CHAPTER- 5

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Work, Energy & Power

WORK

→ Work is a scaler product of force and displacement.

→ Unit - Nm or Joule named after James Prescott Joule.

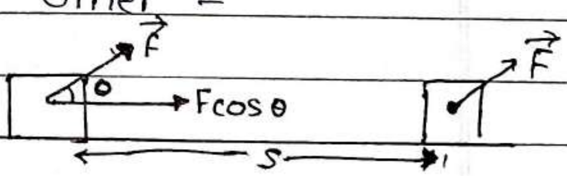
→ Dimension - ML^2T^{-2}

$\boxed{W = \vec{F} \cdot \vec{S}}$ Force is constant.

→ Work done = Force x distance moved in direction of force or component of force.

when force and displacement are inclined to each other -

$\boxed{W = |F||S| \cos\theta}$



Work done can be positive or negetive.

$\theta \rightarrow$ b/w 0° and 90° $\rightarrow W = +FS$ (positive workdone)
$\theta \rightarrow$ b/w 90° and 180° $\rightarrow W = -FS$ (Negative)
$\theta \rightarrow$ 90° $\rightarrow F \perp S \rightarrow W = 0$ (zero work done)

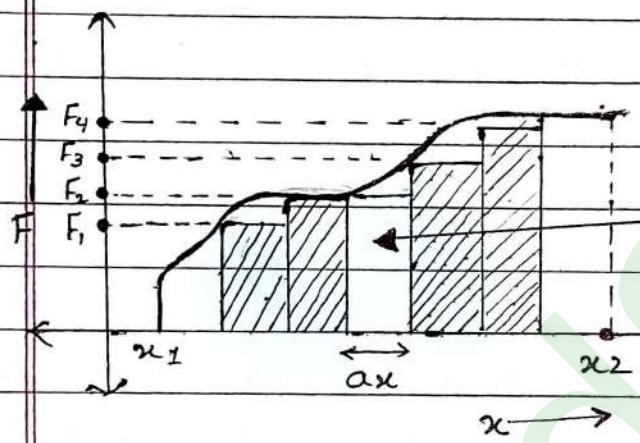
Erg is the absolute unit of work in CGS.

$$1\,erg = 1\,dyne \times 1\,cm$$

Relation b/w joule and erg

$$\boxed{1\ Joule = 10^7\ erg}$$

Work done by a variable force



Area of strip $= F \times dx$

integration is used for adding small quantity

$$\text{Total area} = \int_{x_1}^{x_2} F_1 dx + F_2 dx + \cdots F_n dx$$

$$\boxed{\text{Total work done} = \int_{x_1}^{x_2} \vec{F} \cdot dx}$$

Area of Graph of (F-x) represents work done.

$$\boxed{W = \text{Area under F-x curve}}$$

If displacement is in vector then dx is replaced by $d\vec{r}$.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\boxed{d\vec{r} = dx\hat{i} + dx\hat{j} + dx\hat{k}}$$

$$\boxed{W = \int_{x_1}^{x_2} F_x dx + \int_{x_1}^{x_2} F_y dy + \int_{x_1}^{x_2} F_z dz}$$

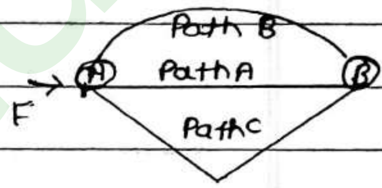
Conservative and Non-Conservative forces

Conservative Force

→ work done by such forces does not depend upon path.

→ It depends only initial and final position.

→ If F is conservative, then, $W_A = W_B = W_C$



→ work done by conservative force in a closed path is alway zero

→ Gravitational force, spring force, electro--static conservative.

Non-conservative force

→ work done depends upon path.

→ In a closed path, $W \neq 0$

e.g friction and viscosity.

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Conditions which should be satisfied for Conservative force.

$$\left.\begin{aligned} &1.\ \frac{\partial f_x}{\partial y} = \frac{\partial f_y}{\partial x} \\ &2.\ \frac{\partial f_y}{\partial z} = \frac{\partial F_z}{\partial y} \\ &3.\ \frac{\partial F_x}{\partial z} = \frac{\partial F_z}{\partial x} \end{aligned}\right\}$$

$\boxed{\text{Energy}}$

→ Energy of a body is its capacity to do work.
mass energy relation $\boxed{E = mc^2}$

→ Scaler quantity, dimensions ⟶ $[ML^2T^{-2}]$

→ K.E with any reference must be positive.

→ kinetic Energy - Energy possessed by motion of a body

⇒ $K.E = \frac{1}{2}mv^2$ ★ $\left(K.E \propto \frac{1}{m}\right)$

⇒ Relation between kinetic energy and linear momentum:—

$$\boxed{P = \sqrt{2mk}}$$

Work - Energy Theorem

- Total work done by all forces on a body = change in K.E of the body (ΔK.E)

$$\boxed{W_{Total} = K.E._f - K.E._i}$$

- work → +ve → K.E increases
  work → −ve → K.E decrease.

