

# Ray Optics And Optical Instruments

\* Light :- Light is a form of energy which produces sensation of sight.

- Light waves are electromagnetic waves, whose nature is transverse.
- The speed of light in vacuum is  $3 \times 10^8$  m/s but it is different in different media.
- The speed and wavelength of light change when it travels from one medium to another but its frequency remains unchanged.

Some Important Terms to know :-

Luminous Objects -

The object which emits its own light, are called luminous objects. e.g.- sun, other stars, an oil lamp etc.

Non-luminous Objects -

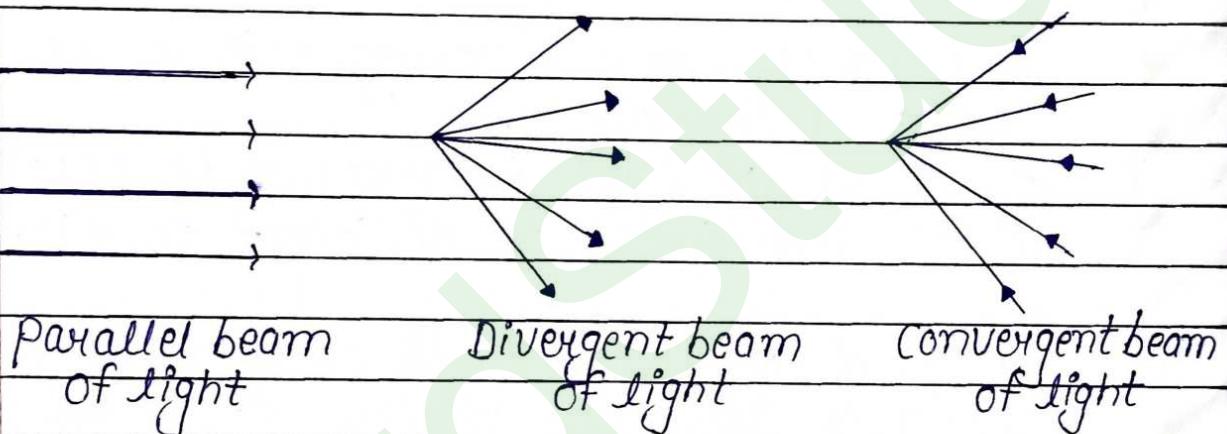
The objects which do not emit its own light but become visible due to the reflection of light falling on them, are called non-luminous objects, e.g- moon, table, chair, tree etc.

## \* Ray of Light :-

A straight line drawn in the direction of propagation of light is called a ray of light.

## \* Beam of light :-

A bundle of the adjacent light rays is called a beam of light.



## \* Image :-

If light rays coming from an object meet or appear to meet at a point after reflection or refraction, then this point is called image of the object.

## Real Image :-

The image obtained by the real meeting of light rays, is called a real image.

Real image can be obtained on a screen.  
Real image is inverted.

## Virtual Image :-

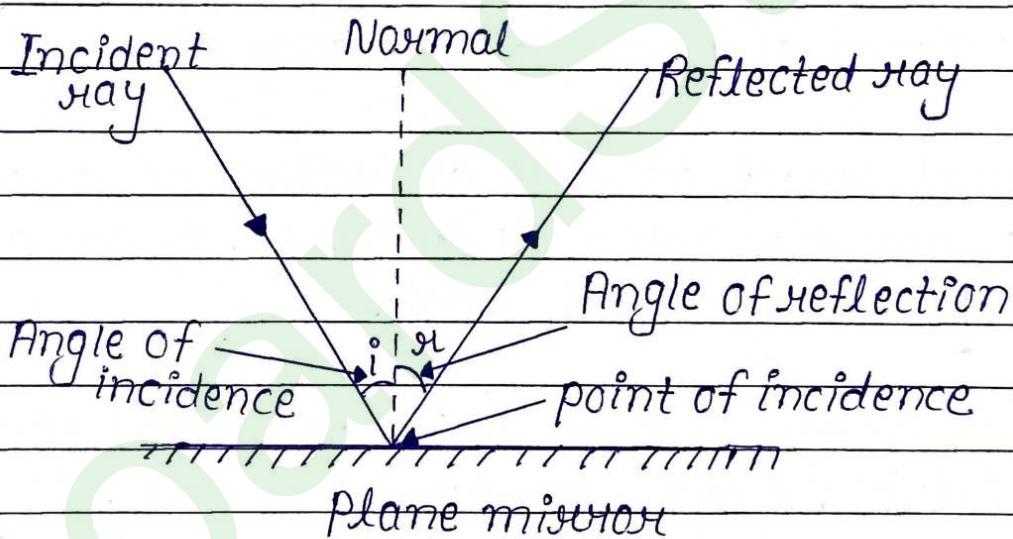
The image obtained when light rays are not really meeting but appears to meet only, is called a virtual image.

## \* Reflection of Light :-

The returning back of light in the same medium after striking to the surface is called reflection of light.

## Law of Reflection :-

There are two laws of reflection.



- (i) The incident ray, the reflected ray and the normal at the point of incidence all three lie in the same plane.
- (ii) The angle of incidence ( $i$ ) is always to the angle of reflection ( $r$ ).

Note:- Due to reflection of light speed, frequency and wavelength will not change but intensity get reduce.

## \* Mirror :-

A smooth and highly polished reflecting surface is called a mirror.

### (1) Plane Mirror —

A highly polished plane surface is called a plane mirror.

Different properties of image formed by plane mirror —

Size of image = Size of object

Magnification = Unity

Distance of image = Distance of object

Focal length as well as radius of curvature of a plane mirror is infinity. Power of a plane mirror is zero.

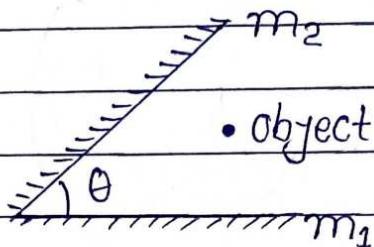
An image formed by a plane mirror is virtual, erect, laterally inverted, of same size as that of object and at the same distance as the object from the mirror.

Speed of image = - (image of object) [but in opposite direction]

### → Concept of Inclined mirror :-

When two plane mirrors are held at an angle  $\theta$ , the number of image of an

Object placed between them is given as below



- (i) Number of images =  $[(360^\circ/\theta) - 1]$ , when  $\theta$  is even.
- (ii) Number of images =  $(360^\circ/\theta)$ , when  $\theta$  is odd.

Spherical Mirrors :-

Mirrors whose reflecting surface is a part of hollow sphere.

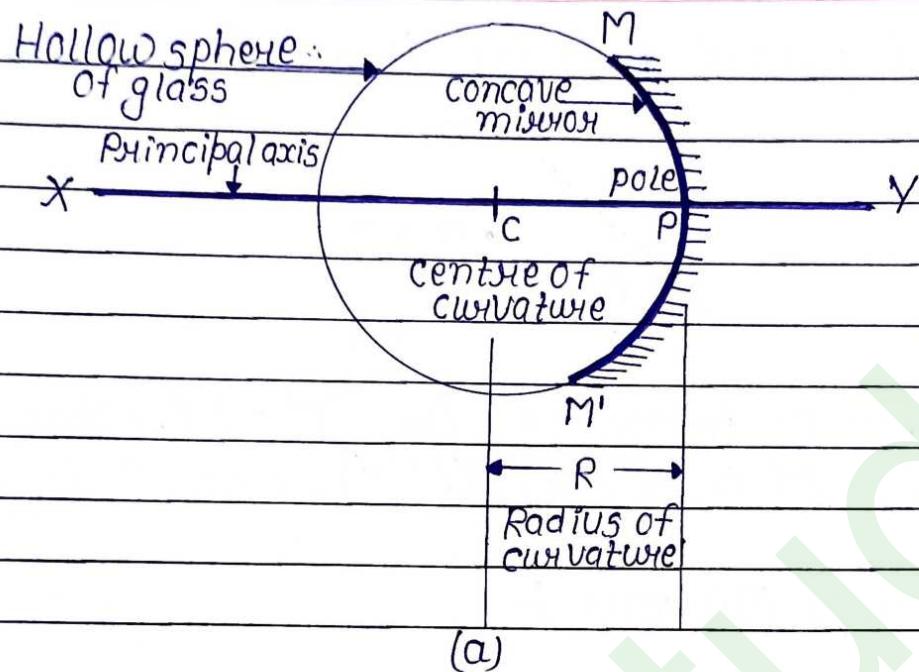
These are of two type :-

- Concave mirror
- Convex mirror

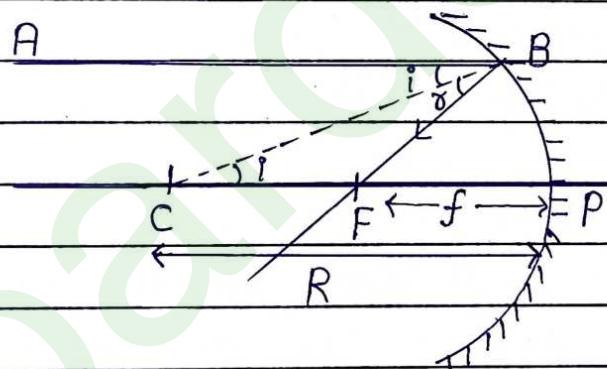
(i) Concave mirror :-

Mirror whose reflecting surface is towards the centre of sphere.

Concave mirror converge all parallel rays falling on its focus so it is also known as converging mirror.



Relation between Radius of curvature ( $R$ ) and focal length ( $f$ ) : For concave mirror :



[By laws of reflection],  $\angle i = \angle r$   
 $\angle i = \angle l$  [Alternate interior angle].

In  $\triangle BCF$

$$\angle i = \angle r$$

$FB = CF$  — ① [side opposite to equal angles are equal]

$\triangle BCF$  is an isosceles triangle.  
 for small aperture, B is very close to the point P. then,

$$FB = FP \quad \text{--- (ii)}$$

from (i) and (ii) :-

$$CF = FP \quad \text{--- (iii)}$$

Now,

$$PC = CF + FP$$

$$PC = FP + FP \quad (\text{from eq (iii)})$$

$$-R = -2FP$$

$$R = 2f$$

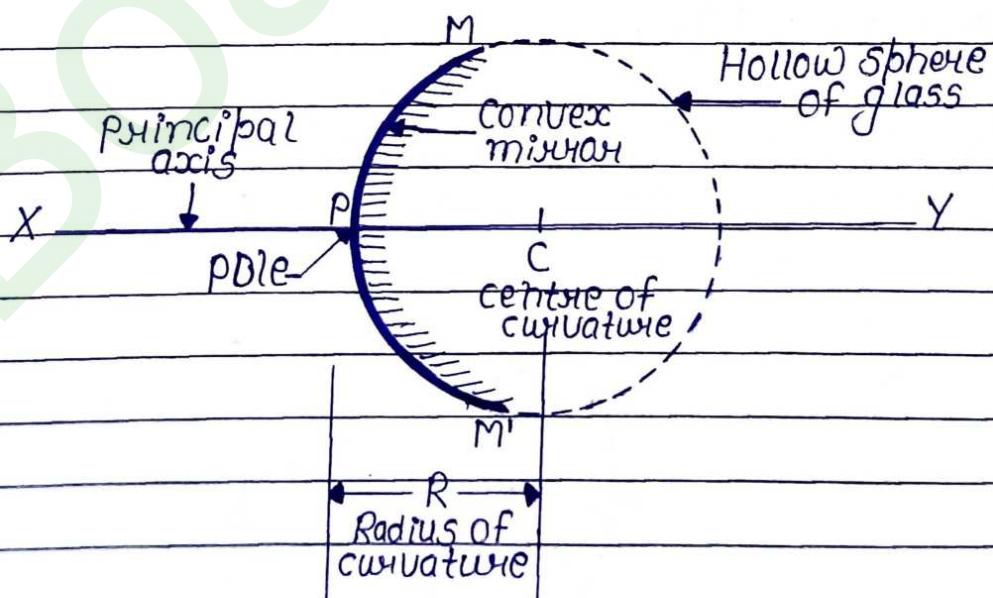
$$\therefore R = 2f$$

$R = 2f$	$f = \frac{R}{2}$
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Note :- Convex means having a surface that curves towards the outside of something.

