Loan Default Risk by Demographics & Finance

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2025-07-16

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Introduction

Imagine applying for a loan without a credit history—it feels like trying to land a job without a resume. You're left thinking, "How can I prove I'm reliable if no one gives me a chance?" This project is about rewriting that

story. By leveraging data analytics, we aim to predict loan defaults more accurately, empowering individuals with limited or no credit history to access loans while minimizing risks for financial institutions. This is about making lending smarter, fairer, and more inclusive for everyone.

This project focuses on Home Credit Group, a global financial institution dedicated to promoting financial inclusion. Our goal is to refine their lending decisions by using advanced data analytics to predict the likelihood of loan defaults. By reducing default risks, we aim to foster financial inclusion and enable fairer access to credit, helping underserved populations while safeguarding institutional sustainability.

Through this project, we take a step toward a financial ecosystem where trust is built not only on history but also on data-driven insights.

Objective

In this dataset, we aim to investigate whether an applicant's social demographics and wealth factors will be important in predicting whether an applicant will repay a loan or not.

Data Sources

The data we used are from Kaggle (the link can be find under the reference) and are called Home Credit Default Risk. The dataset is mainly from 6 countries in the CIS and SEA regions: Kazakhstan, Russia, Vietnam, China, Indonesia, and the Philippines. The company collects and utilizes a variety of data points to assess the creditworthiness of clients with little or no traditional credit history.

Variables: The data includes demographic details, financial information, employment status, housing characteristics, credit bureau inquiries, and a target variable indicating whether a client had payment difficulties.

Usage: Assess loan default risk, helping financial institutions make more informed lending decisions.

Ethical Consideration

Data Ownership and Privacy: We should note that though our dataset is publicly available, it's critical to protect the privacy of individuals and firms, especially in dealing with datasets containing information about communities.

Data Ownership and Usage: The data used in this analysis belongs to the Home Credit Group. It is essential that this report adhere to their terms of use and respect their ownership rights when utilizing this data for analysis.

Community and Individual Welfare: We need to exercise caution during the analysis process in order not to perpetuate discrimination by perpetuating existing biases related to race, gender, etc. In addition, we need to consider the potential influence of policy decisions and market behaviors, the findings of the report may result in or support changes that harm or cause consequences for vulnerable populations.

Project Scope

This project is designed to enhance financial decision-making by leveraging advanced data analytics to predict loan defaults with a focus on fairness and inclusivity, for example, our we will develop models that allow financial institutions to evaluate loan applicants with little or no credit history. This fosters greater access to credit for underserved communities while maintaining a balance between financial inclusion and risk management.

Stakeholders

- Home Credit Group: The organization providing the data and is responsible for delivering financial services to underserved customers.
- Clients (Loan Applicants): The individuals seeking loans and affected by the risk of default, whose financial behaviors are being analyzed.

- Financial Institutions: Other lenders or institutions who may benefit from the insights to improve their loan issuance processes.
- Regulatory Bodies: Government agencies or organizations overseeing financial regulations, responsible lending, and consumer protection.
- Community/Consumers: Broader societal groups who may be impacted by more responsible lending practices, improving financial inclusion and reducing discriminatory practices.

Data Manipulation

Import Library

```
# Import library
library(tidyverse)
library(caret)
library(dplyr)
library(fixest)
library(gtsummary)
library(ggthemes)
library(modelsummary)
library(glmnet)
library(kableExtra)
library(knitr)
library(MASS)
library(smotefamily)
library(class)
library(pROC)
library(scales)
library(reshape2)
library(rpart)
library(rpart.plot)
library(randomForest)
library(gt)
library(factoextra)
library(xgboost)
library(doParallel)
library(ranger)
```

Summary of what each library does:

Data Manipulation and Visualization:

tidyverse: A collection of R packages (e.g., ggplot2, dplyr, tidyr, etc.) designed for data science tasks, including data manipulation, visualization, and analysis.

dplyr: Part of the tidyverse, used for data manipulation tasks such as filtering, summarizing, and joining datasets.

ggthemes: Adds additional themes and scales to ggplot2 for enhanced visualization aesthetics.

scales: Provides tools for handling numeric scaling and transformations in visualizations.

reshape2: Tools for reshaping data between wide and long formats for analysis or visualization.

Statistical Modeling and Machine Learning

caret: A comprehensive package for training and evaluating machine learning models with various preprocessing options and cross-validation methods. glmnet: Implements generalized linear models with penalties like Lasso

and Ridge regression for regularization.

MASS: Includes functions for statistical methods and datasets, including linear and nonlinear modeling.

smotefamily: Provides oversampling methods like SMOTE for dealing with imbalanced datasets.

class: Includes basic functions for classification like k-Nearest Neighbors (k-NN).

randomForest: Implements random forest algorithms for classification and regression tasks.

xgboost: A scalable and efficient gradient-boosting framework for machine learning tasks.

Summarization and Reporting

gtsummary: Generates publication-ready summary tables for statistical analyses.

modelsummary: Creates summary tables for regression models and other statistical outputs.

kableExtra: Enhances knitr::kable() tables with advanced formatting options for reporting.

knitr: Integrates R code into reports and generates dynamic documents (e.g., RMarkdown).

Visualization:

rpart.plot: Visualizes decision tree models created by rpart.

factoextra: Visualizes multivariate data analyses like PCA and clustering.

rpart: Implements recursive partitioning and regression trees.

Statistical Analysis:

pROC: Tools for visualizing and analyzing receiver operating characteristic (ROC) curves.

gt: Constructs publication-quality tables.

Trainning Model:

doParallel: Optimize trainning model.

Import the Data

First, we import our data. Since we have already download the data file from Kaggle, so we are going to use `read.csv` to read our data as dataframe here.

```
# Read csv file and import the data

credit <- read.csv("application_train.csv")
```

Data Overview

Next, we going to start with a summary of this dataset to get an overview of our data.

```
# Create a summry statistic of the dataset before processing
summary_table <- credit %>%
  summarise(
    `Number of rows` = n(),
    `Number of columns` = ncol(.),
    `Character columns` = sum(sapply(., is.character)),
    `Numeric columns` = sum(sapply(., is.numeric))
  ) %>%
  t() %>%
  as.data.frame()
```

```r
colnames(summary_table) <- c("Values")

summary_table <- tibble::rownames_to_column(summary_table, "Metric")

# Create a styled table using `gt`
gt_summary <- summary_table %>%
  gt() %>%
  tab_header(
    title = "Data Summary"
  )

# Display the styled summary
gt_summary
```

Data Summary

Metric	Values
Number of rows	307511
Number of columns	122
Character columns	16
Numeric columns	106

There is a total of 307,511 observations with 122 variables in our dataset, where 16 of these variables have character data type and 106 contain numerical data type, which could be binary or continuous. Since the variable range is diverese, we will categorize variables nicely to avoid multicolinearity or overfitting.

Now we split our data into training and test dataset, since this is a large dataset, we choose to split it into 80/20 ratio so that training data could capture the data well.

```r
# Set a random seed for reproducibility
set.seed(123)
# Split the data (example, assuming 'credit' is your dataset)
train_index <- createDataPartition(credit$TARGET, p = 0.8, list = FALSE)  # 70% training, 30% testing

# Create training and testing datasets
train_data <- credit[train_index, ]
test_data <- credit[-train_index, ]

# Save the training and testing sets to CSV files (for others to download)

#write.csv(train_data, "train_data.csv", row.names = FALSE)
#write.csv(test_data, "test_data.csv", row.names = FALSE)
```

Because we want other users to use the same dataset as we do, we save the training and testing data into `csv` files.

```r
# Separating the data into training and testing datasets.
train <- read.csv("train_data.csv")
test_data <- read.csv("test_data.csv")
```

Next, we going to make a data summary for the missing values in the data.

Missing Values

When working with any dataset, we should handle missing values carefully to provide a dependable and accurate project. With character columns, it is possible that we would encounter empty strings. We would want to transform this into `N/A` value so we can easily fill in missing values.

```
train <- train %>%
  mutate(across(where(~ is.character(.) | is.factor(.)), ~ na_if(., "")))
```

Next, we create a missing value summary from this dataset.

```
# Calculate missing values summary

missing_summary <- train %>%
  summarise(across(everything(), ~ sum(is.na(.)))) %>%
  pivot_longer(cols = everything(), names_to = "Column", values_to = "Missing Count") %>%
  mutate(
    `Total Rows` = nrow(train),
    `Missing Percentage (%)` = (`Missing Count` / `Total Rows`) * 100
  ) %>%
  arrange(desc(`Missing Count`))

# Create a styled table using `gt`
gt_missing_summary <- missing_summary %>%
  gt() %>%
  tab_header(
    title = "Missing Values Summary"
  ) %>%
  fmt_number(
    columns = `Missing Count`,
    decimals = 0
  ) %>%
  fmt_number(
    columns = `Missing Percentage (%)`,
    decimals = 2
  )

# Display the styled table
gt_missing_summary
```

Missing Values Summary

Column	Missing Count	Total Rows	Missing Percentage (%)
COMMONAREA_AVG	171,839	246009	69.85
COMMONAREA_MODE	171,839	246009	69.85
COMMONAREA_MEDI	171,839	246009	69.85
NONLIVINGAPARTMENTS_AVG	170,786	246009	69.42
NONLIVINGAPARTMENTS_MODE	170,786	246009	69.42
NONLIVINGAPARTMENTS_MEDI	170,786	246009	69.42
FONDKAPREMONT_MODE	168,188	246009	68.37
LIVINGAPARTMENTS_AVG	168,144	246009	68.35
LIVINGAPARTMENTS_MODE	168,144	246009	68.35
LIVINGAPARTMENTS_MEDI	168,144	246009	68.35

FLOORSMIN_AVG	166,874	246009	67.83
FLOORSMIN_MODE	166,874	246009	67.83
FLOORSMIN_MEDI	166,874	246009	67.83
YEARS_BUILD_AVG	163,561	246009	66.49
YEARS_BUILD_MODE	163,561	246009	66.49
YEARS_BUILD_MEDI	163,561	246009	66.49
OWN_CAR_AGE	162,155	246009	65.91
LANDAREA_AVG	146,024	246009	59.36
LANDAREA_MODE	146,024	246009	59.36
LANDAREA_MEDI	146,024	246009	59.36
BASEMENTAREA_AVG	143,879	246009	58.49
BASEMENTAREA_MODE	143,879	246009	58.49
BASEMENTAREA_MEDI	143,879	246009	58.49
EXT_SOURCE_1	138,600	246009	56.34
NONLIVINGAREA_AVG	135,710	246009	55.16
NONLIVINGAREA_MODE	135,710	246009	55.16
NONLIVINGAREA_MEDI	135,710	246009	55.16
ELEVATORS_AVG	131,046	246009	53.27
ELEVATORS_MODE	131,046	246009	53.27
ELEVATORS_MEDI	131,046	246009	53.27
WALLSMATERIAL_MODE	125,060	246009	50.84
APARTMENTS_AVG	124,810	246009	50.73
APARTMENTS_MODE	124,810	246009	50.73
APARTMENTS_MEDI	124,810	246009	50.73
ENTRANCES_AVG	123,819	246009	50.33
ENTRANCES_MODE	123,819	246009	50.33
ENTRANCES_MEDI	123,819	246009	50.33
LIVINGAREA_AVG	123,459	246009	50.18
LIVINGAREA_MODE	123,459	246009	50.18
LIVINGAREA_MEDI	123,459	246009	50.18
HOUSETYPE_MODE	123,432	246009	50.17
FLOORSMAX_AVG	122,382	246009	49.75
FLOORSMAX_MODE	122,382	246009	49.75
FLOORSMAX_MEDI	122,382	246009	49.75
YEARS_BEGINEXPLUATATION_AVG	119,950	246009	48.76
YEARS_BEGINEXPLUATATION_MODE	119,950	246009	48.76
YEARS_BEGINEXPLUATATION_MEDI	119,950	246009	48.76
TOTALAREA_MODE	118,708	246009	48.25
EMERGENCYSTATE_MODE	116,568	246009	47.38
OCCUPATION_TYPE	77,082	246009	31.33
EXT_SOURCE_3	48,872	246009	19.87
AMT_REQ_CREDIT_BUREAU_HOUR	33,285	246009	13.53
AMT_REQ_CREDIT_BUREAU_DAY	33,285	246009	13.53
AMT_REQ_CREDIT_BUREAU_WEEK	33,285	246009	13.53
AMT_REQ_CREDIT_BUREAU_MON	33,285	246009	13.53

AMT_REQ_CREDIT_BUREAU_QRT	33,285	246009	13.53
AMT_REQ_CREDIT_BUREAU_YEAR	33,285	246009	13.53
NAME_TYPE_SUITE	1,030	246009	0.42
OBS_30_CNT_SOCIAL_CIRCLE	806	246009	0.33
DEF_30_CNT_SOCIAL_CIRCLE	806	246009	0.33
OBS_60_CNT_SOCIAL_CIRCLE	806	246009	0.33
DEF_60_CNT_SOCIAL_CIRCLE	806	246009	0.33
EXT_SOURCE_2	531	246009	0.22
AMT_GOODS_PRICE	229	246009	0.09
AMT_ANNUITY	12	246009	0.00
CNT_FAM_MEMBERS	2	246009	0.00
DAYS_LAST_PHONE_CHANGE	1	246009	0.00
SK_ID_CURR	0	246009	0.00
TARGET	0	246009	0.00
NAME_CONTRACT_TYPE	0	246009	0.00
CODE_GENDER	0	246009	0.00
FLAG_OWN_CAR	0	246009	0.00
FLAG_OWN_REALTY	0	246009	0.00
CNT_CHILDREN	0	246009	0.00
AMT_INCOME_TOTAL	0	246009	0.00
AMT_CREDIT	0	246009	0.00
NAME_INCOME_TYPE	0	246009	0.00
NAME_EDUCATION_TYPE	0	246009	0.00
NAME_FAMILY_STATUS	0	246009	0.00
NAME_HOUSING_TYPE	0	246009	0.00
REGION_POPULATION_RELATIVE	0	246009	0.00
DAYS_BIRTH	0	246009	0.00
DAYS_EMPLOYED	0	246009	0.00
DAYS_REGISTRATION	0	246009	0.00
DAYS_ID_PUBLISH	0	246009	0.00
FLAG_MOBIL	0	246009	0.00
FLAG_EMP_PHONE	0	246009	0.00
FLAG_WORK_PHONE	0	246009	0.00
FLAG_CONT_MOBILE	0	246009	0.00
FLAG_PHONE	0	246009	0.00
FLAG_EMAIL	0	246009	0.00
REGION_RATING_CLIENT	0	246009	0.00
REGION_RATING_CLIENT_W_CITY	0	246009	0.00
WEEKDAY_APPR_PROCESS_START	0	246009	0.00
HOUR_APPR_PROCESS_START	0	246009	0.00
REG_REGION_NOT_LIVE_REGION	0	246009	0.00
REG_REGION_NOT_WORK_REGION	0	246009	0.00
LIVE_REGION_NOT_WORK_REGION	0	246009	0.00
REG_CITY_NOT_LIVE_CITY	0	246009	0.00
REG_CITY_NOT_WORK_CITY	0	246009	0.00

LIVE_CITY_NOT_WORK_CITY	0	246009	0.00
ORGANIZATION_TYPE	0	246009	0.00
FLAG_DOCUMENT_2	0	246009	0.00
FLAG_DOCUMENT_3	0	246009	0.00
FLAG_DOCUMENT_4	0	246009	0.00
FLAG_DOCUMENT_5	0	246009	0.00
FLAG_DOCUMENT_6	0	246009	0.00
FLAG_DOCUMENT_7	0	246009	0.00
FLAG_DOCUMENT_8	0	246009	0.00
FLAG_DOCUMENT_9	0	246009	0.00
FLAG_DOCUMENT_10	0	246009	0.00
FLAG_DOCUMENT_11	0	246009	0.00
FLAG_DOCUMENT_12	0	246009	0.00
FLAG_DOCUMENT_13	0	246009	0.00
FLAG_DOCUMENT_14	0	246009	0.00
FLAG_DOCUMENT_15	0	246009	0.00
FLAG_DOCUMENT_16	0	246009	0.00
FLAG_DOCUMENT_17	0	246009	0.00
FLAG_DOCUMENT_18	0	246009	0.00
FLAG_DOCUMENT_19	0	246009	0.00
FLAG_DOCUMENT_20	0	246009	0.00
FLAG_DOCUMENT_21	0	246009	0.00

Based on the table above, there are 50 columns that has more than 47% missing data. This is concerning since such a large shortage in data cannot represent the population appropriately. As for the other variables with missing values, we will fill in with appropriate values.

Columns with more than 47% missing data are dropped for several reasons. First, columns with excessive missing values provide limited statistical or predictive utility. The absence of data diminishes their overall relevance and reduces their contribution to meaningful analysis. Second, imputing missing values for columns with such a high proportion of missing data introduces significant challenges. The imputation process is likely to result in considerable bias or inaccuracies, which could compromise the reliability of the dataset. Third, removing these columns enhances computational efficiency and simplifies the dataset. This reduction in complexity can improve the performance of subsequent analyses or predictive models.

For instance, specific columns like COMMONAREA_AVG and YEARS_BUILD_AVG have missing percentages exceeding 65%, indicating that the data for these variables is highly incomplete. Although these columns might contain potentially valuable information, the high proportion of missing values makes reliable imputation impractical.By setting the threshold for missing data at 47%, we aim to balance data retention and quality. This approach ensures that enough information is preserved while minimizing the risk of introducing excessive noise. For example, columns like YEARS_BEGINEXPLUATATION_AVG, which has 48.78% missing data, are excluded to maintain the overall integrity of the analysis.

The cleaned dataset will contains more reliable variables for analysis, ensuring that models built will perform with accurate and robust performance by handling incomplete information.

Data Imputation

For the first step of cleaning the data, we will be dropping all columns with more than 47% missing values. As we have mentioned, columns with over 47% missing values are unreliable to use in our project because of

the huge gap of misinformation. Furthermore, we can't decide which values are suited to fill in to provided an accurate and unbiased result.

```
# Addressing the value of threshold.
missing_threshold <- 47

# Calculate missing percentage for each column
missing_summary <- train %>%
  summarise(across(everything(), ~ sum(is.na(.)) / nrow(train) * 100)) %>%
  pivot_longer(cols = everything(), names_to = "Column", values_to = "Missing_Percentage")

# Identify columns to drop
columns_to_drop <- missing_summary %>%
  filter(Missing_Percentage > missing_threshold) %>%
  pull(Column)

# Drop the identified columns
train_drop <- train %>%
  dplyr::select(-all_of(columns_to_drop))
```

Next up, we deal with numerical missing data. Initially, we made a summary statistics of numerical columns with missing values.

```
# Identify columns with missing data and calculate the percentage of missing values
missing_data <- sapply(train_drop, function(x) sum(is.na(x)) / length(x) * 100)

# Select only numerical columns with missing data
missing_columns <- names(missing_data[missing_data > 0])
numerical_columns <- train[missing_columns] %>% select_if(is.numeric)

# Check that missing_columns are available in the dataset
valid_missing_columns <- intersect(missing_columns, names(numerical_columns))

# Create a table showing the missing data percentage and the data type for these columns
summary_stats <- data.frame(
  Column = valid_missing_columns,
  Missing_Percentage = missing_data[valid_missing_columns],
  Data_Type = sapply(numerical_columns[valid_missing_columns], class),
  Mean = sapply(numerical_columns[valid_missing_columns], function(x) mean(x, na.rm = TRUE)),
  Median = sapply(numerical_columns[valid_missing_columns], function(x) median(x, na.rm = TRUE)),
  Std_Dev = sapply(numerical_columns[valid_missing_columns], function(x) sd(x, na.rm = TRUE))
) %>%
  arrange(desc(Missing_Percentage))

# Display the summary table with gt
summary_stats %>%
  gt() %>%
  tab_header(
    title = "Numerical Columns with Missing Data and Summary Statistics"
  ) %>%
  fmt_number(
    columns = c(Missing_Percentage, Mean, Median, Std_Dev),
    decimals = 2
  ) %>%
  cols_label(
```

```
    Column = "Variable",
    Missing_Percentage = "Percentage (%)",
    Data_Type = "Data Type",
    Mean = "Mean",
    Median = "Median",
    Std_Dev = "Standard Deviation"
  )
```

Numerical Columns with Missing Data and Summary Statistics

Variable	Percentage (%)	Data Type	Mean	Median	St
EXT_SOURCE_3	19.87	numeric	0.51	0.54	
AMT_REQ_CREDIT_BUREAU_HOUR	13.53	integer	0.01	0.00	
AMT_REQ_CREDIT_BUREAU_DAY	13.53	integer	0.01	0.00	
AMT_REQ_CREDIT_BUREAU_WEEK	13.53	integer	0.03	0.00	
AMT_REQ_CREDIT_BUREAU_MON	13.53	integer	0.27	0.00	
AMT_REQ_CREDIT_BUREAU_QRT	13.53	integer	0.26	0.00	
AMT_REQ_CREDIT_BUREAU_YEAR	13.53	integer	1.90	1.00	
OBS_30_CNT_SOCIAL_CIRCLE	0.33	integer	1.43	0.00	
DEF_30_CNT_SOCIAL_CIRCLE	0.33	integer	0.14	0.00	
OBS_60_CNT_SOCIAL_CIRCLE	0.33	integer	1.41	0.00	
DEF_60_CNT_SOCIAL_CIRCLE	0.33	integer	0.10	0.00	
EXT_SOURCE_2	0.22	numeric	0.51	0.57	
AMT_GOODS_PRICE	0.09	numeric	538,582.32	450,000.00	
AMT_ANNUITY	0.00	numeric	27,128.99	24,903.00	
CNT_FAM_MEMBERS	0.00	integer	2.15	2.00	
DAYS_LAST_PHONE_CHANGE	0.00	integer	-962.51	-758.00	

For the numerical data, appropriate imputation strategies will be applied based on the characteristics of the variables and their context.

For columns that represent counts of events, missing values are likely indicative of the absence of such events. In these cases, zero imputation is a logical choice because it aligns with the interpretation of no activity. This approach ensures that the data remains consistent with its intended meaning without introducing bias.

For features with skewed distributions, such as external score variables, median imputation is more suitable. The median is robust against outliers and better reflects the central tendency of the data compared to the mean, making it a reliable method to address missing values while preserving the original distribution.

Columns that track social circle activity or defaults often include many zeros, and missing values may signify no activity or no defaults. For these variables, zero imputation is a natural choice because it aligns with the context of no observed activity. Alternatively, mode imputation could be considered if the mode represents a commonly observed non-zero value.

For monetary values that are highly variable and skewed, median imputation is also appropriate. This method ensures that missing data is replaced in a way that avoids distortion from extreme values, maintaining the integrity of the overall distribution.

Finally, for numeric variables with small percentages of missing data, mean imputation is suitable. This approach works well for data that is normally distributed or nearly normal and is unlikely to introduce significant bias due to the low proportion of missing values. Using the mean ensures that the imputed values remain consistent with the dataset's overall trends.

By tailoring the imputation method to the nature of each variable, this strategy preserves the accuracy and reliability of the dataset for subsequent analysis.

```
# Replace missing values using the median or mean as appropriate
train_fill <- train_drop %>%
  mutate(
    EXT_SOURCE_3 = ifelse(is.na(EXT_SOURCE_3), median(EXT_SOURCE_3, na.rm = TRUE), EXT_SOURCE_3),
    AMT_REQ_CREDIT_BUREAU_HOUR = ifelse(is.na(AMT_REQ_CREDIT_BUREAU_HOUR), 0, AMT_REQ_CREDIT_BUREAU_HOU
    AMT_REQ_CREDIT_BUREAU_DAY = ifelse(is.na(AMT_REQ_CREDIT_BUREAU_DAY), 0, AMT_REQ_CREDIT_BUREAU_DAY),
    AMT_REQ_CREDIT_BUREAU_WEEK = ifelse(is.na(AMT_REQ_CREDIT_BUREAU_WEEK), 0, AMT_REQ_CREDIT_BUREAU_WEE
    AMT_REQ_CREDIT_BUREAU_MON = ifelse(is.na(AMT_REQ_CREDIT_BUREAU_MON), median(AMT_REQ_CREDIT_BUREAU_M
    AMT_REQ_CREDIT_BUREAU_QRT = ifelse(is.na(AMT_REQ_CREDIT_BUREAU_QRT), median(AMT_REQ_CREDIT_BUREAU_Q
    AMT_REQ_CREDIT_BUREAU_YEAR = ifelse(is.na(AMT_REQ_CREDIT_BUREAU_YEAR), median(AMT_REQ_CREDIT_BUREAU_
    OBS_30_CNT_SOCIAL_CIRCLE = ifelse(is.na(OBS_30_CNT_SOCIAL_CIRCLE), median(OBS_30_CNT_SOCIAL_CIRCLE,
    DEF_30_CNT_SOCIAL_CIRCLE = ifelse(is.na(DEF_30_CNT_SOCIAL_CIRCLE), median(DEF_30_CNT_SOCIAL_CIRCLE,
    OBS_60_CNT_SOCIAL_CIRCLE = ifelse(is.na(OBS_60_CNT_SOCIAL_CIRCLE), median(OBS_60_CNT_SOCIAL_CIRCLE,
    DEF_60_CNT_SOCIAL_CIRCLE = ifelse(is.na(DEF_60_CNT_SOCIAL_CIRCLE), median(DEF_60_CNT_SOCIAL_CIRCLE,
    EXT_SOURCE_2 = ifelse(is.na(EXT_SOURCE_2), median(EXT_SOURCE_2, na.rm = TRUE), EXT_SOURCE_2),
    AMT_GOODS_PRICE = ifelse(is.na(AMT_GOODS_PRICE), median(AMT_GOODS_PRICE, na.rm = TRUE), AMT_GOODS_PR
    AMT_ANNUITY = ifelse(is.na(AMT_ANNUITY), median(AMT_ANNUITY, na.rm = TRUE), AMT_ANNUITY),
    CNT_FAM_MEMBERS = ifelse(is.na(CNT_FAM_MEMBERS), median(CNT_FAM_MEMBERS, na.rm = TRUE), CNT_FAM_MEM
    DAYS_LAST_PHONE_CHANGE = ifelse(is.na(DAYS_LAST_PHONE_CHANGE), median(DAYS_LAST_PHONE_CHANGE, na.rm
  )
```

Similar to numerical columns, we address any character columns with missing values with a statistic summary table.

```
# Step 1: Identify character columns
character_columns <- train_drop %>% dplyr::select(where(is.character))

# Step 2: Calculate missing data percentage, number of missing data, and other summary statistics
summary_stats <- character_columns %>%
  reframe(
    Column = names(.),
    Missing_Count = sapply(., function(x) sum(is.na(x))),  # Number of missing values
    Missing_Percentage = sapply(., function(x) sum(is.na(x)) / length(x) * 100),  # Percentage of missi
    Unique_Values = sapply(., function(x) length(unique(x))),  # Count of unique values
    Mode = sapply(., function(x) {
      if (length(unique(x)) == 1) return(unique(x))
      else return(names(sort(table(x), decreasing = TRUE))[1])  # Mode or most frequent value
    })
  ) %>%
  arrange(desc(Missing_Percentage))

# Step 3: Display summary table with gt
summary_stats %>%
  gt() %>%
  tab_header(
    title = "Character Columns Summary"
  ) %>%
  fmt_number(
    columns = c(Missing_Percentage, Missing_Count),
    decimals = 2
  ) %>%
  cols_label(
```

```
    Column = "Variable",
    Missing_Count = "Missing Count",
    Missing_Percentage = "Percentage (%)",
    Unique_Values = "Unique Values",
    Mode = "Most Frequent Value"
  )
```

Character Columns Summary

Variable	Missing Count	Percentage (%)	Unique Values	Most Fre
OCCUPATION_TYPE	77,082.00	31.33	19	Laborers
NAME_TYPE_SUITE	1,030.00	0.42	8	Unaccom
NAME_CONTRACT_TYPE	0.00	0.00	2	Cash loa
CODE_GENDER	0.00	0.00	3	F
FLAG_OWN_CAR	0.00	0.00	2	N
FLAG_OWN_REALTY	0.00	0.00	2	Y
NAME_INCOME_TYPE	0.00	0.00	8	Working
NAME_EDUCATION_TYPE	0.00	0.00	5	Secondar
NAME_FAMILY_STATUS	0.00	0.00	6	Married
NAME_HOUSING_TYPE	0.00	0.00	6	House /
WEEKDAY_APPR_PROCESS_START	0.00	0.00	7	TUESDA
ORGANIZATION_TYPE	0.00	0.00	58	Business

For categorical variables with missing data, appropriate strategies will be applied based on the proportion of missing values and the significance of the feature.

For OCCUPATION_TYPE, the proportion of missing data is significant at 31.35%. Dropping rows with missing values would result in considerable data loss, which is undesirable. Occupation is a key socio-economic variable that is likely predictive of an applicant's ability to repay a loan. Additionally, missing occupation data might represent a specific group of applicants, such as those with irregular jobs or no formal employment, who could exhibit distinct loan repayment behaviors. To address this, missing values will be replaced with the category "Unknown." This approach preserves the data for analysis and treats "Unknown" as a meaningful category, ensuring that the potential predictive power of this feature is retained even when specific occupation information is unavailable.

For NAME_TYPE_SUITE, the proportion of missing data is much lower, at just 0.42%. This feature may reflect the applicant's living situation, such as whether they are unaccompanied, which could have some relevance to loan repayment behavior. Although this variable is likely less critical than occupation, it still contributes to the analysis. Missing values will be replaced with the most frequent category, "Unaccompanied." Since the missing data is minimal, this approach maintains the feature's utility without significantly affecting the overall quality of the dataset.

By using these strategies, the integrity and predictive value of the dataset will be preserved while ensuring that missing data is handled in a meaningful and contextually appropriate manner.

```
# Handling missing values for OCCUPATION_TYPE
train_filled <- train_fill %>%
  mutate(OCCUPATION_TYPE = ifelse(is.na(OCCUPATION_TYPE), "Unknown", OCCUPATION_TYPE)) %>%
  mutate(NAME_TYPE_SUITE = ifelse(is.na(NAME_TYPE_SUITE), "Unaccompanied", NAME_TYPE_SUITE))
```

Next, some of the categorical data have data that is negative, which could be due to the way the data is enter, therefore, we will take the absolute values of those columns.

```r
train_filled <- train_filled %>%
  mutate(across(c(DAYS_BIRTH, DAYS_LAST_PHONE_CHANGE, DAYS_EMPLOYED, DAYS_ID_PUBLISH, DAYS_REGISTRATION
```

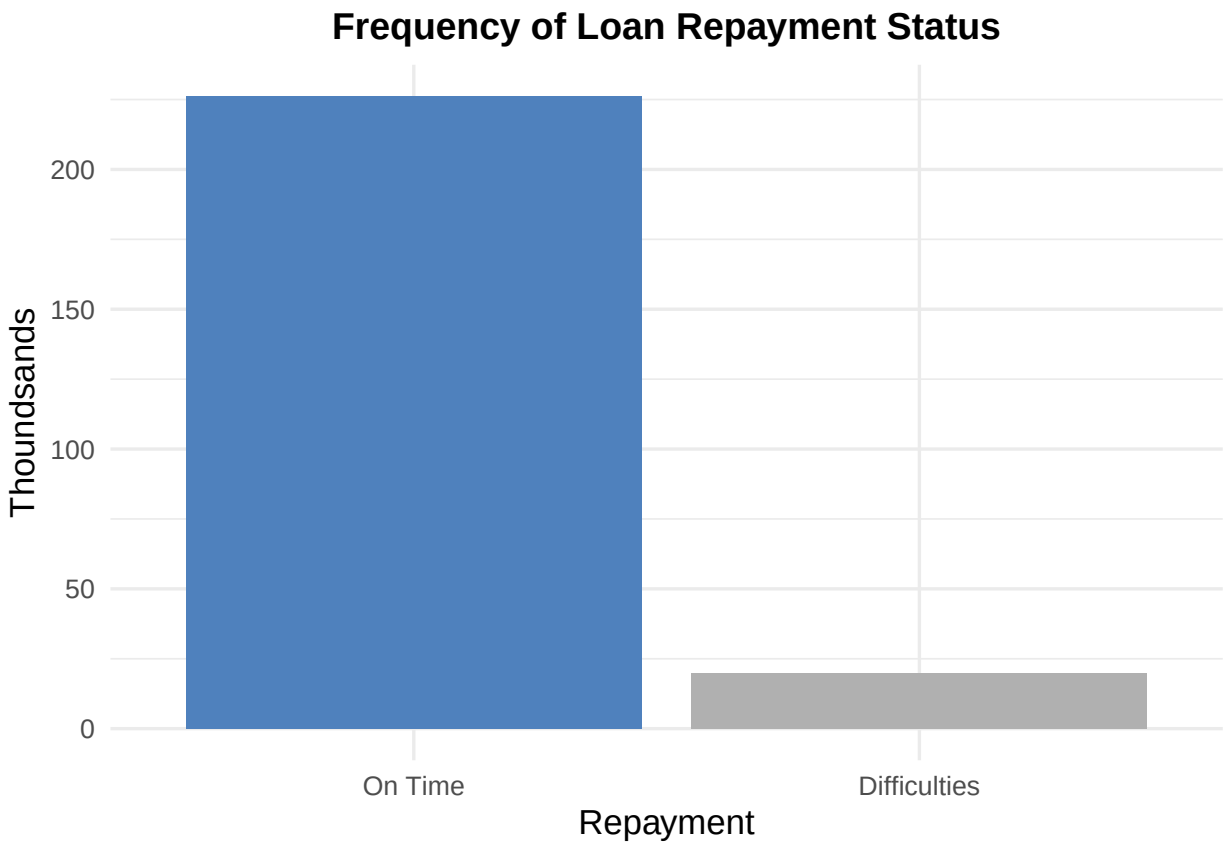
```r
# Reorder and relabel the education categories
train_filled$NAME_EDUCATION_TYPE <- factor(
  train_filled$NAME_EDUCATION_TYPE,
  levels = c("Secondary / secondary special", "Lower secondary", "Incomplete higher",
             "Higher education", "Academic degree"),
  labels = c("Secondary", "Lower Secondary", "Incomplete Higher", "Higher Education", "Academic Degree"
)
```

Exploratory Data Analysis

The goal of our project is to help Home Credit Group make better lending decision and reduce their risk of default. Thus, one of our most concerning aspect/variable is whether the client repay a loan or not. Variable `target` indicates whether a client made a payment on time or they are having difficulties repaying. If `target` is 1 then it indicates a late or missing payment. We choose this variable as our independent variable.

First, we want to see the distribution of the `target` variable

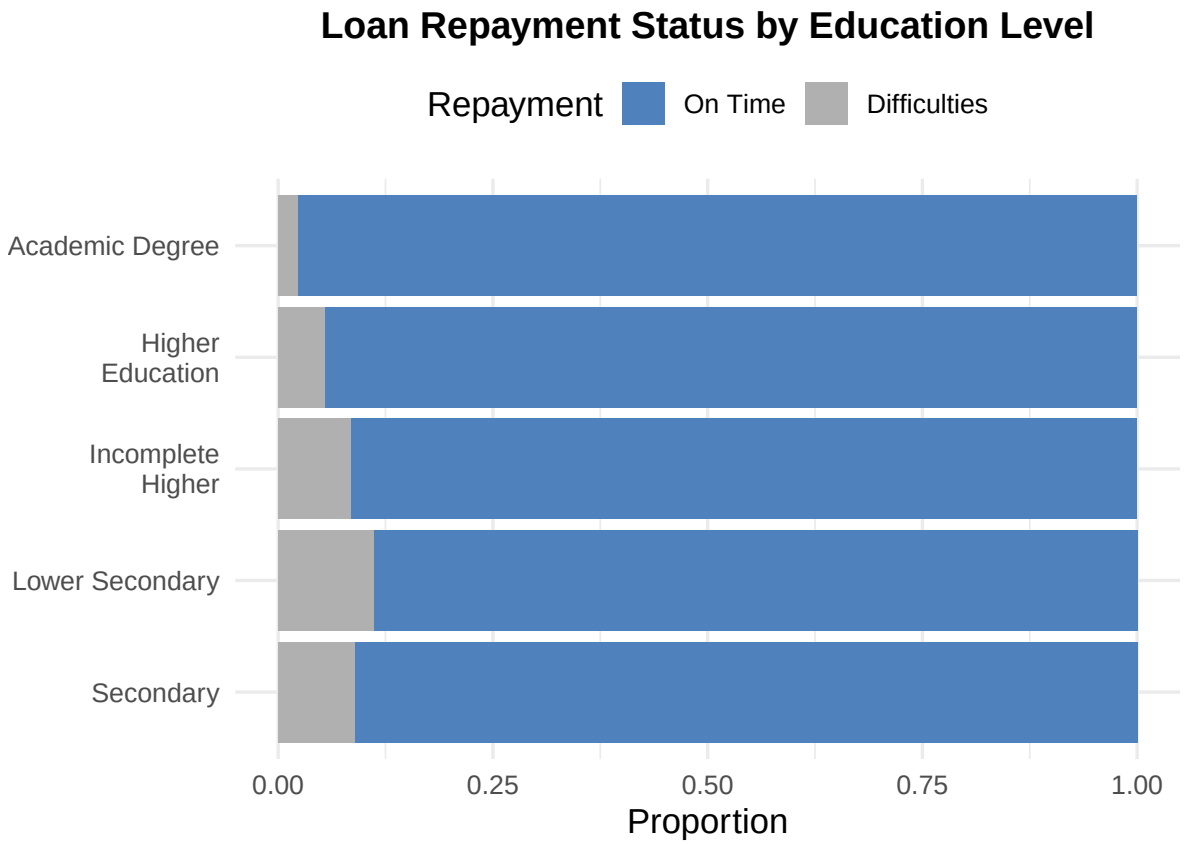
```r
# Distribution of repayment (Loan Repayment Status)
ggplot(train_filled, aes(x = factor(TARGET))) +
  geom_bar(fill = c("#4F81BD", "#B0B0B0")) +
  labs(x = "Repayment", y = "Thoundsands", title = "Frequency of Loan Repayment Status") +
  scale_x_discrete(labels = c("1" = "Difficulties", "0" = "On Time")) +
    scale_y_continuous(labels = scales::label_comma(scale = 1e-3)) +
  theme_minimal(base_size = 13) +
  theme(plot.title = element_text(face = "bold", size = 14, hjust = 0.5))
```



From this information, we see that there is an imbalanced within `target`. There are far more loans that were repaid on time than loans that were not. Once we get into the models, we can weight the classes by their representation in the data to reflect this imbalance.

Since we take social demographic factors into accounting for the effect of default, we want to see if `target` is affected by education level.

```
# Plotting the distribution of repayment by education level
ggplot(train_filled, aes(y = NAME_EDUCATION_TYPE, fill = factor(TARGET))) +
  geom_bar(position = "fill") +  # Stacked percentage bars
scale_fill_manual(values = c("#4F81BD", "#B0B0B0"), labels = c("On Time", "Difficulties")) +
  labs(x = "Proportion", y = "", title = "Loan Repayment Status by Education Level") +
  theme_minimal(base_size = 13) +
  theme(plot.title = element_text(face = "bold", size = 14, hjust = 0.5),
    axis.text.y = element_text(angle = 0, hjust = 1),  # Ensure labels on y-axis are horizontal
    legend.position = "top"  # Move the legend to the top
  ) +
  scale_y_discrete(labels = function(x) str_wrap(x, width = 15)) +  # Wrap long labels
  guides(fill = guide_legend(title = "Repayment"))  # Add title for legend +
```



Accroding to the plot, higher levels of education, such as "Academic Degree" and "Higher Education," show a significantly larger proportion of on-time repayments compared to those in lower educational categories. This trend suggests that education level is a strong predictor of financial reliability in loan repayment.

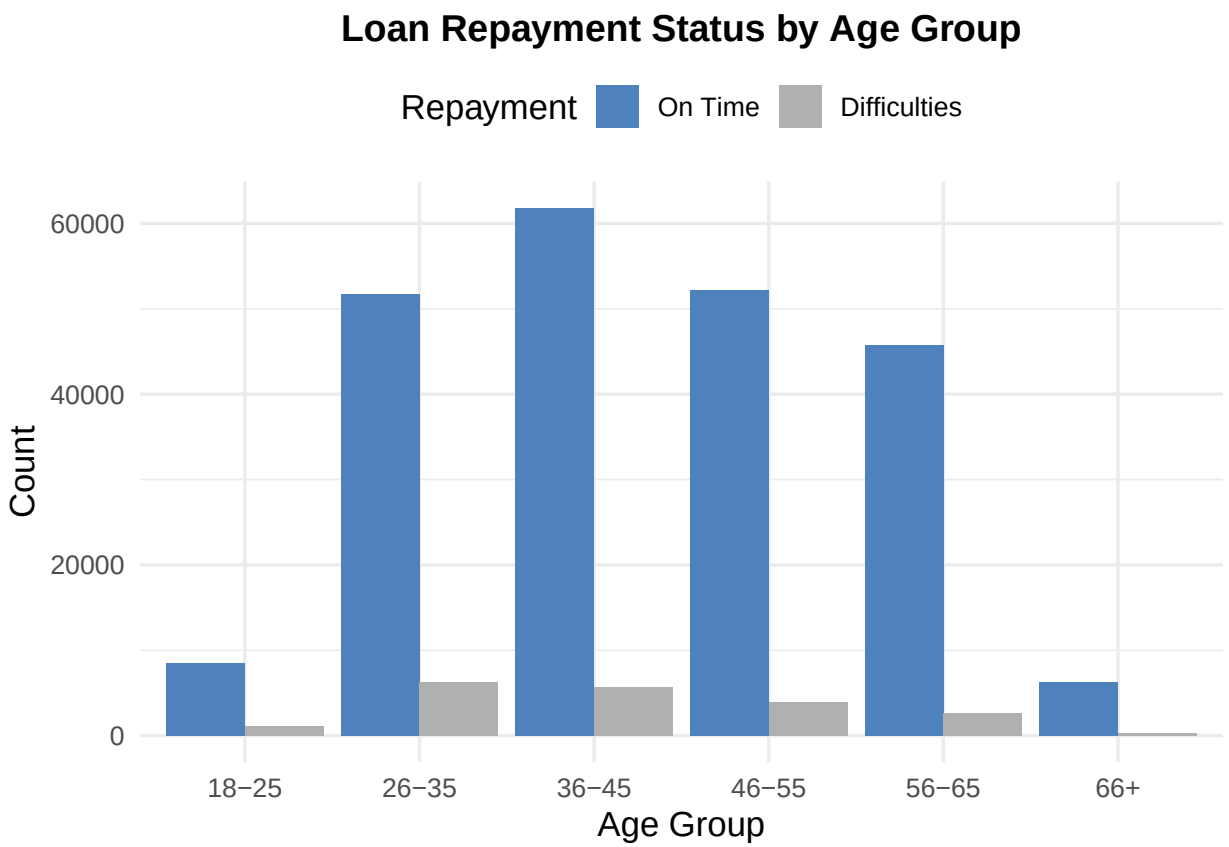
For "Lower Secondary" or "Secondary" education exhibit a visibly higher proportion of getting repayment difficulties, such a ratio could reflect limited financial literacy or employment opportunities, leading to greater challenges in meeting repayment schedules.

Finally, for "Incomplete Higher Education", the individuals in this category exhibit repayment patterns that are between those of lower and higher educational levels.

In many Asian countries, higher education significantly improves job prospects in urban areas, where wages are typically higher. Conversely, those with lower education levels may remain in informal or unstable employment sectors, leading to financial strain and repayment difficulties. Culturally, familial obligations and societal pressures in some Asian countries may also play a role, with educated individuals being better equipped to manage these dynamics.

After looking at the interaction between education level and default proportion, our team think we still need to show another factor that might effect the default rate in our dataset, so we plots the repayment status by age:

```
# Measuring wages
new_data <- train_filled %>%
  mutate(Age_Group = cut(DAYS_BIRTH / 365,
                         breaks = c(0, 25, 35, 45, 55, 65, 100),
                         labels = c("18-25", "26-35", "36-45", "46-55", "56-65", "66+"),
                         right = FALSE)) %>%
  dplyr::select(Age_Group, TARGET)  # Select only the Age Group and TARGET columns

# Bar plot with two bars per age group for Loan Repayment Status
ggplot(new_data, aes(x = Age_Group, fill = factor(TARGET))) +
  geom_bar(position = "dodge") +  # Use dodge for side-by-side bars
  scale_fill_manual(values = c("#4F81BD", "#B0B0B0"), labels = c("On Time", "Difficulties")) +
  labs(x = "Age Group", y = "Count", title = "Loan Repayment Status by Age Group") +
  theme_minimal(base_size = 13) +
  theme(plot.title = element_text(face = "bold", size = 14, hjust = 0.5),
    axis.text.y = element_text(angle = 0, hjust = 1),  # Ensure labels on y-axis are horizontal
    legend.position = "top"  # Move the legend to the top
  ) +
  scale_x_discrete(labels = function(x) str_wrap(x, width = 10)) +
  guides(fill = guide_legend(title = "Repayment"))  # Add title for legend +
```



So according to the plot, the age group 18-25 does not have that many appliers like other groups, but still, repayment difficulties appear, this might be because they often face limited financial literacy, job instability, or irregular income problems, which make them do not know the risk of borrowing. "greater levels of financial literacy decreased one's likelihood of taking out a student loan. Certain socio-demographic groups were also more likely to take out student loans. Concerning student loan repayment, individuals with lower financial literacy were more likely to be delinquent on their student loan"

The age groups 26–35, 36–45, and 46-55 dominate in the number of on-time repayments, with 36–45 having the highest count overall, besides group 26-35 having the highest count of repayment difficulties overall. We can compare people in general between the ages of 26 and 45. This is because people between the ages of 26 and 45 are generally in an age of enthusiasm and active struggle, and generally speaking, they have reached the age of getting married and buying fixed assets, so people in this age group are more likely to make some risky choices, such as borrowing to buy a house or a car, than people in other age groups. However, such behavior also has a greater probability of causing repayment difficulties, because not all people can succeed in their careers, and some people will wrongly estimate their ability, so it will cause more repayment difficulties than other age groups.

Different with two groups at the front, people around age 46-55 have a lower percentage of having difficulties with repayment, because generally, they are richer than other groups of people, although some of them still cannot finish the repayment on time, while the repayment on time count stays the same as the group 26-35, the proportion of getting difficulties in this group is decreasing.

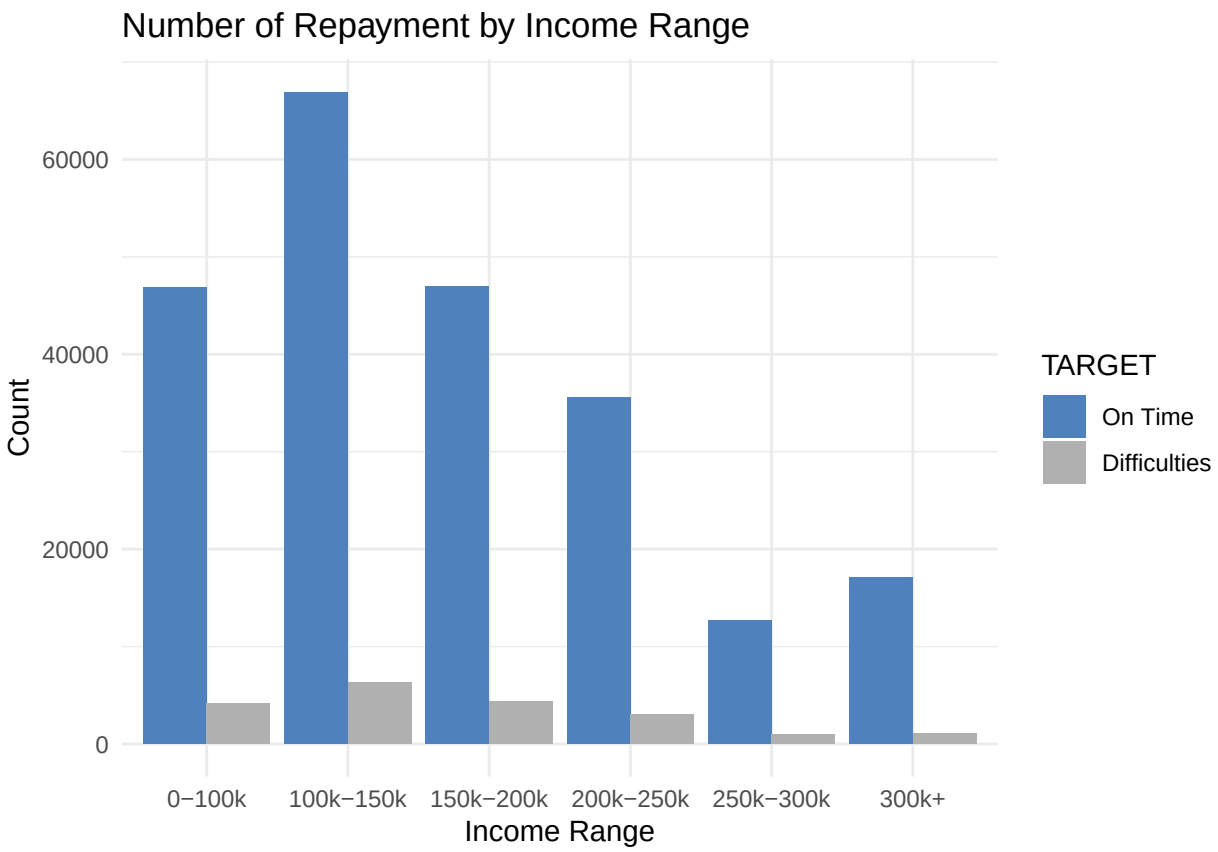
Now comes the last two groups: 56-65 and 66+, or the old people group, typically these two age groups have lost their enthusiasm for work and are beginning to enter retirement age. So for people at this stage, the proportion of people who can't pay their loans has also been much lower than in previous age groups (except for 18-25 years old). And for those older than 66, their lives are rich enough, or enough to live on, to no longer need a loan.