Derivation of work-energy Theorem

Let Force be variable

$$W = \int_{x_1}^{x_2} F\,dx = \int_{x_1}^{x_2} ma\,dx = \int_{x_1}^{x_2} \frac{mv\,dv}{\cancel{dx}}\cancel{dx}$$

$$= m\int_{v_i}^{v_f} v\,dv$$

$$= m\left[\frac{v^2}{2}\right]_{v_i}^{v_f}$$

$$= \frac{1}{2}m\left(v_f^2 - v_i^2\right)$$

$$\int_{x_1}^{x_2} F.dx = K_f - K_i \qquad = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$\boxed{W = KE_f - KE_i}$$

Let Force be constant

$$W = FS$$

$$W = maS$$

$$W = m\left[\frac{v^2 - u^2}{2\cancel{S}}\right]\cancel{S}$$

$$W = m\left[\frac{v^2 - u^2}{2}\right]$$

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

Potential Energy (U) (mutual Energy or energy of configuration)

+ Relative Quatity –

- In physics, change in P.E (ΔU) is defined.

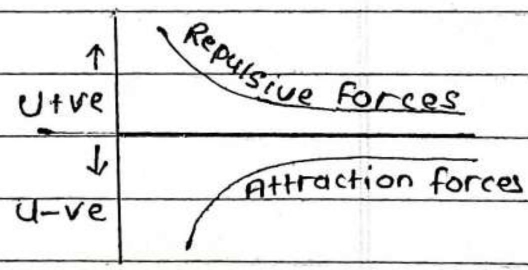
$[U_f = mgh]$

$\Delta U = -$ work done by conservative force

$$\boxed{U_f - U_i = -W_c}$$

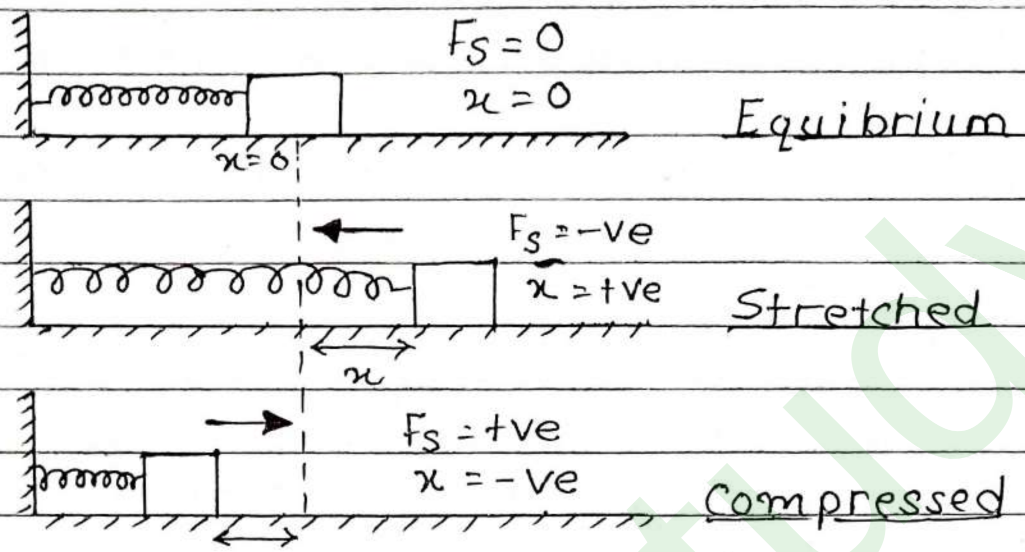
* Δu is not defined for non-conservative forces.

* P.E is invisible form of energy, it is imparted to the system whenever energy is imparted to a system, that energy is stored as potential energy.



* P.E can be positive or negetive.

Potential Energy of a Spring ← Always positive



According to hooke's law,

$$\boxed{F_S = -kx}$$

k → spring constant / force-constant
unit → Nm^{-1}

$F_S - kx$

$$W = \int_{x_1}^{x_2} F dx = \int_{x_1}^{x_2} -kx \cdot dx = -k\left[\frac{x^2}{2}\right]_{x_1}^{x_2}$$

$$W_s = \frac{1}{2}kx^2{}_i - \frac{1}{2}kx^2{}_f$$

$$U_2 - U_1 = -W_c$$

$$U_2 - U_1 = \frac{1}{2}kx_i^2 - \frac{1}{2}kx^2{}_f$$

(At natural length, $F_S = 0$, $x_1 = 0$, $U_1 = 0$)

$$U_2 - 0 = \frac{1}{2}k(x_i)^2 - \frac{1}{2}k(0)^2$$

$\therefore$ $\boxed{W_S = -\frac{1}{2}kx^2}$ $(-W_c = \Delta U)$ $\boxed{U = \frac{1}{2}kx^2}$

