exercicio-07-giuliavieira

December 14, 2023

UNIVERSIDADE FEDERAL DE MINAS GERAIS INSTUTUTO DE CIÊNCIAS EXATAS GRADUAÇÃO EM CIÊNCIA DA COMPUTAÇÃO DISCIPLINA: Introdução a Física Estatística e Computacional

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EXERCÍCIO AVALIATIVO 07: Caminhadas aleatórias

[1]:
```
import numpy as np
from matplotlib import pyplot as plt
from scipy.stats import norm
```

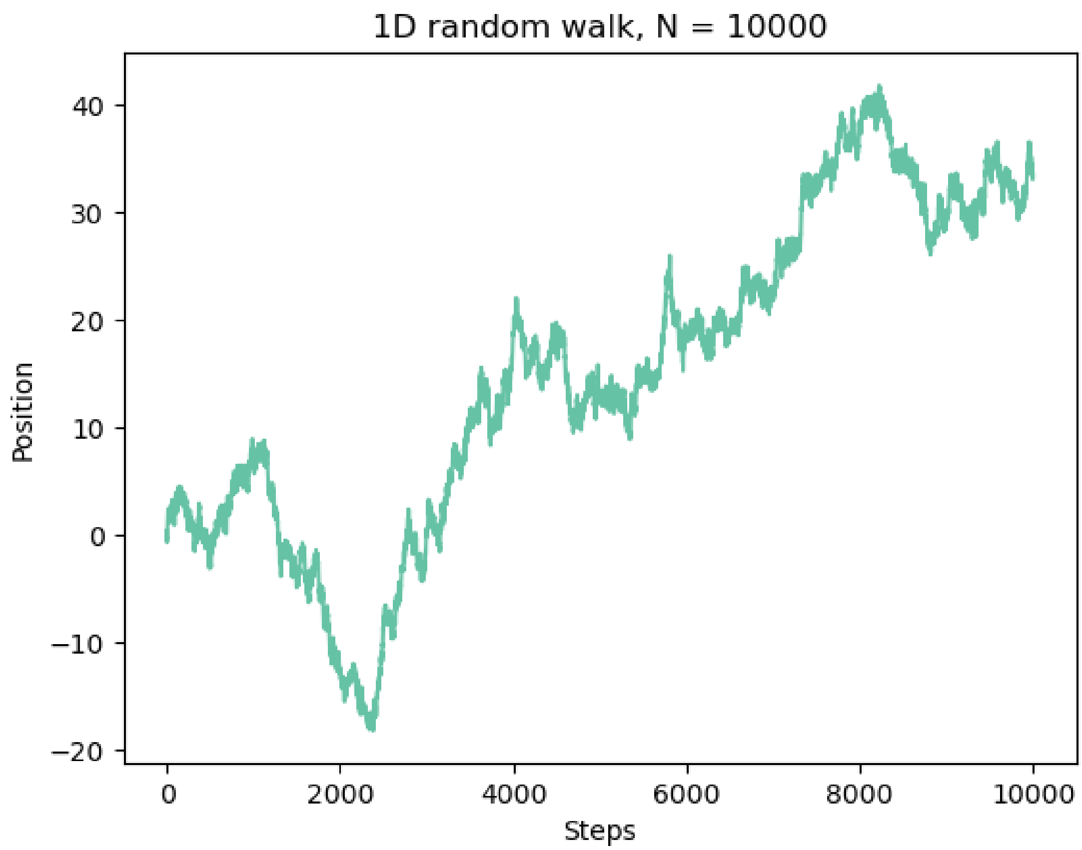
[2]:
```
cm = plt.get_cmap('Set2')
```

[3]:
```
def random_walk(N, d):
    steps = np.random.uniform(-0.5, 0.5, size=(N, d))
    walk = np.cumsum(steps, axis=0)
    return walk
```

[4]:
```
def get_rms_step_size(walk):
    step_sizes = np.diff(walk)
    return np.sqrt(np.mean(step_sizes**2))
```