Additionally, we want to have a peek into the relationship between repayment ability and client's financial

factors. We use `income` and `credit` as our indicators for wealth. To start of, we will separate the income and credit columns based on their values. The threshold is determined appropriately so that the graph provided an equally distributed ranges.

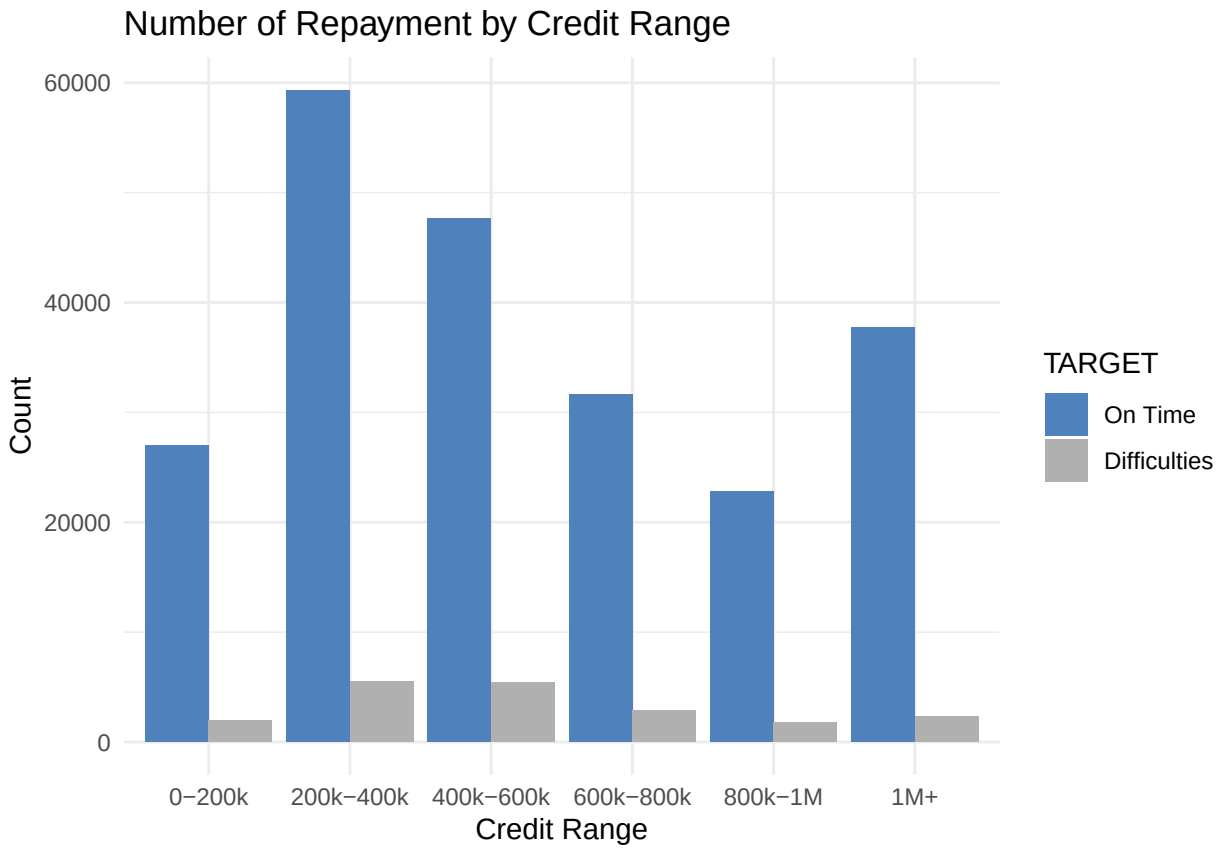
```
# Create bins for each column
train_bins <- train_filled %>%
  mutate(
    AMT_INCOME_TOTAL_BIN = cut(AMT_INCOME_TOTAL, breaks = c(0, 100000, 150000, 200000, 250000, 300000,
                               labels = c("0-100k", "100k-150k", "150k-200k", "200k-250k", "250k-300k",
    AMT_CREDIT_BIN = cut(AMT_CREDIT, breaks = c(0, 200000, 400000, 600000, 800000, 1000000, Inf),
                         labels = c("0-200k", "200k-400k", "400k-600k", "600k-800k", "800k-1M", "1M+"))
  )

# Aggregate data to count TARGET values for each bin
income_target <- train_bins %>%
  group_by(AMT_INCOME_TOTAL_BIN, TARGET) %>%
  summarise(Count = n(), .groups = "drop")

credit_target <- train_bins %>%
  group_by(AMT_CREDIT_BIN, TARGET) %>%
  summarise(Count = n(), .groups = "drop")
```

```
# Income total
ggplot(income_target, aes(x = AMT_INCOME_TOTAL_BIN, y = Count, fill = factor(TARGET))) +
  geom_bar(stat = "identity", position = "dodge") +
  scale_fill_manual(values = c("#4F81BD", "#B0B0B0"), labels = c("On Time", "Difficulties")) +
  labs(title = "Number of Repayment by Income Range",
       x = "Income Range",
       y = "Count",
       fill = "TARGET") +
  theme_minimal()
```



Based on the plot above, we see a lot of people in the income range of 0 to $250k decided to lend money the most. Those with income value higher than 250k are less likely to lend. Additionally, we noticed that the amount of difficulties fluctuates depending on the total number of lending. This suggest a comparison of proportion to further understand the differences between these groups.

```
# Credit
ggplot(credit_target, aes(x = AMT_CREDIT_BIN, y = Count, fill = factor(TARGET))) +
  geom_bar(stat = "identity", position = "dodge") +
  scale_fill_manual(values = c("#4F81BD", "#B0B0B0"), labels = c("On Time", "Difficulties")) +
  labs(title = "Number of Repayment by Credit Range",
       x = "Credit Range",
       y = "Count",
       fill = "TARGET") +
  theme_minimal()
```

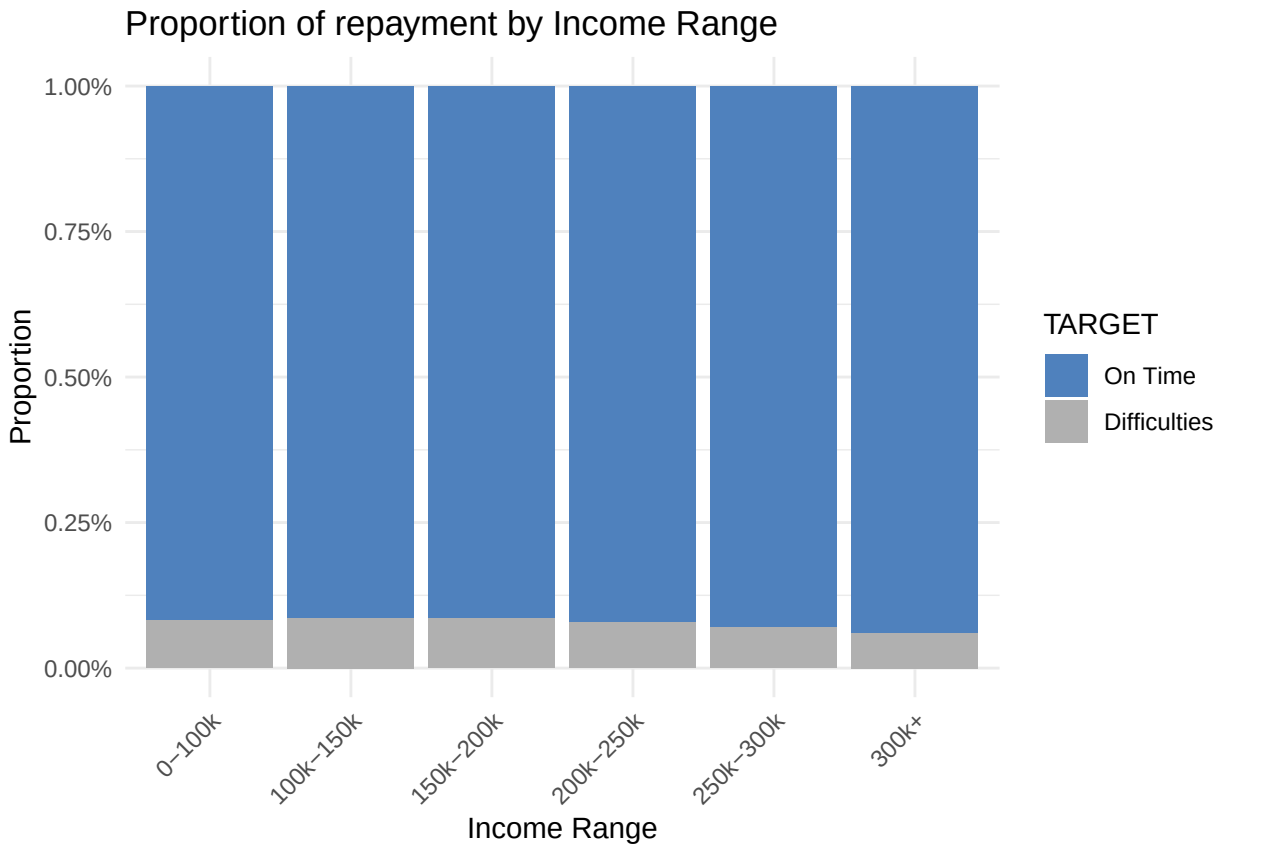


According to the plot, clients in the credit range of 200k to 800k and those with more than $1 million are the one who lend the most. Similar to the credit plot, the amount of difficulties fluctuates depending on the total number of lending for all credit range. This suggest a further look by comparing their proportion.

```
# Calculate proportions for each bin
income_target_prop <- train_bins %>%
  group_by(AMT_INCOME_TOTAL_BIN, TARGET) %>%
  summarise(Count = n(), .groups = "drop") %>%
  group_by(AMT_INCOME_TOTAL_BIN) %>%
  mutate(Proportion = Count / sum(Count))

credit_target_prop <- train_bins %>%
  group_by(AMT_CREDIT_BIN, TARGET) %>%
  summarise(Count = n(), .groups = "drop") %>%
  group_by(AMT_CREDIT_BIN) %>%
  mutate(Proportion = Count / sum(Count))
```

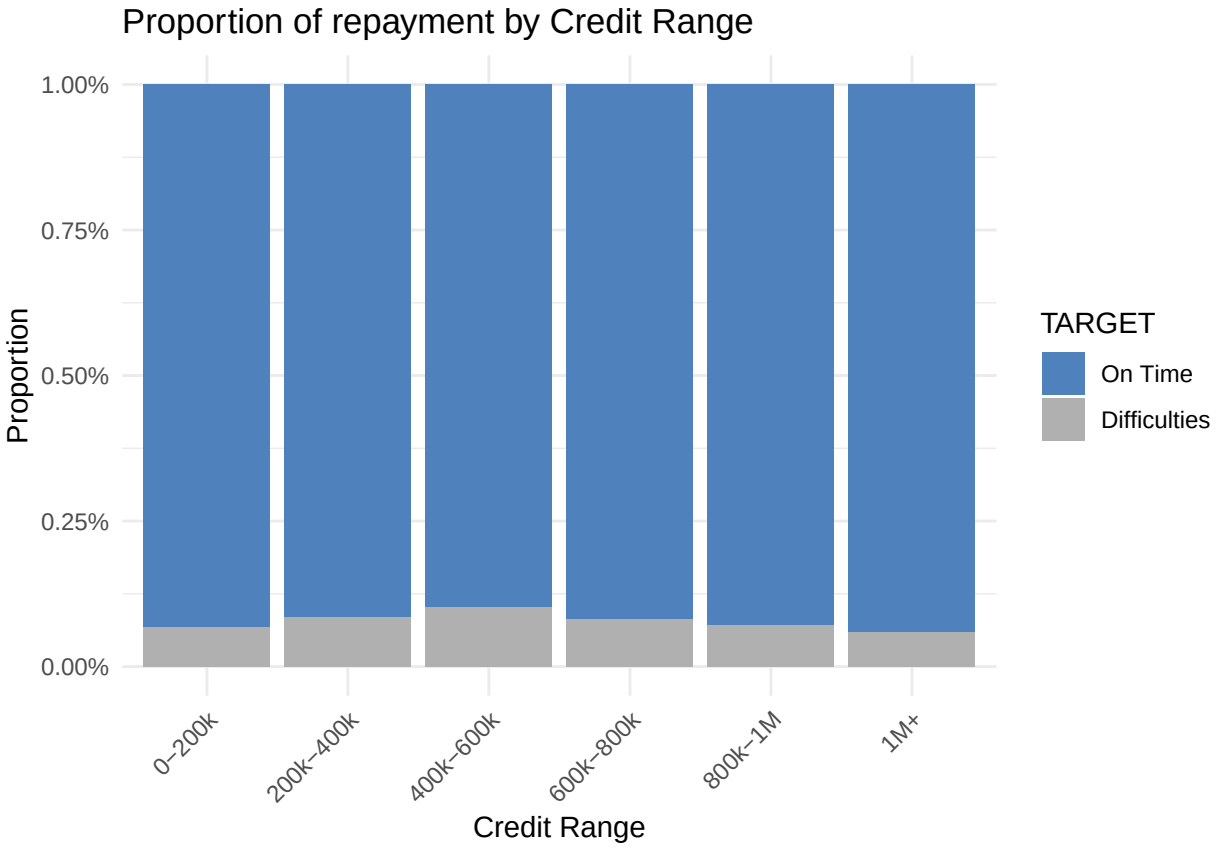
```
# Plotting proportions
# Percentage bar chart for Income Range
ggplot(income_target_prop, aes(fill = factor(TARGET), y = Proportion, x = AMT_INCOME_TOTAL_BIN)) +
  geom_bar(position = "fill", stat = "identity") +
  scale_fill_manual(values = c("#4F81BD", "#B0B0B0"), labels = c("On Time", "Difficulties")) +
  scale_y_continuous(labels = scales::percent_format(scale = 1)) +
  labs(title = "Proportion of repayment by Income Range",
       x = "Income Range",
       y = "Proportion",
       fill = "TARGET") +
  theme_minimal() +
```

```
  theme(axis.text.x = element_text(angle = 45, hjust = 1))
```



Interestingly, when we plot the income range into proportion, we can observe a slight downward trend. This suggest that those with higher income are more likely to pay back the loan. Combining with the insights from the count of loans, it can be imply that people with income lower than 250k are more likely to lend out and less likely to repay the loan on time. On the other, those with income level higher than 250k are less likely to lend and responsibly repay the loan on time.

```
# Percentage bar chart for Credit Range
ggplot(credit_target_prop, aes(fill = factor(TARGET), y = Proportion, x = AMT_CREDIT_BIN)) +
  geom_bar(position = "fill", stat = "identity") +
  scale_fill_manual(values = c("#4F81BD", "#B0B0B0"), labels = c("On Time", "Difficulties")) +
  scale_y_continuous(labels = scales::percent_format(scale = 1)) +
  labs(title = "Proportion of repayment by Credit Range",
       x = "Credit Range",
       y = "Proportion",
       fill = "TARGET") +
  theme_minimal() +
  theme(axis.text.x = element_text(angle = 45, hjust = 1))
```



For credit range proportion, there doesn't seems to be a specific trend in the graph. However, interestingly, when the number of repayment for group 400k-600k decrease, the proportion of difficulties repayment for that group increased. Also, those who has high credit value are more likely to repay the loan on time, aligning with what we have from income graphs' insights.

Variable Selection

After filling in missing values and removing columns with too many missing data, we're left with 54 variables. From the heatmap, we observed that some of these variables are highly correlated. To improve our model's performance and reduce the risk of overfitting, we're using a combination of Elastic Net and Principal Component Analysis (PCA). But before diving into these techniques, we also leveraged domain knowledge to guide our feature engineering process, which helped us create new variables and remove irrelevant ones. Here's how everything fits together:

- Using Domain Knowledge for Feature Engineering

Before applying any advanced techniques, we first used our understanding of the domain to create new variables that might be more meaningful or representative of the underlying patterns in the data. We also removed features that, based on our domain expertise, were unlikely to add value or could introduce noise. This helped us refine the dataset before applying Elastic Net and PCA, ensuring that we weren't working with irrelevant or redundant features from the start.

- Elastic Net: Handling Correlated Variables and Feature Selection

Once we had a cleaner dataset, we turned to Elastic Net. Elastic Net is a regularization method that combines Lasso (L1 regularization) and Ridge (L2 regularization), and it's particularly effective when we have correlated features, as is the case with our dataset. Elastic Net allows us to:

Handle multicollinearity (correlated features) by selecting important features while shrinking the others.

Reduce overfitting by penalizing overly complex models.

Filter out irrelevant variables, ensuring the model only uses the most meaningful features.

This combination of Lasso and Ridge regularization makes Elastic Net especially useful in refining the feature set and focusing on the most impactful variables.

- Principal Component Analysis (PCA): Reducing Dimensionality and Multicollinearity

Even after applying Elastic Net, we still have a fair number of features, which is where PCA comes in. PCA is a dimensionality reduction technique that transforms the original correlated features into uncorrelated principal components, each of which captures a portion of the variance in the data.

PCA is particularly useful in this case for a few key reasons:

Multicollinearity: Since many of our features are correlated, PCA helps by creating new components that are uncorrelated, resolving the issue of multicollinearity and making the model more stable.

Dimensionality Reduction: By focusing on the components that explain the most variance, PCA reduces the number of features we need to work with, simplifying the model and making it more efficient.

Model Efficiency: Reducing the number of features not only makes training faster but also helps in creating a model that can generalize better to new data. How They Work Together

In summary, the combination of Elastic Net and PCA, along with the use of domain knowledge to create and refine features, helps us build a model that is more efficient, generalizable, and accurate. This process ensures that we are working with the most meaningful features and reduces the risk of overfitting, ultimately leading to better performance on real-world data.

Data Cleaning

```r
# Identify all FLAG_DOCUMENT_X columns
flag_columns <- grep("^FLAG_DOCUMENT_", names(train_filled), value = TRUE)

# Create a new column with the count of documents provided (sum of flags)
train_filled$DOCUMENT_COUNT <- rowSums(train_filled[, flag_columns])

# Remove the FLAG_DOCUMENT_X columns from the dataset
train_filled <- train_filled[, !names(train_filled) %in% flag_columns]

# Dropping and mutate column based on domain knowledge
train_clean <-train_filled %>%
  dplyr::select(-AMT_REQ_CREDIT_BUREAU_HOUR, -AMT_REQ_CREDIT_BUREAU_DAY, -AMT_REQ_CREDIT_BUREAU_WEEK, -
  mutate(DAY_EMPLOYED_PERCENT = DAYS_EMPLOYED / DAYS_BIRTH) %>%
  mutate(AGES = DAYS_BIRTH/365)  %>%
  dplyr::select(-SK_ID_CURR, -DAYS_BIRTH) %>%
  mutate(CREDIT_INCOME_PERCENT = AMT_CREDIT / AMT_INCOME_TOTAL) %>%
  filter(CODE_GENDER != "XNA")
```

Here, our goal is to reduce dimensionality and improve the relevance of the features. Instead of using the binary variable for documents, we created a new column, DOCUMENT_COUNT, which represents the total count of documents provided. This could be a more meaningful feature for our model, so we dropped the original document binary variable.

Additionally, we removed variables related to the number of enquiries to the Credit Bureau at different time intervals (e.g., one hour, one day, one week, or a year before the application). Time intervals like “hour” and “day” are too close to each other and may just reflect normal client behavior, while “year” is too distant. We decided to keep only the quarter variable, as it strikes a balance between capturing meaningful action and avoiding overly short or long time frames.

We also created a new feature, DAY_EMPLOYED_PERCENT, by calculating the ratio of DAYS_EMPLOYED to DAYS_BIRTH, which gives a sense of how long the person has been employed relative to their age. This was calculated based on a reasonable assumption that someone with a higher ratio has more work experience relative to their age.

Finally, we removed irrelevant variables like SK_ID_CURR, FLAG_MOBIL (since we have mutiple other variable indicate the same thing) and DAYS_BIRTH, as we now have a new AGE variable, and filtered out the XNA category in the gender variable, which didn't make sense in the context of the data.

```
# Change the categorical column into numerical based on domain knowledge
train_clean$NAME_EDUCATION_TYPE <- as.numeric(factor(train_clean$NAME_EDUCATION_TYPE, labels = c("Secon
train_clean$OCCUPATION_TYPE <- as.numeric(table(train_clean$OCCUPATION_TYPE )[train_clean$OCCUPATION_TY
train_clean$NAME_TYPE_SUITE <- as.numeric(table(train_clean$NAME_TYPE_SUITE )[train_clean$NAME_TYPE_SUI
train_clean$NAME_HOUSING_TYPE <- as.numeric(table(train_clean$NAME_HOUSING_TYPE )[train_clean$NAME_HOUS
train_clean$NAME_CONTRACT_TYPE <- as.numeric(table(train_clean$NAME_CONTRACT_TYPE )[train_clean$NAME_CO
train_clean$ORGANIZATION_TYPE <- as.numeric(table(train_clean$ORGANIZATION_TYPE )[train_clean$ORGANIZAT
train_clean$NAME_FAMILY_STATUS <- as.numeric(table(train_clean$NAME_FAMILY_STATUS )[train_clean$NAME_FA
```

To improve the model's performance, we transformed categorical variables into numerical ones based on domain knowledge. For instance, NAME_EDUCATION_TYPE was converted into a numeric scale representing different education levels. The levels are ordered as follows: "Secondary" (lowest), "Lower Secondary", "Incomplete Higher", "Higher Education", and "Academic Degree" (highest). This transformation reflects the increasing level of education and its potential correlation with income, creditworthiness, or other important factors.

For other categorical variables such as OCCUPATION_TYPE, NAME_TYPE_SUITE, NAME_HOUSING_TYPE, NAME_CONTRACT_TYPE, ORGANIZATION_TYPE, and NAME_FAMILY_STATUS, we transformed them into numeric values based on their frequency in the dataset. This approach helps capture the importance of each category while simplifying the data. The more frequent categories are assigned higher numerical values, which can be interpreted as indicators of commonality or prevalence. By doing this, we ensure that the model can interpret these categorical variables effectively without the need for complex one-hot encoding, while still maintaining important information about their relative occurrence.

```
# Do Hot-one Enconding for the remain variable
dummy_vars <- dummyVars(~ ., data = train_clean, fullRank = TRUE)
train_before <- data.frame(predict(dummy_vars, train_clean))
```

For the remaining categorical variables, we applied One-Hot Encoding to convert them into a format that can be understood by the machine learning model. One-hot encoding creates a binary (0 or 1) column for each category in a variable, where each new column represents the presence or absence of a specific category. This is particularly useful for categorical variables that do not have an inherent order or ranking.

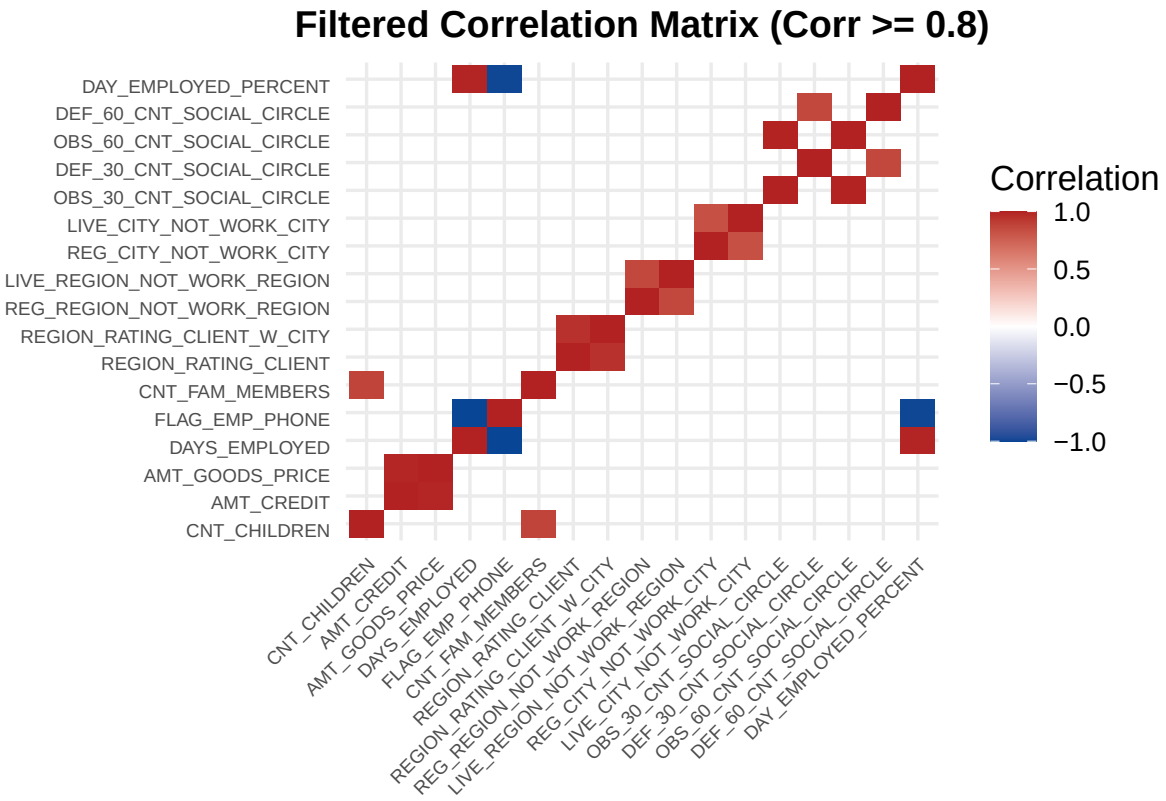
We used the dummyVars function from the caret package to generate these binary variables, specifying the fullRank = TRUE argument to ensure we avoid multicollinearity (by excluding one category per variable). After applying one-hot encoding, we created a new dataset called train_before, which includes these expanded binary columns, making the dataset ready for model selection. This transformation ensures that categorical variables are properly represented, allowing the model to learn from the individual categories without introducing any implicit order or relationship.

Variable Correlation

After we fininsh our exploration on datasets, we realize that we have too much variables in our datasets so here comes up to explain how we choose the variables in the dataset to do our following model buildings and training.

First of all, we decide to use the correlation matrix to find what variables in our dataset can relate with each other:

```r
# Separate the numerical columns
numerical_data <- train_before %>%
  dplyr::select_if(is.numeric)  # Selects only numerical columns

# Calculate the initial correlation matrix
corr_matrix <- cor(numerical_data)

# Set a high correlation threshold
threshold <- 0.8

# Create a copy of the correlation matrix, setting correlations below the threshold to NA
corr_matrix[abs(corr_matrix) < threshold] <- NA  # Set correlations below the threshold to NA

# Function to check if all non-NA values are 1
all_one_or_na <- function(x) all(x[!is.na(x)] == 1)

# Remove columns and rows that contain only 1 and NA
corr_matrix <- corr_matrix[, !apply(corr_matrix, 2, all_one_or_na)]
corr_matrix <- corr_matrix[!apply(corr_matrix, 1, all_one_or_na), ]

# Identify variables with at least one significant correlation
significant_vars <- apply(!is.na(corr_matrix), 1, any)

# Filter out variables without significant correlations
filtered_corr_matrix <- corr_matrix[significant_vars, significant_vars]

# Convert the filtered correlation matrix into long format
cor_long <- melt(filtered_corr_matrix, na.rm = TRUE)

# Plot the heatmap
ggplot(data = cor_long, aes(Var1, Var2, fill = value)) +
  geom_tile() +
  scale_fill_gradient2(low = "#084594", high = "#B22222", mid = "white",
                       limit = c(-1, 1), midpoint = 0,
                       name = "Correlation") +
  labs(title = paste("Filtered Correlation Matrix (Corr >= ", threshold, ")", sep = ""),
       x = "",
       y = "") +
  theme_minimal(base_size = 13) +
  theme(plot.title = element_text(face = "bold", size = 14, hjust = 0.5),
        axis.text.x = element_text(angle = 45, hjust = 1, size = 7, vjust = 1),
        axis.text.y = element_text(hjust = 1, size = 7, vjust = 1))
```



So, after a list of comparations, we decide to set an 0.8 threshold, since either higher or lower than this point, our matrix will excluding many relationships that may still be relevant or will include weaker correlations, making it harder to identify the strongest and most meaningful variable relationships, 0.8 comes to be the best we can set.

And according to the plot, here are some example variables that we can focus on:

AMT_GOODS_PRICE and AMT_CREDIT: These two variables show very high correlation, suggesting that the amount of credit is closely tied to the price of goods.

REGION_RATING_CLIENT and REGION_RATING_CLIENT_W_CITY: These are strongly correlated, reflecting how regional client ratings and those weighted by city are closely linked.

But although we have many high correlation variables, there are still some variables who do have high correlation but we do not need, for example:

OBS_30_CNT_SOCIAL_CIRCLE and OBS_60_CNT_SOCIAL_CIRCLE: These two are highly correlated just because they both showing the same thing, but with different threshold of days past due (30 days and 60 days). So this pair will be pass.

Principal Component Analysis

After that, we proceeded with Principal Component Analysis (PCA) to reduce dimensionality and mitigate multicollinearity. PCA helps by combining correlated features into uncorrelated principal components, allowing the model to focus on the most important underlying patterns. This step aims to enhance the efficiency of model training and prevent overfitting, as it reduces the risk of including redundant features that may not add significant predictive power.

```
# Rename
final_filtered_data <- train_before

# Filter x variable
```

```r
final_filtered_data_x <- final_filtered_data[, -which(names(final_filtered_data) == "TARGET")]

# Apply PCA to the standardized data
pca_result <- prcomp(final_filtered_data_x, center = TRUE, scale. = TRUE)
```

First, we will apply PCA to our data. Before doing so, it is essential to center and scale the data to ensure that each feature contributes equally to the analysis. Centering subtracts the mean of each variable, while scaling normalizes the variables to have unit variance. This step is crucial because PCA is sensitive to the scale of the data, and without it, variables with larger scales could disproportionately influence the results.

```r
# Summary of PCA results
summary(pca_result)
```

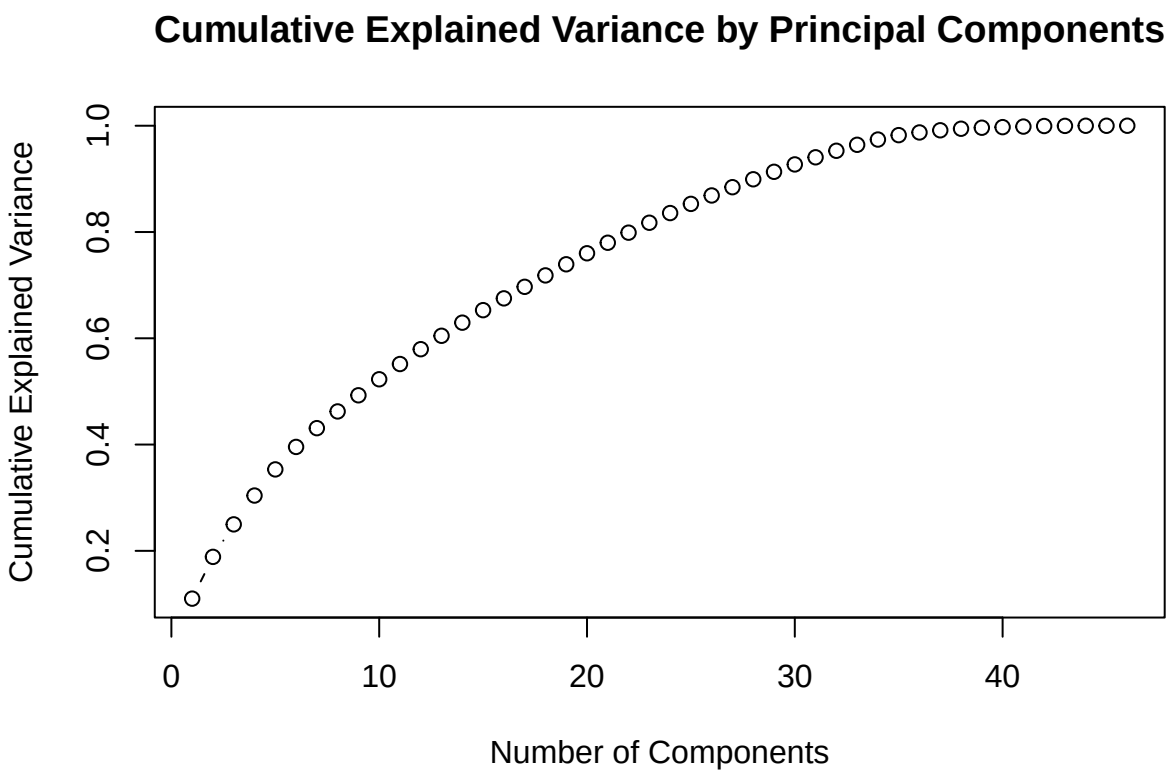
```
## Importance of components:
##                           PC1     PC2     PC3    PC4     PC5     PC6     PC7
## Standard deviation     2.2501 1.90137 1.67763 1.5790 1.50464 1.39435 1.27553
## Proportion of Variance 0.1101 0.07859 0.06118 0.0542 0.04922 0.04227 0.03537
## Cumulative Proportion  0.1101 0.18866 0.24984 0.3040 0.35326 0.39552 0.43089
##                           PC8     PC9    PC10    PC11    PC12    PC13    PC14
## Standard deviation     1.20244 1.18209 1.17760 1.14968 1.13267 1.07700 1.06611
## Proportion of Variance 0.03143 0.03038 0.03015 0.02873 0.02789 0.02522 0.02471
## Cumulative Proportion  0.46232 0.49270 0.52285 0.55158 0.57947 0.60469 0.62940
##                           PC15    PC16    PC17    PC18    PC19    PC20    PC21
## Standard deviation     1.04068 1.01038 1.00005 0.99383 0.98130 0.97609 0.95485
## Proportion of Variance 0.02354 0.02219 0.02174 0.02147 0.02093 0.02071 0.01982
## Cumulative Proportion  0.65294 0.67513 0.69687 0.71834 0.73928 0.75999 0.77981
##                           PC22    PC23    PC24    PC25    PC26    PC27    PC28
## Standard deviation     0.93409 0.92680 0.91565 0.89135 0.85407 0.84464 0.82900
## Proportion of Variance 0.01897 0.01867 0.01823 0.01727 0.01586 0.01551 0.01494
## Cumulative Proportion  0.79878 0.81745 0.83568 0.85295 0.86881 0.88432 0.89926
##                           PC29    PC30   PC31    PC32    PC33    PC34    PC35
## Standard deviation     0.80413 0.79918 0.7851 0.75204 0.71905 0.67395 0.61818
## Proportion of Variance 0.01406 0.01388 0.0134 0.01229 0.01124 0.00987 0.00831
## Cumulative Proportion  0.91331 0.92720 0.9406 0.95289 0.96413 0.97401 0.98231
##                           PC36    PC37    PC38    PC39   PC40    PC41    PC42
## Standard deviation     0.48258 0.43270 0.36690 0.29408 0.2252 0.22114 0.21904
## Proportion of Variance 0.00506 0.00407 0.00293 0.00188 0.0011 0.00106 0.00104
## Cumulative Proportion  0.98738 0.99145 0.99437 0.99625 0.9973 0.99842 0.99946
##                           PC43    PC44    PC45    PC46
## Standard deviation     0.10869 0.10624 0.03847 0.01552
## Proportion of Variance 0.00026 0.00025 0.00003 0.00001
## Cumulative Proportion  0.99972 0.99996 0.99999 1.00000
```

The PCA results indicate the importance of each principal component in explaining the variance in the data. The first few components capture the majority of the variance, with PC1 explaining 11.01% of the variance alone, and the first three components together accounting for 23.55% of the total variance. By PC31, the cumulative proportion of variance reaches 94.06%, meaning that the first 31 components explain most of the variability in the data. Beyond this, the components explain diminishing amounts of variance. This suggests that dimensionality can be effectively reduced by focusing on the first 31 components, retaining significant information while reducing complexity and improving model efficiency.

We would make a plot too check our findings and proceed with our data after PCA

```r
# Get explained variance
explained_variance <- summary(pca_result)$importance[2, ]
```

```r
# Cumulative explained variance
cumulative_variance <- cumsum(explained_variance)

# Plot the cumulative variance
plot(cumulative_variance, type = "b", xlab = "Number of Components", ylab = "Cumulative Explained Varia
     main = "Cumulative Explained Variance by Principal Components")
```



The plot visualizes the cumulative explained variance, with the number of components on the x-axis and the cumulative variance on the y-axis. The resulting plot shows that the first 31 components explain approximately 94% of the variability in the data. As more components are added, the cumulative variance increases at a slower rate, indicating diminishing returns in capturing additional variance. Therefore, we would choose to retain the first 31 components for further analysis, as they capture most of the important information, while reducing dimensionality and simplifying the model without losing significant explanatory power.

```r
# Select the PC for our data
select_train <- pca_result$x[, 1:31]

# Combine the TARGET variable and our principle component
cleaned_matrix <- bind_cols(TARGET = as.factor(final_filtered_data$TARGET), select_train)

# Transform it to DF for trainning
pca_transformed_df <- as.data.frame(cleaned_matrix)

pca_transformed_df$TARGET <- factor(pca_transformed_df$TARGET,
                                    levels = c(0, 1),
                                    labels = c("X0", "X1"))
```

We will create a new dataset using the selected principal components from the PCA transformation and proceed to train our model using this reduced-dimensional data.

Methodology

Next, we will train our model using four different methods: Logistic Regression (LR), Random Forest (RF), and XGBoost (XGB).

- Logistic Regression will serve as a baseline model, as it is particularly effective for capturing linear relationships between the predictors and the target variable. This model will help us understand the basic linear patterns in the data.
- Random Forest, a non-linear model, will allow us to capture more complex relationships and interactions between features, which may not be apparent in linear models. XGBoost, a powerful gradient boosting algorithm, will help us model complex, non-linear relationships through an ensemble of decision trees, focusing on minimizing errors through iterative refinement.
- XGBoost has been shown to perform well in many predictive tasks due to its efficiency, regularization, and ability to handle unstructured data. As demonstrated by Gu et al. (2024) in their study on credit risk assessment for small and micro enterprises, XGBoost outperforms other machine learning models in terms of both accuracy and stability, making it a strong choice for our analysis.

We will train Logistic Regression (LR), Elastic Net (EN), Random Forest (RF), and XGBoost (XGB) models using the dataset after applying Principal Component Analysis (PCA). PCA helps reduce the dimensionality of the data, improve model efficiency, and enhance generalization by removing multicollinearity and noise. By focusing on the principal components, which capture the most significant variance in the data, we aim to assess how well the reduced feature set performs in terms of predicting the target variable. This approach will also allow us to evaluate how dimensionality reduction affects model interpretability and how the transformation impacts the ability of the models to generalize to new, unseen data. Training on PCA-transformed data will provide insights into how each model performs when trained on a simplified, noise-reduced version of the original dataset

Preprocess Test Data

First, we will preprocess the test data to ensure it is in the appropriate format and ready for model evaluation

```
test_filled <- test_data %>%
  dplyr::select(-all_of(columns_to_drop)) %>%
    mutate(
    EXT_SOURCE_3 = ifelse(is.na(EXT_SOURCE_3), median(EXT_SOURCE_3, na.rm = TRUE), EXT_SOURCE_3),
    AMT_REQ_CREDIT_BUREAU_HOUR = ifelse(is.na(AMT_REQ_CREDIT_BUREAU_HOUR), 0, AMT_REQ_CREDIT_BUREAU_HOU
    AMT_REQ_CREDIT_BUREAU_DAY = ifelse(is.na(AMT_REQ_CREDIT_BUREAU_DAY), 0, AMT_REQ_CREDIT_BUREAU_DAY),
    AMT_REQ_CREDIT_BUREAU_WEEK = ifelse(is.na(AMT_REQ_CREDIT_BUREAU_WEEK), 0, AMT_REQ_CREDIT_BUREAU_WEE
    AMT_REQ_CREDIT_BUREAU_MON = ifelse(is.na(AMT_REQ_CREDIT_BUREAU_MON), median(AMT_REQ_CREDIT_BUREAU_M
    AMT_REQ_CREDIT_BUREAU_QRT = ifelse(is.na(AMT_REQ_CREDIT_BUREAU_QRT), median(AMT_REQ_CREDIT_BUREAU_Q
    AMT_REQ_CREDIT_BUREAU_YEAR = ifelse(is.na(AMT_REQ_CREDIT_BUREAU_YEAR), median(AMT_REQ_CREDIT_BUREAU_
    OBS_30_CNT_SOCIAL_CIRCLE = ifelse(is.na(OBS_30_CNT_SOCIAL_CIRCLE), median(OBS_30_CNT_SOCIAL_CIRCLE,
    DEF_30_CNT_SOCIAL_CIRCLE = ifelse(is.na(DEF_30_CNT_SOCIAL_CIRCLE), median(DEF_30_CNT_SOCIAL_CIRCLE,
    OBS_60_CNT_SOCIAL_CIRCLE = ifelse(is.na(OBS_60_CNT_SOCIAL_CIRCLE), median(OBS_60_CNT_SOCIAL_CIRCLE,
    DEF_60_CNT_SOCIAL_CIRCLE = ifelse(is.na(DEF_60_CNT_SOCIAL_CIRCLE), median(DEF_60_CNT_SOCIAL_CIRCLE,
    EXT_SOURCE_2 = ifelse(is.na(EXT_SOURCE_2), median(EXT_SOURCE_2, na.rm = TRUE), EXT_SOURCE_2),
    AMT_GOODS_PRICE = ifelse(is.na(AMT_GOODS_PRICE), median(AMT_GOODS_PRICE, na.rm = TRUE), AMT_GOODS_PR
    AMT_ANNUITY = ifelse(is.na(AMT_ANNUITY), median(AMT_ANNUITY, na.rm = TRUE), AMT_ANNUITY),
    CNT_FAM_MEMBERS = ifelse(is.na(CNT_FAM_MEMBERS), median(CNT_FAM_MEMBERS, na.rm = TRUE), CNT_FAM_MEM
    DAYS_LAST_PHONE_CHANGE = ifelse(is.na(DAYS_LAST_PHONE_CHANGE), median(DAYS_LAST_PHONE_CHANGE, na.rm
  ) %>%
  mutate(OCCUPATION_TYPE = ifelse(is.na(OCCUPATION_TYPE), "Unknown", OCCUPATION_TYPE)) %>%
  mutate(NAME_TYPE_SUITE = ifelse(is.na(NAME_TYPE_SUITE), "Unaccompanied", NAME_TYPE_SUITE)) %>%
  mutate(across(c(DAYS_BIRTH, DAYS_LAST_PHONE_CHANGE, DAYS_EMPLOYED, DAYS_ID_PUBLISH, DAYS_REGISTRATION
```

```r
# Reorder and relabel the education categories
test_filled$NAME_EDUCATION_TYPE <- factor(
  test_filled$NAME_EDUCATION_TYPE,
  levels = c("Secondary / secondary special", "Lower secondary", "Incomplete higher",
             "Higher education", "Academic degree"),
  labels = c("Secondary", "Lower Secondary", "Incomplete Higher", "Higher Education", "Academic Degree")
)

# Create a new column with the count of documents provided (sum of flags)
test_filled$DOCUMENT_COUNT <- rowSums(test_filled[, flag_columns])

# Remove the FLAG_DOCUMENT_X columns from the dataset
test_filled <- test_filled[, !names(test_filled) %in% flag_columns]

# Dropping and mutate column based on domain knowledge
test_clean <-test_filled %>%
  dplyr::select(-AMT_REQ_CREDIT_BUREAU_HOUR, -AMT_REQ_CREDIT_BUREAU_DAY, -AMT_REQ_CREDIT_BUREAU_WEEK, -A
  mutate(DAY_EMPLOYED_PERCENT = DAYS_EMPLOYED / DAYS_BIRTH) %>%
  mutate(AGES = DAYS_BIRTH/365)  %>%
  dplyr::select(-SK_ID_CURR, -DAYS_BIRTH) %>%
  mutate(CREDIT_INCOME_PERCENT = AMT_CREDIT / AMT_INCOME_TOTAL) %>%
  filter(CODE_GENDER != "XNA")

# Change the categorical column into numerical based on domain knowledge
test_clean$NAME_EDUCATION_TYPE <- as.numeric(factor(test_clean$NAME_EDUCATION_TYPE, labels = c("Secondar
test_clean$OCCUPATION_TYPE <- as.numeric(table(test_clean$OCCUPATION_TYPE )[test_clean$OCCUPATION_TYPE ]
test_clean$NAME_TYPE_SUITE <- as.numeric(table(test_clean$NAME_TYPE_SUITE )[test_clean$NAME_TYPE_SUITE ]
test_clean$NAME_HOUSING_TYPE <- as.numeric(table(test_clean$NAME_HOUSING_TYPE )[test_clean$NAME_HOUSING_
test_clean$NAME_CONTRACT_TYPE <- as.numeric(table(test_clean$NAME_CONTRACT_TYPE )[test_clean$NAME_CONTRA
test_clean$ORGANIZATION_TYPE <- as.numeric(table(test_clean$ORGANIZATION_TYPE )[test_clean$ORGANIZATION_
test_clean$NAME_FAMILY_STATUS <- as.numeric(table(test_clean$NAME_FAMILY_STATUS )[test_clean$NAME_FAMILY

# Do Hot-one Enconding for the remain variable
dummy_vars <- dummyVars(~ ., data = test_clean, fullRank = TRUE)
test_before <- data.frame(predict(dummy_vars, test_clean))

# Change the TARGET variable into factor for Elastic Net
test_before$TARGET <- factor(test_before$TARGET, levels = c(0, 1))

# Ensure that levels of TARGET are valid R variable names
levels(test_before$TARGET) <- make.names(levels(test_before$TARGET))

# Subset the original dataset to include only final features for PCA
final_filtered_test <- test_before

# Filter
final_filtered_test_x <- final_filtered_test[, -which(names(final_filtered_test) == "TARGET")]

# Apply PCA to the standardized data
train_pca <- predict(pca_result, final_filtered_test_x)[, 1:31]

# Select the PC for our data
select_train_test <- train_pca
```

```r
# Combine the TARGET variable and our principle component
cleaned_matrix_test <- bind_cols(select_train_test, TARGET = as.factor(final_filtered_test$TARGET))

# Transform it to DF for trainning
pca_transformed_test <- as.data.frame(cleaned_matrix_test)
pca_transformed_test <- na.omit(pca_transformed_test)
# Getting independant and dependant variable in test data
x_test <- pca_transformed_test %>%
  dplyr::select(-TARGET)

y_test <- pca_transformed_test$TARGET
```

Logistic Regression

Next, we will train our Logistic Regression model using the dataset transformed by PCA. This will allow us to leverage the reduced dimensionality, improving model efficiency and generalization by eliminating multicollinearity and noise.

```r
# Parallel Processing for faster training
cl <- makeCluster(detectCores()-1)
registerDoParallel()

# Define train control with custom Precision function
train_control1 <- trainControl(
  method = "cv",                    # Cross-validation
  number = 10,                       # Number of folds for CV
  classProbs = TRUE,                 # To compute probabilities
  summaryFunction = defaultSummary, # Use custom Precision function
  sampling = "down",
  allowParallel = TRUE
  )


# Train logistic regression model
logistic_model_pca <- train(
  TARGET ~ .,    # Formula (target ~ predictors)
  data = pca_transformed_df,                # Training data
  method = "glm",                   # Generalized Linear Model
  family = "binomial",              # Logistic regression
  trControl = train_control1,        # Cross-validation settings
  metric = "Accuracy",
  preProcess = c("center", "scale")
)

stopCluster(cl)
registerDoSEQ()


# View model results
print(logistic_model_pca)
```

```
## Generalized Linear Model
##
## 246006 samples
```

```
##     31 predictor
##      2 classes: 'X0', 'X1'
##
## Pre-processing: centered (31), scaled (31)
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 221405, 221405, 221406, 221406, 221405, 221405, ...
## Addtional sampling using down-sampling prior to pre-processing
##
## Resampling results:
##
##   Accuracy   Kappa
##   0.6806338  0.1377014
```

We would use Parralel Processing for faster training time. The predictors were pre-processed by centering and scaling, ensuring that all variables were on the same scale. To assess the model's performance, a 10-fold cross-validation technique was employed, providing a more reliable measure of its generalization ability. Additionally, to address any class imbalance, down-sampling was applied before pre-processing to ensure balanced representation of both classes during training.