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\+ work done by spring force is always negative.

modified work - Energy Theorem

$W_{Total} = \Delta K.E.$

$W_{conservati\ force} = -\Delta U$

$W_{conservative} + W_{non\text{-}con} = \Delta K.E$

$-\Delta U + W_{non\text{-}con} = \Delta K.E$

$$\boxed{W_{non\text{-}con} = \Delta K.E + \Delta U}$$

(If $W_{NC} = 0$)

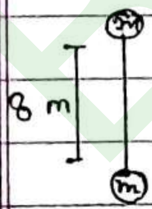
$\Delta K.E + \Delta U = 0$

Δ Mechanic Energy $= 0$

$\Delta E = 0 \leftarrow$ Mechanicle energy conserved.

Ques A 0.5 kg ball is thrown up with an initial speed 14 m/s and reaches a maximum height of 8.0 m. How much energy is dissipated by air drag acting on the balls during the ascent?

Solⁿ



$P.E = mgh = 0.5 \times 9.8 \times 8 = 39.2 J$

$m = 0.5$ kg

$V = 14$ m/s

Air drag loss $= KE - PE$
$= 49 - 39.2$
$= 9.8 J.$

$K.E = \frac{1}{2} \times 0.5 \times 14 \times 14 = 49 J.$

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Ques An object is acted on by a retarding force of 10 N and at a particular instant its kinetic energy is 6J. The object will come to rest after it has travelled a distance of?

Soln

$w = -fs$

$\frac{1}{2} m (0 - v^2) = -fs$

$-\frac{1}{2} mv^2 = + fs$

$\Rightarrow \frac{1}{2} \times 6 = 10 \times 5$

$\Rightarrow \frac{5 \times 10}{6} = \frac{3}{5} m$

Ques When a spring is compressed by 3 cm, the P.E Started in it is U. when it is compressed further by 3cm, the increases in potential energy is-

Soln

$\Delta U = U_2 - U_1$

$U_1 = \frac{1}{2} Kx^2 = \frac{1}{2} k (9) \text{—(I)}$

$U_2 = \frac{1}{2} Kx^2 = \frac{1}{2} k (39) \text{—(II)}$

Divide (I) & (II) we get

$\frac{U_2}{U_1} = 4$

$\frac{U_2}{U_1} - 1 = 3$

$U_2 - U_1 = 3U$

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Einstein's Mass of equivalence

- In 1905, Albert Einstein discovered that mass can be converted into energy and vice versa.

$$E = mc^2$$

[c = Speed of light (3×10^8 m/s)]

→ Energy can neither be created nor destroyed. It may be transformed from one form to another.

$$1\,eV = 1.6 \times 10^{-19}\,J.$$

$$1\,MeV = 10^6\,eV = 1.6 \times 10^{-13}\,J.$$

$$1\,amu = 931\,Mev$$

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POWER

Power - Rate of doing work.

- Scaler quantity

$$\boxed{P_{av} = \frac{\text{Total work}}{\text{Total time}} = \frac{w}{T}} = \frac{\Delta KE}{t}$$

- Dimension - ML^2T^{-3}
- Units - watt → SI unit, erg/sec → CGS

$$1\,kW = 1000 \text{ watt}$$
$$1\,hp = 746 \text{ watt}$$

- Instanteneous power is defined as the limiting Value of average power as time interval aproaches to zero

$$P_{inst.} = \lim_{\Delta t \to 0} \frac{\Delta w}{\Delta t} = \frac{dw}{dt} \text{ or } \frac{dE}{dt} \quad [\text{Energy} = \text{work}]$$

→ Pinst is the dot product of force and velocity.

$$\boxed{P_{inst} = \vec{F} \cdot \vec{V} = FV\cos\theta}$$

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$P \propto \frac{1}{t}$ ($\omega \to$ constant)

Relation b/w kwh (Board of Trade) & joule | $v^2 \propto t$

$$\boxed{1\ kwh = 3.6 \times 10^6\ J}$$

Relation b/w average power and instanteneous power :-

$$\boxed{P_{inst} = 2P_{avg}}$$

* Slope of E-T graph gives Print.
* Area of P-T graph gives Energy

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Efficiency of crane or motor (η)

→ Efficiency (η) is the ratio of output power to input power.

$$\boxed{\eta = \frac{\text{output power}}{\text{input power}} \times 100 = \frac{P_o}{P_i} \times 100}$$

→ η is also defined as the ratio of useful work done to total enegy spend.

$$\eta = \frac{\text{Useful Power}}{\text{Actual Power}}$$

[useful power or input power

$= \frac{W}{T} = \frac{mgh}{t}$

→ If pomp lifts water from depth 'h' and deliver at the rate $\frac{dm}{dt}$ with velocity v, then power delivered.