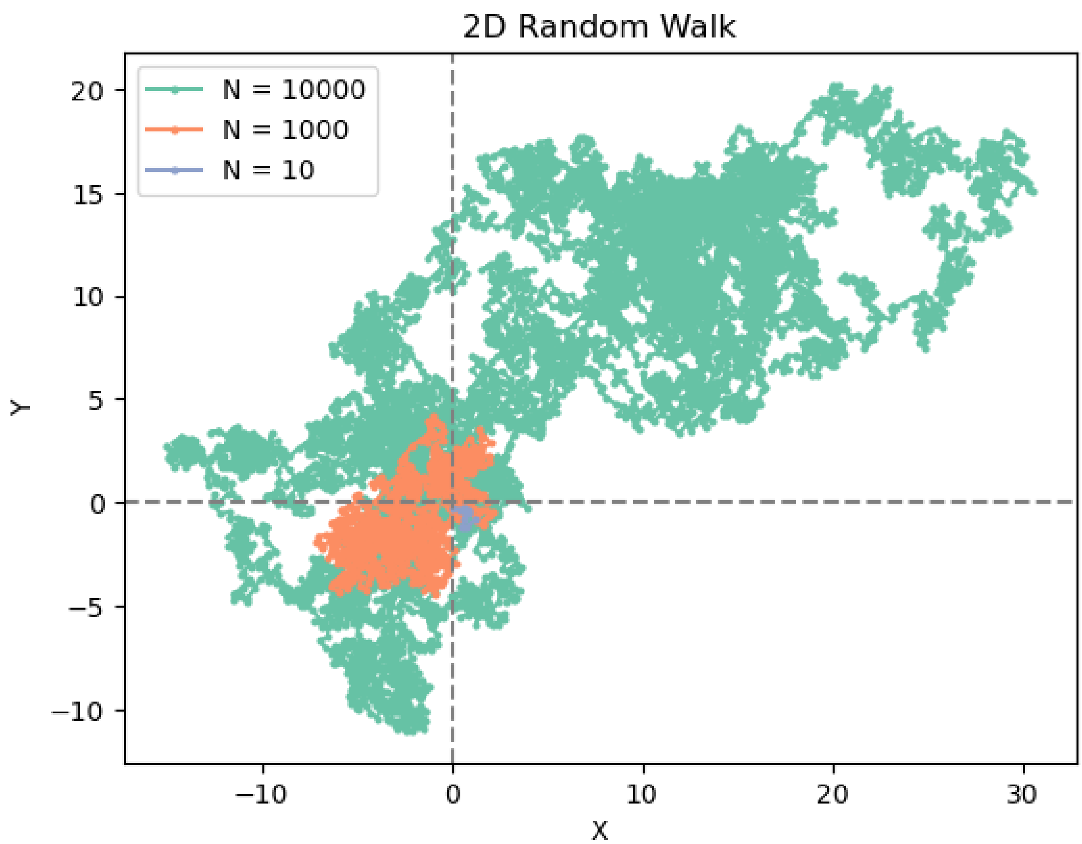
2.5.A.1. 1D random walk

[5]:
```
N = 10000
d = 1

walk_1d = random_walk(N, d)
t = np.arange(len(walk_1d))

plt.plot(t, walk_1d, color=cm(0))
plt.title(f'1D random walk, N = {N}')
plt.xlabel('Steps')
plt.ylabel('Position')
plt.show()
```