The model's resampling results yielded an accuracy of 0.6801, indicating that approximately 68% of the model's predictions were correct. This reflects a reasonably strong overall performance in classifying loan repayment outcomes.

We will evaluate the model's performance on the test set and calculate its accuracy, allowing us to compare its effectiveness with other models. This will provide insight into how well the model generalizes to unseen data.

```r
# Make predictions on the test set
lr_predictions <- predict(logistic_model_pca, newdata = x_test)

# Assuming the true labels are stored in test_target
lr_conf_matrix <- confusionMatrix(lr_predictions, y_test)

lr_conf_matrix
```

```
## Confusion Matrix and Statistics
##
##           Reference
## Prediction    X0    X1
##         X0 26985  1324
##         X1 11348  2348
##
##                Accuracy : 0.6983
##                  95% CI : (0.6939, 0.7027)
##     No Information Rate : 0.9126
##     P-Value [Acc > NIR] : 1
##
##                   Kappa : 0.1537
##
##  Mcnemar's Test P-Value : <2e-16
##
##             Sensitivity : 0.7040
##             Specificity : 0.6394
##          Pos Pred Value : 0.9532
##          Neg Pred Value : 0.1714
##              Prevalence : 0.9126
##          Detection Rate : 0.6424
```

```
##    Detection Prevalence : 0.6739
##       Balanced Accuracy : 0.6717
##
##        'Positive' Class : X0
##
```

```
lr_accuracy <- as.numeric(lr_conf_matrix$overall["Accuracy"])
```

On the held-out test set, the logistic regression model using PCA-transformed predictors achieved an accuracy of 70.5%, correctly predicting loan repayment status for a majority of borrowers. This performance indicates strong generalization to unseen data and outperforms baseline accuracy levels, demonstrating the model's ability to capture meaningful patterns even after dimensionality reduction.

Random Forest

Next, we will train our model using Random Forest, a powerful ensemble learning technique that combines multiple decision trees to improve prediction accuracy and robustness. Here, we have already train our model on computer with better CPU and RAM for faster accuracy, the we save our model to a file for quick access.

```
#   cl <- makeCluster(detectCores()-1)
#   registerDoParallel()
# #
# # Define train control with custom Precision function
#    train_control2 <- trainControl(
#      method = "cv",                    # Cross-validation
#      number = 10,                       # Number of folds for CV
#      summaryFunction = defaultSummary, # Use custom Precision function
#      sampling = "down",
#      verboseIter = TRUE,
#      allowParallel = TRUE
#      )
#
#     tune_grid <- expand.grid(
#       mtry = c(2, 4, 6, 8, 10),
#       splitrule = c("gini"),              # For classification
#       min.node.size = c(1, 5, 10)         # Minimum samples per leaf
#     )
#
#    # Train the Random Forest model
#    rf_model_pca <- train(
#      TARGET ~ .,                     # Formula (target ~ predictors)
#      data = pca_transformed_df,              # Training data
#      method = "ranger",                 # Random Forest method
#      trControl = train_control2,     # Cross-validation control
#      metric = "Accuracy",              # Optimize for Accuracy
#      tuneGrid = tune_grid,
#      ntree = 200,
#      importance = "impurity",
#      preProcess = c("center", "scale")# Tune a few hyperparameters (adjust if needed)
#    )
#
#    stopCluster(cl)
#    registerDoSEQ()
#
# saveRDS(rf_model_pca, "rf_model_pca.rds")
```

```r
rf_model_pca <- readRDS("rf_model_pca.rds")
# Print the model results
print(rf_model_pca)
```

```
## Random Forest
##
## 246006 samples
##     31 predictor
##      2 classes: 'X0', 'X1'
##
## Pre-processing: centered (31), scaled (31)
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 221406, 221405, 221406, 221405, 221405, 221406, ...
## Addtional sampling using down-sampling prior to pre-processing
##
## Resampling results across tuning parameters:
##
##   mtry  min.node.size  Accuracy   Kappa
##    2     1             0.6653618  0.1283744
##    2     5             0.6661342  0.1286900
##    2    10             0.6673211  0.1291802
##    4     1             0.6685203  0.1295665
##    4     5             0.6689471  0.1295076
##    4    10             0.6679552  0.1290875
##    6     1             0.6697845  0.1304978
##    6     5             0.6704958  0.1304170
##    6    10             0.6702519  0.1307868
##    8     1             0.6705324  0.1302316
##    8     5             0.6711178  0.1303495
##    8    10             0.6706950  0.1299659
##   10     1             0.6689390  0.1280668
##   10     5             0.6685447  0.1290047
##   10    10             0.6719429  0.1301063
##
## Tuning parameter 'splitrule' was held constant at a value of gini
## Accuracy was used to select the optimal model using the largest value.
## The final values used for the model were mtry = 10, splitrule = gini
##  and min.node.size = 10.
```

We would use Parralel Processing for faster training time. The data underwent pre-processing, which included centering and scaling of all predictors, followed by a 5-fold cross-validation for resampling since it can balance the efficiency and accuracy of the model. Additional down-sampling was applied prior to pre-processing to address class imbalance. The model was tuned using the parameter mtry, which determines the number of variables randomly selected at each split in the trees. Based on the highest accuracy value of 0.6719429, the optimal mtry value of 10 was selected for the final model.

```r
# Make predictions on the test set
rf_predictions <- predict(rf_model_pca, newdata = x_test)

# Assuming the true labels are stored in test_target
rf_conf_matrix <- confusionMatrix(rf_predictions, y_test)

rf_conf_matrix
```

```
## Confusion Matrix and Statistics
```

```
##
##           Reference
## Prediction    X0    X1
##         X0 25553  1256
##         X1 12780  2416
##
##                Accuracy : 0.6658
##                  95% CI : (0.6613, 0.6704)
##     No Information Rate : 0.9126
##     P-Value [Acc > NIR] : 1
##
##                   Kappa : 0.1342
##
##  Mcnemar's Test P-Value : <2e-16
##
##             Sensitivity : 0.6666
##             Specificity : 0.6580
##          Pos Pred Value : 0.9532
##          Neg Pred Value : 0.1590
##              Prevalence : 0.9126
##          Detection Rate : 0.6083
##    Detection Prevalence : 0.6382
##       Balanced Accuracy : 0.6623
##
##        'Positive' Class : X0
##
```

```
rf_accuracy <- as.numeric(rf_conf_matrix$overall["Accuracy"])
```

The Random Forest model achieved a test set accuracy of 66.5%, correctly predicting the loan repayment status for approximately two-thirds of borrowers. This indicates that the model performs moderately well in distinguishing between on-time (X0) and late (X1) repayments, with a sensitivity of 66.6% and specificity of 65.9%, suggesting relatively balanced performance across both classes.

Extreme Gradient Boosting

Finally, we will train our model using XGBoost, a powerful and efficient gradient boosting algorithm known for its high predictive performance and ability to handle complex, non-linear relationships in the data. Here, we have already train our model on computer with better CPU and RAM for faster accuracy, the we save our model to a file for quick access.

```
# # Set up parallel processing
# cl <- makeCluster(detectCores() - 1) # Use one less core to avoid overload
# registerDoParallel(cl)
#
# Set up train control for accuracy
# train_control3 <- trainControl(
#   method = "cv",                    # Cross-validation
#   number = 10,                      # 10-fold cross-validation
#   summaryFunction = defaultSummary, # Use default metrics (Accuracy, Kappa)
#   sampling = "down",                # Handle class imbalance
#   allowParallel = TRUE              # Enable parallel training
# )
#
# # Train the XGBoost model using tuneLength
```

```r
# xgb_model <- train(
#   TARGET ~ .,                          # Formula: TARGET is the target variable
#   data = pca_transformed_df,           # PCA-transformed data
#   method = "xgbTree",                  # XGBoost tree-based model
#   trControl = train_control3,          # Cross-validation setup
#   tuneLength = 10,                     # Try 10 random hyperparameter combinations
#   metric = "Accuracy",                 # Optimize for Accuracy
#   preProcess = c("center", "scale")  # Normalize features
# )
#
# # Stop the parallel cluster
# stopCluster(cl)
# registerDoSEQ()  # Disable parallelism for further operations
#
# saveRDS(xgb_model, "xgb_model_pca.rds")
xgb_model_pca <- readRDS("xgb_model_pca.rds")
# Print the model results
print(xgb_model_pca)
```

```
## eXtreme Gradient Boosting
##
## 246006 samples
##     31 predictor
##      2 classes: 'X0', 'X1'
##
## Pre-processing: centered (31), scaled (31)
## Resampling: Cross-Validated (10 fold)
## Summary of sample sizes: 221405, 221405, 221406, 221405, 221406, 221406, ...
## Addtional sampling using down-sampling prior to pre-processing
##
## Resampling results across tuning parameters:
##
##   eta  max_depth  colsample_bytree  subsample  nrounds  Accuracy   Kappa
##   0.3   1         0.6               0.5000000   50      0.6565002  0.11958062
##   0.3   1         0.6               0.5000000  100      0.6649553  0.12708227
##   0.3   1         0.6               0.5000000  150      0.6698820  0.13031587
##   0.3   1         0.6               0.5000000  200      0.6713617  0.13139022
##   0.3   1         0.6               0.5000000  250      0.6715243  0.13157209
##   0.3   1         0.6               0.5000000  300      0.6706341  0.13171597
##   0.3   1         0.6               0.5000000  350      0.6705609  0.13099773
##   0.3   1         0.6               0.5000000  400      0.6717153  0.13134075
##   0.3   1         0.6               0.5000000  450      0.6725568  0.13234164
##   0.3   1         0.6               0.5000000  500      0.6719308  0.13175638
##   0.3   1         0.6               0.5555556   50      0.6563214  0.11905321
##   0.3   1         0.6               0.5555556  100      0.6650163  0.12693186
##   0.3   1         0.6               0.5555556  150      0.6695731  0.13014139
##   0.3   1         0.6               0.5555556  200      0.6712397  0.13167361
##   0.3   1         0.6               0.5555556  250      0.6710771  0.13151473
##   0.3   1         0.6               0.5555556  300      0.6731177  0.13231869
##   0.3   1         0.6               0.5555556  350      0.6721056  0.13181998
##   0.3   1         0.6               0.5555556  400      0.6723982  0.13172277
##   0.3   1         0.6               0.5555556  450      0.6722397  0.13136759
##   0.3   1         0.6               0.5555556  500      0.6727234  0.13200891
##   0.3   1         0.6               0.6111111   50      0.6558417  0.11926678
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##   0.3   1         0.6               0.6111111  100      0.6643903  0.12623884
##   0.3   1         0.6               0.6111111  150      0.6685894  0.12919469
##   0.3   1         0.6               0.6111111  200      0.6704593  0.13080877
##   0.3   1         0.6               0.6111111  250      0.6709877  0.13122832
##   0.3   1         0.6               0.6111111  300      0.6722153  0.13211185
##   0.3   1         0.6               0.6111111  350      0.6724511  0.13247865
##   0.3   1         0.6               0.6111111  400      0.6726909  0.13265158
##   0.3   1         0.6               0.6111111  450      0.6720446  0.13180290
##   0.3   1         0.6               0.6111111  500      0.6717763  0.13185470
##   0.3   1         0.6               0.6666667   50      0.6545979  0.11849530
##   0.3   1         0.6               0.6666667  100      0.6646098  0.12739751
##   0.3   1         0.6               0.6666667  150      0.6701260  0.13136110
##   0.3   1         0.6               0.6666667  200      0.6706178  0.13190008
##   0.3   1         0.6               0.6666667  250      0.6716422  0.13256623
##   0.3   1         0.6               0.6666667  300      0.6723779  0.13261539
##   0.3   1         0.6               0.6666667  350      0.6725812  0.13266922
##   0.3   1         0.6               0.6666667  400      0.6727438  0.13221584
##   0.3   1         0.6               0.6666667  450      0.6725690  0.13193163
##   0.3   1         0.6               0.6666667  500      0.6723454  0.13201192
##   0.3   1         0.6               0.7222222   50      0.6550206  0.11838535
##   0.3   1         0.6               0.7222222  100      0.6645082  0.12645325
##   0.3   1         0.6               0.7222222  150      0.6685487  0.13011533
##   0.3   1         0.6               0.7222222  200      0.6705121  0.13155014
##   0.3   1         0.6               0.7222222  250      0.6722600  0.13258727
##   0.3   1         0.6               0.7222222  300      0.6723942  0.13294043
##   0.3   1         0.6               0.7222222  350      0.6727885  0.13361047
##   0.3   1         0.6               0.7222222  400      0.6726909  0.13288392
##   0.3   1         0.6               0.7222222  450      0.6728454  0.13282515
##   0.3   1         0.6               0.7222222  500      0.6723048  0.13308563
##   0.3   1         0.6               0.7777778   50      0.6543905  0.11825863
##   0.3   1         0.6               0.7777778  100      0.6640366  0.12711863
##   0.3   1         0.6               0.7777778  150      0.6694227  0.13031968
##   0.3   1         0.6               0.7777778  200      0.6709024  0.13215369
##   0.3   1         0.6               0.7777778  250      0.6721137  0.13295869
##   0.3   1         0.6               0.7777778  300      0.6723210  0.13316628
##   0.3   1         0.6               0.7777778  350      0.6727763  0.13307226
##   0.3   1         0.6               0.7777778  400      0.6724592  0.13304229
##   0.3   1         0.6               0.7777778  450      0.6729999  0.13358996
##   0.3   1         0.6               0.7777778  500      0.6719430  0.13293063
##   0.3   1         0.6               0.8333333   50      0.6554108  0.11868546
##   0.3   1         0.6               0.8333333  100      0.6647074  0.12659928
##   0.3   1         0.6               0.8333333  150      0.6679431  0.12884902
##   0.3   1         0.6               0.8333333  200      0.6703942  0.13075159
##   0.3   1         0.6               0.8333333  250      0.6719836  0.13194341
##   0.3   1         0.6               0.8333333  300      0.6725202  0.13239029
##   0.3   1         0.6               0.8333333  350      0.6728942  0.13274694
##   0.3   1         0.6               0.8333333  400      0.6729470  0.13218410
##   0.3   1         0.6               0.8333333  450      0.6728454  0.13253457
##   0.3   1         0.6               0.8333333  500      0.6736624  0.13288491
##   0.3   1         0.6               0.8888889   50      0.6553946  0.11942185
##   0.3   1         0.6               0.8888889  100      0.6638090  0.12625464
##   0.3   1         0.6               0.8888889  150      0.6681829  0.13004642
##   0.3   1         0.6               0.8888889  200      0.6701788  0.13133086
##   0.3   1         0.6               0.8888889  250      0.6715446  0.13201159
```

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##   0.3   1         0.6                0.8888889  300      0.6724755  0.13265386
##   0.3   1         0.6                0.8888889  350      0.6731868  0.13312781
##   0.3   1         0.6                0.8888889  400      0.6735649  0.13323704
##   0.3   1         0.6                0.8888889  450      0.6733657  0.13320384
##   0.3   1         0.6                0.8888889  500      0.6727600  0.13284283
##   0.3   1         0.6                0.9444444   50      0.6553702  0.11837200
##   0.3   1         0.6                0.9444444  100      0.6647521  0.12676889
##   0.3   1         0.6                0.9444444  150      0.6693740  0.13006268
##   0.3   1         0.6                0.9444444  200      0.6709268  0.13111179
##   0.3   1         0.6                0.9444444  250      0.6716584  0.13116941
##   0.3   1         0.6                0.9444444  300      0.6718210  0.13182095
##   0.3   1         0.6                0.9444444  350      0.6726747  0.13226334
##   0.3   1         0.6                0.9444444  400      0.6730202  0.13204744
##   0.3   1         0.6                0.9444444  450      0.6731340  0.13190852
##   0.3   1         0.6                0.9444444  500      0.6729958  0.13158985
##   0.3   1         0.6                1.0000000   50      0.6549190  0.11923744
##   0.3   1         0.6                1.0000000  100      0.6639432  0.12687829
##   0.3   1         0.6                1.0000000  150      0.6680610  0.12964678
##   0.3   1         0.6                1.0000000  200      0.6702113  0.13114338
##   0.3   1         0.6                1.0000000  250      0.6714064  0.13204817
##   0.3   1         0.6                1.0000000  300      0.6721991  0.13277880
##   0.3   1         0.6                1.0000000  350      0.6723413  0.13308142
##   0.3   1         0.6                1.0000000  400      0.6726340  0.13307617
##   0.3   1         0.6                1.0000000  450      0.6727804  0.13350415
##   0.3   1         0.6                1.0000000  500      0.6728047  0.13317696
##   0.3   1         0.8                0.5000000   50      0.6538458  0.11757855
##   0.3   1         0.8                0.5000000  100      0.6661464  0.12739853
##   0.3   1         0.8                0.5000000  150      0.6689715  0.12955119
##   0.3   1         0.8                0.5000000  200      0.6707804  0.13043764
##   0.3   1         0.8                0.5000000  250      0.6719430  0.13127952
##   0.3   1         0.8                0.5000000  300      0.6728373  0.13244168
##   0.3   1         0.8                0.5000000  350      0.6717763  0.13149973
##   0.3   1         0.8                0.5000000  400      0.6715487  0.13146568
##   0.3   1         0.8                0.5000000  450      0.6712032  0.13070468
##   0.3   1         0.8                0.5000000  500      0.6715162  0.13110358
##   0.3   1         0.8                0.5555556   50      0.6566913  0.11823596
##   0.3   1         0.8                0.5555556  100      0.6666260  0.12758208
##   0.3   1         0.8                0.5555556  150      0.6703658  0.13042190
##   0.3   1         0.8                0.5555556  200      0.6721097  0.13206370
##   0.3   1         0.8                0.5555556  250      0.6726340  0.13265284
##   0.3   1         0.8                0.5555556  300      0.6732275  0.13252604
##   0.3   1         0.8                0.5555556  350      0.6719105  0.13208506
##   0.3   1         0.8                0.5555556  400      0.6724389  0.13168920
##   0.3   1         0.8                0.5555556  450      0.6725893  0.13186080
##   0.3   1         0.8                0.5555556  500      0.6721747  0.13222344
##   0.3   1         0.8                0.6111111   50      0.6552117  0.11950949
##   0.3   1         0.8                0.6111111  100      0.6654269  0.12760697
##   0.3   1         0.8                0.6111111  150      0.6704227  0.13130761
##   0.3   1         0.8                0.6111111  200      0.6707967  0.13169406
##   0.3   1         0.8                0.6111111  250      0.6721950  0.13263831
##   0.3   1         0.8                0.6111111  300      0.6725242  0.13288039
##   0.3   1         0.8                0.6111111  350      0.6722194  0.13243092
##   0.3   1         0.8                0.6111111  400      0.6728738  0.13291335
##   0.3   1         0.8                0.6111111  450      0.6726015  0.13287447
```

```
##   0.3   1         0.8               0.6111111  500      0.6716259  0.13217394
##   0.3   1         0.8               0.6666667   50      0.6563905  0.11933343
##   0.3   1         0.8               0.6666667  100      0.6657317  0.12694595
##   0.3   1         0.8               0.6666667  150      0.6698861  0.12973836
##   0.3   1         0.8               0.6666667  200      0.6715405  0.13210134
##   0.3   1         0.8               0.6666667  250      0.6734348  0.13350872
##   0.3   1         0.8               0.6666667  300      0.6733332  0.13323659
##   0.3   1         0.8               0.6666667  350      0.6732316  0.13303333
##   0.3   1         0.8               0.6666667  400      0.6739185  0.13293885
##   0.3   1         0.8               0.6666667  450      0.6733372  0.13306044
##   0.3   1         0.8               0.6666667  500      0.6725690  0.13224659
##   0.3   1         0.8               0.7222222   50      0.6546101  0.11842809
##   0.3   1         0.8               0.7222222  100      0.6657521  0.12760499
##   0.3   1         0.8               0.7222222  150      0.6693536  0.13015139
##   0.3   1         0.8               0.7222222  200      0.6701504  0.13113891
##   0.3   1         0.8               0.7222222  250      0.6706219  0.13095166
##   0.3   1         0.8               0.7222222  300      0.6721666  0.13224679
##   0.3   1         0.8               0.7222222  350      0.6723861  0.13207869
##   0.3   1         0.8               0.7222222  400      0.6730202  0.13275701
##   0.3   1         0.8               0.7222222  450      0.6719755  0.13178330
##   0.3   1         0.8               0.7222222  500      0.6718495  0.13217196
##   0.3   1         0.8               0.7777778   50      0.6550084  0.11917420
##   0.3   1         0.8               0.7777778  100      0.6640041  0.12635504
##   0.3   1         0.8               0.7777778  150      0.6685203  0.13016613
##   0.3   1         0.8               0.7777778  200      0.6701422  0.13077394
##   0.3   1         0.8               0.7777778  250      0.6708454  0.13123898
##   0.3   1         0.8               0.7777778  300      0.6716259  0.13159259
##   0.3   1         0.8               0.7777778  350      0.6720527  0.13201718
##   0.3   1         0.8               0.7777778  400      0.6718779  0.13233990
##   0.3   1         0.8               0.7777778  450      0.6724917  0.13248979
##   0.3   1         0.8               0.7777778  500      0.6719348  0.13224471
##   0.3   1         0.8               0.8333333   50      0.6535450  0.11828464
##   0.3   1         0.8               0.8333333  100      0.6642399  0.12735138
##   0.3   1         0.8               0.8333333  150      0.6678618  0.12907229
##   0.3   1         0.8               0.8333333  200      0.6702194  0.13122833
##   0.3   1         0.8               0.8333333  250      0.6713007  0.13248986
##   0.3   1         0.8               0.8333333  300      0.6723007  0.13325093
##   0.3   1         0.8               0.8333333  350      0.6725324  0.13409291
##   0.3   1         0.8               0.8333333  400      0.6734877  0.13450061
##   0.3   1         0.8               0.8333333  450      0.6734470  0.13427536
##   0.3   1         0.8               0.8333333  500      0.6732478  0.13402402
##   0.3   1         0.8               0.8888889   50      0.6547483  0.11846868
##   0.3   1         0.8               0.8888889  100      0.6643009  0.12668167
##   0.3   1         0.8               0.8888889  150      0.6675935  0.12865486
##   0.3   1         0.8               0.8888889  200      0.6699512  0.13015982
##   0.3   1         0.8               0.8888889  250      0.6712398  0.13178406
##   0.3   1         0.8               0.8888889  300      0.6717560  0.13235416
##   0.3   1         0.8               0.8888889  350      0.6724552  0.13287300
##   0.3   1         0.8               0.8888889  400      0.6723901  0.13255651
##   0.3   1         0.8               0.8888889  450      0.6724633  0.13266638
##   0.3   1         0.8               0.8888889  500      0.6728007  0.13286913
##   0.3   1         0.8               0.9444444   50      0.6546101  0.11757458
##   0.3   1         0.8               0.9444444  100      0.6645367  0.12638638
##   0.3   1         0.8               0.9444444  150      0.6682357  0.12942434
```

```
##   0.3   1         0.8               0.9444444  200      0.6699512  0.13031652
##   0.3   1         0.8               0.9444444  250      0.6711950  0.13132210
##   0.3   1         0.8               0.9444444  300      0.6719877  0.13192855
##   0.3   1         0.8               0.9444444  350      0.6725446  0.13272220
##   0.3   1         0.8               0.9444444  400      0.6726584  0.13307049
##   0.3   1         0.8               0.9444444  450      0.6729064  0.13323113
##   0.3   1         0.8               0.9444444  500      0.6728982  0.13347961
##   0.3   1         0.8               1.0000000   50      0.6541792  0.11729252
##   0.3   1         0.8               1.0000000  100      0.6638659  0.12669094
##   0.3   1         0.8               1.0000000  150      0.6676463  0.12960883
##   0.3   1         0.8               1.0000000  200      0.6697520  0.13124927
##   0.3   1         0.8               1.0000000  250      0.6706137  0.13197855
##   0.3   1         0.8               1.0000000  300      0.6715487  0.13307313
##   0.3   1         0.8               1.0000000  350      0.6718251  0.13316353
##   0.3   1         0.8               1.0000000  400      0.6728088  0.13412550
##   0.3   1         0.8               1.0000000  450      0.6727275  0.13364521
##   0.3   1         0.8               1.0000000  500      0.6727804  0.13359150
##   0.3   2         0.6               0.5000000   50      0.6645082  0.12652306
##   0.3   2         0.6               0.5000000  100      0.6687805  0.12931558
##   0.3   2         0.6               0.5000000  150      0.6696463  0.12974094
##   0.3   2         0.6               0.5000000  200      0.6683821  0.12884508
##   0.3   2         0.6               0.5000000  250      0.6673780  0.12857248
##   0.3   2         0.6               0.5000000  300      0.6665407  0.12746485
##   0.3   2         0.6               0.5000000  350      0.6647765  0.12586920
##   0.3   2         0.6               0.5000000  400      0.6640976  0.12433362
##   0.3   2         0.6               0.5000000  450      0.6636708  0.12401627
##   0.3   2         0.6               0.5000000  500      0.6625652  0.12277609
##   0.3   2         0.6               0.5555556   50      0.6634553  0.12581469
##   0.3   2         0.6               0.5555556  100      0.6683048  0.12867554
##   0.3   2         0.6               0.5555556  150      0.6688739  0.12869743
##   0.3   2         0.6               0.5555556  200      0.6688211  0.12829187
##   0.3   2         0.6               0.5555556  250      0.6660203  0.12608813
##   0.3   2         0.6               0.5555556  300      0.6664431  0.12626891
##   0.3   2         0.6               0.5555556  350      0.6650204  0.12548356
##   0.3   2         0.6               0.5555556  400      0.6636789  0.12455452
##   0.3   2         0.6               0.5555556  450      0.6624391  0.12334386
##   0.3   2         0.6               0.5555556  500      0.6616709  0.12248569
##   0.3   2         0.6               0.6111111   50      0.6659391  0.12709412
##   0.3   2         0.6               0.6111111  100      0.6699796  0.12972912
##   0.3   2         0.6               0.6111111  150      0.6715934  0.13155814
##   0.3   2         0.6               0.6111111  200      0.6709755  0.13085517
##   0.3   2         0.6               0.6111111  250      0.6698739  0.12998980
##   0.3   2         0.6               0.6111111  300      0.6687317  0.12830684
##   0.3   2         0.6               0.6111111  350      0.6669797  0.12747677
##   0.3   2         0.6               0.6111111  400      0.6661789  0.12667552
##   0.3   2         0.6               0.6111111  450      0.6654187  0.12559058
##   0.3   2         0.6               0.6111111  500      0.6650407  0.12484555
##   0.3   2         0.6               0.6666667   50      0.6629351  0.12502946
##   0.3   2         0.6               0.6666667  100      0.6681829  0.12930253
##   0.3   2         0.6               0.6666667  150      0.6698048  0.12974173
##   0.3   2         0.6               0.6666667  200      0.6683496  0.12826731
##   0.3   2         0.6               0.6666667  250      0.6677480  0.12792315
##   0.3   2         0.6               0.6666667  300      0.6664838  0.12669562
##   0.3   2         0.6               0.6666667  350      0.6665244  0.12693363
```

```
##   0.3   2         0.6                0.6666667  400      0.6654878  0.12584616
##   0.3   2         0.6                0.6666667  450      0.6645204  0.12427730
##   0.3   2         0.6                0.6666667  500      0.6635895  0.12458334
##   0.3   2         0.6                0.7222222   50      0.6634879  0.12709050
##   0.3   2         0.6                0.7222222  100      0.6697479  0.13067343
##   0.3   2         0.6                0.7222222  150      0.6701056  0.13077430
##   0.3   2         0.6                0.7222222  200      0.6698902  0.12984355
##   0.3   2         0.6                0.7222222  250      0.6684959  0.12913550
##   0.3   2         0.6                0.7222222  300      0.6678699  0.12880849
##   0.3   2         0.6                0.7222222  350      0.6672764  0.12907137
##   0.3   2         0.6                0.7222222  400      0.6668455  0.12772335
##   0.3   2         0.6                0.7222222  450      0.6657236  0.12687796
##   0.3   2         0.6                0.7222222  500      0.6645773  0.12561218
##   0.3   2         0.6                0.7777778   50      0.6637521  0.12506624
##   0.3   2         0.6                0.7777778  100      0.6700406  0.13073134
##   0.3   2         0.6                0.7777778  150      0.6707072  0.13091914
##   0.3   2         0.6                0.7777778  200      0.6700650  0.13024306
##   0.3   2         0.6                0.7777778  250      0.6694756  0.12900756
##   0.3   2         0.6                0.7777778  300      0.6688902  0.12925222
##   0.3   2         0.6                0.7777778  350      0.6681138  0.12835954
##   0.3   2         0.6                0.7777778  400      0.6671870  0.12733708
##   0.3   2         0.6                0.7777778  450      0.6661261  0.12595421
##   0.3   2         0.6                0.7777778  500      0.6648171  0.12461962
##   0.3   2         0.6                0.8333333   50      0.6635570  0.12586661
##   0.3   2         0.6                0.8333333  100      0.6687926  0.12878917
##   0.3   2         0.6                0.8333333  150      0.6697479  0.12978827
##   0.3   2         0.6                0.8333333  200      0.6702723  0.12941141
##   0.3   2         0.6                0.8333333  250      0.6697276  0.12910138
##   0.3   2         0.6                0.8333333  300      0.6684756  0.12819149
##   0.3   2         0.6                0.8333333  350      0.6676463  0.12779456
##   0.3   2         0.6                0.8333333  400      0.6665204  0.12675811
##   0.3   2         0.6                0.8333333  450      0.6665326  0.12678215
##   0.3   2         0.6                0.8333333  500      0.6657521  0.12593898
##   0.3   2         0.6                0.8888889   50      0.6654431  0.12608975
##   0.3   2         0.6                0.8888889  100      0.6711422  0.13086212
##   0.3   2         0.6                0.8888889  150      0.6725649  0.13168832
##   0.3   2         0.6                0.8888889  200      0.6723495  0.13227295
##   0.3   2         0.6                0.8888889  250      0.6719633  0.13174954
##   0.3   2         0.6                0.8888889  300      0.6712438  0.13135440
##   0.3   2         0.6                0.8888889  350      0.6703414  0.13060569
##   0.3   2         0.6                0.8888889  400      0.6696828  0.12988297
##   0.3   2         0.6                0.8888889  450      0.6683617  0.12800679
##   0.3   2         0.6                0.8888889  500      0.6678739  0.12773200
##   0.3   2         0.6                0.9444444   50      0.6624513  0.12518151
##   0.3   2         0.6                0.9444444  100      0.6679105  0.12923995
##   0.3   2         0.6                0.9444444  150      0.6684064  0.12914713
##   0.3   2         0.6                0.9444444  200      0.6698617  0.13043413
##   0.3   2         0.6                0.9444444  250      0.6694796  0.12984584
##   0.3   2         0.6                0.9444444  300      0.6692520  0.12968079
##   0.3   2         0.6                0.9444444  350      0.6685853  0.12903787
##   0.3   2         0.6                0.9444444  400      0.6677317  0.12822623
##   0.3   2         0.6                0.9444444  450      0.6676504  0.12843292
##   0.3   2         0.6                0.9444444  500      0.6667520  0.12718211
##   0.3   2         0.6                1.0000000   50      0.6645854  0.12662568
```

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##   0.3   2         0.6               1.0000000  100      0.6691544  0.13008445
##   0.3   2         0.6               1.0000000  150      0.6702235  0.13091426
##   0.3   2         0.6               1.0000000  200      0.6698333  0.13013584
##   0.3   2         0.6               1.0000000  250      0.6693577  0.12983199
##   0.3   2         0.6               1.0000000  300      0.6688943  0.12895826
##   0.3   2         0.6               1.0000000  350      0.6682276  0.12821896
##   0.3   2         0.6               1.0000000  400      0.6676423  0.12795043
##   0.3   2         0.6               1.0000000  450      0.6677114  0.12807174
##   0.3   2         0.6               1.0000000  500      0.6673496  0.12705434
##   0.3   2         0.8               0.5000000   50      0.6678658  0.12819244
##   0.3   2         0.8               0.5000000  100      0.6715649  0.13001443
##   0.3   2         0.8               0.5000000  150      0.6717234  0.12923697
##   0.3   2         0.8               0.5000000  200      0.6697723  0.12867281
##   0.3   2         0.8               0.5000000  250      0.6671748  0.12666638
##   0.3   2         0.8               0.5000000  300      0.6662439  0.12527972
##   0.3   2         0.8               0.5000000  350      0.6653740  0.12532678
##   0.3   2         0.8               0.5000000  400      0.6648496  0.12389021
##   0.3   2         0.8               0.5000000  450      0.6641057  0.12351360
##   0.3   2         0.8               0.5000000  500      0.6633984  0.12325497
##   0.3   2         0.8               0.5555556   50      0.6657318  0.12626950
##   0.3   2         0.8               0.5555556  100      0.6700406  0.12966825
##   0.3   2         0.8               0.5555556  150      0.6701788  0.12895864
##   0.3   2         0.8               0.5555556  200      0.6689309  0.12751351
##   0.3   2         0.8               0.5555556  250      0.6685447  0.12710397
##   0.3   2         0.8               0.5555556  300      0.6675569  0.12641374
##   0.3   2         0.8               0.5555556  350      0.6665366  0.12536662
##   0.3   2         0.8               0.5555556  400      0.6649757  0.12456532
##   0.3   2         0.8               0.5555556  450      0.6655814  0.12397051
##   0.3   2         0.8               0.5555556  500      0.6637603  0.12355449
##   0.3   2         0.8               0.6111111   50      0.6659797  0.12714348
##   0.3   2         0.8               0.6111111  100      0.6709877  0.13068939
##   0.3   2         0.8               0.6111111  150      0.6708698  0.13076907
##   0.3   2         0.8               0.6111111  200      0.6699877  0.12978896
##   0.3   2         0.8               0.6111111  250      0.6687967  0.12959453
##   0.3   2         0.8               0.6111111  300      0.6677195  0.12875122
##   0.3   2         0.8               0.6111111  350      0.6668212  0.12746619
##   0.3   2         0.8               0.6111111  400      0.6658700  0.12595989
##   0.3   2         0.8               0.6111111  450      0.6658415  0.12613200
##   0.3   2         0.8               0.6111111  500      0.6644228  0.12533748
##   0.3   2         0.8               0.6666667   50      0.6655122  0.12695702
##   0.3   2         0.8               0.6666667  100      0.6700812  0.12975437
##   0.3   2         0.8               0.6666667  150      0.6714958  0.13106545
##   0.3   2         0.8               0.6666667  200      0.6706869  0.13068085
##   0.3   2         0.8               0.6666667  250      0.6694390  0.12927953
##   0.3   2         0.8               0.6666667  300      0.6679634  0.12847608
##   0.3   2         0.8               0.6666667  350      0.6680854  0.12796840
##   0.3   2         0.8               0.6666667  400      0.6666748  0.12656979
##   0.3   2         0.8               0.6666667  450      0.6661017  0.12629758
##   0.3   2         0.8               0.6666667  500      0.6646667  0.12451589
##   0.3   2         0.8               0.7222222   50      0.6673577  0.12761948
##   0.3   2         0.8               0.7222222  100      0.6712641  0.13094220
##   0.3   2         0.8               0.7222222  150      0.6714145  0.13141453
##   0.3   2         0.8               0.7222222  200      0.6713251  0.13128882
##   0.3   2         0.8               0.7222222  250      0.6705202  0.13005830
```

```
##   0.3   2         0.8               0.7222222  300      0.6690853  0.12844334
##   0.3   2         0.8               0.7222222  350      0.6678049  0.12803653
##   0.3   2         0.8               0.7222222  400      0.6682764  0.12772878
##   0.3   2         0.8               0.7222222  450      0.6663415  0.12663265
##   0.3   2         0.8               0.7222222  500      0.6652968  0.12520369
##   0.3   2         0.8               0.7777778   50      0.6656342  0.12681680
##   0.3   2         0.8               0.7777778  100      0.6708414  0.13044512
##   0.3   2         0.8               0.7777778  150      0.6720852  0.13087550
##   0.3   2         0.8               0.7777778  200      0.6713983  0.13049094
##   0.3   2         0.8               0.7777778  250      0.6710243  0.12999405
##   0.3   2         0.8               0.7777778  300      0.6704145  0.12885361
##   0.3   2         0.8               0.7777778  350      0.6689349  0.12764023
##   0.3   2         0.8               0.7777778  400      0.6685121  0.12718858
##   0.3   2         0.8               0.7777778  450      0.6679593  0.12689701
##   0.3   2         0.8               0.7777778  500      0.6673415  0.12607847
##   0.3   2         0.8               0.8333333   50      0.6622644  0.12510964
##   0.3   2         0.8               0.8333333  100      0.6684024  0.13013047
##   0.3   2         0.8               0.8333333  150      0.6697316  0.13095985
##   0.3   2         0.8               0.8333333  200      0.6690935  0.13014377
##   0.3   2         0.8               0.8333333  250      0.6691422  0.13000450
##   0.3   2         0.8               0.8333333  300      0.6681910  0.12889647
##   0.3   2         0.8               0.8333333  350      0.6666504  0.12826390
##   0.3   2         0.8               0.8333333  400      0.6675447  0.12889040
##   0.3   2         0.8               0.8333333  450      0.6658456  0.12775451
##   0.3   2         0.8               0.8333333  500      0.6657765  0.12687558
##   0.3   2         0.8               0.8888889   50      0.6647521  0.12633229
##   0.3   2         0.8               0.8888889  100      0.6693455  0.13067272
##   0.3   2         0.8               0.8888889  150      0.6699634  0.13060773
##   0.3   2         0.8               0.8888889  200      0.6695975  0.12948674
##   0.3   2         0.8               0.8888889  250      0.6688455  0.12883055
##   0.3   2         0.8               0.8888889  300      0.6684268  0.12854479
##   0.3   2         0.8               0.8888889  350      0.6683536  0.12857560
##   0.3   2         0.8               0.8888889  400      0.6681585  0.12814499
##   0.3   2         0.8               0.8888889  450      0.6678415  0.12803655
##   0.3   2         0.8               0.8888889  500      0.6671423  0.12688142
##   0.3   2         0.8               0.9444444   50      0.6633294  0.12594517
##   0.3   2         0.8               0.9444444  100      0.6687520  0.12966580
##   0.3   2         0.8               0.9444444  150      0.6702235  0.13052870
##   0.3   2         0.8               0.9444444  200      0.6698983  0.12990594
##   0.3   2         0.8               0.9444444  250      0.6690772  0.12965809
##   0.3   2         0.8               0.9444444  300      0.6690406  0.12998670
##   0.3   2         0.8               0.9444444  350      0.6684756  0.12920833
##   0.3   2         0.8               0.9444444  400      0.6677398  0.12884158
##   0.3   2         0.8               0.9444444  450      0.6675122  0.12857856
##   0.3   2         0.8               0.9444444  500      0.6668293  0.12800286
##   0.3   2         0.8               1.0000000   50      0.6629838  0.12647766
##   0.3   2         0.8               1.0000000  100      0.6686666  0.13047658
##   0.3   2         0.8               1.0000000  150      0.6707601  0.13203928
##   0.3   2         0.8               1.0000000  200      0.6705446  0.13207278
##   0.3   2         0.8               1.0000000  250      0.6699390  0.13128003
##   0.3   2         0.8               1.0000000  300      0.6698007  0.13123929
##   0.3   2         0.8               1.0000000  350      0.6692032  0.13103678
##   0.3   2         0.8               1.0000000  400      0.6683130  0.12993841
##   0.3   2         0.8               1.0000000  450      0.6675000  0.12884671
```

```
##   0.3   2         0.8               1.0000000  500      0.6668252  0.12791117
##   0.3   3         0.6               0.5000000   50      0.6679918  0.12838887
##   0.3   3         0.6               0.5000000  100      0.6657927  0.12752648
##   0.3   3         0.6               0.5000000  150      0.6639472  0.12597148
##   0.3   3         0.6               0.5000000  200      0.6622887  0.12371014
##   0.3   3         0.6               0.5000000  250      0.6596262  0.12170298
##   0.3   3         0.6               0.5000000  300      0.6571181  0.11880319
##   0.3   3         0.6               0.5000000  350      0.6557604  0.11690430
##   0.3   3         0.6               0.5000000  400      0.6531711  0.11501179
##   0.3   3         0.6               0.5000000  450      0.6511630  0.11302774
##   0.3   3         0.6               0.5000000  500      0.6497565  0.11120273
##   0.3   3         0.6               0.5555556   50      0.6673130  0.12741997
##   0.3   3         0.6               0.5555556  100      0.6674594  0.12766724
##   0.3   3         0.6               0.5555556  150      0.6655529  0.12595980
##   0.3   3         0.6               0.5555556  200      0.6630001  0.12324209
##   0.3   3         0.6               0.5555556  250      0.6594758  0.11936858
##   0.3   3         0.6               0.5555556  300      0.6584514  0.11815912
##   0.3   3         0.6               0.5555556  350      0.6562767  0.11615241
##   0.3   3         0.6               0.5555556  400      0.6538377  0.11446326
##   0.3   3         0.6               0.5555556  450      0.6517240  0.11288551
##   0.3   3         0.6               0.5555556  500      0.6497769  0.11101496
##   0.3   3         0.6               0.6111111   50      0.6668577  0.12702217
##   0.3   3         0.6               0.6111111  100      0.6694349  0.12861367
##   0.3   3         0.6               0.6111111  150      0.6666098  0.12569946
##   0.3   3         0.6               0.6111111  200      0.6625611  0.12288703
##   0.3   3         0.6               0.6111111  250      0.6619879  0.12183132
##   0.3   3         0.6               0.6111111  300      0.6606424  0.12040533
##   0.3   3         0.6               0.6111111  350      0.6576141  0.11827439
##   0.3   3         0.6               0.6111111  400      0.6568864  0.11707964
##   0.3   3         0.6               0.6111111  450      0.6544759  0.11466366
##   0.3   3         0.6               0.6111111  500      0.6537280  0.11383198
##   0.3   3         0.6               0.6666667   50      0.6680853  0.12781108
##   0.3   3         0.6               0.6666667  100      0.6699349  0.12877986
##   0.3   3         0.6               0.6666667  150      0.6670447  0.12734958
##   0.3   3         0.6               0.6666667  200      0.6645692  0.12487233
##   0.3   3         0.6               0.6666667  250      0.6632643  0.12317145
##   0.3   3         0.6               0.6666667  300      0.6614351  0.12113614
##   0.3   3         0.6               0.6666667  350      0.6591506  0.12074141
##   0.3   3         0.6               0.6666667  400      0.6577279  0.11876241
##   0.3   3         0.6               0.6666667  450      0.6564759  0.11695700
##   0.3   3         0.6               0.6666667  500      0.6553011  0.11544819
##   0.3   3         0.6               0.7222222   50      0.6681382  0.12853047
##   0.3   3         0.6               0.7222222  100      0.6680813  0.12817685
##   0.3   3         0.6               0.7222222  150      0.6669268  0.12608413
##   0.3   3         0.6               0.7222222  200      0.6651789  0.12435931
##   0.3   3         0.6               0.7222222  250      0.6638334  0.12335353
##   0.3   3         0.6               0.7222222  300      0.6621831  0.12194889
##   0.3   3         0.6               0.7222222  350      0.6604961  0.12013443
##   0.3   3         0.6               0.7222222  400      0.6581750  0.11774581
##   0.3   3         0.6               0.7222222  450      0.6569637  0.11708370
##   0.3   3         0.6               0.7222222  500      0.6559231  0.11636375
##   0.3   3         0.6               0.7777778   50      0.6679309  0.12997108
##   0.3   3         0.6               0.7777778  100      0.6695650  0.13016876
##   0.3   3         0.6               0.7777778  150      0.6678496  0.12747710
```

```
##   0.3   3         0.6               0.7777778  200      0.6654960  0.12525175
##   0.3   3         0.6               0.7777778  250      0.6644960  0.12363700
##   0.3   3         0.6               0.7777778  300      0.6630977  0.12270849
##   0.3   3         0.6               0.7777778  350      0.6610896  0.12028980
##   0.3   3         0.6               0.7777778  400      0.6594433  0.11924760
##   0.3   3         0.6               0.7777778  450      0.6582888  0.11754332
##   0.3   3         0.6               0.7777778  500      0.6569677  0.11647303
##   0.3   3         0.6               0.8333333   50      0.6689065  0.12935463
##   0.3   3         0.6               0.8333333  100      0.6698129  0.13003599
##   0.3   3         0.6               0.8333333  150      0.6676260  0.12764627
##   0.3   3         0.6               0.8333333  200      0.6658252  0.12608528
##   0.3   3         0.6               0.8333333  250      0.6637155  0.12402928
##   0.3   3         0.6               0.8333333  300      0.6624066  0.12258518
##   0.3   3         0.6               0.8333333  350      0.6603172  0.12055423
##   0.3   3         0.6               0.8333333  400      0.6595286  0.11933374
##   0.3   3         0.6               0.8333333  450      0.6575694  0.11802582
##   0.3   3         0.6               0.8333333  500      0.6562970  0.11704450
##   0.3   3         0.6               0.8888889   50      0.6674797  0.12800022
##   0.3   3         0.6               0.8888889  100      0.6693983  0.12970391
##   0.3   3         0.6               0.8888889  150      0.6679878  0.12792101
##   0.3   3         0.6               0.8888889  200      0.6669025  0.12657832
##   0.3   3         0.6               0.8888889  250      0.6652643  0.12465577
##   0.3   3         0.6               0.8888889  300      0.6633741  0.12373331
##   0.3   3         0.6               0.8888889  350      0.6615327  0.12214243
##   0.3   3         0.6               0.8888889  400      0.6604758  0.12135900
##   0.3   3         0.6               0.8888889  450      0.6594230  0.12015901
##   0.3   3         0.6               0.8888889  500      0.6579921  0.11809569
##   0.3   3         0.6               0.9444444   50      0.6686503  0.12798003
##   0.3   3         0.6               0.9444444  100      0.6703576  0.12883651
##   0.3   3         0.6               0.9444444  150      0.6692804  0.12799463
##   0.3   3         0.6               0.9444444  200      0.6682804  0.12766899
##   0.3   3         0.6               0.9444444  250      0.6667195  0.12613649
##   0.3   3         0.6               0.9444444  300      0.6649391  0.12438789
##   0.3   3         0.6               0.9444444  350      0.6643740  0.12349727
##   0.3   3         0.6               0.9444444  400      0.6627846  0.12157705
##   0.3   3         0.6               0.9444444  450      0.6612644  0.12047868
##   0.3   3         0.6               0.9444444  500      0.6596221  0.11900886
##   0.3   3         0.6               1.0000000   50      0.6682439  0.12813396
##   0.3   3         0.6               1.0000000  100      0.6708048  0.12949792
##   0.3   3         0.6               1.0000000  150      0.6697520  0.12897038
##   0.3   3         0.6               1.0000000  200      0.6688496  0.12829902
##   0.3   3         0.6               1.0000000  250      0.6671138  0.12664057
##   0.3   3         0.6               1.0000000  300      0.6659838  0.12567688
##   0.3   3         0.6               1.0000000  350      0.6646098  0.12388320
##   0.3   3         0.6               1.0000000  400      0.6635895  0.12344022
##   0.3   3         0.6               1.0000000  450      0.6622318  0.12188379
##   0.3   3         0.6               1.0000000  500      0.6608620  0.12037433
##   0.3   3         0.8               0.5000000   50      0.6657521  0.12726330
##   0.3   3         0.8               0.5000000  100      0.6662276  0.12553874
##   0.3   3         0.8               0.5000000  150      0.6628131  0.12277085
##   0.3   3         0.8               0.5000000  200      0.6607197  0.12003882
##   0.3   3         0.8               0.5000000  250      0.6580653  0.11786106
##   0.3   3         0.8               0.5000000  300      0.6553661  0.11442294
##   0.3   3         0.8               0.5000000  350      0.6544434  0.11368764
```