$$\left[P = \frac{dm}{dt}\left[gh + \frac{v^2}{2}\right]\right] \text{ or } \left[P = \frac{mgh + \frac{1}{2}mv^2}{t}\right]$$

If water lifts without velocity ($v=0$)

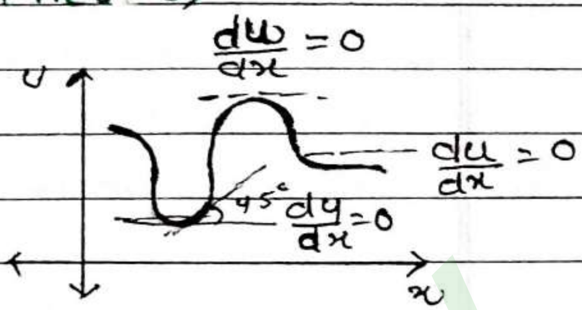
$$\left[P_i = \frac{dm}{dt}(gh\right]$$

* ——— * ——— * ——— * ———

Mechanical Advantage = $\frac{\text{Load}}{\text{Effort}}$ | Efficiency = $\frac{\text{Mech Advant}}{\text{Vel. ratio}}$

velocity ratio = $\frac{\text{Distance travelled by efforts}}{\text{Distance travelled by load}}$

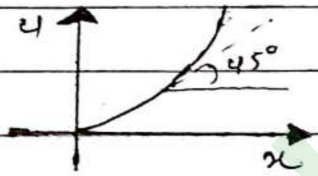
Equilibrium (f_{net} 0)



$dw_c = -dU$

$F \cdot dx = -dU$

$$\boxed{\vec{F_c} = \frac{-dU}{dx}}$$



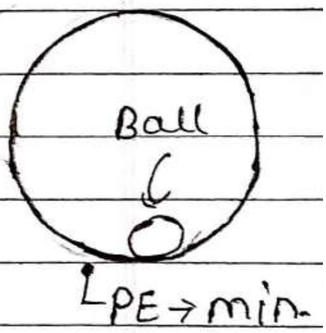
$\text{Slope} = \frac{dU}{dx} = \tan\theta$

↳ $\boxed{F = -\text{Slope}}$ (U vs x Graph)

Types of equilibrium :—

1. Stable equilibrium

- x and F are in opposite direction.
- If we displace the body slightly it returns back to stable equilibrium. $\frac{dU}{dx} = \frac{d\cancel{x}}{dx}$



U → minimum.

- $\frac{d^2U}{dx^2} > 0$

2. Unstable equilibrium

- It we move the body slightly It will move towards stable equilibrium position.

- $\frac{d^2U}{dx^2} < 0$ (U - maximum)

3. Neutral equilibrium :- PE → Constant

$\frac{d^2U}{dx^2} = 0.$

Ques The PE of a particle in a force field is $U = \frac{A}{r^2} - \frac{B}{r}$ where A and B are positive constants and r is the distance of particle from centre of field. for stable equilibrium, the distance of particle is?

Soln $U = \frac{A}{r^2} - \frac{B}{r} = Ar^{-2} - Br^{-1}$

for equilibrim, $\frac{dU}{dr} = 0$

$$\Rightarrow \frac{d(Ar^{-2} - Br^{-1})}{dr} = 0$$

$$\Rightarrow A(-2)r^{-2-1} - B(-1)r^{-1-1} = 0$$

$$\Rightarrow -2Ar^{-3} + Br^{-2} = 0$$

$$\Rightarrow \frac{-2A}{r^3} + \frac{B}{r^2} = 0$$

$$\Rightarrow -2A + Br = 0$$

$$\Rightarrow r = 2A/B$$

Ques The PE b/w two atoms in a molecule is given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are +ve constant and x is is distance b/w atoms. The atoms is in stable eq. when x=?

Soln $\frac{dU}{dx} = 0$

$$\frac{d(ax^{-12} - bx^{-6})}{dx} = 0$$

$$\Rightarrow a(-12)x^{-12-1} - b(-6)x^{-6-1} = 0$$

$$\Rightarrow -12ax^{-13} + 6bx^{-7} = 0$$

$$\Rightarrow \frac{-12a}{x^{-13}} + \frac{6b}{x^{-7}} = 0 \Rightarrow \frac{\cancel{12}^{2}a}{x^{-13}} = \frac{\cancel{6}b}{x^7} = \frac{2a}{x^{13}} = \frac{b}{x^7}$$

$$\frac{2a}{\cancel{x^7} \times x^6} = \frac{b}{\cancel{x^7}}$$

$$2a = x^6 b$$

$$x = \left(\frac{2a}{b}\right)^{1/6}$$

Collisions

- In collision, velocity of a body is affected by others.
- In any type of collision, momentum remains conserved throught collision.

- Heaton or one dimensional collision:- Before and after collision, bodies move along the same straight line.

