1

a

1

$$\begin{aligned}
(1+i)^{-1}(1+i) &= 1 \\
(a+bi)(1+i) &= 1 \\
a+ai+bi-b &= 1 \\
(a-b)+(a+b)i &= 1 \\
|(a-b)+(a+b)i| &= |1| \\
(a-b)^2+(a+b)^2 &= 1 \\
a^2-2ab+b^2+a^2+2ab+b^2 &= 1 \\
2a^2+2b^2 &= 1 \\
a^2+b^2 &= \frac{1}{2} \\
\Rightarrow a &= \frac{1}{2}, b = -\frac{1}{2}
\end{aligned}$$

2

Zunächst $\frac{1}{i}$

$$\begin{aligned}
\frac{1}{i} \cdot i &= 1 \\
(a+bi)i &= 1 \\
ai-b &= 1 \\
b &= -1 \\
\frac{1}{i} &= -i
\end{aligned}$$

$$\begin{aligned}
\frac{1}{i} + 3 \cdot (1+i)^{-1} &= -i + 3 \cdot (0.5-0.5i) \\
&= 1.5-2.5i
\end{aligned}$$

3

$$
\begin{aligned}
\frac{\sqrt{2}}{\sqrt{2}-i} &= a+bi \\
\sqrt{2} &= (\sqrt{2}-i)(a+bi) \\
&= \sqrt{2}a+\sqrt{2}bi-ai+b \\
&= \sqrt{2}a+b+(\sqrt{2}b-a)i \\
1 &= a+\frac{b}{\sqrt{2}}+(b-\frac{a}{\sqrt{2}})i \\
&= (a+\frac{b}{\sqrt{2}})^2+(b-\frac{a}{\sqrt{2}})^2 \\
&= a^2+\frac{b^2}{2}+\frac{2ab}{\sqrt{2}}+\frac{a^2}{2}+b^2-\frac{2ab}{\sqrt{2}} \\
&= \frac{3}{2}a^2+\frac{3}{2}b^2 \\
\frac{2}{3} &= a^2+b^2 \\
&\Rightarrow a=\frac{2}{3}, b=\frac{\sqrt{2}}{3}
\end{aligned}
$$

4

$$
\begin{aligned}
\frac{1+i}{1-i} &= a+bi \\
1+i &= (a+bi)(1-i) \\
&= a-ai+bi+b \\
&= (a+b)+(-a+b)i \\
&\Rightarrow a+b=1, -a+b=1 \\
&\Rightarrow a=0, b=1
\end{aligned}
$$

$$
\begin{aligned}
(i)^{201} &= (i^4)^{50}i \\
&= i^{50}i \\
&= (i^4)^{10}i^{10}i \\
&= i^{20}i \\
&= (i^4)^5i \\
&= i^4i^2 \\
&= -i
\end{aligned}
$$

b

$$
\begin{aligned}
|z+1| &= |z-(1+2i)| \\
|(a+bi)+1| &= |(a+bi)-(1+2i)| \\
\sqrt{(a+1)^2+b^2} &= \sqrt{(a-1)^2+(b-2)^2} \\
(a+1)^2+b^2 &= (a-1)^2+(b-2)^2 \\
a^2+2a+1+b^2 &= a^2-2a+1+b^2-4b+4 \\
4a &= -4b+4 \\
a+b &= 1
\end{aligned}
$$

c

$$
\begin{aligned}
z^2 &= i \\
\Rightarrow \theta &= \frac{\pi}{2} \\
\Rightarrow \theta' &= \frac{\pi}{4}
\end{aligned}
$$

Da $|i| = 1$ ist $z = \cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4})$

$$
\begin{aligned}
\theta &= \pi - \arctan(\frac{\sqrt{3}}{1}) \\
&= \pi - \frac{\pi}{3} = \frac{2}{3}\pi \\
\Rightarrow \theta' &= \frac{2}{3}\pi \cdot \frac{1}{4} \\
&= \frac{\pi}{6}
\end{aligned}
$$

