5 Basic Power Flow Problem

In this chapter the basic power flow problem is formulated and the basic bus types are defined. Also, the conditions for solvability of the problem are discussed

THE POWER FLOW PROBLEM can be formulated as a set of non-linear algebraic equality/inequality constraints. These constraints represent both Kirchhoff's laws and network operation limits. In the basic formulation of the power flow problem, four variables are associated to each bus (network node) k:

- U_k - voltage magnitude
- θ_k - voltage angle
- P_k - net active power (algebraic sum of generation and load)
- Q_k - net reactive power (algebraic sum of generation and load)

5.1 Basic Bus Types

Depending on which of the above four variables are known (given) and which ones are unknown (to be calculated), two basic types of buses can be defined:

- PQ bus: P_k and Q_k are specified; U_k and θ_k are calculated
- PU bus: P_k and U_k are specified; Q_k and θ_k are calculated

PQ buses are normally used to represent load buses without voltage control, and PU buses are used to represent generation buses with voltage control in power flow calculations[1]. Synchronous compensators[2], are also treated as PU buses. A third bus is also needed:

[1]Synchronous machines are often equipped with Automatic Voltage Regulators (AVRs), which controls the excitation of the machine so that the terminal voltage, or some other voltage close to the machine, is kept at the set value.

[2]Synchronous compensators, sometimes also called synchronous condensers, are synchronous machines without any active power generation or load (except for losses) used for reactive power and voltage control.

- Uθ bus: U_k and θ_k are specified; P_k and Q_k are calculated

The Uθ bus, also called reference bus or slack bus, has double functions in the basic formulation of the power flow problem:

1. It serves as the voltage angle reference
2. Since the active power losses are unknown in advance, the active power generation of the Uθ bus is used to balance generation, load, and losses

In "normal" power systems PQ-buses or load buses are the far most common, typically comprising more than 80% of all buses.

Other possible bus types are P, U, and PQU, with obvious definitions. The use of multiple Uθ buses may also be required for certain applications. In more general cases, the given values are not limited to the specific set of buses (P, Q, U, θ), and branch related variables can also be specified.

Example 5.1. *Figure 5.1 shows a 5-bus network with four transmission lines and two transformers. Generators, with voltage control, are connected at buses 1, 3, and 5, and loads are connected at buses 4 and 5, and at bus 4 a shunt is also connected. Classify the buses according to the bus types PU, PQ and Uθ.*

Solution Buses 1,3, and 5 are all candidates for PU or Uθ bus types. Since only one could be Uθ bus, we select (arbitrarily) bus 5 as Uθ. In a practical system usually a generator, or generator station, that could produce power within a large range is selected as reference or slack bus. It should be noted that even if a load is connected to bus 5 it can only be a PU or Uθ bus, since voltage control is available at the bus. The reference angle is set at bus 5, usually to 0. Bus 2 is a transition bus in which both P and Q are equal to zero, and this bus is consequently of type PQ. Bus 4 is a load bus to which is also connected a shunt susceptance: since shunts are modelled as part of the network, see next section, the bus is also classified as a PQ bus. ♦

5.2 Equality and Inequality Constraints

Eqs. (4.11) and (4.12) can be rewritten as follows

$$P_k = \sum_{m \in \Omega_k} P_{km}(U_k, U_m, \theta_k, \theta_m) \tag{5.1}$$

$$Q_k + Q_k^{sh}(U_k) = \sum_{m \in \Omega_k} Q_{km}(U_k, U_m, \theta_k, \theta_m) \tag{5.2}$$