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##   0.3   3         0.8               0.5000000  400      0.6528093  0.11158582
##   0.3   3         0.8               0.5000000  450      0.6511996  0.11082412
##   0.3   3         0.8               0.5000000  500      0.6493744  0.11024513
##   0.3   3         0.8               0.5555556   50      0.6676341  0.12786359
##   0.3   3         0.8               0.5555556  100      0.6657196  0.12662706
##   0.3   3         0.8               0.5555556  150      0.6647155  0.12523934
##   0.3   3         0.8               0.5555556  200      0.6633172  0.12217121
##   0.3   3         0.8               0.5555556  250      0.6600856  0.11921066
##   0.3   3         0.8               0.5555556  300      0.6582157  0.11830298
##   0.3   3         0.8               0.5555556  350      0.6564718  0.11614220
##   0.3   3         0.8               0.5555556  400      0.6532402  0.11345120
##   0.3   3         0.8               0.5555556  450      0.6523540  0.11327645
##   0.3   3         0.8               0.5555556  500      0.6513337  0.11225559
##   0.3   3         0.8               0.6111111   50      0.6682764  0.12819206
##   0.3   3         0.8               0.6111111  100      0.6686788  0.12716568
##   0.3   3         0.8               0.6111111  150      0.6661342  0.12525214
##   0.3   3         0.8               0.6111111  200      0.6638741  0.12305581
##   0.3   3         0.8               0.6111111  250      0.6613131  0.12119259
##   0.3   3         0.8               0.6111111  300      0.6589392  0.11865873
##   0.3   3         0.8               0.6111111  350      0.6570449  0.11667296
##   0.3   3         0.8               0.6111111  400      0.6554718  0.11552389
##   0.3   3         0.8               0.6111111  450      0.6539027  0.11340528
##   0.3   3         0.8               0.6111111  500      0.6517849  0.11194810
##   0.3   3         0.8               0.6666667   50      0.6672724  0.12744972
##   0.3   3         0.8               0.6666667  100      0.6684268  0.12783072
##   0.3   3         0.8               0.6666667  150      0.6657277  0.12552788
##   0.3   3         0.8               0.6666667  200      0.6644269  0.12344959
##   0.3   3         0.8               0.6666667  250      0.6626505  0.12140696
##   0.3   3         0.8               0.6666667  300      0.6601709  0.11895704
##   0.3   3         0.8               0.6666667  350      0.6585165  0.11775322
##   0.3   3         0.8               0.6666667  400      0.6571628  0.11655876
##   0.3   3         0.8               0.6666667  450      0.6552442  0.11443827
##   0.3   3         0.8               0.6666667  500      0.6534312  0.11325183
##   0.3   3         0.8               0.7222222   50      0.6677195  0.12841096
##   0.3   3         0.8               0.7222222  100      0.6684959  0.12791294
##   0.3   3         0.8               0.7222222  150      0.6666098  0.12661349
##   0.3   3         0.8               0.7222222  200      0.6651708  0.12554284
##   0.3   3         0.8               0.7222222  250      0.6626830  0.12355590
##   0.3   3         0.8               0.7222222  300      0.6618253  0.12260448
##   0.3   3         0.8               0.7222222  350      0.6593701  0.11935244
##   0.3   3         0.8               0.7222222  400      0.6577279  0.11799400
##   0.3   3         0.8               0.7222222  450      0.6567523  0.11722806
##   0.3   3         0.8               0.7222222  500      0.6553011  0.11610014
##   0.3   3         0.8               0.7777778   50      0.6663049  0.12699172
##   0.3   3         0.8               0.7777778  100      0.6677357  0.12756401
##   0.3   3         0.8               0.7777778  150      0.6669675  0.12665276
##   0.3   3         0.8               0.7777778  200      0.6648578  0.12504091
##   0.3   3         0.8               0.7777778  250      0.6619026  0.12198395
##   0.3   3         0.8               0.7777778  300      0.6611139  0.12071898
##   0.3   3         0.8               0.7777778  350      0.6590896  0.11913514
##   0.3   3         0.8               0.7777778  400      0.6574026  0.11791681
##   0.3   3         0.8               0.7777778  450      0.6564555  0.11717764
##   0.3   3         0.8               0.7777778  500      0.6543905  0.11509382
##   0.3   3         0.8               0.8333333   50      0.6673414  0.12785877
```

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##   0.3   3         0.8               0.8333333  100      0.6687682  0.12827976
##   0.3   3         0.8               0.8333333  150      0.6667155  0.12619240
##   0.3   3         0.8               0.8333333  200      0.6647927  0.12505152
##   0.3   3         0.8               0.8333333  250      0.6643496  0.12432797
##   0.3   3         0.8               0.8333333  300      0.6625204  0.12299449
##   0.3   3         0.8               0.8333333  350      0.6612197  0.12180552
##   0.3   3         0.8               0.8333333  400      0.6592319  0.11977030
##   0.3   3         0.8               0.8333333  450      0.6575124  0.11840252
##   0.3   3         0.8               0.8333333  500      0.6559596  0.11675468
##   0.3   3         0.8               0.8888889   50      0.6683902  0.12841598
##   0.3   3         0.8               0.8888889  100      0.6699755  0.12909001
##   0.3   3         0.8               0.8888889  150      0.6686056  0.12755534
##   0.3   3         0.8               0.8888889  200      0.6673699  0.12689797
##   0.3   3         0.8               0.8888889  250      0.6654106  0.12481716
##   0.3   3         0.8               0.8888889  300      0.6636423  0.12259781
##   0.3   3         0.8               0.8888889  350      0.6620326  0.12073296
##   0.3   3         0.8               0.8888889  400      0.6605002  0.11974941
##   0.3   3         0.8               0.8888889  450      0.6597888  0.11906342
##   0.3   3         0.8               0.8888889  500      0.6576710  0.11762365
##   0.3   3         0.8               0.9444444   50      0.6667195  0.12793053
##   0.3   3         0.8               0.9444444  100      0.6686951  0.12911465
##   0.3   3         0.8               0.9444444  150      0.6679187  0.12876615
##   0.3   3         0.8               0.9444444  200      0.6663455  0.12744178
##   0.3   3         0.8               0.9444444  250      0.6644147  0.12555707
##   0.3   3         0.8               0.9444444  300      0.6634310  0.12467054
##   0.3   3         0.8               0.9444444  350      0.6619757  0.12312656
##   0.3   3         0.8               0.9444444  400      0.6606058  0.12136856
##   0.3   3         0.8               0.9444444  450      0.6591018  0.11998063
##   0.3   3         0.8               0.9444444  500      0.6586466  0.12017291
##   0.3   3         0.8               1.0000000   50      0.6686381  0.12805255
##   0.3   3         0.8               1.0000000  100      0.6695202  0.12851238
##   0.3   3         0.8               1.0000000  150      0.6690243  0.12785641
##   0.3   3         0.8               1.0000000  200      0.6684146  0.12751039
##   0.3   3         0.8               1.0000000  250      0.6673373  0.12681782
##   0.3   3         0.8               1.0000000  300      0.6662439  0.12568809
##   0.3   3         0.8               1.0000000  350      0.6647439  0.12435924
##   0.3   3         0.8               1.0000000  400      0.6633293  0.12313021
##   0.3   3         0.8               1.0000000  450      0.6620773  0.12208250
##   0.3   3         0.8               1.0000000  500      0.6609229  0.12098409
##   0.3   4         0.6               0.5000000   50      0.6656383  0.12667500
##   0.3   4         0.6               0.5000000  100      0.6605855  0.12130486
##   0.3   4         0.6               0.5000000  150      0.6557280  0.11802630
##   0.3   4         0.6               0.5000000  200      0.6519719  0.11419366
##   0.3   4         0.6               0.5000000  250      0.6492606  0.11166918
##   0.3   4         0.6               0.5000000  300      0.6465209  0.10787261
##   0.3   4         0.6               0.5000000  350      0.6424519  0.10385617
##   0.3   4         0.6               0.5000000  400      0.6417039  0.10266202
##   0.3   4         0.6               0.5000000  450      0.6394234  0.10177815
##   0.3   4         0.6               0.5000000  500      0.6379154  0.10026184
##   0.3   4         0.6               0.5555556   50      0.6651586  0.12566858
##   0.3   4         0.6               0.5555556  100      0.6626505  0.12264410
##   0.3   4         0.6               0.5555556  150      0.6575774  0.11904305
##   0.3   4         0.6               0.5555556  200      0.6535491  0.11493263
##   0.3   4         0.6               0.5555556  250      0.6502361  0.11265168
```

```
##   0.3   4         0.6                 0.5555556  300      0.6473337  0.10906599
##   0.3   4         0.6                 0.5555556  350      0.6451102  0.10724823
##   0.3   4         0.6                 0.5555556  400      0.6427688  0.10389353
##   0.3   4         0.6                 0.5555556  450      0.6399478  0.10139204
##   0.3   4         0.6                 0.5555556  500      0.6384437  0.10056840
##   0.3   4         0.6                 0.6111111   50      0.6659309  0.12525136
##   0.3   4         0.6                 0.6111111  100      0.6630042  0.12292662
##   0.3   4         0.6                 0.6111111  150      0.6593457  0.11834478
##   0.3   4         0.6                 0.6111111  200      0.6561751  0.11527977
##   0.3   4         0.6                 0.6111111  250      0.6526548  0.11259293
##   0.3   4         0.6                 0.6111111  300      0.6494923  0.10964690
##   0.3   4         0.6                 0.6111111  350      0.6475737  0.10878902
##   0.3   4         0.6                 0.6111111  400      0.6449558  0.10616415
##   0.3   4         0.6                 0.6111111  450      0.6425413  0.10327436
##   0.3   4         0.6                 0.6111111  500      0.6409478  0.10159702
##   0.3   4         0.6                 0.6666667   50      0.6657764  0.12611883
##   0.3   4         0.6                 0.6666667  100      0.6628781  0.12346257
##   0.3   4         0.6                 0.6666667  150      0.6597278  0.12101390
##   0.3   4         0.6                 0.6666667  200      0.6558905  0.11675650
##   0.3   4         0.6                 0.6666667  250      0.6536954  0.11398249
##   0.3   4         0.6                 0.6666667  300      0.6512199  0.11230172
##   0.3   4         0.6                 0.6666667  350      0.6481305  0.10848330
##   0.3   4         0.6                 0.6666667  400      0.6469598  0.10754974
##   0.3   4         0.6                 0.6666667  450      0.6446062  0.10466243
##   0.3   4         0.6                 0.6666667  500      0.6427607  0.10417252
##   0.3   4         0.6                 0.7222222   50      0.6680162  0.12843198
##   0.3   4         0.6                 0.7222222  100      0.6659553  0.12555705
##   0.3   4         0.6                 0.7222222  150      0.6618701  0.12161528
##   0.3   4         0.6                 0.7222222  200      0.6595124  0.11812756
##   0.3   4         0.6                 0.7222222  250      0.6564271  0.11602601
##   0.3   4         0.6                 0.7222222  300      0.6541954  0.11390639
##   0.3   4         0.6                 0.7222222  350      0.6519150  0.11129334
##   0.3   4         0.6                 0.7222222  400      0.6486509  0.10899417
##   0.3   4         0.6                 0.7222222  450      0.6461631  0.10738032
##   0.3   4         0.6                 0.7222222  500      0.6444640  0.10575811
##   0.3   4         0.6                 0.7777778   50      0.6682317  0.12811239
##   0.3   4         0.6                 0.7777778  100      0.6657074  0.12531536
##   0.3   4         0.6                 0.7777778  150      0.6632602  0.12258247
##   0.3   4         0.6                 0.7777778  200      0.6608416  0.12025151
##   0.3   4         0.6                 0.7777778  250      0.6574230  0.11721272
##   0.3   4         0.6                 0.7777778  300      0.6545816  0.11503420
##   0.3   4         0.6                 0.7777778  350      0.6526020  0.11282674
##   0.3   4         0.6                 0.7777778  400      0.6505045  0.11094938
##   0.3   4         0.6                 0.7777778  450      0.6485574  0.10947976
##   0.3   4         0.6                 0.7777778  500      0.6464355  0.10821519
##   0.3   4         0.6                 0.8333333   50      0.6660122  0.12687970
##   0.3   4         0.6                 0.8333333  100      0.6648212  0.12515474
##   0.3   4         0.6                 0.8333333  150      0.6624310  0.12305830
##   0.3   4         0.6                 0.8333333  200      0.6587482  0.11980292
##   0.3   4         0.6                 0.8333333  250      0.6564230  0.11716422
##   0.3   4         0.6                 0.8333333  300      0.6541548  0.11390772
##   0.3   4         0.6                 0.8333333  350      0.6521874  0.11268667
##   0.3   4         0.6                 0.8333333  400      0.6502159  0.11002830
##   0.3   4         0.6                 0.8333333  450      0.6485696  0.10918919
```

```
##   0.3   4         0.6               0.8333333  500      0.6474110  0.10818451
##   0.3   4         0.6               0.8888889   50      0.6681626  0.12863952
##   0.3   4         0.6               0.8888889  100      0.6664228  0.12690477
##   0.3   4         0.6               0.8888889  150      0.6648984  0.12566247
##   0.3   4         0.6               0.8888889  200      0.6617400  0.12237580
##   0.3   4         0.6               0.8888889  250      0.6591465  0.11997401
##   0.3   4         0.6               0.8888889  300      0.6566507  0.11768059
##   0.3   4         0.6               0.8888889  350      0.6546589  0.11611696
##   0.3   4         0.6               0.8888889  400      0.6524231  0.11416844
##   0.3   4         0.6               0.8888889  450      0.6502565  0.11258235
##   0.3   4         0.6               0.8888889  500      0.6488013  0.11076596
##   0.3   4         0.6               0.9444444   50      0.6674634  0.12779090
##   0.3   4         0.6               0.9444444  100      0.6664634  0.12604785
##   0.3   4         0.6               0.9444444  150      0.6639798  0.12328163
##   0.3   4         0.6               0.9444444  200      0.6612400  0.12092678
##   0.3   4         0.6               0.9444444  250      0.6592075  0.11854181
##   0.3   4         0.6               0.9444444  300      0.6579515  0.11759883
##   0.3   4         0.6               0.9444444  350      0.6558255  0.11495564
##   0.3   4         0.6               0.9444444  400      0.6538052  0.11296179
##   0.3   4         0.6               0.9444444  450      0.6518215  0.11196822
##   0.3   4         0.6               0.9444444  500      0.6513744  0.11098528
##   0.3   4         0.6               1.0000000   50      0.6681910  0.12925595
##   0.3   4         0.6               1.0000000  100      0.6663293  0.12724950
##   0.3   4         0.6               1.0000000  150      0.6647033  0.12473732
##   0.3   4         0.6               1.0000000  200      0.6622034  0.12246394
##   0.3   4         0.6               1.0000000  250      0.6603050  0.12094020
##   0.3   4         0.6               1.0000000  300      0.6580775  0.11907442
##   0.3   4         0.6               1.0000000  350      0.6557523  0.11685382
##   0.3   4         0.6               1.0000000  400      0.6542767  0.11505456
##   0.3   4         0.6               1.0000000  450      0.6532158  0.11417190
##   0.3   4         0.6               1.0000000  500      0.6518906  0.11365502
##   0.3   4         0.8               0.5000000   50      0.6651382  0.12672026
##   0.3   4         0.8               0.5000000  100      0.6602481  0.12139885
##   0.3   4         0.8               0.5000000  150      0.6554800  0.11655738
##   0.3   4         0.8               0.5000000  200      0.6516142  0.11317025
##   0.3   4         0.8               0.5000000  250      0.6468420  0.10893019
##   0.3   4         0.8               0.5000000  300      0.6447892  0.10712912
##   0.3   4         0.8               0.5000000  350      0.6416998  0.10532941
##   0.3   4         0.8               0.5000000  400      0.6392405  0.10324637
##   0.3   4         0.8               0.5000000  450      0.6364561  0.09973227
##   0.3   4         0.8               0.5000000  500      0.6354113  0.09691606
##   0.3   4         0.8               0.5555556   50      0.6654554  0.12454506
##   0.3   4         0.8               0.5555556  100      0.6610327  0.12013792
##   0.3   4         0.8               0.5555556  150      0.6566873  0.11603310
##   0.3   4         0.8               0.5555556  200      0.6526549  0.11329785
##   0.3   4         0.8               0.5555556  250      0.6486753  0.10894059
##   0.3   4         0.8               0.5555556  300      0.6458420  0.10739551
##   0.3   4         0.8               0.5555556  350      0.6431795  0.10555706
##   0.3   4         0.8               0.5555556  400      0.6413990  0.10343055
##   0.3   4         0.8               0.5555556  450      0.6394031  0.10060414
##   0.3   4         0.8               0.5555556  500      0.6365292  0.09817885
##   0.3   4         0.8               0.6111111   50      0.6660163  0.12593220
##   0.3   4         0.8               0.6111111  100      0.6606994  0.12151774
##   0.3   4         0.8               0.6111111  150      0.6575409  0.11805871
```

```
##   0.3   4         0.8               0.6111111  200      0.6527930  0.11394549
##   0.3   4         0.8               0.6111111  250      0.6510207  0.11232765
##   0.3   4         0.8               0.6111111  300      0.6485127  0.10885191
##   0.3   4         0.8               0.6111111  350      0.6454477  0.10562808
##   0.3   4         0.8               0.6111111  400      0.6429437  0.10444133
##   0.3   4         0.8               0.6111111  450      0.6410697  0.10212739
##   0.3   4         0.8               0.6111111  500      0.6398381  0.10108248
##   0.3   4         0.8               0.6666667   50      0.6663293  0.12579049
##   0.3   4         0.8               0.6666667  100      0.6634350  0.12264795
##   0.3   4         0.8               0.6666667  150      0.6594636  0.11976568
##   0.3   4         0.8               0.6666667  200      0.6561385  0.11665962
##   0.3   4         0.8               0.6666667  250      0.6528784  0.11336689
##   0.3   4         0.8               0.6666667  300      0.6500573  0.11187771
##   0.3   4         0.8               0.6666667  350      0.6476509  0.10840875
##   0.3   4         0.8               0.6666667  400      0.6455940  0.10687392
##   0.3   4         0.8               0.6666667  450      0.6436103  0.10511472
##   0.3   4         0.8               0.6666667  500      0.6415413  0.10298356
##   0.3   4         0.8               0.7222222   50      0.6683943  0.12717039
##   0.3   4         0.8               0.7222222  100      0.6651383  0.12488834
##   0.3   4         0.8               0.7222222  150      0.6608132  0.12109389
##   0.3   4         0.8               0.7222222  200      0.6576100  0.11873034
##   0.3   4         0.8               0.7222222  250      0.6546344  0.11510878
##   0.3   4         0.8               0.7222222  300      0.6528947  0.11324975
##   0.3   4         0.8               0.7222222  350      0.6506305  0.11172735
##   0.3   4         0.8               0.7222222  400      0.6485899  0.10984791
##   0.3   4         0.8               0.7222222  450      0.6457607  0.10745976
##   0.3   4         0.8               0.7222222  500      0.6452078  0.10627416
##   0.3   4         0.8               0.7777778   50      0.6670122  0.12694370
##   0.3   4         0.8               0.7777778  100      0.6638537  0.12354100
##   0.3   4         0.8               0.7777778  150      0.6610245  0.12002296
##   0.3   4         0.8               0.7777778  200      0.6578823  0.11802561
##   0.3   4         0.8               0.7777778  250      0.6560490  0.11579057
##   0.3   4         0.8               0.7777778  300      0.6534271  0.11366916
##   0.3   4         0.8               0.7777778  350      0.6511223  0.11126643
##   0.3   4         0.8               0.7777778  400      0.6499801  0.11011061
##   0.3   4         0.8               0.7777778  450      0.6480614  0.10907311
##   0.3   4         0.8               0.7777778  500      0.6457160  0.10668399
##   0.3   4         0.8               0.8333333   50      0.6682479  0.12749135
##   0.3   4         0.8               0.8333333  100      0.6665163  0.12514811
##   0.3   4         0.8               0.8333333  150      0.6630854  0.12109580
##   0.3   4         0.8               0.8333333  200      0.6598701  0.11904014
##   0.3   4         0.8               0.8333333  250      0.6569962  0.11577402
##   0.3   4         0.8               0.8333333  300      0.6548417  0.11440687
##   0.3   4         0.8               0.8333333  350      0.6530613  0.11319900
##   0.3   4         0.8               0.8333333  400      0.6520938  0.11180184
##   0.3   4         0.8               0.8333333  450      0.6501102  0.11023317
##   0.3   4         0.8               0.8333333  500      0.6482647  0.10912930
##   0.3   4         0.8               0.8888889   50      0.6671260  0.12714054
##   0.3   4         0.8               0.8888889  100      0.6655448  0.12501796
##   0.3   4         0.8               0.8888889  150      0.6618701  0.12138484
##   0.3   4         0.8               0.8888889  200      0.6590205  0.11906791
##   0.3   4         0.8               0.8888889  250      0.6563133  0.11676420
##   0.3   4         0.8               0.8888889  300      0.6543621  0.11547758
##   0.3   4         0.8               0.8888889  350      0.6514679  0.11278336
```

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##   0.3   4         0.8                 0.8888889  400      0.6496427  0.11110461
##   0.3   4         0.8                 0.8888889  450      0.6481996  0.10951788
##   0.3   4         0.8                 0.8888889  500      0.6472566  0.10873404
##   0.3   4         0.8                 0.9444444   50      0.6683455  0.12897892
##   0.3   4         0.8                 0.9444444  100      0.6661382  0.12724617
##   0.3   4         0.8                 0.9444444  150      0.6640854  0.12481671
##   0.3   4         0.8                 0.9444444  200      0.6621505  0.12209337
##   0.3   4         0.8                 0.9444444  250      0.6595937  0.12021348
##   0.3   4         0.8                 0.9444444  300      0.6567482  0.11779087
##   0.3   4         0.8                 0.9444444  350      0.6548987  0.11605721
##   0.3   4         0.8                 0.9444444  400      0.6531385  0.11387292
##   0.3   4         0.8                 0.9444444  450      0.6513825  0.11242328
##   0.3   4         0.8                 0.9444444  500      0.6505858  0.11248099
##   0.3   4         0.8                 1.0000000   50      0.6680528  0.12895879
##   0.3   4         0.8                 1.0000000  100      0.6663821  0.12713037
##   0.3   4         0.8                 1.0000000  150      0.6646423  0.12575566
##   0.3   4         0.8                 1.0000000  200      0.6626302  0.12367720
##   0.3   4         0.8                 1.0000000  250      0.6605286  0.12089500
##   0.3   4         0.8                 1.0000000  300      0.6587807  0.11947112
##   0.3   4         0.8                 1.0000000  350      0.6573457  0.11808231
##   0.3   4         0.8                 1.0000000  400      0.6562075  0.11740055
##   0.3   4         0.8                 1.0000000  450      0.6546710  0.11491712
##   0.3   4         0.8                 1.0000000  500      0.6531345  0.11381842
##   0.3   5         0.6                 0.5000000   50      0.6591222  0.11953407
##   0.3   5         0.6                 0.5000000  100      0.6514232  0.11148845
##   0.3   5         0.6                 0.5000000  150      0.6445249  0.10610156
##   0.3   5         0.6                 0.5000000  200      0.6412080  0.10253119
##   0.3   5         0.6                 0.5000000  250      0.6375373  0.09959722
##   0.3   5         0.6                 0.5000000  300      0.6343301  0.09633013
##   0.3   5         0.6                 0.5000000  350      0.6317976  0.09455524
##   0.3   5         0.6                 0.5000000  400      0.6310334  0.09414312
##   0.3   5         0.6                 0.5000000  450      0.6287977  0.09222283
##   0.3   5         0.6                 0.5000000  500      0.6272043  0.09026031
##   0.3   5         0.6                 0.5555556   50      0.6610286  0.12138765
##   0.3   5         0.6                 0.5555556  100      0.6526467  0.11332444
##   0.3   5         0.6                 0.5555556  150      0.6477891  0.10875158
##   0.3   5         0.6                 0.5555556  200      0.6426876  0.10341246
##   0.3   5         0.6                 0.5555556  250      0.6389316  0.10042718
##   0.3   5         0.6                 0.5555556  300      0.6350861  0.09724776
##   0.3   5         0.6                 0.5555556  350      0.6343748  0.09728845
##   0.3   5         0.6                 0.5555556  400      0.6322000  0.09512124
##   0.3   5         0.6                 0.5555556  450      0.6302448  0.09377235
##   0.3   5         0.6                 0.5555556  500      0.6281717  0.09188719
##   0.3   5         0.6                 0.6111111   50      0.6640733  0.12385685
##   0.3   5         0.6                 0.6111111  100      0.6567401  0.11595776
##   0.3   5         0.6                 0.6111111  150      0.6500736  0.11082315
##   0.3   5         0.6                 0.6111111  200      0.6445778  0.10524529
##   0.3   5         0.6                 0.6111111  250      0.6411876  0.10238934
##   0.3   5         0.6                 0.6111111  300      0.6379194  0.09891841
##   0.3   5         0.6                 0.6111111  350      0.6362975  0.09685500
##   0.3   5         0.6                 0.6111111  400      0.6341756  0.09556647
##   0.3   5         0.6                 0.6111111  450      0.6320943  0.09342383
##   0.3   5         0.6                 0.6111111  500      0.6314318  0.09303561
##   0.3   5         0.6                 0.6666667   50      0.6621018  0.12167931
```

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##   0.3   5         0.6               0.6666667  100      0.6570775  0.11761017
##   0.3   5         0.6               0.6666667  150      0.6514841  0.11237591
##   0.3   5         0.6               0.6666667  200      0.6470655  0.10815137
##   0.3   5         0.6               0.6666667  250      0.6431429  0.10611507
##   0.3   5         0.6               0.6666667  300      0.6396917  0.10259939
##   0.3   5         0.6               0.6666667  350      0.6383991  0.10145960
##   0.3   5         0.6               0.6666667  400      0.6372975  0.10089735
##   0.3   5         0.6               0.6666667  450      0.6354480  0.09914234
##   0.3   5         0.6               0.6666667  500      0.6349927  0.09903625
##   0.3   5         0.6               0.7222222   50      0.6637277  0.12417702
##   0.3   5         0.6               0.7222222  100      0.6583945  0.11918159
##   0.3   5         0.6               0.7222222  150      0.6530938  0.11378331
##   0.3   5         0.6               0.7222222  200      0.6493663  0.10995636
##   0.3   5         0.6               0.7222222  250      0.6452038  0.10658716
##   0.3   5         0.6               0.7222222  300      0.6429477  0.10394845
##   0.3   5         0.6               0.7222222  350      0.6410738  0.10226581
##   0.3   5         0.6               0.7222222  400      0.6384885  0.10064142
##   0.3   5         0.6               0.7222222  450      0.6367853  0.09856667
##   0.3   5         0.6               0.7222222  500      0.6354804  0.09831698
##   0.3   5         0.6               0.7777778   50      0.6657439  0.12433219
##   0.3   5         0.6               0.7777778  100      0.6604189  0.12002191
##   0.3   5         0.6               0.7777778  150      0.6555206  0.11580367
##   0.3   5         0.6               0.7777778  200      0.6519597  0.11259564
##   0.3   5         0.6               0.7777778  250      0.6486427  0.10956856
##   0.3   5         0.6               0.7777778  300      0.6456672  0.10638254
##   0.3   5         0.6               0.7777778  350      0.6438014  0.10439621
##   0.3   5         0.6               0.7777778  400      0.6419965  0.10312545
##   0.3   5         0.6               0.7777778  450      0.6396145  0.10100156
##   0.3   5         0.6               0.7777778  500      0.6394438  0.10058638
##   0.3   5         0.6               0.8333333   50      0.6628822  0.12398673
##   0.3   5         0.6               0.8333333  100      0.6582604  0.11854399
##   0.3   5         0.6               0.8333333  150      0.6533215  0.11452092
##   0.3   5         0.6               0.8333333  200      0.6505695  0.11204965
##   0.3   5         0.6               0.8333333  250      0.6481102  0.11035615
##   0.3   5         0.6               0.8333333  300      0.6457485  0.10845807
##   0.3   5         0.6               0.8333333  350      0.6434437  0.10659223
##   0.3   5         0.6               0.8333333  400      0.6424762  0.10595130
##   0.3   5         0.6               0.8333333  450      0.6405901  0.10396885
##   0.3   5         0.6               0.8333333  500      0.6393462  0.10228457
##   0.3   5         0.6               0.8888889   50      0.6654960  0.12547825
##   0.3   5         0.6               0.8888889  100      0.6612888  0.12124074
##   0.3   5         0.6               0.8888889  150      0.6585409  0.11925541
##   0.3   5         0.6               0.8888889  200      0.6542402  0.11523219
##   0.3   5         0.6               0.8888889  250      0.6512199  0.11176544
##   0.3   5         0.6               0.8888889  300      0.6487403  0.10931032
##   0.3   5         0.6               0.8888889  350      0.6467932  0.10676982
##   0.3   5         0.6               0.8888889  400      0.6451388  0.10619537
##   0.3   5         0.6               0.8888889  450      0.6444680  0.10566367
##   0.3   5         0.6               0.8888889  500      0.6431103  0.10448985
##   0.3   5         0.6               0.9444444   50      0.6635529  0.12443504
##   0.3   5         0.6               0.9444444  100      0.6612115  0.12121553
##   0.3   5         0.6               0.9444444  150      0.6579677  0.11793152
##   0.3   5         0.6               0.9444444  200      0.6545369  0.11525962
##   0.3   5         0.6               0.9444444  250      0.6525410  0.11380125
```

```
##   0.3   5         0.6                 0.9444444  300      0.6492159  0.10999306
##   0.3   5         0.6                 0.9444444  350      0.6467607  0.10826904
##   0.3   5         0.6                 0.9444444  400      0.6456184  0.10722826
##   0.3   5         0.6                 0.9444444  450      0.6435250  0.10556369
##   0.3   5         0.6                 0.9444444  500      0.6429884  0.10528495
##   0.3   5         0.6                 1.0000000   50      0.6672683  0.12608215
##   0.3   5         0.6                 1.0000000  100      0.6640936  0.12297162
##   0.3   5         0.6                 1.0000000  150      0.6614636  0.12062350
##   0.3   5         0.6                 1.0000000  200      0.6577116  0.11770241
##   0.3   5         0.6                 1.0000000  250      0.6549027  0.11484512
##   0.3   5         0.6                 1.0000000  300      0.6532280  0.11250468
##   0.3   5         0.6                 1.0000000  350      0.6505126  0.11093894
##   0.3   5         0.6                 1.0000000  400      0.6492931  0.10935737
##   0.3   5         0.6                 1.0000000  450      0.6476997  0.10841593
##   0.3   5         0.6                 1.0000000  500      0.6470737  0.10778883
##   0.3   5         0.8                 0.5000000   50      0.6577441  0.11893757
##   0.3   5         0.8                 0.5000000  100      0.6512687  0.11299080
##   0.3   5         0.8                 0.5000000  150      0.6443176  0.10572101
##   0.3   5         0.8                 0.5000000  200      0.6396511  0.10144491
##   0.3   5         0.8                 0.5000000  250      0.6356552  0.09771785
##   0.3   5         0.8                 0.5000000  300      0.6333260  0.09544525
##   0.3   5         0.8                 0.5000000  350      0.6309927  0.09351060
##   0.3   5         0.8                 0.5000000  400      0.6293464  0.09247848
##   0.3   5         0.8                 0.5000000  450      0.6266026  0.09040344
##   0.3   5         0.8                 0.5000000  500      0.6256717  0.08871014
##   0.3   5         0.8                 0.5555556   50      0.6592807  0.12051157
##   0.3   5         0.8                 0.5555556  100      0.6532321  0.11421393
##   0.3   5         0.8                 0.5555556  150      0.6476469  0.10855170
##   0.3   5         0.8                 0.5555556  200      0.6432404  0.10493779
##   0.3   5         0.8                 0.5555556  250      0.6393544  0.10149087
##   0.3   5         0.8                 0.5555556  300      0.6357854  0.09757180
##   0.3   5         0.8                 0.5555556  350      0.6341065  0.09674083
##   0.3   5         0.8                 0.5555556  400      0.6315984  0.09501880
##   0.3   5         0.8                 0.5555556  450      0.6312489  0.09511722
##   0.3   5         0.8                 0.5555556  500      0.6290985  0.09311927
##   0.3   5         0.8                 0.6111111   50      0.6628782  0.12309870
##   0.3   5         0.8                 0.6111111  100      0.6557523  0.11617277
##   0.3   5         0.8                 0.6111111  150      0.6516833  0.11156836
##   0.3   5         0.8                 0.6111111  200      0.6459152  0.10559723
##   0.3   5         0.8                 0.6111111  250      0.6431104  0.10268755
##   0.3   5         0.8                 0.6111111  300      0.6408177  0.10148607
##   0.3   5         0.8                 0.6111111  350      0.6377162  0.09808741
##   0.3   5         0.8                 0.6111111  400      0.6357853  0.09503364
##   0.3   5         0.8                 0.6111111  450      0.6348382  0.09416944
##   0.3   5         0.8                 0.6111111  500      0.6333057  0.09345229
##   0.3   5         0.8                 0.6666667   50      0.6639025  0.12247330
##   0.3   5         0.8                 0.6666667  100      0.6577116  0.11838794
##   0.3   5         0.8                 0.6666667  150      0.6523052  0.11288548
##   0.3   5         0.8                 0.6666667  200      0.6472160  0.10836097
##   0.3   5         0.8                 0.6666667  250      0.6443746  0.10541194
##   0.3   5         0.8                 0.6666667  300      0.6406226  0.10275631
##   0.3   5         0.8                 0.6666667  350      0.6389641  0.10135652
##   0.3   5         0.8                 0.6666667  400      0.6373219  0.09937252
##   0.3   5         0.8                 0.6666667  450      0.6360861  0.09771544
```

```
##   0.3   5         0.8               0.6666667  500      0.6348545  0.09765228
##   0.3   5         0.8               0.7222222   50      0.6626546  0.12318662
##   0.3   5         0.8               0.7222222  100      0.6584433  0.11833229
##   0.3   5         0.8               0.7222222  150      0.6525735  0.11299721
##   0.3   5         0.8               0.7222222  200      0.6484517  0.10965434
##   0.3   5         0.8               0.7222222  250      0.6443420  0.10598876
##   0.3   5         0.8               0.7222222  300      0.6419519  0.10414276
##   0.3   5         0.8               0.7222222  350      0.6398706  0.10261439
##   0.3   5         0.8               0.7222222  400      0.6374723  0.09974033
##   0.3   5         0.8               0.7222222  450      0.6368341  0.09902891
##   0.3   5         0.8               0.7222222  500      0.6356227  0.09905990
##   0.3   5         0.8               0.7777778   50      0.6647521  0.12504262
##   0.3   5         0.8               0.7777778  100      0.6595734  0.12017611
##   0.3   5         0.8               0.7777778  150      0.6540085  0.11482724
##   0.3   5         0.8               0.7777778  200      0.6496102  0.11102483
##   0.3   5         0.8               0.7777778  250      0.6469924  0.10891656
##   0.3   5         0.8               0.7777778  300      0.6443542  0.10605381
##   0.3   5         0.8               0.7777778  350      0.6422161  0.10405822
##   0.3   5         0.8               0.7777778  400      0.6411632  0.10268002
##   0.3   5         0.8               0.7777778  450      0.6391999  0.10156715
##   0.3   5         0.8               0.7777778  500      0.6379153  0.10048708
##   0.3   5         0.8               0.8333333   50      0.6655285  0.12422526
##   0.3   5         0.8               0.8333333  100      0.6600083  0.12054609
##   0.3   5         0.8               0.8333333  150      0.6541385  0.11438669
##   0.3   5         0.8               0.8333333  200      0.6511345  0.11188475
##   0.3   5         0.8               0.8333333  250      0.6480452  0.10893107
##   0.3   5         0.8               0.8333333  300      0.6455371  0.10733209
##   0.3   5         0.8               0.8333333  350      0.6424966  0.10514973
##   0.3   5         0.8               0.8333333  400      0.6410291  0.10374609
##   0.3   5         0.8               0.8333333  450      0.6390820  0.10126806
##   0.3   5         0.8               0.8333333  500      0.6373788  0.09945270
##   0.3   5         0.8               0.8888889   50      0.6657805  0.12483043
##   0.3   5         0.8               0.8888889  100      0.6615245  0.12153395
##   0.3   5         0.8               0.8888889  150      0.6578214  0.11759387
##   0.3   5         0.8               0.8888889  200      0.6547848  0.11470008
##   0.3   5         0.8               0.8888889  250      0.6516914  0.11155317
##   0.3   5         0.8               0.8888889  300      0.6496590  0.10873070
##   0.3   5         0.8               0.8888889  350      0.6469476  0.10689116
##   0.3   5         0.8               0.8888889  400      0.6454233  0.10478509
##   0.3   5         0.8               0.8888889  450      0.6443542  0.10421499
##   0.3   5         0.8               0.8888889  500      0.6434396  0.10408575
##   0.3   5         0.8               0.9444444   50      0.6646992  0.12438865
##   0.3   5         0.8               0.9444444  100      0.6615164  0.12211454
##   0.3   5         0.8               0.9444444  150      0.6574149  0.11739599
##   0.3   5         0.8               0.9444444  200      0.6536914  0.11490226
##   0.3   5         0.8               0.9444444  250      0.6515736  0.11312152
##   0.3   5         0.8               0.9444444  300      0.6501671  0.11209469
##   0.3   5         0.8               0.9444444  350      0.6485248  0.11044987
##   0.3   5         0.8               0.9444444  400      0.6459843  0.10916865
##   0.3   5         0.8               0.9444444  450      0.6446510  0.10783328
##   0.3   5         0.8               0.9444444  500      0.6430819  0.10630752
##   0.3   5         0.8               1.0000000   50      0.6658212  0.12660940
##   0.3   5         0.8               1.0000000  100      0.6629920  0.12355856
##   0.3   5         0.8               1.0000000  150      0.6584027  0.11945016
```

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##   0.3   5         0.8               1.0000000  200      0.6559637  0.11699968
##   0.3   5         0.8               1.0000000  250      0.6529312  0.11368208
##   0.3   5         0.8               1.0000000  300      0.6507727  0.11172583
##   0.3   5         0.8               1.0000000  350      0.6488175  0.10951317
##   0.3   5         0.8               1.0000000  400      0.6474476  0.10845868
##   0.3   5         0.8               1.0000000  450      0.6461753  0.10749592
##   0.3   5         0.8               1.0000000  500      0.6446997  0.10581982
##   0.3   6         0.6               0.5000000   50      0.6532524  0.11500322
##   0.3   6         0.6               0.5000000  100      0.6441388  0.10538305
##   0.3   6         0.6               0.5000000  150      0.6365658  0.09874557
##   0.3   6         0.6               0.5000000  200      0.6327529  0.09593126
##   0.3   6         0.6               0.5000000  250      0.6286188  0.09260639
##   0.3   6         0.6               0.5000000  300      0.6250213  0.09063475
##   0.3   6         0.6               0.5000000  350      0.6239279  0.08898172
##   0.3   6         0.6               0.5000000  400      0.6225742  0.08776523
##   0.3   6         0.6               0.5000000  450      0.6225824  0.08775881
##   0.3   6         0.6               0.5000000  500      0.6207572  0.08663652
##   0.3   6         0.6               0.5555556   50      0.6551141  0.11685988
##   0.3   6         0.6               0.5555556  100      0.6454192  0.10681942
##   0.3   6         0.6               0.5555556  150      0.6402730  0.10294873
##   0.3   6         0.6               0.5555556  200      0.6360943  0.09802950
##   0.3   6         0.6               0.5555556  250      0.6307163  0.09464896
##   0.3   6         0.6               0.5555556  300      0.6280660  0.09247025
##   0.3   6         0.6               0.5555556  350      0.6282977  0.09244543
##   0.3   6         0.6               0.5555556  400      0.6259441  0.09000285
##   0.3   6         0.6               0.5555556  450      0.6254685  0.08947413
##   0.3   6         0.6               0.5555556  500      0.6250092  0.08947220
##   0.3   6         0.6               0.6111111   50      0.6564271  0.11723118
##   0.3   6         0.6               0.6111111  100      0.6480086  0.10899457
##   0.3   6         0.6               0.6111111  150      0.6419965  0.10389689
##   0.3   6         0.6               0.6111111  200      0.6367934  0.09998050
##   0.3   6         0.6               0.6111111  250      0.6341512  0.09650911
##   0.3   6         0.6               0.6111111  300      0.6321675  0.09386456
##   0.3   6         0.6               0.6111111  350      0.6296350  0.09262196
##   0.3   6         0.6               0.6111111  400      0.6284725  0.09161662
##   0.3   6         0.6               0.6111111  450      0.6279481  0.09183952
##   0.3   6         0.6               0.6111111  500      0.6275619  0.09159849
##   0.3   6         0.6               0.6666667   50      0.6575653  0.11820297
##   0.3   6         0.6               0.6666667  100      0.6499638  0.11072515
##   0.3   6         0.6               0.6666667  150      0.6439193  0.10608648
##   0.3   6         0.6               0.6666667  200      0.6396795  0.10232254
##   0.3   6         0.6               0.6666667  250      0.6371633  0.10094830
##   0.3   6         0.6               0.6666667  300      0.6335171  0.09789132
##   0.3   6         0.6               0.6666667  350      0.6328586  0.09718882
##   0.3   6         0.6               0.6666667  400      0.6311960  0.09528157
##   0.3   6         0.6               0.6666667  450      0.6314683  0.09471702
##   0.3   6         0.6               0.6666667  500      0.6307895  0.09419255
##   0.3   6         0.6               0.7222222   50      0.6584270  0.11824163
##   0.3   6         0.6               0.7222222  100      0.6509679  0.11174249
##   0.3   6         0.6               0.7222222  150      0.6459558  0.10821945
##   0.3   6         0.6               0.7222222  200      0.6409234  0.10382654
##   0.3   6         0.6               0.7222222  250      0.6387405  0.10161865
##   0.3   6         0.6               0.7222222  300      0.6369317  0.09979581
##   0.3   6         0.6               0.7222222  350      0.6353992  0.09864598
```

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##   0.3   6         0.6                0.7222222  400      0.6342488  0.09777360
##   0.3   6         0.6                0.7222222  450      0.6338911  0.09734590
##   0.3   6         0.6                0.7222222  500      0.6331309  0.09726589
##   0.3   6         0.6                0.7777778   50      0.6593294  0.11990288
##   0.3   6         0.6                0.7777778  100      0.6517158  0.11384728
##   0.3   6         0.6                0.7777778  150      0.6470777  0.10882894
##   0.3   6         0.6                0.7777778  200      0.6433258  0.10573878
##   0.3   6         0.6                0.7777778  250      0.6406998  0.10431999
##   0.3   6         0.6                0.7777778  300      0.6376958  0.10190384
##   0.3   6         0.6                0.7777778  350      0.6369154  0.10076736
##   0.3   6         0.6                0.7777778  400      0.6357040  0.09993258
##   0.3   6         0.6                0.7777778  450      0.6339967  0.09877267
##   0.3   6         0.6                0.7777778  500      0.6341512  0.09907668
##   0.3   6         0.6                0.8333333   50      0.6618538  0.12156567
##   0.3   6         0.6                0.8333333  100      0.6552645  0.11643923
##   0.3   6         0.6                0.8333333  150      0.6503378  0.11224301
##   0.3   6         0.6                0.8333333  200      0.6464477  0.10868779
##   0.3   6         0.6                0.8333333  250      0.6424031  0.10471713
##   0.3   6         0.6                0.8333333  300      0.6407974  0.10388243
##   0.3   6         0.6                0.8333333  350      0.6389275  0.10220355
##   0.3   6         0.6                0.8333333  400      0.6375780  0.10149616
##   0.3   6         0.6                0.8333333  450      0.6368788  0.10086727
##   0.3   6         0.6                0.8333333  500      0.6371755  0.10136090
##   0.3   6         0.6                0.8888889   50      0.6609432  0.12101358
##   0.3   6         0.6                0.8888889  100      0.6561141  0.11628488
##   0.3   6         0.6                0.8888889  150      0.6518947  0.11149190
##   0.3   6         0.6                0.8888889  200      0.6479151  0.10901578
##   0.3   6         0.6                0.8888889  250      0.6455656  0.10629751
##   0.3   6         0.6                0.8888889  300      0.6431510  0.10376204
##   0.3   6         0.6                0.8888889  350      0.6411551  0.10271943
##   0.3   6         0.6                0.8888889  400      0.6392974  0.10130157
##   0.3   6         0.6                0.8888889  450      0.6392039  0.10165170
##   0.3   6         0.6                0.8888889  500      0.6389072  0.10097028
##   0.3   6         0.6                0.9444444   50      0.6610977  0.12167370
##   0.3   6         0.6                0.9444444  100      0.6555613  0.11683470
##   0.3   6         0.6                0.9444444  150      0.6504151  0.11232244
##   0.3   6         0.6                0.9444444  200      0.6470086  0.10828994
##   0.3   6         0.6                0.9444444  250      0.6455900  0.10672314
##   0.3   6         0.6                0.9444444  300      0.6432161  0.10489160
##   0.3   6         0.6                0.9444444  350      0.6417649  0.10306725
##   0.3   6         0.6                0.9444444  400      0.6411267  0.10292785
##   0.3   6         0.6                0.9444444  450      0.6403950  0.10180080
##   0.3   6         0.6                0.9444444  500      0.6399194  0.10183354
##   0.3   6         0.6                1.0000000   50      0.6615367  0.12152325
##   0.3   6         0.6                1.0000000  100      0.6564393  0.11713544
##   0.3   6         0.6                1.0000000  150      0.6527768  0.11350277
##   0.3   6         0.6                1.0000000  200      0.6488948  0.10982845
##   0.3   6         0.6                1.0000000  250      0.6472525  0.10857295
##   0.3   6         0.6                1.0000000  300      0.6454640  0.10822771
##   0.3   6         0.6                1.0000000  350      0.6441876  0.10670312
##   0.3   6         0.6                1.0000000  400      0.6429559  0.10539340
##   0.3   6         0.6                1.0000000  450      0.6416876  0.10447476
##   0.3   6         0.6                1.0000000  500      0.6405698  0.10325937
##   0.3   6         0.8                0.5000000   50      0.6527076  0.11334764
```

```
##   0.3   6         0.8               0.5000000  100      0.6433299  0.10462830
##   0.3   6         0.8               0.5000000  150      0.6369519  0.09860187
##   0.3   6         0.8               0.5000000  200      0.6325171  0.09435092
##   0.3   6         0.8               0.5000000  250      0.6278871  0.09126114
##   0.3   6         0.8               0.5000000  300      0.6267286  0.09048283
##   0.3   6         0.8               0.5000000  350      0.6240254  0.08857238
##   0.3   6         0.8               0.5000000  400      0.6237815  0.08806358
##   0.3   6         0.8               0.5000000  450      0.6232165  0.08804911
##   0.3   6         0.8               0.5000000  500      0.6231881  0.08781624
##   0.3   6         0.8               0.5555556   50      0.6555775  0.11648475
##   0.3   6         0.8               0.5555556  100      0.6460981  0.10794859
##   0.3   6         0.8               0.5555556  150      0.6375170  0.09963236
##   0.3   6         0.8               0.5555556  200      0.6340658  0.09756446
##   0.3   6         0.8               0.5555556  250      0.6312692  0.09570433
##   0.3   6         0.8               0.5555556  300      0.6293139  0.09459295
##   0.3   6         0.8               0.5555556  350      0.6277815  0.09335446
##   0.3   6         0.8               0.5555556  400      0.6272774  0.09279772
##   0.3   6         0.8               0.5555556  450      0.6267937  0.09205858
##   0.3   6         0.8               0.5555556  500      0.6262977  0.09210112
##   0.3   6         0.8               0.6111111   50      0.6551995  0.11585284
##   0.3   6         0.8               0.6111111  100      0.6463013  0.10742616
##   0.3   6         0.8               0.6111111  150      0.6391918  0.10270862
##   0.3   6         0.8               0.6111111  200      0.6357406  0.09833478
##   0.3   6         0.8               0.6111111  250      0.6328220  0.09599085
##   0.3   6         0.8               0.6111111  300      0.6300781  0.09388787
##   0.3   6         0.8               0.6111111  350      0.6295334  0.09295228
##   0.3   6         0.8               0.6111111  400      0.6285944  0.09254479
##   0.3   6         0.8               0.6111111  450      0.6278953  0.09202158
##   0.3   6         0.8               0.6111111  500      0.6281839  0.09269540
##   0.3   6         0.8               0.6666667   50      0.6580531  0.11836320
##   0.3   6         0.8               0.6666667  100      0.6507768  0.11106079
##   0.3   6         0.8               0.6666667  150      0.6452891  0.10609725
##   0.3   6         0.8               0.6666667  200      0.6406714  0.10163515
##   0.3   6         0.8               0.6666667  250      0.6377771  0.09864561
##   0.3   6         0.8               0.6666667  300      0.6350943  0.09668320
##   0.3   6         0.8               0.6666667  350      0.6339480  0.09552127
##   0.3   6         0.8               0.6666667  400      0.6338626  0.09564751
##   0.3   6         0.8               0.6666667  450      0.6330537  0.09491678
##   0.3   6         0.8               0.6666667  500      0.6323748  0.09469459
##   0.3   6         0.8               0.7222222   50      0.6591831  0.11928848
##   0.3   6         0.8               0.7222222  100      0.6515735  0.11227954
##   0.3   6         0.8               0.7222222  150      0.6453054  0.10668240
##   0.3   6         0.8               0.7222222  200      0.6406348  0.10262169
##   0.3   6         0.8               0.7222222  250      0.6389072  0.10117704
##   0.3   6         0.8               0.7222222  300      0.6371836  0.09989427
##   0.3   6         0.8               0.7222222  350      0.6354967  0.09867054
##   0.3   6         0.8               0.7222222  400      0.6334276  0.09695819
##   0.3   6         0.8               0.7222222  450      0.6336837  0.09630437
##   0.3   6         0.8               0.7222222  500      0.6335333  0.09613474
##   0.3   6         0.8               0.7777778   50      0.6588173  0.11931896
##   0.3   6         0.8               0.7777778  100      0.6519394  0.11209099
##   0.3   6         0.8               0.7777778  150      0.6476224  0.10850643
##   0.3   6         0.8               0.7777778  200      0.6434111  0.10509870
##   0.3   6         0.8               0.7777778  250      0.6404071  0.10308870
```

```
##   0.3   6         0.8               0.7777778  300      0.6378625  0.10083652
##   0.3   6         0.8               0.7777778  350      0.6366715  0.10029072
##   0.3   6         0.8               0.7777778  400      0.6357569  0.09915414
##   0.3   6         0.8               0.7777778  450      0.6353301  0.09901536
##   0.3   6         0.8               0.7777778  500      0.6353545  0.09814963
##   0.3   6         0.8               0.8333333   50      0.6595490  0.11931516
##   0.3   6         0.8               0.8333333  100      0.6537727  0.11530571
##   0.3   6         0.8               0.8333333  150      0.6483745  0.10896328
##   0.3   6         0.8               0.8333333  200      0.6447526  0.10635138
##   0.3   6         0.8               0.8333333  250      0.6415372  0.10310776
##   0.3   6         0.8               0.8333333  300      0.6394316  0.10268160
##   0.3   6         0.8               0.8333333  350      0.6386755  0.10199045
##   0.3   6         0.8               0.8333333  400      0.6376471  0.10021078
##   0.3   6         0.8               0.8333333  450      0.6364682  0.09918046
##   0.3   6         0.8               0.8333333  500      0.6356227  0.09834859
##   0.3   6         0.8               0.8888889   50      0.6615692  0.12265995
##   0.3   6         0.8               0.8888889  100      0.6565938  0.11834355
##   0.3   6         0.8               0.8888889  150      0.6515735  0.11302476
##   0.3   6         0.8               0.8888889  200      0.6480940  0.11009292
##   0.3   6         0.8               0.8888889  250      0.6451672  0.10656870
##   0.3   6         0.8               0.8888889  300      0.6439315  0.10581263
##   0.3   6         0.8               0.8888889  350      0.6427608  0.10477576
##   0.3   6         0.8               0.8888889  400      0.6420291  0.10445471
##   0.3   6         0.8               0.8888889  450      0.6412852  0.10385398
##   0.3   6         0.8               0.8888889  500      0.6407568  0.10352457
##   0.3   6         0.8               0.9444444   50      0.6614432  0.12252917
##   0.3   6         0.8               0.9444444  100      0.6551751  0.11672308
##   0.3   6         0.8               0.9444444  150      0.6508500  0.11120581
##   0.3   6         0.8               0.9444444  200      0.6467566  0.10825343
##   0.3   6         0.8               0.9444444  250      0.6442404  0.10541950
##   0.3   6         0.8               0.9444444  300      0.6424518  0.10472598
##   0.3   6         0.8               0.9444444  350      0.6408828  0.10274455
##   0.3   6         0.8               0.9444444  400      0.6403706  0.10274918
##   0.3   6         0.8               0.9444444  450      0.6393096  0.10213190
##   0.3   6         0.8               0.9444444  500      0.6385698  0.10123395
##   0.3   6         0.8               1.0000000   50      0.6643781  0.12316126
##   0.3   6         0.8               1.0000000  100      0.6584921  0.11795735
##   0.3   6         0.8               1.0000000  150      0.6539312  0.11318795
##   0.3   6         0.8               1.0000000  200      0.6506508  0.11042952
##   0.3   6         0.8               1.0000000  250      0.6477728  0.10808269
##   0.3   6         0.8               1.0000000  300      0.6460209  0.10714127
##   0.3   6         0.8               1.0000000  350      0.6443746  0.10546573
##   0.3   6         0.8               1.0000000  400      0.6430372  0.10423455
##   0.3   6         0.8               1.0000000  450      0.6427364  0.10383666
##   0.3   6         0.8               1.0000000  500      0.6418909  0.10228114
##   0.3   7         0.6               0.5000000   50      0.6453420  0.10677615
##   0.3   7         0.6               0.5000000  100      0.6360048  0.09910410
##   0.3   7         0.6               0.5000000  150      0.6315090  0.09450704
##   0.3   7         0.6               0.5000000  200      0.6283343  0.09101825
##   0.3   7         0.6               0.5000000  250      0.6259034  0.08891498
##   0.3   7         0.6               0.5000000  300      0.6241514  0.08717673
##   0.3   7         0.6               0.5000000  350      0.6249400  0.08756540
##   0.3   7         0.6               0.5000000  400      0.6245864  0.08737850
##   0.3   7         0.6               0.5000000  450      0.6245864  0.08774521
```

```
##   0.3   7         0.6               0.5000000  500      0.6254603  0.08839701
##   0.3   7         0.6               0.5555556   50      0.6493825  0.11075359
##   0.3   7         0.6               0.5555556  100      0.6388543  0.10334710
##   0.3   7         0.6               0.5555556  150      0.6333951  0.09850541
##   0.3   7         0.6               0.5555556  200      0.6297245  0.09471435
##   0.3   7         0.6               0.5555556  250      0.6265498  0.09229541
##   0.3   7         0.6               0.5555556  300      0.6250214  0.09143120
##   0.3   7         0.6               0.5555556  350      0.6251392  0.09178874
##   0.3   7         0.6               0.5555556  400      0.6255823  0.09171962
##   0.3   7         0.6               0.5555556  450      0.6260945  0.09147325
##   0.3   7         0.6               0.5555556  500      0.6271189  0.09276017
##   0.3   7         0.6               0.6111111   50      0.6496508  0.11052912
##   0.3   7         0.6               0.6111111  100      0.6404884  0.10232792
##   0.3   7         0.6               0.6111111  150      0.6351837  0.09763178
##   0.3   7         0.6               0.6111111  200      0.6328301  0.09528232
##   0.3   7         0.6               0.6111111  250      0.6318870  0.09470973
##   0.3   7         0.6               0.6111111  300      0.6299481  0.09363814
##   0.3   7         0.6               0.6111111  350      0.6298424  0.09337196
##   0.3   7         0.6               0.6111111  400      0.6294237  0.09313641
##   0.3   7         0.6               0.6111111  450      0.6294928  0.09340081
##   0.3   7         0.6               0.6111111  500      0.6296757  0.09293583
##   0.3   7         0.6               0.6666667   50      0.6532036  0.11407658
##   0.3   7         0.6               0.6666667  100      0.6443786  0.10540867
##   0.3   7         0.6               0.6666667  150      0.6389966  0.10007698
##   0.3   7         0.6               0.6666667  200      0.6352162  0.09710180
##   0.3   7         0.6               0.6666667  250      0.6327285  0.09573937
##   0.3   7         0.6               0.6666667  300      0.6320781  0.09572583
##   0.3   7         0.6               0.6666667  350      0.6316756  0.09549215
##   0.3   7         0.6               0.6666667  400      0.6314643  0.09571510
##   0.3   7         0.6               0.6666667  450      0.6310984  0.09527207
##   0.3   7         0.6               0.6666667  500      0.6323342  0.09661299
##   0.3   7         0.6               0.7222222   50      0.6537767  0.11650197
##   0.3   7         0.6               0.7222222  100      0.6451672  0.10772002
##   0.3   7         0.6               0.7222222  150      0.6388056  0.10185954
##   0.3   7         0.6               0.7222222  200      0.6356959  0.09929467
##   0.3   7         0.6               0.7222222  250      0.6339805  0.09758685
##   0.3   7         0.6               0.7222222  300      0.6332447  0.09695661
##   0.3   7         0.6               0.7222222  350      0.6335008  0.09704381
##   0.3   7         0.6               0.7222222  400      0.6329805  0.09708112
##   0.3   7         0.6               0.7222222  450      0.6331594  0.09694222
##   0.3   7         0.6               0.7222222  500      0.6329033  0.09680281
##   0.3   7         0.6               0.7777778   50      0.6530166  0.11382270
##   0.3   7         0.6               0.7777778  100      0.6461265  0.10574066
##   0.3   7         0.6               0.7777778  150      0.6407974  0.10159266
##   0.3   7         0.6               0.7777778  200      0.6384885  0.09927526
##   0.3   7         0.6               0.7777778  250      0.6367568  0.09876686
##   0.3   7         0.6               0.7777778  300      0.6353016  0.09640365
##   0.3   7         0.6               0.7777778  350      0.6352162  0.09696329
##   0.3   7         0.6               0.7777778  400      0.6348544  0.09694264
##   0.3   7         0.6               0.7777778  450      0.6352569  0.09765684
##   0.3   7         0.6               0.7777778  500      0.6349967  0.09770616
##   0.3   7         0.6               0.8333333   50      0.6562076  0.11872387
##   0.3   7         0.6               0.8333333  100      0.6495086  0.11115624
##   0.3   7         0.6               0.8333333  150      0.6435900  0.10648070
```

