MAGNETISM & MATTER

(CHAPTER-4)

The phenomenon of attraction of small bits of iron, steel, cobalt, nickel etc. towards the ore is called magnetism.

CHARACTERSTICS OF MAGNET:-

1. Monopole doesnot exist.
2. Repulsion is a sure test of magnetisation
3. The distance between two poles of a magnet is called magnetic length and distance between two end of magnet is called geographic length.

$$\boxed{M \cdot L = \frac{7}{8} \times G \cdot L}$$

4. If we break a magnet, $\perp^r$ to the axis then pole strength remains unchanged.
5. If we break the magnet, along the axis into two equal part, pole strength becomes half.

MAGNETIC FIELD LINES:-

Their properties are given below:-

1. Two magnetic field line cannot intersect each other.
2. They form continous closed loops.
3. The tangent at any point on the magnetic field represents the direction of the net magnetic field.
4. The larger the number of field lines crossing per unit area, the stronger is the magnitude of the magnetic field.

MAGNETIC DIPOLE:-

An arrangement of two equal and opposite magnetic pole separated by a small distance.

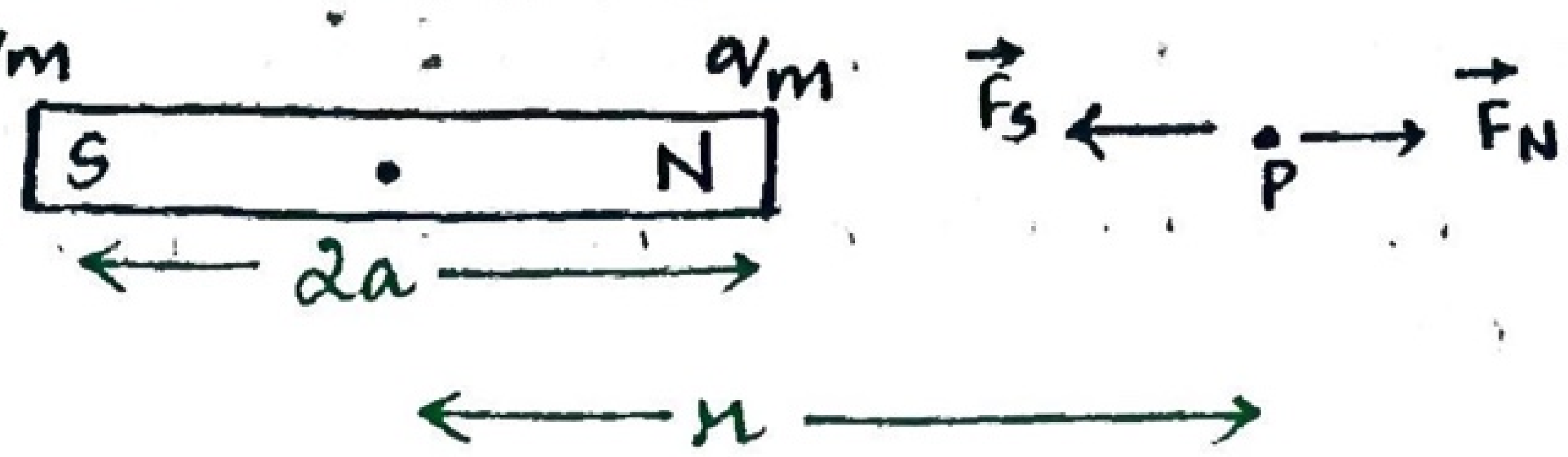
MAGNETIC DIPOLE MOMENT:-(M)

It is defined as the product of its pole strength with the magnetic length of the magnet.

$$\boxed{M = q_m \cdot 2l}$$

* SI unit - A/m^2 or J/T
* It's direction is from south to north.

MAGNETIC FIELD AT AXIAL POSITION :-



P is any point on the axial line at a distance r from the centre.

$$\vec{F_N} = \frac{K q_m q_{m_0}}{(r-a)^2} (\hat{i})$$

$$\vec{F_S} = \frac{K q_m \cdot q_{m_0}}{(r+a)^2} (-\hat{i})$$

$$\vec{F}_{net} = (\vec{F_N} - \vec{F_S}) (\hat{i})$$

$$= \left[\frac{K q_m q_{m_0}}{(r-a)^2} - \frac{K q_m q_{m_0}}{(r+a)^2}\right] (\hat{i})$$

$$= K q_m q_{m_0} \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right] (\hat{i})$$

