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Electromagnetic Induction

* Magnetic Flux :-

The total number of magnetic field lines crossing through any surface normally when it is placed in a magnetic field is known as the magnetic flux of that surface.

$$\phi = \vec{B} \cdot \vec{A} \ \hat{n}$$

$$\phi = BA \cos\theta \ \hat{n}$$

where θ is the angle between area vector and magnetic field.

→ It is a scalar quantity.
→ SI unit is weber (Wb) or Tm^2.
→ CGS unit is Maxwell.

$$1\ Wb = 10^8 \text{ maxwell}$$

→ Dimension of magnetic flux :-

$$\phi = B \cdot A$$
$$= [ML^0T^{-2}A^{-1}][L^2]$$
$$[\phi] = [ML^2T^{-2}A^{-1}]$$

$$[\phi] = [ML^2T^{-2}A^{-1}]$$

* Electromagnetic Induction :-

The phenomenon of producing an induced e.m.f in a conductor or conducting coil due to changing magnetic flux is called electromagnetic induction.

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(i) Faraday's first Law of Electromagnetic Induction :-

Whenever magnetic flux linked with a closed conductor changes, an e.m.f. is induced in it. This induced e.m.f. lasts as long as the change in magnetic flux continues in the coil.

From Experiment -

→ When the magnet is moved towards the coil, the magnetic flux through the coil changes. Due to this change in flux, an electromotive force (emf) is induced in the coil, causing the galvanometer to show a deflection.

→ When the magnet is moved away from the coil, the magnetic flux through the coil changes again. Due to this charge in magnetic flux, an emf is induced in the coil, and the galvanometer shows a deflection.

→ If the magnet does not move, the magnetic flux through the coil remains constant. As a result, no emf is induced, and the galvanometer shows no deflection.

(ii) Faraday's Second Law :-

The magnetic of the induced e.m.f. in a closed conductor or a coil is directly proportional to the rate of change of magnetic flux linked with the coil.

$$\mathcal{E} \propto \frac{d\phi}{dt}$$

$$\mathcal{E} = \frac{K d\phi}{dt}$$

K is constant of proportionality. Let K = 1.

$$\boxed{\mathcal{E} = -\frac{d\phi}{dt}}$$

→ When the magnet is move faster, more flux will change and hence more E.mf will induced.

→ When the magnet is move slower, less flux will change and hence less emf will induced.

Note :- Faraday laws are in accordance with conservation of energy.

For N number of times

$$\boxed{\mathcal{E} = -\frac{N d\phi}{dt}}$$

The negative sign show that emf induced will always oppose the change in flux.

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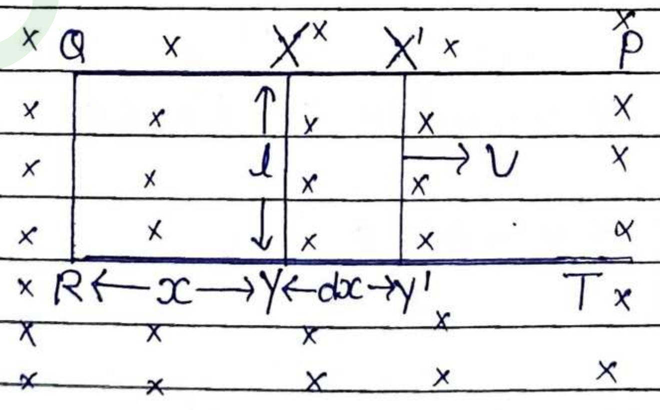
Methods of induced Emf :-

(i) By changing magnetic field :- If we change magnetic field, flux will change and due to change in magnetic flux, Emf will induce.

(ii) By changing Area :- If we change area, magnetic flux will change and due to change in magnetic flux, Emf will induce.

Derivation : Motional Emf — When a conductor is moved in MF then due to change in MF emf is induced across its end. This emf is called motional emf.

Let PQRT is a rectangular metallic frame. let XY is a wire which can slide over it. let v is the velocity given to wire having length l let dx is the displacement provided to wire.