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##   0.3   7         0.6               0.8333333  200      0.6403259  0.10397605
##   0.3   7         0.6               0.8333333  250      0.6395047  0.10364028
##   0.3   7         0.6               0.8333333  300      0.6384600  0.10285238
##   0.3   7         0.6               0.8333333  350      0.6381145  0.10221901
##   0.3   7         0.6               0.8333333  400      0.6380698  0.10184305
##   0.3   7         0.6               0.8333333  450      0.6381430  0.10122936
##   0.3   7         0.6               0.8333333  500      0.6380779  0.10236504
##   0.3   7         0.6               0.8888889   50      0.6562686  0.11729365
##   0.3   7         0.6               0.8888889  100      0.6488297  0.11080275
##   0.3   7         0.6               0.8888889  150      0.6440291  0.10570664
##   0.3   7         0.6               0.8888889  200      0.6417323  0.10388247
##   0.3   7         0.6               0.8888889  250      0.6400047  0.10299312
##   0.3   7         0.6               0.8888889  300      0.6388340  0.10235709
##   0.3   7         0.6               0.8888889  350      0.6387893  0.10122542
##   0.3   7         0.6               0.8888889  400      0.6385007  0.10104544
##   0.3   7         0.6               0.8888889  450      0.6385373  0.10187140
##   0.3   7         0.6               0.8888889  500      0.6389194  0.10113335
##   0.3   7         0.6               0.9444444   50      0.6566303  0.11747123
##   0.3   7         0.6               0.9444444  100      0.6501508  0.11222267
##   0.3   7         0.6               0.9444444  150      0.6465005  0.10818477
##   0.3   7         0.6               0.9444444  200      0.6445128  0.10684767
##   0.3   7         0.6               0.9444444  250      0.6423461  0.10501383
##   0.3   7         0.6               0.9444444  300      0.6414234  0.10442747
##   0.3   7         0.6               0.9444444  350      0.6410291  0.10333507
##   0.3   7         0.6               0.9444444  400      0.6408909  0.10403356
##   0.3   7         0.6               0.9444444  450      0.6407893  0.10388765
##   0.3   7         0.6               0.9444444  500      0.6408015  0.10354553
##   0.3   7         0.6               1.0000000   50      0.6584108  0.11868985
##   0.3   7         0.6               1.0000000  100      0.6533865  0.11426381
##   0.3   7         0.6               1.0000000  150      0.6495045  0.11053106
##   0.3   7         0.6               1.0000000  200      0.6467119  0.10852005
##   0.3   7         0.6               1.0000000  250      0.6443786  0.10658800
##   0.3   7         0.6               1.0000000  300      0.6441469  0.10656373
##   0.3   7         0.6               1.0000000  350      0.6435941  0.10579508
##   0.3   7         0.6               1.0000000  400      0.6426998  0.10502078
##   0.3   7         0.6               1.0000000  450      0.6425616  0.10475204
##   0.3   7         0.6               1.0000000  500      0.6422567  0.10451756
##   0.3   7         0.8               0.5000000   50      0.6459070  0.10815044
##   0.3   7         0.8               0.5000000  100      0.6345821  0.09753967
##   0.3   7         0.8               0.5000000  150      0.6292611  0.09342512
##   0.3   7         0.8               0.5000000  200      0.6271676  0.09065317
##   0.3   7         0.8               0.5000000  250      0.6246230  0.08851180
##   0.3   7         0.8               0.5000000  300      0.6238140  0.08788046
##   0.3   7         0.8               0.5000000  350      0.6229198  0.08758304
##   0.3   7         0.8               0.5000000  400      0.6231759  0.08762155
##   0.3   7         0.8               0.5000000  450      0.6235295  0.08763129
##   0.3   7         0.8               0.5000000  500      0.6240132  0.08899004
##   0.3   7         0.8               0.5555556   50      0.6475493  0.10907514
##   0.3   7         0.8               0.5555556  100      0.6383300  0.10027536
##   0.3   7         0.8               0.5555556  150      0.6322570  0.09673863
##   0.3   7         0.8               0.5555556  200      0.6294074  0.09407017
##   0.3   7         0.8               0.5555556  250      0.6284765  0.09339751
##   0.3   7         0.8               0.5555556  300      0.6269969  0.09111687
##   0.3   7         0.8               0.5555556  350      0.6258343  0.09000783
```

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##   0.3   7         0.8               0.5555556  400      0.6260376  0.09043766
##   0.3   7         0.8               0.5555556  450      0.6272611  0.09177035
##   0.3   7         0.8               0.5555556  500      0.6278383  0.09228414
##   0.3   7         0.8               0.6111111   50      0.6498866  0.11293075
##   0.3   7         0.8               0.6111111  100      0.6408055  0.10429781
##   0.3   7         0.8               0.6111111  150      0.6354439  0.09969793
##   0.3   7         0.8               0.6111111  200      0.6322204  0.09669624
##   0.3   7         0.8               0.6111111  250      0.6316798  0.09578343
##   0.3   7         0.8               0.6111111  300      0.6305456  0.09492761
##   0.3   7         0.8               0.6111111  350      0.6305212  0.09492624
##   0.3   7         0.8               0.6111111  400      0.6303220  0.09509981
##   0.3   7         0.8               0.6111111  450      0.6311391  0.09530452
##   0.3   7         0.8               0.6111111  500      0.6309725  0.09551619
##   0.3   7         0.8               0.6666667   50      0.6501508  0.11181427
##   0.3   7         0.8               0.6666667  100      0.6407283  0.10271079
##   0.3   7         0.8               0.6666667  150      0.6364845  0.09855247
##   0.3   7         0.8               0.6666667  200      0.6327285  0.09633123
##   0.3   7         0.8               0.6666667  250      0.6313545  0.09542574
##   0.3   7         0.8               0.6666667  300      0.6301147  0.09521584
##   0.3   7         0.8               0.6666667  350      0.6298017  0.09506214
##   0.3   7         0.8               0.6666667  400      0.6302407  0.09542166
##   0.3   7         0.8               0.6666667  450      0.6311106  0.09573356
##   0.3   7         0.8               0.6666667  500      0.6310212  0.09590072
##   0.3   7         0.8               0.7222222   50      0.6527483  0.11408067
##   0.3   7         0.8               0.7222222  100      0.6448664  0.10619653
##   0.3   7         0.8               0.7222222  150      0.6398015  0.10242047
##   0.3   7         0.8               0.7222222  200      0.6370007  0.10027104
##   0.3   7         0.8               0.7222222  250      0.6340414  0.09700933
##   0.3   7         0.8               0.7222222  300      0.6333138  0.09636051
##   0.3   7         0.8               0.7222222  350      0.6337406  0.09717994
##   0.3   7         0.8               0.7222222  400      0.6330374  0.09671121
##   0.3   7         0.8               0.7222222  450      0.6333829  0.09709156
##   0.3   7         0.8               0.7222222  500      0.6332651  0.09620888
##   0.3   7         0.8               0.7777778   50      0.6537117  0.11384341
##   0.3   7         0.8               0.7777778  100      0.6456224  0.10583999
##   0.3   7         0.8               0.7777778  150      0.6405047  0.10302185
##   0.3   7         0.8               0.7777778  200      0.6380170  0.10057902
##   0.3   7         0.8               0.7777778  250      0.6367690  0.09932762
##   0.3   7         0.8               0.7777778  300      0.6350374  0.09869916
##   0.3   7         0.8               0.7777778  350      0.6346553  0.09756087
##   0.3   7         0.8               0.7777778  400      0.6349032  0.09815450
##   0.3   7         0.8               0.7777778  450      0.6350618  0.09829345
##   0.3   7         0.8               0.7777778  500      0.6354927  0.09861437
##   0.3   7         0.8               0.8333333   50      0.6536305  0.11574467
##   0.3   7         0.8               0.8333333  100      0.6464314  0.10845234
##   0.3   7         0.8               0.8333333  150      0.6417202  0.10407013
##   0.3   7         0.8               0.8333333  200      0.6384479  0.10104593
##   0.3   7         0.8               0.8333333  250      0.6365130  0.09927323
##   0.3   7         0.8               0.8333333  300      0.6359398  0.09964961
##   0.3   7         0.8               0.8333333  350      0.6353545  0.09883661
##   0.3   7         0.8               0.8333333  400      0.6352528  0.09822364
##   0.3   7         0.8               0.8333333  450      0.6360293  0.09903403
##   0.3   7         0.8               0.8333333  500      0.6360292  0.09979465
##   0.3   7         0.8               0.8888889   50      0.6542198  0.11572407
```

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##   0.3   7         0.8               0.8888889  100      0.6476062  0.10988656
##   0.3   7         0.8               0.8888889  150      0.6429843  0.10509250
##   0.3   7         0.8               0.8888889  200      0.6409884  0.10369028
##   0.3   7         0.8               0.8888889  250      0.6385535  0.10208882
##   0.3   7         0.8               0.8888889  300      0.6379072  0.10245680
##   0.3   7         0.8               0.8888889  350      0.6372934  0.10232097
##   0.3   7         0.8               0.8888889  400      0.6377812  0.10223664
##   0.3   7         0.8               0.8888889  450      0.6377446  0.10224030
##   0.3   7         0.8               0.8888889  500      0.6380332  0.10250565
##   0.3   7         0.8               0.9444444   50      0.6566182  0.11668557
##   0.3   7         0.8               0.9444444  100      0.6503053  0.11048347
##   0.3   7         0.8               0.9444444  150      0.6460046  0.10688980
##   0.3   7         0.8               0.9444444  200      0.6426998  0.10472845
##   0.3   7         0.8               0.9444444  250      0.6411998  0.10355271
##   0.3   7         0.8               0.9444444  300      0.6405820  0.10329674
##   0.3   7         0.8               0.9444444  350      0.6398625  0.10220079
##   0.3   7         0.8               0.9444444  400      0.6393422  0.10201185
##   0.3   7         0.8               0.9444444  450      0.6389275  0.10071940
##   0.3   7         0.8               0.9444444  500      0.6389194  0.10098579
##   0.3   7         0.8               1.0000000   50      0.6571222  0.11917873
##   0.3   7         0.8               1.0000000  100      0.6525166  0.11401934
##   0.3   7         0.8               1.0000000  150      0.6472525  0.10966741
##   0.3   7         0.8               1.0000000  200      0.6447607  0.10775259
##   0.3   7         0.8               1.0000000  250      0.6427811  0.10597779
##   0.3   7         0.8               1.0000000  300      0.6413340  0.10395080
##   0.3   7         0.8               1.0000000  350      0.6406511  0.10368484
##   0.3   7         0.8               1.0000000  400      0.6401592  0.10295262
##   0.3   7         0.8               1.0000000  450      0.6407405  0.10449890
##   0.3   7         0.8               1.0000000  500      0.6406714  0.10416198
##   0.3   8         0.6               0.5000000   50      0.6414966  0.10288319
##   0.3   8         0.6               0.5000000  100      0.6301757  0.09321624
##   0.3   8         0.6               0.5000000  150      0.6271392  0.09018466
##   0.3   8         0.6               0.5000000  200      0.6258872  0.08847822
##   0.3   8         0.6               0.5000000  250      0.6252693  0.08850080
##   0.3   8         0.6               0.5000000  300      0.6257368  0.08876274
##   0.3   8         0.6               0.5000000  350      0.6264197  0.08946612
##   0.3   8         0.6               0.5000000  400      0.6264725  0.08968366
##   0.3   8         0.6               0.5000000  450      0.6265294  0.08901560
##   0.3   8         0.6               0.5000000  500      0.6277570  0.09095017
##   0.3   8         0.6               0.5555556   50      0.6420616  0.10401641
##   0.3   8         0.6               0.5555556  100      0.6328707  0.09548396
##   0.3   8         0.6               0.5555556  150      0.6293668  0.09279651
##   0.3   8         0.6               0.5555556  200      0.6284887  0.09135551
##   0.3   8         0.6               0.5555556  250      0.6271798  0.09069257
##   0.3   8         0.6               0.5555556  300      0.6273343  0.09157843
##   0.3   8         0.6               0.5555556  350      0.6276229  0.09181729
##   0.3   8         0.6               0.5555556  400      0.6289156  0.09253638
##   0.3   8         0.6               0.5555556  450      0.6296676  0.09344216
##   0.3   8         0.6               0.5555556  500      0.6298627  0.09366151
##   0.3   8         0.6               0.6111111   50      0.6429437  0.10535834
##   0.3   8         0.6               0.6111111  100      0.6341065  0.09727172
##   0.3   8         0.6               0.6111111  150      0.6305172  0.09507428
##   0.3   8         0.6               0.6111111  200      0.6276514  0.09304646
##   0.3   8         0.6               0.6111111  250      0.6277774  0.09270639
```

```
##   0.3   8         0.6               0.6111111  300      0.6288180  0.09282484
##   0.3   8         0.6               0.6111111  350      0.6284115  0.09254590
##   0.3   8         0.6               0.6111111  400      0.6298058  0.09457497
##   0.3   8         0.6               0.6111111  450      0.6307854  0.09458364
##   0.3   8         0.6               0.6111111  500      0.6313748  0.09514327
##   0.3   8         0.6               0.6666667   50      0.6457363  0.10651928
##   0.3   8         0.6               0.6666667  100      0.6373463  0.09936170
##   0.3   8         0.6               0.6666667  150      0.6322122  0.09574687
##   0.3   8         0.6               0.6666667  200      0.6317407  0.09489286
##   0.3   8         0.6               0.6666667  250      0.6318789  0.09580104
##   0.3   8         0.6               0.6666667  300      0.6324114  0.09672767
##   0.3   8         0.6               0.6666667  350      0.6337041  0.09744437
##   0.3   8         0.6               0.6666667  400      0.6338667  0.09812574
##   0.3   8         0.6               0.6666667  450      0.6342203  0.09834090
##   0.3   8         0.6               0.6666667  500      0.6342569  0.09817834
##   0.3   8         0.6               0.7222222   50      0.6472607  0.10891998
##   0.3   8         0.6               0.7222222  100      0.6396511  0.10240696
##   0.3   8         0.6               0.7222222  150      0.6367284  0.09944406
##   0.3   8         0.6               0.7222222  200      0.6347406  0.09890398
##   0.3   8         0.6               0.7222222  250      0.6341960  0.09762946
##   0.3   8         0.6               0.7222222  300      0.6347691  0.09837795
##   0.3   8         0.6               0.7222222  350      0.6349317  0.09968968
##   0.3   8         0.6               0.7222222  400      0.6358748  0.10023612
##   0.3   8         0.6               0.7222222  450      0.6370089  0.10136929
##   0.3   8         0.6               0.7222222  500      0.6371349  0.10186978
##   0.3   8         0.6               0.7777778   50      0.6475248  0.11021570
##   0.3   8         0.6               0.7777778  100      0.6400616  0.10450497
##   0.3   8         0.6               0.7777778  150      0.6374235  0.10191223
##   0.3   8         0.6               0.7777778  200      0.6360008  0.10120181
##   0.3   8         0.6               0.7777778  250      0.6359154  0.10143839
##   0.3   8         0.6               0.7777778  300      0.6359520  0.10129619
##   0.3   8         0.6               0.7777778  350      0.6356756  0.10097166
##   0.3   8         0.6               0.7777778  400      0.6366796  0.10177335
##   0.3   8         0.6               0.7777778  450      0.6370251  0.10274424
##   0.3   8         0.6               0.7777778  500      0.6374479  0.10265443
##   0.3   8         0.6               0.8333333   50      0.6496061  0.11096730
##   0.3   8         0.6               0.8333333  100      0.6422770  0.10418857
##   0.3   8         0.6               0.8333333  150      0.6386430  0.10187707
##   0.3   8         0.6               0.8333333  200      0.6376796  0.10084769
##   0.3   8         0.6               0.8333333  250      0.6373503  0.10075207
##   0.3   8         0.6               0.8333333  300      0.6377284  0.10144702
##   0.3   8         0.6               0.8333333  350      0.6385454  0.10238871
##   0.3   8         0.6               0.8333333  400      0.6388666  0.10237345
##   0.3   8         0.6               0.8333333  450      0.6389804  0.10258476
##   0.3   8         0.6               0.8333333  500      0.6397040  0.10321764
##   0.3   8         0.6               0.8888889   50      0.6495980  0.11086261
##   0.3   8         0.6               0.8888889  100      0.6425209  0.10470565
##   0.3   8         0.6               0.8888889  150      0.6398868  0.10237927
##   0.3   8         0.6               0.8888889  200      0.6381674  0.10123676
##   0.3   8         0.6               0.8888889  250      0.6384966  0.10082983
##   0.3   8         0.6               0.8888889  300      0.6383422  0.10125404
##   0.3   8         0.6               0.8888889  350      0.6386064  0.10126643
##   0.3   8         0.6               0.8888889  400      0.6388503  0.10164617
##   0.3   8         0.6               0.8888889  450      0.6392446  0.10140307
```

```
##   0.3   8         0.6               0.8888889  500      0.6393177  0.10195724
##   0.3   8         0.6               0.9444444   50      0.6515654  0.11175673
##   0.3   8         0.6               0.9444444  100      0.6448501  0.10592172
##   0.3   8         0.6               0.9444444  150      0.6423624  0.10385770
##   0.3   8         0.6               0.9444444  200      0.6405576  0.10229167
##   0.3   8         0.6               0.9444444  250      0.6395129  0.10090063
##   0.3   8         0.6               0.9444444  300      0.6403909  0.10184904
##   0.3   8         0.6               0.9444444  350      0.6405576  0.10176476
##   0.3   8         0.6               0.9444444  400      0.6411673  0.10270723
##   0.3   8         0.6               0.9444444  450      0.6410372  0.10287912
##   0.3   8         0.6               0.9444444  500      0.6412161  0.10263464
##   0.3   8         0.6               1.0000000   50      0.6526548  0.11308365
##   0.3   8         0.6               1.0000000  100      0.6481183  0.10932346
##   0.3   8         0.6               1.0000000  150      0.6440615  0.10557509
##   0.3   8         0.6               1.0000000  200      0.6427526  0.10515568
##   0.3   8         0.6               1.0000000  250      0.6414762  0.10414576
##   0.3   8         0.6               1.0000000  300      0.6417323  0.10444473
##   0.3   8         0.6               1.0000000  350      0.6420169  0.10460650
##   0.3   8         0.6               1.0000000  400      0.6423827  0.10519655
##   0.3   8         0.6               1.0000000  450      0.6431754  0.10514331
##   0.3   8         0.6               1.0000000  500      0.6427648  0.10476051
##   0.3   8         0.8               0.5000000   50      0.6385739  0.10047338
##   0.3   8         0.8               0.5000000  100      0.6294034  0.09268172
##   0.3   8         0.8               0.5000000  150      0.6253831  0.08957965
##   0.3   8         0.8               0.5000000  200      0.6242815  0.08847583
##   0.3   8         0.8               0.5000000  250      0.6236189  0.08816019
##   0.3   8         0.8               0.5000000  300      0.6237002  0.08836792
##   0.3   8         0.8               0.5000000  350      0.6244604  0.08908841
##   0.3   8         0.8               0.5000000  400      0.6259197  0.09013885
##   0.3   8         0.8               0.5000000  450      0.6272571  0.09118958
##   0.3   8         0.8               0.5000000  500      0.6274034  0.09121030
##   0.3   8         0.8               0.5555556   50      0.6419721  0.10371106
##   0.3   8         0.8               0.5555556  100      0.6319439  0.09544040
##   0.3   8         0.8               0.5555556  150      0.6279318  0.09078298
##   0.3   8         0.8               0.5555556  200      0.6263221  0.09036210
##   0.3   8         0.8               0.5555556  250      0.6262489  0.09036313
##   0.3   8         0.8               0.5555556  300      0.6267896  0.09087379
##   0.3   8         0.8               0.5555556  350      0.6272449  0.09147114
##   0.3   8         0.8               0.5555556  400      0.6273302  0.09183734
##   0.3   8         0.8               0.5555556  450      0.6278709  0.09282292
##   0.3   8         0.8               0.5555556  500      0.6290985  0.09347237
##   0.3   8         0.8               0.6111111   50      0.6425697  0.10436333
##   0.3   8         0.8               0.6111111  100      0.6336715  0.09750324
##   0.3   8         0.8               0.6111111  150      0.6298383  0.09380294
##   0.3   8         0.8               0.6111111  200      0.6305009  0.09360236
##   0.3   8         0.8               0.6111111  250      0.6305700  0.09347862
##   0.3   8         0.8               0.6111111  300      0.6301839  0.09349252
##   0.3   8         0.8               0.6111111  350      0.6300050  0.09333730
##   0.3   8         0.8               0.6111111  400      0.6311960  0.09461171
##   0.3   8         0.8               0.6111111  450      0.6322936  0.09578611
##   0.3   8         0.8               0.6111111  500      0.6331919  0.09671346
##   0.3   8         0.8               0.6666667   50      0.6446103  0.10688176
##   0.3   8         0.8               0.6666667  100      0.6380007  0.10071326
##   0.3   8         0.8               0.6666667  150      0.6331431  0.09616199
```

```
##   0.3   8         0.8               0.6666667  200      0.6319033  0.09494692
##   0.3   8         0.8               0.6666667  250      0.6317488  0.09518851
##   0.3   8         0.8               0.6666667  300      0.6322123  0.09525940
##   0.3   8         0.8               0.6666667  350      0.6335659  0.09683893
##   0.3   8         0.8               0.6666667  400      0.6344358  0.09801767
##   0.3   8         0.8               0.6666667  450      0.6353097  0.09864424
##   0.3   8         0.8               0.6666667  500      0.6347772  0.09844227
##   0.3   8         0.8               0.7222222   50      0.6465818  0.10773390
##   0.3   8         0.8               0.7222222  100      0.6389113  0.10183450
##   0.3   8         0.8               0.7222222  150      0.6354276  0.09873516
##   0.3   8         0.8               0.7222222  200      0.6339154  0.09725449
##   0.3   8         0.8               0.7222222  250      0.6342122  0.09777797
##   0.3   8         0.8               0.7222222  300      0.6348748  0.09851026
##   0.3   8         0.8               0.7222222  350      0.6354886  0.09943999
##   0.3   8         0.8               0.7222222  400      0.6360414  0.09930290
##   0.3   8         0.8               0.7222222  450      0.6366633  0.09949811
##   0.3   8         0.8               0.7222222  500      0.6371146  0.09968185
##   0.3   8         0.8               0.7777778   50      0.6457566  0.10720130
##   0.3   8         0.8               0.7777778  100      0.6399153  0.10249325
##   0.3   8         0.8               0.7777778  150      0.6359520  0.09876505
##   0.3   8         0.8               0.7777778  200      0.6351918  0.09882471
##   0.3   8         0.8               0.7777778  250      0.6363219  0.09950079
##   0.3   8         0.8               0.7777778  300      0.6372650  0.10001941
##   0.3   8         0.8               0.7777778  350      0.6373625  0.10041714
##   0.3   8         0.8               0.7777778  400      0.6378137  0.10044266
##   0.3   8         0.8               0.7777778  450      0.6382202  0.10108031
##   0.3   8         0.8               0.7777778  500      0.6384600  0.10114165
##   0.3   8         0.8               0.8333333   50      0.6498216  0.11098324
##   0.3   8         0.8               0.8333333  100      0.6420738  0.10436711
##   0.3   8         0.8               0.8333333  150      0.6395413  0.10162244
##   0.3   8         0.8               0.8333333  200      0.6379641  0.10052161
##   0.3   8         0.8               0.8333333  250      0.6375373  0.10010721
##   0.3   8         0.8               0.8333333  300      0.6376349  0.10024570
##   0.3   8         0.8               0.8333333  350      0.6383788  0.10111447
##   0.3   8         0.8               0.8333333  400      0.6388056  0.10183451
##   0.3   8         0.8               0.8333333  450      0.6396958  0.10250211
##   0.3   8         0.8               0.8333333  500      0.6394316  0.10236336
##   0.3   8         0.8               0.8888889   50      0.6513622  0.11337849
##   0.3   8         0.8               0.8888889  100      0.6442404  0.10669544
##   0.3   8         0.8               0.8888889  150      0.6403299  0.10341199
##   0.3   8         0.8               0.8888889  200      0.6389763  0.10183779
##   0.3   8         0.8               0.8888889  250      0.6381552  0.10135005
##   0.3   8         0.8               0.8888889  300      0.6386430  0.10131212
##   0.3   8         0.8               0.8888889  350      0.6386836  0.10128161
##   0.3   8         0.8               0.8888889  400      0.6397405  0.10267824
##   0.3   8         0.8               0.8888889  450      0.6398747  0.10235336
##   0.3   8         0.8               0.8888889  500      0.6404112  0.10334275
##   0.3   8         0.8               0.9444444   50      0.6521996  0.11307604
##   0.3   8         0.8               0.9444444  100      0.6460656  0.10788839
##   0.3   8         0.8               0.9444444  150      0.6425128  0.10428322
##   0.3   8         0.8               0.9444444  200      0.6405007  0.10263825
##   0.3   8         0.8               0.9444444  250      0.6396511  0.10218897
##   0.3   8         0.8               0.9444444  300      0.6394275  0.10208414
##   0.3   8         0.8               0.9444444  350      0.6398299  0.10270774
```

```
##   0.3   8         0.8                 0.9444444  400      0.6403259  0.10282985
##   0.3   8         0.8                 0.9444444  450      0.6412445  0.10367355
##   0.3   8         0.8                 0.9444444  500      0.6413543  0.10342404
##   0.3   8         0.8                 1.0000000   50      0.6527402  0.11466410
##   0.3   8         0.8                 1.0000000  100      0.6463135  0.10795426
##   0.3   8         0.8                 1.0000000  150      0.6442363  0.10696558
##   0.3   8         0.8                 1.0000000  200      0.6423136  0.10516138
##   0.3   8         0.8                 1.0000000  250      0.6416388  0.10405149
##   0.3   8         0.8                 1.0000000  300      0.6417242  0.10420071
##   0.3   8         0.8                 1.0000000  350      0.6418624  0.10395037
##   0.3   8         0.8                 1.0000000  400      0.6421592  0.10447398
##   0.3   8         0.8                 1.0000000  450      0.6423787  0.10424641
##   0.3   8         0.8                 1.0000000  500      0.6424315  0.10457188
##   0.3   9         0.6                 0.5000000   50      0.6338870  0.09775682
##   0.3   9         0.6                 0.5000000  100      0.6261067  0.09128096
##   0.3   9         0.6                 0.5000000  150      0.6242978  0.08908729
##   0.3   9         0.6                 0.5000000  200      0.6246514  0.08959477
##   0.3   9         0.6                 0.5000000  250      0.6248669  0.09009987
##   0.3   9         0.6                 0.5000000  300      0.6268749  0.09200803
##   0.3   9         0.6                 0.5000000  350      0.6274888  0.09264069
##   0.3   9         0.6                 0.5000000  400      0.6277774  0.09310978
##   0.3   9         0.6                 0.5000000  450      0.6291960  0.09395512
##   0.3   9         0.6                 0.5000000  500      0.6297082  0.09430430
##   0.3   9         0.6                 0.5555556   50      0.6352813  0.09698366
##   0.3   9         0.6                 0.5555556  100      0.6284603  0.09171282
##   0.3   9         0.6                 0.5555556  150      0.6259278  0.09002099
##   0.3   9         0.6                 0.5555556  200      0.6268343  0.09115037
##   0.3   9         0.6                 0.5555556  250      0.6270254  0.08987453
##   0.3   9         0.6                 0.5555556  300      0.6289156  0.09245321
##   0.3   9         0.6                 0.5555556  350      0.6307123  0.09426743
##   0.3   9         0.6                 0.5555556  400      0.6306025  0.09346729
##   0.3   9         0.6                 0.5555556  450      0.6308912  0.09387413
##   0.3   9         0.6                 0.5555556  500      0.6311310  0.09411191
##   0.3   9         0.6                 0.6111111   50      0.6380658  0.10034677
##   0.3   9         0.6                 0.6111111  100      0.6311797  0.09449958
##   0.3   9         0.6                 0.6111111  150      0.6303139  0.09406559
##   0.3   9         0.6                 0.6111111  200      0.6296147  0.09291762
##   0.3   9         0.6                 0.6111111  250      0.6305212  0.09428135
##   0.3   9         0.6                 0.6111111  300      0.6315252  0.09494732
##   0.3   9         0.6                 0.6111111  350      0.6332650  0.09627085
##   0.3   9         0.6                 0.6111111  400      0.6338219  0.09694691
##   0.3   9         0.6                 0.6111111  450      0.6347813  0.09733134
##   0.3   9         0.6                 0.6111111  500      0.6349845  0.09719354
##   0.3   9         0.6                 0.6666667   50      0.6386023  0.09997931
##   0.3   9         0.6                 0.6666667  100      0.6323017  0.09631970
##   0.3   9         0.6                 0.6666667  150      0.6305212  0.09525446
##   0.3   9         0.6                 0.6666667  200      0.6305415  0.09612315
##   0.3   9         0.6                 0.6666667  250      0.6321634  0.09683272
##   0.3   9         0.6                 0.6666667  300      0.6326756  0.09787773
##   0.3   9         0.6                 0.6666667  350      0.6337407  0.09798590
##   0.3   9         0.6                 0.6666667  400      0.6349236  0.09888436
##   0.3   9         0.6                 0.6666667  450      0.6350861  0.09945616
##   0.3   9         0.6                 0.6666667  500      0.6355252  0.09965057
##   0.3   9         0.6                 0.7222222   50      0.6415169  0.10404154
```

```
##   0.3   9         0.6               0.7222222  100      0.6352284  0.09808533
##   0.3   9         0.6               0.7222222  150      0.6346715  0.09793784
##   0.3   9         0.6               0.7222222  200      0.6335984  0.09639101
##   0.3   9         0.6               0.7222222  250      0.6338829  0.09671736
##   0.3   9         0.6               0.7222222  300      0.6351146  0.09665008
##   0.3   9         0.6               0.7222222  350      0.6359520  0.09770689
##   0.3   9         0.6               0.7222222  400      0.6361268  0.09845098
##   0.3   9         0.6               0.7222222  450      0.6375739  0.10015235
##   0.3   9         0.6               0.7222222  500      0.6375983  0.09990435
##   0.3   9         0.6               0.7777778   50      0.6428421  0.10566004
##   0.3   9         0.6               0.7777778  100      0.6374601  0.10017704
##   0.3   9         0.6               0.7777778  150      0.6363016  0.09990350
##   0.3   9         0.6               0.7777778  200      0.6358178  0.09963257
##   0.3   9         0.6               0.7777778  250      0.6362487  0.10026805
##   0.3   9         0.6               0.7777778  300      0.6370414  0.10097404
##   0.3   9         0.6               0.7777778  350      0.6381023  0.10246092
##   0.3   9         0.6               0.7777778  400      0.6387446  0.10291449
##   0.3   9         0.6               0.7777778  450      0.6390454  0.10348061
##   0.3   9         0.6               0.7777778  500      0.6394275  0.10339482
##   0.3   9         0.6               0.8333333   50      0.6450981  0.10629664
##   0.3   9         0.6               0.8333333  100      0.6396552  0.10219413
##   0.3   9         0.6               0.8333333  150      0.6386308  0.10117486
##   0.3   9         0.6               0.8333333  200      0.6378666  0.10104410
##   0.3   9         0.6               0.8333333  250      0.6388869  0.10236268
##   0.3   9         0.6               0.8333333  300      0.6397974  0.10361067
##   0.3   9         0.6               0.8333333  350      0.6404803  0.10404768
##   0.3   9         0.6               0.8333333  400      0.6416998  0.10507349
##   0.3   9         0.6               0.8333333  450      0.6424234  0.10541838
##   0.3   9         0.6               0.8333333  500      0.6425087  0.10502431
##   0.3   9         0.6               0.8888889   50      0.6460696  0.10710546
##   0.3   9         0.6               0.8888889  100      0.6405047  0.10188128
##   0.3   9         0.6               0.8888889  150      0.6385211  0.10049029
##   0.3   9         0.6               0.8888889  200      0.6390495  0.10098927
##   0.3   9         0.6               0.8888889  250      0.6396552  0.10189079
##   0.3   9         0.6               0.8888889  300      0.6401104  0.10232117
##   0.3   9         0.6               0.8888889  350      0.6408096  0.10297195
##   0.3   9         0.6               0.8888889  400      0.6416388  0.10427725
##   0.3   9         0.6               0.8888889  450      0.6425657  0.10461868
##   0.3   9         0.6               0.8888889  500      0.6427120  0.10472838
##   0.3   9         0.6               0.9444444   50      0.6457323  0.10744920
##   0.3   9         0.6               0.9444444  100      0.6416063  0.10368781
##   0.3   9         0.6               0.9444444  150      0.6400738  0.10309196
##   0.3   9         0.6               0.9444444  200      0.6397243  0.10148910
##   0.3   9         0.6               0.9444444  250      0.6404356  0.10230242
##   0.3   9         0.6               0.9444444  300      0.6409600  0.10296319
##   0.3   9         0.6               0.9444444  350      0.6419274  0.10489822
##   0.3   9         0.6               0.9444444  400      0.6426795  0.10504870
##   0.3   9         0.6               0.9444444  450      0.6437933  0.10586788
##   0.3   9         0.6               0.9444444  500      0.6438380  0.10568046
##   0.3   9         0.6               1.0000000   50      0.6469720  0.10999188
##   0.3   9         0.6               1.0000000  100      0.6436591  0.10627393
##   0.3   9         0.6               1.0000000  150      0.6414478  0.10420788
##   0.3   9         0.6               1.0000000  200      0.6414884  0.10514674
##   0.3   9         0.6               1.0000000  250      0.6422283  0.10549116
```

```
##   0.3   9         0.6               1.0000000  300      0.6422079  0.10586404
##   0.3   9         0.6               1.0000000  350      0.6428177  0.10651349
##   0.3   9         0.6               1.0000000  400      0.6428177  0.10650620
##   0.3   9         0.6               1.0000000  450      0.6434518  0.10712943
##   0.3   9         0.6               1.0000000  500      0.6437648  0.10765769
##   0.3   9         0.8               0.5000000   50      0.6343707  0.09682050
##   0.3   9         0.8               0.5000000  100      0.6265254  0.09059368
##   0.3   9         0.8               0.5000000  150      0.6241840  0.08830558
##   0.3   9         0.8               0.5000000  200      0.6254888  0.09000718
##   0.3   9         0.8               0.5000000  250      0.6268140  0.09105823
##   0.3   9         0.8               0.5000000  300      0.6278384  0.09263902
##   0.3   9         0.8               0.5000000  350      0.6290782  0.09351571
##   0.3   9         0.8               0.5000000  400      0.6293668  0.09402801
##   0.3   9         0.8               0.5000000  450      0.6300456  0.09436709
##   0.3   9         0.8               0.5000000  500      0.6308423  0.09519599
##   0.3   9         0.8               0.5555556   50      0.6347650  0.09830494
##   0.3   9         0.8               0.5555556  100      0.6285619  0.09134836
##   0.3   9         0.8               0.5555556  150      0.6275701  0.09193917
##   0.3   9         0.8               0.5555556  200      0.6288180  0.09295465
##   0.3   9         0.8               0.5555556  250      0.6283505  0.09330099
##   0.3   9         0.8               0.5555556  300      0.6296310  0.09344974
##   0.3   9         0.8               0.5555556  350      0.6308301  0.09548249
##   0.3   9         0.8               0.5555556  400      0.6318748  0.09670272
##   0.3   9         0.8               0.5555556  450      0.6319724  0.09636679
##   0.3   9         0.8               0.5555556  500      0.6319968  0.09680501
##   0.3   9         0.8               0.6111111   50      0.6387852  0.10078577
##   0.3   9         0.8               0.6111111  100      0.6327447  0.09578422
##   0.3   9         0.8               0.6111111  150      0.6310862  0.09443777
##   0.3   9         0.8               0.6111111  200      0.6313139  0.09503918
##   0.3   9         0.8               0.6111111  250      0.6326228  0.09626546
##   0.3   9         0.8               0.6111111  300      0.6338016  0.09683078
##   0.3   9         0.8               0.6111111  350      0.6346796  0.09734611
##   0.3   9         0.8               0.6111111  400      0.6359885  0.09831473
##   0.3   9         0.8               0.6111111  450      0.6363950  0.09884717
##   0.3   9         0.8               0.6111111  500      0.6369397  0.09961394
##   0.3   9         0.8               0.6666667   50      0.6393015  0.10186841
##   0.3   9         0.8               0.6666667  100      0.6345171  0.09700561
##   0.3   9         0.8               0.6666667  150      0.6325903  0.09503129
##   0.3   9         0.8               0.6666667  200      0.6325618  0.09607187
##   0.3   9         0.8               0.6666667  250      0.6334724  0.09692470
##   0.3   9         0.8               0.6666667  300      0.6344398  0.09788421
##   0.3   9         0.8               0.6666667  350      0.6358463  0.09825704
##   0.3   9         0.8               0.6666667  400      0.6364926  0.09984686
##   0.3   9         0.8               0.6666667  450      0.6366512  0.09974383
##   0.3   9         0.8               0.6666667  500      0.6369967  0.10012682
##   0.3   9         0.8               0.7222222   50      0.6421347  0.10369834
##   0.3   9         0.8               0.7222222  100      0.6352812  0.09734468
##   0.3   9         0.8               0.7222222  150      0.6345170  0.09754879
##   0.3   9         0.8               0.7222222  200      0.6348707  0.09715705
##   0.3   9         0.8               0.7222222  250      0.6353138  0.09796897
##   0.3   9         0.8               0.7222222  300      0.6370373  0.09998211
##   0.3   9         0.8               0.7222222  350      0.6372365  0.10002114
##   0.3   9         0.8               0.7222222  400      0.6380657  0.09989043
##   0.3   9         0.8               0.7222222  450      0.6383543  0.10038886
```