$$= K q_m q_{m_0} \left[\frac{4ar}{(r^2-a^2)^2}\right] (\hat{i})$$

$$= \frac{2Kmr q_{m_0}}{(r^2-a^2)^2} (\hat{i})$$

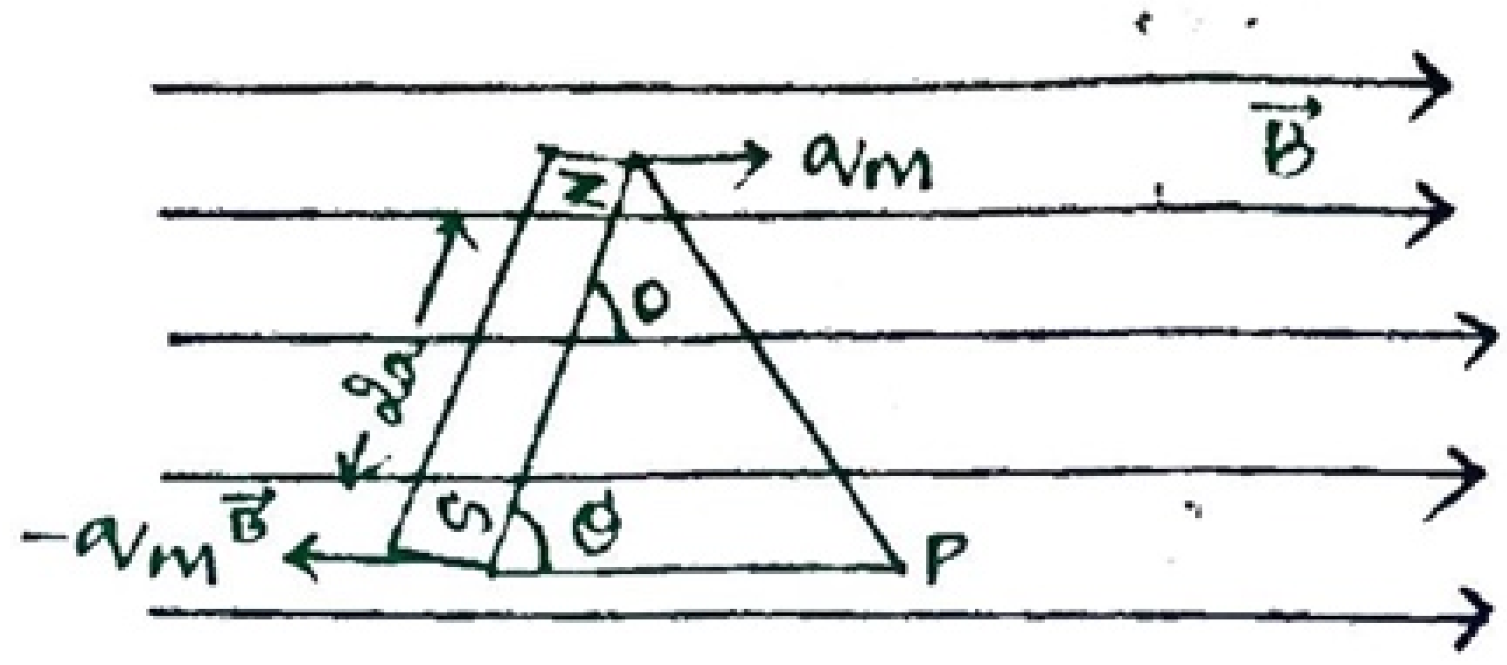
$$\vec{F}_{net} = \frac{2K\vec{m} r q_{m_0}}{(r^2-a^2)^2} (\hat{i})$$

$$\boxed{\vec{B}_{net} = \frac{\vec{F}_{net}}{q_{m_0}} = \frac{2K\vec{m} r}{(r^2-a^2)^2} (\hat{i})}$$

When the magnet is short, $a <<< r$, a^2 can be neglected.

$$\boxed{\vec{B}_{net} = \frac{2K\vec{m} r}{r^4} = \frac{2K\vec{m}}{r^3}}$$

MAGNETIC FIELD AT AN EQUATORIAL POINT:-



In Δ SNP,

$$\sin\theta = \frac{NP}{NS}$$

$$\Rightarrow PN = 2a\sin\theta$$

The force acting on the south pole is towards left. The force acting on the north pole is towards right.

$$\vec{F}_{net} = q_m\vec{B} - q_m\vec{B} = 0$$

As, the force are not in same line of action. So net $\tau \neq 0$. So they constitute a couple due to which the dipole rotates.

$$\tau = \text{magnitude of force} \times \perp^r \text{dist.}$$

$$= q_m B \cdot 2a\sin\theta$$

$$= B(q_m \cdot 2a\sin\theta)$$

$$\tau = Bm\sin\theta$$

$$\vec{\tau} = \vec{m} \times \vec{B}$$

$\vec{\tau} \perp \vec{m}$ and $\vec{\tau} \perp \vec{B}$.

Case-1

when $\theta = 0$

$\boxed{\tau = 0}$ $\Rightarrow$ Stable equilibrium.

