MOVING CHARGES AND MAGNETISM

(CHAPTER-4)

MAGNETIC FIELD:-

It is a region surrounding to a magnet within which another magnet experiences a force.

SI unit :- T (tesla)

CGS unit :- G (gauss)

$$\boxed{1T = 10^4 G}$$

BIOT-SAVART'S LAW:-

The magnitude of the magnetic field at any point around the current carrying conductor (i) is directly proportional to current.

(ii) is directly proportional to current element.

(iii) is directly proportional to sine of angle between current element and position vector.

(iv) and is inversely proportional to the square of distance between them.

$$dB \propto I$$
$$dB \propto dl$$
$$dB \propto \sin\theta$$
$$dB \propto \frac{1}{r^2}$$
$$dB \propto \frac{Idl\sin\theta}{r^2}$$
$$\Rightarrow \boxed{dB = \frac{\mu_0}{4\pi}\frac{Idl\sin\theta}{r^2}}$$

μ_0 = permeability of free space / magnetic permeability of vacuum.

$$\boxed{\mu_0 = 4\pi \times 10^{-7}\ Tm/A}$$

Case-I

when $\theta = 0°$ or $180°$

$\Rightarrow \boxed{dB = 0}$ (Along the conductor magnetic field is 0)

Case-II

when $\theta = 90°$

$\Rightarrow \boxed{dB = \frac{\mu_0}{4\pi}\frac{Idl}{r^2}}$ (Maximum)

In vector form,

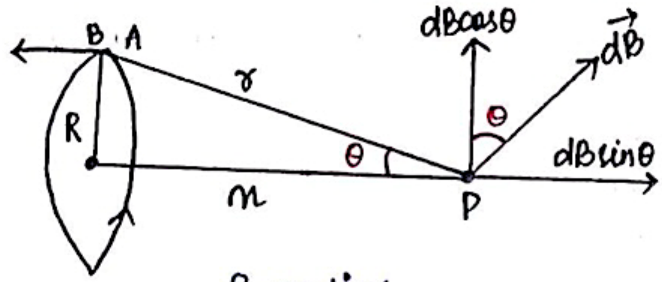
$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl\sin\theta\, r}{r^3}$$

$$\Rightarrow dB = \frac{\mu_0 I}{4\pi} \frac{|\vec{dl} \times \vec{r}|}{r^3}$$

$$\Rightarrow \boxed{\vec{dB} = \frac{\mu_0 I}{4\pi} \frac{(\vec{dl} \times \vec{r})}{r^3}}$$

$\vec{dB} \perp^r \vec{dl}$ and $\vec{dB} \perp^r \vec{r}$.

MAGNETIC FIELD AT ANY POINT ON THE AXIS OF CIRCULAR COIL:-



R = radius

P = any point on the axis at a dist x from the centre

AB = current element.

According to Biot-Savart's law,

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl\sin\theta}{r^2}$$

$$= \frac{\mu_0}{4\pi} \frac{I\,dl\sin 90^\circ}{r^2}$$

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2}$$

$\vec{dB}$ can be resolved into 2 rectangular component. $dB\cos\theta$ is cancelled due to symmetry.

$$B = \int dB\sin\theta$$

$$= \int \frac{\mu_0}{4\pi} \frac{I\,dl}{r^2} \sin\theta$$

$$= \frac{\mu_0}{4\pi} \frac{I\sin\theta}{r^2} \int dl$$

$$= \frac{\mu_0}{4\pi} \cdot \frac{I}{r^2} \cdot \frac{R}{r} \times 2\pi R$$

$$= \frac{\mu_0}{4\pi} \frac{2I\pi R^2}{r^3}$$

$$\boxed{B = \frac{\mu_0}{4\pi} \frac{2IA}{(R^2+x^2)^{3/2}}}$$

($A = \pi R^2$ = area of the coil)

For N no. of turns,

$$\boxed{B = \frac{\mu_0}{4\pi} \frac{2NIA}{(R^2+x^2)^{3/2}}}$$

Case-I

If $x >>> R$, $\boxed{B = \frac{\mu_0}{4\pi} \frac{2NIA}{x^3} = \frac{\mu_0}{4\pi} \frac{2M}{x^3}}$

This expression is similar to the expression of magnetic field due to magnetic dipole where dipole moment $M = NIA$.