### (ii) Convex Mirror :-

Mirror whose reflecting surface is always from the centre of sphere.



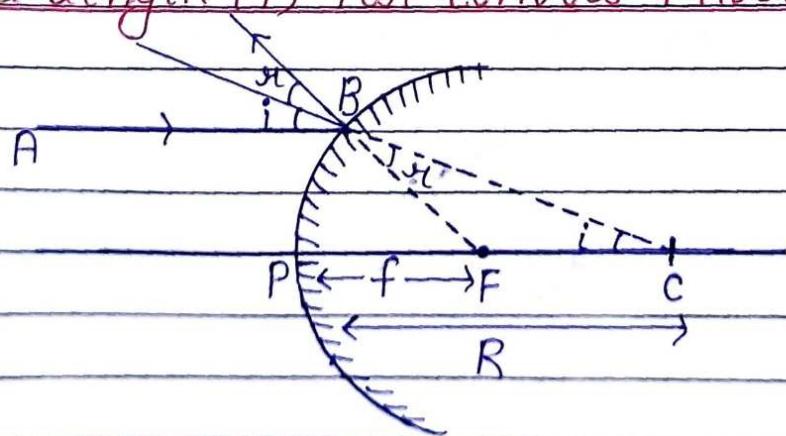
(b)

A convex mirror is one which diverges all the parallel rays falling on it. When these rays are back produced, they appear to meet at one point which is called focus. It is also known as Diverging mirror.

### Sign Convention for spherical Mirrors :-

- All distances are measured from the pole of the mirror.
- Distances measured in the direction of incident light rays are taken as positive.
- Distances measured in opposite direction to the incident light rays are taken as negative.
- Distances measured above the principal axis are positive.
- Distances measured below the principal axis are negative.

### Relation between Radius of curvature ( $R$ ) and focal length ( $f$ ) for convex Mirror :-



$\angle i = \angle r$  [By laws of reflection]

$\angle r = \angle h$  [Vertically opposite angle]

$\angle i = \angle h$  [corresponding angle]

In  $\triangle BCF$

$$\angle i = \angle h$$

$FB = FC - 0$  [side opposite to equal angle are equal]

For small aperture B point is very close to P, then

$$FB = FP \quad \text{--- (ii)}$$

from (i) and (ii)

$$FC = FP \quad \text{--- (iii)}$$

Now,

$$PC = FP + FC$$

$$PC = FP + FP$$

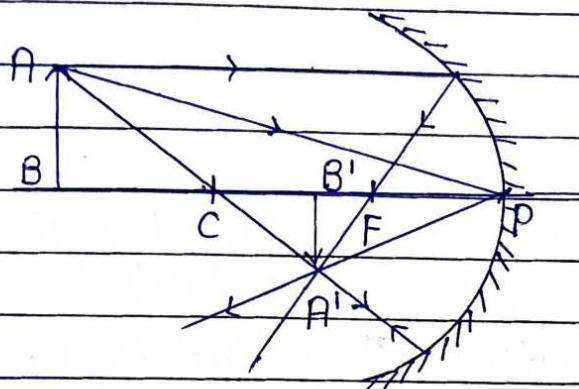
$$PC = 2FP$$

$$\therefore R = 2f$$

Note :- Concave mirror can produce both real as well as virtual image but convex mirror produce only virtual image.

Note :- All derivations are done by assuming light is parallel and remains close to the pole of the mirror i.e., small angle approximation.

\* Mінчай Formula for concave mirror : (Real image) :-



Hence,  $\triangle ABC \not\sim A'B'C$

$$\angle ABC = \angle A'B'C = 90^\circ$$

$\angle ABC = \angle A'B'C$  (opposite angle)

$\triangle ABC$  and  $\triangle A'B'C$  are similar

$$\frac{AB}{A'B'} = \frac{BC}{B'C} \quad \text{--- (i)}$$

$\triangle ABP$  and  $A'B'P$  are similar

$$\frac{AB}{A'B'} = \frac{BP}{B'P} \quad \text{--- (ii)}$$

from (i) and (ii) -

$$\frac{BC}{B'C} = \frac{BP}{B'P}$$

$$\frac{BP}{B'P} = \frac{PB - PC}{PC - PB'}$$

$$\frac{-u}{-V} = \frac{-u - (-R)}{-R - (-V)}$$

$\triangle ABC$  and  $\triangle A'B'C'$  are similar :

$$\frac{AB}{A'B'} = \frac{BC}{B'C'} \quad \text{--- (i)}$$

$\triangle ABP$  and  $\triangle A'B'P$  are similar :

$$\frac{AB}{A'B'} = \frac{BP}{B'P} \quad \text{--- (ii)}$$

From (i) and (ii) -

$$\frac{BP}{B'P} = \frac{BC}{B'C'}$$

Now measuring all distance from pole :

$$\frac{BP}{B'P} = \frac{PC - PB}{PC + PB'}$$

$$\frac{-u}{+v} = \frac{-R - (-u)}{-R + (+v)}$$

$$\frac{-u}{v} = \frac{-R + u}{-R + v}$$

$$-VR + UV = UR - UV$$

$$UV + UV = UR + VR$$

$$2UV = UR + VR$$

Divide the whole eqn by  $UVR$  -

$$\frac{2UV}{UVR} = \frac{UR}{UVR} + \frac{VR}{UVR}$$

$$\frac{2}{R} = \frac{1}{V} + \frac{1}{U}$$

$$\text{as } R = 2f$$

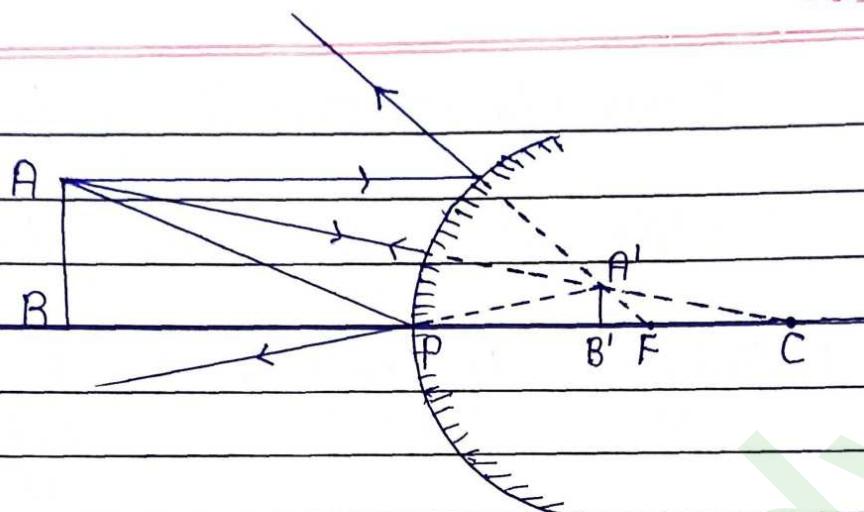
$$\text{so, } \frac{2}{2f} = \frac{1}{V} + \frac{1}{U}$$

$$\boxed{\frac{1}{f} = \frac{1}{V} + \frac{1}{U}}$$

→ Concave Mirror Object and Image Positions :-

Object Position	Image Position	Nature of Image
At Infinity	At the principal focus.	Real, inverted and extremely diminished.
Beyond the centre of curvature	Between the centre of curvature and focus	Real, inverted and diminished.
At the centre of curvature	At the centre of curvature	Real, inverted object and image of the same size.
Between focus and centre of curvature	Beyond the centre of curvature	Real, inverted, enlarged image
At the principal focus	At infinity	Extremely magnified
Between the pole and principal axis	Behind the mirror	Virtual, erect and magnified

→ Mirror Formula for convex mirror : virtual image —



$\triangle ABC$  and  $\triangle A'B'C$  are similar

$$\frac{AB}{A'B'} = \frac{BC}{B'C} \quad \text{--- (1)}$$

$\triangle ABP$  and  $\triangle A'B'P$  are similar

$$\frac{AB}{A'B'} = \frac{BP}{B'P} \quad \text{--- (2)}$$

From (1) and (2) —

$$\frac{BC}{B'C} = \frac{BP}{B'P}$$

All distance measure from pole.

$$\frac{BP+PC}{PC-PB'} = \frac{BP}{B'P}$$

$$\frac{-U+R}{R-(+V)} = -\frac{U}{V}$$

$$-UV + VR = -UR + UV$$

$$UR + VR = -UR \quad UV + UV$$

$$UR + VR = 2UV$$

Divide both side by  $UVR$

$$\frac{UR}{UVR} + \frac{VR}{UVR} = \frac{2UV}{UVR}$$

$$\frac{1}{v} + \frac{1}{u} = \frac{2}{2f} \quad \therefore R = 2f$$

So,  $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

## Convex Mirror Object and Image Positions :-

Object Position	Image position	Nature of Image
At Infinity	At the principal focus	Virtual, erect and extremely diminished.
Between infinity and pole	Appear between focus and pole	Virtual, erect and diminished.

### \* Magnification :-

It is defined as the ratio of size of image to the size of object.

$$m = \frac{\text{size of image}}{\text{size of object}}$$

$$m = \frac{h'}{h} = -\frac{v}{u}$$

if,  $m = +ve \rightarrow$  Virtual and erect

$m = -ve \rightarrow$  Real and Inverted

$m > 1 \rightarrow$  Enlarged

$m < 1 \rightarrow$  diminished

Note: In concave 'm' can be positive and negative but in convex 'm' is always positive only.

Note: For spherical mirrors,  
speed of image = magnification  $\times$  speed of object.

