1

a

1

$$
\begin{aligned}
(1+i)^{-1}(1+i) &= 1 \\
(a+bi)(1+i) &= 1 \\
a+ai+bi-b &= 1 \\
(a-b)+(a+b)i &= 1 \\
|(a-b)+(a+b)i| &= |1| \\
(a-b)^2+(a+b)^2 &= 1 \\
a^2-2ab+b^2+a^2+2ab+b^2 &= 1 \\
2a^2+2b^2 &= 1 \\
a^2+b^2 &= \frac{1}{2} \\
\Rightarrow a = \frac{1}{2}, b &= -\frac{1}{2}
\end{aligned}
$$

2

Zunächst $\frac{1}{i}$

$$
\begin{aligned}
\frac{1}{i} \cdot i &= 1 \\
(a+bi)i &= 1 \\
ai-b &= 1 \\
b &= -1 \\
\frac{1}{i} &= -i
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{i} + 3 \cdot (1+i)^{-1} &= -i + 3 \cdot (0.5 - 0.5i) \\
&= 1.5 - 2.5i
\end{aligned}
$$

3

$$\begin{aligned}
\frac{\sqrt{2}}{\sqrt{2}-i} &= a+bi \\
\sqrt{2} &= (\sqrt{2}-i)(a+bi) \\
&= \sqrt{2}a+\sqrt{2}bi-ai+b \\
&= \sqrt{2}a+b+(\sqrt{2}b-a)i \\
1 &= a+\frac{b}{\sqrt{2}}+(b-\frac{a}{\sqrt{2}})i \\
&= (a+\frac{b}{\sqrt{2}})^2+(b-\frac{a}{\sqrt{2}})^2 \\
&= a^2+\frac{b^2}{2}+\frac{2ab}{\sqrt{2}}+\frac{a^2}{2}+b^2-\frac{2ab}{\sqrt{2}} \\
&= \frac{3}{2}a^2+\frac{3}{2}b^2 \\
\frac{2}{3} &= a^2+b^2 \\
&\Rightarrow a=\frac{2}{3}, b=\frac{\sqrt{2}}{3}
\end{aligned}$$

4

$$\begin{aligned}
\frac{1+i}{1-i} &= a+bi \\
1+i &= (a+bi)(1-i) \\
&= a-ai+bi+b \\
&= (a+b)+(-a+b)i \\
&\Rightarrow a+b=1, -a+b=1 \\
&\Rightarrow a=0, b=1
\end{aligned}$$

$$\begin{aligned}
(i)^{201} &= (i^4)^{50}i \\
&= i^{50}i \\
&= (i^4)^{10}i^{10}i \\
&= i^{20}i \\
&= (i^4)^5 i \\
&= i^4 i^2 \\
&= -i
\end{aligned}$$

b

$$
\begin{aligned}
|z+1| &= |z-(1+2i)| \\
|(a+bi)+1| &= |(a+bi)-(1+2i)| \\
\sqrt{(a+1)^2+b^2} &= \sqrt{(a-1)^2+(b-2)^2} \\
(a+1)^2+b^2 &= (a-1)^2+(b-2)^2 \\
a^2+2a+1+b^2 &= a^2-2a+1+b^2-4b+4 \\
4a &= -4b+4 \\
a+b &= 1
\end{aligned}
$$

c

$$
\begin{aligned}
z^2 &= i \\
\Rightarrow \theta &= \frac{\pi}{2} \\
\Rightarrow \theta' &= \frac{\pi}{4}
\end{aligned}
$$

Da $|i| = 1$ ist $z = \cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4})$

$$
\begin{aligned}
\theta &= \pi - \arctan(\frac{\sqrt{3}}{1}) \\
&= \pi - \frac{\pi}{3} = \frac{2}{3}\pi \\
\Rightarrow \theta' &= \frac{2}{3}\pi \cdot \frac{1}{4} \\
&= \frac{\pi}{6}
\end{aligned}
$$

$\sqrt[4]{|-1+\sqrt{3}|} = \sqrt[4]{2} \Rightarrow z = \sqrt[4]{2}\cos(\frac{\pi}{6}) + i\sin(\frac{\pi}{6})$

3

$$A = \begin{bmatrix} 0 & 1 & 3 & 0 \\ 8 & 5 & 0 & 6 \\ 0 & 0 & -1 & -2 \end{bmatrix} \; B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 3 & 3 \\ 9 & 5 \end{bmatrix} \; C = \begin{bmatrix} -1 & 1 & -3 \\ 8 & 5 & -4 \\ 0 & 0 & 6 \end{bmatrix} \; D = \begin{bmatrix} 1 & -3 \\ 5 & 0 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 \\ 5 \\ 3 \end{bmatrix} \; F = \begin{bmatrix} -2 & 3 & 0 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 9 & 9 \\ 54 & 38 \\ -21 & -13 \end{bmatrix}$$

$$B \cdot D = \begin{bmatrix} 5 & 0 \\ 0 & 0 \\ 18 & -9 \\ 34 & -27 \end{bmatrix}$$

$$C \cdot A = \begin{bmatrix} 8 & 8 & 0 & 12 \\ 40 & 33 & 28 & 38 \\ 0 & 0 & -6 & -12 \end{bmatrix} \; C \cdot C = \begin{bmatrix} 9 & 4 & -19 \\ 32 & 33 & -68 \\ 0 & 0 & 36 \end{bmatrix} \; C \cdot E = \begin{bmatrix} -5 \\ 21 \\ 18 \end{bmatrix}$$

$$D \cdot D = \begin{bmatrix} -14 & -3 \\ 5 & -15 \end{bmatrix}$$

$$E \cdot F = \begin{bmatrix} 2 & 3 & 0 \\ 10 & 15 & 0 \\ 6 & 9 & 0 \end{bmatrix}$$

$$F \cdot A = \begin{bmatrix} 24 & 17 & 6 & 18 \end{bmatrix} \; F \cdot C = \begin{bmatrix} 22 & 17 & -18 \end{bmatrix} \; F \cdot E = \begin{bmatrix} 17 \end{bmatrix}$$

b

$$A^1 = \begin{bmatrix} i & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -i \end{bmatrix}$$

$$A^2 = \begin{bmatrix} -1 & 1+i & 1 \\ 0 & 1 & 1-i \\ 0 & 0 & -1 \end{bmatrix}$$

$$A^3 = \begin{bmatrix} -i & i & 1 \\ 0 & 1 & -i \\ 0 & 0 & i \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} i & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & -i \end{bmatrix} = A \Rightarrow A^n = A^n - 4$$

4

a

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 3 & 5 \\ 4 & 4 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 3 & 3 & 3 \end{bmatrix}$$

$$A \cdot A = \begin{bmatrix} 4 & 5 & 5 \\ 2 & 5 & 2 \\ 0 & 2 & 3 \end{bmatrix} \quad B \cdot A = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

b

$$A = \begin{bmatrix} 4 & 0 & 0 & 4 & 5 \\ 5 & 5 & 0 & 2 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 5 & 2 \\ 3 & 0 \\ 3 & 4 \\ 4 & 0 \\ 1 & 0 \end{bmatrix}$$

$$A \cdot B = \begin{bmatrix} 5 & 2 \\ 2 & 4 \end{bmatrix}$$

$$B \cdot A = \begin{bmatrix} 0 & 4 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 3 \\ 2 & 2 & 0 & 2 & 5 \\ 4 & 0 & 0 & 4 & 2 \\ 4 & 0 & 0 & 4 & 5 \end{bmatrix}$$