- oblique or two dimensional collision:- Do not move along same line.

On the basis of loss in KE, there are mainly two types of collision :-

1. Elastic collision

Deformation (Deformation completely over) → perfectly elastic collision.

- Momentum of system remains conserved throught collision

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \quad \text{——} (1).$$

- $KE_i = KE_f$ (not conserved throughout elastic collisio)

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \quad \text{——} (11).$$

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- If mass of two bodies are same (identicle mass), then velocity is exchanged.

$\boxed{V_1 = U_2 \mid V_2 = U_2}$

- If $m_2 >>>> m_1$ and m_2 is at rest initially. (Truck and cycle)

$m_1 \to 0$ (as compare to m_2)

$$V_1 = -\frac{\cancel{m_2} U_1}{\cancel{m_2}} + \frac{2\cancel{m_2} U_2}{\cancel{m_2}} = 0$$

$\boxed{V_1 = -U_1}$ Rebound.

- If $m_1 >>>> m_2$ and m_2 is at Rest $(U_2 = 0)$

$\boxed{V_1 = U_1}$ $\boxed{V_2 = 2U_1}$

Elastic collision in one dimension

from ①

$$m_1 U_1 + m_2 U_2 = m_1 V_1 + m_1 V_2$$

$$m_1 U_1 - m_1 V_1 = m_2 V_2 - m_2 U_2$$

$$m_1 (U_1 - V_1) = m_2 (V_2 - U_2) \quad \text{— (a)}$$

from ⑪

$$\frac{1}{2} m_1 U_1^2 + \frac{1}{2} m_2 U_2^2 = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2$$

$$m_1 U_1^2 - m_1 V_1^2 = m_2 V_2^2 - m_2 U_2^2$$

$$m_1 (U_1^2 - V_1^2) = m_2 (V_2^2 - U_2^2) \quad \text{— (b)}$$

Devide (b) by (a)

$$\frac{\cancel{m_1}(u_1^2 - v_1^2)}{\cancel{m_1}(u_1 - v_1)} = \frac{\cancel{m_2}(v_2^2 - u_2^2)}{\cancel{m_2}(v_2 - u_2)}$$

$$\frac{\cancel{(u_1 - v_1)}(u_1 + v_1)}{\cancel{(u_1 - v_1)}} = \frac{\cancel{(v_2 - u_2)}(v_2 + u_2)}{\cancel{(v_2 - u_2)}}$$

$$u_1 + v_1 = v_2 + u_2$$

$$\boxed{u_1 - u_2 = v_2 - v_1} \rightarrow \text{ideal elastic collision}$$

Relative velocity before collision or velocity of aproach.

Relative velocity after collision OR velocity of separation.

As we know now,

$$u_1 - u_2 = v_2 - v_1$$

$$v_2 = u_1 - u_2 + v_1 \quad \text{— (c)}$$

putting this value of v_2 in eq(1), we get—

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 (u_1 - u_2 + v_1)$$

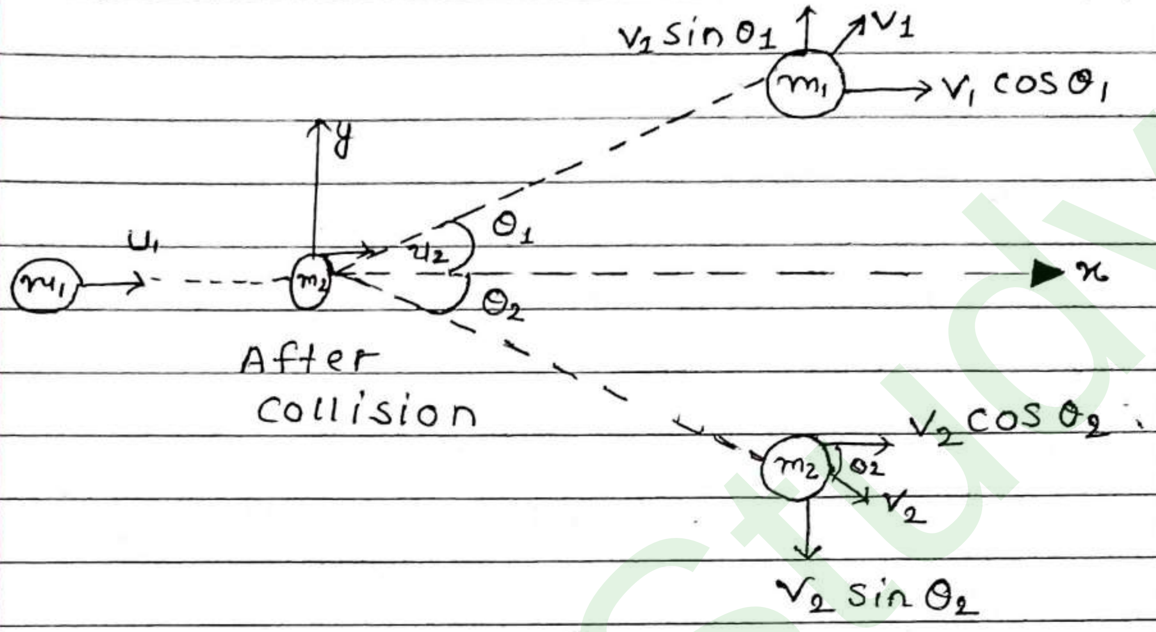
$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 u_1 - m_2 u_2 + m_2 v_1$$