2.5.A.2. 2D random walks

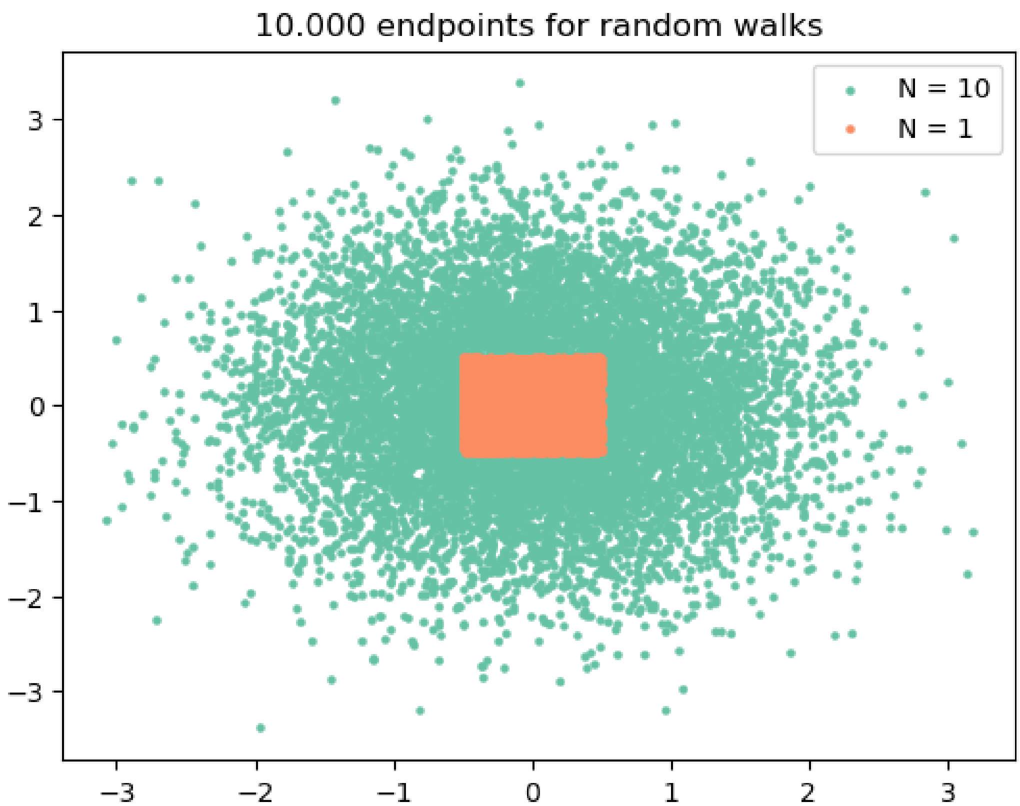
[6]:
```
N_values = [10000, 1000, 10]
d = 2

for i, N in enumerate(N_values):
    walk_2d = random_walk(N, d)
    t = np.arange(len(walk_2d))

    plt.plot(walk_2d[:, 0], walk_2d[:, 1], marker='o', linestyle='-',␣
  ↪markersize=2, color=cm(i), label=f'N = {N}')
    plt.xlabel('X')
    plt.ylabel('Y')
    plt.title(f'2D Random Walk')
    plt.legend(loc='best')

plt.axhline(y=0, linestyle="dashed", color='gray')
plt.axvline(x=0, linestyle="dashed", color='gray')

plt.show()
```



2.5.A.3. Distances changes based on number of steps A distância líquida percorrida em uma caminhada aleatória geralmente é escalada pela raiz quadrada do número de passos. Sendo assim, se multiplicarmos o número de passos por 100 vemos a distância líquida aumentar aproximadamente 10x. **2.5.B.**

[7]:
```
W = 10000
walks_1 = [random_walk(1, d) for _ in range(W)]
endpoints_1 = np.array([walk[-1] for walk in walks_1])

walks_10 = [random_walk(10, d) for _ in range(W)]
endpoints_10 = np.array([walk[-1] for walk in walks_10])

plt.scatter(endpoints_10[:, 0], endpoints_10[:, 1], s=5, color=cm(0), label=f'N␣
↪= 10')
plt.scatter(endpoints_1[:, 0], endpoints_1[:, 1], s=5, color=cm(1), label=f'N =␣
↪1')

plt.title("10.000 endpoints for random walks")
plt.legend()
```