Flux linked with area QXYR

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$$\phi_1 = BA\cos\theta$$
$$\phi_1 = BA \qquad \theta = 0^\circ$$
$$\phi_1 = Bxl \quad \text{—— (1)}$$

Flux linked with area $QX'Y'R$

$$\phi_2 = BA\cos\theta$$
$$\phi_2 = BA \qquad \theta = 0^\circ$$
$$\phi_2 = B(x+dx)l$$

Change in flux, $d\phi = \phi_2 - \phi_1$

$$d\phi = B(x+dx)l - Bxl$$
$$d\phi = Bxl + Bdxl - Bxl$$
$$d\phi = Bdxl$$

Dividing both side by dt :

$$\frac{d\phi}{dt} = \frac{Bldx}{dt}$$

$$|\varepsilon| = BlV$$

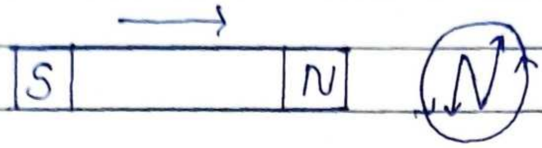
(iii) By changing the orientation : If we change orientation of coil, flux will change and due to change in flux, emf will induce.

* Lenz's law :-

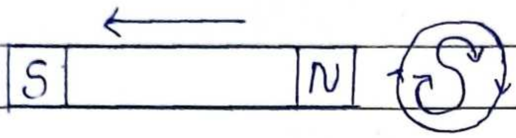
It is stated that the direction of induced e.m.f. is always in such direction that it opposes the change in magnetic flux.

$$\varepsilon = \frac{-d\phi}{dt} \quad \text{or} \quad \varepsilon = \frac{-Nd\phi}{dt}$$

Direction of Induced current :-



Anticlock wise

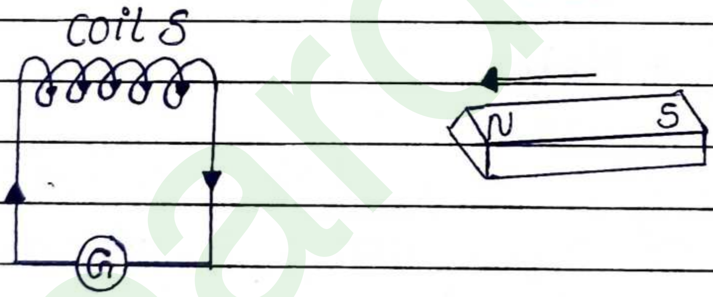


clockwise

→ If the induced pole is north, then the direction of current is Anticlock wise.

→ If the induced pole is south, then the direction of current is clockwise.

Lenz's law and energy conservation :-



Lenz law is in accordance with the law of conservation of energy. In the above experiment, when N-pole of magnet is moved towards the coil, the right face of the coil acquires North polarity. Thus, work has to be done against the force of repulsion in bringing the magnet closer to the coil.

When N pole of magnet is moved away. South pole develops on the right face of the coil. Therefore, work has to be done against the force of attraction in taking the magnet away from the coil.

This mechanical work in moving the magnet with respect to the coil changes into electrical energy producing induced current. Hence, energy transformation takes place. Hence, we can say that Lenz's law is a consequence of conservation of energy.

* Self Induction :-

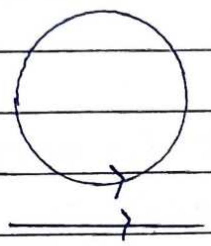
Self-induction is the phenomenon wher a charging current in a coil induces an electromotive force (EMF) within the same coil. This EMF opposes the charge in current.

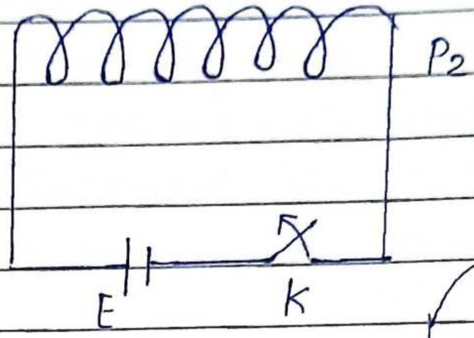
The tendency of a coil to resist changes in its own current is called self-inductiance. The induced EMF is proportional to the rate of change of current.

