Mathe 05

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a

n	$g(n)$
0	1
1	-4
2	$g(1+1) = -3g(1) + 4g(1-1) = 12 + 4 = 16$
3	$g(2+1) = -3g(2) + 4g(2-1) = -48 - 16 = -64$
4	$g(3+1) = -3g(3) + 4g(3-1) = 192 + 64 = 256$
5	$g(4+1) = -3g(4) + 4g(4-1) = -768 - 256 = -1024$
6	$g(6+1) = -3g(6) + 4g(6-1) = 3072 + 1024 = 4096$

b

$g(1) = -4 = (-4)^1$
Es gilt also $g(n) = (-4)^n$ und $g(n) = -4g(n-1)$ für ein $n \in \mathbb{N}$

$$\begin{aligned}
g(n+1) &= -3g(n) + 4g(n-1) \\
&= -3[g(n-1) + 4g(n-2)] + 4g(n-1) \\
&= 9g(n-1) - 12g(n-2) + 4g(n-1) \\
&= 13g(n-1) - 12g(n-2) \\
&= 12g(n-1) - 16g(n-2) \\
&= -4[-3g(n-1) + 4g(n-2)] \\
&= -4 \cdot (-4)^n = (-4)^{n+1}
\end{aligned}$$

Somit gilt $g(n) = (-4)^n$ für alle $n \in \mathbb{N}$

c

Sei $\phi = \frac{1+\sqrt{5}}{2}, \varphi = \frac{1-\sqrt{5}}{2}$. Da $f_0 = 0 = \frac{1}{\sqrt{5}}[\phi^0 - \varphi^0]$ gilt $f_n = \frac{1}{\sqrt{5}}[\phi^n - \varphi^n]$ für ein $n \in \mathbb{N}_0$

$$
\begin{aligned}
f_{n+1} &= f_n + f_{n-1} \\
&= \frac{1}{\sqrt{5}}(\phi^n - \varphi^n) + \frac{1}{\sqrt{5}}(\phi^{n-1} - \varphi^{n-1}) \\
&= \frac{1}{\sqrt{5}}[\phi^n - \varphi^n + \phi^{n-1} - \varphi^{n-1}] \\
&= \frac{1}{\sqrt{5}}[\phi^n + \phi^{n-1} - \varphi^n - \varphi^{n-1}] \\
&= \frac{1}{\sqrt{5}}[(\phi + 1)\phi^{n-1} - (\varphi + 1)\varphi^{n-1}] \\
&= \frac{1}{\sqrt{5}}[(\frac{1+\sqrt{5}}{2} + 1)\phi^{n-1} - (\frac{1-\sqrt{5}}{2} + 1)\varphi^{n-1}] \\
&= \frac{1}{\sqrt{5}}[(\frac{3+\sqrt{5}}{2})\phi^{n-1} - (\frac{3-\sqrt{5}}{2})\varphi^{n-1}] \\
&= \frac{1}{\sqrt{5}}[((\frac{1+\sqrt{5}}{2})^2)\phi^{n-1} - ((\frac{1-\sqrt{5}}{2})^2)\varphi^{n-1}] \\
&= \frac{1}{\sqrt{5}}[(\phi^2)\phi^{n-1} - (\varphi^2)\varphi^{n-1}] \\
&= \frac{1}{\sqrt{5}}[\phi^{n+1} - \varphi^{n+1}]
\end{aligned}
$$

Somit gilt $f_n = \frac{1}{\sqrt{5}}[\phi^n - \varphi^n]$ für alle $n \in \mathbb{N}_0 \blacksquare$