CHAPTER - 3

Date
Page No.

Motion in a Plane

+ motion in a plane is a 2- dimensional (2-D) motion.

Scalar quantity

+ It is a quantity with magnitude (numerical value) only.

Eg. Distance, mass, temperature, time etc.

- It can be added subtracted, multiplied and divided by simple rules of algebra.

vector quantity

+ It has both magnitude as well as direction.

- It obeys the triangle law of addition and parallelogram law of addition

- e.g. Displacement, velocity, acceleration force, etc.

Date
Page No.

- The magnitude of vector is often called its absolute value $|V|$.

Representation of vector :-

Either represented by bold face type or arrow over letter
e.g $\mathbf{V}$ or $\vec{v}$

Types of vectors :-

1. Equal vector (same direction, same magnitude)
2. Unequal vector (any one is different)
3. Parallel Vector (Two vector are in same direction, $\theta = 0°$)
4. Antiparallel vector [opp. direction, $\theta = 180°$]
5. Co-linear vector [Two vector fall in same line]
6. Coplaner Vector [Any two vector are always parallel]
7. Concurrent vector [A vector acting at same point]

Date
Page No.

8. Zero vector [magnitude = 0, direction is orbitary.]

9. unit vector [Magnitude = 1 it gives direction of vector]

Physical Quantity = magnitude x unit

$\vec{A} = |A| \times \hat{A}$ ← unit vector

$$\text{unit vector } (\hat{A}) = \frac{\vec{A}}{|A|}$$

Addition and Subtraction of Vectors - Graphical Method

→ Head - Tail Method

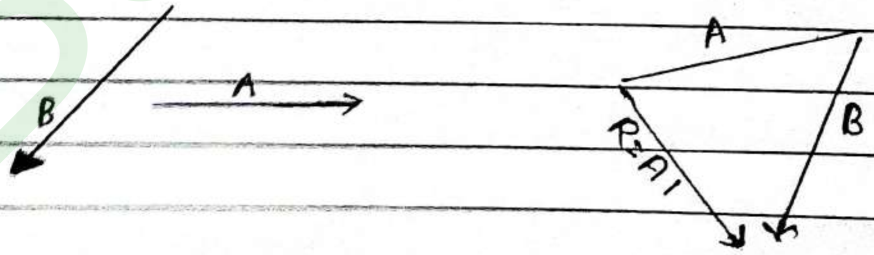
→ Parallelogram law of addition

→ Triangle law.

Head-Tail Method

To find sum of $\vec{A} + \vec{B}$, we place vector $\vec{B}$ so that its tail is at the head of the vector $\vec{A}$.
Join the tail of $\vec{A}$ to the head of $\vec{B}$

Resultant (R) = $\vec{A} + \vec{B}$



The addition of vector is commutative.

$A + B = B + A.$

Date
Page No.

Ques what is the result of adding two equal and opposite vector.

Sol. Since the magnitudes of the two vector are same,
but direction is opp., the resultant vector has zero magnitude (0) called a zero vector or a null vector.

$$A + (-A) = 0$$
$$A - A = 0$$

- Direction of a null vector is not specified.

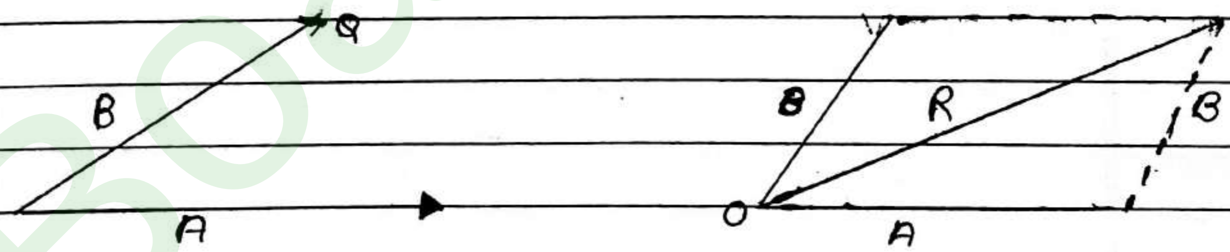
Subtraction of vector :-

The difference of two vectors $\vec{A}$ and $\vec{B}$ is the sum of two vector $\vec{A}$ and $-\vec{B}$.

$$\vec{A} - \vec{B} = \vec{A} + (-\vec{B})$$

Parallelogram method

- Join two vectors from tail to tail as two adjacent side of a parallelogram and complete the imaginary llgm.
- Join the common point of intersection to O, to get R.

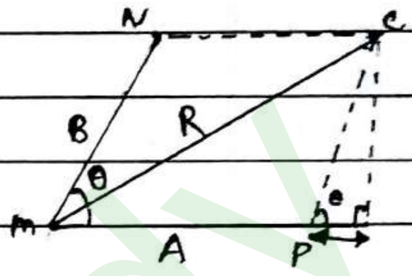


R = diagonal of the llgm from common point

$$\boxed{R^2 = A^2 + B^2 + 2AB\cos\theta}$$

All the value are positive, because it represents magnitude of vector.

Derivation of magnitude of vector



In ΔPOQ

$\cos\theta = \frac{B}{H} = \frac{PQ}{PO}$

$PO\cos\theta = PQ$

$(PO = B)$

$\boxed{PQ = B\cos\theta}$

In ΔPOQ

$\sin\theta = \frac{P}{H} = \frac{OQ}{PO}$

$\sin\theta = \frac{OQ}{B}$

$\boxed{OQ = B\sin\theta}$

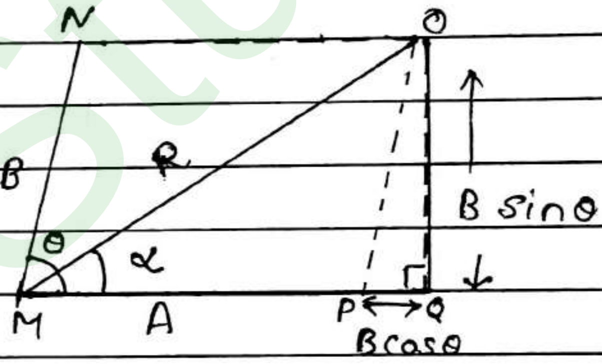
Date
Page No.