Case-II

At the centre of circular coil, $x=0$

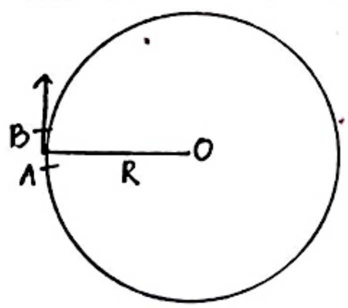
$$B = \frac{\mu_0}{4\pi} \frac{2NIA}{R^3}$$

$$= \frac{\mu_0}{4\pi} \frac{2I\pi R^2}{R^3} \quad (n=1)$$

$$= \frac{\mu_0}{4\pi} \frac{2I\pi}{R}$$

$$\boxed{B = \frac{\mu_0 I}{2R}}$$

MAGNETIC FIELD AT THE CENTRE OF CIRCULAR CURRENT LOOP:-



R = radius

I = current passing through coil

using biot-savart's law:-

$$dB = \frac{\mu_0}{4\pi} \frac{I\,dl\sin 90^\circ}{R^2} = \frac{\mu_0}{4\pi} \frac{I\,dl}{R^2}$$

$$B = \int dB$$

$$= \frac{\mu_0 I}{4\pi R^2} \int dl$$

$$= \frac{\mu_0 I}{4\pi R^2} \times 2\pi R$$

$$B = \frac{\mu_0 I}{2R}$$

For N turns, $\boxed{B = \frac{\mu_0 NI}{2R}}$

CASE-I

For semicircle

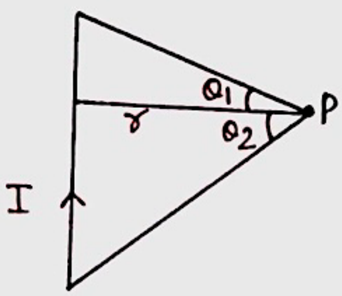
$$\boxed{B = \frac{\mu_0 I}{4R}}$$

CASE-II

for an arc,

$$\boxed{B = \frac{\mu_0 I}{2R} \cdot \frac{\theta}{360^\circ}}$$

MAGNETIC FIELD DUE TO LONG STRAIGHT CURRENT CARRYING CONDUCTOR:-



$$B = \frac{\mu_0 I}{4\pi r}(\sin\phi_1 + \sin\phi_2)$$

CASE-I

Infinite straight

$\phi_1 = \phi_2 = 90^\circ$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

CASE-II

Semi-infinite

$$\boxed{B = \frac{\mu_0 I}{4\pi r}}$$

AMPERE'S CIRCUITAL LAW:-

It states that the line integral of magnetic field over a closed loop is μ_0 times current enclosed by the loop.

$$\boxed{\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{en}}$$

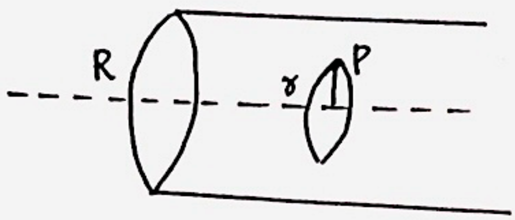
PROOF:-

r = radius of amperian loop.

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\oint \vec{B}\cdot\vec{dl} = \oint B\,dl\cos 0$$
$$= B\oint dl$$
$$= \frac{\mu_0 I}{2\pi r} \times 2\pi r$$
$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

Q:- A straight thick long wire of uniform cross-section of radius 'R' is carrying stedy current I. Use Ampere's circuital law to obtain a relation showing the variation of magnetic field inside and outside the wire with distance 'r'. ($r \le R$) and ($r > R$) of the field point from the centre of its cross-section. Plot a graph showing the variation of field B with distance r.



R = radius
I = current passing through conductor.