```
##   0.3   9         0.8               0.7222222  500      0.6381348  0.10074046
##   0.3   9         0.8               0.7777778   50      0.6432526  0.10606426
##   0.3   9         0.8               0.7777778  100      0.6394844  0.10229219
##   0.3   9         0.8               0.7777778  150      0.6377080  0.10105436
##   0.3   9         0.8               0.7777778  200      0.6379276  0.10162247
##   0.3   9         0.8               0.7777778  250      0.6391511  0.10225031
##   0.3   9         0.8               0.7777778  300      0.6393421  0.10285408
##   0.3   9         0.8               0.7777778  350      0.6405007  0.10329625
##   0.3   9         0.8               0.7777778  400      0.6405494  0.10346041
##   0.3   9         0.8               0.7777778  450      0.6411958  0.10347908
##   0.3   9         0.8               0.7777778  500      0.6419112  0.10430523
##   0.3   9         0.8               0.8333333   50      0.6443868  0.10598198
##   0.3   9         0.8               0.8333333  100      0.6384763  0.10243702
##   0.3   9         0.8               0.8333333  150      0.6379520  0.10093405
##   0.3   9         0.8               0.8333333  200      0.6380251  0.10189010
##   0.3   9         0.8               0.8333333  250      0.6391593  0.10251853
##   0.3   9         0.8               0.8333333  300      0.6395536  0.10326751
##   0.3   9         0.8               0.8333333  350      0.6403137  0.10461108
##   0.3   9         0.8               0.8333333  400      0.6411023  0.10475398
##   0.3   9         0.8               0.8333333  450      0.6417364  0.10446131
##   0.3   9         0.8               0.8333333  500      0.6423218  0.10510544
##   0.3   9         0.8               0.8888889   50      0.6466347  0.10775871
##   0.3   9         0.8               0.8888889  100      0.6415291  0.10353710
##   0.3   9         0.8               0.8888889  150      0.6403828  0.10210577
##   0.3   9         0.8               0.8888889  200      0.6407161  0.10277949
##   0.3   9         0.8               0.8888889  250      0.6408950  0.10326352
##   0.3   9         0.8               0.8888889  300      0.6417974  0.10387042
##   0.3   9         0.8               0.8888889  350      0.6423583  0.10472388
##   0.3   9         0.8               0.8888889  400      0.6428705  0.10574227
##   0.3   9         0.8               0.8888889  450      0.6434437  0.10569853
##   0.3   9         0.8               0.8888889  500      0.6434193  0.10578762
##   0.3   9         0.8               0.9444444   50      0.6479842  0.10930037
##   0.3   9         0.8               0.9444444  100      0.6424843  0.10645450
##   0.3   9         0.8               0.9444444  150      0.6411267  0.10380641
##   0.3   9         0.8               0.9444444  200      0.6407730  0.10351254
##   0.3   9         0.8               0.9444444  250      0.6413014  0.10374607
##   0.3   9         0.8               0.9444444  300      0.6422282  0.10470536
##   0.3   9         0.8               0.9444444  350      0.6420697  0.10440324
##   0.3   9         0.8               0.9444444  400      0.6430819  0.10542340
##   0.3   9         0.8               0.9444444  450      0.6431876  0.10529167
##   0.3   9         0.8               0.9444444  500      0.6441063  0.10675890
##   0.3   9         0.8               1.0000000   50      0.6480858  0.10924392
##   0.3   9         0.8               1.0000000  100      0.6437770  0.10599877
##   0.3   9         0.8               1.0000000  150      0.6415332  0.10368240
##   0.3   9         0.8               1.0000000  200      0.6417608  0.10373769
##   0.3   9         0.8               1.0000000  250      0.6419559  0.10388060
##   0.3   9         0.8               1.0000000  300      0.6424356  0.10451934
##   0.3   9         0.8               1.0000000  350      0.6434640  0.10557112
##   0.3   9         0.8               1.0000000  400      0.6437526  0.10630845
##   0.3   9         0.8               1.0000000  450      0.6433624  0.10583676
##   0.3   9         0.8               1.0000000  500      0.6433542  0.10608678
##   0.3  10         0.6               0.5000000   50      0.6272043  0.09036970
##   0.3  10         0.6               0.5000000  100      0.6235458  0.08780877
##   0.3  10         0.6               0.5000000  150      0.6237816  0.08839365
```

```
##   0.3  10         0.6               0.5000000  200      0.6246352  0.08900944
##   0.3  10         0.6               0.5000000  250      0.6264400  0.09115908
##   0.3  10         0.6               0.5000000  300      0.6281880  0.09154915
##   0.3  10         0.6               0.5000000  350      0.6292733  0.09243476
##   0.3  10         0.6               0.5000000  400      0.6295050  0.09316654
##   0.3  10         0.6               0.5000000  450      0.6296513  0.09323543
##   0.3  10         0.6               0.5000000  500      0.6304277  0.09366608
##   0.3  10         0.6               0.5555556   50      0.6324683  0.09545378
##   0.3  10         0.6               0.5555556  100      0.6287367  0.09195122
##   0.3  10         0.6               0.5555556  150      0.6277977  0.09269899
##   0.3  10         0.6               0.5555556  200      0.6302651  0.09374105
##   0.3  10         0.6               0.5555556  250      0.6305781  0.09505391
##   0.3  10         0.6               0.5555556  300      0.6323017  0.09617985
##   0.3  10         0.6               0.5555556  350      0.6334886  0.09696598
##   0.3  10         0.6               0.5555556  400      0.6341878  0.09809177
##   0.3  10         0.6               0.5555556  450      0.6347122  0.09865100
##   0.3  10         0.6               0.5555556  500      0.6354601  0.09907953
##   0.3  10         0.6               0.6111111   50      0.6333871  0.09400340
##   0.3  10         0.6               0.6111111  100      0.6301269  0.09218502
##   0.3  10         0.6               0.6111111  150      0.6308871  0.09326473
##   0.3  10         0.6               0.6111111  200      0.6322691  0.09432058
##   0.3  10         0.6               0.6111111  250      0.6343545  0.09700576
##   0.3  10         0.6               0.6111111  300      0.6346024  0.09628949
##   0.3  10         0.6               0.6111111  350      0.6353179  0.09773794
##   0.3  10         0.6               0.6111111  400      0.6362243  0.09869850
##   0.3  10         0.6               0.6111111  450      0.6366512  0.09851967
##   0.3  10         0.6               0.6111111  500      0.6376186  0.09934365
##   0.3  10         0.6               0.6666667   50      0.6350821  0.09825147
##   0.3  10         0.6               0.6666667  100      0.6312610  0.09590397
##   0.3  10         0.6               0.6666667  150      0.6321960  0.09634413
##   0.3  10         0.6               0.6666667  200      0.6332650  0.09751170
##   0.3  10         0.6               0.6666667  250      0.6351634  0.09874876
##   0.3  10         0.6               0.6666667  300      0.6366146  0.10027382
##   0.3  10         0.6               0.6666667  350      0.6367406  0.10001458
##   0.3  10         0.6               0.6666667  400      0.6382974  0.10108896
##   0.3  10         0.6               0.6666667  450      0.6381918  0.10098501
##   0.3  10         0.6               0.6666667  500      0.6387446  0.10174531
##   0.3  10         0.6               0.7222222   50      0.6381715  0.10049741
##   0.3  10         0.6               0.7222222  100      0.6346878  0.09723195
##   0.3  10         0.6               0.7222222  150      0.6346471  0.09799001
##   0.3  10         0.6               0.7222222  200      0.6352772  0.09911334
##   0.3  10         0.6               0.7222222  250      0.6372203  0.09995895
##   0.3  10         0.6               0.7222222  300      0.6379723  0.10061292
##   0.3  10         0.6               0.7222222  350      0.6392893  0.10166388
##   0.3  10         0.6               0.7222222  400      0.6397974  0.10173685
##   0.3  10         0.6               0.7222222  450      0.6402202  0.10231740
##   0.3  10         0.6               0.7222222  500      0.6399072  0.10170135
##   0.3  10         0.6               0.7777778   50      0.6381592  0.10042905
##   0.3  10         0.6               0.7777778  100      0.6349560  0.09862269
##   0.3  10         0.6               0.7777778  150      0.6350496  0.09852958
##   0.3  10         0.6               0.7777778  200      0.6375373  0.09982699
##   0.3  10         0.6               0.7777778  250      0.6393868  0.10165559
##   0.3  10         0.6               0.7777778  300      0.6395047  0.10192957
##   0.3  10         0.6               0.7777778  350      0.6406185  0.10316139
```

```
##   0.3  10         0.6               0.7777778  400      0.6410779  0.10360745
##   0.3  10         0.6               0.7777778  450      0.6405535  0.10317907
##   0.3  10         0.6               0.7777778  500      0.6412079  0.10401068
##   0.3  10         0.6               0.8333333   50      0.6427364  0.10506961
##   0.3  10         0.6               0.8333333  100      0.6386715  0.10138492
##   0.3  10         0.6               0.8333333  150      0.6383259  0.10122286
##   0.3  10         0.6               0.8333333  200      0.6392324  0.10162095
##   0.3  10         0.6               0.8333333  250      0.6402487  0.10230599
##   0.3  10         0.6               0.8333333  300      0.6413909  0.10329813
##   0.3  10         0.6               0.8333333  350      0.6424153  0.10453339
##   0.3  10         0.6               0.8333333  400      0.6426673  0.10443697
##   0.3  10         0.6               0.8333333  450      0.6433177  0.10551746
##   0.3  10         0.6               0.8333333  500      0.6439030  0.10622153
##   0.3  10         0.6               0.8888889   50      0.6423624  0.10501256
##   0.3  10         0.6               0.8888889  100      0.6395007  0.10349927
##   0.3  10         0.6               0.8888889  150      0.6395495  0.10451393
##   0.3  10         0.6               0.8888889  200      0.6403299  0.10451064
##   0.3  10         0.6               0.8888889  250      0.6414966  0.10585443
##   0.3  10         0.6               0.8888889  300      0.6425535  0.10624050
##   0.3  10         0.6               0.8888889  350      0.6435331  0.10677206
##   0.3  10         0.6               0.8888889  400      0.6433258  0.10641210
##   0.3  10         0.6               0.8888889  450      0.6438339  0.10714148
##   0.3  10         0.6               0.8888889  500      0.6439559  0.10667032
##   0.3  10         0.6               0.9444444   50      0.6433543  0.10586808
##   0.3  10         0.6               0.9444444  100      0.6404763  0.10356053
##   0.3  10         0.6               0.9444444  150      0.6403421  0.10322776
##   0.3  10         0.6               0.9444444  200      0.6413868  0.10432754
##   0.3  10         0.6               0.9444444  250      0.6417771  0.10445454
##   0.3  10         0.6               0.9444444  300      0.6434762  0.10592120
##   0.3  10         0.6               0.9444444  350      0.6431348  0.10520008
##   0.3  10         0.6               0.9444444  400      0.6439315  0.10609529
##   0.3  10         0.6               0.9444444  450      0.6437526  0.10587615
##   0.3  10         0.6               0.9444444  500      0.6438014  0.10589441
##   0.3  10         0.6               1.0000000   50      0.6452973  0.10696079
##   0.3  10         0.6               1.0000000  100      0.6433461  0.10577830
##   0.3  10         0.6               1.0000000  150      0.6439924  0.10657646
##   0.3  10         0.6               1.0000000  200      0.6438664  0.10683464
##   0.3  10         0.6               1.0000000  250      0.6441144  0.10699298
##   0.3  10         0.6               1.0000000  300      0.6452607  0.10815592
##   0.3  10         0.6               1.0000000  350      0.6452770  0.10804440
##   0.3  10         0.6               1.0000000  400      0.6462322  0.10889336
##   0.3  10         0.6               1.0000000  450      0.6465981  0.10992257
##   0.3  10         0.6               1.0000000  500      0.6460005  0.10969127
##   0.3  10         0.8               0.5000000   50      0.6275416  0.08933082
##   0.3  10         0.8               0.5000000  100      0.6236230  0.08756752
##   0.3  10         0.8               0.5000000  150      0.6233100  0.08751183
##   0.3  10         0.8               0.5000000  200      0.6255376  0.08945854
##   0.3  10         0.8               0.5000000  250      0.6274116  0.09110013
##   0.3  10         0.8               0.5000000  300      0.6285416  0.09204056
##   0.3  10         0.8               0.5000000  350      0.6297489  0.09269866
##   0.3  10         0.8               0.5000000  400      0.6305131  0.09312761
##   0.3  10         0.8               0.5000000  450      0.6313545  0.09420123
##   0.3  10         0.8               0.5000000  500      0.6315903  0.09430029
##   0.3  10         0.8               0.5555556   50      0.6333788  0.09558400
```

```
##   0.3  10         0.8               0.5555556  100      0.6282286  0.09314751
##   0.3  10         0.8               0.5555556  150      0.6291066  0.09297355
##   0.3  10         0.8               0.5555556  200      0.6303180  0.09428621
##   0.3  10         0.8               0.5555556  250      0.6310944  0.09443365
##   0.3  10         0.8               0.5555556  300      0.6324643  0.09585591
##   0.3  10         0.8               0.5555556  350      0.6337366  0.09717330
##   0.3  10         0.8               0.5555556  400      0.6342772  0.09802314
##   0.3  10         0.8               0.5555556  450      0.6337610  0.09757886
##   0.3  10         0.8               0.5555556  500      0.6343382  0.09758781
##   0.3  10         0.8               0.6111111   50      0.6332894  0.09710839
##   0.3  10         0.8               0.6111111  100      0.6301066  0.09631517
##   0.3  10         0.8               0.6111111  150      0.6306350  0.09638636
##   0.3  10         0.8               0.6111111  200      0.6315131  0.09641349
##   0.3  10         0.8               0.6111111  250      0.6334642  0.09797191
##   0.3  10         0.8               0.6111111  300      0.6340984  0.09902414
##   0.3  10         0.8               0.6111111  350      0.6354764  0.10086280
##   0.3  10         0.8               0.6111111  400      0.6354642  0.10031774
##   0.3  10         0.8               0.6111111  450      0.6362975  0.10062690
##   0.3  10         0.8               0.6111111  500      0.6359967  0.10061161
##   0.3  10         0.8               0.6666667   50      0.6354032  0.09762843
##   0.3  10         0.8               0.6666667  100      0.6322773  0.09536966
##   0.3  10         0.8               0.6666667  150      0.6330008  0.09648245
##   0.3  10         0.8               0.6666667  200      0.6344276  0.09795541
##   0.3  10         0.8               0.6666667  250      0.6359479  0.09917965
##   0.3  10         0.8               0.6666667  300      0.6368422  0.10014793
##   0.3  10         0.8               0.6666667  350      0.6382324  0.10057398
##   0.3  10         0.8               0.6666667  400      0.6381673  0.10089858
##   0.3  10         0.8               0.6666667  450      0.6392120  0.10207060
##   0.3  10         0.8               0.6666667  500      0.6394478  0.10158509
##   0.3  10         0.8               0.7222222   50      0.6360008  0.09912204
##   0.3  10         0.8               0.7222222  100      0.6333017  0.09685281
##   0.3  10         0.8               0.7222222  150      0.6347813  0.09765465
##   0.3  10         0.8               0.7222222  200      0.6362568  0.09907854
##   0.3  10         0.8               0.7222222  250      0.6368259  0.09974718
##   0.3  10         0.8               0.7222222  300      0.6374153  0.10052984
##   0.3  10         0.8               0.7222222  350      0.6383869  0.10123847
##   0.3  10         0.8               0.7222222  400      0.6391633  0.10163895
##   0.3  10         0.8               0.7222222  450      0.6396999  0.10221947
##   0.3  10         0.8               0.7222222  500      0.6406551  0.10331645
##   0.3  10         0.8               0.7777778   50      0.6388218  0.10050032
##   0.3  10         0.8               0.7777778  100      0.6359195  0.09797991
##   0.3  10         0.8               0.7777778  150      0.6361064  0.09910945
##   0.3  10         0.8               0.7777778  200      0.6383503  0.10082888
##   0.3  10         0.8               0.7777778  250      0.6386674  0.10167208
##   0.3  10         0.8               0.7777778  300      0.6396470  0.10202518
##   0.3  10         0.8               0.7777778  350      0.6396429  0.10172237
##   0.3  10         0.8               0.7777778  400      0.6405088  0.10303910
##   0.3  10         0.8               0.7777778  450      0.6401673  0.10240407
##   0.3  10         0.8               0.7777778  500      0.6410494  0.10390650
##   0.3  10         0.8               0.8333333   50      0.6405860  0.10299507
##   0.3  10         0.8               0.8333333  100      0.6369113  0.10046785
##   0.3  10         0.8               0.8333333  150      0.6371024  0.10127227
##   0.3  10         0.8               0.8333333  200      0.6390779  0.10233256
##   0.3  10         0.8               0.8333333  250      0.6399519  0.10346099
```

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##   0.3  10         0.8              0.8333333  300      0.6415494  0.10474811
##   0.3  10         0.8              0.8333333  350      0.6420331  0.10490965
##   0.3  10         0.8              0.8333333  400      0.6421063  0.10545549
##   0.3  10         0.8              0.8333333  450      0.6419722  0.10496783
##   0.3  10         0.8              0.8333333  500      0.6420819  0.10498661
##   0.3  10         0.8              0.8888889   50      0.6407771  0.10380935
##   0.3  10         0.8              0.8888889  100      0.6388218  0.10234382
##   0.3  10         0.8              0.8888889  150      0.6397486  0.10299035
##   0.3  10         0.8              0.8888889  200      0.6407852  0.10452735
##   0.3  10         0.8              0.8888889  250      0.6417892  0.10524312
##   0.3  10         0.8              0.8888889  300      0.6427486  0.10613510
##   0.3  10         0.8              0.8888889  350      0.6432689  0.10693192
##   0.3  10         0.8              0.8888889  400      0.6431795  0.10693357
##   0.3  10         0.8              0.8888889  450      0.6436673  0.10735292
##   0.3  10         0.8              0.8888889  500      0.6439437  0.10796132
##   0.3  10         0.8              0.9444444   50      0.6440128  0.10650234
##   0.3  10         0.8              0.9444444  100      0.6416063  0.10524626
##   0.3  10         0.8              0.9444444  150      0.6413583  0.10408761
##   0.3  10         0.8              0.9444444  200      0.6433949  0.10659197
##   0.3  10         0.8              0.9444444  250      0.6438054  0.10745505
##   0.3  10         0.8              0.9444444  300      0.6446631  0.10772951
##   0.3  10         0.8              0.9444444  350      0.6450493  0.10822681
##   0.3  10         0.8              0.9444444  400      0.6459070  0.10927019
##   0.3  10         0.8              0.9444444  450      0.6459192  0.10967631
##   0.3  10         0.8              0.9444444  500      0.6462566  0.11042089
##   0.3  10         0.8              1.0000000   50      0.6458095  0.10867757
##   0.3  10         0.8              1.0000000  100      0.6430372  0.10522451
##   0.3  10         0.8              1.0000000  150      0.6436510  0.10551834
##   0.3  10         0.8              1.0000000  200      0.6436591  0.10560213
##   0.3  10         0.8              1.0000000  250      0.6439356  0.10705695
##   0.3  10         0.8              1.0000000  300      0.6444315  0.10714819
##   0.3  10         0.8              1.0000000  350      0.6456469  0.10836002
##   0.3  10         0.8              1.0000000  400      0.6454558  0.10853180
##   0.3  10         0.8              1.0000000  450      0.6455940  0.10851372
##   0.3  10         0.8              1.0000000  500      0.6461265  0.10898618
##   0.4   1         0.6              0.5000000   50      0.6597319  0.12192792
##   0.4   1         0.6              0.5000000  100      0.6686138  0.12969383
##   0.4   1         0.6              0.5000000  150      0.6711341  0.13140410
##   0.4   1         0.6              0.5000000  200      0.6706544  0.13131604
##   0.4   1         0.6              0.5000000  250      0.6708333  0.13147251
##   0.4   1         0.6              0.5000000  300      0.6724430  0.13224804
##   0.4   1         0.6              0.5000000  350      0.6714674  0.13121285
##   0.4   1         0.6              0.5000000  400      0.6707032  0.13109436
##   0.4   1         0.6              0.5000000  450      0.6713780  0.13104721
##   0.4   1         0.6              0.5000000  500      0.6694512  0.13024243
##   0.4   1         0.6              0.5555556   50      0.6585815  0.12127161
##   0.4   1         0.6              0.5555556  100      0.6680325  0.12914903
##   0.4   1         0.6              0.5555556  150      0.6700487  0.13060271
##   0.4   1         0.6              0.5555556  200      0.6720242  0.13160077
##   0.4   1         0.6              0.5555556  250      0.6722804  0.13178622
##   0.4   1         0.6              0.5555556  300      0.6734917  0.13210293
##   0.4   1         0.6              0.5555556  350      0.6718820  0.13138851
##   0.4   1         0.6              0.5555556  400      0.6712235  0.13088637
##   0.4   1         0.6              0.5555556  450      0.6700446  0.13047570
```

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##   0.4   1         0.6                 0.5555556  500      0.6715934  0.13089150
##   0.4   1         0.6                 0.6111111   50      0.6589351  0.12054799
##   0.4   1         0.6                 0.6111111  100      0.6678740  0.12826840
##   0.4   1         0.6                 0.6111111  150      0.6701910  0.12991497
##   0.4   1         0.6                 0.6111111  200      0.6713576  0.13135356
##   0.4   1         0.6                 0.6111111  250      0.6720121  0.13114446
##   0.4   1         0.6                 0.6111111  300      0.6717925  0.13121190
##   0.4   1         0.6                 0.6111111  350      0.6709918  0.13088029
##   0.4   1         0.6                 0.6111111  400      0.6705487  0.13019019
##   0.4   1         0.6                 0.6111111  450      0.6711137  0.13042838
##   0.4   1         0.6                 0.6111111  500      0.6705365  0.12935888
##   0.4   1         0.6                 0.6666667   50      0.6584189  0.12160820
##   0.4   1         0.6                 0.6666667  100      0.6670854  0.12837384
##   0.4   1         0.6                 0.6666667  150      0.6703292  0.13079617
##   0.4   1         0.6                 0.6666667  200      0.6718332  0.13194510
##   0.4   1         0.6                 0.6666667  250      0.6716381  0.13180150
##   0.4   1         0.6                 0.6666667  300      0.6711666  0.13155914
##   0.4   1         0.6                 0.6666667  350      0.6717113  0.13164872
##   0.4   1         0.6                 0.6666667  400      0.6717479  0.13174825
##   0.4   1         0.6                 0.6666667  450      0.6717722  0.13221823
##   0.4   1         0.6                 0.6666667  500      0.6714836  0.13127454
##   0.4   1         0.6                 0.7222222   50      0.6575490  0.12151636
##   0.4   1         0.6                 0.7222222  100      0.6659228  0.12827308
##   0.4   1         0.6                 0.7222222  150      0.6697926  0.13073218
##   0.4   1         0.6                 0.7222222  200      0.6717804  0.13251962
##   0.4   1         0.6                 0.7222222  250      0.6728779  0.13274212
##   0.4   1         0.6                 0.7222222  300      0.6719023  0.13223020
##   0.4   1         0.6                 0.7222222  350      0.6713983  0.13165167
##   0.4   1         0.6                 0.7222222  400      0.6714999  0.13133154
##   0.4   1         0.6                 0.7222222  450      0.6716340  0.13135178
##   0.4   1         0.6                 0.7222222  500      0.6708007  0.13088992
##   0.4   1         0.6                 0.7777778   50      0.6598335  0.12177545
##   0.4   1         0.6                 0.7777778  100      0.6674431  0.12774236
##   0.4   1         0.6                 0.7777778  150      0.6710690  0.13075652
##   0.4   1         0.6                 0.7777778  200      0.6718414  0.13128737
##   0.4   1         0.6                 0.7777778  250      0.6724186  0.13139012
##   0.4   1         0.6                 0.7777778  300      0.6729348  0.13164781
##   0.4   1         0.6                 0.7777778  350      0.6727844  0.13148934
##   0.4   1         0.6                 0.7777778  400      0.6727234  0.13164597
##   0.4   1         0.6                 0.7777778  450      0.6721340  0.13117439
##   0.4   1         0.6                 0.7777778  500      0.6722194  0.13097480
##   0.4   1         0.6                 0.8333333   50      0.6576710  0.12074540
##   0.4   1         0.6                 0.8333333  100      0.6665203  0.12808194
##   0.4   1         0.6                 0.8333333  150      0.6696178  0.13010528
##   0.4   1         0.6                 0.8333333  200      0.6713576  0.13173314
##   0.4   1         0.6                 0.8333333  250      0.6711991  0.13178501
##   0.4   1         0.6                 0.8333333  300      0.6723373  0.13199006
##   0.4   1         0.6                 0.8333333  350      0.6726584  0.13282999
##   0.4   1         0.6                 0.8333333  400      0.6732885  0.13339839
##   0.4   1         0.6                 0.8333333  450      0.6719186  0.13216528
##   0.4   1         0.6                 0.8333333  500      0.6722601  0.13203430
##   0.4   1         0.6                 0.8888889   50      0.6586344  0.12219082
##   0.4   1         0.6                 0.8888889  100      0.6662968  0.12821634
##   0.4   1         0.6                 0.8888889  150      0.6687804  0.13012734
```

```
##   0.4   1         0.6                 0.8888889  200      0.6706463  0.13155188
##   0.4   1         0.6                 0.8888889  250      0.6709633  0.13171076
##   0.4   1         0.6                 0.8888889  300      0.6715771  0.13208812
##   0.4   1         0.6                 0.8888889  350      0.6721218  0.13229401
##   0.4   1         0.6                 0.8888889  400      0.6719958  0.13247687
##   0.4   1         0.6                 0.8888889  450      0.6711747  0.13169476
##   0.4   1         0.6                 0.8888889  500      0.6716015  0.13248183
##   0.4   1         0.6                 0.9444444   50      0.6580612  0.12146975
##   0.4   1         0.6                 0.9444444  100      0.6666138  0.12804747
##   0.4   1         0.6                 0.9444444  150      0.6698048  0.13093112
##   0.4   1         0.6                 0.9444444  200      0.6711625  0.13128898
##   0.4   1         0.6                 0.9444444  250      0.6714389  0.13157853
##   0.4   1         0.6                 0.9444444  300      0.6716625  0.13129949
##   0.4   1         0.6                 0.9444444  350      0.6720324  0.13173336
##   0.4   1         0.6                 0.9444444  400      0.6724552  0.13197617
##   0.4   1         0.6                 0.9444444  450      0.6723698  0.13194242
##   0.4   1         0.6                 0.9444444  500      0.6721625  0.13150494
##   0.4   1         0.6                 1.0000000   50      0.6571588  0.12069511
##   0.4   1         0.6                 1.0000000  100      0.6655935  0.12643751
##   0.4   1         0.6                 1.0000000  150      0.6688130  0.12897717
##   0.4   1         0.6                 1.0000000  200      0.6704268  0.13034671
##   0.4   1         0.6                 1.0000000  250      0.6713007  0.13118550
##   0.4   1         0.6                 1.0000000  300      0.6720243  0.13152391
##   0.4   1         0.6                 1.0000000  350      0.6720812  0.13196012
##   0.4   1         0.6                 1.0000000  400      0.6724633  0.13192121
##   0.4   1         0.6                 1.0000000  450      0.6723942  0.13194842
##   0.4   1         0.6                 1.0000000  500      0.6724714  0.13202624
##   0.4   1         0.8                 0.5000000   50      0.6584148  0.12152584
##   0.4   1         0.8                 0.5000000  100      0.6661342  0.12778760
##   0.4   1         0.8                 0.5000000  150      0.6704918  0.13026232
##   0.4   1         0.8                 0.5000000  200      0.6714633  0.13031481
##   0.4   1         0.8                 0.5000000  250      0.6698455  0.13010603
##   0.4   1         0.8                 0.5000000  300      0.6696016  0.12928142
##   0.4   1         0.8                 0.5000000  350      0.6690813  0.12933860
##   0.4   1         0.8                 0.5000000  400      0.6687601  0.12868098
##   0.4   1         0.8                 0.5000000  450      0.6677683  0.12903663
##   0.4   1         0.8                 0.5000000  500      0.6700650  0.12995946
##   0.4   1         0.8                 0.5555556   50      0.6587807  0.12161158
##   0.4   1         0.8                 0.5555556  100      0.6672073  0.12867775
##   0.4   1         0.8                 0.5555556  150      0.6699837  0.13071888
##   0.4   1         0.8                 0.5555556  200      0.6736787  0.13245405
##   0.4   1         0.8                 0.5555556  250      0.6723494  0.13173256
##   0.4   1         0.8                 0.5555556  300      0.6716909  0.13142619
##   0.4   1         0.8                 0.5555556  350      0.6715812  0.13099832
##   0.4   1         0.8                 0.5555556  400      0.6709918  0.13056286
##   0.4   1         0.8                 0.5555556  450      0.6711910  0.13047226
##   0.4   1         0.8                 0.5555556  500      0.6723738  0.13121494
##   0.4   1         0.8                 0.6111111   50      0.6573092  0.12065054
##   0.4   1         0.8                 0.6111111  100      0.6658415  0.12838541
##   0.4   1         0.8                 0.6111111  150      0.6697520  0.13098618
##   0.4   1         0.8                 0.6111111  200      0.6708333  0.13117659
##   0.4   1         0.8                 0.6111111  250      0.6704308  0.13133442
##   0.4   1         0.8                 0.6111111  300      0.6696138  0.13019933
##   0.4   1         0.8                 0.6111111  350      0.6707194  0.13140155
```

```
##   0.4   1         0.8               0.6111111  400      0.6698983  0.13053006
##   0.4   1         0.8               0.6111111  450      0.6705162  0.13059648
##   0.4   1         0.8               0.6111111  500      0.6704877  0.13109216
##   0.4   1         0.8               0.6666667   50      0.6586303  0.12204582
##   0.4   1         0.8               0.6666667  100      0.6670285  0.12870596
##   0.4   1         0.8               0.6666667  150      0.6705121  0.13121479
##   0.4   1         0.8               0.6666667  200      0.6724308  0.13212225
##   0.4   1         0.8               0.6666667  250      0.6722397  0.13211603
##   0.4   1         0.8               0.6666667  300      0.6712519  0.13172190
##   0.4   1         0.8               0.6666667  350      0.6718251  0.13171755
##   0.4   1         0.8               0.6666667  400      0.6721706  0.13220823
##   0.4   1         0.8               0.6666667  450      0.6713495  0.13170278
##   0.4   1         0.8               0.6666667  500      0.6713779  0.13137490
##   0.4   1         0.8               0.7222222   50      0.6592279  0.12193271
##   0.4   1         0.8               0.7222222  100      0.6659838  0.12815025
##   0.4   1         0.8               0.7222222  150      0.6700894  0.13042297
##   0.4   1         0.8               0.7222222  200      0.6711544  0.13158279
##   0.4   1         0.8               0.7222222  250      0.6705853  0.13141188
##   0.4   1         0.8               0.7222222  300      0.6720121  0.13160683
##   0.4   1         0.8               0.7222222  350      0.6723535  0.13173119
##   0.4   1         0.8               0.7222222  400      0.6714186  0.13094157
##   0.4   1         0.8               0.7222222  450      0.6709918  0.13073930
##   0.4   1         0.8               0.7222222  500      0.6706300  0.13047254
##   0.4   1         0.8               0.7777778   50      0.6599798  0.12217169
##   0.4   1         0.8               0.7777778  100      0.6679797  0.12911301
##   0.4   1         0.8               0.7777778  150      0.6710731  0.13130921
##   0.4   1         0.8               0.7777778  200      0.6717926  0.13283702
##   0.4   1         0.8               0.7777778  250      0.6731991  0.13334209
##   0.4   1         0.8               0.7777778  300      0.6743901  0.13350224
##   0.4   1         0.8               0.7777778  350      0.6728942  0.13249093
##   0.4   1         0.8               0.7777778  400      0.6730690  0.13262546
##   0.4   1         0.8               0.7777778  450      0.6725202  0.13207211
##   0.4   1         0.8               0.7777778  500      0.6723617  0.13144152
##   0.4   1         0.8               0.8333333   50      0.6590327  0.12263263
##   0.4   1         0.8               0.8333333  100      0.6673293  0.12855906
##   0.4   1         0.8               0.8333333  150      0.6698048  0.13063605
##   0.4   1         0.8               0.8333333  200      0.6715365  0.13122121
##   0.4   1         0.8               0.8333333  250      0.6724796  0.13175450
##   0.4   1         0.8               0.8333333  300      0.6725649  0.13216351
##   0.4   1         0.8               0.8333333  350      0.6721056  0.13218446
##   0.4   1         0.8               0.8333333  400      0.6723210  0.13185184
##   0.4   1         0.8               0.8333333  450      0.6723292  0.13188377
##   0.4   1         0.8               0.8333333  500      0.6715568  0.13134344
##   0.4   1         0.8               0.8888889   50      0.6583417  0.12053035
##   0.4   1         0.8               0.8888889  100      0.6672155  0.12779893
##   0.4   1         0.8               0.8888889  150      0.6700203  0.13049586
##   0.4   1         0.8               0.8888889  200      0.6716422  0.13194030
##   0.4   1         0.8               0.8888889  250      0.6722560  0.13246967
##   0.4   1         0.8               0.8888889  300      0.6720283  0.13183599
##   0.4   1         0.8               0.8888889  350      0.6724064  0.13199771
##   0.4   1         0.8               0.8888889  400      0.6719633  0.13169679
##   0.4   1         0.8               0.8888889  450      0.6723982  0.13207086
##   0.4   1         0.8               0.8888889  500      0.6716137  0.13148919
##   0.4   1         0.8               0.9444444   50      0.6596953  0.12223857
```

```
##   0.4   1         0.8               0.9444444  100      0.6668333  0.12821911
##   0.4   1         0.8               0.9444444  150      0.6694268  0.13010956
##   0.4   1         0.8               0.9444444  200      0.6705121  0.13184481
##   0.4   1         0.8               0.9444444  250      0.6712560  0.13188749
##   0.4   1         0.8               0.9444444  300      0.6715121  0.13207115
##   0.4   1         0.8               0.9444444  350      0.6723129  0.13241719
##   0.4   1         0.8               0.9444444  400      0.6720446  0.13234729
##   0.4   1         0.8               0.9444444  450      0.6720365  0.13243369
##   0.4   1         0.8               0.9444444  500      0.6720039  0.13188714
##   0.4   1         0.8               1.0000000   50      0.6590693  0.12171511
##   0.4   1         0.8               1.0000000  100      0.6661667  0.12743427
##   0.4   1         0.8               1.0000000  150      0.6691992  0.12902797
##   0.4   1         0.8               1.0000000  200      0.6710690  0.13050630
##   0.4   1         0.8               1.0000000  250      0.6715040  0.13096254
##   0.4   1         0.8               1.0000000  300      0.6716584  0.13116885
##   0.4   1         0.8               1.0000000  350      0.6721666  0.13172429
##   0.4   1         0.8               1.0000000  400      0.6718007  0.13135292
##   0.4   1         0.8               1.0000000  450      0.6718332  0.13157950
##   0.4   1         0.8               1.0000000  500      0.6719958  0.13177870
##   0.4   2         0.6               0.5000000   50      0.6663781  0.12734158
##   0.4   2         0.6               0.5000000  100      0.6670244  0.12811357
##   0.4   2         0.6               0.5000000  150      0.6645895  0.12517544
##   0.4   2         0.6               0.5000000  200      0.6636342  0.12362891
##   0.4   2         0.6               0.5000000  250      0.6613375  0.12176769
##   0.4   2         0.6               0.5000000  300      0.6611424  0.12091735
##   0.4   2         0.6               0.5000000  350      0.6611099  0.11912476
##   0.4   2         0.6               0.5000000  400      0.6588863  0.11767349
##   0.4   2         0.6               0.5000000  450      0.6570287  0.11662266
##   0.4   2         0.6               0.5000000  500      0.6555328  0.11483917
##   0.4   2         0.6               0.5555556   50      0.6656139  0.12648233
##   0.4   2         0.6               0.5555556  100      0.6673577  0.12742861
##   0.4   2         0.6               0.5555556  150      0.6665204  0.12667701
##   0.4   2         0.6               0.5555556  200      0.6650570  0.12495158
##   0.4   2         0.6               0.5555556  250      0.6634839  0.12365011
##   0.4   2         0.6               0.5555556  300      0.6620408  0.12213568
##   0.4   2         0.6               0.5555556  350      0.6613050  0.12101025
##   0.4   2         0.6               0.5555556  400      0.6601140  0.12029663
##   0.4   2         0.6               0.5555556  450      0.6587400  0.11863722
##   0.4   2         0.6               0.5555556  500      0.6569880  0.11736368
##   0.4   2         0.6               0.6111111   50      0.6660854  0.12693871
##   0.4   2         0.6               0.6111111  100      0.6683699  0.12844529
##   0.4   2         0.6               0.6111111  150      0.6671748  0.12708941
##   0.4   2         0.6               0.6111111  200      0.6660082  0.12570464
##   0.4   2         0.6               0.6111111  250      0.6651179  0.12425124
##   0.4   2         0.6               0.6111111  300      0.6643862  0.12374604
##   0.4   2         0.6               0.6111111  350      0.6620082  0.12145606
##   0.4   2         0.6               0.6111111  400      0.6614392  0.12069922
##   0.4   2         0.6               0.6111111  450      0.6590652  0.11918361
##   0.4   2         0.6               0.6111111  500      0.6589799  0.11980081
##   0.4   2         0.6               0.6666667   50      0.6651423  0.12653168
##   0.4   2         0.6               0.6666667  100      0.6684959  0.12902668
##   0.4   2         0.6               0.6666667  150      0.6682479  0.12888637
##   0.4   2         0.6               0.6666667  200      0.6675610  0.12781717
##   0.4   2         0.6               0.6666667  250      0.6662399  0.12574623
```

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##   0.4   2         0.6               0.6666667  300      0.6659269  0.12496166
##   0.4   2         0.6               0.6666667  350      0.6639472  0.12379463
##   0.4   2         0.6               0.6666667  400      0.6620733  0.12138582
##   0.4   2         0.6               0.6666667  450      0.6614554  0.12100172
##   0.4   2         0.6               0.6666667  500      0.6604798  0.11992963
##   0.4   2         0.6               0.7222222   50      0.6672195  0.12847787
##   0.4   2         0.6               0.7222222  100      0.6701585  0.13041876
##   0.4   2         0.6               0.7222222  150      0.6691951  0.12959788
##   0.4   2         0.6               0.7222222  200      0.6673740  0.12776824
##   0.4   2         0.6               0.7222222  250      0.6672764  0.12785240
##   0.4   2         0.6               0.7222222  300      0.6658821  0.12603705
##   0.4   2         0.6               0.7222222  350      0.6643293  0.12468062
##   0.4   2         0.6               0.7222222  400      0.6629594  0.12336193
##   0.4   2         0.6               0.7222222  450      0.6632724  0.12296273
##   0.4   2         0.6               0.7222222  500      0.6615448  0.12154561
##   0.4   2         0.6               0.7777778   50      0.6647724  0.12582404
##   0.4   2         0.6               0.7777778  100      0.6686056  0.12915807
##   0.4   2         0.6               0.7777778  150      0.6695365  0.12949547
##   0.4   2         0.6               0.7777778  200      0.6687601  0.12822874
##   0.4   2         0.6               0.7777778  250      0.6674918  0.12676711
##   0.4   2         0.6               0.7777778  300      0.6658537  0.12576671
##   0.4   2         0.6               0.7777778  350      0.6639513  0.12393127
##   0.4   2         0.6               0.7777778  400      0.6631424  0.12281517
##   0.4   2         0.6               0.7777778  450      0.6619026  0.12252149
##   0.4   2         0.6               0.7777778  500      0.6614513  0.12051638
##   0.4   2         0.6               0.8333333   50      0.6653496  0.12672890
##   0.4   2         0.6               0.8333333  100      0.6684105  0.12919205
##   0.4   2         0.6               0.8333333  150      0.6691219  0.12919217
##   0.4   2         0.6               0.8333333  200      0.6677154  0.12800854
##   0.4   2         0.6               0.8333333  250      0.6664268  0.12647480
##   0.4   2         0.6               0.8333333  300      0.6654391  0.12469289
##   0.4   2         0.6               0.8333333  350      0.6641342  0.12339404
##   0.4   2         0.6               0.8333333  400      0.6627399  0.12251524
##   0.4   2         0.6               0.8333333  450      0.6618985  0.12209749
##   0.4   2         0.6               0.8333333  500      0.6610083  0.12129020
##   0.4   2         0.6               0.8888889   50      0.6660610  0.12670417
##   0.4   2         0.6               0.8888889  100      0.6689390  0.12947015
##   0.4   2         0.6               0.8888889  150      0.6685650  0.12873822
##   0.4   2         0.6               0.8888889  200      0.6677480  0.12786231
##   0.4   2         0.6               0.8888889  250      0.6662073  0.12682271
##   0.4   2         0.6               0.8888889  300      0.6655895  0.12581283
##   0.4   2         0.6               0.8888889  350      0.6650407  0.12513649
##   0.4   2         0.6               0.8888889  400      0.6639391  0.12342570
##   0.4   2         0.6               0.8888889  450      0.6635407  0.12389177
##   0.4   2         0.6               0.8888889  500      0.6625448  0.12284815
##   0.4   2         0.6               0.9444444   50      0.6662886  0.12702219
##   0.4   2         0.6               0.9444444  100      0.6704959  0.12969280
##   0.4   2         0.6               0.9444444  150      0.6703820  0.12991426
##   0.4   2         0.6               0.9444444  200      0.6688699  0.12770734
##   0.4   2         0.6               0.9444444  250      0.6683658  0.12729939
##   0.4   2         0.6               0.9444444  300      0.6673211  0.12635641
##   0.4   2         0.6               0.9444444  350      0.6664106  0.12571166
##   0.4   2         0.6               0.9444444  400      0.6657155  0.12508056
##   0.4   2         0.6               0.9444444  450      0.6647277  0.12375844
```

```
##   0.4   2         0.6                 0.9444444  500      0.6632440  0.12335564
##   0.4   2         0.6                 1.0000000   50      0.6646586  0.12605475
##   0.4   2         0.6                 1.0000000  100      0.6683455  0.12775524
##   0.4   2         0.6                 1.0000000  150      0.6682398  0.12792987
##   0.4   2         0.6                 1.0000000  200      0.6682927  0.12786955
##   0.4   2         0.6                 1.0000000  250      0.6674309  0.12748059
##   0.4   2         0.6                 1.0000000  300      0.6671016  0.12672589
##   0.4   2         0.6                 1.0000000  350      0.6661464  0.12604448
##   0.4   2         0.6                 1.0000000  400      0.6653740  0.12588657
##   0.4   2         0.6                 1.0000000  450      0.6642765  0.12484384
##   0.4   2         0.6                 1.0000000  500      0.6636180  0.12459355
##   0.4   2         0.8                 0.5000000   50      0.6653009  0.12571952
##   0.4   2         0.8                 0.5000000  100      0.6682317  0.12772827
##   0.4   2         0.8                 0.5000000  150      0.6671708  0.12622423
##   0.4   2         0.8                 0.5000000  200      0.6642034  0.12420375
##   0.4   2         0.8                 0.5000000  250      0.6633212  0.12275722
##   0.4   2         0.8                 0.5000000  300      0.6619839  0.12171409
##   0.4   2         0.8                 0.5000000  350      0.6593051  0.11919179
##   0.4   2         0.8                 0.5000000  400      0.6574718  0.11799830
##   0.4   2         0.8                 0.5000000  450      0.6554312  0.11595522
##   0.4   2         0.8                 0.5000000  500      0.6556385  0.11601085
##   0.4   2         0.8                 0.5555556   50      0.6660041  0.12643426
##   0.4   2         0.8                 0.5555556  100      0.6682764  0.12784802
##   0.4   2         0.8                 0.5555556  150      0.6681545  0.12726505
##   0.4   2         0.8                 0.5555556  200      0.6652196  0.12520190
##   0.4   2         0.8                 0.5555556  250      0.6637440  0.12357594
##   0.4   2         0.8                 0.5555556  300      0.6632887  0.12346057
##   0.4   2         0.8                 0.5555556  350      0.6615286  0.12092165
##   0.4   2         0.8                 0.5555556  400      0.6600774  0.11946709
##   0.4   2         0.8                 0.5555556  450      0.6576344  0.11811281
##   0.4   2         0.8                 0.5555556  500      0.6573010  0.11727513
##   0.4   2         0.8                 0.6111111   50      0.6653862  0.12630092
##   0.4   2         0.8                 0.6111111  100      0.6680935  0.12817174
##   0.4   2         0.8                 0.6111111  150      0.6673415  0.12728664
##   0.4   2         0.8                 0.6111111  200      0.6653415  0.12564693
##   0.4   2         0.8                 0.6111111  250      0.6633578  0.12440447
##   0.4   2         0.8                 0.6111111  300      0.6625448  0.12338283
##   0.4   2         0.8                 0.6111111  350      0.6613579  0.12164876
##   0.4   2         0.8                 0.6111111  400      0.6581791  0.11884544
##   0.4   2         0.8                 0.6111111  450      0.6577401  0.11771232
##   0.4   2         0.8                 0.6111111  500      0.6567320  0.11797420
##   0.4   2         0.8                 0.6666667   50      0.6659838  0.12721517
##   0.4   2         0.8                 0.6666667  100      0.6685162  0.12755729
##   0.4   2         0.8                 0.6666667  150      0.6668130  0.12697983
##   0.4   2         0.8                 0.6666667  200      0.6659228  0.12567481
##   0.4   2         0.8                 0.6666667  250      0.6640123  0.12385032
##   0.4   2         0.8                 0.6666667  300      0.6637196  0.12261774
##   0.4   2         0.8                 0.6666667  350      0.6622928  0.12174030
##   0.4   2         0.8                 0.6666667  400      0.6611099  0.12089926
##   0.4   2         0.8                 0.6666667  450      0.6598173  0.11953696
##   0.4   2         0.8                 0.6666667  500      0.6587644  0.11764545
##   0.4   2         0.8                 0.7222222   50      0.6660000  0.12701010
##   0.4   2         0.8                 0.7222222  100      0.6690609  0.12970267
##   0.4   2         0.8                 0.7222222  150      0.6697804  0.12892558
```

```
##   0.4   2         0.8               0.7222222  200      0.6673333  0.12676898
##   0.4   2         0.8               0.7222222  250      0.6659675  0.12561288
##   0.4   2         0.8               0.7222222  300      0.6659513  0.12492824
##   0.4   2         0.8               0.7222222  350      0.6645529  0.12364160
##   0.4   2         0.8               0.7222222  400      0.6633944  0.12313869
##   0.4   2         0.8               0.7222222  450      0.6621058  0.12137976
##   0.4   2         0.8               0.7222222  500      0.6617318  0.12061625
##   0.4   2         0.8               0.7777778   50      0.6658537  0.12690306
##   0.4   2         0.8               0.7777778  100      0.6696829  0.12957379
##   0.4   2         0.8               0.7777778  150      0.6693617  0.12982196
##   0.4   2         0.8               0.7777778  200      0.6673293  0.12904425
##   0.4   2         0.8               0.7777778  250      0.6660448  0.12718643
##   0.4   2         0.8               0.7777778  300      0.6646342  0.12587807
##   0.4   2         0.8               0.7777778  350      0.6633538  0.12425442
##   0.4   2         0.8               0.7777778  400      0.6621708  0.12267861
##   0.4   2         0.8               0.7777778  450      0.6616262  0.12244829
##   0.4   2         0.8               0.7777778  500      0.6609229  0.12236162
##   0.4   2         0.8               0.8333333   50      0.6657318  0.12581430
##   0.4   2         0.8               0.8333333  100      0.6686016  0.12815673
##   0.4   2         0.8               0.8333333  150      0.6677073  0.12676636
##   0.4   2         0.8               0.8333333  200      0.6673252  0.12641250
##   0.4   2         0.8               0.8333333  250      0.6664268  0.12495371
##   0.4   2         0.8               0.8333333  300      0.6657561  0.12465909
##   0.4   2         0.8               0.8333333  350      0.6636058  0.12286353
##   0.4   2         0.8               0.8333333  400      0.6623131  0.12133283
##   0.4   2         0.8               0.8333333  450      0.6614757  0.12120939
##   0.4   2         0.8               0.8333333  500      0.6603741  0.12009795
##   0.4   2         0.8               0.8888889   50      0.6669472  0.12756588
##   0.4   2         0.8               0.8888889  100      0.6701707  0.12894483
##   0.4   2         0.8               0.8888889  150      0.6698211  0.12916538
##   0.4   2         0.8               0.8888889  200      0.6699634  0.12895564
##   0.4   2         0.8               0.8888889  250      0.6686463  0.12830200
##   0.4   2         0.8               0.8888889  300      0.6677114  0.12717184
##   0.4   2         0.8               0.8888889  350      0.6667317  0.12646071
##   0.4   2         0.8               0.8888889  400      0.6651058  0.12499473
##   0.4   2         0.8               0.8888889  450      0.6643985  0.12375323
##   0.4   2         0.8               0.8888889  500      0.6636180  0.12323737
##   0.4   2         0.8               0.9444444   50      0.6670447  0.12754560
##   0.4   2         0.8               0.9444444  100      0.6699836  0.13002479
##   0.4   2         0.8               0.9444444  150      0.6698414  0.12949124
##   0.4   2         0.8               0.9444444  200      0.6688495  0.12889286
##   0.4   2         0.8               0.9444444  250      0.6675528  0.12758205
##   0.4   2         0.8               0.9444444  300      0.6657439  0.12607724
##   0.4   2         0.8               0.9444444  350      0.6651464  0.12512874
##   0.4   2         0.8               0.9444444  400      0.6643862  0.12490387
##   0.4   2         0.8               0.9444444  450      0.6634188  0.12335235
##   0.4   2         0.8               0.9444444  500      0.6624594  0.12219432
##   0.4   2         0.8               1.0000000   50      0.6649675  0.12596369
##   0.4   2         0.8               1.0000000  100      0.6692113  0.13017309
##   0.4   2         0.8               1.0000000  150      0.6695772  0.12946842
##   0.4   2         0.8               1.0000000  200      0.6690569  0.12949319
##   0.4   2         0.8               1.0000000  250      0.6684878  0.12913883
##   0.4   2         0.8               1.0000000  300      0.6680000  0.12924056
##   0.4   2         0.8               1.0000000  350      0.6673293  0.12846855
```

```
##   0.4   2         0.8               1.0000000  400      0.6663415  0.12683820
##   0.4   2         0.8               1.0000000  450      0.6645610  0.12490295
##   0.4   2         0.8               1.0000000  500      0.6643700  0.12510847
##   0.4   3         0.6               0.5000000   50      0.6629635  0.12354221
##   0.4   3         0.6               0.5000000  100      0.6602481  0.12153082
##   0.4   3         0.6               0.5000000  150      0.6551913  0.11685074
##   0.4   3         0.6               0.5000000  200      0.6523052  0.11370732
##   0.4   3         0.6               0.5000000  250      0.6487199  0.11049580
##   0.4   3         0.6               0.5000000  300      0.6465736  0.10824150
##   0.4   3         0.6               0.5000000  350      0.6447404  0.10724950
##   0.4   3         0.6               0.5000000  400      0.6426673  0.10519686
##   0.4   3         0.6               0.5000000  450      0.6400332  0.10345687
##   0.4   3         0.6               0.5000000  500      0.6385983  0.10252819
##   0.4   3         0.6               0.5555556   50      0.6642561  0.12529082
##   0.4   3         0.6               0.5555556  100      0.6615692  0.12262393
##   0.4   3         0.6               0.5555556  150      0.6574718  0.11861909
##   0.4   3         0.6               0.5555556  200      0.6549068  0.11482504
##   0.4   3         0.6               0.5555556  250      0.6526954  0.11291868
##   0.4   3         0.6               0.5555556  300      0.6501711  0.10945312
##   0.4   3         0.6               0.5555556  350      0.6481346  0.10742678
##   0.4   3         0.6               0.5555556  400      0.6453745  0.10570344
##   0.4   3         0.6               0.5555556  450      0.6429030  0.10335566
##   0.4   3         0.6               0.5555556  500      0.6413380  0.10212609
##   0.4   3         0.6               0.6111111   50      0.6654553  0.12695704
##   0.4   3         0.6               0.6111111  100      0.6640733  0.12393678
##   0.4   3         0.6               0.6111111  150      0.6596709  0.12027722
##   0.4   3         0.6               0.6111111  200      0.6571019  0.11861110
##   0.4   3         0.6               0.6111111  250      0.6537768  0.11557718
##   0.4   3         0.6               0.6111111  300      0.6516549  0.11304012
##   0.4   3         0.6               0.6111111  350      0.6487850  0.10970464
##   0.4   3         0.6               0.6111111  400      0.6456184  0.10700777
##   0.4   3         0.6               0.6111111  450      0.6439640  0.10546846
##   0.4   3         0.6               0.6111111  500      0.6423258  0.10359157
##   0.4   3         0.6               0.6666667   50      0.6656220  0.12643821
##   0.4   3         0.6               0.6666667  100      0.6640448  0.12452836
##   0.4   3         0.6               0.6666667  150      0.6599433  0.12109263
##   0.4   3         0.6               0.6666667  200      0.6572807  0.11869056
##   0.4   3         0.6               0.6666667  250      0.6540288  0.11579250
##   0.4   3         0.6               0.6666667  300      0.6523540  0.11455045
##   0.4   3         0.6               0.6666667  350      0.6498744  0.11218559
##   0.4   3         0.6               0.6666667  400      0.6477200  0.11043765
##   0.4   3         0.6               0.6666667  450      0.6463907  0.10971685
##   0.4   3         0.6               0.6666667  500      0.6452079  0.10776354
##   0.4   3         0.6               0.7222222   50      0.6663943  0.12713010
##   0.4   3         0.6               0.7222222  100      0.6656057  0.12520243
##   0.4   3         0.6               0.7222222  150      0.6625082  0.12148832
##   0.4   3         0.6               0.7222222  200      0.6592888  0.11856157
##   0.4   3         0.6               0.7222222  250      0.6575490  0.11721260
##   0.4   3         0.6               0.7222222  300      0.6554800  0.11545458
##   0.4   3         0.6               0.7222222  350      0.6538337  0.11446677
##   0.4   3         0.6               0.7222222  400      0.6507362  0.11143462
##   0.4   3         0.6               0.7222222  450      0.6486630  0.10958240
##   0.4   3         0.6               0.7222222  500      0.6471915  0.10867054
##   0.4   3         0.6               0.7777778   50      0.6667073  0.12722416
```

