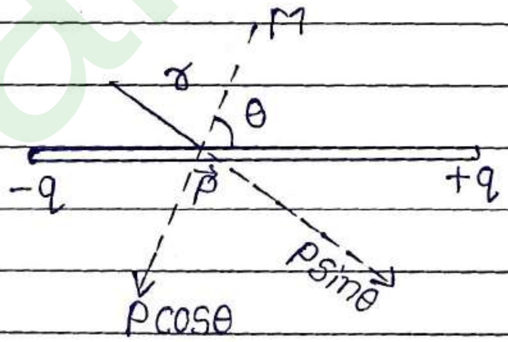
$$V = 0$$

$$\therefore \boxed{V_{equatorial} = 0}$$

Case iii) Electric potential at any general point due to a dipole. (r, θ)

Consider an electric dipole consisting of two point charges $-q$ and $+q$ and separated by distance $2l$. We have to find the V at point M at a distance r from O centre. OM makes angle θ with dipole moment $\vec{P}$.

Then, we will divide dipole P into two components.



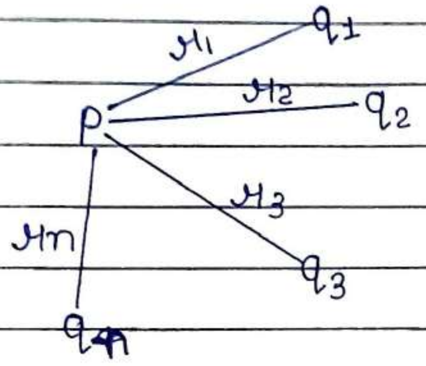
Net potential at point M due to dipole is —

$$V_M = \underset{(axis)}{V_{P\cos\theta}} + \underset{(\perp \text{ bisector})}{V_{P\sin\theta}}$$

$$V_M = \frac{KP\cos\theta}{x^2} + 0$$

$$\boxed{V_M = \frac{KP\cos\theta}{r^2}}$$

Electric potential due to a system of charges:-



Let N point charges $q_1, q_2, q_3 ---- q_n$ at distance $r_1, r_2, r_3 ---- r_n$ from a point P.

As electric potential is a scalar quantity, so total potential at point P will be equal to algebraic sum of all the individual potential.

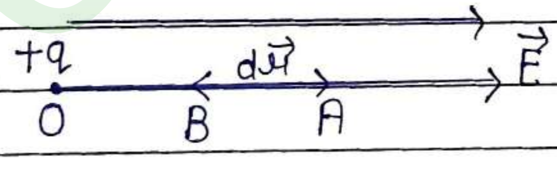
$$V = V_1 + V_2 + V_3 + ---- V_N$$

$$V = K\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} ---- \frac{q_n}{r_n}\right]$$

or $$\boxed{V = K\sum_{i=1}^{N}\frac{q_i}{r_i}}$$

Relation between electric field and potential:-

Case (i) For uniform electric field :-



Consider an electric field due to +q located at origin O. Let A and B adjecent point separated dr.

The force required to move a test charge q_0 against electric field $\vec{E}$ is-

Date / /
Page

$$\vec{F} = -q_0\vec{E}$$

The workdone to move a test charge from A to B is

$$W = F \cdot dr$$
$$W = -q_0 E \cdot dr \quad \text{—— (i)}$$

also, W = charge x potential difference

$$W = q_0 (V_B - V_A) = q_0 dV \quad \text{—— (ii)}$$

On equating the two work :

$$-q_0 E \cdot dr = \cancel{q_0} dV$$

$$dV = -\vec{E} \cdot d\vec{r}$$

or

$$\boxed{E = -\frac{dV}{dr}}$$

Case 2. For non-uniform electric field :-

$$\boxed{\Delta V = \int_{r_1}^{r_2} dV = -\int_{r_1}^{r_2} E \cdot dr}$$

→ SI unit of E is NC^{-1} or Vm^{-1}

or $E_x = \frac{-\delta V}{\delta x}$, $E_y = \frac{-\delta V}{\delta y}$ and $E_z = \frac{-\delta V}{\delta z}$

Note :- Electric field is in direction in which potential decreases steepest.

→ Its magnitude is given by the change in magnitude of potential per unit displacement normal to the equipotential surface at the point.

Potential due to spheres :

Case(1) Potential due to hollow / solid conducting sphere.

(i) Outside at distance r from centre : $(R < r)$

$$\vec{E_p} = \frac{KQ}{r^2}$$

$$\Delta V = -\int E \cdot dr_p$$

$$V_p - V_\infty = -\int_\infty^p E \cdot dr = -\int_\infty^p \frac{KQ}{r^2} \cdot dr$$

$$V_p - V_\infty = -KQ \int_\infty^r \frac{1}{r^2} \cdot dr = -KQ \left[\frac{-1}{r}\right]_\infty^r$$

$$V_p - 0 = -KQ \left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$\boxed{V_p = \frac{KQ}{r}}$$

(ii) On the surface $(r = R)$

$$\boxed{V_S = \frac{KQ}{R}}$$

(iii) inside at distance r from centre $(R > r)$

$$\vec{E_p} = 0$$

$$\Delta V = -\int_S^P E \cdot dr$$

$$V_p - V_S = -\int_S^P 0 \cdot dr$$

Date / /
Page

$$V_P - V_S = 0$$
$$V_P = V_S$$
$$\boxed{V_P = \frac{KQ}{R}} \text{ constant.}$$

Case 2. Potential due to Non-conducting sphere:

(i) Outside the sphere at distance r from centre.

$$\vec{E}_P = \frac{KQ}{r^2}$$

$$\Delta V = -\int_{\infty}^{P} \varepsilon \cdot dr$$

$$V_P - V_\infty = -\int_{\infty}^{P} \frac{KQ}{r^2} \cdot dr \cos 0°$$

$$V_P - 0 = -KQ \int_{\infty}^{r} \frac{1}{r^2} \cdot dr$$

$$V_P = -KQ \left[\frac{-1}{r} \right]_{\infty}^{r}$$

$$V_P = -KQ \left[-\frac{1}{r} - \frac{1}{\infty} \right]$$

$$\boxed{V_P = \frac{KQ}{r}}$$

(ii) At surface $(r = R)$

$$\boxed{V_S = \frac{KQ}{R}}$$

(iii) inside the sphere at distance r from centre.

$$\Delta V = -\int_{S}^{P} \varepsilon \cdot dr \qquad \vec{E}_{P\,\text{inside}} = \frac{KQr}{R^3}$$

$$V_P - V_S = -\int_{R}^{r} \frac{KQr}{R^3} \cdot dr \cos 0°$$

$$V_P - V_S = -\frac{KQ}{R^3} \int_{R}^{r} r \cdot dr$$

$$V_p - V_s = \frac{-KQ}{R^3}\left[\frac{x^2}{2}\right]_R^r$$

$$V_p - V_s = \frac{-KQ}{R^3}\left[\frac{r^2}{2} - \frac{R^2}{2}\right]$$

$$V_p - \frac{KQ}{R} = \frac{-KQr^2}{2R^3} + \frac{KQR^2}{2R^3}$$

$$V_p - \frac{KQ}{R} = \frac{-KQr^2}{2R^3} + \frac{KQ}{2R}$$

$$V_p = \frac{KQ}{2R} + \frac{KQ}{R} - \frac{KQr^2}{2R^3}$$

$$V_p = \frac{3KQ}{2R} - \frac{KQ}{2R} \cdot \frac{r^2}{R^2}$$

$$\boxed{V_p = \frac{KQ}{R}\left[\frac{3}{2} - \frac{r^2}{R^2}\right]}$$

V at centre, $r = 0$

$$V_c = \frac{KQ}{R} \times \frac{3}{2}$$

$$\boxed{V_c = \frac{3V_s}{2}}$$

Equipotential surface :-

An equipotential surface is a surface with a constant value of potential at all point on the surface.

For a point charge q,

- Concentric sphere with charge as centre of sphere are equipotential surface.

Date / /
Page

- For linear charge,
Concentric cylinder with linear charge as axis of cylinder are equipotential surface.

Properties of equipotential surface :-

1) The potential difference between any two point on equipotential surface is zero.

2) No work is done by electric force in moving a charge on equipotential surface.

3) The direction of electric field is always (Iar) perpendicular to equipotential surface.

$$dV = -\vec{E} \cdot d\vec{r}$$
$$0 = -\vec{E} \cdot d\vec{r}$$

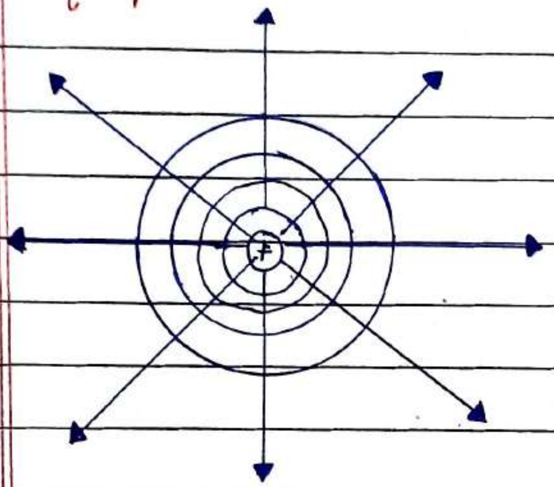
$\because$ if dot product is zero then value of ($\cos\theta = 90°$)

4) No two equipotential surface can intersect each other.

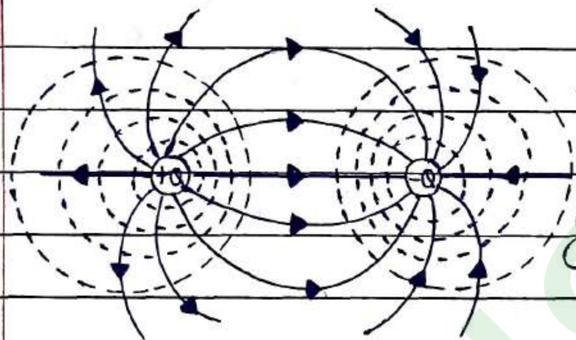
5) Equipotential surfaces are closer in direction (region) of strong field and farther a part in regions of weak electric field.

i.e; $\boxed{E = \dfrac{-dV}{dr}}$ or $\boxed{dr = \dfrac{-dV}{E}}$

Equipotent Surface of various System :-

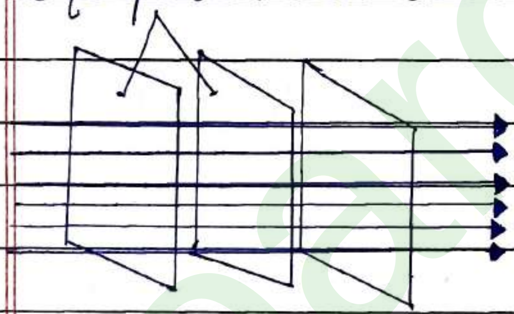
(a) 

for a single charge q equipotential surface are spherical surfaces Centred at charge and $\vec{E}$ field line are radially outwards.

