

1

a

1

$$\begin{aligned}(1+i)^{-1}(1+i) &= 1 \\(a+bi)(1+i) &= 1 \\a+ai+bi-b &= 1 \\(a-b) + (a+b)i &= 1 \\|(a-b) + (a+b)i| &= |1| \\(a-b)^2 + (a+b)^2 &= 1 \\a^2 - 2ab + b^2 + a^2 + 2ab + b^2 &= 1 \\2a^2 + 2b^2 &= 1 \\a^2 + b^2 &= \frac{1}{2} \\\Rightarrow a = \frac{1}{2}, b = -\frac{1}{2}\end{aligned}$$

2

Zunächst $\frac{1}{i}$

$$\begin{aligned}\frac{1}{i} \cdot i &= 1 \\(a+bi)i &= 1 \\ai - b &= 1 \\b &= -1 \\\frac{1}{i} &= -i\end{aligned}$$

$$\begin{aligned}\frac{1}{i} + 3 \cdot (1+i)^{-1} &= -i + 3 \cdot (0.5 - 0.5i) \\&= 1.5 - 2.5i\end{aligned}$$

3

$$\begin{aligned}
 \frac{\sqrt{2}}{\sqrt{2}-i} &= a+bi \\
 \sqrt{2} &= (\sqrt{2}-i)(a+bi) \\
 &= \sqrt{2}a + \sqrt{2}bi - ai + b \\
 &= \sqrt{2}a + b + (\sqrt{2}b - a)i \\
 1 &= a + \frac{b}{\sqrt{2}} + (b - \frac{a}{\sqrt{2}})i \\
 &= (a + \frac{b}{\sqrt{2}})^2 + (b - \frac{a}{\sqrt{2}})^2 \\
 &= a^2 + \frac{b^2}{2} + \frac{2ab}{\sqrt{2}} + \frac{a^2}{2} + b^2 - \frac{2ab}{\sqrt{2}} \\
 &= \frac{3}{2}a^2 + \frac{3}{2}b^2 \\
 \frac{2}{3} &= a^2 + b^2 \\
 \Rightarrow a &= \frac{2}{3}, b = \frac{\sqrt{2}}{3}
 \end{aligned}$$

4

$$\begin{aligned}
 \frac{1+i}{1-i} &= a+bi \\
 1+i &= (a+bi)(1-i) \\
 &= a - ai + bi + b \\
 &= (a+b) + (-a+b)i \\
 \Rightarrow a+b &= 1, -a+b = 1 \\
 \Rightarrow a &= 0, b = 1
 \end{aligned}$$

$$\begin{aligned}
 (i)^{201} &= (i^4)^{50}i \\
 &= i^{50}i \\
 &= (i^4)^{10}i^{10}i \\
 &= i^{20}i \\
 &= (i^4)^5i \\
 &= i^4i^2 \\
 &= -i
 \end{aligned}$$

b

$$\begin{aligned}|z+1| &= |z-(1+2i)| \\ |(a+bi)+1| &= |(a+bi)-(1+2i)| \\ \sqrt{(a+1)^2+b^2} &= \sqrt{(a-1)^2+(b-2)^2} \\ (a+1)^2+b^2 &= (a-1)^2+(b-2)^2 \\ a^2+2a+1+b^2 &= a^2-2a+1+b^2-4b+4 \\ 4a &= -4b+4 \\ a+b &= 1\end{aligned}$$

c

$$\begin{aligned}z^2 &= i \\ \Rightarrow \theta &= \frac{\pi}{2} \\ \Rightarrow \theta' &= \frac{\pi}{4}\end{aligned}$$

Da $|i| = 1$ ist $z = \cos(\frac{\pi}{4}) + i \sin(\frac{\pi}{4})$

$$\begin{aligned}\theta &= \pi - \arctan\left(\frac{\sqrt{3}}{1}\right) \\ &= \pi - \frac{\pi}{3} = \frac{2}{3}\pi \\ \Rightarrow \theta' &= \frac{2}{3}\pi \cdot \frac{1}{4} \\ &= \frac{\pi}{6}\end{aligned}$$

$$\sqrt[4]{|-1+\sqrt{3}|} = \sqrt[4]{2} \Rightarrow z = \sqrt[4]{2} \cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right)$$

