Shri Mata Vaishno Devi University

School Of Mathematics

B.Tech.- I Semester (Civil, Computer Science, Electrical, Electronics, Mechanical, Mathematics & Computing)

Mid Term 2024-2025

Entry No.: 24BEC055

Date:

Total No. Pages: [2]

Total No. Questions: [3]

Course Title: Engineering Mathematics I (MTL BS-101)

Time alloted : 1.5 Hours

Total marks: [20]

1. Do any two of the followings, each carry two marks: [Marks 4]

   (i) Define concavity and convexity. Give one example for each.

   (ii) Find the asymptotes parallel to the coordinate axes of the curve $\frac{a^2}{x^2}+\frac{b^2}{y^2}=1$

   (iii) Find the curvature of the curve $x = a\cos t, y = a\sin t$.

2. Do any two of the followings, each carry three marks: [Marks 6]

   (i) If $z = x\log(x+r) - r$ where $x^2+y^2=r^2$, then prove that $\frac{\partial^3 z}{\partial x^3} = -\frac{x}{r^3}$.

   (ii) Show that the curve $x = \log(\frac{y}{x})$ has a point of inflexion at $(-2, -2e^{-2})$.

   (iii) Find the value of Jacobian of u, v with respect to r, θ where $u = x^2-y^2$, $v = 2xy$ and $x = r\cos\theta$, $y = r\sin\theta$.

   (iv) Evalute $\int_0^{\frac{\pi}{4}} \log(1+\tan x)dx$.

3. Do any two of the followings: [Marks 10]

   (i) If $u = \tan^{-1}(\frac{y^2}{x})$, show that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = -\sin 2u \sin^2 u$.

   (ii) Examine the extreme values for the function $f(x,y) = x^4+y^4-2x^2+4xy-2y^2$.

   (iii) Find all the asymptotes for the curve $(x^2-y^2)(y^2-4x^2)-6x^3+5x^2y+3xy^2-2y^3-x^2+3xy-1=0$.

COURSE OUTCOMES

CO 1. Introduce the basic concept of differential calculus to understand the different subjects of engineering as well as basic sciences.

CO 2. Enable the students to develop the concept of partial differentiation to understand their applications in engineering.

CO 3. Understand the fundamentals of Integral calculus to understand their applications to length, area, volume, surface of revolution, moments and centre of gravity.

CO 4. Understand the improper integrals and Beta and Gamma functions and their applications.

CO 5. Understand the idea of Linear Algebra which are useful to all branches of engineering.

CO	Questions Mapping	Total Marks
CO 1	1(i,ii,iv), 2(i)	8
CO 2	1(iii,v), 2(ii,iii,iv)	17
CO 3		
CO 4		
CO 5		

350

Shri Mata Vaishno Devi University

School Of Mathematics

B.Tech.- I Semester (Computer Science, Mathematic and Computing, Electrical, Electronics, Mechanical, civil)

Major Exam: Odd Sem 2024-2025

Entry No.: 24BEC055

Date: 16.12.2024

Total No. Pages: [2]

Total No. Questions: [3]

Course Title: Engineering Mathematics I (MTLBS101)

Time alloted : 3 Hours

Total marks: [40]

Attempt all questions.

1. Do any four parts. Each carry equal marks: [Marks 08]

(i) Define Beta function. State two properties of Beta function.

(ii) Define rank of a matrix. Give one example.

(iii) Define eigen values and eigen vectors. Give one example.

(iv) Evaluate the integral $\int \sin^3 x dx$.

(v) Define gamma function. State two properties of gamma function.

(vi) Write the formula of curvature in cartesian and polar form.

2. Do any four parts. Each carry equal marks: [Marks 12]

(i) Find the eigen values of A^5, where $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$.

(ii) Using Cayley-Hamilton theorem, find the value of A^8, where $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

(iii) Examine the function $x^3 + y^3 = 3axy$ for extreme values.

(iv) Evaluate the integral $\int_0^{\pi/2} \sin^5 \theta \cos^4 \theta d\theta$.