### ↳ Uses of concave mirrors :-

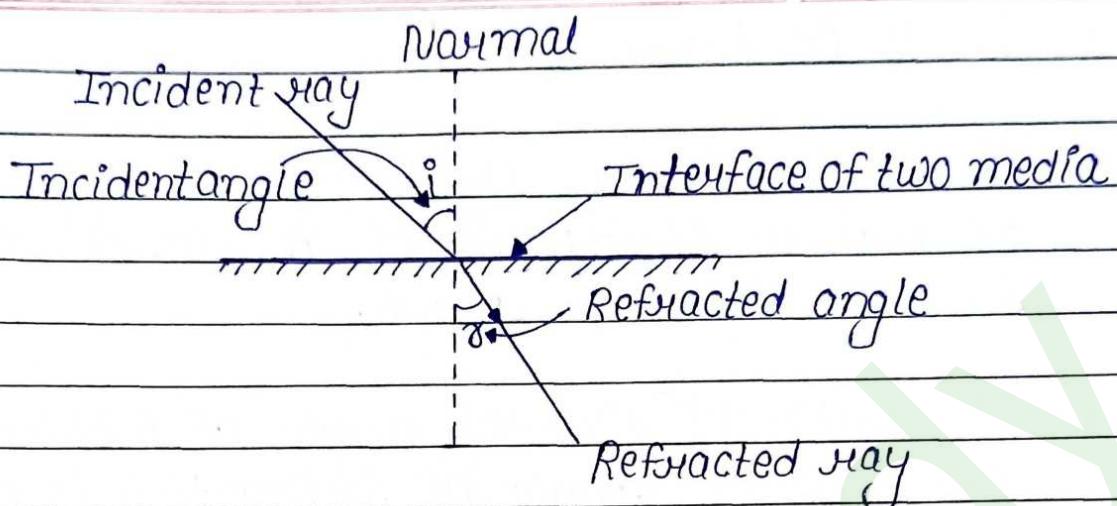
- Concave mirrors used as reflector in search lights.
- Use in head light of vehicle.
- Concave mirrors are used to see large image of face.
- Solar cooker.

### ↳ Uses of convex mirrors :-

- Used as reflector in street light.
- Used in vehicles as rear view mirror.
- Safety mirror.

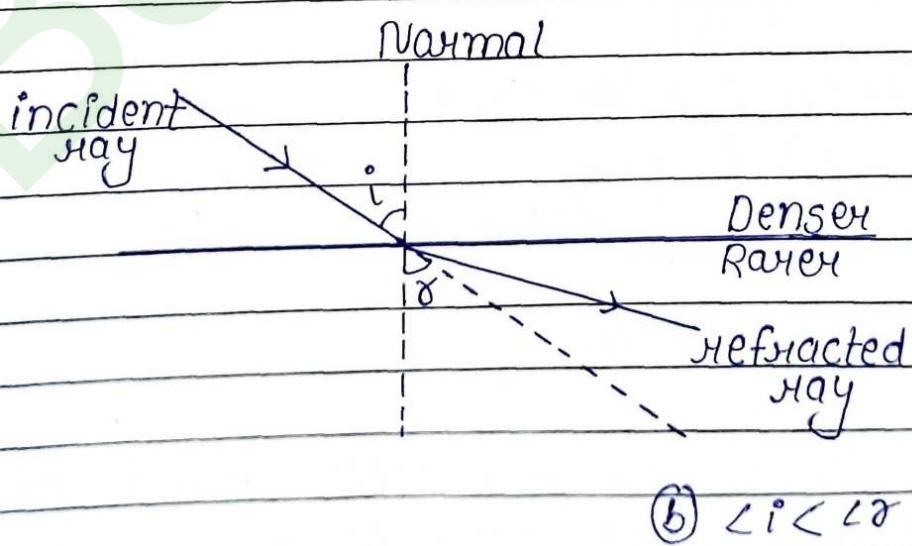
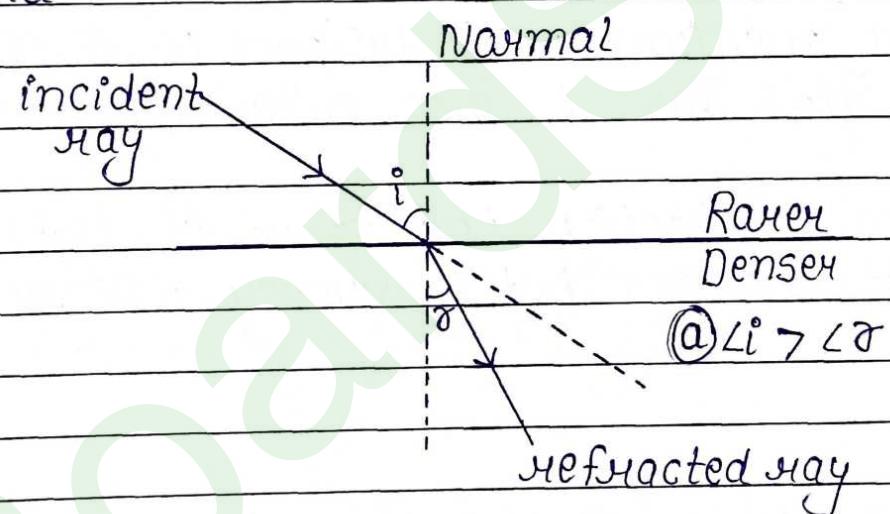
### \* Refraction of Light :-

The deviation of light rays from its path when it travels from one transparent medium to another transparent medium, is called refraction of light.



### Cause of Refraction -

The speed of light is different in different media.



## → Laws of Refraction :-

- (i) The incident ray, the refracted ray and the normal at the point of incidence, all three lies in the same plane.
- (ii) The ratio of sine of angle of incidence to the sine of angle of refraction is constant for a pair of two media,

$$\text{i.e., } \frac{\sin i}{\sin r} = \text{constant } (\mu_2)$$

where  $\mu_2$  is called refractive index of second medium with respect to first medium. This law is also called Snell's law.

Note :- During refraction the speed of light, wave length changes but frequency will remain same.

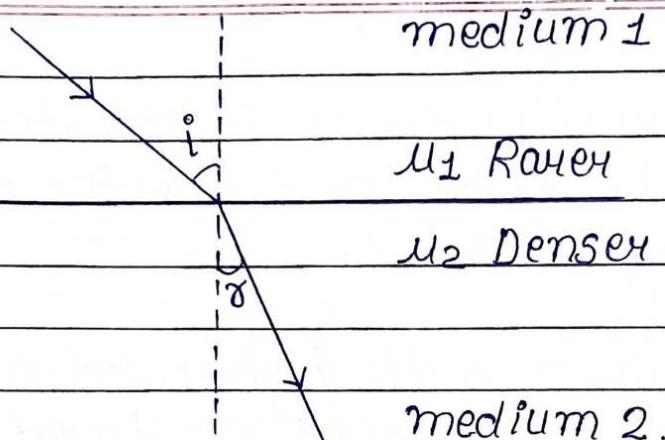
## → How to Apply Snell's law :-

$$\mu_1 \sin i = \mu_2 \sin r$$

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$$

$$\frac{\sin i}{\sin r} = \mu_2'$$

where  $\mu_2'$  means refractive index of medium 2 with respect to 1.



### \* Refractive Index :-

The ratio of speed of light in vacuum ( $c$ ) to the speed of light in any medium ( $v$ ) is called refractive index of the medium.

Refractive index of a medium,

$$\mu = c/v$$

Refractive index of water =  $4/3 = 1.33$ ;

Refractive index of glass =  $3/2 = 1.50$ .

### Relative Refractive Index :-

When light passes from one medium to another medium, the refractive index of medium 2 with respect to medium 1 is written as -

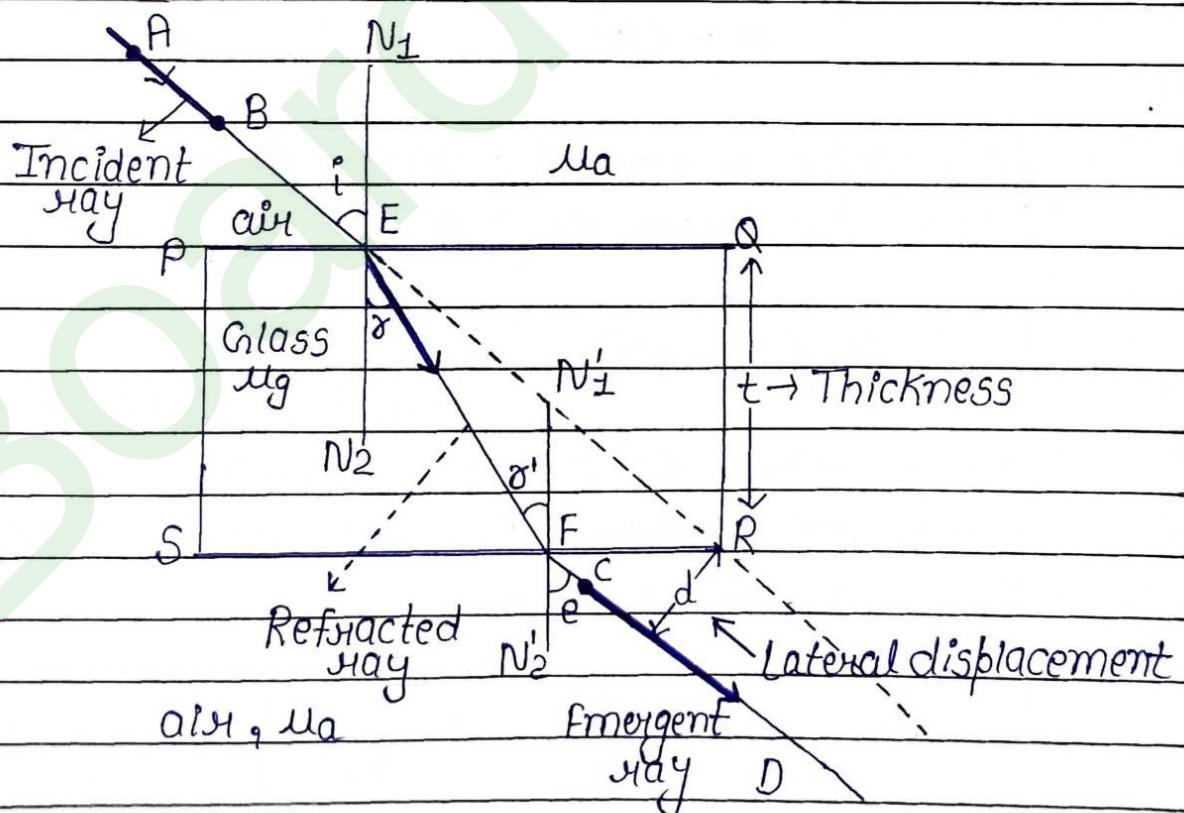
$${}^1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{\frac{c}{v_2}}{\frac{c}{v_1}} = \frac{c}{v_2} \times \frac{v_1}{c} = \frac{v_1}{v_2} = \frac{\lambda_{1f}}{\lambda_{2f}} = \frac{\lambda_1}{\lambda_2}$$

$${}^1\mu_2 = \frac{\text{speed of light in medium } ①}{\text{speed of light in medium } ②}$$

Note :-

- (1) If refractive index is greater than lesser will the speed and greater will be the bending of light.
- (2) A medium with higher refractive index is said to be denser compared to the medium with lower refractive index.
- (3) Angle of refraction  $\propto \frac{1}{\text{Bending of light}}$

→ Refraction through glass slab / rectangular prism :-



To prove :  $\angle i = \angle e$

Proof : Apply Snell's law at A

$$\mu_a \sin i = \mu_g \sin r$$

$$\frac{\sin i}{\sin r} = \frac{\mu_g}{\mu_a} \quad \text{--- (1)}$$

Apply Snell's law at B

$$\mu_g \sin r = \mu_a \sin e$$

$$\frac{\sin r}{\sin e} = \frac{\mu_a}{\mu_g} \quad \text{--- (2)}$$

Multiply (1) and (2) :-

$$\frac{\sin i}{\sin r} \times \frac{\sin r}{\sin e} = \frac{\mu_g}{\mu_a} \times \frac{\mu_a}{\mu_g}$$

$$\frac{\sin i}{\sin e} = 1$$

$$\sin i = \sin e$$

$$[i = e] \quad \therefore \angle i = \angle e$$

Hence proved.

