4
Nodal Formulation of the Network Equations

In this chapter the basic network equations are derived from Kirchhoff's Current Law (KCL) and put into forms that are suitable for the formulation of the power flow equations in the subsequent chapter

THE NET COMPLEX current injection at a network bus, see Figure 4.1, is related to the current flows in the branches incident to the bus. Applying Kirchhoff's Current Law (KCL) yields

$$I_k + I_k^{sh} = \sum_{m \in \Omega_k} I_{km}; \text{ for } k = 1, \ldots, N \tag{4.1}$$

where k is a generic node, I_k is the net current injection from generators and loads, I_k^{sh} is the current injection from shunts, m is a node adjacent to k, Ω_k is the set of nodes adjacent to k, and N is the number of nodes in the network.

The complex current I_{km} in the unified branch model, Figure 2.9, is

$$I_{km} = (a_{km}^2 E_k - t_{km}^* t_{mk} E_m) y_{km} + y_{km}^{sh} a_{km}^2 E_k \tag{4.2}$$

where $t_{km} = a_{km} e^{j\varphi_{km}}$ and $t_{mk} = a_{mk} e^{j\varphi_{mk}}$.

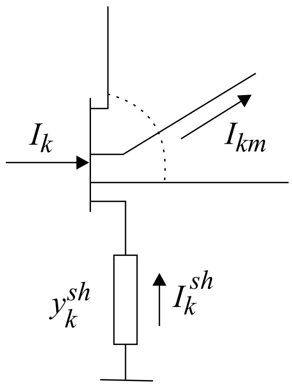

Figure 4.1. Generic bus with sign conventions for currents and power flows.

Equations (4.1) and (4.2) yield

$$I_k = \Big(y_k^{sh} + \sum_{m\in\Omega_k} a_{km}^2 (y_{km}^{sh} + y_{km})\Big) E_k - \sum_{m\in\Omega_k} (t_{km}^* t_{mk} y_{km}) E_m \tag{4.3}$$

for $k = 1, \ldots, N$. This expression can be written as

$$\mathbf{I} = \mathbf{Y}\mathbf{E} \tag{4.4}$$

where

- $\mathbf{I}$ is the injection vector with elements $I_k,\ k = 1, \ldots, N$
- $\mathbf{E}$ is the nodal voltage vector with elements $E_k = U_k e^{j\theta_k}$
- $\mathbf{Y} = \mathbf{G} + j\mathbf{B}$ is the nodal admittance matrix, with the following elements

$$Y_{km} = -t_{km}^* t_{mk} y_{km} \tag{4.5}$$

$$Y_{kk} = y_k^{sh} + \sum_{m\in\Omega_k} a_{km}^2 (y_{km}^{sh} + y_{km}) \tag{4.6}$$

We see that the nodal admittance matrix defined by eqs. (4.5) and (4.6) is modified as compared with the nodal admittance matrix without transformers. Particularly it should be noted that $\mathbf{Y}$ as defined above is not necessarily symmetric.

For large practical networks this matrix is usually very sparse. The degree of sparsity (percentage of zero elements) normally increases with the dimensions of the network: e.g., a network with 1000 buses and 1500 branches, typically presents a degree of sparsity greater than 99 %, i.e. less than 1 % of the matrix elements have non-zero values.

The kth component of $\mathbf{I}$, I_k, defined in eq. (4.3)can by using eqs. (4.5) and (4.6) be written as

$$I_k = Y_{kk} E_k + \sum_{m\in\Omega_k} Y_{km} E_m = \sum_{m\in K} Y_{km} E_m \tag{4.7}$$

where K is the set of buses adjacent to bus k, including bus k, and Ω_k is the set of buses adjacent to bus k, excluding bus k. Now considering that $Y_{km} = G_{km} + jB_{km}$ and $E_m = U_m e^{j\theta_m}$, eq. (4.7) can be rewritten as

$$I_k = \sum_{m\in K} (G_{km} + jB_{km})(U_m e^{j\theta_m}) \tag{4.8}$$

The complex power injection at bus k is

$$S_k = P_k + jQ_k = E_k I_k^* \tag{4.9}$$

and by applying eqs. (4.8) and (4.9) this gives

$$S_k = U_k e^{j\theta_k} \sum_{m \in K} (G_{km} - jB_{km})(U_m e^{-j\theta_m}) \tag{4.10}$$

The expressions for active and reactive power injections are obtained by identifying the real and imaginaty parts of eq. (4.10), yielding

$$P_k = U_k \sum_{m \in K} U_m (G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km}) \tag{4.11}$$

$$Q_k = U_k \sum_{m \in K} U_m (G_{km} \sin\theta_{km} - B_{km} \cos\theta_{km}) \tag{4.12}$$