In ΔOQM

$$(MO)^2 = (OQ)^2 + (MQ)^2$$
$$R^2 = (B\sin\theta)^2 + (A + B\cos\theta)^2$$
$$R^2 = B^2\sin^2\theta + A^2 + B^2\cos^2\theta + 2AB\cos\theta$$
$$R^2 = A^2 + B^2(\sin^2\theta + \cos^2\theta) + 2AB\cos\theta$$

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

Derivation of direction of Resultant

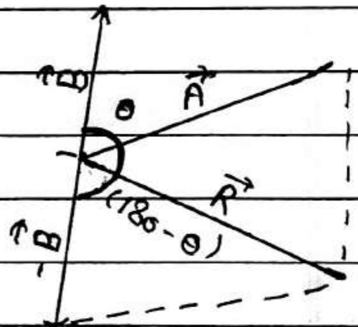


In ΔMOQ :-

$$\tan\alpha = \frac{P}{B} = \frac{OQ}{MQ}$$

$$\boxed{\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}}$$

Subtraction of vectors



$$\vec{R} = \vec{A} - \vec{B}$$
$$\vec{R} = \vec{A} + (-\vec{B})$$

$\cos(180-\theta) = -\cos\theta$
$\sin(180-\theta) = +\sin\theta$

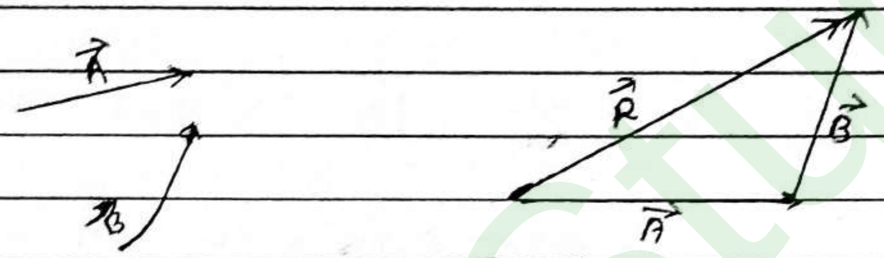
$$R^2 = A^2 + B^2 + 2AB\cos(180-\theta)$$

$$\boxed{R^2 = A^2 + B^2 - 2AB\cos\theta}$$

$$\tan\alpha = \frac{B\sin(180-\theta)}{A + B\cos(180-\theta)} = \frac{B\sin\theta}{A +- B\cos\theta}$$

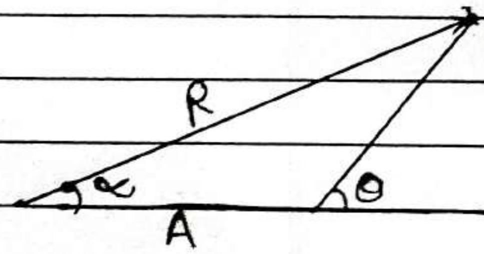
Triangle Law of Addition

If two vectors are represented by two side of a triangle taken in same order then the resultant is give by third side of a triagle taken in opposite order.



$$\boxed{\vec{R} = \vec{A} + \vec{B}}$$

- Triangle law is same as head-to-tail method.



$$\boxed{R^2 = A^2 + B^2 + 2AB\cos\theta}$$

magnitude of vector

$$\boxed{\tan\alpha = \frac{B\sin\theta}{A + B\cos\theta}}$$

Direction of vector.

* The two methods (llgm method and triangle law) Yields the same result.

Thus, the two methods are equivalent.

Date
Page No.

Minimum and Maximum Value of Resultant

$-1 \leftarrow \cos\theta \rightarrow +1$

$-1 \leftarrow \sin\theta \rightarrow +1$

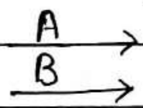
(min.) (max.)

- Value of R is maximum when value of $\cos\theta$ is maximum.

$$\boxed{\cos 0^\circ = 1}$$

$$R^2 = A^2 + B^2 + 2AB\cos\theta$$

$$R^2 = A^2 + B^2 + 2AB\,(1) \quad (0^\circ \text{ mean parallel})$$

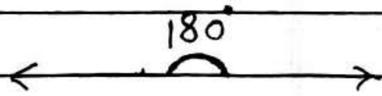


$$R^2 = (A+B)^2$$

$$\boxed{R_{max.} = |A+B|}$$

$$\boxed{\cos 180^\circ = -1}$$

$$R^2 = A^2 + B^2 + 2AB\cos 180^\circ$$



$$R^2 = A^2 + B^2 + 2AB\,(-1)$$

$$R^2 = A^2 + B^2 - 2AB$$

$$R^2 = (A-B)^2$$

$$\boxed{R_{min.} = |A-B|}$$

We have two vector 3 and 4 Then the resultant cannot be

a. 2 Range ⇒ $|A-B|$ to $|A+B|$

b. 6 $|3-4|$ to $|3+4|$

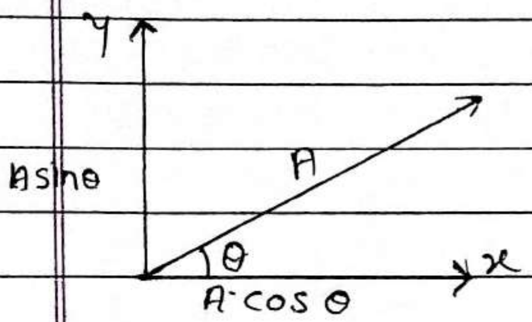
c. 8 ✓ 1 to 7.

d. 4

Date

Page No.

Resolution of vectors



we take cos θ where θ makes angle with A.

* component of vector can be +ve, -ve or 0.

* Component of $\vec{A}$ are $A\cos\theta$ and $A\sin\theta$

* $\boxed{|A| = \sqrt{Ax^2 + Ay^2}}$

Que If $\vec{A} = 10$, then find the component Ax and Ay.

Soln

$Ax = A\cos\theta$
$= 10\cos 30°$
$= 10 \times \frac{\sqrt{3}}{2}$

$Ay = A\sin\theta$
$= 10\sin 30°$
$= 10 \times \frac{1}{2}$

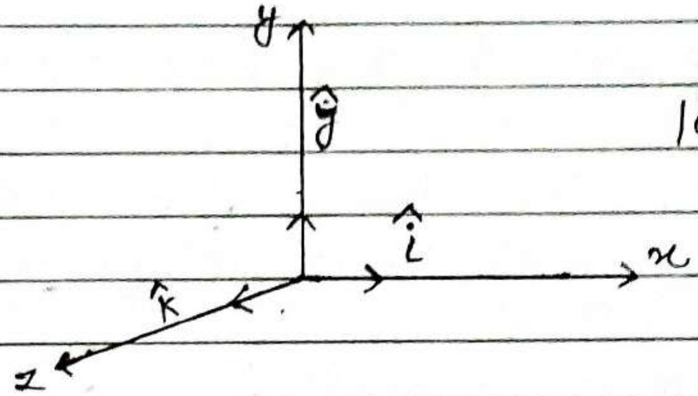
~~$Ax = A\cos\theta$~~

~~$= 1$~~

$Ay = 5$

$\boxed{Ax = 5\sqrt{2}}$

Orthogonal (Perpendincular) unit vector



$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$

+ unit vector has magnitude 1 and has no dimension and unit, it only gives direction

+ If we multiply a unit vector by a scalar say $\hat{n}$ the rescult is a vector $\boxed{\vec{A} = |A| \times \hat{n}}$

Date
Page No.