Case-2

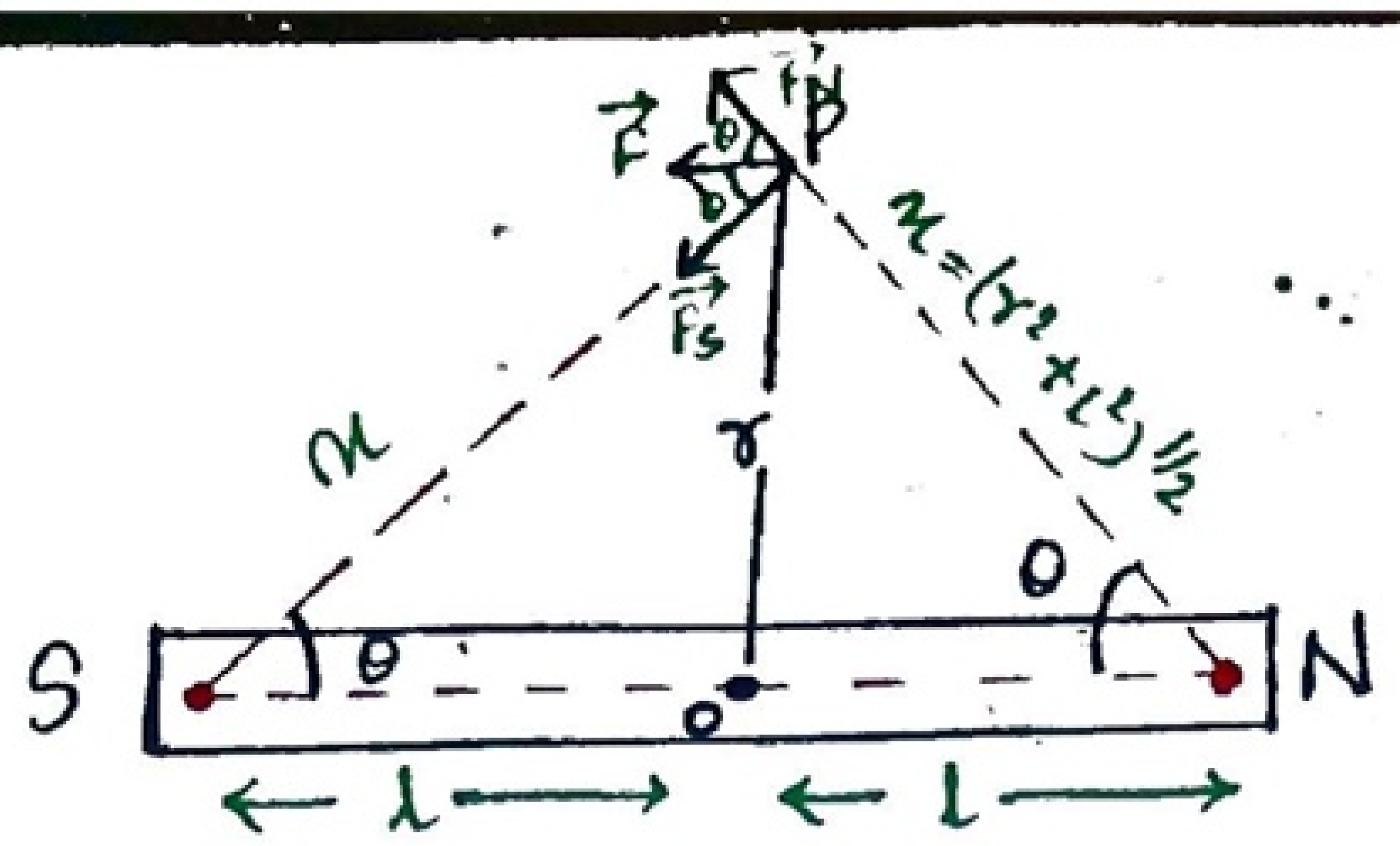
when $\theta = 180°$,

$\boxed{\tau = 0}$ $\Rightarrow$ unstable equilibrium.

Case-3

when $\theta = 90°$,

$\boxed{\tau = Bm}$ $\Rightarrow$ Maximum torque.



P is any point on the equatorial line at a distance r from the centre.

$$F_N = \frac{Kq_m}{x^2}, \text{ along } NP$$

$$F_S = \frac{Kq_m}{x^2}, \text{ along } PS$$

As the magnitudes of F_N and F_S are equal, so their vertical components get cancelled while the horizontal components add up.

$$B_{eq} = F_N \cos\theta + F_S \cos\theta$$

$$= 2F_N \cos\theta$$

$$= \frac{2Kq_m}{x^2} \cdot \frac{l}{x}$$

$$= \frac{Km}{x^3}$$

$$\boxed{B_{eq} = \frac{Km}{(r^2+l^2)^{3/2}}}$$

For short magnet, $l << r$, so l^2 can be neglected.

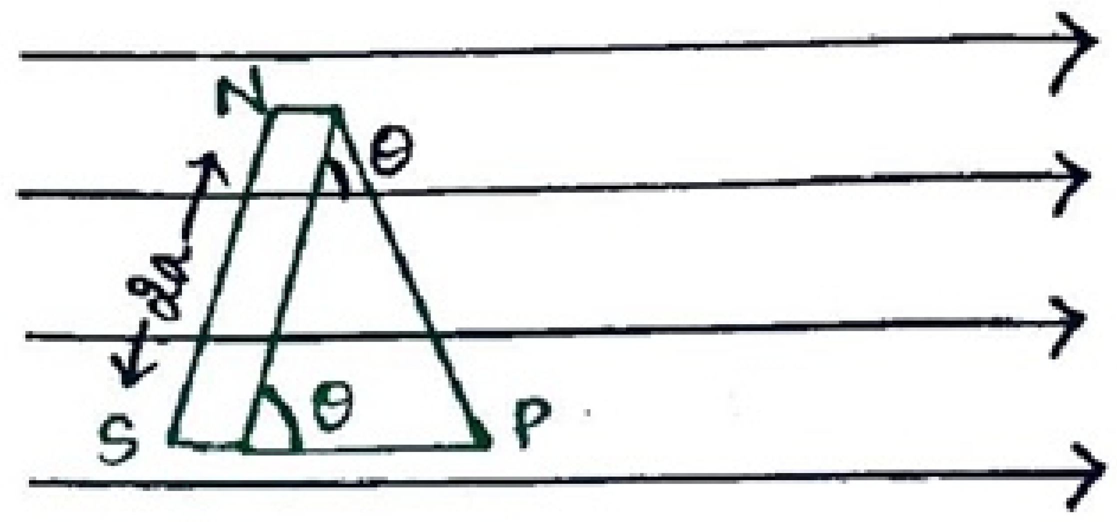
$$B_{eq} = \frac{Km}{r^3} \text{ along } PR$$

The magnetic field at any equitorial point of a magnetic dipole is in the direction opposite to that of its magnetic dipole moment.

$$\boxed{\vec{B}_{eq} = \frac{-K\vec{m}}{r^3}}$$

TORQUE ON A MAGNETIC DIPOLE IN A MAGNETIC FIELD:-

POTENTIAL ENERGY OF A MAGNETIC DIPOLE IN A UNIFORM MAGNETIC FIELD:-



Let the magnetic dipole moved through a small angle $d\theta$ and torque acting on dipole is τ. Then the small work done in moving dipole $dW = \tau d\theta$

$$\Rightarrow \int_0^W dW = \int_{\theta_1}^{\theta_2} \tau d\theta$$

$$\Rightarrow W = \int_{\theta_1}^{\theta_2} mB\sin\theta \, d\theta$$

$$= mB \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$= mB(-\cos\theta)_{\theta_1}^{\theta_2}$$

$$= -mB(\cos\theta_2 - \cos\theta_1)$$

$$\boxed{\begin{aligned} W &= -mB(\cos\theta_2 - \cos\theta_1) \\ W &= mB(\cos\theta_1 - \cos\theta_2) \end{aligned}}$$