$$m_1 u_1 - m_2 u_1 + m_2 u_2 + m_2 u_2 = m_1 v_1 + m_2 v_1$$

$$u_1 (m_1 - m_2) + 2 m_2 u_2 = v_1 (m_1 + m_2)$$

$$\boxed{v_1 = \left[\frac{m_1 - m_2}{m_1 + m_2}\right] u_1 + \left[\frac{2 m_2}{m_1 + m_2}\right] u_2}$$

$$\boxed{v_2 = \left[\frac{m_2 - m_1}{m_1 + m_2}\right] u_2 + \left[\frac{2m_1}{m_1 + m_2}\right] u_1}$$



→ x-direction ($P_i = P_f$)

$$m_1 u_1 + m_2 u_2 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2 \quad \text{— (I)}$$

→ y-direction ($P_i = P_f = 0$)

$$m_1 v_1 \sin\theta_1 = m_2 v_2 \sin\theta_2 \quad \text{— (II)}$$

- $KE_i = KE_f$

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad \text{— (III)}$$

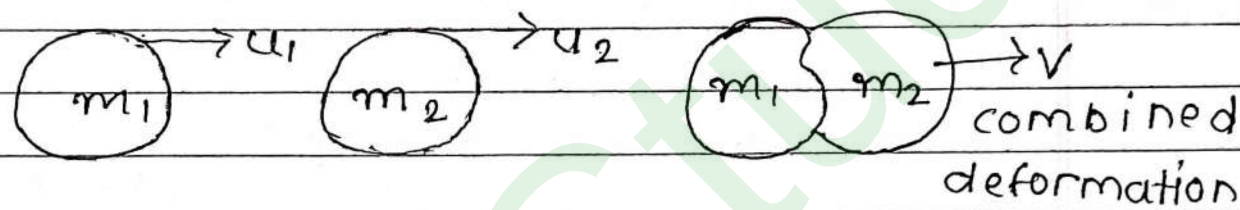
► use these three equations for solving numericals of elastic collision in two dimension.

2 Inelastic Collision

→ After collision both bodies stick together and move.

→ momentum remains conserved ($P_i = P_f$)

→ $KE_i > KE_f$ (some energy is lost)



$$\boxed{m_1 u_1 + m_2 u_2 = (m_1 + m_2) V}$$

Coefficient of Restitution (e)

- e is the ratio of velocity of separation to the velocity of approch.

$$e = \frac{\text{velocity of seperation}}{\text{velocity of aproach}} = \frac{\text{Relative velocity before collision}}{\text{Relativ vel. after collision}}$$

- $\boxed{e = \frac{v_2 - v_1}{u_1 - u_2}}$ → velocities should be along line of incidence.

- for perfectly elastic collision → $e = 1$
  for perfectly inelastic collision → $e = 0$

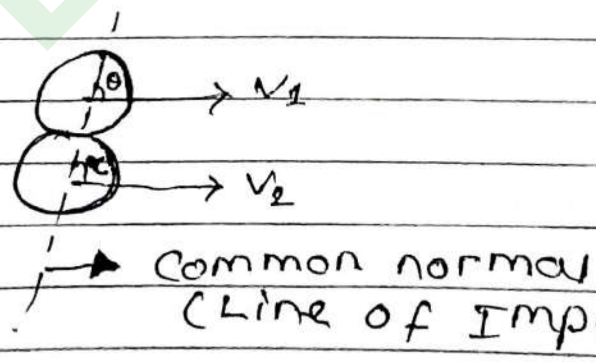
- for practical collision → $0 < e < 1$

- value of e depends on materials of colliding bodies.

oblique collision → Inelastic, Elastic, Partial.

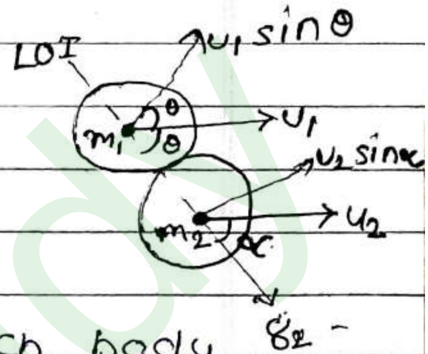
→ Head on Head

velocity are not along LOI they are at same angle to LOI.