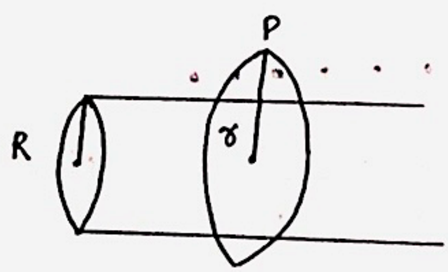
P is any point at a distance r from the axis.

Case-1 ($r < R$)

using ampere's circuital law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_{in}$$
$$\Rightarrow B\,dl\cos 0 = \frac{\mu_0 I \pi r^2}{\pi R^2}$$
$$\Rightarrow B\cdot 2\pi r = \frac{\mu_0 I r^2}{R^2}$$
$$\Rightarrow B = \frac{\mu_0 I r^2}{2\pi r R^2}$$
$$\Rightarrow \boxed{B = \frac{\mu_0 I r}{2\pi R^2}} \qquad B \propto r$$

Case-2 ($r > R$)



using ampere's circuital law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_n$$

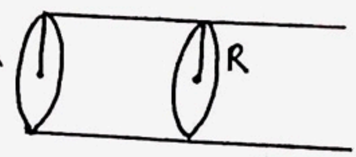
$$\Rightarrow B dl \cos 0 = \mu_0 I$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi r}}$$

$$B \propto \frac{1}{r}$$

Case-III

(on the conductor, $r = R$)

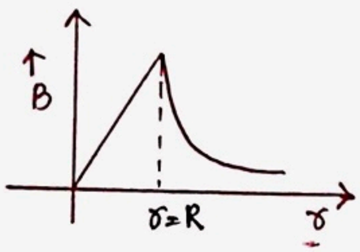


using ampere's circuital law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_n$$

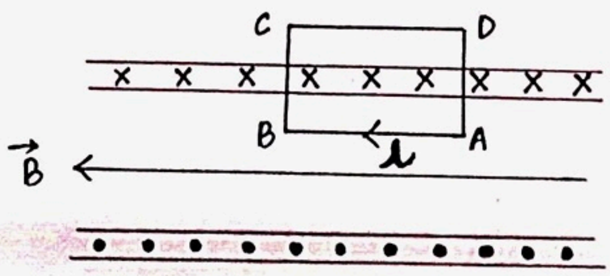
$$\Rightarrow B\cdot 2\pi R = \mu_0 I$$

$$\Rightarrow \boxed{B = \frac{\mu_0 I}{2\pi R}} \text{ (Max)}$$



MAGNETIC FIELD INSIDE A STRAIGHT SOLENOID:-

Inside solenoid, magnetic field is uniform but just outside is 0.



n = no. of turns per unit length = N/l

I = current passing through each turn.

ABCD is the amperian loop of length l

N no of turns passes through it.

Using ampere's circuital law,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_n$$

$$\Rightarrow \int_A^B B\,dl\cos 0^\circ + \int_B^C B\,dl\cos 90^\circ + \int_D^A B\,dl\cos 90^\circ + 0 = \mu_0 NI$$

$$\Rightarrow BL = \mu_0 NI$$

$$\Rightarrow B = \frac{\mu_0 NI}{l}$$

$$\Rightarrow \boxed{B = \mu_0 nI}$$

CASE-I

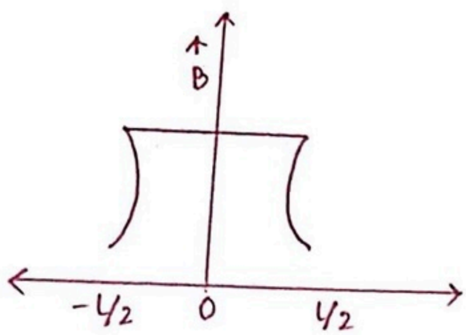
If the core contains material of relative magnetic permeability μ_r.

$$\boxed{B = \mu_0 \mu_r nI}$$

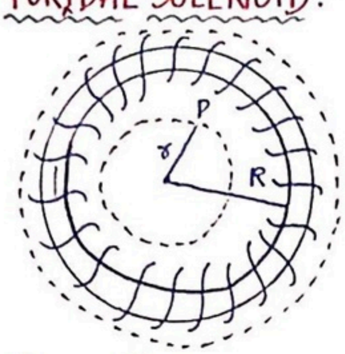
CASE-II

End of solenoid,

$$\boxed{B = \frac{\mu_0 nI}{2}}$$



MAGNETIC FIELD DUE TO TOROIDAL SOLENOID:-



R = radius

I = current passing through each turn

N = total no. of turns.