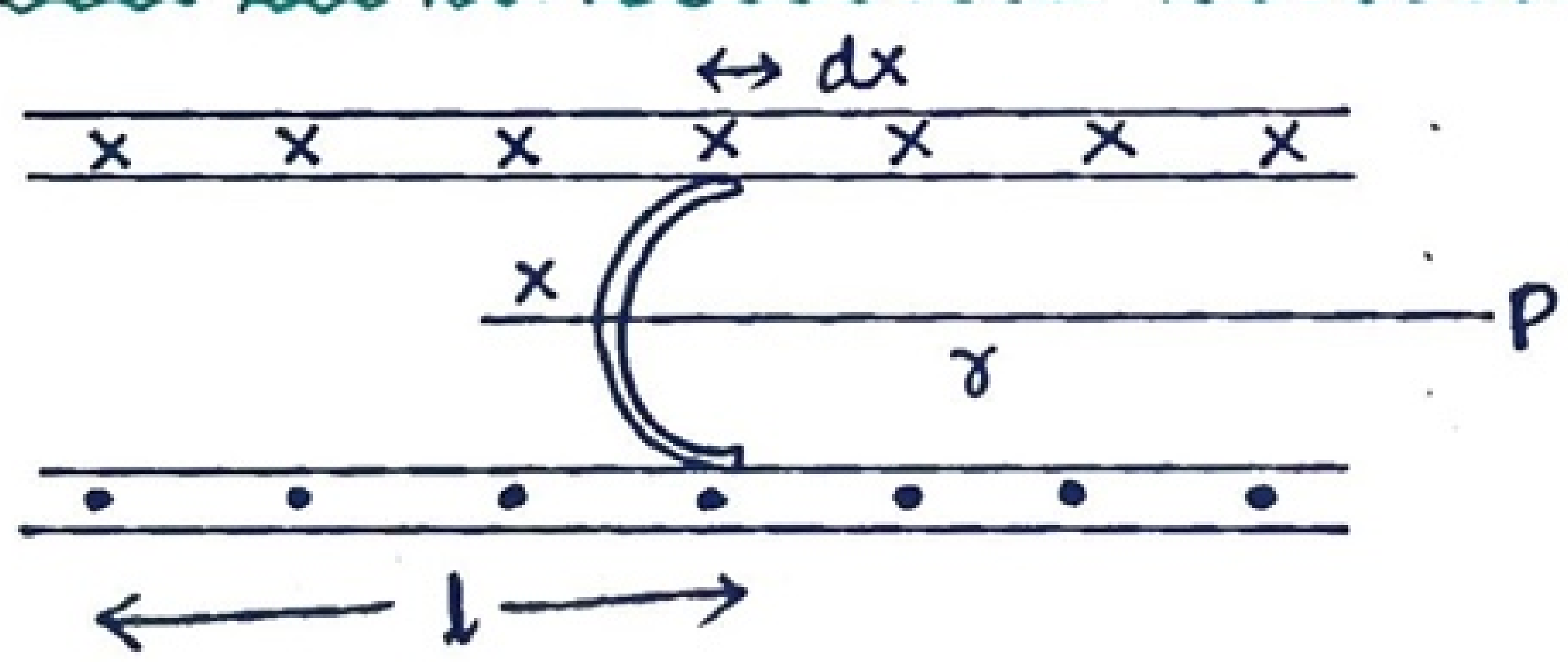
If initial angle, $\theta_1 = \pi/2$ and $\theta_2 = \theta$

$$W = mB(-\cos\theta)$$

$$\Rightarrow U = -mB\cos\theta$$

$$\Rightarrow \boxed{U = -\vec{m} \cdot \vec{B}}$$

BAR MAGNET AS AN EQUIVALENT SOLENOID:-



n = no. of turns per unit length
l = length of solenoid
R = radius of solenoid
r = dist. of point P from the centre of the solenoid

Consider a circular loop at a dist x from the solenoid.

Magnetic field at point P due to circular coil,

$$dB = \frac{\mu_0}{4\pi} \frac{2n\,dx \cdot IA}{\{R^2 + (r-x)^2\}^{3/2}}$$

Assuming, the point far away, $r >>> R$ and $r >>> x$

$$dB = \frac{\mu_0}{4\pi} \frac{2n\,dx\,IA}{r^3}$$

Integrating both sides with appropriate limits,

$$\int dB = \frac{\mu_0}{4\pi} \frac{2nIA}{r^3} \int_{-l/2}^{l/2} dx$$

$$\Rightarrow B = \frac{\mu_0}{4\pi} \frac{2nIA}{r^3} [x]_{-l/2}^{l/2}$$

$$= \frac{\mu_0}{4\pi} \frac{2nIA}{r^3} \left[\frac{l}{2} + \frac{l}{2}\right]$$

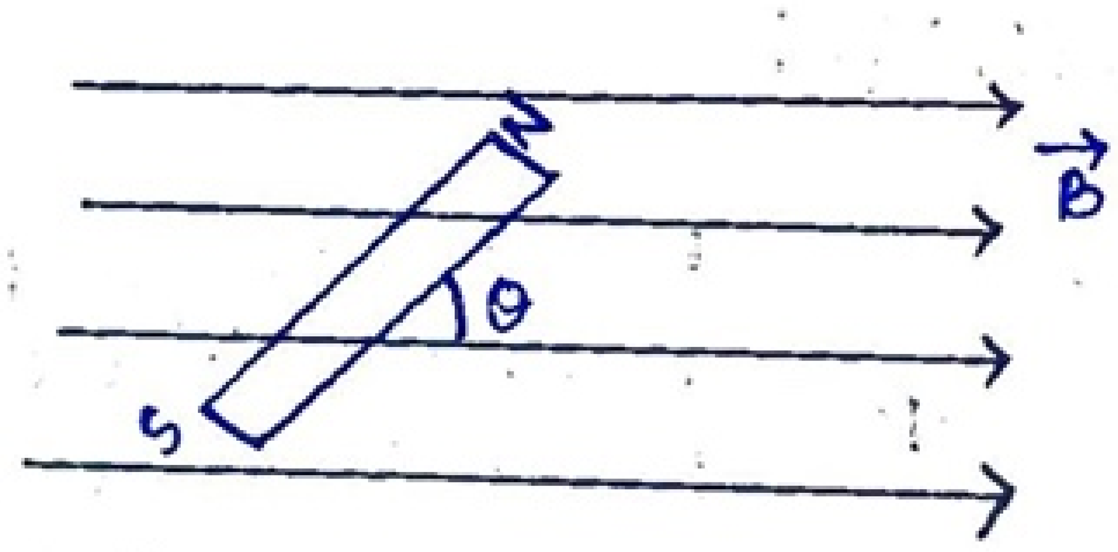
$$= \frac{\mu_0}{4\pi} \frac{2nIAl}{r^3}$$

$$= \frac{\mu_0}{4\pi} \frac{2NIA}{r^3}$$

$$\Rightarrow \boxed{B = \frac{\mu_0}{4\pi} \frac{2M}{r^3}}$$

This expression is similar to the expression of magnetic field due to bar magnet. So solenoid behaves like a bar magnet.