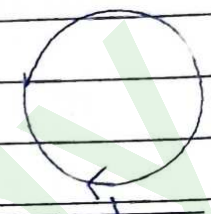
Experiment :-

Let a coil is connected with a cell or a battery with a taping key.

R. Finger curl → current
R. Thumb → Magnetic field







I (decrease)   (I) increase

As the key is switched on current starts flowing in the circuit from P_1 and P_2, as current never reaches it maximum value intantaneoulsly. i.e. takes certain finite time. During this time flux linked with the coil changes and hence current get induce in the coil.

According to Lenz's law, the direction of induced current ~~oppose the increase in current flowing in the e~~ is form P_2 to P_1. This induced current oppose the increase in current flowing in the current.

As the key is switched off, current starts, reducing in the circuit it can never reaches its minimum value instantaneously i.e, takes certain finite time, during this time flux linked with the coil changes and hence current get induced in the circuit. This Induced current

will oppose any decay of current in the circuit.
The direction of induced current in circuit is from P_1 to P_2.

(L) Coefficient of Self Induction or self Induction:-

We know flux linked with coil is always directly proportional to current flowing through coil.

$$\phi \propto I$$
$$\phi = LI$$

where L is constant of proportionality called self inductance.

also, $$\varepsilon = -\frac{d\phi}{dt}$$

$$\varepsilon = -\frac{d[LI]}{dt}$$

$$\varepsilon = -\frac{L dI}{dt}$$

$$\frac{dt \times \varepsilon}{dI} = -L$$

$$\left\{\begin{array}{l} \varepsilon = \frac{d\phi}{dt} \\ \text{volt} \times \text{sec} = d\phi \\ \text{volt} \times \text{sec} = \text{weber} \end{array}\right.$$

Or

$$\boxed{L = \frac{-\varepsilon}{dI/dt}}$$

→ SI unit is Henry or weber/Ampere.

So, self Inductance is defined as emf induced in the coil. if rate of change of current in coil is unity.

Define 1 Henry :-

$$1H = \frac{1V}{1A/1S}$$

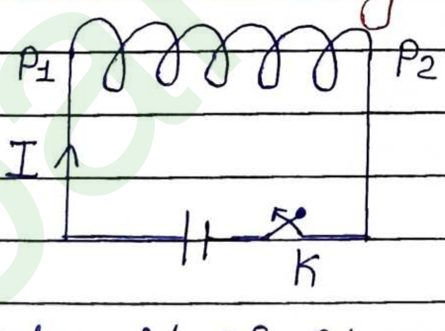
Self inductance is said to be 1 henry if Emf of 1 volt is induced in the coil provided rate of change of current in the coil is 1 Ampere per second.

Dimension of L :

$$L = \frac{\varepsilon}{\frac{dI}{dt}} = \frac{\left[\frac{W}{q}\right]}{\left[\frac{A}{T}\right]} = \frac{\frac{[ML^2T^{-2}]}{[AT]}}{[A]/[T]}$$

$$L = [ML^2T^{-2}A^{-2}]$$

Self Induction of long solenoid :-



Let a solenoid of N number of turns and let I is the current flowing through a solenoid. let B is the magnetic field inside the solenoid.

$$\phi \propto I$$

$$\phi = LI \quad \text{---} \; (1)$$

also, $\phi = NBA$ --- (2)

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Magnetic field at the centre of solenoid

$$B = \mu_0 n I$$

$$B = \mu_0 \frac{N}{l} I \quad \text{— ③} \qquad \left\{ \because n = \frac{N}{l} \right\}$$