Methods to solve numericals of oblique collision.

1. Conservation of momentum (elastic/inelastic/partial) can be applied.



$$m_1 u_1 \cos\theta + m_2 u_2 \cos\alpha = m_1 v_1 + m_2 v_2$$

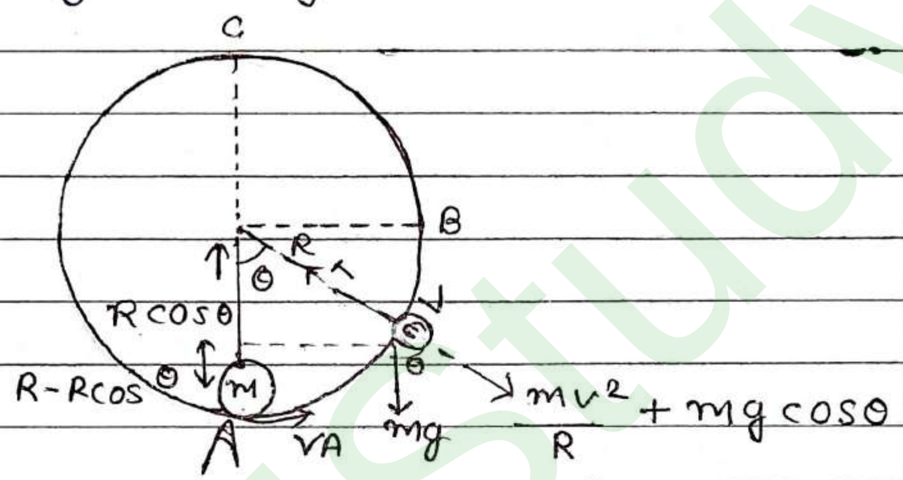
2. The component of velocity of each body ⊥ to LOI remains same.

3. We will consider final velocity along line of Impact.

4. $e = \dfrac{\text{Velocity of seperation}}{\text{velocity of approach}} = \dfrac{v_2 - v_1}{u_1 \cos\theta - u_2 \cos\alpha}$ → Almost LOI.

Motion in a Vertical Circle

* Vertical circular motion is non-uniform because under gravity its speed constantly changes.



Tension at any point of circle: $\boxed{T = \frac{mv^2}{R} + mg\cos\theta}$

Here, $f = 0$, work done by $T = 0$

By conservation of energy

$$U_i + K_i = U_f + K_f$$

$$0 + \frac{1}{2} m V_A^2 = mgh + \frac{1}{2} mv^2$$

$$\frac{1}{2} m V_A^2 = mg(R - R\cos\theta) + \frac{1}{2} mv^2$$

$$\cancel{m} V_A^2 = 2\cancel{m} gR(1 - \cos\theta) + \cancel{m} v^2$$

$$V_A^2 = 2gR(1 - \cos\theta) + v^2$$

$$v^2 = V_A^2 - 2gR(1 - \cos\theta)$$

$$\boxed{v = \sqrt{V_A^2 - 2gh}}$$

→ v at any point.

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$$T = \frac{mv^2}{R} + mg\cos\theta$$

$$= \frac{m\left(v_A^2 - 2g(R - R\cos\theta)\right)}{R} + mg\cos\theta$$

$$= \frac{mv_A^2}{R} - \frac{2mg\cancel{R}(1-\cos\theta)}{\cancel{R}} + mg\cos\theta$$

$$= \frac{mv_A^2}{R} - 2mg + 2mg\cos\theta + mg\cos\theta$$

$$\Rightarrow \boxed{T = \frac{mv_A^2}{R} - 2mg + 3mg\cos\theta}$$ → This eq^n can be used in whole circle.

→ which one is correct

a. $T_A > T_B > T_C$ ✓
b. $T_C > T_B > T_A$
c. $T_A = T_B = T_C$
d. None

($T_A \rightarrow \theta = 0^\circ$; $\cos 0^\circ = 1$)

$$T_A = \frac{mv_A^2}{R} + mg$$

$(\cos 90^\circ = 0)$ $T_B = \frac{Mv^2_A}{R} - 2mg$

$(\cos 180^\circ = -1)$ $T_C = \frac{Mv^2_A}{R} - 5mg$

$T_A > T_B > T_C$

* At lower point T is maximum, as it goes upwards. T will decreases.

* Particle can never leave the circle b/w A and B because b/w these points $\theta \rightarrow +ve$, $T = +ve$.

* If $T \leq 0$, then string slacks → Particle leaves circular Path.