OSCILLATIONS OF A FREELY SUSPENDED MAGNET:- / DIPOLE IN A UNIFORM MAGNETIC FIELD:-



When magnetic dipole is left in uniform magnetic field at any angle, it execute S.H.M

Proof:- When θ is the angle between dipole moment and $\vec{B}$, Restoring torque,

$$\tau = -MB\sin\theta$$

$$\Rightarrow I\alpha = -MB\sin\theta$$

$$\Rightarrow \alpha = -\frac{MB}{I}\theta \text{ (taking } \theta \text{ very small)}$$

$$\alpha \propto (-\theta)$$

So, dipole execute s.h.m, where,

$$\omega^2 = \frac{MB}{I}$$

$$\Rightarrow \omega = \sqrt{\frac{MB}{I}}$$

$$\Rightarrow \frac{2\pi}{T} = \sqrt{\frac{MB}{I}}$$

$$\Rightarrow T = \frac{\sqrt{\frac{MB}{I}}}{2\pi}$$

$$\Rightarrow \boxed{T = 2\pi\sqrt{\frac{I}{MB}}}$$

THE ELECTROSTATIC ANALOG:-

Physical Quantity	Electrostatics	Magnetism
Dipole moment	$P = q \times 2l$	$m = q_m \times 2l$
Axial field	$E_{axial} = \frac{2KP}{r^3}$	$B_{axial} = \frac{2KM}{r^3}$
Equatorial field	$E_{equa} = -\frac{KP}{r^3}$	$B_{eq} = -\frac{KM}{r^3}$
Torque in external field.	$\tau = PE\sin\theta$	$\tau = mB\sin\theta$
P.E in external field.	$U = -PE\cos\theta$	$U = -mB\cos\theta$

GAUSS'S LAW IN MAGNETISM :-

$$\Phi_B = \oint \vec{B} \cdot \vec{ds} = 0$$

* The surface integral of magnetic field over a closed surface is always 0 as magnetic monopole never exist.
* The magnetic flux around a closed surface is 0.

SOME DEFINATIONS IN CONNECTION WITH EARTH'S MAGNETISM:-

① GEOGRAPHIC AXIS:- The straight line passing through the geographic north and geographic south.

② MAGNETIC AXIS :- The straight line passing through magnetic north and south pole of the earth.

③ MAGNETIC EQUATOR :- It is a great circle on the earth perpendicular to the magnetic axis.

④ GEOGRAPHIC MEREDIAN :- The vertical plane passing through geographic north and south pole.

⑤ MAGNETIC MEREDIAN:- The vertical plane passing through magnetic axis of a freely suspended small magnet.

ELEMENTS OF EARTH'S MAGNETIC FIELD:-

① Angle of dip / Angle of Inclination :- (δ)

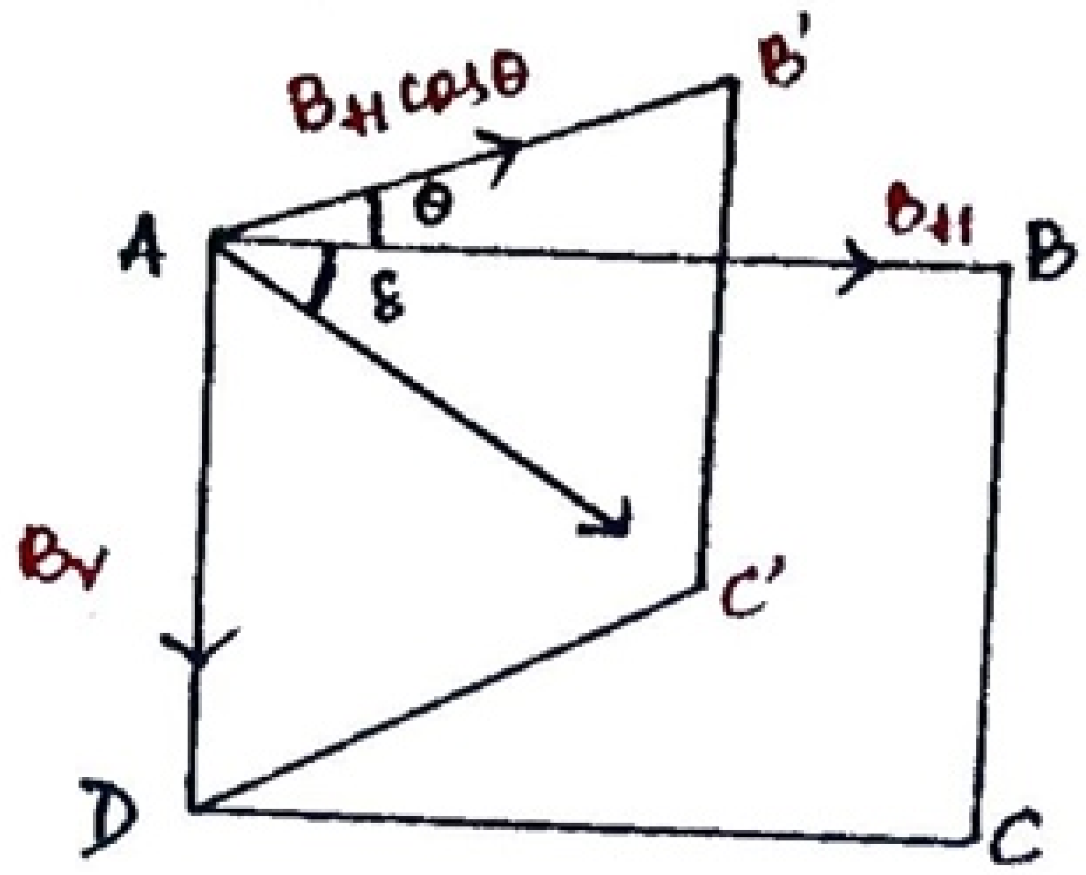
* It is the angle made by the resultant magnetic field of earth with horizontal in magnetic meredian.
* Its value is zero at the equator and 90° at the pole.

② HORIZONTAL COMPONENT:-

* It is the component of earth magnetic field along horizontal.
* It is zero at the pole and 90° at the equator.

③ DECLINATION/MAGNETIC DECLINATION:-

* It is the angle between Geographic meredian and magnetic meredian.
* It is measured as θ° east or θ° west.



ABCD → magnetic meredian
θ → declination
δ → angle of dip

$B_H = B \cos\delta$

$B_V = B \sin\delta$

$$B = \sqrt{B_H^2 + B_V^2}$$

$$\boxed{\tan\delta = \frac{B_V}{B_H}}$$

SOME IMPORTANT TERMS USED TO DESCRIBE MAGNETIC PROPERTIES OF MATERIALS:-

① Intensity of magnetisation :- (I)

* It is defined as dipole moment of substance per unit volume.