put equation ③ in ② —

$$LI = N \mu_0 \frac{N I A}{l}$$

* L is independent of current

$$\boxed{L = \frac{\mu_0 N^2 A}{l}}$$

For medium insted of air, $L = \frac{\mu_m N^2 A}{l}$

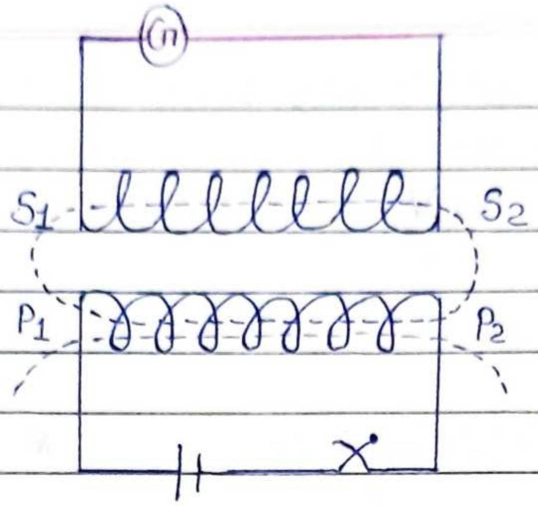
also $\mu_r = \frac{\mu_m}{\mu_0}$, $\boxed{L = \frac{\mu_0 \mu_r N^2 A}{l}}$

Mutual Induction :-

It is the property of two coil by virtue of which current flows to primary coil and hence due to change in flux accross secondary coil, current get induced in that secondary coil.

Experiment :-

Let two coil P_1, P_2 primary and S_1, S_2 as secondary coil. let S_1, S_2 is connected with a galvanometer where as P_1, P_2 is connected with a cell along with a key.



→ As current flow through primary coil P_1, P_2 flux linked with secondary coil changes and hence current get induced in secondary coil.

(M) Coefficient of Mutual Induction :-

Flux linked with secondary coil is directly proportional to current flowing through primary coil.

$$\phi_2 \propto I_1$$
$$\phi_2 = MI_1$$

where ~~m~~ M is constant of proportionality called mutual induction.

$$\mathcal{E}_2 = -d\phi_2$$
$$\mathcal{E}_2 = \frac{-d(MI_1)}{dt}$$
$$\mathcal{E}_2 = \frac{-MdI_1}{dt}$$
$$\frac{-\mathcal{E}_2}{dI_1/dt} = M$$
$$\boxed{\therefore\ M = \frac{-\mathcal{E}}{dI_1/dt}}$$

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→ SI unit is Henry.

So, the mutual induction is defined as Emf induced in secondary provided rate of change of current in primary coil is unity.

→ Define 1 Henry :-

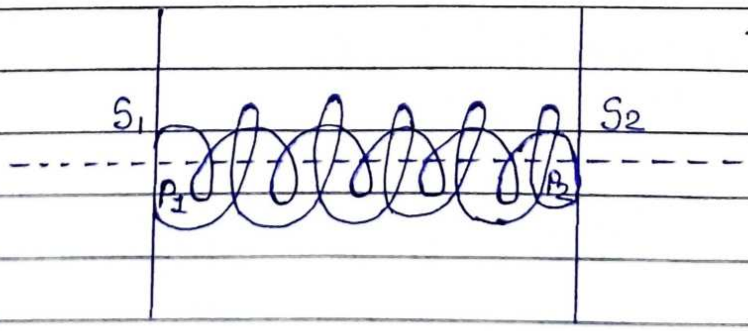
$$1H = \frac{1V}{1A/1S}$$

Hence mutual inductance is said to the 1 Henry if emf of 1V is induced in the secondary coil provided rate of change of current in the primary coil is 1A/1S.

Matual Induction of two long coaxial coil :-

Let two coaxial coil having each of length l. let P_1, P_2 are primary coil and S_1, S_2 are Secondary coil. let N_1, N_2 are no. of turns of primary and secondary coil respectively.

Let primary coil having area of cross-section A.



We know that flux linked with secondary coil is directly proportional to current flowing in primary coil.

As we know that,

$$\phi_2 = MI_1 \quad \text{—— (1)}$$

and, $$\phi_2 = N_2 B_1 A \quad \text{—— (2)}$$

Magnetic Field at centre of solenoid –

$$B_1 = \mu_0 n_1 I_1$$

$$B_1 = \frac{\mu_0 N_1 I_1}{l} \quad \text{—— (3)}$$

Put (3) in (2) equation –

$$\phi_2 = \frac{N_2 \mu_0 N_1 I_1 A}{l} \quad \text{—— (4)}$$

From (1) and (4) equation –

$$MI_1 = \frac{N_2 \mu_0 N_1 I_1 A}{l}$$

$$\boxed{M = \frac{\mu_0 N_1 N_2 A}{l}}$$

* Emf across Rotating Rod :–

Let a rod of length l is rotating in a given magnetic field.