(v) Prove that $\int_0^a x^2(a^2 - x^2)^{3/2} dx = \frac{\pi}{32} a^6$.

(vi) Find all the asymptotes of the curve $y^3 + x^2y + 2xy^2 - y + 1 = 0$.

3. Do any four parts. Each carry equal marks: [Marks 20]

(i) Trace the curve $3ay^2 = x^2(a - x)$.

(ii) Diagonalize the matrix $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$.

(iii) Find the length of the arc of the curve $ay^2 = x^3$ from vertex to (a, a).

(iv) Prove that $\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$.

(v) Find the rank of the matrix $A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$ by reducing it to normal form.

(vi) State Cayley-Hamilton theorem and verify for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$.

(vii) Solve the following system of equations $x+y+z=6,\ 5x+2y+z=12,\ 2x+y+3z=13$ by using Gauss elimination method.

OURSE OUTCOMES

1. Introduce the basic concept of differential calculus to understand the different subjects of engineering as well as basic sciences.

2. Enable the students to develop the concept of partial differentiation to understand their applications in engineering.

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4. Understand the improper integrals and Beta and Gamma functions and their applications.

5. Understand the idea of Linear Algebra which are useful to all branches of engineering.

CO	Questions Mapping	Total Marks
CO 1	1(vi). 2(vi), 3(i)	10
CO 2	2(iii)	3
CO 3	1(iv), 3(iii)	7
CO 4	1(i. v), 2(iv, v). 3(iv)	15
CO 5	1(ii. iii), 2(i. ii), 3 (ii, v vi, vii)	30

Shri Mata Vaishno Devi University

School Of Mathematics
B.Tech.- I Semester (Civil, Computer Science, Electrical, Electronics, Mechanical)
Minor Exam 1: Odd Sem 2023-2024

Entry No.:——
Date:——

Total No. Pages: [2]
Total No. Questions: [2]

Course Title: Engineering Mathematics I (MTL-1025)

Time alloted : 1 Hours — Total marks: [20]

Attempt all questions.

1. Choose the correct answer in followings: [Marks 5]

(i) Degree of the function $\dfrac{x^{\frac{1}{3}}+y^{\frac{1}{3}}}{x^{\frac{1}{4}}-y^{\frac{1}{4}}}$ is

(a) -1/12 (b) 1/12 (c) 1/6 (d) -1/6 [CO 1]

(ii) For all real value of x, the minimum value of $\dfrac{1-x+x^2}{1+x+x^2}$ is equal to [CO 1]

(a) 0 (b) 1 (c) 3 (d) 1/3

(iii) If $f(x,y,z)=x^2+xyz+z$, then the value of f_x at $(1,1,1)$ [CO 2]

(a) 0 (b) 1 (c) 3 (d) -1

(iv) If $f(x,y)=x^2+y^2$, $x=t^2+t^3$, $y=t^3+t^9$, then find $\dfrac{df}{dt}$ at $t=1$ [CO 1]

(a) 154 (b) 122 (c) 100 (d) 164

(v) Which among the followings is the Jacobian of u,v with respect to x,y [CO 2]

(a) $J\left(\dfrac{(x,y)}{(u,v)}\right)$ (b) $J\left(\dfrac{(u,v)}{(x,y)}\right)$ (c) $\dfrac{\partial(x,y)}{\partial(u,v)}$ (d) $\dfrac{\partial(u,x)}{\partial(v,y)}$.