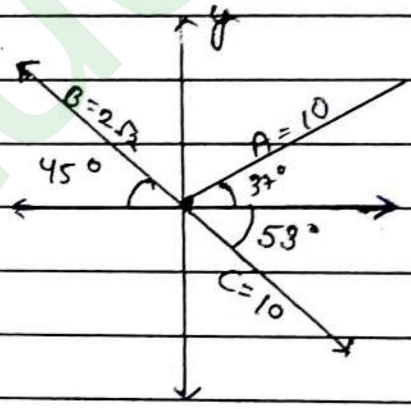
Cartesian form of vectors

$$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_x\hat{k}$$

$$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

magnitude of vector

Addition of three or more vectors



$$A_x = A\cos 37^\circ = 10 \times \frac{4}{5} = +8$$

$$A_y = A\sin 37^\circ = 10 \times \frac{3}{5} = +6$$

$$B_x = B\cos 45^\circ = 2\sqrt{2} \times \frac{1}{\sqrt{2}} = -2$$

$$B_y = B\sin 45^\circ = 2\sqrt{2} \times \frac{1}{\sqrt{2}} = +2$$

$$C_x = C\cos 53^\circ = 10 \times \frac{3}{5} = +6$$

Date
Page No.

$C_y = C\sin 53^\circ = 10 \times \frac{4}{5} = -8$

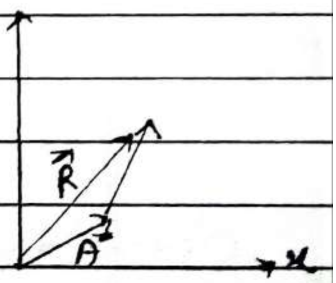
$R_x = A_x + B_x + C_x$
$= 8 + (-2) + 6$
$= 12$

$R_y = A_y + B_y + C_y$
$= 6 + 2 + (-8)$
$= 0$

$\vec{R} = \vec{R_x} + \vec{R_y}$
$= 12 + 0$
$\boxed{\vec{R} = 12\hat{i}}$

Add $\vec{A} = A_x\hat{i} + A_y\hat{j}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j}$

$R_x = A_x + B_x$
$R_y = A_y + B_y$
$\vec{R} = R_x\hat{i} + R_y\hat{j}$
$\Rightarrow (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$



Date

Page No.

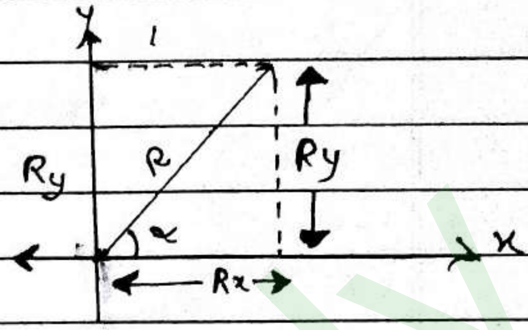
$$R_x = A_x + B_x + C_x$$

$$R_y = A_y + B_y + C_y$$

$$\vec{R} = R_x\hat{i} + R_y\hat{j}$$

$$|R| = \sqrt{R_x^2 + R_y^2}$$

$$\vec{R} = |R| \times \hat{R}$$



$$\boxed{\tan\alpha = \frac{R_y}{R_x}}$$

Multiplication of Vector

Vector x Vector = Scalar [Scalar products]

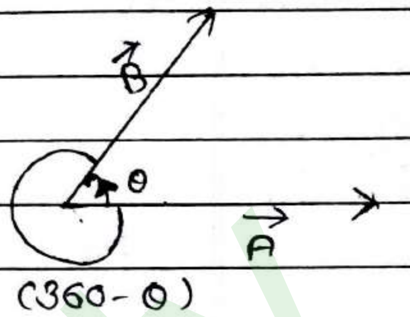
e.g. work = $\vec{F} \times \vec{S}$

Vector x Vector = vector [cross product]

Scalar Product / Dot Product

$$\vec{A} \bullet \vec{B} = C$$

$$\boxed{\vec{A}\cdot\vec{B} = |A| \times |B| \times \cos\theta}$$



Is $\vec{A}\cdot\vec{B}$ is commucative

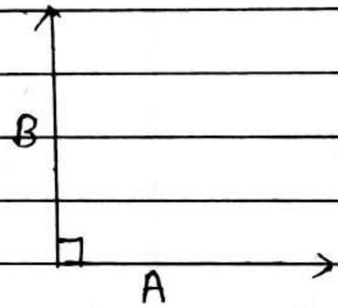
⇒ yes

$$\vec{A}\cdot\vec{B} = |A||B|\cos\theta$$

$$\vec{B}\cdot\vec{A} = |B||A|\cos(360-\theta) \quad \boxed{\cos(360-\theta) = +\cos\theta}$$

$$\vec{B}\cdot\vec{A} = |B||A|\cos\theta$$

* when A is perpendicular to B
 then $\theta = 90^\circ$



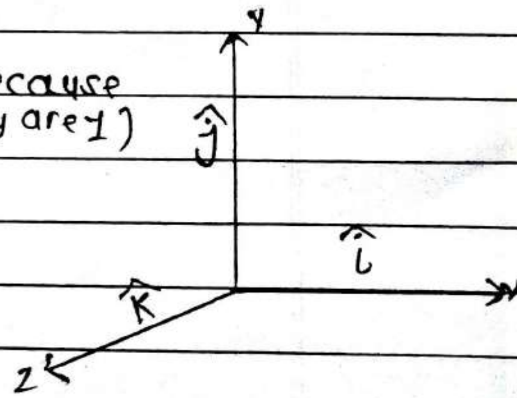
$$\vec{A}\cdot\vec{B} = |A||B|\cos 90^\circ$$

$$\boxed{\vec{A}\cdot\vec{B} = 0}$$

Orthogonal (Perpendicular) unit vector

$$\boxed{\begin{gathered}\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{i}\cdot\hat{k} = 0 \\ \hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1\end{gathered}}$$

(because they are ⊥)



Date
Page No.