(b) 

for a dipole equipotential surface are close together in region between the two charges.

(c) Equipotential surface for uniform electric field, the line of force are parallel straight line and perpendicular to equipotential surface.

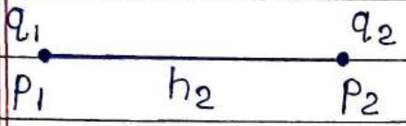


Electric potential energy :-

The electric potential energy of a system of point charges may be defined as the amount of work done in assembling the charges at their locations by bringing them in from infinity.

Potential Energy of a system of two point charges :-

Let a charge q_1 at a point P_1 in space. Electric potential due to q_1 and Point P_2 at r_{12} distance from P_1 will be –



$V_1 = \frac{Kq_1}{r_{12}}$, If charge q_2 is moved in from infinity to point P_2, the work required is –

$W_2 = V_1 \times q_2$ also, $W_1 = 0$

$W_2 = \frac{Kq_1}{r_{12}} \times q_2 = \frac{Kq_1 q_2}{r_{12}}$

As the workdone is stored as potential energy U of the system $(q_1 + q_2)$ so,

$$\boxed{U = W_1 + W_2 = \frac{Kq_1 q_2}{r_{12}}}$$

potential energy of a system of N point charge

$$\boxed{U = K \sum_{\substack{i=1 \\ j=1 \\ i \neq j}}^{N} \frac{q_i q_j}{r_{ij}}}$$

Potential energy in an external field :-

Electric potential of a given point in an external field as the potential energy of a unit positive charge at that point.

we know that,

Let, workdone in bringing q_1 from ∞ to $\vec{r_1}$ against the external field $= q_1 V(\vec{r_1})$.

workdone in bringing q_2 from ∞ to $\vec{r_2}$ against external field $= q_2 V(\vec{r_2})$.

workdone on q_2 against the force extented by q_1

$V = \frac{W}{q}$

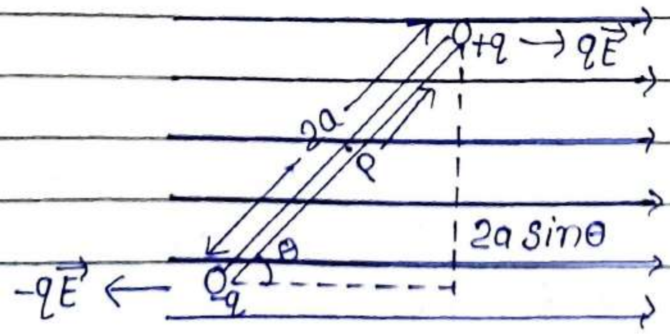
$$= Vq$$
$$= \frac{Kq_1}{r_{12}} \times q_2$$
$$= \frac{Kq_1 q_2}{r_{12}}$$

Total potential energy of a system = the work done in assembling the two charges.

$$\boxed{U = q_1 V(\vec{r_1}) + q_2 V(\vec{r_2}) + \frac{Kq_1 q_2}{r_{12}}}$$

Potential energy of a dipole in uniform electric field :-

Consides an electric dipole in uniform electric field $\vec{E}$ with dipole moment $\vec{P}$ making an angle θ with the field.



Date / /
Page

Two equal and opposite forces form a couple. The torque exerted by couple will be

$$\tau = \text{force} \times \perp^{ar} \text{distance}$$

$$\tau = qE \times 2a\sin\theta$$

$$\tau = PE\sin\theta$$

If dipole is rotate to a small angle $d\theta$ against torque acting on it, then the small workdone is

$$dw = \tau\, d\theta$$

$$dw = PE\sin\theta \cdot d\theta$$

the workdone in rotating dipole from θ_1 to θ_2 will be :

$$w = \int dw = \int_{\theta_1}^{\theta_2} PE\sin\theta \cdot d\theta$$

$$= PE\left[-\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$= -PE(\cos\theta_2 - \cos\theta_1)$$

$$= -PE(\cos\theta_2 - \cos\theta_1)$$

$$= PE(\cos\theta_1 - \cos\theta_2)$$

The workdone is stored as potential energy U of dipole

$$\therefore \boxed{U = PE(\cos\theta_1 - \cos\theta_2)}$$

If initially dipole is oriented perpendicular to direct^n of field ($\theta_1 = 90°$) and then makes an angle θ with field ($\theta_2 = \theta$), the potential energy –

$$U = PE(\cos 90° - \cos\theta) = PE(0 - \cos\theta)$$

$$U = -PE\cos\theta$$

$\because U = -PE\cos\theta = -\vec{p}\cdot\vec{E}$

Special cases :-

(1) position of stable equilibrium

$\theta = 0°$, then $U = -PE\cos\theta$

$\boxed{U = -PE}$

Thus, P. Energy is minimum when $\theta = 0°$ (parallel)

