Introduction

Many observations indicate that seismicity could be triggered by fluid operations at depth (Ellsworth, 2013). Recent studies have furthermore shown that such operations not only trigger earthquakes but also a significant amount of aseismic slip on preexisting faults (Cornet et al., 1997; Bourouis and Bernard, 2007; Guglielmi et al., 2015; Wei et al., 2015). This propagating aseismic slip in turn loads remote seismogenic asperities on the fault, which results in a complex pattern of seismic events (Schaff et al., 1998; Bourouis and Bernard, 2007). Understanding the dynamics of such propagating slow slip phenomena is thus of primarily importance to better assess the induced seismicity hazard. Here we model a typical aseismic fault by a planar interface between elastic solids, where slip is resisted by Dieterich-Ruina friction. We present numerical results about the slow slip events generated by a local fluid injection on such a fault. In a second step, we demonstrate the existence of a self similar solution for slip rate along this fault of the form $V = t^{-1} f(x/t)$. This solution allows to identify the mechanical parameters controlling the evolution of slow slip in response to a localized pressure increase.

Method

We consider the mode III fault system depicted in Figure 1. A linear infinite fault lies at the interface between 2D semi-infinite elastic half-spaces ($y > 0$ and $y < 0$). Slip $\delta(x,t)$ (and slip rate $v = \dot{\delta}$) on the fault occurs in the z direction and varies with distance along the fault x and time t. It is resisted by a Dieterich-Ruina rate-and-state friction law (Dieterich, 1979; Ruina, 1983), where the friction coefficient evolves with the whole slip history on the fault. The far-field stresses σ and τ_b are constant. At time $t = 0$, a fluid is injected inside the fault at a pressure p, so that the effective normal stress within the fault is given by $\sigma - p$.

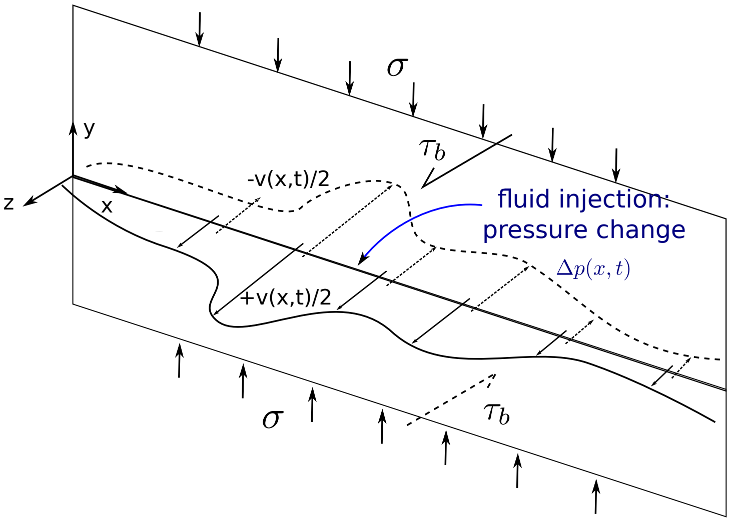

***Figure 1** Mode III fault system studied. Arrows along $y = 0$ indicate the orientation of slip rate V. σ and τ_b are the normal and shear stress boundary conditions.*

In this framework, the equations governing the evolution of slip along the fault are:

$$\tau_f(x,t) = f(x,t)\,[\sigma - p(x,t)]\,, \qquad (1)$$

where τ_f is the frictional stress acting in the z direction along the fault, f is the friction coefficient, and $\sigma - p$ the effective normal stress. Assuming a Dieterich-Ruina rate-and-state friction leads to the following equations for the evolution of f:

$$f(x,t) = f_0 + a\ln\frac{V(x,t)}{V_\infty} + b\ln\frac{\theta(x,t)V_\infty}{d_c}, \quad (2)$$

where f_0 is a reference friction coefficient, a and b are non-dimensional rate-and-state parameters, d_c is the critical slip, V is the slip rate and θ the state variable. f_0, a, b, d_c and V_∞ are constant along the fault. The state variable θ is assumed to evolve according to the ageing law (Ruina, 1983) defined by:

$$\dot{\theta}(x,t) = 1 - \frac{V\theta}{d_c}(x,t). \quad (3)$$

The friction parameters a, b and d_c control the stability of slip on the fault (Rice and Ruina, 1983; Gu et al., 1984). In the following, we will assume stable friction conditions which will be ensured by $a-b>0$. The fault is initially slipping at steady-state, i.e. at a constant slip rate V_∞ and a constant state variable $\theta_\infty = d_c/V_\infty$. The far field shear stress τ_b necessary to sustain these conditions is written τ_∞. When the fault is sheared by a non-uniform slip, the frictional stress τ_f is balanced by the elastostatic stress τ_e acting on $y=0$ and given by:

$$\tau_e(x,t) = \tau_\infty - \frac{\mu}{2}\mathscr{H}\left[\delta'\right](x,t), \quad (4)$$

where μ is the shear modulus of the elastic medium, and $\mathscr{H}$ is the Hilbert transform of the slip gradient δ'. For any prescribed pressure history $p(x,t)$, the equations (1), (2), (3) and (4) can be solved numerically, to compute the slip rate $V(x,t)$ and the state history $\theta(x,t)$ along the fault. Since the fault is infinite along the x axis, this requires the evaluation of the Hilbert transform operator on the entire real line. For that, we follow (Viesca, 2016) and use a Weideman expansion of the Hilbert transform (Weideman, 1995).

Results

In Figure 2 we show the results obtained for a localized pressure step $p(x,t)$ of the form:

$$p(x,t) = \Delta p H(t)\left[H(x+L) - H(x-L)\right], \quad (5)$$

where Δp is the amplitude of the pressure increase, L is the half length of the pressurized patch and H is the heaviside function. For simplicity, we assumed that this pressure perturbation does not evolve with time, as expected for a fluid diffusing within a permeable fault. This assumption is reasonable as long as the characteristic time for state evolution d_c/V_∞ is much smaller that the charatceristic time for pore pressure change L/D^2, D being the typical diffusivity within the permeable fault. This pressure step triggers a slow slip event over the entire fault: slip rate is first instantaneously increased over the pressurized zone (direct effect of the rate-and-state friction), and then decays and expands through the entire fault region. The sate variable displays a very similar behavior. Such an expanding and decaying solution had already been noted by (Perfettini and Ampuero, 2008) in response to a broad instantaneous shear stress perturbation. The solution depicted in Figure 2 is qualitatively very similar to the diffusion process of a unit localized mass.

We have found that, for small pressure increase $\Delta p < \sigma$, the asymptotic behavior of the system could be written in the form:

$$V(x,t) = V_\infty + \Delta V(x,t) = V_\infty\left[1 + \frac{C}{t(1+\eta^2)}\right] \quad (6)$$

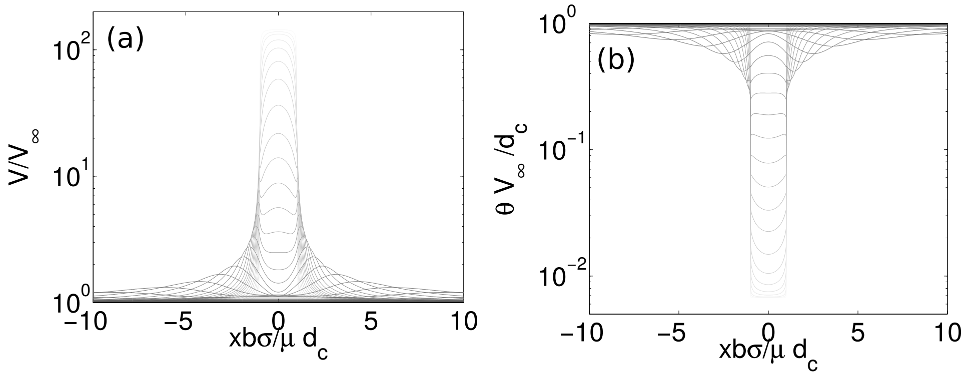

Figure 2 Normalized slip rate V/V_∞ (a) and normalized state variable $\theta V_\infty/d_c$ (b) after a pressure injection $\Delta p = 0.01\sigma$, on a fault characterized by $a/b = 1.2$. Profiles are plotted every 50 percent increase of time. Darker colors correspond to later times.