Que IF $\vec{A} = 2\hat{i} + 3\hat{j}$ and $\vec{B} = 4\hat{i} + 5\hat{j}$. then Find $\vec{A}\cdot\vec{B}$

Sol- $\vec{A}\cdot\vec{B} = (2\hat{i}+3\hat{j})\,(4\hat{i}+5\hat{j})$

$= 2\hat{i}(4\hat{i}+5\hat{j}) + 3\hat{j}(4\hat{i}+5\hat{j})$

$= 8(1) + 10(0) + 12(0) + 15(1)$

$= 8 + 15$

$= \underline{23}$

or (Shortcut)

↓

$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y$

$= (2\times 4) + (3\times 5)$

$= 8 + 15$

$= \underline{23}$

Application of dot product → To Find angle b/w two Vector

$\vec{A}\cdot\vec{B} = |A||B|\cos\theta$

$$\boxed{\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|A||B|}}$$

Date
Page No.

Que: If a vector $2\hat{i} + 3\hat{j} + 8\hat{k}$ is perpendicular to the vector $4\hat{i} - 4\hat{j} + \alpha\hat{k}$, then the value of α is?

Solve: If $\vec{A} \perp \vec{B}$, then $\vec{A} \cdot \vec{B} = 0$

$$(2\hat{i} + 3\hat{j} + 8\hat{k})(4\hat{i} - 4\hat{j} + \alpha\hat{k}) = 0$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$
$$0 = (2 \times 4) + (3 \times (-4)) + 8 \times (\alpha)$$
$$0 = 8 - 12 + 8\alpha$$
$$0 = -4 + 8\alpha$$
$$\frac{4}{8} = \alpha$$

$$\boxed{\alpha = 1/2}$$

Que: If $\vec{P} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{Q} = \hat{i} - \hat{j}$, then find θ.

Sol: $\vec{P} \cdot \vec{Q} = 2 + (1) + 1(-1) \times 0$
$= 2 - 1$
$= 1.$

$$|P| = \sqrt{Px^2 + Py^2 + Pz^2} = \sqrt{4+1+1} = \sqrt{6}$$

$$|Q| = \sqrt{1^2 + 1^2} = \sqrt{2}.$$

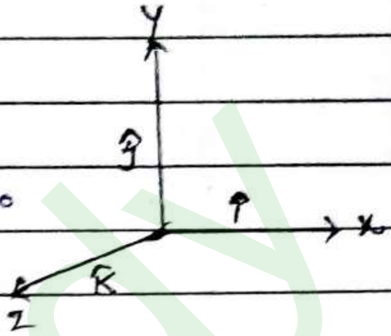
$$\cos\theta = \frac{\vec{P} \cdot \vec{Q}}{|P||Q|} = \frac{1}{\sqrt{6} \times \sqrt{2}} = \frac{1}{\sqrt{12}} \text{ or } \frac{1}{2\sqrt{3}}$$

Que. Find the angle that $\hat{i} + \hat{j}$ makes with x-axis.

Sol. $\vec{A} = \hat{i} + \hat{j}$

$\vec{B} = \hat{i}$

$\cos\theta = \dfrac{\vec{A}\cdot\vec{B}}{|A||B|} = \dfrac{1}{\sqrt{2}}$ $\quad \theta = 45°$



Vector Cross Product

$\vec{A} \times \vec{B} = \vec{C}$

e.g. Torque / moment of force $= \vec{F} \times \overrightarrow{disp}$

$\vec{A} \times \vec{B} = |A||B| \sin\theta$

Is $\vec{A} \times \vec{B}$ is communicative?

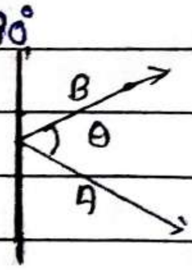
→ No

e.g. $\vec{A} = 5$, $\vec{B}, 2$, $\theta = 30°$, find $\vec{A} \times \vec{B}$ and $\vec{B} \times \vec{A}$.

$\vec{A} \times \vec{B} = |A||B| \sin\theta$ $\qquad$ $\vec{B} \times \vec{A} = |B||A| \sin 30°$

$= 5 \times 2 \sin 30°$ $\qquad$ $= 2 \times 5 \times \frac{1}{2}$

$= 10 \times \frac{1}{2}$

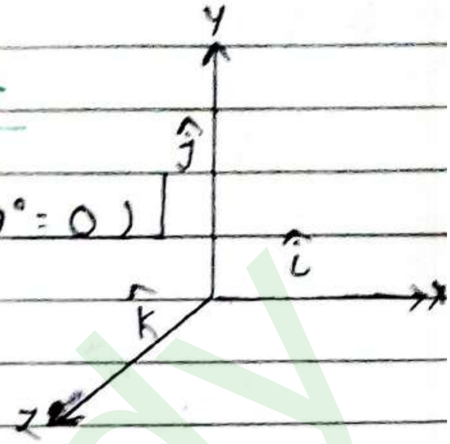


$= 5$ (upward) $\qquad$ $= 5$ (downward)

⇒ direction is opposite.

$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ $\Rightarrow$ $\boxed{\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}}$

Orthogonal unit Vector

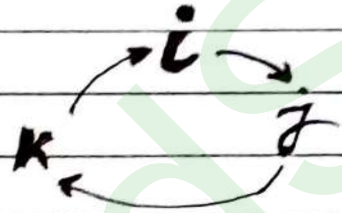


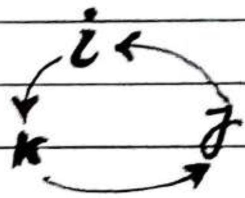
$\boxed{\hat{i}\times\hat{i} = \hat{k}\times\hat{k} = \hat{j}\times\hat{j} = 0 \ (\sin 0^\circ = 0)}$

$\hat{i}\times\hat{j} = |i||j| \sin 90^\circ$
$= 1\times1\times1 \ \hat{n}$

$\boxed{\hat{i}\times\hat{j} = \hat{k}}$

$\boxed{\hat{j}\times\hat{i} = -\hat{k}}$





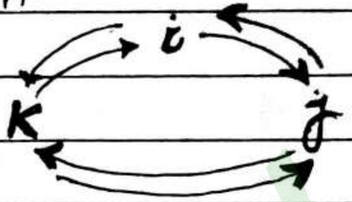
Clockwise → Answer = +ve

Anti clockwise → Answer = -ve.