Position of zero energy

when $\theta = 90°$

$U = -PE\cos 90° = 0$

$\boxed{U = 0}$ Thus, PE is zero when $\theta = 90°$ (perpendicular)

(2) Position of unstable equilibrium

when $\theta = 180°$

$U = -PE\cos 180° = PE$

$\boxed{U = PE}$ Thus, P.E is maximum when $\theta = 180°$ (antiparallel)

* Electric capacitance of a conductor :-

It is defined as the measure of its ability to hold charge.

$$\text{Capacitance} = \frac{\text{charge}}{\text{potential}}$$

Capacitance of a conductor may be defined as the charge required to increase the potential of conductor by unit amount.

Date / /
Page

It depends on the following :-

(i) Size and shape of conductor.
(ii) Nature of surrounding medium.
(iii) presence of other conductor in it neighbour.

- SI unit is farad (F).
- Dimension $[M^{-1}L^{-2}T^{4}A^{2}]$

Capacitance of an isolated spherical capacitor :-

Let an isolated spherical conductor of radius R and charge Q distributed on its surface. The potential at any point on surface

$q = \frac{KQ}{R}$, then capacitance will be

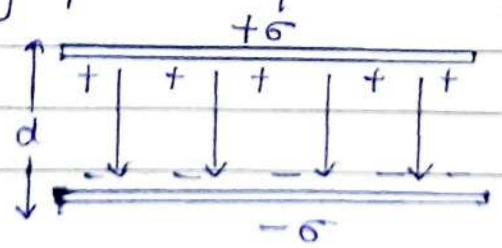
$$C = \frac{Q}{V}$$

$$C = \frac{\cancel{Q}}{\frac{K\cancel{Q}}{R}} = \frac{R}{K} = 4\pi\varepsilon_0 R$$

$2r$ $\boxed{C = 4\pi\varepsilon_0 R}$

Parallel plate capacitor :-

- Simplest and most widely used capacitor.
- It consist of two large plane parallel conducting plate separated by a small distance.



Date / /
Page

let the area of each plate A, d is the distance between two plate, $\pm\sigma$ is uniform charge density and $\pm Q$ is charge on each plate.

In outer region, $E = \frac{\sigma}{2\varepsilon_0} - \frac{\sigma}{2\varepsilon_0} = 0$

In inner region, $E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$

potential difference between the plate is

$$V = E \cdot d$$
$$= \frac{\sigma d}{\varepsilon_0}$$

also, $C = \frac{Q}{V}$

$$C = \frac{\sigma A}{\frac{\sigma d}{\varepsilon_0}} = \frac{\varepsilon_0 A}{d}$$

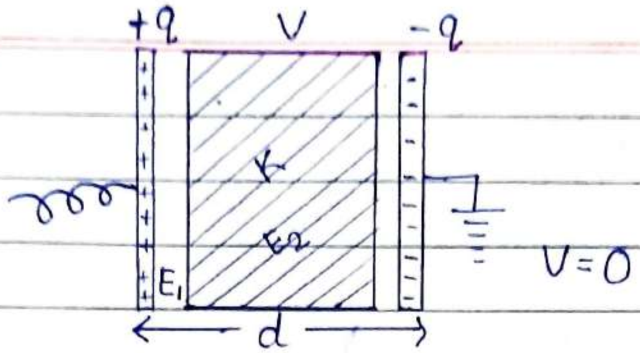
$\because \boxed{C = \frac{\varepsilon_0 A}{d}}$

Factors on which capacitance of parallel plate capacitor depends :-

i) Area of plate
ii) Distance between plate
iii) permitivity of medium b/w plate.

Capacitor of a partially filled capacitor :-

Consider a parallel plate capacitor with plate area A and separated between distance 'd'.



Consider 'q' charge is given to the capacitor and potential between 'V' then by defination capacitor of parallel plate, $C = \frac{q}{V}$ — (1)

The surface charge density on plate becomes
$$\sigma = \frac{q}{A} \quad \text{— (2)}$$

Let a dielectric slab of thickness 't' is introduced between plates partically with dielectric constant 'k'. Now, electric field inside plate in air.
$$E_1 = \frac{\sigma}{\varepsilon_0}$$

Electric field inside plate within dielectric slab,
$$E_2 = \frac{\sigma}{\varepsilon \cdot k}$$

also, we know $E = \frac{V}{d}$

$V = E \cdot d$
$= E_1 (d-t) + E_2 \cdot t$

using above equation

$$V = \frac{q}{A\varepsilon_0} \times \left[(d-t) + \frac{t}{k}\right]$$

putting the value in equation (1)

$$C = \frac{q}{\frac{q}{\varepsilon_0 A}\left[(d-t)+\frac{t}{K}\right]}$$

$$\therefore \boxed{C = \frac{A\varepsilon_0}{\left[(d-t)+\frac{t}{K}\right]}}$$

Case ① If $t = 0$ (not filled)

then, $C = \frac{A\varepsilon_0}{d}$

Case ② If $t = d$ (fully filled)