$$\mathcal{E} = B l V_{avg}$$

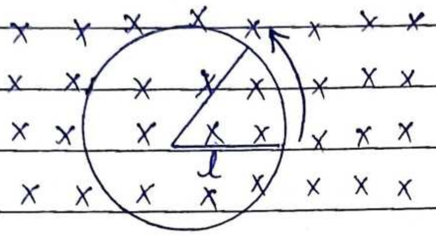
$$V_{avg} = \frac{u+v}{2} = \frac{0+v}{2} = \frac{v}{2}$$

$$\mathcal{E} = \frac{Blv}{2}$$

also, $V = l\omega$ [relation between linear velocity and angular velocity]

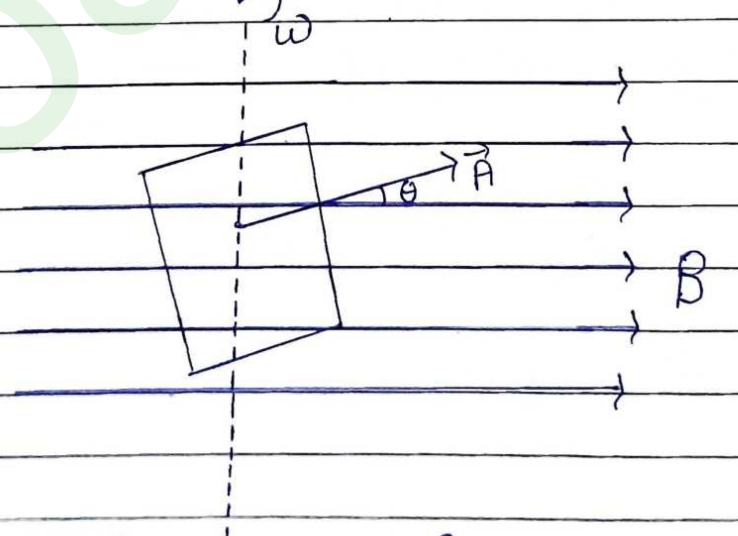
$$\mathcal{E} = \frac{Bl(l\omega)}{2} = \boxed{\frac{Bl^2\omega}{2} = \mathcal{E}}$$

$\left\{\omega = \frac{2\pi}{T} \text{ or } 2\pi\nu\right.$



⇒ By changing Orientation :
Emf induced due to change in orientation :
Derivation.

Let a rectangular metallic frame having N no. of turns and area 'A' is rotated with angular velocity 'ω' in unifrom magnetic field B. let normal drawn on plane of coil makes an angle θ with 'B'.



$\phi = NBA\cos\theta$ —— (I)

$\omega = \frac{\theta}{t}$ $\left\{\text{angular velocity } (\omega) = \frac{\theta}{t}\right\}$

$\theta = \omega t$ —— (II)

Put (ii) in (i) —

$$\phi = NBA \cos \omega t$$

differenciation on both side w.r.to t

$$\frac{d\phi}{dt} = NBA \frac{d(\cos \omega t)}{dt}$$

$$\frac{d\phi}{dt} = NBA(-\sin \omega t)\,\omega$$

$$\frac{d\phi}{dt} = -NBA\omega \sin \omega t \quad \text{—— (3)}$$

Now, according to Lenz's law —

$$\varepsilon = -\frac{d\phi}{dt} \quad \text{or} \quad -\varepsilon = \frac{d\phi}{dt} \quad \text{—— (4)}$$

From (3) and (4) —

$$-\varepsilon = -NBA\omega \sin \omega t$$

$$\varepsilon = NBA\omega \sin \omega t \quad \text{—— (5)}$$

Now, $\varepsilon = \varepsilon_0$ (max) if $\sin \omega t = 1$.

$$\varepsilon_0 = NBA\omega \quad \text{—— (6)}$$

put (6) equation in (5) —

$$\varepsilon = \underbrace{NBA\omega}\sin \omega t$$

$$\varepsilon = \varepsilon_0 \sin \omega t$$

So, $\boxed{\varepsilon = \varepsilon_0 \sin \omega t}$