2

a

$$\begin{aligned} |z+w|^2 + |z-w|^2 &= |a+bi+c+di|^2 + |a+bi-(c+di)|^2 \\ &= |(a+c) + (b+d)i|^2 + |(a-c) + (b-d)i|^2 \\ &= (a+c)^2 + (b+d)^2 + (a-c)^2 + (b-d)^2 \\ &= a^2 + 2ac + c^2 + b^2 + 2bd + d^2 + a^2 - 2ac + c^2 + b^2 - 2bd + d^2 \\ &= 2a^2 + 2c^2 + 2b^2 + 2d^2 \\ &= 2(a^2 + b^2) + 2(c^2 + d^2) \\ &= 2\sqrt{a^2 + b^2}^2 + 2\sqrt{c^2 + d^2}^2 \\ &= 2|z|^2 + 2|w|^2 \end{aligned}$$

3

$$A = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 8 & 5 & 0 & 6 \\ 0 & 0 & -1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 3 & 3 \\ 9 & 5 \end{bmatrix} \quad C = \begin{bmatrix} -1 & 1 & -3 \\ 8 & 5 & -4 \\ 0 & 0 & 6 \end{bmatrix} \quad D = \begin{bmatrix} 1 & -3 \\ 5 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix} \quad F = \begin{bmatrix} -2 & 3 & 0 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 9 & 9 \\ 54 & 38 \\ -21 & -13 \end{bmatrix}$$

$$B \cdot D = \begin{bmatrix} 5 & 0 \\ 0 & 0 \\ 18 & -9 \\ 34 & -27 \end{bmatrix}$$

$$C \cdot A = \begin{bmatrix} 8 & 8 & 0 & 12 \\ 40 & 33 & 28 & 38 \\ 0 & 0 & -6 & -12 \end{bmatrix} \quad C \cdot C = \begin{bmatrix} 9 & 4 & -19 \\ 32 & 33 & -68 \\ 0 & 0 & 36 \end{bmatrix} \quad C \cdot E = \begin{bmatrix} -5 \\ 21 \\ 18 \end{bmatrix}$$

$$D \cdot D = \begin{bmatrix} -14 & -3 \\ 5 & -15 \end{bmatrix}$$

$$E \cdot F = \begin{bmatrix} 2 & 3 & 0 \\ 10 & 15 & 0 \\ 6 & 9 & 0 \end{bmatrix}$$

$$F \cdot A = \begin{bmatrix} 24 & 17 & 6 & 18 \end{bmatrix} \quad F \cdot C = \begin{bmatrix} 22 & 17 & -18 \end{bmatrix} \quad F \cdot E = \begin{bmatrix} 17 \end{bmatrix}$$

b

$$\begin{aligned}A^1 &= \begin{bmatrix} i & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -i \end{bmatrix} \\A^2 &= \begin{bmatrix} -1 & 1+i & 1 \\ 0 & 1 & 1-i \\ 0 & 0 & -1 \end{bmatrix} \\A^3 &= \begin{bmatrix} -i & i & 1 \\ 0 & 1 & -i \\ 0 & 0 & i \end{bmatrix} \\A^4 &= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\A^5 &= \begin{bmatrix} i & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -i \end{bmatrix} = A \Rightarrow A^n = A^{n-4}\end{aligned}$$

4

a

$$\begin{aligned}A &= \begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & 5 \\ 4 & 4 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 3 & 3 \end{bmatrix} \\A \cdot A &= \begin{bmatrix} 4 & 5 & 5 \\ 2 & 5 & 2 \\ 0 & 2 & 3 \end{bmatrix} \quad B \cdot A = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}\end{aligned}$$

b

$$\begin{aligned}A &= \begin{bmatrix} 4 & 0 & 0 & 4 & 5 \\ 5 & 5 & 0 & 2 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 2 \\ 3 & 0 \\ 3 & 4 \\ 4 & 0 \\ 1 & 0 \end{bmatrix} \\A \cdot B &= \begin{bmatrix} 5 & 2 \\ 2 & 4 \end{bmatrix} \\B \cdot A &= \begin{bmatrix} 0 & 4 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 3 \\ 2 & 2 & 0 & 2 & 5 \\ 4 & 0 & 0 & 4 & 2 \\ 4 & 0 & 0 & 4 & 5 \end{bmatrix}\end{aligned}$$