then, $C = \frac{A\varepsilon_0 K}{d}$

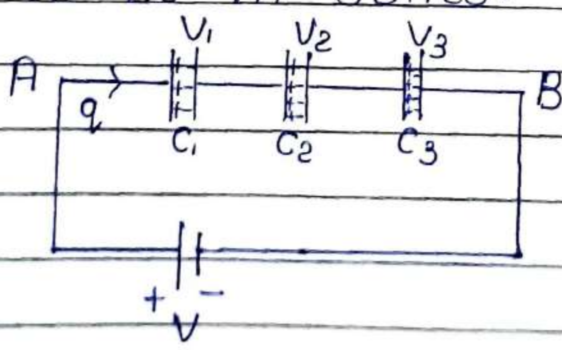
Case ③ If many slabs of different thickness kept

$$C = \frac{\varepsilon_0 A}{d-(t_1+t_2+t_3)+\frac{t_1}{K_1}+\frac{t_2}{K_2}+\frac{t_3}{K_3}}$$

Combination of capacitor :-

1) capacitor in series —

When negative plate of one capacitor is connected to the positive plate of second and negative plate of second to positive of third and so on, thus is said to be in series.



Date / /
Page

2. Consider three capacitor C_1, C_2 and C_3 all connected in series two point A and B from figure, $V = V_1 + V_2 + V_3$ — ①

As all capacitor all connected in series, then
$q_1 = q_2 = q_3 = q$

Now,
$$\left.\begin{aligned} q &= C_1 V_1 \\ q &= C_2 V_2 \\ q &= C_3 V_3 \end{aligned}\right\} \text{— (A)}$$

putting this equation in ①
$$V = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$
$$V = q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right] \text{— ②}$$

If equivalent capacitance is 'c' then,
$$q = CV$$
$$V = \frac{q}{C} \text{ — (B)}$$
$$\because \frac{q}{C} = q\left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]$$
$$\boxed{\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

For series combination :-

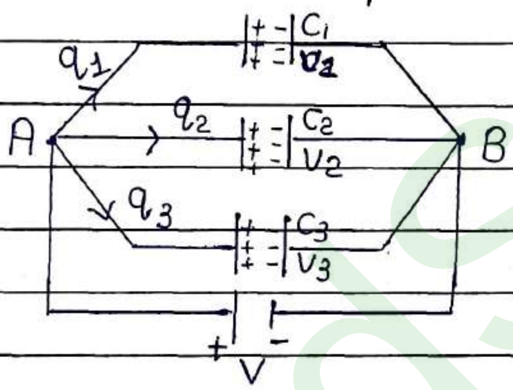
1. The reciprocal of equivalent capacitance is equal to the sum of the reciprocal of the individual capacitance.

2. The equivalent capacitance is smaller than the smallest individual capacitance.

3. The charge on each capacitance is same.
4. The potential difference across any capacitor is inversly proportional to its capacitance.

② Capacitance in parallel :-

When the positive plate of all capacitors are connected to one common point and negative plate to another common point, the capacitor is said to be in parallel.



from figure, $q = q_1 + q_2 + q_3$ —— ①
Since, all capacitor are connected between same point then, $V_1 = V_2 = V_3 = V$ —— ②
Now, $q_1 = C_1 V$
$q_2 = C_2 V$
$q_3 = C_3 V$
putting the value in equation ①
$$q = C_1 V + C_2 V + C_3 V$$
$$q = V(C_1 + C_2 + C_3) \text{ —— ③}$$

If equivalent capacitance is C, then
$$q = CV$$
Using equation — ③

Date / /
Page

$$CV = V(C_1 + C_2 + C_3)$$

$$C = C_1 + C_2 + C_3$$

$$\boxed{C_{eq} = C_1 + C_2 + C_3}$$

For parallel combination of capacitor —

(i) C_{eq} is equal to sum of individual capacitance.

(ii) The equivalent capacitance is larger than largest individual capacitance.

(iii) Potential difference across each capacitor is same.

(iv) The charge on each capacitor is proportional to its capacitance.

Energy stored in capacitor :-

The workdone in charging a capacitor is called energy of capacitor.

Let us consider a capacitor of any instant has charge 'q' and potential 'v' than to charge capacitor to 'dq' amount of charge work is done.

$$dW = dq \times V$$

$$W = \int_0^q dq \times \frac{q}{C}$$

$$= \frac{1}{C}\int_0^q q \cdot dq$$

$$= \frac{1}{C}\left[\frac{q^2}{2}\right]_0^q$$

$$= \frac{1}{C}\left[\frac{q^2}{2} - \frac{0^2}{2}\right]$$

$$W = \frac{q^2}{2C}$$

$$W = \frac{1}{2C}(CV)^2$$

$$W = \frac{1C^2V^2}{2C}$$

$$W = \frac{CV^2}{2}$$

$$W = \frac{1}{2}CV^2$$, This workdone stored in form of potential energy

$$\boxed{U = \frac{1}{2}CV^2}$$

in terms of Q and V $\Rightarrow$ $\boxed{U = \frac{1}{2}QV}$

in terms of Q and C $\Rightarrow$ $\boxed{U = \frac{Q^2}{2C}}$

Behaviour of conductor in electrostatic field :-

1. Net electrostatic fields is zero in interior of a conductor.

2. Just outside the surface of a charged conductor electric field is normal to the surface.

3. The net charge in the interior of a conductor is zero and any excess charge resides on its surface.

Date / /
Page

4. potential is constant within and on the surface of conductor.
5. Electric field at the surface of a charge conductor is proportional to the surface charge density.
6. Electric field is zero in the cavity of a hallow charge conductor.