This shows a ray of light displaced parallel to itself i.e. it goes lateral shift or lateral displacement.

→ Expression for Lateral displacement :-

From B, draw  $BN' \perp KA$  produced

$BN'$  = lateral displacement

$$\angle BAN' = \delta$$

In  $\triangle ABN'$

$$\sin s = \frac{p}{H} = \frac{BN'}{AB}$$

$$BN' = AB \sin s \quad \text{--- (i)}$$

In  $\triangle AA'B$

$$\cos \alpha = \frac{B}{H} = \frac{AA'}{AB}$$

$$AB = \frac{AA'}{\cos \alpha}$$

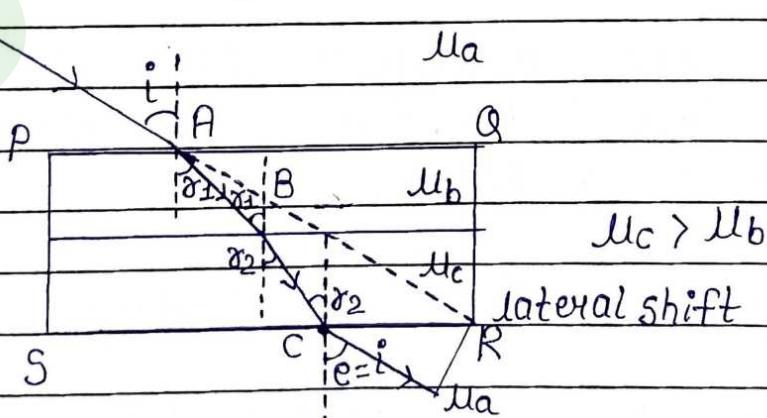
$$AB = \frac{t}{\cos \alpha} \quad \text{--- (ii)} \quad t = \text{thickness of glass slab.}$$

Put (ii) in (i) :-

$$BN' = \frac{t \sin s}{\cos \alpha}$$

$$BN' = \frac{t \sin(i - \alpha)}{\cos \alpha}$$

→ Refraction through compound glass slab :-



Apply Snell's Law at A,

$$\mu_a \sin i = \mu_b \sin r_1$$

$$\frac{\sin i}{\sin r_1} = \frac{a_m}{n_a}$$

$$\frac{\sin i}{\sin r_1} = \frac{a_m}{n_a} \quad \text{--- (1)}$$

Apply snell's law at B,

$$a_m \sin r_1 = n_c \sin r_2$$

$$\frac{\sin r_1}{\sin r_2} = \frac{n_c}{a_m} \quad \text{--- (2)}$$

Apply snell's law at C,

$$n_c \sin r_2 = n_a \sin i$$

$$\frac{\sin r_2}{\sin i} = \frac{n_a}{n_c}$$

$$\frac{\sin r_2}{\sin i} = \frac{c_m}{n_a} \quad \text{--- (3)}$$

Multiply (1), (2) and (3) —

$$\frac{\sin i}{\sin r_1} \times \frac{\sin r_1}{\sin r_2} \times \frac{\sin r_2}{\sin i} = \frac{a_m}{n_a} \times \frac{n_c}{a_m} \times \frac{c_m}{n_a}$$

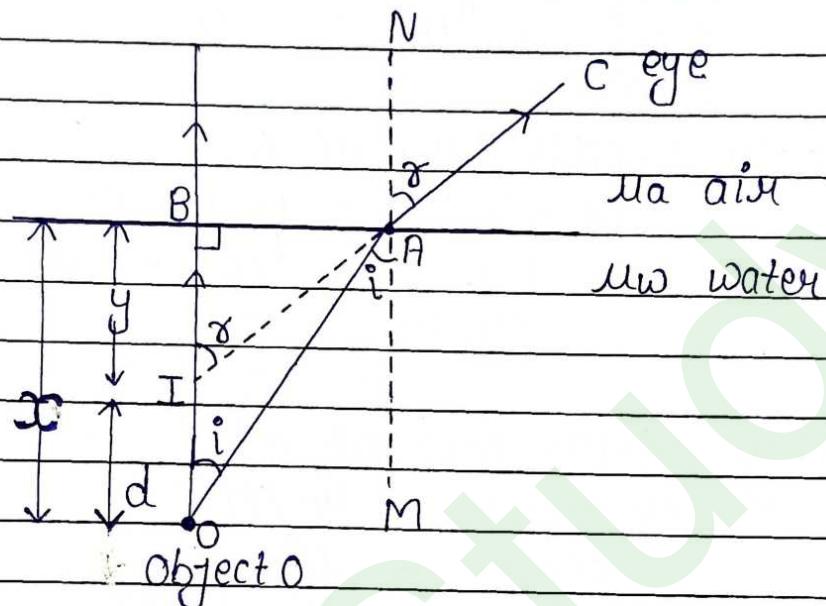
$$a_m \times b_m \times c_m = 1$$

$$a_m \times b_m = \frac{1}{c_m}$$

$$\therefore a_m = \frac{1}{c_m}$$

$a_m \times b_m = a_m$

## Real depth and Apparent depth :-



$$\text{Apparent depth} = IB = y$$

$$\text{Real depth} = OB = x$$

$$\angle OAM = \angle AOB = i \quad [\text{alternate } \angle]$$

$$\angle CAN = \angle AIR = r \quad [\text{corresponding } \angle]$$

Apply Snell's law at A

$$n_w \sin i = n_a \sin r$$

$$\frac{\sin i}{\sin r} = \frac{n_a}{n_w} = \frac{\omega n_a}{n_w} \quad \text{--- (1)}$$

In  $\triangle AOB$ ,

$$\frac{\sin i}{H} = \frac{BA}{OA} \quad \text{--- (2)}$$

In  $\triangle AIB$ ,

$$\frac{\sin r}{IA} = \frac{BA}{IA} \quad \text{--- (3)}$$

Putting the value of (2) and (3) in (1)

$$\frac{u_a}{u_w} = \frac{BA/OA}{BA/IA}$$

$$u_a = IA \\ OA$$

As angle  $i$  is very small,  $A$  is close to  $B$ .

$$u_a = \frac{IB}{OB} = \frac{y}{x}$$

$$\frac{1}{u_w} = \frac{y}{x} \quad (\text{principle of reversibility})$$

$u_w = \frac{x}{y}$	= real depth apparent depth
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or	$u_w = \frac{x}{y}$
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Normal shift ;  $d$

$$d = \text{real depth} - \text{apparent depth}$$

$$d = x - y$$

$$d = x - \frac{x}{u_w}$$

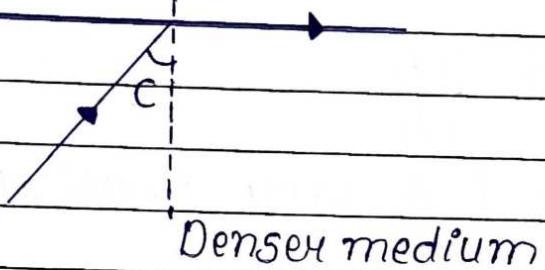
$$d = x \left[ 1 - \frac{1}{u_w} \right]$$

$d = x \left[ 1 - \frac{1}{u_w} \right]$
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\* Critical Angle :

The angle of incidence in a denser medium for which the angle of refraction in rarer medium becomes  $90^\circ$ , is called critical angle ( $c$ ).

Rarer medium



Apply Snell's law at A,

$$\mu_w \sin i = \mu_a \sin r$$

$$\frac{\sin i}{\sin r} = \frac{\mu_a}{\mu_w}$$

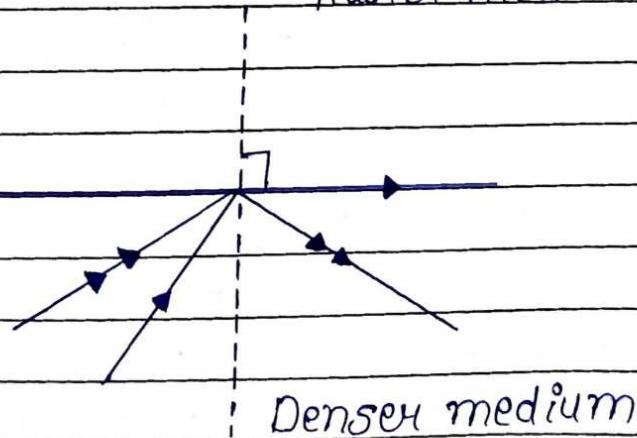
also,  $i = i_c$  and  $r = 90^\circ$

$$i_c = \sin^{-1} \left( \frac{\mu_a}{\mu_w} \right)$$

where  $\mu_a$  and  $\mu_w$  are refractive indices of rarer and denser medium.

Critical angle increases with temperature :-

Rarer medium



The refractive index is maximum for violet colour of light and minimum for red colour of light. i.e.,  $n_V > n_R$  therefore critical angle is maximum for red colour of light and minimum for violet colour of light, i.e.,  $C_V < C_R$ .

### \* Total Internal Reflection (TIR) :-

When a light ray travelling from a denser medium towards a rarer medium is incident at the interface at an angle of incidence greater than critical angle, then light rays reflected back in to the denser medium. This phenomena is called T.I.R.

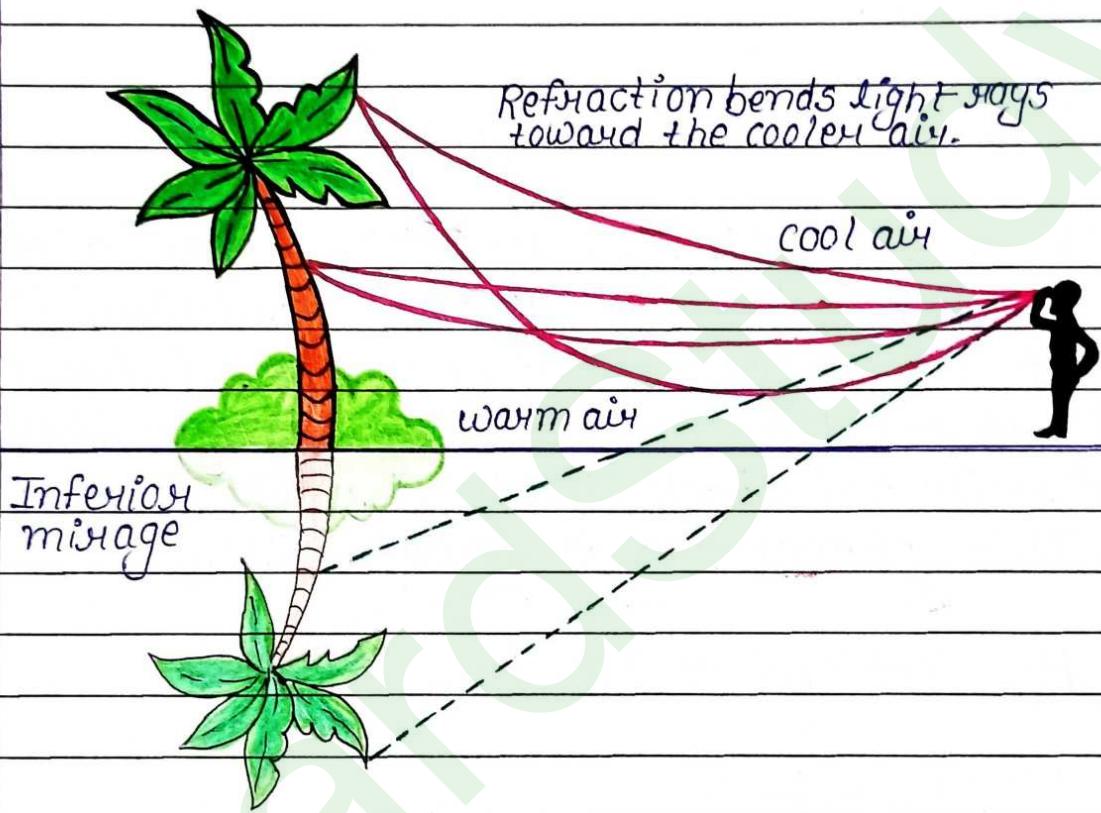
### Applications of TIR :-

#### (1) MIRAGE :-

Mirage is an optical illusion of desert on hot summer days due to very high temperature the air near the surface of earth become rarer and above the surface of earth will be denser as compare to near the surface of earth.