$\sqrt[4]{|-1+\sqrt{3}|} = \sqrt[4]{2} \Rightarrow z = \sqrt[4]{2}\cos(\frac{\pi}{6}) + i\sin(\frac{\pi}{6})$

2

a

$$
\begin{aligned}
|z+w|^2+|z-w|^2 &= |a+bi+c+di|^2+|a+bi-(c+di)|^2 \\
&= |(a+c)+(b+d)i|^2+|(a-c)+(b-d)i|^2 \\
&= (a+c)^2+(b+d)^2+(a-c)^2+(b-d)^2 \\
&= a^2+2ac+c^2+b^2+2bd+d^2+a^2-2ac+c^2+b^2-2bd+d^2 \\
&= 2a^2+2c^2+2b^2+2d^2 \\
&= 2(a^2+b^2)+2(c^2+d^2) \\
&= 2\sqrt{a^2+b^2}^2+2\sqrt{c^2+d^2}^2 \\
&= 2|z|^2+2|w|^2
\end{aligned}
$$

3

$$
A=\begin{bmatrix}0&1&3&0\\8&5&0&6\\0&0&-1&-2\end{bmatrix}\ B=\begin{bmatrix}0&1\\0&0\\3&3\\9&5\end{bmatrix}\ C=\begin{bmatrix}-1&1&-3\\8&5&-4\\0&0&6\end{bmatrix}\ D=\begin{bmatrix}1&-3\\5&0\end{bmatrix}
$$

$$
E=\begin{bmatrix}1\\5\\3\end{bmatrix}\ F=\begin{bmatrix}-2&3&0\end{bmatrix}
$$

$$
A\cdot B=\begin{bmatrix}9&9\\54&38\\-21&-13\end{bmatrix}
$$

$$
B\cdot D=\begin{bmatrix}5&0\\0&0\\18&-9\\34&-27\end{bmatrix}
$$

$$
C\cdot A=\begin{bmatrix}8&8&0&12\\40&33&28&38\\0&0&-6&-12\end{bmatrix}\ C\cdot C=\begin{bmatrix}9&4&-19\\32&33&-68\\0&0&36\end{bmatrix}\ C\cdot E=\begin{bmatrix}-5\\21\\18\end{bmatrix}
$$

$$
D\cdot D=\begin{bmatrix}-14&-3\\5&-15\end{bmatrix}
$$

$$
E\cdot F=\begin{bmatrix}2&3&0\\10&15&0\\6&9&0\end{bmatrix}
$$

$$
F\cdot A=\begin{bmatrix}24&17&6&18\end{bmatrix}\ F\cdot C=\begin{bmatrix}22&17&-18\end{bmatrix}\ F\cdot E=\begin{bmatrix}17\end{bmatrix}
$$

b

$$A^1 = \begin{bmatrix} i & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -i \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 1+i & 1 \\ 0 & 1 & 1-i \\ 0 & 0 & -1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} -i & i & 1 \\ 0 & 1 & -i \\ 0 & 0 & i \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} i & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -i \end{bmatrix} = A \Rightarrow A^n = A^n - 4$$

4

a

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & 5 \\ 4 & 4 & 1 \end{bmatrix} \; B = \begin{bmatrix} 3 & 3 & 3 \end{bmatrix}$$

$$A \cdot A = \begin{bmatrix} 4 & 5 & 5 \\ 2 & 5 & 2 \\ 0 & 2 & 3 \end{bmatrix} \; B \cdot A = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

b

$$A = \begin{bmatrix} 4 & 0 & 0 & 4 & 5 \\ 5 & 5 & 0 & 2 & 2 \end{bmatrix} \; B = \begin{bmatrix} 5 & 2 \\ 3 & 0 \\ 3 & 4 \\ 4 & 0 \\ 1 & 0 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 5 & 2 \\ 2 & 4 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 0 & 4 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 3 \\ 2 & 2 & 0 & 2 & 5 \\ 4 & 0 & 0 & 4 & 2 \\ 4 & 0 & 0 & 4 & 5 \end{bmatrix}$$