Energy density of an electric field :-

The presence of an electric field implies stored energy or the energy is stored in the electric field,

Consider a parallel plate capacitor having plate area A and plate separation d, Then the Capacitance of the parallel plate —

$$C = \frac{\varepsilon_0 A}{d}$$

If σ is the surface charge density on the capacitor, then the electric field between plate —

$$E = \frac{\sigma}{\varepsilon_0}$$

$$\sigma = E \cdot \varepsilon_0$$

charge on either plate of capacitor is

$$q = \sigma A = \varepsilon_0 E A$$

∵ Energy stored in capacitor $U = \frac{Q^2}{2C}$

$$U = \frac{(\varepsilon_0 E A)^2}{\frac{2\varepsilon_0 A}{d}} = \frac{1}{2}\varepsilon_0 E^2 A d$$

Date / /
Page

The energy stored per unit volume or electric field's energy density is given by-

$$u = \frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2$$

$$u = \frac{1}{2}\varepsilon_0 E^2$$

Redistribution of charge :
[when two conductors are connected].
Let us consider two capacitance or conductor q_1, C_1, V_1 and q_2, C_2, V_2 has parameter then we connected both conductor with a negligible resistance wire.

Before connection

$$q_1 = C_1V_1$$
$$q_2 = C_2V_2$$

After connection

total charge $= q_1 + q_2$

total capacitance $= C_1 + C_2$

$$V_{common} = \frac{\text{total charge}}{\text{total capacitance}} = \frac{q_1+q_2}{C_1+C_2}$$

$$\boxed{V_{common} = \frac{C_1V_1 + C_2V_2}{C_1+C_2}}$$

Dielectric :- A dielectric is a substance which does not allow the flow of charge through it but permits them to exert electrostatic force on another through it.
eg:- Glass, wax, water, air, rubber etc.

Dielectric are of two types—
i) polar dielectric
ii) Non-polar dielectric

Polar Dielectric :-
A molecule in which centre of mass of positive charge does not coinside the with the centre of mass of negative charge.

eg :- HCL, NH_3, H_2O etc.

Non-polar Dielectric :- A molecule in which the Centre of mass of positive charge coinside with centre of mass of negative charge.

eg :- N_2, H_2, C_2, CH_4, benzene etc.

Polarisation of dielectrics :-
Both polar and non polar dielectric develop a net dipole moment in presence of an external electric field. This fact is called polarisation of dielectrics.

Polarisation density :- (P)

The incluced dipole moment developed per unit volume of a dielectric when place in on external field is called polarisation density.

Date __/__/__
Page ____

Electric Susceptibility [χ] –

The polarisation $\vec{P}$ is proportional to the resultant electric field. $\vec{E}$ existing in the dielectric.

$$\vec{P} \propto \vec{E}$$
$$\vec{P} = \varepsilon_0 \chi \vec{E}$$

where χ is proportionality constant and called electric susceptibility.

$$\chi = \frac{\vec{P}}{\varepsilon_0 \vec{E}}$$

Thus the ratio of polarisation density to ε_0 times the electric field is called the electric susceptibility of the dielectric.

Dielectric Constant (K) or relative permitivity (ε_r)

The ratio of original dielectric field $\vec{E_0}$ and reduced electric field $\vec{E} = \vec{E} - E_{in}$ in the dielectric is called dielectric constant.

$$K = \frac{\vec{E_0}}{\vec{E_0} - E_{in}} \qquad \text{Or} \quad K = \varepsilon_r$$

$$\text{Or} \quad K = \frac{\vec{E_0}}{\vec{E}}$$

Dielectric Strength :- The maximum electric field that can exist in a dielectric without causing the breakdown of its insulating property is called dielectric strength of the material.

The unit of Dielectric strength –
Vm^{-1} or $kVmm^{-1}$.

Q.) Two charges 5×10^{-8} C and -3×10^{-8} C are located 16 cm apart. At what point (s) on the line joining the two charges is the electric potential zero ? Take the potential at infinity to be zero.

Solⁿ:- A/q

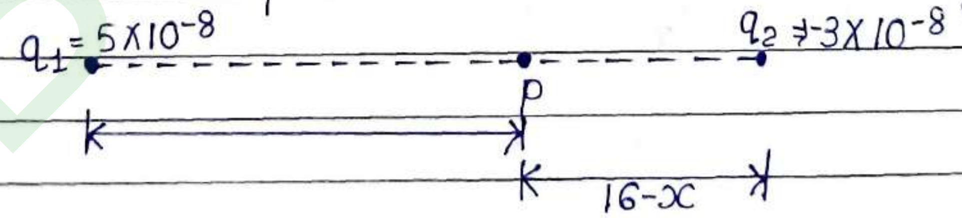
$q_1 = 5\times10^{-8}$ C

$q_2 = -3\times10^{-8}$ C

$r_{12} = 16$ cm

Case - I

When point is between the charges



For zero potential

$V = 0$

$\Rightarrow V_1 + V_2 = 0$

$\Rightarrow \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{r_2} = 0$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) = 0$$