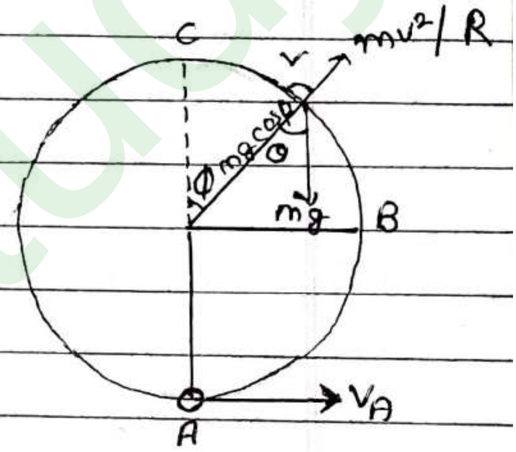
* T at $c \geq 0$, so that c it is max. chance when a particle can leave the circle.

Note:- At C, even if T is zero, velocity will drive the circle due to inertia.

Critical velocity

Velocity at any point such that string doesn't slack at that point, $T \geq 0$.

At C,



$$mg\cos\phi + T = \frac{mv^2}{R}$$

$$T = \frac{mv^2}{R} - mg\cos\phi$$

$$T \geq 0$$

$$\frac{mv^2}{R} - mg\cos\phi \geq 0$$

$$v^2 \geq gR\cos\phi$$

$$v \geq \sqrt{gR\cos\phi}$$

Critical velocity at C, $\phi = 180°$

$$v \geq \sqrt{rg\cos\phi}$$

$$v \geq \sqrt{gR}$$

Note:- If v at c is $\geq \sqrt{gR}$, then T at ≥ 0 and T everywhere is ≥ 0 and the circle can be compleated.

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Critical velocity at B ($\phi = 90°$, $\cos 90° = 0$)

$$V \geq \sqrt{gR\cos\phi}$$
$$\boxed{v \geq 0}$$

→ Find VA such that particle reaches C (or complete circle)

Conservation of energy

$A = C$

$U_A + K_A = U_C + K_C$

$0 + \frac{1}{2} m v_A^2 = mg2R + \frac{1}{2} mv^2$

$\cancel{m} v_A^2 = 2\cancel{m}g\, 2R + \cancel{m}gR$ ($V_{min} = \sqrt{gR}$ at C)

$V_A^2 = 5gR$

★ $\boxed{V_A \geq \sqrt{5gR}}$

Find VA such that particle reaches B.

$A = B$

$U_A + K_A = U_B + K_B$

$0 + \frac{1}{2} mV_A^2 = mgR + \frac{1}{2} mv^2$

($v_B \geq 0$)

$\cancel{m} v_A^2 = 2\cancel{m}gR + 0$

$$\boxed{v_A = \sqrt{2gR}}$$

* If $v_A < \sqrt{2gR}$, particle moves in circular arc b/w A and B, velocity will became zero before B.
  $T \neq 0$, string never slacks.

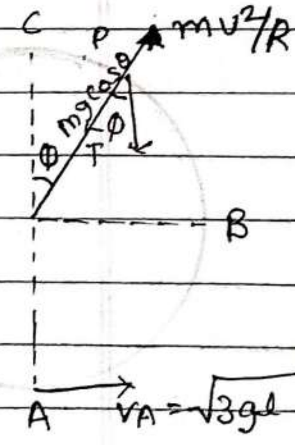
* If $v_A = \sqrt{2gR}$, particle moves in circular arc from A & B. $v_B = 0$, $T = 0$, string slacks.

* If $v_A > \sqrt{5gR}$, v never became zero, $T \neq 0 \rightarrow$ circles completes.

* If $v_A - \sqrt{5gR}$, at c, $v = \sqrt{gR}$, $T = 0$ but still string will not slack $\rightarrow$ circle complete.

* If $\sqrt{2gR} < v_A < \sqrt{5gR}$, then particle leaves circle b/w B and C, $v \neq 0$, instead $T = 0 \leftarrow$ sting slacks, particle takes parabolic path at that point.

* At what angle the particle will leave the circle.



$\sqrt{2gl} < v < \sqrt{5gl}$, $T = 0$ $v \neq 0$
string slacks

$$T + mg\cos\phi = \frac{mv^2}{l}$$

$$T = 0$$

$$\cancel{m}g\cos\theta = \frac{\cancel{m}v^2}{l}$$

$$\cos\phi = \frac{v^2}{g l}$$

$$v_P = \sqrt{g l \cos\phi}$$

$$A = P$$

$$U_A + K_A = U_P + K_P$$

$$0 + \frac{1}{\cancel{2}} \cancel{m} 3 \cancel{g} l = \cancel{m}\cancel{g}(l + l\cos\phi) + \frac{1}{\cancel{2}} \cancel{m}\cancel{g} l \cos\phi$$

$$\frac{3l}{2} = 2l(1 + \cos\phi) + l\cos\phi$$

$$3l = 2l + 2l\cos\phi + l\cos\phi$$

$$3l - 2l = 3l\cos\phi$$

$$1\cancel{l} = 3\cancel{l}\cos\phi$$

★ $\boxed{\cos\phi = 1/3}$ → At this angle ϕ, particle will leave the circle