Date

Page No.

Que $\vec{A} = 4\hat{j}$, $\vec{B} = 3\hat{i}$, $\vec{C} = 2\hat{k}$. Find $\vec{A} * \vec{B}$, $\vec{B} * \vec{A}$ and $\vec{A} \times \vec{C}$, $\vec{C} \times \vec{A}$.



Solve

$\vec{A} \times \vec{B} = 4\hat{j} \times 3\hat{i} = -12\hat{k}$

$\vec{B} \times \vec{A} = 3\hat{i} \times 4\hat{j} = +12k$

$\vec{A} \times \vec{C} = 4\hat{j} \times 2\hat{k} = +8\hat{i}$

$\vec{C} \times \vec{A} = 2R \times 4\hat{j} = -8\hat{i}$

Que $\vec{A} = 2\hat{i} + 3\hat{j} + 4\hat{k}$, $\vec{B} = 3\hat{i} + 2\hat{j} + 3\hat{k}$, Find $\vec{A} \times \vec{B}$.

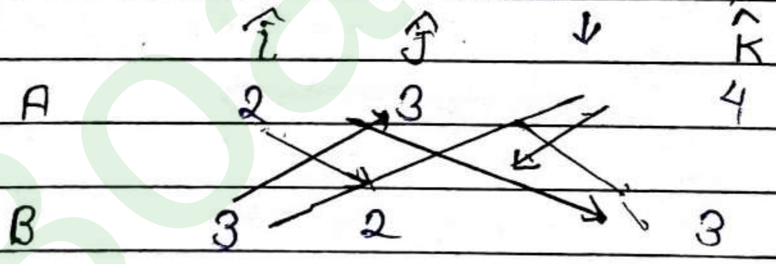
Solve $\vec{A} \times \vec{B} = 2\hat{i}(3\hat{i} + 2\hat{j} + 3\hat{k}) + 3\hat{j}(3\hat{i} + 2\hat{j} + 3\hat{k}) + 4\hat{k}(3\hat{i} + 2\hat{j} + 3\hat{k})$

$= \cancel{6}$

$= 6(0) + 4\hat{k} + (-6\hat{j}) + 9(-\hat{k}) + 6(0) + 9\hat{i} + 12\hat{j} + 8(-\hat{i}) + 12(0)$

$\vec{A} \times \vec{B} = \hat{i} \times 6\hat{j} - 5\hat{k}$

or (shortcut)



$\vec{A} \times \vec{B} = \hat{i}(3\times3 - 4\times2) + \hat{j}(4\times3 - 2\times3) + \hat{k}(2\times2 - 3\times3)$

$= \hat{i}(9-8) + \hat{j}(12-6) + k(4-9)$

$= \hat{i} + 6\hat{j} - 5\hat{k}$

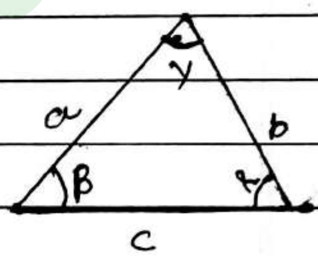
If angle b/w two vector is zero then
$\vec{A} \times \vec{B} = 0$.

$\sin(180 - \theta) = \sin\theta$
$\cos(180 - \theta) = -\cos\theta$

+ $\vec{A} \times \vec{B} = 0$
  ($\theta = 0$ or $180°$)
+ $\vec{A} \cdot \vec{B} = 0$
  ($\theta = 90°$)

Cosine law

To determine diff. angles and side in any triangle.



$$\boxed{\begin{aligned} \cos\alpha &= \frac{b^2 + c^2 - a^2}{2bc} \\ \cos\beta &= \frac{a^2 + c^2 - b^2}{2ac} \\ \cos\gamma &= \frac{a^2 + b^2 + c^2}{2ab} \end{aligned}}$$

⇒ Valid in any triangle.

Sine Law

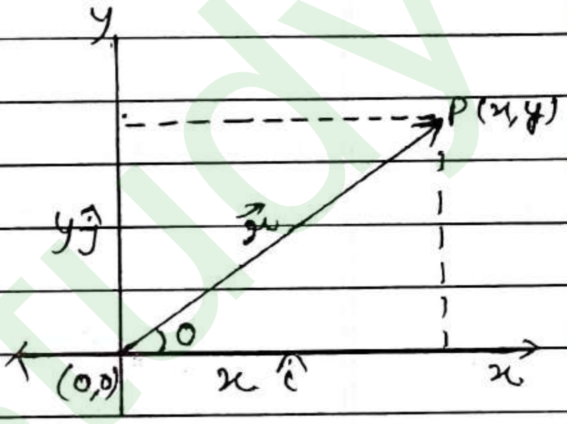
$$\boxed{\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}}$$

⇒ Valid in any triangle

Date
Page No.

Position Vector (2-D)

Position vector is used to specify position of a point, by taking a reference point called origin



$\vec{r} = x\hat{i} + y\hat{j}$
(r vectors)

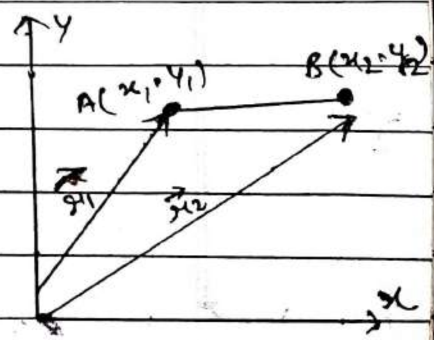
$|r| = \sqrt{x^2 + y^2}$
(magnitude)

$\tan\theta = \dfrac{y\hat{j}}{x\hat{i}}$
(direction)

$\hat{r} = \dfrac{\vec{r}}{|r|} = \dfrac{x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}}$
(direction)

Displacement Vector

It is vector joining intial position to final position.



$\vec{AB} = r_2 - r_1$

$\vec{AB} = (x_2\hat{i} + y_2\hat{j}) - (x_1\hat{i} + y_1\hat{j})$

$\vec{AB} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}$

$|\vec{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
(magnitude)

$\hat{AB} = \dfrac{\vec{AB}}{|A|} = \dfrac{(x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j}}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$
(direction)

disp. vector
$\boxed{\vec{AB} = r_2 - r_1}$

Date
Page No.