Whenever ray comming from tree undergo refraction and at point, the ray make an angle greater than critical angle. Then it undergoes in total internal reflection and when it goes in to the eye of person

an inverted image of tree is seen by the person which appears as there is water but actually there is no water.



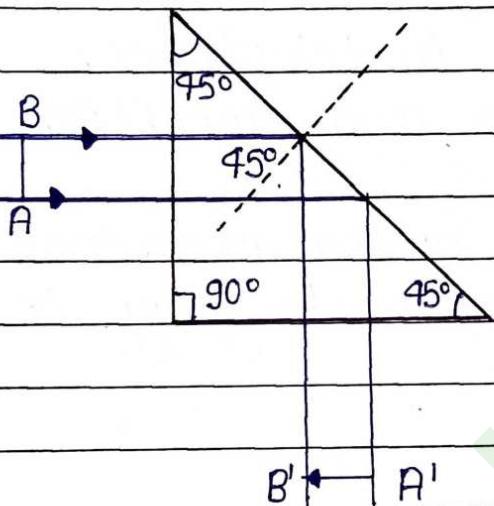
## (2) Sparkly Diamonds :-

Diamonds shine so much because of TIR! when light enters a diamond, it bounces around inside many times before coming out, making it sparkle. Diamonds have the perfect properties for lots of TIR.

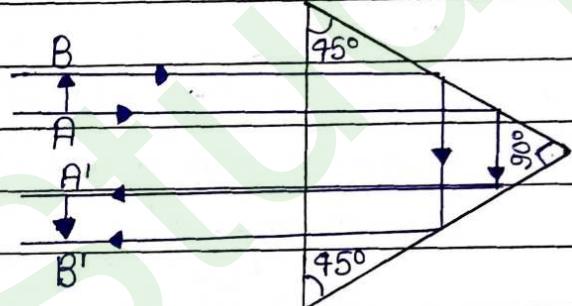
## (3) Prisms as Mirrors :-

Instead of using

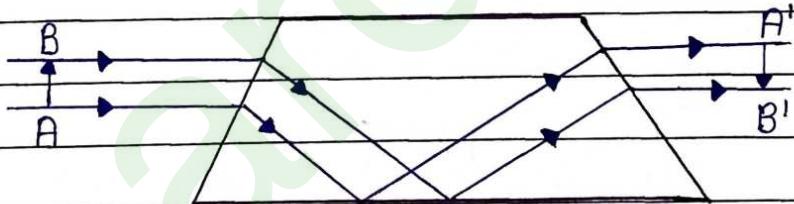
In regular mirrors, some devices like binoculars use special glass shapes called prisms. Light entering these prisms can bounce off the inside surface using TIR, giving a very clear and bright reflection.



(a)



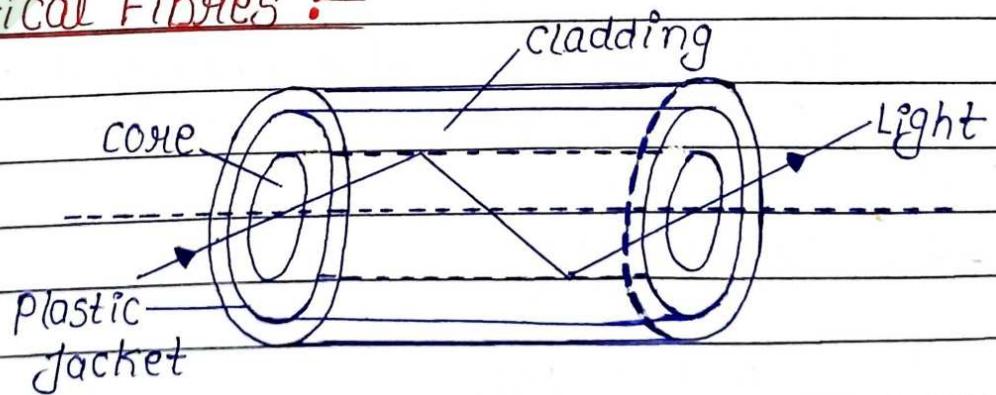
(b)



(c)

Fig :- Prism designed to bend rays by 90 and 180 degrees to invert image without changing its size make use of total internal reflection.

#### (4) Optical Fibres :-



The repeated TIR at the core-cladding interface keeps the light confined within the core, allowing it to travel long distances. (as  $\mu$  of cladding is lesser than  $\mu$  of core)

### USES :-

- Optical fibres are used in telephones.
- Used in transmission and reception of Signal.
- Used in medical and optical examination of inaccessible part of human body eg in endoscopy.

### \* Refraction through spherical surfaces :-

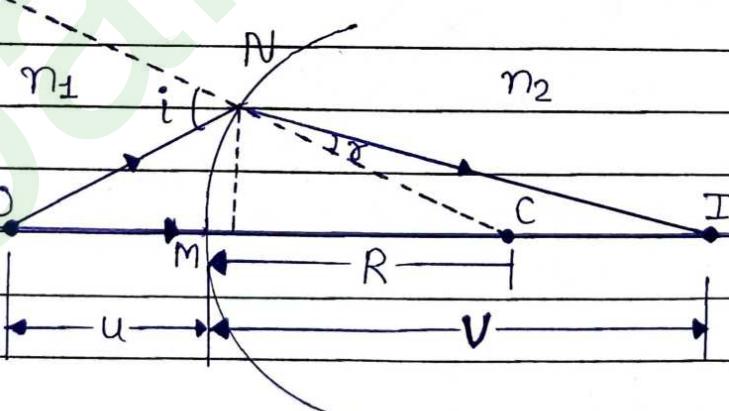


fig :- Refraction at a spherical surface separating two media.

$$\tan \angle NOM = \frac{MN}{OM}$$

$$\tan \angle NCM = \frac{MN}{MC}$$

$$\tan \angle NIM = \frac{MN}{MI}$$

Now, for  $\triangle NOC$ ,  $i$  is the exterior angle. Therefore,  $i = \angle NOM + \angle NCM$

$$i = \frac{MN}{OM} + \frac{MN}{MC} \quad \text{--- (1)}$$

Similarly,

$$r = \angle NCM - \angle NIM$$

$$\text{i.e., } r = \frac{MN}{MC} - \frac{MN}{MI} \quad \text{--- (2)}$$

Now, by Snell's law

$$n_1 \sin i = n_2 \sin r$$

or for small angles

$$n_1 i = n_2 r$$

Substituting  $i$  and  $r$  from eqn (1) & (2), we get

$$\frac{n_1 + n_2}{OM} = \frac{n_2 - n_1}{MC}$$

Hence,  $OM$ ,  $MI$  and  $MC$  represent magnitudes of distances. Applying the Cartesian sign convention,

$$OM = -u, MI = +v, MC = +R$$

Substituting these in eqn 3, we get

$$\frac{n_2 - n_1}{v} = \frac{n_2 - n_1}{R}$$

Formula :

$$\frac{n_2 - n_1}{V} = \frac{n_2 - n_1}{R}$$

Note :- use proper sign convention.

Example :-

- Q) Light from a point source in air falls on a spherical glass surface ( $n = 1.5$  and radius of curvature = 20 cm). The distance of the light source from the glass surface is 100 cm. At what position the image is formed?

SOLN: Here,

$$u = -100 \text{ cm}$$

$$v = ?$$

$$R = +20 \text{ cm}$$

$$n_1 = 1, n_2 = 1.5.$$

We then have

$$\frac{1.5 - 1}{V} = \frac{0.5}{20}$$

$$\frac{1.5}{V} = \frac{5}{200} - \frac{1}{100}$$

$$\frac{0.5}{V} = \frac{8}{200}$$

$$\frac{5}{V \times 10} = \frac{1}{200}$$

$$V = +100 \text{ cm}$$

The image is formed at a distance of 100 cm from the glass surface, in the direction of incident light.

### \* Thin Lenses :-

A transparent medium, typically glass, with two spherical surfaces that refract light.

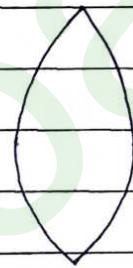
We mainly focus on two type of lenses :-

(1) Convex or converging lens

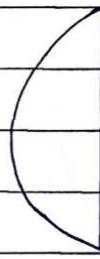
(2) Concave or diverging lens

### Convex lens :-

A lens which is thicker at the centre and thinner at its ends.



double convex  
lens



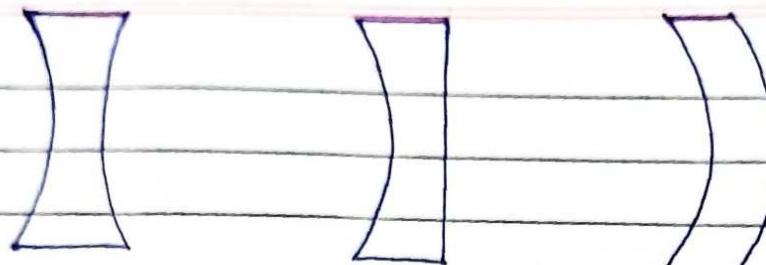
plano  
convex lens



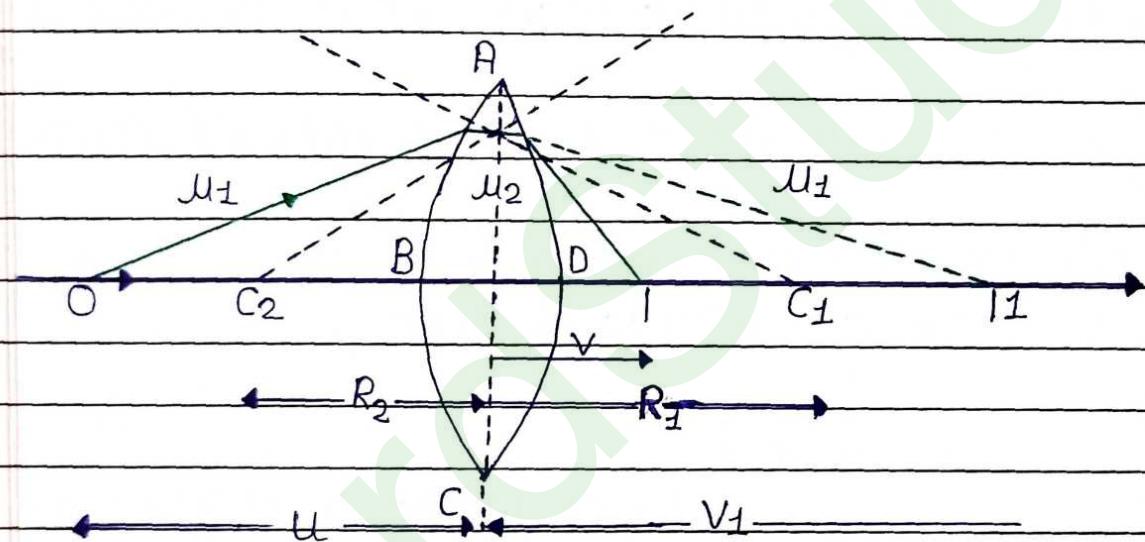
concave  
convex lens

### Concave lens :-

A lens which is thinner at the centre and thicker at its ends.