N.B:- SOLENOID:- It is defined as insulated copper wire wound in the form of helix.

TOROID:- when two ends of solenoid are joined then toroid is formed.

CASE-I

Space enclosed by toroid,

using ampere's circuital law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_n \;(I_n=0)$$

$$\Rightarrow \boxed{B=0}$$

CASE-II

Outside the toroid,

using ampere's circuital law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_n \;(I_n=0)$$

$$\Rightarrow \boxed{B=0}$$

CASE-III

Inside the toroid,

using ampere's circuital law,

$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I_n$$

$$\Rightarrow B dl \cos 0 = \mu_0 N I$$

$$\Rightarrow B = \frac{\mu_0 N I}{2\pi R}$$

$$\Rightarrow \boxed{B=\mu_0 n I}$$

SPECIAL CASE:-

If the core contain material of relative permeability μ_r.

$$\boxed{B=\mu_0 \mu_r n I}$$

FORCE ON A MOVING CHARGE :-

$$\vec{F} = q(\vec{v}\times\vec{B})$$

$$\Rightarrow |\vec{F}| = qvB\sin\theta$$

$$\vec{F}\perp\vec{v} \text{ and } \vec{F}\perp\vec{B}$$

Cause :- Moving charge is equivalent to current. Current produces its magnetic field and is affected by external magnetic field.

CASE-I

$\theta = 0^\circ$ or 180°

$\boxed{F=0}$ (min)

When charge moves either along or opposite to field then F is 0.

CASE-II

$\theta = 90^\circ$

$\boxed{F=qvB}$ (max)

The direction of force can be determined by Fleming's left hand rule.

CASE-III

$v=0$

$\Rightarrow \boxed{F=0}$

Static charge experiences no force.

TRAJECTORY OF CHARGE PARTICLE:-

CASE-I

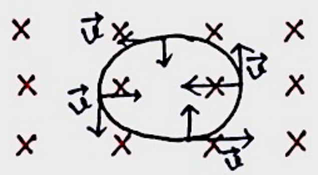
when $\theta = 0^\circ$ or 180°

$\Rightarrow \boxed{F=0}$

Trajectory is a straight line along the field or opposite to the field.

CASE-II

when $\theta = 90^\circ$



The necessary centripetal force is provided by $\vec{B}$.

$$qvB = \frac{mv^2}{r} \Rightarrow \boxed{r = \frac{mv}{qB}}$$

$$\text{Time period, } T = \frac{2\pi r}{v} = \boxed{\frac{2\pi m}{Bq}}$$

$$f = \frac{1}{T} = \boxed{\frac{Bq}{2\pi m}}$$

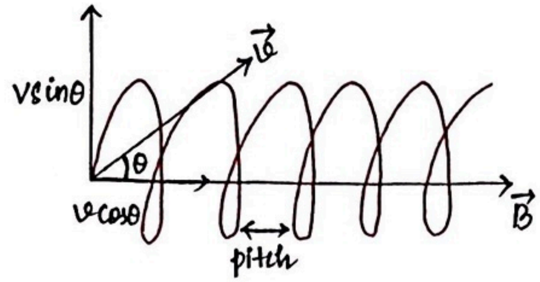
Trajectory is circular.

Case-III

when $\theta \neq 90^\circ$

θ is the angle between $\vec{v}$ and $\vec{B}$

$\vec{v}$ can be resolved into 2 components.



(i) $v\cos\theta$ along the $\vec{B}$ which drag the particle in the forward direction.

(ii) $v\sin\theta$ $\perp^r$ to the $\vec{B}$ which make the particle to move in circular path.