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##   0.4   3         0.6               0.7777778  100      0.6657887  0.12602967
##   0.4   3         0.6               0.7777778  150      0.6635448  0.12383357
##   0.4   3         0.6               0.7777778  200      0.6607685  0.12161026
##   0.4   3         0.6               0.7777778  250      0.6581262  0.11871547
##   0.4   3         0.6               0.7777778  300      0.6553377  0.11595671
##   0.4   3         0.6               0.7777778  350      0.6525776  0.11211835
##   0.4   3         0.6               0.7777778  400      0.6512402  0.11077059
##   0.4   3         0.6               0.7777778  450      0.6484598  0.10898840
##   0.4   3         0.6               0.7777778  500      0.6475899  0.10715826
##   0.4   3         0.6               0.8333333   50      0.6670488  0.12710752
##   0.4   3         0.6               0.8333333  100      0.6659960  0.12600016
##   0.4   3         0.6               0.8333333  150      0.6641749  0.12435437
##   0.4   3         0.6               0.8333333  200      0.6626465  0.12256610
##   0.4   3         0.6               0.8333333  250      0.6602888  0.12019211
##   0.4   3         0.6               0.8333333  300      0.6573702  0.11799639
##   0.4   3         0.6               0.8333333  350      0.6548702  0.11501792
##   0.4   3         0.6               0.8333333  400      0.6526874  0.11316703
##   0.4   3         0.6               0.8333333  450      0.6512402  0.11208208
##   0.4   3         0.6               0.8333333  500      0.6492159  0.11062244
##   0.4   3         0.6               0.8888889   50      0.6687114  0.12823287
##   0.4   3         0.6               0.8888889  100      0.6678984  0.12803780
##   0.4   3         0.6               0.8888889  150      0.6658375  0.12546893
##   0.4   3         0.6               0.8888889  200      0.6634391  0.12260604
##   0.4   3         0.6               0.8888889  250      0.6600693  0.11968624
##   0.4   3         0.6               0.8888889  300      0.6574027  0.11709478
##   0.4   3         0.6               0.8888889  350      0.6559637  0.11584696
##   0.4   3         0.6               0.8888889  400      0.6539353  0.11417447
##   0.4   3         0.6               0.8888889  450      0.6527077  0.11218863
##   0.4   3         0.6               0.8888889  500      0.6502809  0.11040699
##   0.4   3         0.6               0.9444444   50      0.6677845  0.12727853
##   0.4   3         0.6               0.9444444  100      0.6681179  0.12691826
##   0.4   3         0.6               0.9444444  150      0.6656626  0.12537586
##   0.4   3         0.6               0.9444444  200      0.6636627  0.12340769
##   0.4   3         0.6               0.9444444  250      0.6613335  0.12091893
##   0.4   3         0.6               0.9444444  300      0.6594067  0.11871583
##   0.4   3         0.6               0.9444444  350      0.6575734  0.11693926
##   0.4   3         0.6               0.9444444  400      0.6559677  0.11511244
##   0.4   3         0.6               0.9444444  450      0.6536792  0.11297509
##   0.4   3         0.6               0.9444444  500      0.6523499  0.11233026
##   0.4   3         0.6               1.0000000   50      0.6677764  0.12747080
##   0.4   3         0.6               1.0000000  100      0.6673862  0.12599818
##   0.4   3         0.6               1.0000000  150      0.6655000  0.12478054
##   0.4   3         0.6               1.0000000  200      0.6638091  0.12308769
##   0.4   3         0.6               1.0000000  250      0.6613904  0.12145370
##   0.4   3         0.6               1.0000000  300      0.6591953  0.11924367
##   0.4   3         0.6               1.0000000  350      0.6575002  0.11780139
##   0.4   3         0.6               1.0000000  400      0.6560491  0.11640587
##   0.4   3         0.6               1.0000000  450      0.6542930  0.11431151
##   0.4   3         0.6               1.0000000  500      0.6533296  0.11302697
##   0.4   3         0.8               0.5000000   50      0.6663008  0.12640259
##   0.4   3         0.8               0.5000000  100      0.6611872  0.12154250
##   0.4   3         0.8               0.5000000  150      0.6564434  0.11727004
##   0.4   3         0.8               0.5000000  200      0.6541304  0.11406130
##   0.4   3         0.8               0.5000000  250      0.6512727  0.11058263
```

```
##   0.4   3         0.8                 0.5000000  300      0.6488622  0.10944094
##   0.4   3         0.8                 0.5000000  350      0.6459070  0.10555070
##   0.4   3         0.8                 0.5000000  400      0.6439233  0.10514103
##   0.4   3         0.8                 0.5000000  450      0.6415697  0.10232172
##   0.4   3         0.8                 0.5000000  500      0.6388544  0.09923431
##   0.4   3         0.8                 0.5555556   50      0.6661057  0.12523765
##   0.4   3         0.8                 0.5555556  100      0.6631180  0.12358883
##   0.4   3         0.8                 0.5555556  150      0.6593742  0.11952726
##   0.4   3         0.8                 0.5555556  200      0.6551792  0.11675639
##   0.4   3         0.8                 0.5555556  250      0.6527280  0.11341055
##   0.4   3         0.8                 0.5555556  300      0.6506752  0.11147570
##   0.4   3         0.8                 0.5555556  350      0.6481712  0.10898125
##   0.4   3         0.8                 0.5555556  400      0.6465452  0.10736438
##   0.4   3         0.8                 0.5555556  450      0.6430250  0.10468620
##   0.4   3         0.8                 0.5555556  500      0.6420128  0.10363156
##   0.4   3         0.8                 0.6111111   50      0.6654553  0.12695989
##   0.4   3         0.8                 0.6111111  100      0.6618416  0.12310684
##   0.4   3         0.8                 0.6111111  150      0.6581263  0.11959507
##   0.4   3         0.8                 0.6111111  200      0.6563417  0.11683425
##   0.4   3         0.8                 0.6111111  250      0.6519150  0.11324213
##   0.4   3         0.8                 0.6111111  300      0.6507687  0.11148230
##   0.4   3         0.8                 0.6111111  350      0.6486671  0.10949658
##   0.4   3         0.8                 0.6111111  400      0.6465005  0.10763931
##   0.4   3         0.8                 0.6111111  450      0.6446754  0.10623615
##   0.4   3         0.8                 0.6111111  500      0.6426307  0.10393965
##   0.4   3         0.8                 0.6666667   50      0.6642927  0.12538277
##   0.4   3         0.8                 0.6666667  100      0.6630448  0.12295265
##   0.4   3         0.8                 0.6666667  150      0.6594636  0.12023645
##   0.4   3         0.8                 0.6666667  200      0.6567685  0.11745229
##   0.4   3         0.8                 0.6666667  250      0.6547076  0.11633420
##   0.4   3         0.8                 0.6666667  300      0.6514638  0.11268497
##   0.4   3         0.8                 0.6666667  350      0.6492972  0.11079023
##   0.4   3         0.8                 0.6666667  400      0.6477321  0.10931170
##   0.4   3         0.8                 0.6666667  450      0.6454639  0.10685736
##   0.4   3         0.8                 0.6666667  500      0.6444314  0.10585018
##   0.4   3         0.8                 0.7222222   50      0.6661260  0.12577522
##   0.4   3         0.8                 0.7222222  100      0.6647806  0.12510832
##   0.4   3         0.8                 0.7222222  150      0.6609067  0.12143455
##   0.4   3         0.8                 0.7222222  200      0.6583701  0.11819464
##   0.4   3         0.8                 0.7222222  250      0.6566954  0.11689430
##   0.4   3         0.8                 0.7222222  300      0.6535491  0.11406816
##   0.4   3         0.8                 0.7222222  350      0.6511671  0.11167716
##   0.4   3         0.8                 0.7222222  400      0.6502037  0.11052571
##   0.4   3         0.8                 0.7222222  450      0.6480208  0.10866199
##   0.4   3         0.8                 0.7222222  500      0.6462200  0.10756788
##   0.4   3         0.8                 0.7777778   50      0.6674268  0.12578774
##   0.4   3         0.8                 0.7777778  100      0.6651830  0.12425562
##   0.4   3         0.8                 0.7777778  150      0.6629107  0.12201395
##   0.4   3         0.8                 0.7777778  200      0.6603538  0.11966354
##   0.4   3         0.8                 0.7777778  250      0.6575612  0.11722549
##   0.4   3         0.8                 0.7777778  300      0.6547930  0.11515220
##   0.4   3         0.8                 0.7777778  350      0.6528459  0.11379638
##   0.4   3         0.8                 0.7777778  400      0.6510532  0.11263705
##   0.4   3         0.8                 0.7777778  450      0.6491712  0.11064777
```

```
##   0.4   3         0.8                 0.7777778  500       0.6484842  0.11053224
##   0.4   3         0.8                 0.8333333   50       0.6646220  0.12507495
##   0.4   3         0.8                 0.8333333  100       0.6638822  0.12372550
##   0.4   3         0.8                 0.8333333  150       0.6606953  0.12111654
##   0.4   3         0.8                 0.8333333  200       0.6584921  0.11910937
##   0.4   3         0.8                 0.8333333  250       0.6564596  0.11729223
##   0.4   3         0.8                 0.8333333  300       0.6546223  0.11490636
##   0.4   3         0.8                 0.8333333  350       0.6520370  0.11291641
##   0.4   3         0.8                 0.8333333  400       0.6505858  0.11206901
##   0.4   3         0.8                 0.8333333  450       0.6492443  0.11036293
##   0.4   3         0.8                 0.8333333  500       0.6477078  0.10998088
##   0.4   3         0.8                 0.8888889   50       0.6665935  0.12696829
##   0.4   3         0.8                 0.8888889  100       0.6658618  0.12583708
##   0.4   3         0.8                 0.8888889  150       0.6640570  0.12475503
##   0.4   3         0.8                 0.8888889  200       0.6609148  0.12104624
##   0.4   3         0.8                 0.8888889  250       0.6593823  0.11957508
##   0.4   3         0.8                 0.8888889  300       0.6572564  0.11758467
##   0.4   3         0.8                 0.8888889  350       0.6549434  0.11568440
##   0.4   3         0.8                 0.8888889  400       0.6537524  0.11361713
##   0.4   3         0.8                 0.8888889  450       0.6508907  0.11099721
##   0.4   3         0.8                 0.8888889  500       0.6495411  0.11003008
##   0.4   3         0.8                 0.9444444   50       0.6677886  0.12872874
##   0.4   3         0.8                 0.9444444  100       0.6658171  0.12659922
##   0.4   3         0.8                 0.9444444  150       0.6634920  0.12471966
##   0.4   3         0.8                 0.9444444  200       0.6614148  0.12240375
##   0.4   3         0.8                 0.9444444  250       0.6598742  0.12022680
##   0.4   3         0.8                 0.9444444  300       0.6581791  0.11905124
##   0.4   3         0.8                 0.9444444  350       0.6566385  0.11811257
##   0.4   3         0.8                 0.9444444  400       0.6542442  0.11585516
##   0.4   3         0.8                 0.9444444  450       0.6526833  0.11425994
##   0.4   3         0.8                 0.9444444  500       0.6511183  0.11322721
##   0.4   3         0.8                 1.0000000   50       0.6658049  0.12488620
##   0.4   3         0.8                 1.0000000  100       0.6653253  0.12447169
##   0.4   3         0.8                 1.0000000  150       0.6638131  0.12299629
##   0.4   3         0.8                 1.0000000  200       0.6618497  0.12054180
##   0.4   3         0.8                 1.0000000  250       0.6608294  0.11996605
##   0.4   3         0.8                 1.0000000  300       0.6586303  0.11850689
##   0.4   3         0.8                 1.0000000  350       0.6568498  0.11645449
##   0.4   3         0.8                 1.0000000  400       0.6551426  0.11470147
##   0.4   3         0.8                 1.0000000  450       0.6537402  0.11316458
##   0.4   3         0.8                 1.0000000  500       0.6518540  0.11233466
##   0.4   4         0.6                 0.5000000   50       0.6613172  0.12073113
##   0.4   4         0.6                 0.5000000  100       0.6544637  0.11380798
##   0.4   4         0.6                 0.5000000  150       0.6465168  0.10721437
##   0.4   4         0.6                 0.5000000  200       0.6422120  0.10410372
##   0.4   4         0.6                 0.5000000  250       0.6376837  0.09998866
##   0.4   4         0.6                 0.5000000  300       0.6350455  0.09703684
##   0.4   4         0.6                 0.5000000  350       0.6313749  0.09447602
##   0.4   4         0.6                 0.5000000  400       0.6294521  0.09324883
##   0.4   4         0.6                 0.5000000  450       0.6275701  0.09059818
##   0.4   4         0.6                 0.5000000  500       0.6243507  0.08890724
##   0.4   4         0.6                 0.5555556   50       0.6626546  0.12288580
##   0.4   4         0.6                 0.5555556  100       0.6555979  0.11575308
##   0.4   4         0.6                 0.5555556  150       0.6488907  0.10951475
```

```
##   0.4   4         0.6               0.5555556  200      0.6458298  0.10730465
##   0.4   4         0.6               0.5555556  250      0.6412405  0.10315338
##   0.4   4         0.6               0.5555556  300      0.6389316  0.10116155
##   0.4   4         0.6               0.5555556  350      0.6358138  0.09916383
##   0.4   4         0.6               0.5555556  400      0.6318586  0.09411154
##   0.4   4         0.6               0.5555556  450      0.6293058  0.09305094
##   0.4   4         0.6               0.5555556  500      0.6275945  0.09149844
##   0.4   4         0.6               0.6111111   50      0.6603904  0.12094085
##   0.4   4         0.6               0.6111111  100      0.6548499  0.11544868
##   0.4   4         0.6               0.6111111  150      0.6507077  0.11150683
##   0.4   4         0.6               0.6111111  200      0.6454640  0.10709342
##   0.4   4         0.6               0.6111111  250      0.6407365  0.10250099
##   0.4   4         0.6               0.6111111  300      0.6391633  0.10133672
##   0.4   4         0.6               0.6111111  350      0.6360170  0.09852136
##   0.4   4         0.6               0.6111111  400      0.6335578  0.09560378
##   0.4   4         0.6               0.6111111  450      0.6326025  0.09493560
##   0.4   4         0.6               0.6111111  500      0.6292367  0.09185933
##   0.4   4         0.6               0.6666667   50      0.6626099  0.12382308
##   0.4   4         0.6               0.6666667  100      0.6569311  0.11816259
##   0.4   4         0.6               0.6666667  150      0.6516833  0.11294256
##   0.4   4         0.6               0.6666667  200      0.6471225  0.10902114
##   0.4   4         0.6               0.6666667  250      0.6430697  0.10528504
##   0.4   4         0.6               0.6666667  300      0.6411389  0.10263229
##   0.4   4         0.6               0.6666667  350      0.6387934  0.10057973
##   0.4   4         0.6               0.6666667  400      0.6358910  0.09822045
##   0.4   4         0.6               0.6666667  450      0.6344114  0.09678183
##   0.4   4         0.6               0.6666667  500      0.6333748  0.09596020
##   0.4   4         0.6               0.7222222   50      0.6652805  0.12494832
##   0.4   4         0.6               0.7222222  100      0.6589067  0.11825271
##   0.4   4         0.6               0.7222222  150      0.6531345  0.11220687
##   0.4   4         0.6               0.7222222  200      0.6496508  0.10934429
##   0.4   4         0.6               0.7222222  250      0.6460208  0.10600110
##   0.4   4         0.6               0.7222222  300      0.6426591  0.10304721
##   0.4   4         0.6               0.7222222  350      0.6399234  0.10056393
##   0.4   4         0.6               0.7222222  400      0.6385820  0.10000735
##   0.4   4         0.6               0.7222222  450      0.6361836  0.09757402
##   0.4   4         0.6               0.7222222  500      0.6343870  0.09623879
##   0.4   4         0.6               0.7777778   50      0.6635977  0.12356529
##   0.4   4         0.6               0.7777778  100      0.6583661  0.11903395
##   0.4   4         0.6               0.7777778  150      0.6546141  0.11502738
##   0.4   4         0.6               0.7777778  200      0.6495249  0.11107236
##   0.4   4         0.6               0.7777778  250      0.6465859  0.10713222
##   0.4   4         0.6               0.7777778  300      0.6439071  0.10522580
##   0.4   4         0.6               0.7777778  350      0.6417161  0.10355205
##   0.4   4         0.6               0.7777778  400      0.6396958  0.10105096
##   0.4   4         0.6               0.7777778  450      0.6378950  0.09953128
##   0.4   4         0.6               0.7777778  500      0.6372325  0.09878701
##   0.4   4         0.6               0.8333333   50      0.6651708  0.12438408
##   0.4   4         0.6               0.8333333  100      0.6605164  0.11992932
##   0.4   4         0.6               0.8333333  150      0.6558092  0.11590159
##   0.4   4         0.6               0.8333333  200      0.6520898  0.11243789
##   0.4   4         0.6               0.8333333  250      0.6486265  0.10989120
##   0.4   4         0.6               0.8333333  300      0.6453298  0.10674112
##   0.4   4         0.6               0.8333333  350      0.6434559  0.10557863
```

```
##   0.4   4         0.6                 0.8333333  400       0.6407852  0.10332958
##   0.4   4         0.6                 0.8333333  450       0.6395047  0.10226609
##   0.4   4         0.6                 0.8333333  500       0.6388706  0.10220361
##   0.4   4         0.6                 0.8888889   50       0.6651667  0.12479458
##   0.4   4         0.6                 0.8888889  100       0.6602929  0.12010810
##   0.4   4         0.6                 0.8888889  150       0.6556548  0.11635347
##   0.4   4         0.6                 0.8888889  200       0.6528296  0.11355819
##   0.4   4         0.6                 0.8888889  250       0.6498988  0.11043783
##   0.4   4         0.6                 0.8888889  300       0.6470046  0.10797283
##   0.4   4         0.6                 0.8888889  350       0.6448664  0.10670661
##   0.4   4         0.6                 0.8888889  400       0.6429152  0.10483116
##   0.4   4         0.6                 0.8888889  450       0.6416145  0.10410407
##   0.4   4         0.6                 0.8888889  500       0.6399356  0.10223728
##   0.4   4         0.6                 0.9444444   50       0.6652439  0.12494908
##   0.4   4         0.6                 0.9444444  100       0.6616546  0.12103929
##   0.4   4         0.6                 0.9444444  150       0.6578376  0.11762353
##   0.4   4         0.6                 0.9444444  200       0.6541344  0.11329166
##   0.4   4         0.6                 0.9444444  250       0.6516589  0.11074139
##   0.4   4         0.6                 0.9444444  300       0.6486630  0.10792755
##   0.4   4         0.6                 0.9444444  350       0.6464883  0.10617833
##   0.4   4         0.6                 0.9444444  400       0.6452038  0.10523909
##   0.4   4         0.6                 0.9444444  450       0.6426998  0.10291904
##   0.4   4         0.6                 0.9444444  500       0.6409152  0.10152242
##   0.4   4         0.6                 1.0000000   50       0.6654025  0.12530606
##   0.4   4         0.6                 1.0000000  100       0.6627562  0.12248513
##   0.4   4         0.6                 1.0000000  150       0.6585286  0.11890905
##   0.4   4         0.6                 1.0000000  200       0.6557564  0.11676118
##   0.4   4         0.6                 1.0000000  250       0.6523418  0.11379303
##   0.4   4         0.6                 1.0000000  300       0.6494679  0.11047325
##   0.4   4         0.6                 1.0000000  350       0.6474801  0.10857382
##   0.4   4         0.6                 1.0000000  400       0.6455615  0.10695892
##   0.4   4         0.6                 1.0000000  450       0.6442322  0.10599038
##   0.4   4         0.6                 1.0000000  500       0.6427445  0.10409596
##   0.4   4         0.8                 0.5000000   50       0.6625976  0.12235367
##   0.4   4         0.8                 0.5000000  100       0.6537239  0.11421939
##   0.4   4         0.8                 0.5000000  150       0.6480573  0.10960284
##   0.4   4         0.8                 0.5000000  200       0.6406023  0.10383192
##   0.4   4         0.8                 0.5000000  250       0.6377365  0.10008183
##   0.4   4         0.8                 0.5000000  300       0.6339154  0.09696150
##   0.4   4         0.8                 0.5000000  350       0.6299440  0.09275642
##   0.4   4         0.8                 0.5000000  400       0.6281351  0.09163891
##   0.4   4         0.8                 0.5000000  450       0.6265823  0.08996415
##   0.4   4         0.8                 0.5000000  500       0.6242165  0.08759823
##   0.4   4         0.8                 0.5555556   50       0.6612196  0.12107836
##   0.4   4         0.8                 0.5555556  100       0.6537767  0.11449677
##   0.4   4         0.8                 0.5555556  150       0.6482850  0.10948744
##   0.4   4         0.8                 0.5555556  200       0.6437729  0.10485368
##   0.4   4         0.8                 0.5555556  250       0.6395332  0.10034016
##   0.4   4         0.8                 0.5555556  300       0.6363626  0.09894572
##   0.4   4         0.8                 0.5555556  350       0.6334968  0.09697319
##   0.4   4         0.8                 0.5555556  400       0.6304968  0.09351227
##   0.4   4         0.8                 0.5555556  450       0.6279643  0.09156490
##   0.4   4         0.8                 0.5555556  500       0.6272083  0.09079412
##   0.4   4         0.8                 0.6111111   50       0.6616546  0.12164230
```

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##   0.4   4         0.8               0.6111111  100      0.6547686  0.11460922
##   0.4   4         0.8               0.6111111  150      0.6476874  0.10870621
##   0.4   4         0.8               0.6111111  200      0.6438095  0.10518859
##   0.4   4         0.8               0.6111111  250      0.6402608  0.10221848
##   0.4   4         0.8               0.6111111  300      0.6381511  0.09983533
##   0.4   4         0.8               0.6111111  350      0.6350496  0.09742463
##   0.4   4         0.8               0.6111111  400      0.6330740  0.09530992
##   0.4   4         0.8               0.6111111  450      0.6306269  0.09412191
##   0.4   4         0.8               0.6111111  500      0.6288668  0.09257685
##   0.4   4         0.8               0.6666667   50      0.6637033  0.12233234
##   0.4   4         0.8               0.6666667  100      0.6574433  0.11672271
##   0.4   4         0.8               0.6666667  150      0.6517971  0.11211000
##   0.4   4         0.8               0.6666667  200      0.6473135  0.10769479
##   0.4   4         0.8               0.6666667  250      0.6443989  0.10460909
##   0.4   4         0.8               0.6666667  300      0.6414600  0.10264196
##   0.4   4         0.8               0.6666667  350      0.6384885  0.10003910
##   0.4   4         0.8               0.6666667  400      0.6356227  0.09776097
##   0.4   4         0.8               0.6666667  450      0.6352243  0.09763265
##   0.4   4         0.8               0.6666667  500      0.6318098  0.09444672
##   0.4   4         0.8               0.7222222   50      0.6633904  0.12165139
##   0.4   4         0.8               0.7222222  100      0.6582726  0.11761421
##   0.4   4         0.8               0.7222222  150      0.6521711  0.11243708
##   0.4   4         0.8               0.7222222  200      0.6483460  0.10908231
##   0.4   4         0.8               0.7222222  250      0.6446266  0.10534139
##   0.4   4         0.8               0.7222222  300      0.6410372  0.10264441
##   0.4   4         0.8               0.7222222  350      0.6387893  0.10013939
##   0.4   4         0.8               0.7222222  400      0.6368259  0.09898734
##   0.4   4         0.8               0.7222222  450      0.6352000  0.09776215
##   0.4   4         0.8               0.7222222  500      0.6345455  0.09772792
##   0.4   4         0.8               0.7777778   50      0.6636099  0.12408977
##   0.4   4         0.8               0.7777778  100      0.6589718  0.11925481
##   0.4   4         0.8               0.7777778  150      0.6548540  0.11519383
##   0.4   4         0.8               0.7777778  200      0.6513500  0.11252210
##   0.4   4         0.8               0.7777778  250      0.6468257  0.10819382
##   0.4   4         0.8               0.7777778  300      0.6436307  0.10560335
##   0.4   4         0.8               0.7777778  350      0.6409234  0.10305897
##   0.4   4         0.8               0.7777778  400      0.6392812  0.10141898
##   0.4   4         0.8               0.7777778  450      0.6376552  0.09992813
##   0.4   4         0.8               0.7777778  500      0.6365739  0.09912071
##   0.4   4         0.8               0.8333333   50      0.6636993  0.12318178
##   0.4   4         0.8               0.8333333  100      0.6591222  0.11923251
##   0.4   4         0.8               0.8333333  150      0.6553661  0.11561188
##   0.4   4         0.8               0.8333333  200      0.6509476  0.11166710
##   0.4   4         0.8               0.8333333  250      0.6481143  0.10872823
##   0.4   4         0.8               0.8333333  300      0.6455127  0.10671422
##   0.4   4         0.8               0.8333333  350      0.6432852  0.10463854
##   0.4   4         0.8               0.8333333  400      0.6414112  0.10303440
##   0.4   4         0.8               0.8333333  450      0.6392527  0.10163259
##   0.4   4         0.8               0.8333333  500      0.6380251  0.10018803
##   0.4   4         0.8               0.8888889   50      0.6646342  0.12465198
##   0.4   4         0.8               0.8888889  100      0.6607400  0.12069383
##   0.4   4         0.8               0.8888889  150      0.6559352  0.11640801
##   0.4   4         0.8               0.8888889  200      0.6518866  0.11273764
##   0.4   4         0.8               0.8888889  250      0.6485127  0.10988643
```

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##   0.4   4         0.8               0.8888889  300      0.6458054  0.10708172
##   0.4   4         0.8               0.8888889  350      0.6432607  0.10455687
##   0.4   4         0.8               0.8888889  400      0.6410210  0.10290851
##   0.4   4         0.8               0.8888889  450      0.6397161  0.10232826
##   0.4   4         0.8               0.8888889  500      0.6379154  0.10137740
##   0.4   4         0.8               0.9444444   50      0.6650285  0.12540353
##   0.4   4         0.8               0.9444444  100      0.6606099  0.12175682
##   0.4   4         0.8               0.9444444  150      0.6570450  0.11833203
##   0.4   4         0.8               0.9444444  200      0.6538865  0.11589779
##   0.4   4         0.8               0.9444444  250      0.6504069  0.11244743
##   0.4   4         0.8               0.9444444  300      0.6466794  0.10895336
##   0.4   4         0.8               0.9444444  350      0.6448623  0.10736529
##   0.4   4         0.8               0.9444444  400      0.6435128  0.10641199
##   0.4   4         0.8               0.9444444  450      0.6418258  0.10475495
##   0.4   4         0.8               0.9444444  500      0.6398421  0.10283910
##   0.4   4         0.8               1.0000000   50      0.6662073  0.12568497
##   0.4   4         0.8               1.0000000  100      0.6625082  0.12214574
##   0.4   4         0.8               1.0000000  150      0.6584758  0.11829830
##   0.4   4         0.8               1.0000000  200      0.6557482  0.11597776
##   0.4   4         0.8               1.0000000  250      0.6534475  0.11436567
##   0.4   4         0.8               1.0000000  300      0.6505248  0.11158795
##   0.4   4         0.8               1.0000000  350      0.6489110  0.10987513
##   0.4   4         0.8               1.0000000  400      0.6472403  0.10812523
##   0.4   4         0.8               1.0000000  450      0.6449070  0.10504861
##   0.4   4         0.8               1.0000000  500      0.6435900  0.10449937
##   0.4   5         0.6               0.5000000   50      0.6522686  0.11402985
##   0.4   5         0.6               0.5000000  100      0.6428665  0.10373223
##   0.4   5         0.6               0.5000000  150      0.6346146  0.09742026
##   0.4   5         0.6               0.5000000  200      0.6289277  0.09197713
##   0.4   5         0.6               0.5000000  250      0.6250254  0.08942484
##   0.4   5         0.6               0.5000000  300      0.6231068  0.08712410
##   0.4   5         0.6               0.5000000  350      0.6196190  0.08463159
##   0.4   5         0.6               0.5000000  400      0.6171476  0.08309233
##   0.4   5         0.6               0.5000000  450      0.6156639  0.08117011
##   0.4   5         0.6               0.5000000  500      0.6151313  0.08121571
##   0.4   5         0.6               0.5555556   50      0.6536386  0.11313519
##   0.4   5         0.6               0.5555556  100      0.6452282  0.10597833
##   0.4   5         0.6               0.5555556  150      0.6384113  0.10053986
##   0.4   5         0.6               0.5555556  200      0.6333464  0.09647364
##   0.4   5         0.6               0.5555556  250      0.6294684  0.09170968
##   0.4   5         0.6               0.5555556  300      0.6270416  0.08959448
##   0.4   5         0.6               0.5555556  350      0.6247937  0.08777922
##   0.4   5         0.6               0.5555556  400      0.6225174  0.08664292
##   0.4   5         0.6               0.5555556  450      0.6225133  0.08597480
##   0.4   5         0.6               0.5555556  500      0.6209198  0.08394050
##   0.4   5         0.6               0.6111111   50      0.6560978  0.11649611
##   0.4   5         0.6               0.6111111  100      0.6452648  0.10669811
##   0.4   5         0.6               0.6111111  150      0.6388177  0.10178056
##   0.4   5         0.6               0.6111111  200      0.6345414  0.09835655
##   0.4   5         0.6               0.6111111  250      0.6296594  0.09472644
##   0.4   5         0.6               0.6111111  300      0.6282773  0.09262805
##   0.4   5         0.6               0.6111111  350      0.6253831  0.09068238
##   0.4   5         0.6               0.6111111  400      0.6238059  0.08902117
##   0.4   5         0.6               0.6111111  450      0.6225864  0.08727782
```

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##   0.4   5         0.6         0.6111111  500      0.6216312  0.08652365
##   0.4   5         0.6         0.6666667   50      0.6571303  0.11699694
##   0.4   5         0.6         0.6666667  100      0.6486020  0.10944659
##   0.4   5         0.6         0.6666667  150      0.6437932  0.10468238
##   0.4   5         0.6         0.6666667  200      0.6386185  0.09939249
##   0.4   5         0.6         0.6666667  250      0.6350820  0.09757542
##   0.4   5         0.6         0.6666667  300      0.6315374  0.09364541
##   0.4   5         0.6         0.6666667  350      0.6298708  0.09319159
##   0.4   5         0.6         0.6666667  400      0.6273790  0.09202540
##   0.4   5         0.6         0.6666667  450      0.6265945  0.09163886
##   0.4   5         0.6         0.6666667  500      0.6252408  0.08935768
##   0.4   5         0.6         0.7222222   50      0.6571628  0.11680246
##   0.4   5         0.6         0.7222222  100      0.6482565  0.11010653
##   0.4   5         0.6         0.7222222  150      0.6433583  0.10579449
##   0.4   5         0.6         0.7222222  200      0.6392893  0.10179781
##   0.4   5         0.6         0.7222222  250      0.6341024  0.09674145
##   0.4   5         0.6         0.7222222  300      0.6321065  0.09548312
##   0.4   5         0.6         0.7222222  350      0.6307488  0.09432564
##   0.4   5         0.6         0.7222222  400      0.6290782  0.09208205
##   0.4   5         0.6         0.7222222  450      0.6273628  0.09047528
##   0.4   5         0.6         0.7222222  500      0.6260051  0.08918957
##   0.4   5         0.6         0.7777778   50      0.6606140  0.12021769
##   0.4   5         0.6         0.7777778  100      0.6519231  0.11155602
##   0.4   5         0.6         0.7777778  150      0.6455737  0.10559524
##   0.4   5         0.6         0.7777778  200      0.6409437  0.10158248
##   0.4   5         0.6         0.7777778  250      0.6367772  0.09815607
##   0.4   5         0.6         0.7777778  300      0.6340537  0.09692911
##   0.4   5         0.6         0.7777778  350      0.6316391  0.09453167
##   0.4   5         0.6         0.7777778  400      0.6310497  0.09459316
##   0.4   5         0.6         0.7777778  450      0.6303342  0.09313354
##   0.4   5         0.6         0.7777778  500      0.6296269  0.09303176
##   0.4   5         0.6         0.8333333   50      0.6605042  0.12062577
##   0.4   5         0.6         0.8333333  100      0.6539434  0.11396160
##   0.4   5         0.6         0.8333333  150      0.6475289  0.10941762
##   0.4   5         0.6         0.8333333  200      0.6427038  0.10557132
##   0.4   5         0.6         0.8333333  250      0.6393625  0.10279556
##   0.4   5         0.6         0.8333333  300      0.6367894  0.10083968
##   0.4   5         0.6         0.8333333  350      0.6352081  0.09830905
##   0.4   5         0.6         0.8333333  400      0.6330862  0.09639859
##   0.4   5         0.6         0.8333333  450      0.6320415  0.09559936
##   0.4   5         0.6         0.8333333  500      0.6309806  0.09457148
##   0.4   5         0.6         0.8888889   50      0.6584758  0.11868994
##   0.4   5         0.6         0.8888889  100      0.6537646  0.11379199
##   0.4   5         0.6         0.8888889  150      0.6491753  0.10941508
##   0.4   5         0.6         0.8888889  200      0.6448095  0.10630004
##   0.4   5         0.6         0.8888889  250      0.6399600  0.10174269
##   0.4   5         0.6         0.8888889  300      0.6371796  0.09948441
##   0.4   5         0.6         0.8888889  350      0.6357934  0.09826650
##   0.4   5         0.6         0.8888889  400      0.6337284  0.09590431
##   0.4   5         0.6         0.8888889  450      0.6318626  0.09418299
##   0.4   5         0.6         0.8888889  500      0.6313342  0.09411243
##   0.4   5         0.6         0.9444444   50      0.6608822  0.12089878
##   0.4   5         0.6         0.9444444  100      0.6547442  0.11527988
##   0.4   5         0.6         0.9444444  150      0.6502077  0.11103035
```

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##   0.4   5         0.6                 0.9444444  200      0.6464842  0.10740782
##   0.4   5         0.6                 0.9444444  250      0.6438664  0.10546211
##   0.4   5         0.6                 0.9444444  300      0.6417567  0.10389460
##   0.4   5         0.6                 0.9444444  350      0.6394681  0.10136625
##   0.4   5         0.6                 0.9444444  400      0.6380535  0.10052825
##   0.4   5         0.6                 0.9444444  450      0.6351634  0.09802917
##   0.4   5         0.6                 0.9444444  500      0.6347406  0.09765969
##   0.4   5         0.6                 1.0000000   50      0.6610164  0.12083095
##   0.4   5         0.6                 1.0000000  100      0.6564840  0.11676696
##   0.4   5         0.6                 1.0000000  150      0.6519638  0.11280346
##   0.4   5         0.6                 1.0000000  200      0.6479476  0.10922917
##   0.4   5         0.6                 1.0000000  250      0.6445493  0.10568495
##   0.4   5         0.6                 1.0000000  300      0.6421185  0.10348950
##   0.4   5         0.6                 1.0000000  350      0.6411876  0.10253524
##   0.4   5         0.6                 1.0000000  400      0.6391470  0.10102915
##   0.4   5         0.6                 1.0000000  450      0.6374398  0.09992983
##   0.4   5         0.6                 1.0000000  500      0.6370699  0.09929244
##   0.4   5         0.8                 0.5000000   50      0.6495126  0.11047666
##   0.4   5         0.8                 0.5000000  100      0.6426266  0.10362110
##   0.4   5         0.8                 0.5000000  150      0.6358788  0.09650629
##   0.4   5         0.8                 0.5000000  200      0.6300171  0.09150294
##   0.4   5         0.8                 0.5000000  250      0.6257815  0.08802270
##   0.4   5         0.8                 0.5000000  300      0.6230498  0.08668733
##   0.4   5         0.8                 0.5000000  350      0.6228100  0.08593734
##   0.4   5         0.8                 0.5000000  400      0.6196678  0.08525308
##   0.4   5         0.8                 0.5000000  450      0.6174768  0.08380526
##   0.4   5         0.8                 0.5000000  500      0.6175053  0.08290402
##   0.4   5         0.8                 0.5555556   50      0.6548580  0.11700385
##   0.4   5         0.8                 0.5555556  100      0.6441673  0.10628110
##   0.4   5         0.8                 0.5555556  150      0.6381186  0.10191977
##   0.4   5         0.8                 0.5555556  200      0.6310781  0.09459686
##   0.4   5         0.8                 0.5555556  250      0.6274156  0.09249947
##   0.4   5         0.8                 0.5555556  300      0.6237897  0.08889724
##   0.4   5         0.8                 0.5555556  350      0.6218751  0.08684748
##   0.4   5         0.8                 0.5555556  400      0.6200946  0.08571015
##   0.4   5         0.8                 0.5555556  450      0.6200377  0.08556626
##   0.4   5         0.8                 0.5555556  500      0.6190865  0.08444857
##   0.4   5         0.8                 0.6111111   50      0.6574962  0.11688661
##   0.4   5         0.8                 0.6111111  100      0.6475980  0.10724235
##   0.4   5         0.8                 0.6111111  150      0.6386145  0.10021224
##   0.4   5         0.8                 0.6111111  200      0.6339683  0.09698047
##   0.4   5         0.8                 0.6111111  250      0.6303952  0.09375857
##   0.4   5         0.8                 0.6111111  300      0.6268790  0.09052248
##   0.4   5         0.8                 0.6111111  350      0.6244807  0.08875178
##   0.4   5         0.8                 0.6111111  400      0.6231352  0.08718603
##   0.4   5         0.8                 0.6111111  450      0.6225742  0.08750872
##   0.4   5         0.8                 0.6111111  500      0.6215458  0.08664806
##   0.4   5         0.8                 0.6666667   50      0.6563296  0.11709786
##   0.4   5         0.8                 0.6666667  100      0.6479964  0.10809172
##   0.4   5         0.8                 0.6666667  150      0.6403543  0.09981436
##   0.4   5         0.8                 0.6666667  200      0.6349439  0.09639821
##   0.4   5         0.8                 0.6666667  250      0.6323545  0.09463488
##   0.4   5         0.8                 0.6666667  300      0.6295904  0.09282968
##   0.4   5         0.8                 0.6666667  350      0.6279075  0.09030906
```

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##   0.4   5         0.8              0.6666667  400      0.6265823  0.08966587
##   0.4   5         0.8              0.6666667  450      0.6256962  0.08911411
##   0.4   5         0.8              0.6666667  500      0.6248506  0.08856898
##   0.4   5         0.8              0.7222222   50      0.6581954  0.11778644
##   0.4   5         0.8              0.7222222  100      0.6502159  0.11084799
##   0.4   5         0.8              0.7222222  150      0.6438949  0.10530458
##   0.4   5         0.8              0.7222222  200      0.6393584  0.10169765
##   0.4   5         0.8              0.7222222  250      0.6355374  0.09774414
##   0.4   5         0.8              0.7222222  300      0.6328058  0.09573424
##   0.4   5         0.8              0.7222222  350      0.6309562  0.09458767
##   0.4   5         0.8              0.7222222  400      0.6288180  0.09437207
##   0.4   5         0.8              0.7222222  450      0.6286961  0.09323659
##   0.4   5         0.8              0.7222222  500      0.6267937  0.09121317
##   0.4   5         0.8              0.7777778   50      0.6578986  0.11765542
##   0.4   5         0.8              0.7777778  100      0.6491834  0.11063666
##   0.4   5         0.8              0.7777778  150      0.6430046  0.10468159
##   0.4   5         0.8              0.7777778  200      0.6384153  0.10161085
##   0.4   5         0.8              0.7777778  250      0.6353626  0.09872782
##   0.4   5         0.8              0.7777778  300      0.6325212  0.09663049
##   0.4   5         0.8              0.7777778  350      0.6310253  0.09485032
##   0.4   5         0.8              0.7777778  400      0.6297204  0.09484447
##   0.4   5         0.8              0.7777778  450      0.6285131  0.09388148
##   0.4   5         0.8              0.7777778  500      0.6281595  0.09376524
##   0.4   5         0.8              0.8333333   50      0.6590896  0.11911800
##   0.4   5         0.8              0.8333333  100      0.6514923  0.11218036
##   0.4   5         0.8              0.8333333  150      0.6467160  0.10855812
##   0.4   5         0.8              0.8333333  200      0.6425006  0.10462541
##   0.4   5         0.8              0.8333333  250      0.6383381  0.10039077
##   0.4   5         0.8              0.8333333  300      0.6365089  0.09847093
##   0.4   5         0.8              0.8333333  350      0.6350821  0.09767283
##   0.4   5         0.8              0.8333333  400      0.6328057  0.09685139
##   0.4   5         0.8              0.8333333  450      0.6320090  0.09587603
##   0.4   5         0.8              0.8333333  500      0.6313749  0.09423787
##   0.4   5         0.8              0.8888889   50      0.6604148  0.12030737
##   0.4   5         0.8              0.8888889  100      0.6532889  0.11409580
##   0.4   5         0.8              0.8888889  150      0.6477281  0.10888533
##   0.4   5         0.8              0.8888889  200      0.6437445  0.10603738
##   0.4   5         0.8              0.8888889  250      0.6403665  0.10254907
##   0.4   5         0.8              0.8888889  300      0.6381308  0.10118965
##   0.4   5         0.8              0.8888889  350      0.6359601  0.09900108
##   0.4   5         0.8              0.8888889  400      0.6340618  0.09753844
##   0.4   5         0.8              0.8888889  450      0.6324846  0.09705470
##   0.4   5         0.8              0.8888889  500      0.6324236  0.09703154
##   0.4   5         0.8              0.9444444   50      0.6601262  0.12000271
##   0.4   5         0.8              0.9444444  100      0.6545247  0.11480746
##   0.4   5         0.8              0.9444444  150      0.6503988  0.11034834
##   0.4   5         0.8              0.9444444  200      0.6471306  0.10757589
##   0.4   5         0.8              0.9444444  250      0.6431429  0.10389655
##   0.4   5         0.8              0.9444444  300      0.6409844  0.10280339
##   0.4   5         0.8              0.9444444  350      0.6390861  0.10000528
##   0.4   5         0.8              0.9444444  400      0.6381836  0.09983168
##   0.4   5         0.8              0.9444444  450      0.6370699  0.09890452
##   0.4   5         0.8              0.9444444  500      0.6358341  0.09674031
##   0.4   5         0.8              1.0000000   50      0.6619026  0.12289014
```