Average velocity

Average velocity is given by

$$\vec{v}_{avg} = \frac{\Delta \vec{r}}{\Delta t} = \frac{\Delta x \hat{i} + \Delta y \hat{j}}{\Delta t}$$

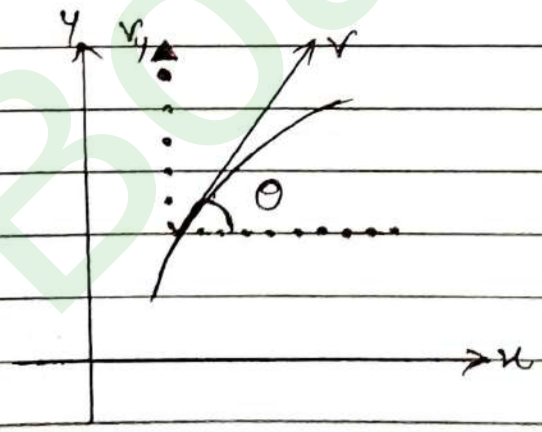
Note:- Direction of the Average velocity is the same as that of $\Delta \vec{r}$.

Instantaneous velocity

Instantaneous velocity is given by

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\Rightarrow \vec{v} = v_x \hat{i} + v_y \hat{j}$$



Date
Page No.

Here,

$$v_x = \frac{dx}{dt} \text{ and } v_y = \frac{dy}{dt}$$

$$\Rightarrow |\vec{v}| = \sqrt{v_x^2 + v_y^2}$$

Also,

$$\tan\theta = \frac{v_y}{v_x}$$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Note – The direction of instantaneous velocity at any point on the path of an object is the tangent to the path at that at that point and is in the direction of motion.

Average Acceleration

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

Average Acceleration is given by,

$$\vec{a}_{avg} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\Delta v_x}{\Delta t}\hat{i} + \frac{\Delta v_y}{\Delta t}\hat{j}$$

$$\Rightarrow \vec{a}_{avg} = a_x\hat{i} + a_y\hat{j}$$

Instantaneous acceleration

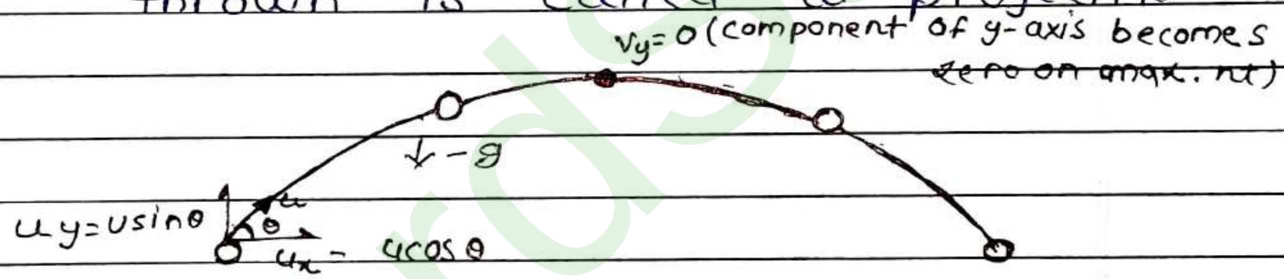
Instantaneous acceleration is given by

$$\vec{a} = \frac{dv}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j}$$

$$\Rightarrow \vec{a} = a_x\hat{i} + a_y\hat{j}$$

Projectile Motion

An object that is in flight after being thrown is called a projectile.



+ $a_x = 0$, $a_y = -g$ (constant always) (acceleration)

+ components of intial velocity (u)

$u_x = u\cos\theta$, $u_y = u\sin\theta$

$\Downarrow$ $\Downarrow$

Remains constant changes

Date
Page No.

Horizontal Axis	Vertical Axis
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = 0$	$a_y = -g$
(In the absence of external force, a_x would be assumed to be zero.	$S_y = u_y t + \frac{1}{2} a_y t^2$
	$\Rightarrow 0-0 = u\sin\theta t - \frac{1}{2}gt^2$ $\boxed{T = \frac{2u\sin\theta}{g} = \frac{2u_y}{g}}$
$S_x = u_x t + \frac{1}{2}a_x t^2$	$v_y = u_y + a_y t$
$\Rightarrow x - 0 = u\cos\theta\, t$	$\boxed{v_y = u\sin\theta - gt}$
$\Rightarrow x = u\cos\theta \times 2u \sqrt{g}$	It depends on time 't'
$\Rightarrow x = \frac{2u^2\cos\theta\sin\theta}{g}$	It is not constant
$\Rightarrow R = \frac{u^2\sin 2\theta}{g}$	Its magnitude first decreases becomes zero and the increases.
$(\because 2\cos\theta\sin\theta = \sin 2\theta$	
Horizontal distance covered is known as Range (R).	

Date
Page No.

$v_x = u_x + a_x t$

$v_x = u\cos\theta$

It is independent of t.

It is constant.

Time of ascent and time of descent :-

At top most point $v_y = 0$

$v_y = u_y + a_y t$

$\Rightarrow 0 = u\sin\theta - gt$

$\Rightarrow t_1 = \frac{u\sin\theta}{g}$

$\Rightarrow t_2 = T - t_1 = \frac{u\sin\theta}{g}$

$\Rightarrow t_1 = t_2 = \frac{T}{2} = \frac{u\sin\theta}{g}$

maximum height obtained by the particle.

method 1: using time of ascent

$S_y = u_y t_1 + \frac{1}{2} a_y t_1^2$

$\Rightarrow \boxed{H = \frac{u^2\sin^2\theta}{2g}}$

method 2:- using third equation of motion

$V_y^2 - u_y^2 = 2a_y S_y$

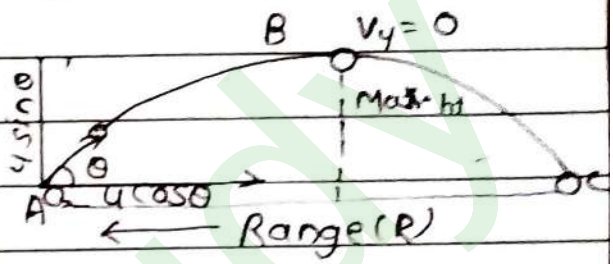
$\Rightarrow 0 - u^2\sin^2\theta = -2gS_y$

$\Rightarrow H = \frac{u^2\sin^2\theta}{2g}$

Date
Page No.