So, ultimately trajectory is helical.

$$r = \frac{mv\sin\theta}{Bq}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{Bq}$$

Pitch:- The distance travelled by the particle between its period of revolution in the direction of magnetic field.

$$\text{Pitch} = v\cos\theta \times T = \frac{v\cos\theta \times 2\pi m}{Bq}$$

SIMILARITIES AND DIFFERENCES BETWEEN BIOT-SAVART'S LAW AND COULOMB'S LAW:-

Similarities:-

1. Both are long range forces.
2. Both obey inverse square law.
3. Both obey superposition principle.

Differences:-

ELECTRIC FIELD	MAGNETIC FIELD
① apply force on static charge	cannot apply force on static charge
② force is along the electric field	force is perpendicular to magnetic field.
③ force is independent of angle	force depends on angle.
④ It can change the speed of the particle.	It cannot change the speed of the particle.

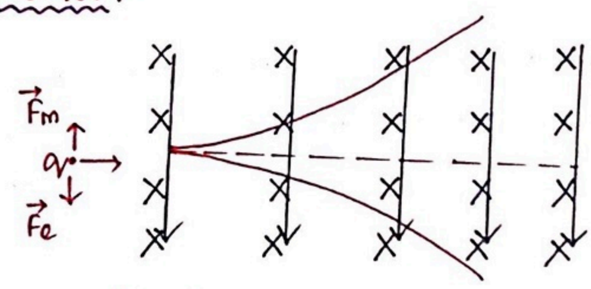
LORENTZ FORCE:-

It is the net force experienced by a charge particle when it is in region of both electric field and magnetic field.

$$\vec{F_L} = \vec{F_e} + \vec{F_m}$$

$$\Rightarrow \boxed{\vec{F_L} = q\vec{E} + q(\vec{v} \times \vec{B})}$$

VELOCITY SELECTOR:-



Particle gets undeflected if,

$$F_e = F_m$$

$$\Rightarrow qE = qvB$$

$$\Rightarrow \boxed{\frac{E}{B} = v}$$

CASE-I

$v > E/B$

$\Rightarrow F_m > F_e$

Particle is deflected upward.

CASE-II

$v < E/B$

$\Rightarrow F_m < F_e$

Particle is deflected downward

N.B :- Work done on the charge due to magnetic field.

$$W = Fs\cos\theta$$
$$= Fs\cos 90^\circ$$
$$\boxed{W = 0}$$

$\vec{F} \perp \vec{v}$ and $\vec{F} \perp \vec{s}$.

So, it cannot change K.E. i.e speed of particle.

CYCLOTRON :-

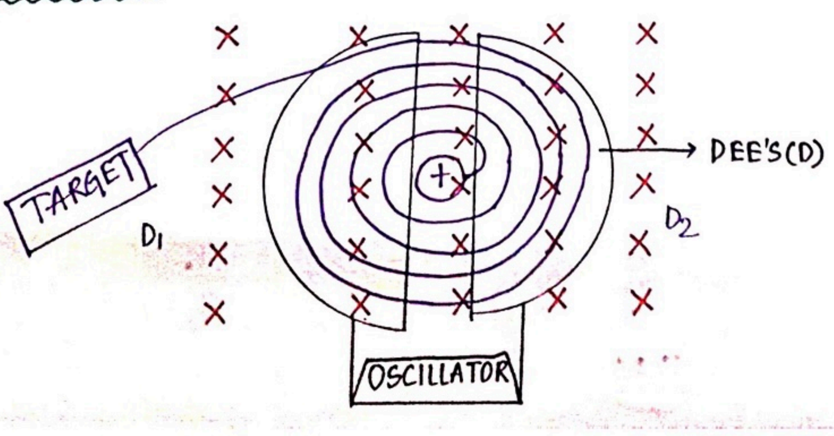
It is a device which is used to accelerate the charged particle like proton, α-particle except electron and neutron.

PRINCIPLE :-

① frequency of revolution of charge particle is independent of radius and velocity.

② when charged particle enter into crossed electric and magnetic field. Electric field increases Kinetic energy and Magnetic field make it to move in a circular path.