2. Do any three of the followings: [Marks 15]

(i) Find all the asymptotes of the curve $x^3+3x^2y-4y^3-x+y+3=0$. [CO 1]

(ii) If $u=\sin^{-1}\left(\dfrac{x^{\frac{1}{3}}+y^{\frac{1}{3}}}{x^{\frac{1}{2}}-y^{\frac{1}{2}}}\right)^{\frac{1}{2}}$, then prove that $x^2\dfrac{\partial^2u}{\partial x^2}+2xy\dfrac{\partial^2u}{\partial x\partial y}+y^2\dfrac{\partial^2u}{\partial y^2}=$ $n(n-1)u$ $\dfrac{\tan u}{144}(\tan^2 u+13)$. [CO 2]

(iii) Determine the extremum values of the function $f(x,y)=x^3+3xy^2-15x^2-15y^2+72x$. [CO 2]

$\frac{dy}{dt}=3t^2+9t^8$

$2(t^2+t^3)(2t+3t^2)+2(t^3+t^9)(3t^2+9t^8)$

(iv) If $u = \dfrac{y-x}{1+xy}$, $v = \tan^{-1} y - \tan^{-1} x$, then find the value of Jacobian of u, v with respect to x, y. [CO 2]

$$\begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$$

COURSE OUTCOMES

CO 1. Introduce the basic concept of differential calculus to understand the diffe subjects of engineering as well as basic sciences.

CO 2. Enable the students to develop the concept of partial differentiation to unde their applications in engineering.

CO 3. Understand the fundamentals of Integral calculus to understand their app to length, area, volume, surface of revolution, moments and centre of g

CO 4. Understand the improper integrals and Beta and Gamma function

School Of Mathematics

B.Tech.- I Semester (Civil, Computer Science, Electrical, Electronics, Mechanical)

Major Exam: Odd Sem 2023-2024

Entry No.:——

Date:——

Total No. Pages: [2]

Total No. Questions: [3]

Course Title: Engineering Mathematics I (MTL-1025)

Time alloted : 3 Hours | Total marks: [50]

Attempt all questions.

1. Solve the followings: [Marks 10]

$\partial(u,v)$

(i) Find the asymptotes parallel to coordinate axes for the curve

$$a^2x^2 + b^2x^2 - x^2y^2 = 0.$$

(ii) If $u = x^2 - y^2, v = 2xy$, then find the value of $\dfrac{\partial(u,v)}{\partial(x,y)}$.

(iii) Prove that $\displaystyle\int_0^{\frac{\pi}{6}} \sin^7 3x\, dx = \frac{16}{105}$.

(iv) Evaluate the integral $\displaystyle\int_0^1 (1-x^2)^{-\frac{1}{2}} dx$.

(v) Explain the rank of a matrix.

2. Do the followings: [Marks 15]

(i) If $u = u(y-z, z-x, x-y)$, then prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$.

(ii) Find the interval of concave upward and concave downword for curve $y = 3x^5 - 40x^3 + 3x - 20$.

$-1, 1$

(iii) Prove the following by using the reduction formula

$\log x = t$

$$\int_0^{\infty} \frac{dx}{(1+x^2)^5} = \frac{7\cdot5\cdot3\cdot1}{8\cdot6\cdot4\cdot2}\frac{\pi}{2}.$$

(iv) Evaluate the integral $\int_0^1 (x \log x)^4 dx$.

(v) Prove that $\beta(p.q) = \beta(p+1, q) + \beta(p, q+1)$.

3. Do any five of the following: [Marks 25].

(i) If $u = \sec^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, then show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\cot u$ and find the value of $x^2\frac{\partial^2 u}{\partial x^2} + y^2\frac{\partial^2 u}{\partial y^2} + 2xy\frac{\partial^2 u}{\partial x \partial y}$.

(ii) Find the position and nature of the double points of the curve

$$(x+y)^3 - \sqrt{2}(y-x+2)^2 = 0.$$

(iii) In a plane triangle ABC, find the maximum value of $\cos A \cos B . \cos C$.

(iv) Prove that

$$\Gamma(m)\Gamma(m+\frac{1}{2}) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m).$$

Further, deduce that

$$\beta(m,m) = 2^{1-2m}\beta(m,\frac{1}{2}).$$

(V) Find the rank of the matrix $A = \begin{bmatrix} 1 & a & b & 0 \\ 0 & c & d & 1 \\ 1 & a & b & 0 \\ 0 & c & d & 1 \end{bmatrix}$ by reducing it to normal form.