double concave  
lensplano  
concave  
lensconvex concave  
lens

### ↳ Lens maker formula :-



For the first surface,

$$\frac{u_2 - u_1}{v_1} = \frac{u_2 - u_1}{R_1} \quad \text{--- (1)}$$

For the second surface,

$$\frac{u_1 - u_2}{v} = \frac{u_1 - u_2}{R_2} \quad \text{--- (2)}$$

Now adding equation (1) and (2),

$$\frac{u_1 - u_2}{v} - \frac{u_1 - u_2}{u} = (u_2 - u_1) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right]$$

$$\frac{1}{V} - \frac{1}{U} = \left( \frac{U_2 - 1}{U_1} \right) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right]$$

when  $U = \infty$  and  $V = f$

$$\frac{1}{f} = \left( \frac{U_2 - 1}{U_1} \right) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right]$$

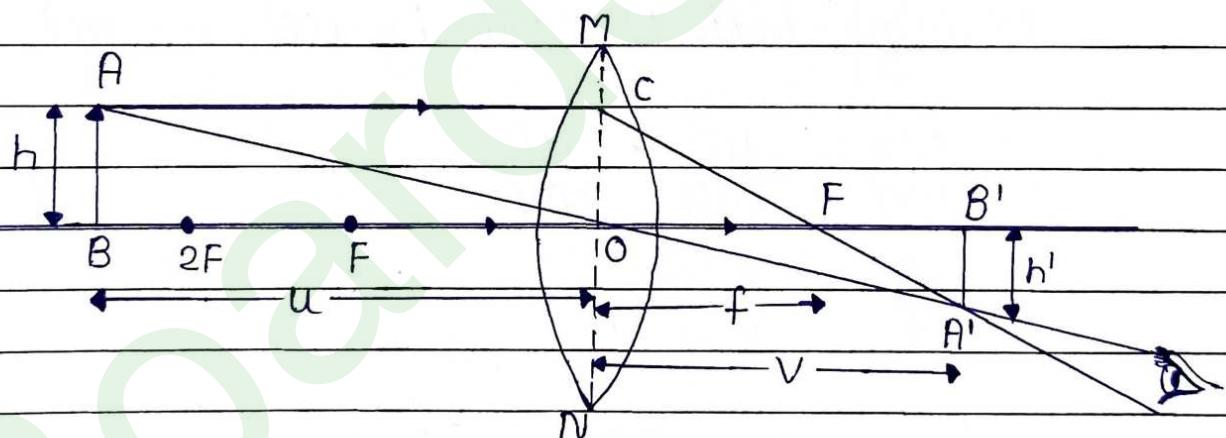
But also,

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{f}$$

Therefore, we can say that,

$$\frac{1}{f} = (U-1) \left[ \frac{1}{R_1} - \frac{1}{R_2} \right]$$

### \* Lense Formula :-



$$\frac{A'B'}{AB} = \frac{OB'}{OB} \quad (1)$$

Similarly,  $\triangle A'B'F$  and  $\triangle OCF$  are similar, hence

$$\frac{A'B'}{OC} = \frac{FB'}{OF}$$

But

$$OC = AB$$

Hence,

$$\frac{A'B'}{AB} = \frac{FB'}{OF} \quad \text{--- (2)}$$

Equating eq (1) and (2), we get

$$\frac{OB'}{OB} = \frac{FB'}{OF} = \frac{OB' - OF}{OF}$$

Substituting the sign convention, we get

$$OB = -u, OB' = v \text{ and } OF = f$$

$$\frac{v}{-u} = \frac{v-f}{f}$$

$$vf = -uv + uf$$

$$uv = uf - vf$$

Dividing both the side by  $uvf$ , we get

$$\frac{uv}{uvf} = \frac{uf}{uvf} - \frac{vf}{uvf}$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$

### Magnification of Lens :-

The ratio of size of image to the size of object.

$$m = \frac{\text{size of image}}{\text{size of object}} = \frac{h'}{h} = \frac{v}{u}$$

Note :  $m \rightarrow +ve$  virtual and erect

$m \rightarrow -ve$  Real and inverted

$m > 1$  enlarged

$m < 1$  diminished

## Power of Lense :

It is defined as the ability of the lens to converge or diverge a beam of light.

$$P = \frac{1}{f} \text{ (in m)}$$

SI unit of power is Dioptric 'D'.

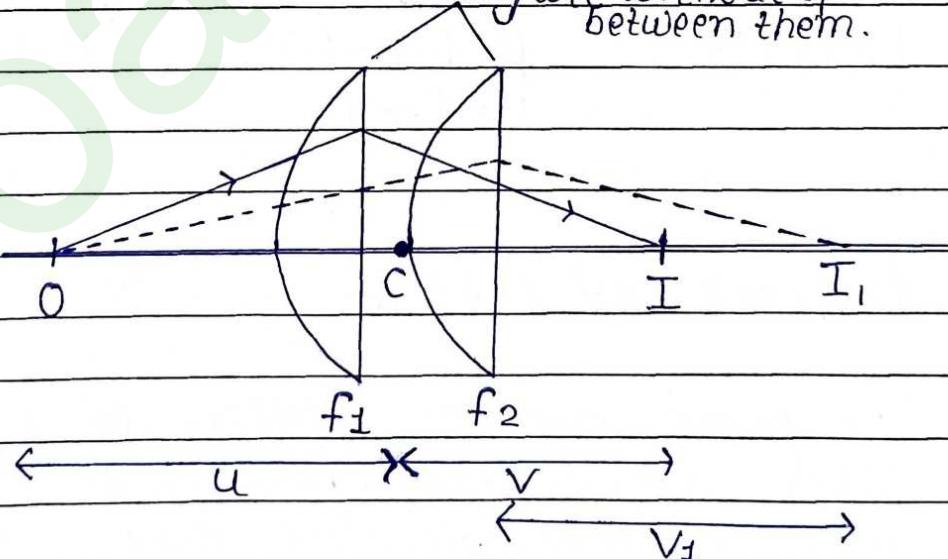
**Note:** For convex lens, power is +ve as  $f$  is +ve.  
For concave lens, power is -ve as focal length is -ve.

## Combination of Lens :

Refraction on lens 1

$$\frac{1}{V_1} - \frac{1}{U} = \frac{1}{f} \Rightarrow \frac{1}{V_1} - \frac{1}{U} = \frac{1}{f_1} \quad \text{--- (1)}$$

They are without space between them.



Refraction on lens 2

Apply lens formula

$$\frac{1}{V} - \frac{1}{U} = \frac{1}{f}$$

$$\frac{1}{V} - \frac{1}{V_1} = \frac{1}{f_2} \quad \text{--- (2)}$$

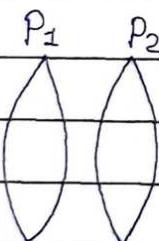
Adding ① and ② —

$$\frac{1}{V_1} - \frac{1}{U} + \frac{1}{V} - \frac{1}{V_1} = \frac{1}{f_1} + \frac{1}{f_2}$$

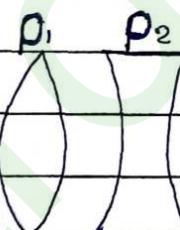
$$\frac{1}{V} - \frac{1}{U} = \frac{1}{f_1} + \frac{1}{f_2}$$

$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$
---

$$P = P_1 + P_2$$

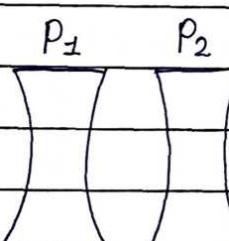


$$P = P_1 + P_2$$



$$P = P_1 + (-P_2)$$

$$P = P_1 - P_2$$

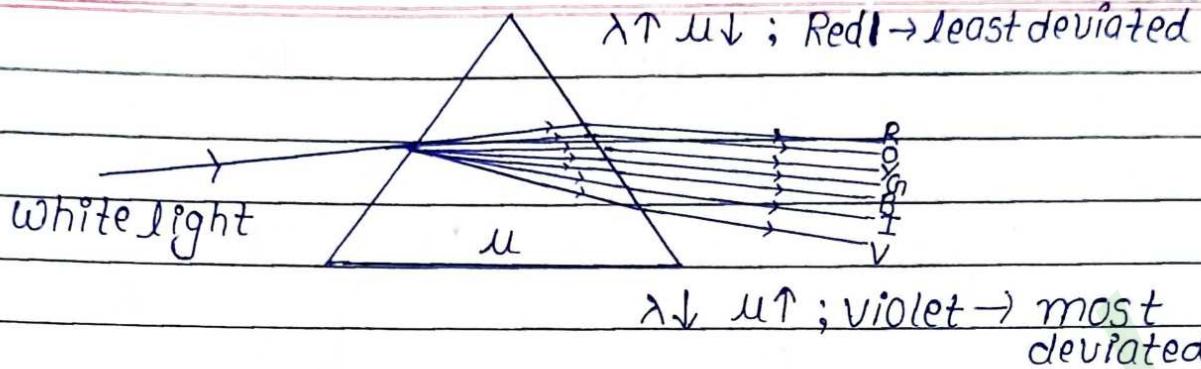


$$P = (-P_1) + (-P_2)$$

$$P = -P_1 - P_2$$

## \* Dispersion of light :-

The phenomenon of splitting of light into seven colours after passing through prism is called dispersion of light.



### Cause of Dispersion :-

The basic cause of dispersion is that refractive index of material of prism is different for different colour or wavelength.