$$\frac{q_1}{r_1} + \frac{q_2}{r_2} = 0$$

$$\frac{5 \times 10^{-8}}{x} + \frac{(-3 \times 10^{-8})}{16 - x} = 0$$

$$\frac{5 \times \cancel{10^{-8}}}{x} = \frac{3 \times \cancel{10^{-8}}}{16 - x}$$

$$3x = 80 - 5x$$

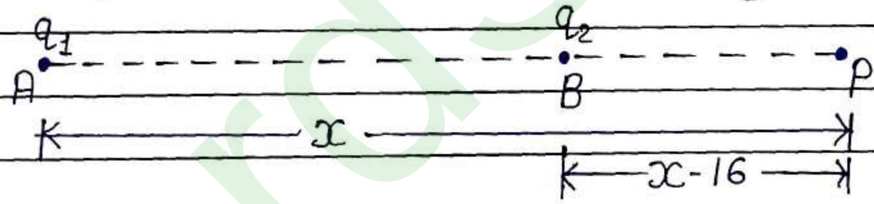
$$3x + 5x = 80$$

$$\cancel{8}x = \cancel{80}$$

$$x = 10 \text{ cm.}$$

Case - II

When point is out of the charges,



For zero potential

$$V = 0$$

$$\Rightarrow V_1 + V_2 = 0$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{q_1} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_2} = 0$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0} \left(\frac{q_1 + q_2}{r_1 + r_2}\right) = 0$$

$$\Rightarrow \frac{q_1}{r_1} + \frac{q_2}{r_2} = 0$$

$$\Rightarrow \frac{5 \times 10^{-8}}{x} + \frac{(-3 \times 10^{-8})}{x - 16} = 0$$

$$\Rightarrow \frac{5\times10^{-8}}{x} = \frac{3\times10^{-8}}{x-16}$$

$$3x = 5x - 80$$

$$3x - 5x = -80$$

$$-2x = -180$$

$$x = \frac{\cancel{-}180}{\cancel{-}2}$$

$$x = 40 \text{ ans}$$

Q) Two charges 3×10^{-8}C and -2×10^{-8}C are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

Soln :- A/c to question

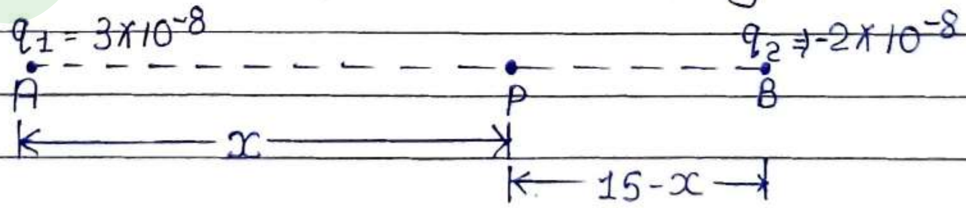
$$q_1 = 3\times10^{-8}\,C$$

$$q_2 = -2\times10^{-8}\,C$$

$$r_{12} = 15\text{ cm}$$

Case-I :-

when point is between the charges.



For zero potential

$$V = 0$$

$$\Rightarrow V_1 + V_2 = 0$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{r_2} = 0$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0} \cdot \left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) = 0$$

$$\Rightarrow \frac{q_1}{r_1} + \frac{q_2}{r_2} = 0$$

$$\Rightarrow \frac{3\times10^{-8}}{x} + \frac{(-2\times10^{-8})}{15-x} = 0$$

$$\Rightarrow \frac{3\times\cancel{10^{-8}}}{x} = \frac{2\times\cancel{10^{-8}}}{15-x}$$

$$\Rightarrow 2x = 45 - 3x$$

$$\Rightarrow 2x + 3x = 45$$

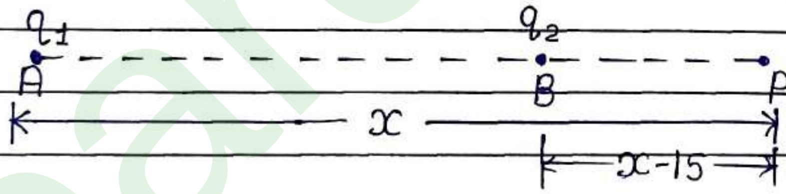
$$\Rightarrow 5x = 45$$

$$\Rightarrow x = \frac{45}{5}$$

$$\Rightarrow x = 9$$

Case-II :-

when point is out of the charge



For zero potential

$$V = 0$$

$$\Rightarrow V_1 + V_2 = 0$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_2}{r_2} = 0$$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) = 0$$

$$\Rightarrow \frac{q_1}{r_1} + \frac{q_2}{r_2} = 0$$

Date / /
Page

$$\Rightarrow \frac{3\times10^{-8}}{x} + \frac{(-2\times10^{-8})}{x-15} = 0$$

$$\Rightarrow \frac{3\times\cancel{10^{-8}}}{x} = \frac{2\times\cancel{10^{-8}}}{x-15}$$

$$\Rightarrow 2x = 3x - 45$$

$$\Rightarrow 2x - 3x = -45$$

$$\Rightarrow \cancel{-}x = \cancel{-}45$$

$$\Rightarrow x = 45 \text{ ans}$$

Q.) A parallel plate capacitor with air between the plates has a capacitance of 8 pF (1pF $= 10^{-12}$ F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6 ?