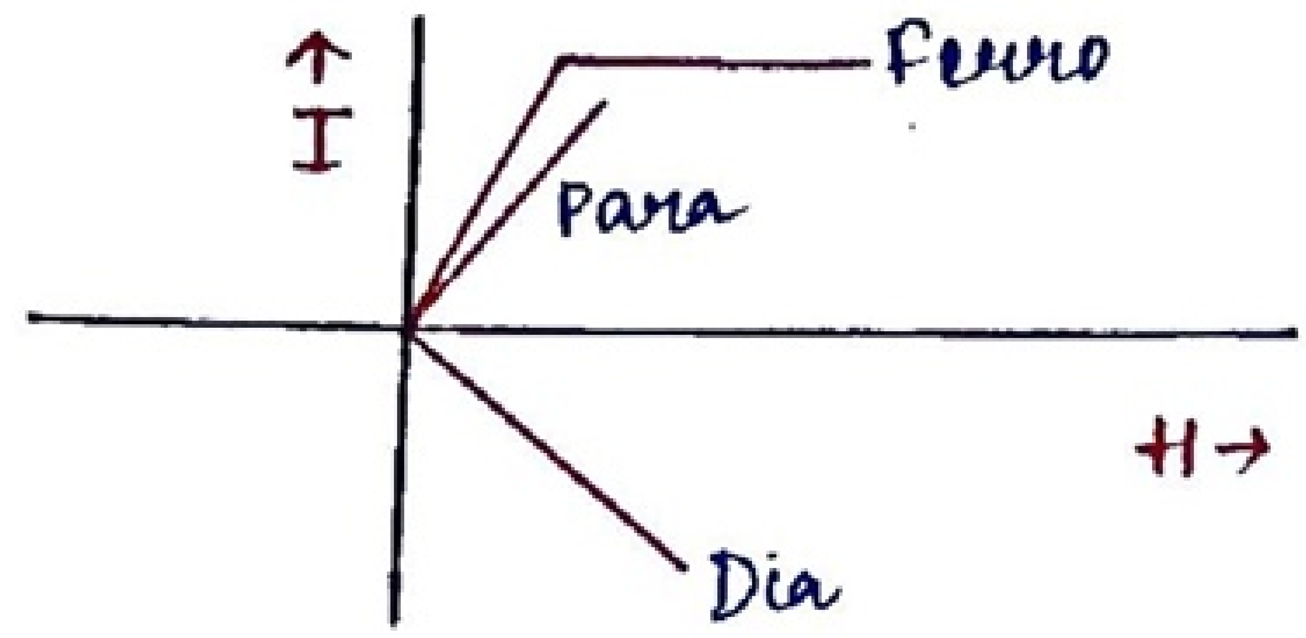
$$I = \frac{m}{\text{Volume}} = \frac{q_m \times 2l}{A \times 2l} = \frac{q_m}{A}$$

For diamagnetic substance, I is −ve
For paramagnetic substance, I is +ve
For ferromagnetic substance, I is highly +ve.

② Intensity of magnetic field :- ($\vec{H}$)

* It is defined as the ratio between the external applied magnetic field to the permeability.

The variation of intensity of magnetisation and magnetising field :-



③ MAGNETIC SUSCEPTIBILITY :- (χ)

* It is defined as the ratio of intensity of magnetisation to magnetising field

$$\boxed{\chi = \frac{I}{H}}$$

- For dia, it is -ve
- For paramagnetic substance, it is +ve
- For ferromagnetic substance, it is highly +ve

④ RELATIVE MAGNETIC PERMEABILITY :- (μ_r)

* It is defined as the ratio of magnetic field inside a substance to applied magnetic field.

$$\boxed{\mu_r = \frac{B}{H}}$$

* It is unitless and dimensionless.

- For diamagnetic substance, $B < H$ so $\mu_r < 1$
- For paramagnetic substance, $B > H$, so $\mu_r > 1$
- For ferromagnetic substance, $B >> H$, so $\mu_r >> 1$.

Relation between susceptibility and magnetic permeability :-

$$B = B_0 + B_m$$

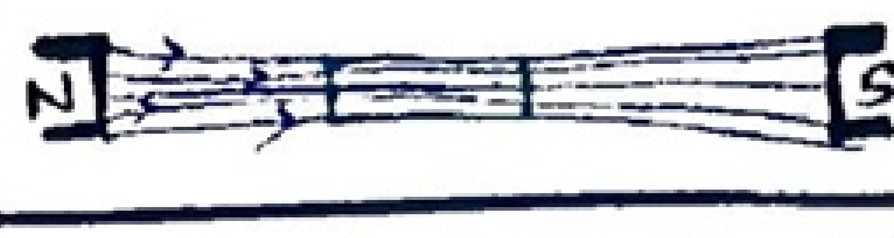
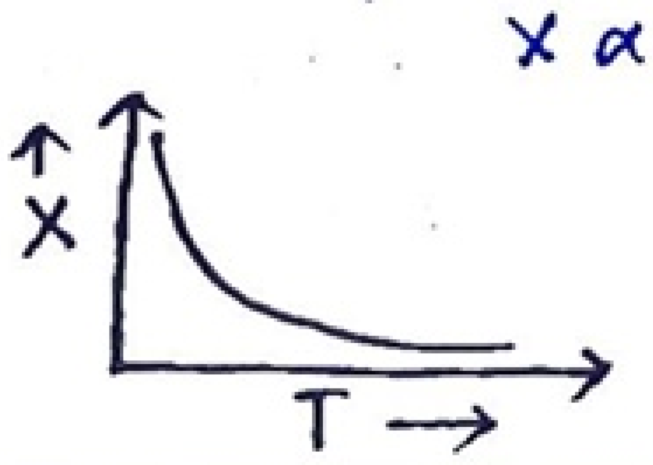
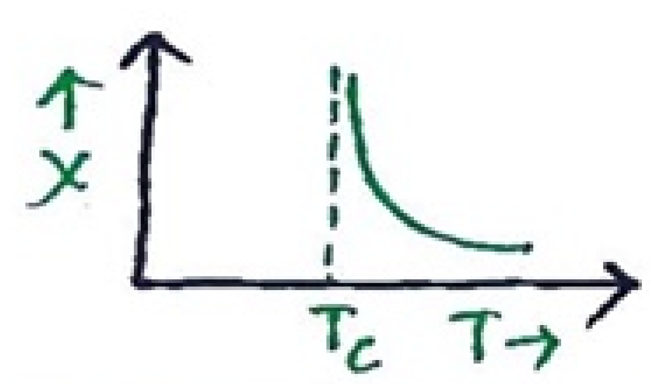
$$\Rightarrow \mu H = \mu_0 H + \mu_0 I$$