From Cauchy's formula,  $\mu \propto \frac{1}{\lambda^2}$

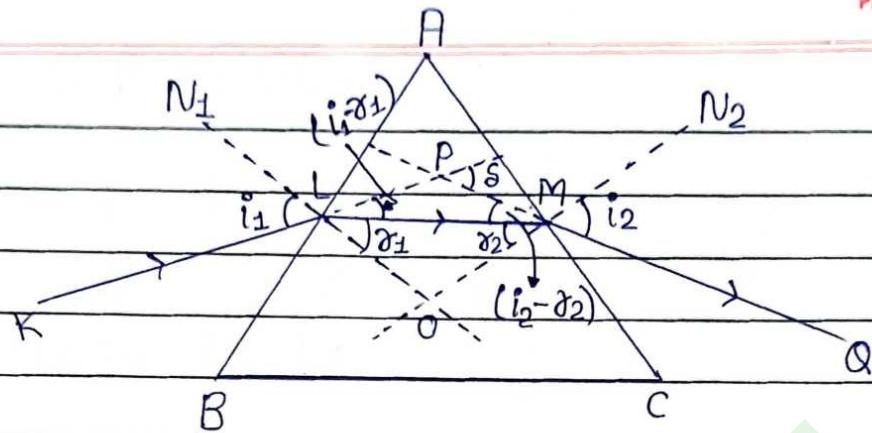
Due to different refractive index their deviation is different.

$$\delta \propto \mu$$

Example : Red colour has maximum wavelength. Therefore its refractive index is minimum and hence deviation is minimum.

### Refraction through a prism :-

(a) Calculation of angle of deviation :-



In  $\triangle PLM$

$\angle A$  = Angle of prism  
 $\delta$  = Angle of deviation

$$\begin{aligned}\delta &= i_1 - \alpha_1 + i_2 - \alpha_2 \\ &= i_1 + i_2 - \alpha_1 - \alpha_2 \\ &= i_1 + i_2 - (\alpha_1 + \alpha_2) \quad \text{--- (1)}\end{aligned}$$

In  $\triangle OLM$

$$\angle O + \angle \alpha_1 + \angle \alpha_2 = 180^\circ \quad \text{--- (2)}$$

In quadrilateral OLAM :

$$\angle O + \angle A + \angle L + \angle M = 360^\circ$$

$$\angle O + \angle A + 90^\circ + 90^\circ = 360^\circ$$

$$\angle O + \angle A = 360 - 180^\circ$$

$$\angle O + \angle A = 180^\circ \quad \text{--- (3)}$$

from (2) and (3) —

$$\angle O + \angle A = \angle O + \angle \alpha_1 + \angle \alpha_2$$

$$\angle A = \angle \alpha_1 + \angle \alpha_2$$

$$A = \alpha_1 + \alpha_2 \quad \text{--- (4)}$$

Put equation (2) in (1) :-

$$\delta = i_1 + i_2 - A \quad \text{--- (5)}$$

for thin prism -

According to Snell's law

$$\mu = \frac{\sin i}{\sin \delta}$$

for small angle,  $\sin i = i$   
 $\sin \delta = \delta$

$$\text{So, } \mu = \frac{i}{\delta}$$

$$i = \mu \delta$$

$$i_1 = \mu \delta_1$$

$$i_2 = \mu \delta_2$$

Put value of  $i_1$  and  $i_2$  in eqn-⑤

$$S = \mu \delta_1 + \mu \delta_2 - A$$

$$S = \mu (\delta_1 + \delta_2) - A$$

$$S = \mu A - A$$

$$S = A (\mu - 1)$$

$$S = A (\mu - 1)$$

(b) Angle of minimum deviation and prism formula :-

Prism are designed in such a way that deviation should be minimum.

So, condition of minimum deviation -

$$\text{we know, } S = i_1 + i_2 - A$$

$$S = (\sqrt{i_1})^2 + (\sqrt{i_2})^2 - A + 2\sqrt{i_1 i_2} - 2\sqrt{i_1 i_2}$$

$$S = (\sqrt{i_1} - \sqrt{i_2})^2 - A + 2\sqrt{i_1 i_2}$$

for  $\delta$  to be minimum

$$(\sqrt{i_1} - \sqrt{i_2})^2 = 0$$

$$\sqrt{i_1} - \sqrt{i_2} = 0$$

$$\sqrt{i_1} = \sqrt{i_2} \Rightarrow i_1 = i_2$$

$$\therefore i_1 = i_2 = i$$

$$\alpha_1 = \alpha_2 = \alpha$$

$$\text{Now, } A = \alpha_1 + \alpha_2$$

$$A = \alpha + \alpha$$

$$A = 2\alpha$$

$$\therefore \alpha = \frac{A}{2}$$

$$\text{Similarly, } \delta = i_1 + i_2 - A$$

$$\delta_m = i + i - A$$

$$\delta_m = 2i - A$$

$$\frac{\delta_m + A}{2} = i$$

According to Snell's law,

$$n = \frac{\sin i}{\sin \alpha}$$

$$n = \frac{\sin(\delta_m + A)}{\sin(\frac{A}{2})}$$

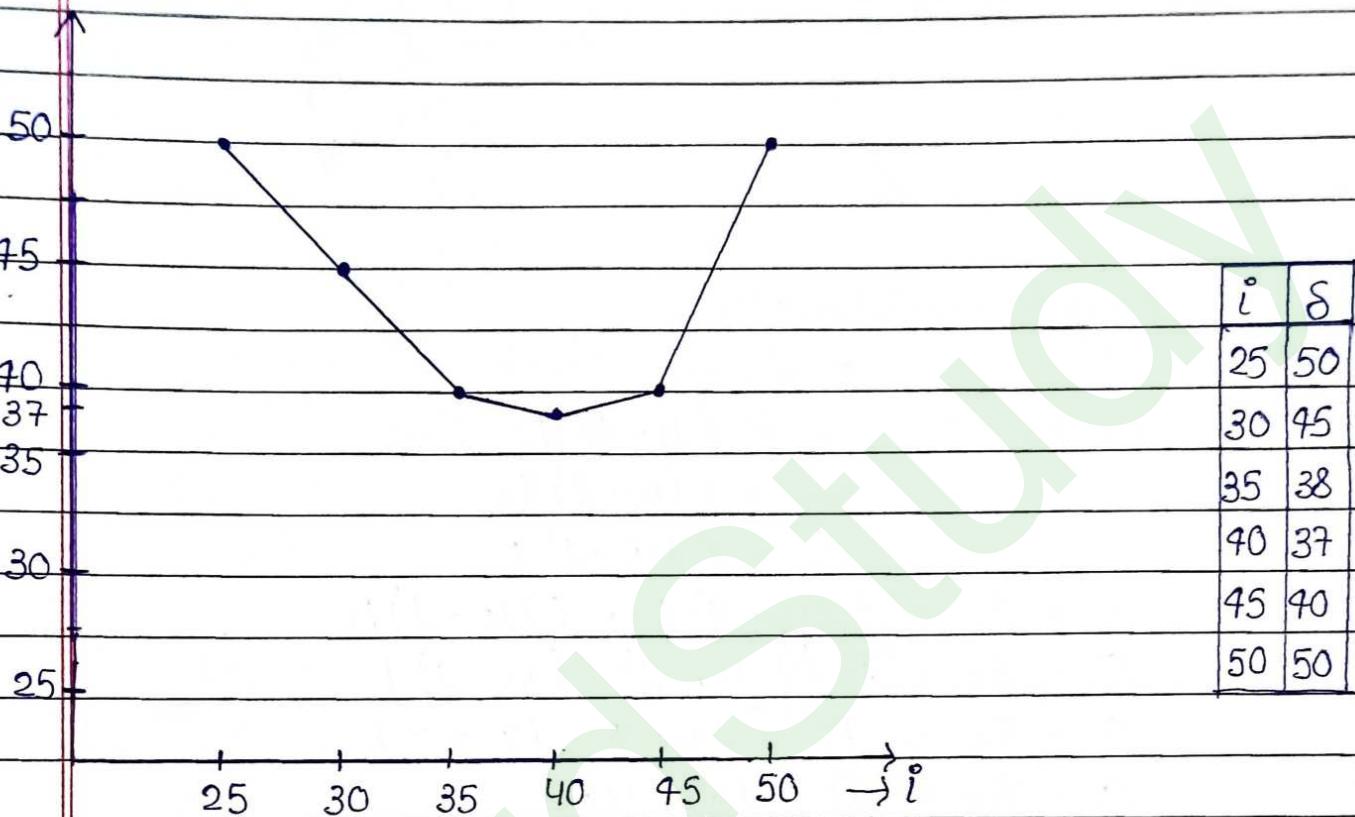
$$\sin(\frac{A}{2})$$

'This called prism formula'.

→ The graph between Angle of Incidence and Angle of Deviation

The angle of deviation first decreases and becomes minimum for a particular angle

of incidence and then increases to the maximum.



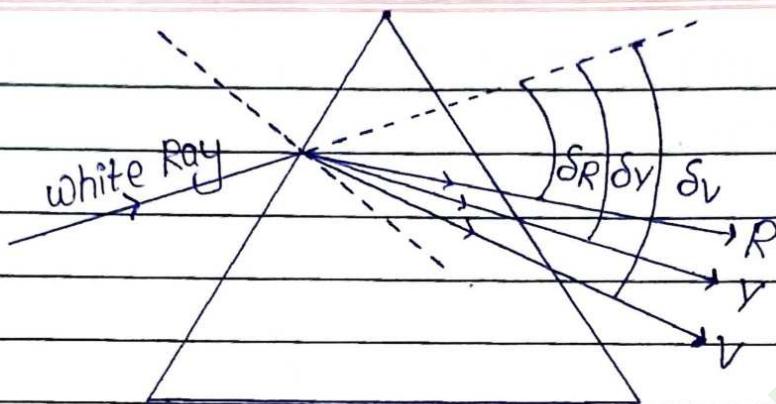
When prism is set in condition of minimum deviation, it is observed that

$$i_1 = i_2 = i$$

$$\text{so, } \delta_1 = \delta_2 = \delta$$

### Angular Dispersion ( $\theta$ )

Angular dispersion is the difference between the deviations of the extreme colours, red and violet, after passing through the prism.



Angular dispersion,

$$= \delta_V - \delta_R$$

$$\text{we know, } \delta = (\mu - 1) A$$

$$\delta_V = (\mu_V - 1) A$$

$$\delta_R = (\mu_R - 1) A$$

$$\delta_V - \delta_R = (\mu_V - 1) A - (\mu_R - 1) A$$

$$\delta_V - \delta_R = A [(\mu_V - 1) - (\mu_R - 1)]$$

$$\delta_V - \delta_R = A [\mu_V - 1 - \mu_R + 1]$$

$$\delta_V - \delta_R = A (\mu_V - \mu_R)$$

$$\boxed{\delta_V - \delta_R = A (\mu_V - \mu_R)}$$

### \* Dispersive Power ( $V$ ) :

- Dispersive power is the ratio of angular dispersion between two colours to the deviation of the mean ray (typically yellow) caused by the prism;  $V = \frac{\Delta\theta}{\delta_m}$ , where

$\Delta\theta$  is the angular dispersion and  $\delta_m$  is the deviation of the mean ray.

- For small prism angles ( $A$ ), dispersive power depends only on the material of the prism, not on its angle.

- Dispersive power ( $\nu$ ) is always zero or greater. It is zero in a vacuum, nearly zero in air, and greater than zero in other refractive materials.