Soln:- A/q

$$C_1 = \frac{\varepsilon_0 A}{d} = 8pF$$

$$\varepsilon_{r_1} = 1$$

$$d_1 = d$$

$$\varepsilon_{r_2} = 6$$

$$d_2 = d/2$$

$$C_2 = ?$$

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$C_1 = \frac{\varepsilon_0 \varepsilon_{r_1} A}{d_1} \text{ ——— (I)}$$

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d/2}$$

$$C_2 = \frac{\varepsilon_0 \varepsilon_{r_2} A}{d_2} \text{ ——— (II)}$$

Date / /
Page

(i)/(ii)

$$\frac{C_1}{C_2} = \frac{\varepsilon_0 \varepsilon_r A}{d_1} \Big/ \frac{\varepsilon_0 \varepsilon_{r2} \cdot A}{d_2}$$

$$\frac{C_1}{C_2} = \frac{\varepsilon_{r1}}{d_1} \times \frac{d_2}{\varepsilon_{r2}}$$

$$\Rightarrow \frac{8}{C_2} = \frac{1}{d} \times \frac{d/2}{6}$$

$$\Rightarrow \frac{8}{C_2} = \frac{1}{12}$$

$$\Rightarrow C_2 = 96 \text{ pF ans}$$

Q) Explain what would happen if in the capacitor given in exercise 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates.

(a) while the voltage supply remained connected.

Soln:-

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$\Rightarrow C = \frac{\varepsilon_0 A}{d} \times \varepsilon_r$$

$$\Rightarrow C = 18 \text{ pF} \times 6$$

$$\Rightarrow C = 108 \text{ pF}$$

$$V = 100 \text{ V}$$

$$Q = CV$$

$$\Rightarrow Q = 108 \times 10^{-12} \times 10^{-2}$$

$$= 108 \times 10^{-10}$$

$$= 1.08 \times 10^{-8} \text{ C ans}$$

(b) After the supply was disconnected.

Soln:- $C = 108\,pF$

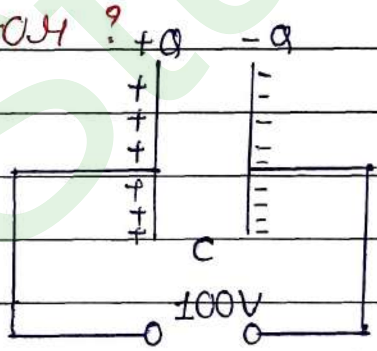
$Q = 1.8 \times 10^{-9}$

$V = \dfrac{Q}{C}$

$= \dfrac{1.8 \times 10^{-9}}{108 \times 10^{-12}}$

$= 16.6\,V$ ans

Q.) A 900 pF capacitor is charged by 100 V battery. How much electrostatic energy is stored by the capacitor?



Soln:- $C = 900\,pF$

$= 900 \times 10^{-12}\,F$

$= 9 \times 10^{-10}\,F$

$V = 100\,V$

$U = ?$

$\therefore \quad U = \dfrac{1}{2} CV^2$

$= \dfrac{1}{2} \times 9 \times 10^{-10} \times 100^2$

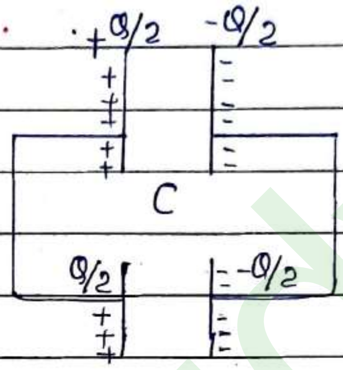
$= \dfrac{1}{2} \times 9 \times 10^{-10} \times 10^4$

$= 4.5 \times 10^{-10} \times 10^4$

$= 4.5 \times 10^{-6}\,J$ ans

Date / /
Page

Q) The capacitor is disconnected from the battery and connected to another 900 pF capacitor. What is the electrostatic energy stored by the system.



Soln

$C_1 = 900 pF$
$= 9 \times 10^{-0} F$
$V_1 = 100 V$
$C_2 = 900 pF = 9 \times 10^{-10} F$
$V_2 = 0$
Rest energy = ?

$$\Delta V = \frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} (V_1 - V_2)^2$$

$$= \frac{1}{2} \times \frac{9 \times 10^{-10} \times 9 \times 10^{-10}}{9 \times 10^{-10} + 9 \times 10^{-10}} \times (100-0)^2$$

$$= \frac{1}{2} \times \frac{\cancel{9} \times 9 \times 10^{-10}}{\cancel{18}_{2} \times 10^{-10}} \times 10^4$$

$$= \frac{9 \times 10^{-10} \times 10^4}{2 \times 2 \times 10^{-10}}$$

$$= 2.25 \times 10^{-6} J$$

Rest energy $= 4.5 \times 10^{-6} - 2.25 \times 10^{-6} J$
$= 2.25 \times 10^{-6} J$ ans