$$\Rightarrow \mu H = \mu_0 (H + I)$$

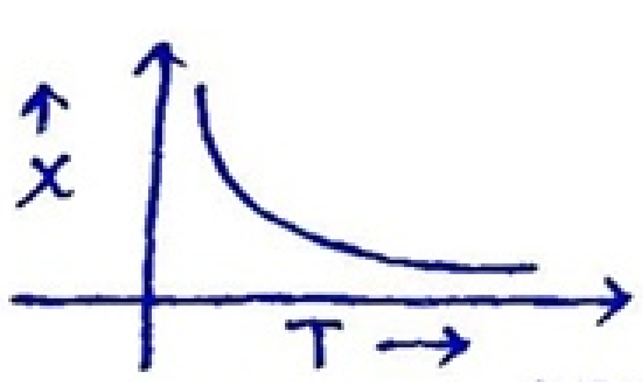
$$\Rightarrow \frac{\mu}{\mu_0} = 1 + \frac{I}{H}$$

$$\Rightarrow \boxed{\mu_r = 1 + \chi}$$

COMPARATIVE STUDY OF MAGNETIC MATERIALS:-

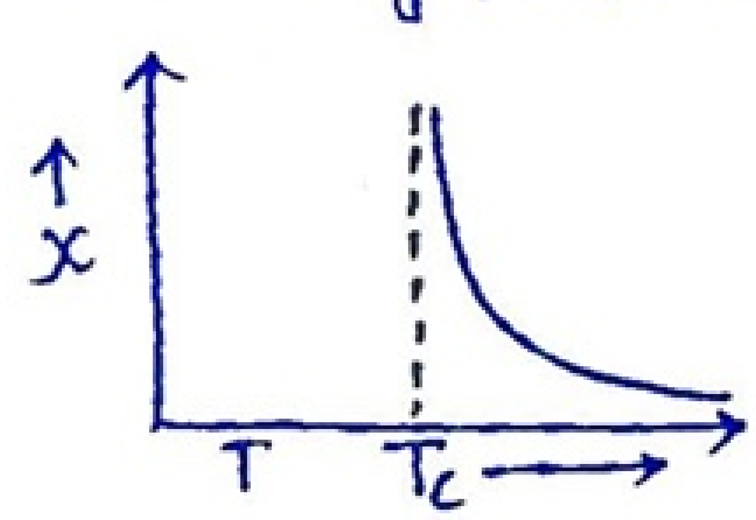
PROPERTY	DIAMAGNETIC	PARAMAGNETIC	FERROMAGNETIC
① Effect of magnet	Feebly repelled 	Feebly attracted 	Strongly attracted 
② External magnetic field	Acquire feeble magnetism opp. to the direction of magnetising field. $\rightarrow H$, M ←	Acquire feeble magnetism in the direction of magnetising field $\rightarrow H$, → M	They are strongly magnetised in the directn of magnetising field $\rightarrow H$, → M
③ Non uniform magnetic field	Tend to move slowly from stronger to weaker magnetic field.	Tend to move slowly from weaker to stronger magnetic field.	Tend to move rapidly from weaker to stronger magnetic field.
④ Susceptibility (χ)	Negative	Positive	Highly positive
⑤ Permeability (μ_r)	$\mu_r < 1$	$\mu_r > 1$	$\mu_r >> 1$
⑥ Effect of temp.	It is independent of temp. 	It is inversely prop to temp $\chi \propto \frac{1}{T}$ 	It follow Curie-Weiss law. 
⑦ Examples	Bi, Cu, Pb, Si, H_2O, NaCl, N_2 (STP)	Al, Na, Ca, O_2 (STP) etc.	Fe, Co, Ni, Alnico, Ticonal

CURIE-WEISS LAW :- It states that susceptibility is inversely proportional to temp for paramagnetic substance.