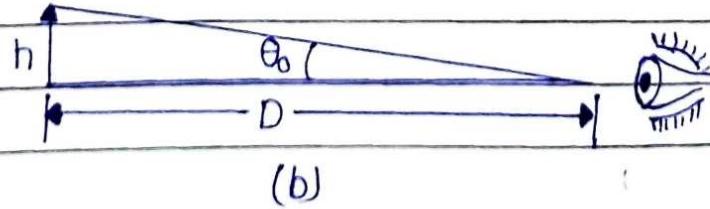
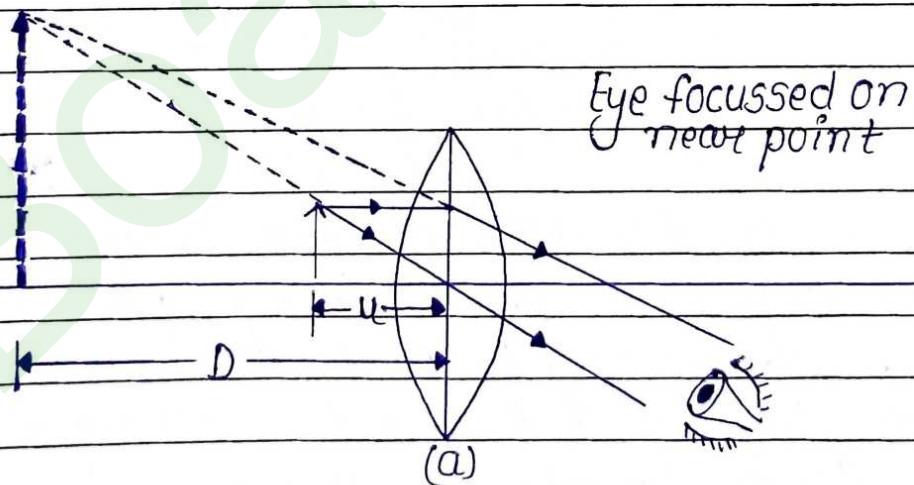
$$D.P = \frac{\text{Angular dispersion}}{\text{mean deviation}} = \frac{\delta_v - \delta_R}{\delta}$$

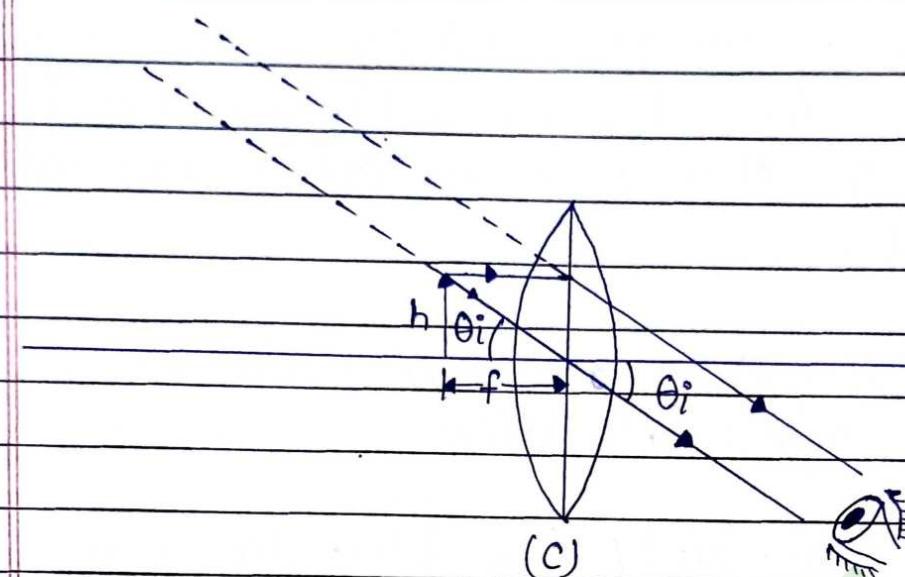
$$\omega = \frac{(\mu_v - \mu_R) A}{(\mu - 1) A} = \frac{\mu_v - \mu_R}{\mu - 1} = \omega$$

## \* Optical Instruments :-

### Simple Microscope -

It is used for observing magnified images of objects. It consists of a converging lens of small focal length.





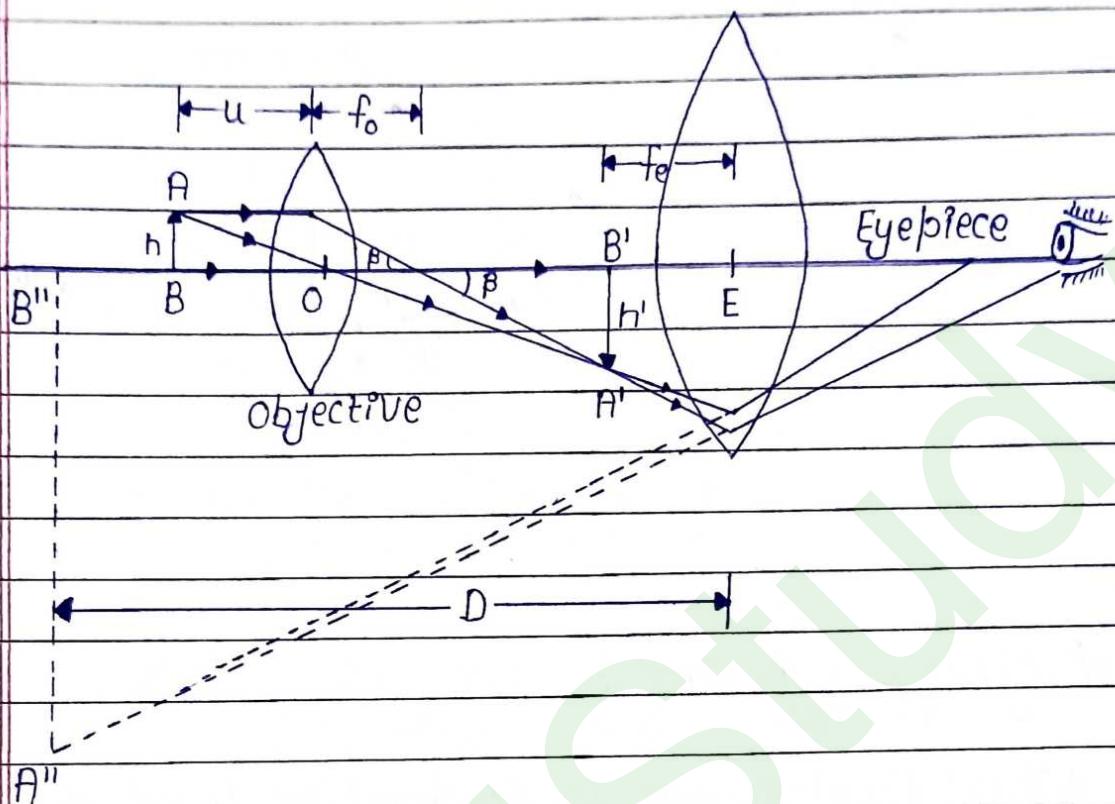
Eye focussed at infinity

### Magnifying Power :-

- When final image is formed at least distance of distinct vision ( $D$ ), then  $M = 1 + d/f$   
where,  $f$  = focal length of the lens.
- When final image is formed at infinity, then  $M = D/f$

### Compound Microscope :-

It is a combination of two convex lenses, called objective lens and eye piece separated by a distance. Both lenses are of small focal lengths but  $f_o < f_e$ , where  $f_o$  and  $f_e$  are focal lengths of objective lens and eye piece respectively.



Magnifying power :-

$$M = V_o / U_o \{ 1 + (D/f_o) \}$$

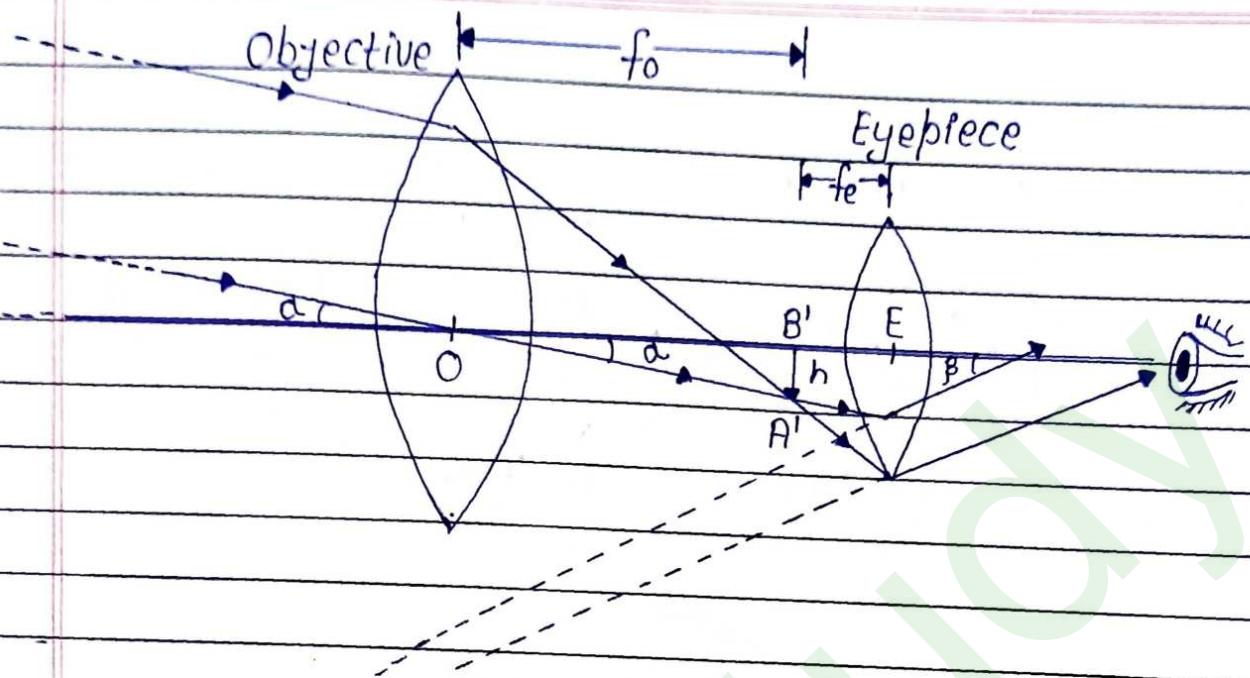
where  $V_o$  = distance of image, formed by objective lens and  $U_o$  = distance of object from the objective.

(ii) When final image is formed at infinity, then

$$M = V_o / U_o \cdot D / f_e$$

Astronomical Telescope :-

It is also a combination of two lenses, called objective lens and eye piece, separated by a distance. It is used for observing distinct images of heavenly bodies like stars, planets etc.



### Magnifying power :

- (i) When final image is formed at least distance of distinct vision ( $D$ ), then  $M = f_o/f_e \{1 + D/f_e\}$   
where  $f_o$  and  $f_e$  are focal lengths of objective and eyepiece respectively.

Length of the telescope ( $L$ ) =  $(f_o + u_e)$

where,  $u_e$  = distance of object from the eyepiece.

- (ii) When final image is formed at infinity,  
then  $M = f_o/f_e$ .

Length of the telescope ( $L$ ) =  $f_o + f_e$

For large magnifying power of a telescope  
 $f_o$  should be large and  $f_e$  should be small.

For large magnifying power of a microscope ;  $f_o < f_e$  should be small.