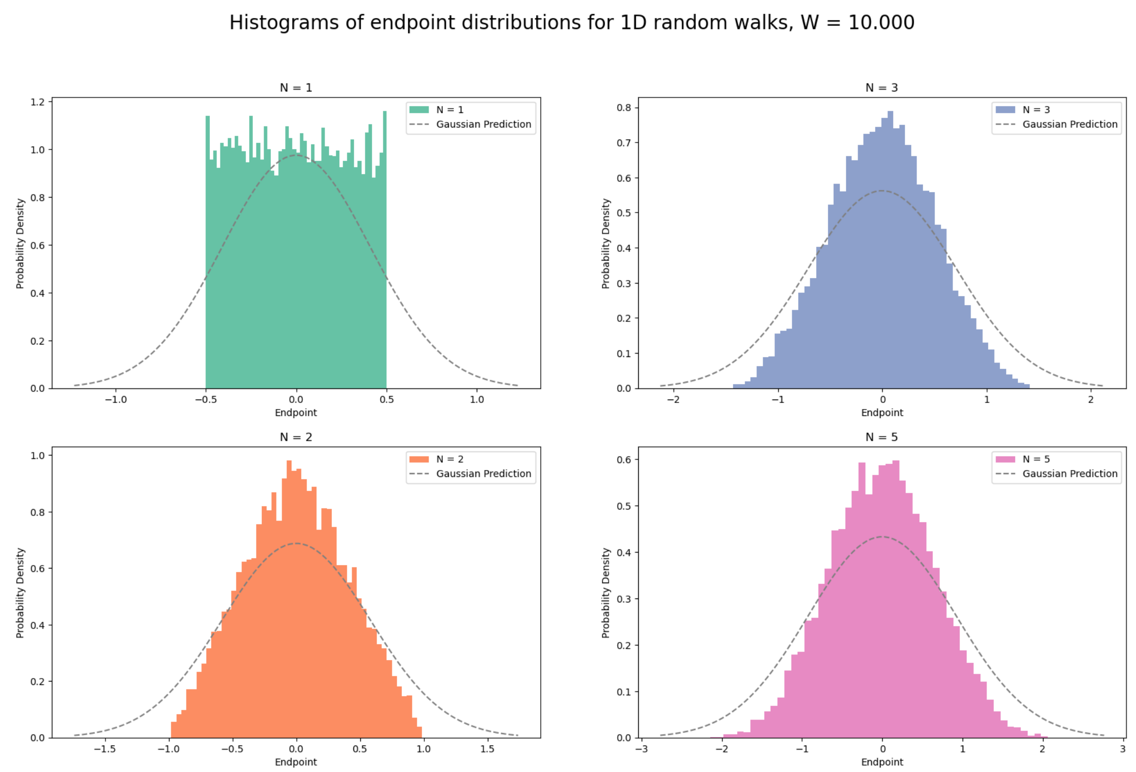
```
plt.show()
```



2.5.C

[21]:
```
# Parameters
W = 10000
d = 1
N_vec = [1, 2, 3, 5]  # Different values of N

fig, axes = plt.subplots(2, 2, figsize=(20, 12))
fig.suptitle("Histograms of endpoint distributions for 1D random walks, W = 10.
↪000", fontsize=20)

col = 0
# Generate and plot histograms for different values of N
for i, N in enumerate(N_vec):
    walks = [random_walk(N, d) for _ in range(W)]
    endpoints = np.array([walk[-1] for walk in walks])

    row = i%2
    if i > (len(N_vec)/2)-1:
```

```
        col = 1

    axes[row][col].hist(endpoints, bins=50, density=True, color=cm(i), label=f'N = {N}')

    rms_step_size = get_rms_step_size(endpoints.flatten())

    # Plot the predicted Gaussian distribution
    x = np.linspace(-3 * rms_step_size, 3 * rms_step_size, 100)
    y = 1 / (np.sqrt(2 * np.pi) * rms_step_size) * np.exp(-0.5 * (x / rms_step_size)**2)
    axes[row][col].plot(x, y, '--', color='gray', label='Gaussian Prediction')

    axes[row][col].set_title(f'N = {N}')
    axes[row][col].set_xlabel('Endpoint')
    axes[row][col].set_ylabel('Probability Density')
    axes[row][col].legend()

plt.show()
```



Podemos perceber que a partir de N = 2 a distribuição Gaussiana já começa a representar a distribuição de alguma forma.

Isso se deve ao Teorema do Limite Central, que diz que a distribuição da soma(ou médias) de um grande número de variáveis aleatórias idênticamente distribuídas e independentes se aproxima da distribuição normal, independentemente do formato da distribuição inicial.

Em caminhadas aleatórias, os passos podem ser definidos como este tipo de variável. O acúmulo de passos para encontrar o endpoint é a soma dos passos, portanto, a distribuição se aproxima da distribuição Gaussiana

The reason why a Gaussian distribution starts to emerge in the distribution of endpoints when a random walk has more than one step is related to the Central Limit Theorem (CLT). The Central Limit Theorem states that the distribution of the sum (or average) of a large number of independent, identically distributed random variables approaches a normal (Gaussian) distribution, regardless of the shape of the original distribution.

In the context of a random walk, each step is usually considered to be an independent and identically distributed random variable. When you perform multiple steps and accumulate them to get the endpoint, you are effectively summing these random variables. According to the Central Limit Theorem, as the number of steps increases, the distribution of the endpoints tends to become Gaussian.

The key conditions for the CLT to apply are that the random variables must be independent and have a finite mean and variance. In the case of a random walk with steps that satisfy these conditions, the emergence of a Gaussian distribution for the endpoints is expected.

To illustrate, consider a simple random walk where each step is sampled from a uniform distribution. As you accumulate more steps, the distribution of the endpoints converges towards a Gaussian distribution. This is a manifestation of the Central Limit Theorem in action.

[ ]:

```

```