CONSTRUCTION :-



Dee's:- It should be two small hollow metallic half cylinder in the shape of D.

Magnetic Field:- Dee's has to be kept in a plane perpendicular to the magnetic field. The whole device is in high vacuum and maintained at high pressure inside it

Osillator:- Osillator is a device that convert DC to AC. Here, it provide necessary electric field.

WORKING:-

Suppose a +ve ion enters the gap between two dee's and found D_2 to be negative and D_1 +ve. It gets accelarated towards D_2. As it enters the D_2, it doesnot experience any electric field due to the shielding effect. The perpendicular magnetic field throws it into a circular path. At the instant it comes out of D_2, it finds D_1 to be -ve and D_2 to be +ve. It now gets accelarated towards D_1 and the process is repeated. Every time the particle process the gap between the two dee's, its velocity increases. Hence, it moves with a greater radius and finally acquires a very large energy. The trajectory of the charged particle is spiral.

THEORY:-

Centripetal force,

$$\frac{mv^2}{r} = qvB$$

$$\Rightarrow \boxed{r = \frac{mv}{qB}}$$

More is the speed, more is the radius.

time taken to complete a semi-circle,

$$t = \frac{\pi r}{v} = \frac{\pi m v}{v q B} = \boxed{\frac{\pi m}{qB}}$$

According to resonance condition,

time taken to complete a semi-circle = half of the time period of the osillator.

$$T = 2 \times t$$

$$\boxed{T = \frac{2\pi m}{Bq}}$$

Frequency,

$$\boxed{f = \frac{1}{T} = \frac{Bq}{2\pi m}}$$ It is called cyclotron frequency.

Max. K.E gained by the charged particle

$$K.E_{max} = \frac{1}{2} m v_{max}^2$$

$$= \frac{1}{2} m \left(\frac{qBr_{max}}{m}\right)^2$$

$$\boxed{K.E_{max} = \frac{q^2 B^2 r_{max}^2}{2m}}$$

LIMITATION :-

① electrons cannot be accelerated as it has lighter mass.

② neutrons cannot be accelerated as it has no charge.

USES :-

① atomic reactors.

② nuclear reactors.

③ synthesize of new particle.

④ used in hospital for diagnosis of cancer.

FORCE ON A CURRENT CARRYING CONDUCTOR :-

When current passes through a conductor, it behaves like a magnet and is affected by external magnetic field.

$$\vec{F} = I(\vec{l} \times \vec{B})$$

$$|\vec{F}| = BIl\sin\theta$$

$$\vec{F} \perp \vec{l} \text{ and } \vec{F} \perp \vec{B}$$

CASE-I

when $\theta = 0°$ or $180°$

$\Rightarrow \boxed{F = 0}$

CASE-II

when $\theta = 90°$

$\boxed{F = BIL}$.

The direction of force can be determined by Fleming's left hand rule.

CASE-III

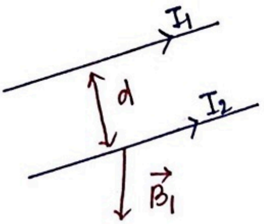
when $I = 0$

$\Rightarrow \boxed{F = 0}$

FORCE BETWEEN TWO PARALLEL CURRENT CARRYING CONDUCTOR:-

Two parallel current apply force on each other because in the magnetic field of one current other current is present.

I_1 and I_2 are two || currents separated by distance d.



$\vec{B}$ due to I_1 on I_2,

$$B_1 = \frac{\mu_0 I_1}{2\pi d}$$

Force on I_2 due to I_1.

$$F_{21} = B_1 I_2 l \sin 90^\circ$$
$$= \frac{\mu_0 I_1 I_2}{2\pi d} \cdot l$$
$$\Rightarrow \frac{F_{21}}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

Similarly force on I_1 due to I_2

$$\frac{F_{12}}{l} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

using fleming's left hand rule it is clear that, Two || currents attract each other and two anti || current repel each other.