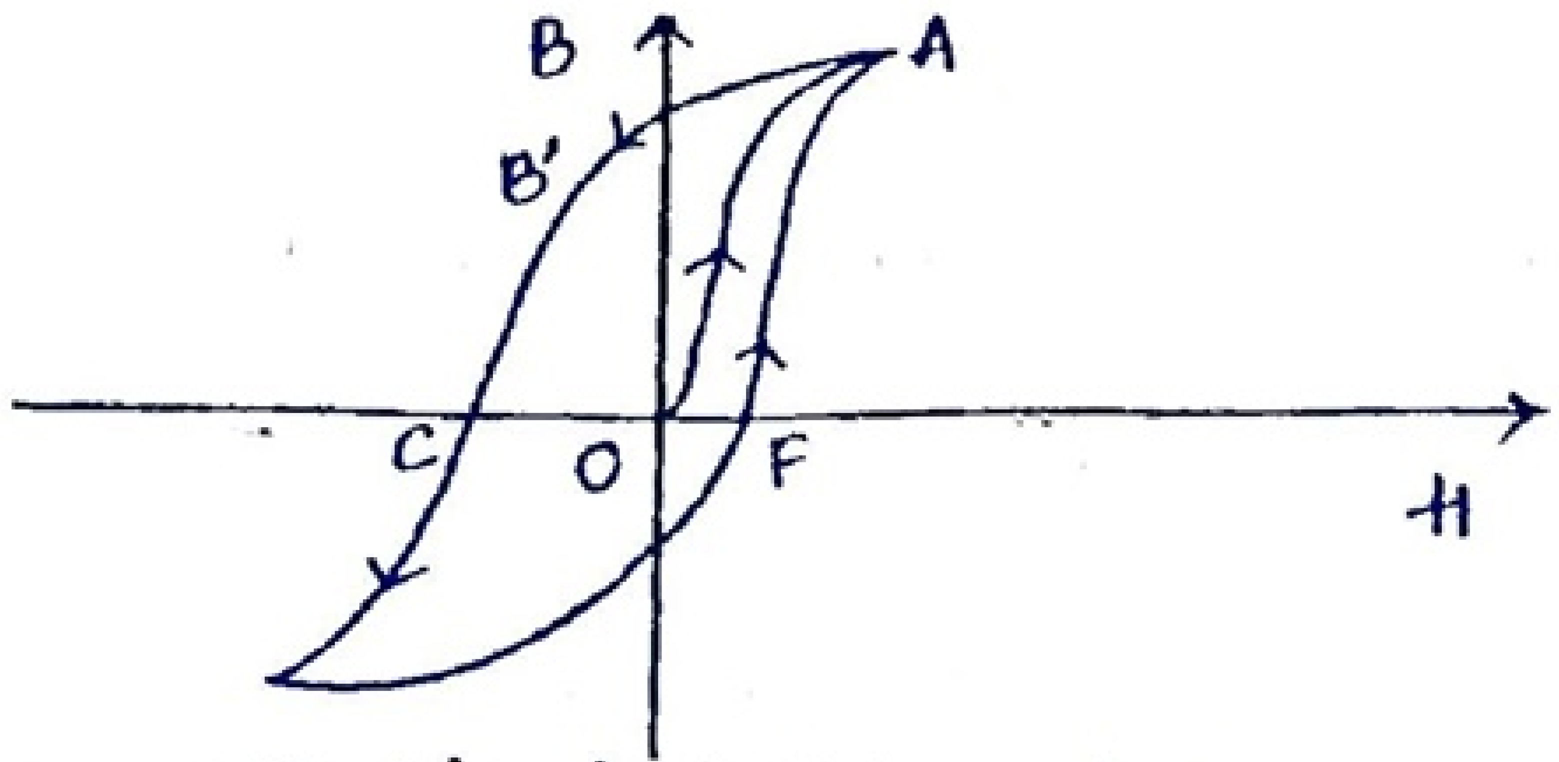
$\chi \propto \frac{1}{T}$ (Paramagnetic)

$\Rightarrow \boxed{\chi = \frac{C}{T}}$, C = curie's constant



T_c (curie's temp) - temp above which ferromagnetic substance behaves like paramagnetic substance

HYSTERESIS :-



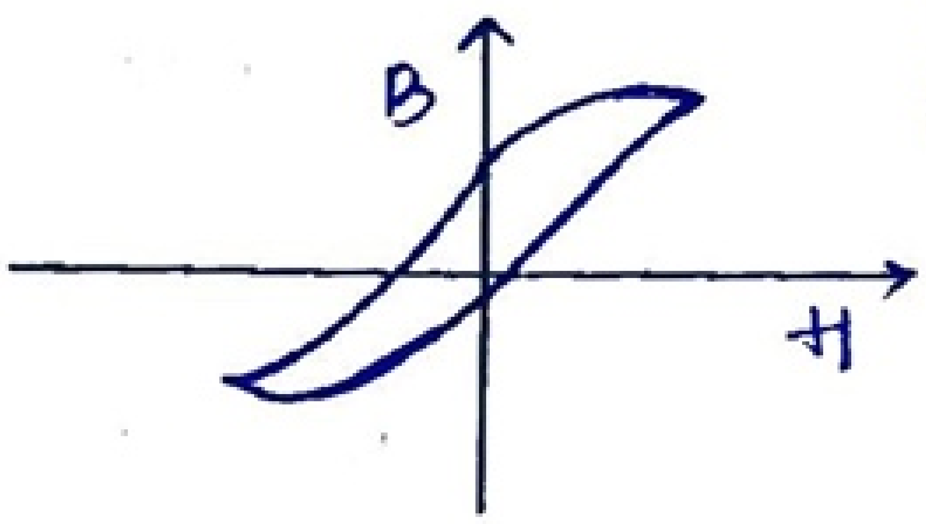
OB' = retentivity
OC = coercivity

The variation of magnetic field inside a ferromagnetic substance and applied magnetic field for a complete cycle of magnetisation and demagnetisation is called hysteresis loop.

Depending on graph :-

Ferromagnetic substances are classified into :-

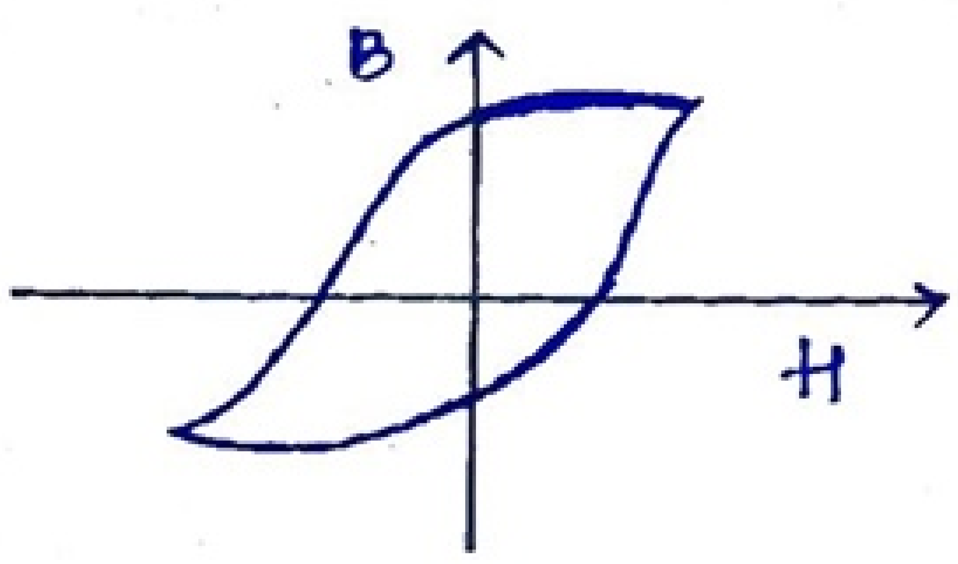
① Soft iron :-



* low retentivity
* low coercivity
* high permeability

used for making electromagnet.

② hard iron :-



* high retentivity
* high coercivity
* high permeability

used for making permanent magnet