where

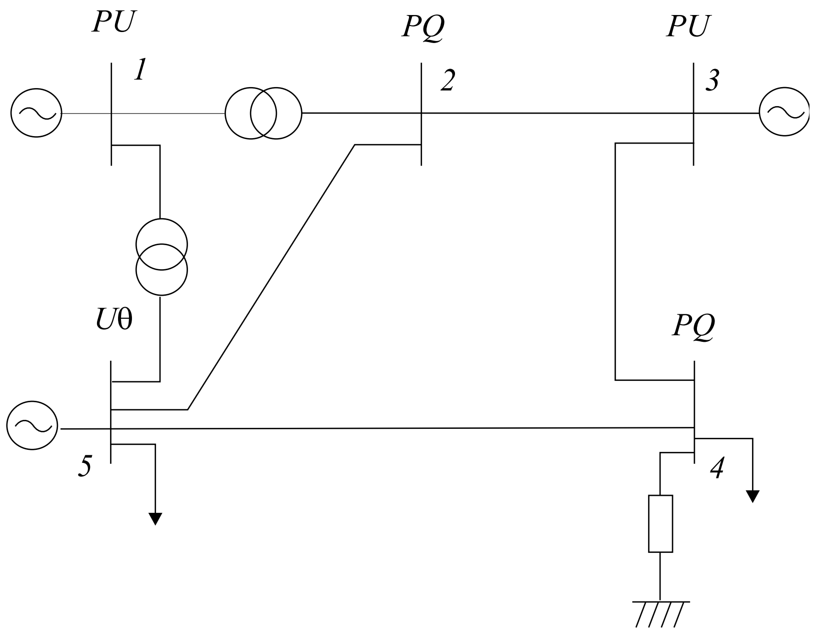

Figure 5.1. 5-bus system

- $k = 1, \ldots, N$ (N is the number of buses in the network)
- Ω_k: set of buses adjacent to k
- $U_k,\ U_m$: voltage magnitudes at the terminal buses of branch $k - m$
- $\theta_k,\ \theta_m$: voltage angles at the terminal buses of branch $k - m$
- P_{km}: active power flow from bus k to bus m
- Q_{km}: reactive power flow from bus k to bus m
- Q_k^{sh}: component of reactive power injection due to the shunt element at bus k ($Q_k^{sh} = b_k^{sh} U_m^2$), where b_k^{sh} is the shunt impedance[3]

A set of inequality constraints imposes operating limits on variables such as the reactive power injections at PU buses (generator buses), se section 2.5, and voltage magnitudes at PQ buses:

$$U_k^{min} \leq U_k \leq U_k^{max} \tag{5.3}$$

$$Q_k^{min} \leq Q_k \leq Q_k^{max} \tag{5.4}$$

[3]It is here assumed that all shunts are reactive without losses. If shunts with resistive components should be included, then eq. (5.1) must be modified accordingly.

When no inequality constraints are violated, nothing is affected in the power flow equations, but if a limit is violated, the bus status is changed and it is enforced as an equality constraint at the limiting value. This normally requires a change in bus type: if, for example, a Q limit of a PU bus is violated, the bus is transformed into an PQ bus (Q is specified and the U becomes a problem unknown). A similar procedure is adopted for backing-off when ever appropriate. What is crucial is that bus type changes must not affect solvability. Various other types of limits are also considered in practical implementations, including branch current flows, branch power flows, active power generation levels, transformer taps, phase shifter angles, and area interchanges.

5.3 Problem Solvability

One problem in the definition of bus type (bus classification) is to guarantee that the resulting set of power flow equations contains the same number of equations as unknowns, as are normally necessary for solvability, although not always sufficient. Consider a system with N buses, where N_{PU} are of type PU, N_{PQ} are of type PQ, and one is of type Uθ. To fully specify the state of the system we need to know the voltage magnitudes and voltage angles of all buses, i.e. in total $2N$ quantities. But the voltage angle and voltage magnitude of the slack bus are given together with the voltage magnitudes of N_{PU} buses. Unknown are thus the voltage magnitudes of the PQ buses, and the voltage angles of the PU and the PQ buses, giving a total of $N_{PU} + 2N_{PQ}$ unknown states. From the PU buses we get N_{PU} balance equations regarding active power injections, and from the PQ buses $2N_{PQ}$ equations regarding active and reactive power injections, thus in total $N_{PU} + 2N_{PQ}$ equations, and hence equal to the number of unknowns, and the necessary condition for solvability has been established.

Similar necessary conditions for solvability can be established when other types of buses, such as P, U, and PQU buses are used in the formulation of the power flow problem.

Example 5.2. *Consider again the 5-bus in Figure 5.1. Formulate the equality constraints of the system and the inequality constraints for the generator buses.*

Solution In this case $N = 5$, $N_{PQ} = 2$, $N_{PU} = 2$, and of course $N_{U\theta} = 1$. The number of equations are thus: $N_{PU} + 2N_{PQ} = 2 + 2 \cdot 2 = 6$, and these are:

$$
\begin{aligned}
P_1 &= P_{12} + P_{15} \\
P_2 &= P_{21} + P_{23} + P_{25} \\
Q_2 &= Q_{21} + Q_{23} + Q_{25} \\
P_3 &= P_{32} + P_{34} \\
P_4 &= P_{43} + P_{45} \\
Q_4 + Q_4^{sh} &= Q_{43} + Q_{45}
\end{aligned}
$$

In the above equations P_1, P_2, P_3, P_4, Q_2, and Q_4 are given. All the other quantities are functions of the bus voltage magnitudes and phase angles, of which U_1, U_3, and U_5 and θ_5 are given. The other six, i.e. U_2, U_4, θ_1, θ_2, θ_3, θ_4, in total 6 unknowns, can be solved from the above equations, and from these all power flows and injections can be calculated.

The inequality constraints of the generator buses are:

$$
\begin{aligned}
Q_1^{min} \leq Q_1 \leq Q_1^{max} \\
Q_3^{min} \leq Q_3 \leq Q_3^{max} \\
Q_5^{min} \leq Q_5 \leq Q_5^{max}
\end{aligned}
$$

The reactive limits above are derived from the generator capability curves as explained in section 2.5. For the slack bus it must also be checked that the injected active and reactive powers are within the range of the generator, if not the power generation of the other generators must be changed or the voltage settings of these. ⧫