1 AMPERE :- $I_1 = I_2 = 1A$, $d = 1m$

$$\frac{F}{l} = \frac{\mu_0}{2\pi} = \frac{10^{-7} \times 4\pi}{2\pi} = 2\times 10^{-7}\ N/m$$

One ampere is defined as that current which when passes along two || conductors in same direction separated by 1m in vacuum exert force of attraction of 2×10^{-7} N/m on each other.

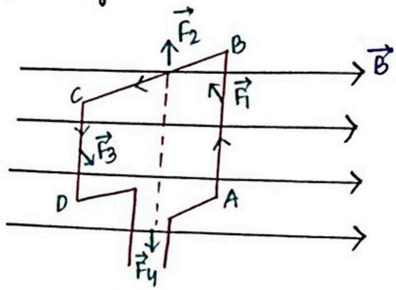
TORQUE ON A CURRENT CARRYING LOOP:-

ABCD is a current carrying loop of length l and breadth b

I = current passing through it

area of the loop $A = l \times b$

B = magnetic field strength



Force on AB,

$\vec{F_1} = I(\vec{l} \times \vec{B})$ inward

Force on BC,

$\vec{F_2} = I(\vec{b} \times \vec{B})$ along the axis upward.

Force on CD,

$\vec{F_3} = I(\vec{l} \times \vec{B})$ outward

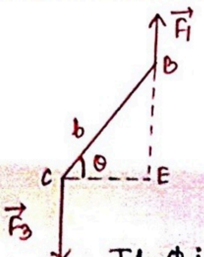
Force on AD,

$\vec{F_4} = I(\vec{b} \times \vec{B})$ downward

$\vec{F_1}$ and $\vec{F_2}$ are cancelled by $\vec{F_3}$ and $\vec{F_4}$ respectively.

So net force = 0.

but $\vec{F_1}$ and $\vec{F_3}$ are not in same line of action. So they constitute a couple due to which loop rotate.



$$\tau = (BIl \sin 90^\circ) \times CE$$

$$= BIl \times b\cos\theta$$

$$= BIA\cos\theta.$$

for N turns, $\boxed{\tau = BINA\cos\theta}$

If ϕ is the angle made by axial vector of coil with $\vec{B}$

$$\theta + \phi = 90^\circ \Rightarrow \theta = 90^\circ - \phi$$

$$\tau = BINA\cos\theta = BINA\cos(90^\circ - \phi) = BINA\sin\phi$$

$$\tau = BM\sin\phi$$

$$\boxed{\vec{\tau} = \vec{M} \times \vec{B}}$$

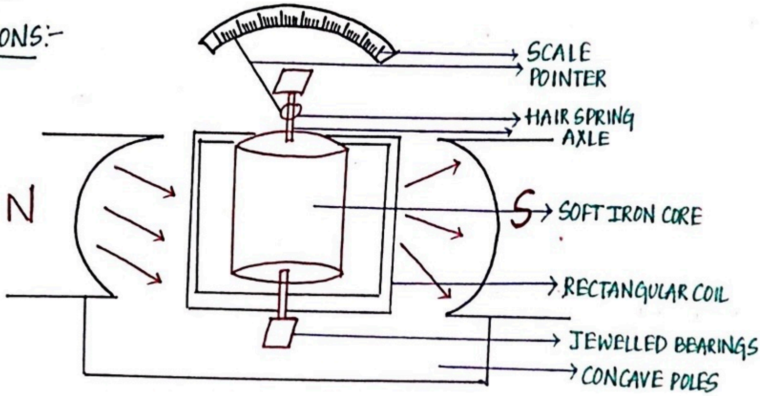
MOVING COIL GALVANOMETER:-

GALVANOMETER:- It is a device that is used to detect small current in the circuit.

PRINCIPLE:-

When a current carrying coil is placed in uniform magnetic field, it experiences torque.

CONSTRUCTIONS:-



The moving coil galvanometer consists of a coil with many turns free to rotate about a fixed axis, in a uniform radial magnetic field to maintain the plane of the coil always remain parallel to field $\vec{B}$ and to have maximum torque. The soft iron cylinder is used as a core material due to its high permeability which intensifies the $\vec{B}$ and hence increases the sensitivity of the galvanometer. When a current flows through the coil, a torque acts on it.

