1

a

1

$$\begin{aligned}
(1+i)^{-1}(1+i) &= 1 \\
(a+bi)(1+i) &= 1 \\
a + ai + bi - b &= 1 \\
(a-b) + (a+b)i &= 1 \\
|(a-b) + (a+b)i| &= |1| \\
(a-b)^2 + (a+b)^2 &= 1 \\
a^2 - 2ab + b^2 + a^2 + 2ab + b^2 &= 1 \\
2a^2 + 2b^2 &= 1 \\
a^2 + b^2 &= \frac{1}{2} \\
\Rightarrow a &= \frac{1}{2}, b = -\frac{1}{2}
\end{aligned}$$

2

Zunächst $\frac{1}{i}$

$$\begin{aligned}
\frac{1}{i} \cdot i &= 1 \\
(a+bi)i &= 1 \\
ai - b &= 1 \\
b &= -1 \\
\frac{1}{i} &= -i
\end{aligned}$$

$$\begin{aligned}
\frac{1}{i} + 3 \cdot (1+i)^{-1} &= -i + 3 \cdot (0.5 - 0.5i) \\
&= 1.5 - 2.5i
\end{aligned}$$

3

$$
\begin{aligned}
\frac{\sqrt{2}}{\sqrt{2}-i} &= a+bi \\
\sqrt{2} &= (\sqrt{2}-i)(a+bi) \\
&= \sqrt{2}a+\sqrt{2}bi-ai+b \\
&= \sqrt{2}a+b+(\sqrt{2}b-a)i \\
1 &= a+\frac{b}{\sqrt{2}}+(b-\frac{a}{\sqrt{2}})i \\
&= (a+\frac{b}{\sqrt{2}})^2+(b-\frac{a}{\sqrt{2}})^2 \\
&= a^2+\frac{b^2}{2}+\frac{2ab}{\sqrt{2}}+\frac{a^2}{2}+b^2-\frac{2ab}{\sqrt{2}} \\
&= \frac{3}{2}a^2+\frac{3}{2}b^2 \\
\frac{2}{3} &= a^2+b^2 \\
&\Rightarrow a=\frac{2}{3}, b=\frac{\sqrt{2}}{3}
\end{aligned}
$$

4

$$
\begin{aligned}
\frac{1+i}{1-i} &= a+bi \\
1+i &= (a+bi)(1-i) \\
&= a-ai+bi+b \\
&= (a+b)+(-a+b)i \\
&\Rightarrow a+b=1, -a+b=1 \\
&\Rightarrow a=0, b=1
\end{aligned}
$$

$$
\begin{aligned}
(i)^{201} &= (i^4)^{50}i \\
&= i^{50}i \\
&= (i^4)^{10}i^{10}i \\
&= i^{20}i \\
&= (i^4)^5 i \\
&= i^4 i^2 \\
&= -i
\end{aligned}
$$

b

$$\begin{aligned}
|z+1| &= |z-(1+2i)| \\
|(a+bi)+1| &= |(a+bi)-(1+2i)| \\
\sqrt{(a+1)^2+b^2} &= \sqrt{(a-1)^2+(b-2)^2} \\
(a+1)^2+b^2 &= (a-1)^2+(b-2)^2 \\
a^2+2a+1+b^2 &= a^2-2a+1+b^2-4b+4 \\
4a &= -4b+4 \\
a+b &= 1
\end{aligned}$$

c