Derivation of Time of flight

consider motion in y-direction (A → C)



$u_y = u\sin\theta$

$a_y = -g$

$t - T$

S_y (displacement) = 0

$S = ut + \frac{1}{2} at^2$

$0 = u\sin\theta\, T + \frac{1}{2}(-g)T^2$

$0 = T\left(u\sin\theta - \frac{gT}{2}\right)$

$u\sin\theta = \frac{gT}{2} \Rightarrow \boxed{\frac{2u\sin\theta}{g} = T}$

Maximum height of a Projectile

Consider motion in y-direction (A → B)

$u_y = u\sin\theta$
$v_y = 0$
$a_y = -g$

$S_y = H$

$v^2 = u^2 + 2as$
$0 = u^2\sin^2\theta - 2gH$

$2gH = u^2\sin^2\theta$

$$\boxed{H = \frac{u^2\sin^2\theta}{2g}}$$

Horizontal Range of a Projectile

Consider motion in x-direction (A-C)

$u_x = u\cos\theta$
$a_x = 0$
$s_y = R$
$t = T$
$s = ut + \frac{1}{2}at^2$

* Range is same for two diff. angles of projection if u is same.

$R = u\cos\theta \times \frac{2u\sin\theta}{g} + 0$

$\boxed{R = \frac{u^2 2\sin\theta . \cos\theta}{g}}$ or $\boxed{R = \frac{u^2 \sin 2\theta}{g}}$

when the horizontal range is maximum

when $\theta = 45°$, the horizontal range is maximum

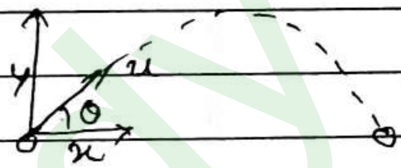
$R = \frac{u^2 \sin^2\theta}{g}$ $\qquad \theta = 45°$
$\qquad 2\theta = 90°$

$R = \frac{u^2 \sin 90°}{g}$ $\qquad (\sin 90° = 1)$

$\boxed{R_{max} = \frac{u^2}{g}}$

Equation of Trajectory (Path)

* The path of projectile is a parabola



x - component	y - component
$u_x = u\cos\theta$	$u_y = u\sin\theta$
$a_x = 0$	$a_y = -g$
t	t

$s = ut + \frac{1}{2}at^2$ $\qquad$ $s = ut + \frac{1}{2}at^2$

$x = u\cos\theta\, t + 0$ $\qquad$ $y = u\sin\theta\, t + \frac{1}{2}(-g)t^2$ —— (a)

$$\boxed{t = \frac{x}{u\cos\theta}}$$

Put value of t in eqn (a)

$$y = u\sin\theta \times \frac{x}{u\cos\theta} - \frac{1}{2}\frac{g x^2}{u^2\cos^2\theta}$$

$$y = x\tan\theta - \frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$$

Date
Page No.

$$y = x \tan\theta - \frac{g x^2}{2u^2 \cos^2\theta}$$

Multiply and divide by $\sin\theta$

$$y = x \tan\theta - \frac{g x^2 \sin\theta}{2u^2 \cos^2\theta \cdot \sin\theta}$$

$$y = x \tan\theta - \frac{g x^2 \tan\theta}{2u^2 \cos\theta \cdot \sin\theta}$$

$$y = x \tan\theta - \frac{g x^2 \tan\theta}{u^2\, 2\sin\theta \cdot \cos\theta}$$

$$y = x \tan\theta - \frac{x^2 \tan\theta}{\frac{u^2 \sin 2\theta}{g}} \qquad \frac{u^2 \sin 2\theta}{g} = R$$

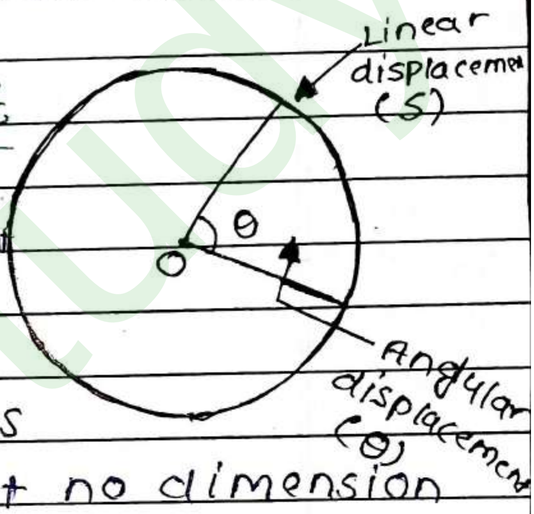
$$y = x \tan\theta - \frac{x^2 \tan\theta}{R}$$

$$\boxed{y = x \tan\theta \left[1 - \frac{x}{R}\right]}$$

Uniform Circular Motion (2-D)

* When an Object follows a circular path at a constant speed the motion of the object is called uniform circular motion.

Angular displacement



+ 'θ' is the Angular displacement
+ Dimension → $M^0L^0T^0$ → dimensionless
+ It has unit → radian, but no dimension

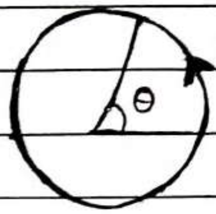
'θ' is measures in :-

degree	Radian (SI unit)
360°	2π
180°	π
90°	$\pi/2$
45°	$\pi/4$
60°	$\pi/3$
30°	$\pi/6$

* 'θ' is a vector quantity (axial vector)

Date
Page No.

* Direction of angular displ. is given by Right hand Thumb Rule.



→ outwards

Angular displacement (θ) — Linear dis (S)

(1 complete rotation) 2π (360°) — $2\pi r$ (circumferences)

1π

$\frac{\cancel{2\pi} r}{\cancel{2\pi}} = R$

$\theta \longrightarrow R\theta$

Relation b/w angular displacement and linear displ.

$\boxed{S = R\theta}$ OR $\boxed{\theta = \frac{S}{R} = \frac{\text{arc}}{\text{radius}}}$

Angular velocity ($\omega \rightarrow$ omega)

$\text{Velocity} = \frac{\Delta S}{\Delta t}$

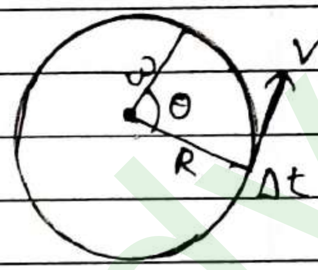
$\boxed{\text{Ang. vel } (\omega_{avg}) = \frac{\Delta \theta}{\Delta t}}$ → unit = Radian/sec.