THEORY:-

N = no. of turns
A = area of the coil
B = magnetic field strength

When current I passes through the coil, deflecting torque,

$$\boxed{\tau = BINA\cos\theta}$$

As the $\vec{B}$ is radial i.e angle between plane of the coil and $\vec{B}$ is 0

$$\boxed{\tau = BINA}$$

If K is restoring torque per unit angle of twist,
ϕ is the unit angle of twist.

In equilibrium,

$$BINA = K\phi$$

$$\Rightarrow \phi = \left(\frac{BAN}{K}\right) I$$

$$\Rightarrow \phi \propto I$$

When current passes through the coil, it deflects.

SENSITIVITY OF GALVANOMETER :- (I_S) → Current sensitivity
(V_S) → Voltage sensitivity

(i) CURRENT SENSITIVITY (I_S) :-

It is defined as angle of deflection per unit current.

$$I_S = \frac{\phi}{I} = \boxed{\frac{BAN}{K}}$$

It can be increased :-

(a) ↑ no of turns
(b) ↑ area of coil
(c) increasing $\vec{B}$
(d) ↓ restoring torque (torsion constant)

(ii) VOLTAGE SENSITIVITY (V_S) :-

It is defined as angle of deflection per unit voltage.

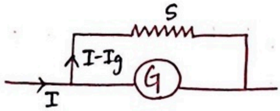
$$V_S = \frac{\phi}{V} = \boxed{\frac{BAN}{KR}}$$

It can be increased :-

(a) increasing $\vec{B}$
(b) increasing area of coil
(c) decreasing torsion constant.

CONVERSION OF GALVANOMETER :-

(a) AMMETER



Galvanometer can be converted to ammeter by connecting a small resistance (shunt) ∥ to it

I_g → galvanometer range
I → Ammeter range
G → resistance of galvanometer.

As the connection is ∥,

$$I_g G = (I - I_g) S$$

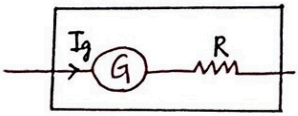
$$\Rightarrow \boxed{S = \frac{I_g G}{I - I_g}}$$

Resistance of ammeter.

$$\boxed{R = \frac{GS}{G+S}}$$

(b) VOLTMETER:-

Galvanometer can be converted to voltmeter by connecting high resistance in series.



I_g = galvanometer range

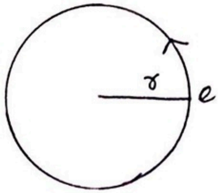
G = galvanometer resistance

R = high resistance.

Range of voltmeter, $\boxed{V = I_g(R+G)}$

Resistance of voltmeter = $\boxed{R+G}$

MAGNETIC MOMENT OF REVOLVING ELECTRON:-



$$I = \frac{e}{t} = \frac{e}{2\pi r/v} = \frac{ev}{2\pi r}$$

Magnetic dipole moment,

$$M_L = I \times A \times 1$$

$$= \frac{ev}{2\pi r} \times \pi r^2$$

$$M_L = \frac{evr}{2}$$

$$M_L = \frac{emvr}{2m}$$

$$\Rightarrow M_L = \frac{eL}{2m}$$

According to Bohr's postulate,

$$M_L = \frac{e}{2m} = \frac{nh}{2\pi}$$

$$\boxed{M_L = \frac{neh}{4\pi m}}$$

So, magnetic moment of revolving electron is quantised

$$(M_L)_{min} = \frac{eh}{4\pi m} \text{ (Bohr's magneton)}$$

$$= \frac{1.6 \times 10^{-19} \times 6.6 \times 10^{-34}}{4 \times 3.14 \times 9.1 \times 10^{-31}} = 9.27 \times 10^{-24} \text{ A m}^2$$

GYROMAGNETIC RATIO:- Ratio of magnetic dipole moment of revolving e^- to its angular momentum

$$\frac{M_L}{L} = \frac{e}{2m} = \text{constant} = 8.8 \times 10^{10}$$