(vi) Verify Cayley-Hamilton Theorem for the matrix $A = \begin{bmatrix} 7 & -1 & 3 \\ 6 & 1 & 4 \\ 2 & 4 & 8 \end{bmatrix}$, and hence find the inverse of A.

SHRI MATA VAISHNO DEVI UNIVERSITY, KATRA

School of Mathematics

B.Tech.- I Semester (Civil, Computer Science, Electrical, Electronics, Mechanical) Minor Exam II: Even Sem 2023-2024

Entry No: [ ] **Total Number of Pages:[01]**

Date: **Total Number of Questions:[04]**

Course Title:Engineering Mathematics I

Course Code: MTL-1025

Time Allowed: 1 Hours **Max Marks:[20]**

NOTE: All questions are complusory.

Q1. Choose the correct answer in followings 5 Marks

(i) The value of the $\int_0^{\frac{\pi}{2}} sin^5 x dx$ is

(a) 8/15 (b) 1 (c) 3 (d) 15/8

(ii) If the two tangents are imaginary, then the origin is a

(a) Node (b) Cusp (c) Conjugate (d) None

(iii) The value of $\int_0^{\frac{\pi}{2}} sin^4 x cos^6 x dx$ is given by

(a) $\frac{3\pi}{512}$ (b) $\frac{5\pi}{128}$ (c) 0 (d) $\frac{3\pi}{32}$

$\frac{3 \times 1 \times}{\ldots}$

(iv) Which of the following is true for the curve $y = f(x)$ is concave upward

(a) $\frac{d^2y}{dx^2} < 0$ (b) $\frac{d^2y}{dx^2} = 0$ (c) $\frac{d^2y}{dx^2} > 0$ (d) None

(v) Which of the following curves is concave upwards in every intervals.

(a) $y = e^x$ (b) $y = log(x)$ (c) Both (d) None

Q2. Find the position and nature of the double points of the curve $x^4 - 2ay^3 - 3a^2y^2 - 2a^2x^2 + a^4 = 0$. 5 Marks

Q3. Find the points of inflexion of the curve x = a $tan\theta$, y = a $sin\theta cos\theta$. 5 Marks

Q4. Find the Reduction formula for $\int sin^n x dx$. Hence evaluate $\int_0^1 x^4(1-x^2)^{3/2} dx$. 5 Marks

... of Mathematics

B.Tech. - I Semester, Minor II : Odd Sem (2023-2024)

Course Title : Engineering. Mathematics I (MTL 1025)

(1) Choose the correct answer in followings:

(i) The value $\int_0^{\pi/2} \sin^5 x \cos x \, dx$ is

(a) $1/12$ (b) $\pi/12$ (c) $1/6$ (d) $\pi/6$

(ii) For the double point (a,b) of the curve $f(x,y)=0$ we have $\left(\frac{\partial^2 f}{\partial x^2}\right)_{(a,b)} \left(\frac{\partial^2 f}{\partial y^2}\right)_{(a,b)} < \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2_{(a,b)}$, then point (a,b) is

(a) node (b) cusp (c) conjugate (d) None of these

(iii) The curve $y = x^4$ at the origin is

(a) concave upward (b) concave downward (c) point of inflexion (d) none of these

(iv) The point of inflexion for the curve ~~$x=(\log y)^3$ is~~ $y^2 = x(x+1)^2$ is

(a) $x = 1/2$ (b) $x = 1/3$ (c) $x = 1$ (d) $x=-1$.

(v) The interval where $y=(x-b)^3$ is concave downward is

(a) (b,∞) (b) (a,∞) (c) $(-\infty,b)$ (d) $(-\infty,a)$

Q.(2) Find the point of inflexion on the curve $y = \frac{x(x^2-1)}{3x^2+1}$

Q.(3) Prove that the curve $y^2=(x-a)^2(x-b)$ has at $x=a$, a node if $a>b$, a cusp if $a=b$ and a conjugate point if $a<b$.

Q.(4) State and prove Wallis' formula and find the integr $\int_0^5 \frac{x^6 \, dx}{\sqrt{25-x^2}}$.