for slip rate and:

$$\theta(x,t) = \frac{d_c}{V_\infty} - \Delta\theta(x,t) = \frac{d_c}{V_\infty}\left[1 - \frac{C}{t(1+\eta^2)}\right], \tag{7}$$

for the state variable, where the similarity variable is given by:

$$\eta = 2\frac{(a-b)\sigma}{\mu V_\infty}\frac{x}{t}, \tag{8}$$

and C is a constant determined by a net force balance (the total traction acting on the fault $y = 0$ has to balance the external traction generated by the pressure increase). We find:

$$C = \frac{2}{\pi}\frac{L}{V_\infty}\left(1 - \frac{b}{a}\right)\frac{f_0\Delta p}{\mu}. \tag{9}$$

The rescaled profiles depicted in figure 3 all collapse to the solutions (6) and (7) at large times. The main difference between this solution and the classical diffusion process is that the slip rate expands as t, whereas it would expand as $\sqrt{t}$ for the classical diffusion.

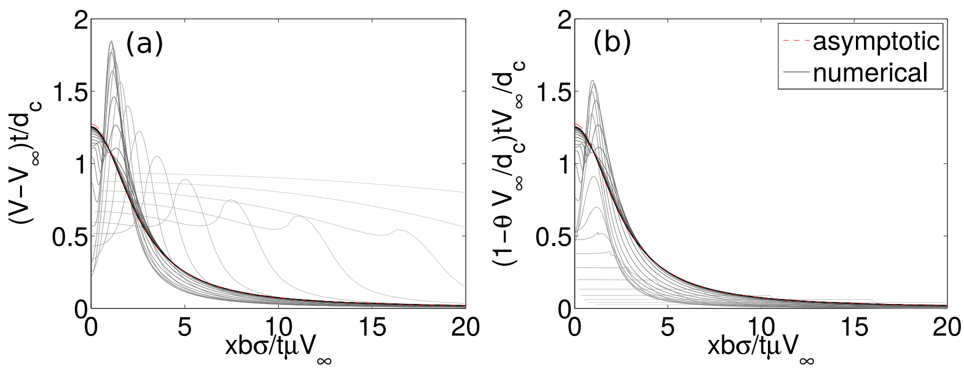

Figure 3 Renormalized slip rate V (a) and state variable θ (b) half profiles, initially shown in figure 2. Profiles are plotted every 50 percent increase of time. Darker colors correspond to later times. Red dashed curves correspond to the solutions (6) and (7)

The main parameters controlling the evolution of slow slip in response to a fluid pressure increase are first the amplitude of the pressure increase Δp, the size of the pressurized fault patch L, and the ratio a/b of the rate-and-state parameters. We note that the slow slip is stronger for a/b close to 1, which indicates that faults close to critical (defined by $a/b = 1$) are more sensitive to pressurization. Finally, similar solutions could be found for different fault rheologies, such as the linear viscous rheology where τ_f would be proportional to slip rate, or purely rate-strengthening rheology, where τ_f scales with $\ln V$. This makes the solution (6) quite general.

Conclusions

Localized fluid pressure increase on a rate-strengthening Dieterich-Ruina frictional fault triggers an aseismic slip event that decays in amplitude as inverse of time, and spreads out from the pressurized fault patch. This aseismic event could be considered as the diffusion of accelerated slip along a fault. We have found that the equations governing the evolution of this slow slip admit a self-similar solution of the form $t^{-1}f(x/t)$ for the slip rate. This solution is not only valid for a rate-and-state type rheology, but could also be found for a linear viscous, or a purely rate-strengthening rheology, which are usually assumed for aseismic faults. Finally, our solution allows to estimate the slip rate and therefore the stress conditions on potential seismogenic asperities embedded in the fault, from a given pressure history. It therefore provides a very simple, efficient, and general way to estimate the extent of the perturbed region associated with a fluid injection at depth.

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