$V_{inst} = \frac{dS}{dt}$ $(t \rightarrow 0)$

$\boxed{\omega_{inst.} = \frac{d\theta}{dt}}$

Relation between linear and angular velocity :-

Linear velocity $(v) = \frac{\Delta S}{\Delta t}$,

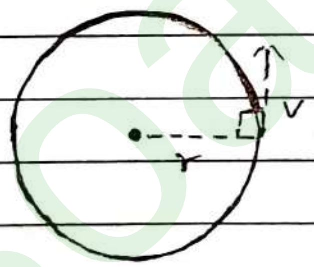


$S = R\theta \Rightarrow \boxed{\theta = \frac{S}{R}}$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\Delta\left[\frac{S}{R}\right]}{\Delta t} = \frac{1}{R}\underbrace{\left[\frac{\Delta S}{\Delta t}\right]}_{\to v}$$

$\boxed{\omega = \frac{v}{R}}$ or $\boxed{v = \omega R}$

* $v = \omega R$ and $S = R\theta$ → This relation is only true for circular motion.
* If linear velocity is fixed then the angular velocity can be change.



$v_{\perp} = R\omega$

$\boxed{\omega = \frac{v_{\perp}}{R}}$ ← This is always correct.

Vector Relation b/w linear & angular velocity.

$\boxed{\vec{v} = \vec{\omega} \times \vec{R}}$

Date

Page No.

Time period → Time taken by the object to make one revolution.

Frequency ⇒ No. of Revolution make in 1 second.
(v or f)

$$f = \boxed{f(\text{or } v) = \frac{1}{T}}$$

* unit of frequency → 1/sec
* Dimension → $M^0L^0T^{-1}$

Relation between angular velocity (ω) and frequency (f)

1 in revol. (in 1 sec) → $\theta = 2\pi$

in $\frac{1}{f}$ revolution → $\theta = (2\pi) \times f$

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi f}{1 \text{ sec}}$$

$$\boxed{\omega = 2\pi f \text{ or } 2\pi v}$$

$(T = 1/f)$

$$\boxed{\omega = \frac{2\pi}{T}}$$

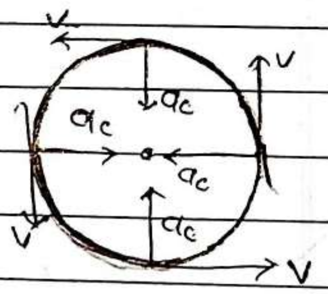
In circular motion, these are 4 types of acceleration.

1. Centripetal Acceleration [a_c]
2. Tangential Acceleration [a_t]
3. Net Acceleration [a_{net}]
4. Angular Acceleration [α]

Centripetal Acceleration

+ Acceleration produced due to the change in direction of velocity.

+ a_c gives change in direction of velocities.
+ It is a sure condition for any circular motion.
+ a_c is always towards the centre of (along the radius)



$$a_c = \frac{V^2}{R^2} \begin{matrix} \leftarrow \text{speed} \\ \leftarrow \text{radius} \end{matrix}$$

$V = r\omega$

$$a_c = \frac{V^2}{R^2} = \frac{r^2 \omega^2}{\cancel{r}} = R\omega^2$$

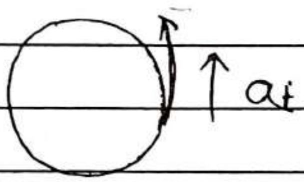
$$\boxed{a_c = \frac{V^2}{R} = R\omega^2}$$

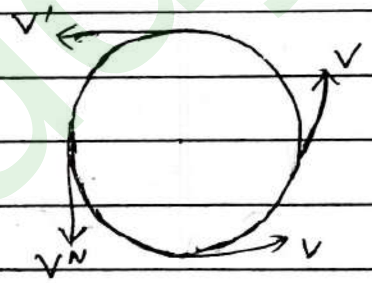
Date
Page No.

Tangential Acceleration (a_t)

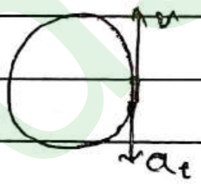
+ Acceleration produced due to change in magnitude.

+ Velocity increase → Acceleration → Same direction



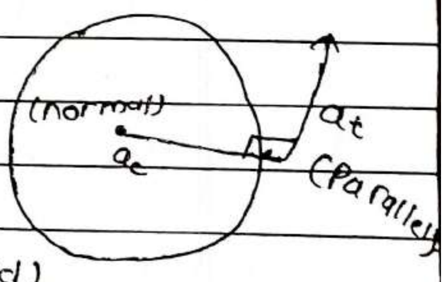


+ Velocity (v) → accel. → opp direction



+ a_t and a_c are perpendicular to each other

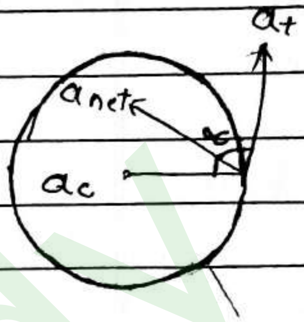
+ $\boxed{a_t = \frac{d|v|}{dt}}$ | v → velocity
|v| → mag. of velocity (speed)



+ If speed is constant, $a_t = 0$

Net Acceleration [a_{net}]

Total change in velocity



$$\boxed{a_{net} = \left|\frac{d\vec{v}}{dt}\right|}$$

$\Downarrow$

(change in velocity → mag. + accel.)

Magnitude of a_{net} :-

Direction

$$a_{net} = \sqrt{a_c^2 + a_t^2 + 2\underbrace{a_c a_t \cos 90^\circ}_{=0}}$$

$$\boxed{a_{net} = \sqrt{a_c^2 + a_t^2}}$$

$$\boxed{\tan\alpha = \frac{a_c}{a_t}}$$

Angular Acceleration (α)

$$\boxed{\alpha_{ang} = \frac{\Delta\omega}{\Delta t}} \qquad \boxed{\alpha_{inst} = \frac{d\omega}{dt}}$$

Que. If $\omega = t^2 + 1$, then find α in 0-2 sec and at 2 sec.

Sol. $\alpha_{ang} = \frac{\Delta\omega}{\Delta t} = \frac{\omega_2 - \omega_0}{2} = \frac{5-1}{2} = \frac{4}{2}$

$\alpha_{inst} = \frac{d\omega}{dt} = 2t + 0 = 2 \times 2 = \underline{4\ rad/s^2}$