1

1.1

Nach Annahme gilt

$$\limsup_{n\to\infty}\frac{f(n)}{g(n)} =: A < \infty$$
$$\Leftrightarrow \lim_{n\to\infty}\sup(\frac{f(n)}{g(n)}) < \infty$$
$$\limsup_{n\to\infty}\frac{g(n)}{h(n)} =: B < \infty$$
$$\Leftrightarrow \lim_{n\to\infty}\sup(\frac{f(g)}{g(h)}) < \infty$$

Es gilt also

$$\Leftrightarrow \lim_{n\to\infty}\sup(\frac{f(n)}{g(n)}) \cdot \lim_{n\to\infty}\sup(\frac{f(g)}{g(h)})$$
$$= \lim_{n\to\infty}\sup(\frac{f(n)}{g(n)}) \cdot \sup(\frac{g(n)}{h(n)}) = A \cdot B$$
$$\Leftrightarrow \lim_{n\to\infty}\sup(\frac{f(n)}{g(n)} \cdot \frac{g(n)}{h(n)}) \leq A \cdot B$$
$$\Leftrightarrow \lim_{n\to\infty}\sup(\frac{f(n)}{h(n)}) \leq A \cdot B < \infty$$
$$\Leftrightarrow \limsup_{n\to\infty}\frac{f(n)}{h(n)} < \infty$$
$$f \in O(h)\blacksquare$$

1.2

Nach Annahme gilt

$$\limsup_{n\to\infty}\frac{f_1(n)}{g_1(n)} < \infty$$
$$\Leftrightarrow \lim_{n\to\infty}\sup(\frac{f_1(n)}{g_1(n)}) < \infty$$
$$\limsup_{n\to\infty}\frac{f_2(n)}{g_2(n)} < \infty$$
$$\Leftrightarrow \lim_{n\to\infty}\sup(\frac{f_2(n)}{g_2(n)}) < \infty$$

$$
\begin{aligned}
\infty > f_1 \cdot f_2 &= \lim_{n\to\infty} \sup(\frac{f_1}{g_1}) \cdot \lim_{n\to\infty} \sup(\frac{f_2}{g_2}) \\
&\geq \lim_{n\to\infty} \sup(\frac{f_1}{g_1} \cdot \frac{f_1}{g_2}) \\
&= \limsup_{n\to\infty} \frac{f_1 \cdot f_2}{g_1 \cdot g_2} < \infty \\
&\Leftrightarrow f_1 \cdot f_2 \in O(g_1 \cdot g_2) \blacksquare
\end{aligned}
$$

2

2.1

$$
\begin{aligned}
&\limsup_{n\to\infty} \frac{n^k}{a^n} \\
&= \limsup_{n\to\infty} \frac{k \cdot n^{(n-1)}}{\ln(a)a^n} \\
\vdots \\
&= \limsup_{n\to\infty} \frac{\frac{2(k-1)}{2} n^0}{\ln^k(a)a^n} \\
&= \limsup_{n\to\infty} \frac{1}{a^n} = 0 \\
&\Rightarrow n^k \in o(a^n)
\end{aligned}
$$

2.2

$\lim_{n\to\infty} \frac{\log^r n}{n^l} = 0$

3

3.1

Induktionsvorraussetzung

$$\sum_{i=1}^{1} i = 1 = \frac{1(1+1)}{2}$$

Induktionsannahme

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

Induktionsschritt

$$\begin{aligned}
&\sum_{i=1}^{n+1} i \\
&= \sum_{i=1}^{n} i + n + 1 \\
&= \frac{n(n+1)}{2} + n + 1 \\
&= \frac{n(n+1)}{2} + \frac{2(n+1)}{2} \\
&= \frac{(n+1)((n+1)+1)}{2}
\end{aligned}$$

3.2

$n^{0.4} < \sqrt{n} < 3n < n \log n < \sqrt{3}^n < 2^n < n!$

4

4.1 f1

$$N = \sum_{i=1}^{n} \sum_{j=1}^{n} 1 = n^2$$

4.2 f2

$$N = \sum_{i=1}^{n} \sum_{j=i}^{n} 1 = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$$

4.3 f3

$$N = \sum_{i=1}^{n} \sum_{j=1}^{i} 1 = \frac{n(n+1)}{2}$$

4.4 f4

$$N = \sum_{i=1}^{1000n} \sum_{j=1}^{1000} 1 = 10^6 n$$

4.5 f5

$$N = \sum_{i=1}^{n-1} \sum_{i=1}^{n-2} \ldots = \prod_{i=1}^{n} i = (n-1)!$$

4.6 f6

$$N = n(\log n + 1)$$

4.7 f7

$$N = 2 \cdot n - 1$$

4.8 f8

$$N = 2 \cdot n - 1$$

4.9 f9

$$N = \sum_{1}^{n} \sum_{1}^{n-1} \ldots = \prod_{1}^{n} i = n!$$

4.10 f10

$$N = 99999$$

4.11 f11

$$N = \log n + 1$$

4.12 f12

$$N = \frac{\log_4(n)}{2}$$