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##   0.4   5         0.8               1.0000000  100      0.6569108  0.11761144
##   0.4   5         0.8               1.0000000  150      0.6518865  0.11339437
##   0.4   5         0.8               1.0000000  200      0.6482768  0.10993738
##   0.4   5         0.8               1.0000000  250      0.6446672  0.10729789
##   0.4   5         0.8               1.0000000  300      0.6420697  0.10414368
##   0.4   5         0.8               1.0000000  350      0.6409640  0.10334904
##   0.4   5         0.8               1.0000000  400      0.6384682  0.10245861
##   0.4   5         0.8               1.0000000  450      0.6370373  0.10086470
##   0.4   5         0.8               1.0000000  500      0.6366796  0.09998528
##   0.4   6         0.6               0.5000000   50      0.6412486  0.10330187
##   0.4   6         0.6               0.5000000  100      0.6319074  0.09540259
##   0.4   6         0.6               0.5000000  150      0.6257409  0.08875116
##   0.4   6         0.6               0.5000000  200      0.6222125  0.08619209
##   0.4   6         0.6               0.5000000  250      0.6185175  0.08403848
##   0.4   6         0.6               0.5000000  300      0.6160826  0.08162633
##   0.4   6         0.6               0.5000000  350      0.6159688  0.08185270
##   0.4   6         0.6               0.5000000  400      0.6151558  0.08161367
##   0.4   6         0.6               0.5000000  450      0.6149200  0.08104846
##   0.4   6         0.6               0.5000000  500      0.6150541  0.07987940
##   0.4   6         0.6               0.5555556   50      0.6455208  0.10688759
##   0.4   6         0.6               0.5555556  100      0.6348666  0.09694716
##   0.4   6         0.6               0.5555556  150      0.6277489  0.09058109
##   0.4   6         0.6               0.5555556  200      0.6237084  0.08773530
##   0.4   6         0.6               0.5555556  250      0.6219767  0.08696044
##   0.4   6         0.6               0.5555556  300      0.6201190  0.08554933
##   0.4   6         0.6               0.5555556  350      0.6196678  0.08477767
##   0.4   6         0.6               0.5555556  400      0.6190947  0.08422964
##   0.4   6         0.6               0.5555556  450      0.6187004  0.08378275
##   0.4   6         0.6               0.5555556  500      0.6192532  0.08455516
##   0.4   6         0.6               0.6111111   50      0.6482241  0.10890655
##   0.4   6         0.6               0.6111111  100      0.6372406  0.09941681
##   0.4   6         0.6               0.6111111  150      0.6313667  0.09468556
##   0.4   6         0.6               0.6111111  200      0.6260579  0.09113267
##   0.4   6         0.6               0.6111111  250      0.6229970  0.09005568
##   0.4   6         0.6               0.6111111  300      0.6218344  0.08757072
##   0.4   6         0.6               0.6111111  350      0.6204483  0.08664250
##   0.4   6         0.6               0.6111111  400      0.6195865  0.08602978
##   0.4   6         0.6               0.6111111  450      0.6198670  0.08643949
##   0.4   6         0.6               0.6111111  500      0.6201109  0.08644294
##   0.4   6         0.6               0.6666667   50      0.6486915  0.10939692
##   0.4   6         0.6               0.6666667  100      0.6391999  0.10078214
##   0.4   6         0.6               0.6666667  150      0.6305944  0.09261435
##   0.4   6         0.6               0.6666667  200      0.6281067  0.08965373
##   0.4   6         0.6               0.6666667  250      0.6253506  0.08845327
##   0.4   6         0.6               0.6666667  300      0.6236312  0.08731897
##   0.4   6         0.6               0.6666667  350      0.6237368  0.08816148
##   0.4   6         0.6               0.6666667  400      0.6231759  0.08672463
##   0.4   6         0.6               0.6666667  450      0.6238506  0.08718497
##   0.4   6         0.6               0.6666667  500      0.6231596  0.08649627
##   0.4   6         0.6               0.7222222   50      0.6501590  0.11210356
##   0.4   6         0.6               0.7222222  100      0.6407324  0.10229733
##   0.4   6         0.6               0.7222222  150      0.6346797  0.09688281
##   0.4   6         0.6               0.7222222  200      0.6310619  0.09498707
##   0.4   6         0.6               0.7222222  250      0.6291066  0.09307195
```

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##   0.4   6         0.6                0.7222222  300      0.6270173  0.09246174
##   0.4   6         0.6                0.7222222  350      0.6251799  0.08996483
##   0.4   6         0.6                0.7222222  400      0.6258791  0.09025233
##   0.4   6         0.6                0.7222222  450      0.6245620  0.08896470
##   0.4   6         0.6                0.7222222  500      0.6247450  0.08883075
##   0.4   6         0.6                0.7777778   50      0.6519922  0.11255485
##   0.4   6         0.6                0.7777778  100      0.6427729  0.10430289
##   0.4   6         0.6                0.7777778  150      0.6361471  0.09971811
##   0.4   6         0.6                0.7777778  200      0.6344520  0.09841586
##   0.4   6         0.6                0.7777778  250      0.6315171  0.09634263
##   0.4   6         0.6                0.7777778  300      0.6302651  0.09452273
##   0.4   6         0.6                0.7777778  350      0.6289847  0.09395838
##   0.4   6         0.6                0.7777778  400      0.6285091  0.09303584
##   0.4   6         0.6                0.7777778  450      0.6281676  0.09255580
##   0.4   6         0.6                0.7777778  500      0.6287042  0.09301818
##   0.4   6         0.6                0.8333333   50      0.6546588  0.11503181
##   0.4   6         0.6                0.8333333  100      0.6447323  0.10645382
##   0.4   6         0.6                0.8333333  150      0.6390047  0.10174671
##   0.4   6         0.6                0.8333333  200      0.6347569  0.09799252
##   0.4   6         0.6                0.8333333  250      0.6317488  0.09562104
##   0.4   6         0.6                0.8333333  300      0.6304684  0.09381476
##   0.4   6         0.6                0.8333333  350      0.6295741  0.09318018
##   0.4   6         0.6                0.8333333  400      0.6296107  0.09395377
##   0.4   6         0.6                0.8333333  450      0.6295578  0.09350542
##   0.4   6         0.6                0.8333333  500      0.6291473  0.09294800
##   0.4   6         0.6                0.8888889   50      0.6547402  0.11507198
##   0.4   6         0.6                0.8888889  100      0.6467607  0.10810050
##   0.4   6         0.6                0.8888889  150      0.6407567  0.10310700
##   0.4   6         0.6                0.8888889  200      0.6375820  0.09962329
##   0.4   6         0.6                0.8888889  250      0.6341797  0.09687267
##   0.4   6         0.6                0.8888889  300      0.6329033  0.09549692
##   0.4   6         0.6                0.8888889  350      0.6323830  0.09498953
##   0.4   6         0.6                0.8888889  400      0.6321310  0.09501895
##   0.4   6         0.6                0.8888889  450      0.6317610  0.09492039
##   0.4   6         0.6                0.8888889  500      0.6315253  0.09489697
##   0.4   6         0.6                0.9444444   50      0.6548214  0.11557035
##   0.4   6         0.6                0.9444444  100      0.6475696  0.10830682
##   0.4   6         0.6                0.9444444  150      0.6416673  0.10344093
##   0.4   6         0.6                0.9444444  200      0.6372162  0.09941778
##   0.4   6         0.6                0.9444444  250      0.6355495  0.09894715
##   0.4   6         0.6                0.9444444  300      0.6342366  0.09767116
##   0.4   6         0.6                0.9444444  350      0.6337122  0.09745444
##   0.4   6         0.6                0.9444444  400      0.6325984  0.09622334
##   0.4   6         0.6                0.9444444  450      0.6325781  0.09565642
##   0.4   6         0.6                0.9444444  500      0.6323017  0.09544202
##   0.4   6         0.6                1.0000000   50      0.6567238  0.11687599
##   0.4   6         0.6                1.0000000  100      0.6499313  0.11016455
##   0.4   6         0.6                1.0000000  150      0.6450534  0.10603835
##   0.4   6         0.6                1.0000000  200      0.6407324  0.10252881
##   0.4   6         0.6                1.0000000  250      0.6381836  0.09972438
##   0.4   6         0.6                1.0000000  300      0.6365739  0.09859439
##   0.4   6         0.6                1.0000000  350      0.6358870  0.09847813
##   0.4   6         0.6                1.0000000  400      0.6354967  0.09831143
##   0.4   6         0.6                1.0000000  450      0.6350658  0.09749796
```

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##   0.4   6         0.6                 1.0000000  500      0.6348219  0.09714003
##   0.4   6         0.8                 0.5000000   50      0.6432363  0.10426188
##   0.4   6         0.8                 0.5000000  100      0.6296757  0.09276159
##   0.4   6         0.8                 0.5000000  150      0.6225905  0.08659777
##   0.4   6         0.8                 0.5000000  200      0.6184199  0.08317568
##   0.4   6         0.8                 0.5000000  250      0.6162411  0.08172416
##   0.4   6         0.8                 0.5000000  300      0.6148996  0.07955624
##   0.4   6         0.8                 0.5000000  350      0.6139200  0.07995790
##   0.4   6         0.8                 0.5000000  400      0.6139362  0.07939848
##   0.4   6         0.8                 0.5000000  450      0.6138996  0.07934728
##   0.4   6         0.8                 0.5000000  500      0.6142045  0.07913363
##   0.4   6         0.8                 0.5555556   50      0.6470574  0.10751887
##   0.4   6         0.8                 0.5555556  100      0.6365536  0.09762036
##   0.4   6         0.8                 0.5555556  150      0.6280172  0.09072010
##   0.4   6         0.8                 0.5555556  200      0.6223304  0.08848061
##   0.4   6         0.8                 0.5555556  250      0.6220540  0.08645517
##   0.4   6         0.8                 0.5555556  300      0.6212085  0.08679410
##   0.4   6         0.8                 0.5555556  350      0.6204605  0.08584843
##   0.4   6         0.8                 0.5555556  400      0.6195378  0.08442099
##   0.4   6         0.8                 0.5555556  450      0.6191394  0.08418336
##   0.4   6         0.8                 0.5555556  500      0.6191719  0.08432810
##   0.4   6         0.8                 0.6111111   50      0.6471875  0.10832688
##   0.4   6         0.8                 0.6111111  100      0.6356308  0.09769216
##   0.4   6         0.8                 0.6111111  150      0.6291066  0.09162901
##   0.4   6         0.8                 0.6111111  200      0.6251636  0.08858291
##   0.4   6         0.8                 0.6111111  250      0.6216922  0.08590899
##   0.4   6         0.8                 0.6111111  300      0.6210784  0.08513783
##   0.4   6         0.8                 0.6111111  350      0.6204564  0.08486437
##   0.4   6         0.8                 0.6111111  400      0.6198751  0.08498389
##   0.4   6         0.8                 0.6111111  450      0.6192857  0.08440173
##   0.4   6         0.8                 0.6111111  500      0.6199117  0.08541495
##   0.4   6         0.8                 0.6666667   50      0.6506468  0.11092922
##   0.4   6         0.8                 0.6666667  100      0.6393300  0.09947753
##   0.4   6         0.8                 0.6666667  150      0.6323139  0.09449961
##   0.4   6         0.8                 0.6666667  200      0.6299968  0.09232920
##   0.4   6         0.8                 0.6666667  250      0.6259644  0.08957425
##   0.4   6         0.8                 0.6666667  300      0.6256148  0.08964065
##   0.4   6         0.8                 0.6666667  350      0.6254969  0.08947783
##   0.4   6         0.8                 0.6666667  400      0.6249116  0.08961133
##   0.4   6         0.8                 0.6666667  450      0.6248344  0.08953403
##   0.4   6         0.8                 0.6666667  500      0.6249685  0.08935486
##   0.4   6         0.8                 0.7222222   50      0.6508744  0.11071892
##   0.4   6         0.8                 0.7222222  100      0.6405006  0.10151344
##   0.4   6         0.8                 0.7222222  150      0.6335984  0.09650182
##   0.4   6         0.8                 0.7222222  200      0.6303627  0.09389221
##   0.4   6         0.8                 0.7222222  250      0.6260376  0.09015163
##   0.4   6         0.8                 0.7222222  300      0.6252287  0.08960712
##   0.4   6         0.8                 0.7222222  350      0.6250417  0.08869502
##   0.4   6         0.8                 0.7222222  400      0.6247978  0.08938803
##   0.4   6         0.8                 0.7222222  450      0.6248913  0.08927122
##   0.4   6         0.8                 0.7222222  500      0.6250701  0.08913531
##   0.4   6         0.8                 0.7777778   50      0.6521467  0.11251994
##   0.4   6         0.8                 0.7777778  100      0.6413787  0.10316166
##   0.4   6         0.8                 0.7777778  150      0.6362609  0.09928362
```

```
##   0.4   6         0.8               0.7777778  200      0.6324602  0.09602596
##   0.4   6         0.8               0.7777778  250      0.6316025  0.09562107
##   0.4   6         0.8               0.7777778  300      0.6297205  0.09376212
##   0.4   6         0.8               0.7777778  350      0.6282123  0.09184652
##   0.4   6         0.8               0.7777778  400      0.6280742  0.09181633
##   0.4   6         0.8               0.7777778  450      0.6279319  0.09246372
##   0.4   6         0.8               0.7777778  500      0.6277530  0.09295320
##   0.4   6         0.8               0.8333333   50      0.6539882  0.11374388
##   0.4   6         0.8               0.8333333  100      0.6436429  0.10503819
##   0.4   6         0.8               0.8333333  150      0.6386877  0.09990325
##   0.4   6         0.8               0.8333333  200      0.6348951  0.09671068
##   0.4   6         0.8               0.8333333  250      0.6332366  0.09631703
##   0.4   6         0.8               0.8333333  300      0.6319114  0.09529532
##   0.4   6         0.8               0.8333333  350      0.6316309  0.09451319
##   0.4   6         0.8               0.8333333  400      0.6314399  0.09451930
##   0.4   6         0.8               0.8333333  450      0.6303180  0.09411170
##   0.4   6         0.8               0.8333333  500      0.6305009  0.09403893
##   0.4   6         0.8               0.8888889   50      0.6546751  0.11567619
##   0.4   6         0.8               0.8888889  100      0.6458095  0.10730151
##   0.4   6         0.8               0.8888889  150      0.6400901  0.10214917
##   0.4   6         0.8               0.8888889  200      0.6373381  0.10026866
##   0.4   6         0.8               0.8888889  250      0.6346065  0.09864486
##   0.4   6         0.8               0.8888889  300      0.6326960  0.09730214
##   0.4   6         0.8               0.8888889  350      0.6322814  0.09649130
##   0.4   6         0.8               0.8888889  400      0.6314358  0.09550890
##   0.4   6         0.8               0.8888889  450      0.6308058  0.09535493
##   0.4   6         0.8               0.8888889  500      0.6315497  0.09612452
##   0.4   6         0.8               0.9444444   50      0.6548905  0.11549089
##   0.4   6         0.8               0.9444444  100      0.6479395  0.10858793
##   0.4   6         0.8               0.9444444  150      0.6419925  0.10341808
##   0.4   6         0.8               0.9444444  200      0.6388178  0.10131428
##   0.4   6         0.8               0.9444444  250      0.6357243  0.09882051
##   0.4   6         0.8               0.9444444  300      0.6341919  0.09609050
##   0.4   6         0.8               0.9444444  350      0.6330252  0.09591904
##   0.4   6         0.8               0.9444444  400      0.6326391  0.09604253
##   0.4   6         0.8               0.9444444  450      0.6317936  0.09527359
##   0.4   6         0.8               0.9444444  500      0.6322448  0.09544699
##   0.4   6         0.8               1.0000000   50      0.6540410  0.11471658
##   0.4   6         0.8               1.0000000  100      0.6474558  0.10823632
##   0.4   6         0.8               1.0000000  150      0.6427079  0.10509084
##   0.4   6         0.8               1.0000000  200      0.6393950  0.10230090
##   0.4   6         0.8               1.0000000  250      0.6367650  0.09921033
##   0.4   6         0.8               1.0000000  300      0.6356187  0.09879706
##   0.4   6         0.8               1.0000000  350      0.6348300  0.09786831
##   0.4   6         0.8               1.0000000  400      0.6347609  0.09738261
##   0.4   6         0.8               1.0000000  450      0.6343260  0.09800039
##   0.4   6         0.8               1.0000000  500      0.6349723  0.09805361
##   0.4   7         0.6               0.5000000   50      0.6350048  0.09789134
##   0.4   7         0.6               0.5000000  100      0.6215905  0.08716606
##   0.4   7         0.6               0.5000000  150      0.6168752  0.08235766
##   0.4   7         0.6               0.5000000  200      0.6146151  0.08036869
##   0.4   7         0.6               0.5000000  250      0.6133062  0.07999277
##   0.4   7         0.6               0.5000000  300      0.6133062  0.07964178
##   0.4   7         0.6               0.5000000  350      0.6136842  0.08056983
```

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##   0.4   7         0.6                 0.5000000  400      0.6141883  0.08072792
##   0.4   7         0.6                 0.5000000  450      0.6160135  0.08174412
##   0.4   7         0.6                 0.5000000  500      0.6167858  0.08215178
##   0.4   7         0.6                 0.5555556   50      0.6377162  0.09906255
##   0.4   7         0.6                 0.5555556  100      0.6260173  0.08946207
##   0.4   7         0.6                 0.5555556  150      0.6212288  0.08666080
##   0.4   7         0.6                 0.5555556  200      0.6180622  0.08321052
##   0.4   7         0.6                 0.5555556  250      0.6189808  0.08364339
##   0.4   7         0.6                 0.5555556  300      0.6186272  0.08387375
##   0.4   7         0.6                 0.5555556  350      0.6197897  0.08448992
##   0.4   7         0.6                 0.5555556  400      0.6205702  0.08459220
##   0.4   7         0.6                 0.5555556  450      0.6209239  0.08493643
##   0.4   7         0.6                 0.5555556  500      0.6217328  0.08625110
##   0.4   7         0.6                 0.6111111   50      0.6393990  0.10195607
##   0.4   7         0.6                 0.6111111  100      0.6276311  0.09256629
##   0.4   7         0.6                 0.6111111  150      0.6240295  0.08843878
##   0.4   7         0.6                 0.6111111  200      0.6213263  0.08680392
##   0.4   7         0.6                 0.6111111  250      0.6212044  0.08656428
##   0.4   7         0.6                 0.6111111  300      0.6206353  0.08683512
##   0.4   7         0.6                 0.6111111  350      0.6219035  0.08726467
##   0.4   7         0.6                 0.6111111  400      0.6217166  0.08780314
##   0.4   7         0.6                 0.6111111  450      0.6233669  0.08807834
##   0.4   7         0.6                 0.6111111  500      0.6236637  0.08856363
##   0.4   7         0.6                 0.6666667   50      0.6408990  0.10155517
##   0.4   7         0.6                 0.6666667  100      0.6305212  0.09279550
##   0.4   7         0.6                 0.6666667  150      0.6257571  0.08913103
##   0.4   7         0.6                 0.6666667  200      0.6236068  0.08836276
##   0.4   7         0.6                 0.6666667  250      0.6224076  0.08657688
##   0.4   7         0.6                 0.6666667  300      0.6227287  0.08665497
##   0.4   7         0.6                 0.6666667  350      0.6231555  0.08726324
##   0.4   7         0.6                 0.6666667  400      0.6242937  0.08809266
##   0.4   7         0.6                 0.6666667  450      0.6251799  0.08850292
##   0.4   7         0.6                 0.6666667  500      0.6261189  0.08936011
##   0.4   7         0.6                 0.7222222   50      0.6439030  0.10473307
##   0.4   7         0.6                 0.7222222  100      0.6337813  0.09609127
##   0.4   7         0.6                 0.7222222  150      0.6306554  0.09335279
##   0.4   7         0.6                 0.7222222  200      0.6284278  0.09122934
##   0.4   7         0.6                 0.7222222  250      0.6270863  0.09036623
##   0.4   7         0.6                 0.7222222  300      0.6271839  0.09068964
##   0.4   7         0.6                 0.7222222  350      0.6276392  0.09077595
##   0.4   7         0.6                 0.7222222  400      0.6275172  0.09100780
##   0.4   7         0.6                 0.7222222  450      0.6280375  0.09141632
##   0.4   7         0.6                 0.7222222  500      0.6297286  0.09382050
##   0.4   7         0.6                 0.7777778   50      0.6461834  0.10723625
##   0.4   7         0.6                 0.7777778  100      0.6365576  0.09947969
##   0.4   7         0.6                 0.7777778  150      0.6322407  0.09680532
##   0.4   7         0.6                 0.7777778  200      0.6290538  0.09367870
##   0.4   7         0.6                 0.7777778  250      0.6285172  0.09335079
##   0.4   7         0.6                 0.7777778  300      0.6280253  0.09332361
##   0.4   7         0.6                 0.7777778  350      0.6284481  0.09351083
##   0.4   7         0.6                 0.7777778  400      0.6289115  0.09393363
##   0.4   7         0.6                 0.7777778  450      0.6296391  0.09393467
##   0.4   7         0.6                 0.7777778  500      0.6307976  0.09513788
##   0.4   7         0.6                 0.8333333   50      0.6469802  0.10897336
```

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##   0.4   7         0.6               0.8333333  100      0.6379479  0.10160259
##   0.4   7         0.6               0.8333333  150      0.6344561  0.09800391
##   0.4   7         0.6               0.8333333  200      0.6318058  0.09598682
##   0.4   7         0.6               0.8333333  250      0.6313708  0.09593508
##   0.4   7         0.6               0.8333333  300      0.6317651  0.09603106
##   0.4   7         0.6               0.8333333  350      0.6318139  0.09667780
##   0.4   7         0.6               0.8333333  400      0.6322488  0.09694202
##   0.4   7         0.6               0.8333333  450      0.6328992  0.09805932
##   0.4   7         0.6               0.8333333  500      0.6331634  0.09775426
##   0.4   7         0.6               0.8888889   50      0.6482891  0.10998738
##   0.4   7         0.6               0.8888889  100      0.6407324  0.10296330
##   0.4   7         0.6               0.8888889  150      0.6360089  0.09905812
##   0.4   7         0.6               0.8888889  200      0.6343585  0.09794557
##   0.4   7         0.6               0.8888889  250      0.6328504  0.09613940
##   0.4   7         0.6               0.8888889  300      0.6333464  0.09649347
##   0.4   7         0.6               0.8888889  350      0.6332976  0.09627956
##   0.4   7         0.6               0.8888889  400      0.6333789  0.09660233
##   0.4   7         0.6               0.8888889  450      0.6341268  0.09694360
##   0.4   7         0.6               0.8888889  500      0.6345496  0.09769835
##   0.4   7         0.6               0.9444444   50      0.6493500  0.11025401
##   0.4   7         0.6               0.9444444  100      0.6418136  0.10311630
##   0.4   7         0.6               0.9444444  150      0.6379560  0.09992755
##   0.4   7         0.6               0.9444444  200      0.6352162  0.09780680
##   0.4   7         0.6               0.9444444  250      0.6344276  0.09631374
##   0.4   7         0.6               0.9444444  300      0.6339601  0.09688766
##   0.4   7         0.6               0.9444444  350      0.6343422  0.09760359
##   0.4   7         0.6               0.9444444  400      0.6345780  0.09793446
##   0.4   7         0.6               0.9444444  450      0.6354317  0.09974200
##   0.4   7         0.6               0.9444444  500      0.6356308  0.09990360
##   0.4   7         0.6               1.0000000   50      0.6505248  0.11206145
##   0.4   7         0.6               1.0000000  100      0.6429396  0.10428843
##   0.4   7         0.6               1.0000000  150      0.6375170  0.10004265
##   0.4   7         0.6               1.0000000  200      0.6348179  0.09786957
##   0.4   7         0.6               1.0000000  250      0.6346634  0.09693836
##   0.4   7         0.6               1.0000000  300      0.6344032  0.09721948
##   0.4   7         0.6               1.0000000  350      0.6342325  0.09732805
##   0.4   7         0.6               1.0000000  400      0.6349967  0.09816673
##   0.4   7         0.6               1.0000000  450      0.6360211  0.09922548
##   0.4   7         0.6               1.0000000  500      0.6361715  0.09955089
##   0.4   7         0.8               0.5000000   50      0.6333382  0.09600730
##   0.4   7         0.8               0.5000000  100      0.6229239  0.08754000
##   0.4   7         0.8               0.5000000  150      0.6175703  0.08306654
##   0.4   7         0.8               0.5000000  200      0.6145298  0.08085530
##   0.4   7         0.8               0.5000000  250      0.6145419  0.08050791
##   0.4   7         0.8               0.5000000  300      0.6140541  0.08087409
##   0.4   7         0.8               0.5000000  350      0.6147249  0.08230614
##   0.4   7         0.8               0.5000000  400      0.6163630  0.08322076
##   0.4   7         0.8               0.5000000  450      0.6170866  0.08332927
##   0.4   7         0.8               0.5000000  500      0.6174281  0.08451520
##   0.4   7         0.8               0.5555556   50      0.6371267  0.09775097
##   0.4   7         0.8               0.5555556  100      0.6245214  0.08789447
##   0.4   7         0.8               0.5555556  150      0.6206881  0.08483445
##   0.4   7         0.8               0.5555556  200      0.6186150  0.08330840
##   0.4   7         0.8               0.5555556  250      0.6177939  0.08305539
```

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##   0.4   7         0.8               0.5555556  300      0.6183305  0.08342292
##   0.4   7         0.8               0.5555556  350      0.6187451  0.08392106
##   0.4   7         0.8               0.5555556  400      0.6195621  0.08432228
##   0.4   7         0.8               0.5555556  450      0.6208995  0.08605237
##   0.4   7         0.8               0.5555556  500      0.6211353  0.08641478
##   0.4   7         0.8               0.6111111   50      0.6389682  0.10042097
##   0.4   7         0.8               0.6111111  100      0.6282327  0.09045790
##   0.4   7         0.8               0.6111111  150      0.6233100  0.08729618
##   0.4   7         0.8               0.6111111  200      0.6214198  0.08592341
##   0.4   7         0.8               0.6111111  250      0.6212491  0.08605470
##   0.4   7         0.8               0.6111111  300      0.6220255  0.08714772
##   0.4   7         0.8               0.6111111  350      0.6232734  0.08860588
##   0.4   7         0.8               0.6111111  400      0.6248100  0.08884212
##   0.4   7         0.8               0.6111111  450      0.6251433  0.08937363
##   0.4   7         0.8               0.6111111  500      0.6261433  0.09063740
##   0.4   7         0.8               0.6666667   50      0.6410860  0.10285894
##   0.4   7         0.8               0.6666667  100      0.6313098  0.09494429
##   0.4   7         0.8               0.6666667  150      0.6254400  0.08941873
##   0.4   7         0.8               0.6666667  200      0.6248506  0.08848348
##   0.4   7         0.8               0.6666667  250      0.6243710  0.08839538
##   0.4   7         0.8               0.6666667  300      0.6240092  0.08928071
##   0.4   7         0.8               0.6666667  350      0.6252409  0.08927956
##   0.4   7         0.8               0.6666667  400      0.6254644  0.08942388
##   0.4   7         0.8               0.6666667  450      0.6269278  0.09069806
##   0.4   7         0.8               0.6666667  500      0.6270782  0.09092004
##   0.4   7         0.8               0.7222222   50      0.6421632  0.10354292
##   0.4   7         0.8               0.7222222  100      0.6328708  0.09618252
##   0.4   7         0.8               0.7222222  150      0.6294806  0.09311225
##   0.4   7         0.8               0.7222222  200      0.6270822  0.09196570
##   0.4   7         0.8               0.7222222  250      0.6264725  0.09175662
##   0.4   7         0.8               0.7222222  300      0.6280375  0.09213726
##   0.4   7         0.8               0.7222222  350      0.6287367  0.09220462
##   0.4   7         0.8               0.7222222  400      0.6285213  0.09306879
##   0.4   7         0.8               0.7222222  450      0.6287408  0.09255595
##   0.4   7         0.8               0.7222222  500      0.6302286  0.09410884
##   0.4   7         0.8               0.7777778   50      0.6441876  0.10605464
##   0.4   7         0.8               0.7777778  100      0.6357406  0.09827930
##   0.4   7         0.8               0.7777778  150      0.6310700  0.09512262
##   0.4   7         0.8               0.7777778  200      0.6290132  0.09336361
##   0.4   7         0.8               0.7777778  250      0.6288668  0.09319002
##   0.4   7         0.8               0.7777778  300      0.6277408  0.09226998
##   0.4   7         0.8               0.7777778  350      0.6289156  0.09294812
##   0.4   7         0.8               0.7777778  400      0.6291473  0.09317672
##   0.4   7         0.8               0.7777778  450      0.6293343  0.09358296
##   0.4   7         0.8               0.7777778  500      0.6304643  0.09434556
##   0.4   7         0.8               0.8333333   50      0.6455859  0.10783702
##   0.4   7         0.8               0.8333333  100      0.6372934  0.10009318
##   0.4   7         0.8               0.8333333  150      0.6324643  0.09661999
##   0.4   7         0.8               0.8333333  200      0.6312001  0.09582166
##   0.4   7         0.8               0.8333333  250      0.6304684  0.09650885
##   0.4   7         0.8               0.8333333  300      0.6311635  0.09745929
##   0.4   7         0.8               0.8333333  350      0.6314683  0.09636920
##   0.4   7         0.8               0.8333333  400      0.6320171  0.09720968
##   0.4   7         0.8               0.8333333  450      0.6329114  0.09820145
```

```
##   0.4   7         0.8               0.8333333  500      0.6328748  0.09808294
##   0.4   7         0.8               0.8888889   50      0.6448176  0.10656079
##   0.4   7         0.8               0.8888889  100      0.6366430  0.10030543
##   0.4   7         0.8               0.8888889  150      0.6332894  0.09731555
##   0.4   7         0.8               0.8888889  200      0.6315781  0.09651039
##   0.4   7         0.8               0.8888889  250      0.6311757  0.09495297
##   0.4   7         0.8               0.8888889  300      0.6315740  0.09612945
##   0.4   7         0.8               0.8888889  350      0.6318626  0.09657967
##   0.4   7         0.8               0.8888889  400      0.6324480  0.09712316
##   0.4   7         0.8               0.8888889  450      0.6332935  0.09797263
##   0.4   7         0.8               0.8888889  500      0.6332854  0.09798286
##   0.4   7         0.8               0.9444444   50      0.6480899  0.10886686
##   0.4   7         0.8               0.9444444  100      0.6398177  0.10167451
##   0.4   7         0.8               0.9444444  150      0.6352812  0.09725778
##   0.4   7         0.8               0.9444444  200      0.6345048  0.09614724
##   0.4   7         0.8               0.9444444  250      0.6324032  0.09550372
##   0.4   7         0.8               0.9444444  300      0.6328463  0.09584965
##   0.4   7         0.8               0.9444444  350      0.6336227  0.09774278
##   0.4   7         0.8               0.9444444  400      0.6331349  0.09706811
##   0.4   7         0.8               0.9444444  450      0.6338341  0.09718725
##   0.4   7         0.8               0.9444444  500      0.6341349  0.09749755
##   0.4   7         0.8               1.0000000   50      0.6489476  0.10975933
##   0.4   7         0.8               1.0000000  100      0.6419640  0.10406842
##   0.4   7         0.8               1.0000000  150      0.6385170  0.10156701
##   0.4   7         0.8               1.0000000  200      0.6357934  0.09874959
##   0.4   7         0.8               1.0000000  250      0.6344480  0.09808261
##   0.4   7         0.8               1.0000000  300      0.6342203  0.09864318
##   0.4   7         0.8               1.0000000  350      0.6346675  0.09847706
##   0.4   7         0.8               1.0000000  400      0.6351797  0.09911330
##   0.4   7         0.8               1.0000000  450      0.6350984  0.09949498
##   0.4   7         0.8               1.0000000  500      0.6351675  0.09997590
##   0.4   8         0.6               0.5000000   50      0.6298302  0.09226597
##   0.4   8         0.6               0.5000000  100      0.6182248  0.08426329
##   0.4   8         0.6               0.5000000  150      0.6144362  0.08146885
##   0.4   8         0.6               0.5000000  200      0.6142289  0.08077122
##   0.4   8         0.6               0.5000000  250      0.6153509  0.08210381
##   0.4   8         0.6               0.5000000  300      0.6167289  0.08261002
##   0.4   8         0.6               0.5000000  350      0.6180134  0.08433920
##   0.4   8         0.6               0.5000000  400      0.6191272  0.08585647
##   0.4   8         0.6               0.5000000  450      0.6197654  0.08566391
##   0.4   8         0.6               0.5000000  500      0.6208426  0.08593799
##   0.4   8         0.6               0.5555556   50      0.6310659  0.09316284
##   0.4   8         0.6               0.5555556  100      0.6226068  0.08611183
##   0.4   8         0.6               0.5555556  150      0.6198304  0.08442859
##   0.4   8         0.6               0.5555556  200      0.6200093  0.08445240
##   0.4   8         0.6               0.5555556  250      0.6209971  0.08568771
##   0.4   8         0.6               0.5555556  300      0.6218954  0.08582671
##   0.4   8         0.6               0.5555556  350      0.6228832  0.08764367
##   0.4   8         0.6               0.5555556  400      0.6238994  0.08779374
##   0.4   8         0.6               0.5555556  450      0.6251962  0.08883195
##   0.4   8         0.6               0.5555556  500      0.6260010  0.08985973
##   0.4   8         0.6               0.6111111   50      0.6309643  0.09396691
##   0.4   8         0.6               0.6111111  100      0.6239075  0.08820747
##   0.4   8         0.6               0.6111111  150      0.6208670  0.08511882
```

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##   0.4   8         0.6               0.6111111  200      0.6213344  0.08608217
##   0.4   8         0.6               0.6111111  250      0.6226230  0.08675250
##   0.4   8         0.6               0.6111111  300      0.6233547  0.08732318
##   0.4   8         0.6               0.6111111  350      0.6244116  0.08812592
##   0.4   8         0.6               0.6111111  400      0.6259929  0.08956229
##   0.4   8         0.6               0.6111111  450      0.6268180  0.09077683
##   0.4   8         0.6               0.6111111  500      0.6283912  0.09169339
##   0.4   8         0.6               0.6666667   50      0.6337894  0.09720939
##   0.4   8         0.6               0.6666667  100      0.6267530  0.09237114
##   0.4   8         0.6               0.6666667  150      0.6244848  0.08996560
##   0.4   8         0.6               0.6666667  200      0.6234848  0.08933910
##   0.4   8         0.6               0.6666667  250      0.6246840  0.08971066
##   0.4   8         0.6               0.6666667  300      0.6261677  0.09046629
##   0.4   8         0.6               0.6666667  350      0.6270457  0.09236630
##   0.4   8         0.6               0.6666667  400      0.6280294  0.09304203
##   0.4   8         0.6               0.6666667  450      0.6290823  0.09373091
##   0.4   8         0.6               0.6666667  500      0.6302448  0.09420333
##   0.4   8         0.6               0.7222222   50      0.6359276  0.09987629
##   0.4   8         0.6               0.7222222  100      0.6287733  0.09371305
##   0.4   8         0.6               0.7222222  150      0.6268425  0.09135252
##   0.4   8         0.6               0.7222222  200      0.6270701  0.09174823
##   0.4   8         0.6               0.7222222  250      0.6283709  0.09362144
##   0.4   8         0.6               0.7222222  300      0.6285579  0.09340812
##   0.4   8         0.6               0.7222222  350      0.6296188  0.09464998
##   0.4   8         0.6               0.7222222  400      0.6300863  0.09435511
##   0.4   8         0.6               0.7222222  450      0.6309318  0.09498270
##   0.4   8         0.6               0.7222222  500      0.6306554  0.09461422
##   0.4   8         0.6               0.7777778   50      0.6384804  0.10161326
##   0.4   8         0.6               0.7777778  100      0.6305293  0.09437991
##   0.4   8         0.6               0.7777778  150      0.6289196  0.09321000
##   0.4   8         0.6               0.7777778  200      0.6293912  0.09401965
##   0.4   8         0.6               0.7777778  250      0.6302936  0.09508211
##   0.4   8         0.6               0.7777778  300      0.6313992  0.09605682
##   0.4   8         0.6               0.7777778  350      0.6327895  0.09788366
##   0.4   8         0.6               0.7777778  400      0.6333545  0.09747622
##   0.4   8         0.6               0.7777778  450      0.6338504  0.09808442
##   0.4   8         0.6               0.7777778  500      0.6334358  0.09731814
##   0.4   8         0.6               0.8333333   50      0.6422079  0.10361039
##   0.4   8         0.6               0.8333333  100      0.6330577  0.09591509
##   0.4   8         0.6               0.8333333  150      0.6309277  0.09468137
##   0.4   8         0.6               0.8333333  200      0.6312285  0.09571453
##   0.4   8         0.6               0.8333333  250      0.6315862  0.09559273
##   0.4   8         0.6               0.8333333  300      0.6329399  0.09751252
##   0.4   8         0.6               0.8333333  350      0.6334927  0.09837835
##   0.4   8         0.6               0.8333333  400      0.6343301  0.09846519
##   0.4   8         0.6               0.8333333  450      0.6356065  0.09990704
##   0.4   8         0.6               0.8333333  500      0.6354967  0.10001001
##   0.4   8         0.6               0.8888889   50      0.6425047  0.10523487
##   0.4   8         0.6               0.8888889  100      0.6345618  0.09891130
##   0.4   8         0.6               0.8888889  150      0.6327407  0.09666014
##   0.4   8         0.6               0.8888889  200      0.6326757  0.09587396
##   0.4   8         0.6               0.8888889  250      0.6323830  0.09499842
##   0.4   8         0.6               0.8888889  300      0.6338423  0.09742204
##   0.4   8         0.6               0.8888889  350      0.6344236  0.09750558
```

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##   0.4   8         0.6                 0.8888889  400      0.6353544  0.09835690
##   0.4   8         0.6                 0.8888889  450      0.6363300  0.09896875
##   0.4   8         0.6                 0.8888889  500      0.6365861  0.09901681
##   0.4   8         0.6                 0.9444444   50      0.6433786  0.10564306
##   0.4   8         0.6                 0.9444444  100      0.6367202  0.10108783
##   0.4   8         0.6                 0.9444444  150      0.6342284  0.09872333
##   0.4   8         0.6                 0.9444444  200      0.6341919  0.09866334
##   0.4   8         0.6                 0.9444444  250      0.6341553  0.09865725
##   0.4   8         0.6                 0.9444444  300      0.6356349  0.09973827
##   0.4   8         0.6                 0.9444444  350      0.6363951  0.10073259
##   0.4   8         0.6                 0.9444444  400      0.6359560  0.10067356
##   0.4   8         0.6                 0.9444444  450      0.6370170  0.10159798
##   0.4   8         0.6                 0.9444444  500      0.6370617  0.10150576
##   0.4   8         0.6                 1.0000000   50      0.6430290  0.10429624
##   0.4   8         0.6                 1.0000000  100      0.6379682  0.09937409
##   0.4   8         0.6                 1.0000000  150      0.6361634  0.09811875
##   0.4   8         0.6                 1.0000000  200      0.6357569  0.09782111
##   0.4   8         0.6                 1.0000000  250      0.6357122  0.09831188
##   0.4   8         0.6                 1.0000000  300      0.6362528  0.09940813
##   0.4   8         0.6                 1.0000000  350      0.6366187  0.09923160
##   0.4   8         0.6                 1.0000000  400      0.6370618  0.10032670
##   0.4   8         0.6                 1.0000000  450      0.6379398  0.10076843
##   0.4   8         0.6                 1.0000000  500      0.6387609  0.10139050
##   0.4   8         0.8                 0.5000000   50      0.6251189  0.09035067
##   0.4   8         0.8                 0.5000000  100      0.6170297  0.08234446
##   0.4   8         0.8                 0.5000000  150      0.6149850  0.08115340
##   0.4   8         0.8                 0.5000000  200      0.6152045  0.08157719
##   0.4   8         0.8                 0.5000000  250      0.6157289  0.08307893
##   0.4   8         0.8                 0.5000000  300      0.6170215  0.08509384
##   0.4   8         0.8                 0.5000000  350      0.6190662  0.08646884
##   0.4   8         0.8                 0.5000000  400      0.6204076  0.08685844
##   0.4   8         0.8                 0.5000000  450      0.6206800  0.08756762
##   0.4   8         0.8                 0.5000000  500      0.6218060  0.08829858
##   0.4   8         0.8                 0.5555556   50      0.6284115  0.09123727
##   0.4   8         0.8                 0.5555556  100      0.6206597  0.08627046
##   0.4   8         0.8                 0.5555556  150      0.6195784  0.08535008
##   0.4   8         0.8                 0.5555556  200      0.6192898  0.08423635
##   0.4   8         0.8                 0.5555556  250      0.6205825  0.08635153
##   0.4   8         0.8                 0.5555556  300      0.6216800  0.08689591
##   0.4   8         0.8                 0.5555556  350      0.6222247  0.08697366
##   0.4   8         0.8                 0.5555556  400      0.6238954  0.08849749
##   0.4   8         0.8                 0.5555556  450      0.6248385  0.09001742
##   0.4   8         0.8                 0.5555556  500      0.6252206  0.08985426
##   0.4   8         0.8                 0.6111111   50      0.6306513  0.09509333
##   0.4   8         0.8                 0.6111111  100      0.6228141  0.08900560
##   0.4   8         0.8                 0.6111111  150      0.6208466  0.08598848
##   0.4   8         0.8                 0.6111111  200      0.6216800  0.08695593
##   0.4   8         0.8                 0.6111111  250      0.6232450  0.08847000
##   0.4   8         0.8                 0.6111111  300      0.6248872  0.08966306
##   0.4   8         0.8                 0.6111111  350      0.6255579  0.09104001
##   0.4   8         0.8                 0.6111111  400      0.6261961  0.09098201
##   0.4   8         0.8                 0.6111111  450      0.6276351  0.09257442
##   0.4   8         0.8                 0.6111111  500      0.6278505  0.09275982
##   0.4   8         0.8                 0.6666667   50      0.6342650  0.09806476
```

```
##   0.4   8         0.8               0.6666667  100      0.6254481  0.09115087
##   0.4   8         0.8               0.6666667  150      0.6221393  0.08748253
##   0.4   8         0.8               0.6666667  200      0.6237897  0.08969428
##   0.4   8         0.8               0.6666667  250      0.6239441  0.08948236
##   0.4   8         0.8               0.6666667  300      0.6254157  0.09091667
##   0.4   8         0.8               0.6666667  350      0.6264603  0.09297820
##   0.4   8         0.8               0.6666667  400      0.6274766  0.09410458
##   0.4   8         0.8               0.6666667  450      0.6285213  0.09463551
##   0.4   8         0.8               0.6666667  500      0.6295578  0.09517097
##   0.4   8         0.8               0.7222222   50      0.6377650  0.10031516
##   0.4   8         0.8               0.7222222  100      0.6294115  0.09348229
##   0.4   8         0.8               0.7222222  150      0.6267164  0.09129753
##   0.4   8         0.8               0.7222222  200      0.6280579  0.09285021
##   0.4   8         0.8               0.7222222  250      0.6294806  0.09337150
##   0.4   8         0.8               0.7222222  300      0.6305294  0.09378478
##   0.4   8         0.8               0.7222222  350      0.6304399  0.09355822
##   0.4   8         0.8               0.7222222  400      0.6307732  0.09440141
##   0.4   8         0.8               0.7222222  450      0.6322895  0.09545427
##   0.4   8         0.8               0.7222222  500      0.6324602  0.09591254
##   0.4   8         0.8               0.7777778   50      0.6377609  0.09958190
##   0.4   8         0.8               0.7777778  100      0.6302651  0.09354961
##   0.4   8         0.8               0.7777778  150      0.6288546  0.09283292
##   0.4   8         0.8               0.7777778  200      0.6294522  0.09195940
##   0.4   8         0.8               0.7777778  250      0.6305538  0.09330784
##   0.4   8         0.8               0.7777778  300      0.6311025  0.09428428
##   0.4   8         0.8               0.7777778  350      0.6315375  0.09496495
##   0.4   8         0.8               0.7777778  400      0.6326838  0.09585466
##   0.4   8         0.8               0.7777778  450      0.6342081  0.09723189
##   0.4   8         0.8               0.7777778  500      0.6340089  0.09704953
##   0.4   8         0.8               0.8333333   50      0.6398950  0.10199006
##   0.4   8         0.8               0.8333333  100      0.6330537  0.09662604
##   0.4   8         0.8               0.8333333  150      0.6310497  0.09493692
##   0.4   8         0.8               0.8333333  200      0.6311431  0.09446018
##   0.4   8         0.8               0.8333333  250      0.6324277  0.09488350
##   0.4   8         0.8               0.8333333  300      0.6333382  0.09682649
##   0.4   8         0.8               0.8333333  350      0.6342691  0.09734991
##   0.4   8         0.8               0.8333333  400      0.6352528  0.09808111
##   0.4   8         0.8               0.8333333  450      0.6359276  0.09924025
##   0.4   8         0.8               0.8333333  500      0.6362040  0.09972075
##   0.4   8         0.8               0.8888889   50      0.6413787  0.10379838
##   0.4   8         0.8               0.8888889  100      0.6336269  0.09605561
##   0.4   8         0.8               0.8888889  150      0.6324643  0.09622285
##   0.4   8         0.8               0.8888889  200      0.6325944  0.09600918
##   0.4   8         0.8               0.8888889  250      0.6330090  0.09665403
##   0.4   8         0.8               0.8888889  300      0.6338301  0.09782095
##   0.4   8         0.8               0.8888889  350      0.6348016  0.09899386
##   0.4   8         0.8               0.8888889  400      0.6354113  0.09981891
##   0.4   8         0.8               0.8888889  450      0.6362650  0.10011697
##   0.4   8         0.8               0.8888889  500      0.6364926  0.10024806
##   0.4   8         0.8               0.9444444   50      0.6435575  0.10569426
##   0.4   8         0.8               0.9444444  100      0.6363463  0.09868719
##   0.4   8         0.8               0.9444444  150      0.6344561  0.09704840
##   0.4   8         0.8               0.9444444  200      0.6335130  0.09739365
##   0.4   8         0.8               0.9444444  250      0.6346471  0.09852131
```

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##   0.4   8         0.8               0.9444444  300      0.6357691  0.09990827
##   0.4   8         0.8               0.9444444  350      0.6368097  0.10082613
##   0.4   8         0.8               0.9444444  400      0.6373544  0.10167374
##   0.4   8         0.8               0.9444444  450      0.6377690  0.10153287
##   0.4   8         0.8               0.9444444  500      0.6389194  0.10265003
##   0.4   8         0.8               1.0000000   50      0.6433136  0.10684900
##   0.4   8         0.8               1.0000000  100      0.6377365  0.10214762
##   0.4   8         0.8               1.0000000  150      0.6356552  0.10068199
##   0.4   8         0.8               1.0000000  200      0.6348382  0.09963997
##   0.4   8         0.8               1.0000000  250      0.6354439  0.09997727
##   0.4   8         0.8               1.0000000  300      0.6357528  0.09976636
##   0.4   8         0.8               1.0000000  350      0.6373300  0.10102042
##   0.4   8         0.8               1.0000000  400      0.6379885  0.10220356
##   0.4   8         0.8               1.0000000  450      0.6375658  0.10198837
##   0.4   8         0.8               1.0000000  500      0.6381715  0.10259021
##   0.4   9         0.6               0.5000000   50      0.6205336  0.08424387
##   0.4   9         0.6               0.5000000  100      0.6140907  0.07939325
##   0.4   9         0.6               0.5000000  150      0.6128713  0.07852409
##   0.4   9         0.6               0.5000000  200      0.6155541  0.08092782
##   0.4   9         0.6               0.5000000  250      0.6183305  0.08310097
##   0.4   9         0.6               0.5000000  300      0.6189117  0.08399230
##   0.4   9         0.6               0.5000000  350      0.6215458  0.08619506
##   0.4   9         0.6               0.5000000  400      0.6229035  0.08708438
##   0.4   9         0.6               0.5000000  450      0.6232653  0.08708660
##   0.4   9         0.6               0.5000000  500      0.6226027  0.08649133
##   0.4   9         0.6               0.5555556   50      0.6240173  0.08665545
##   0.4   9         0.6               0.5555556  100      0.6193345  0.08336136
##   0.4   9         0.6               0.5555556  150      0.6182694  0.08300225
##   0.4   9         0.6               0.5555556  200      0.6210905  0.08472698
##   0.4   9         0.6               0.5555556  250      0.6232734  0.08857722
##   0.4   9         0.6               0.5555556  300      0.6252408  0.08921459
##   0.4   9         0.6               0.5555556  350      0.6259197  0.09048479
##   0.4   9         0.6               0.5555556  400      0.6262652  0.09045475
##   0.4   9         0.6               0.5555556  450      0.6269644  0.09112762
##   0.4   9         0.6               0.5555556  500      0.6279562  0.09167341
##   0.4   9         0.6               0.6111111   50      0.6253506  0.09100684
##   0.4   9         0.6               0.6111111  100      0.6211841  0.08763519
##   0.4   9         0.6               0.6111111  150      0.6227978  0.08923207
##   0.4   9         0.6               0.6111111  200      0.6246230  0.08979677
##   0.4   9         0.6               0.6111111  250      0.6256351  0.09098493
##   0.4   9         0.6               0.6111111  300      0.6274115  0.09198247
##   0.4   9         0.6               0.6111111  350      0.6286473  0.09412338
##   0.4   9         0.6               0.6111111  400      0.6293464  0.09447003
##   0.4   9         0.6               0.6111111  450      0.6298017  0.09460102
##   0.4   9         0.6               0.6111111  500      0.6308302  0.09562476
##   0.4   9         0.6               0.6666667   50      0.6286757  0.09218953
##   0.4   9         0.6               0.6666667  100      0.6240661  0.08756207
##   0.4   9         0.6               0.6666667  150      0.6235295  0.08663242
##   0.4   9         0.6               0.6666667  200      0.6266880  0.09033915
##   0.4   9         0.6               0.6666667  250      0.6281066  0.09120454
##   0.4   9         0.6               0.6666667  300      0.6297529  0.09312944
##   0.4   9         0.6               0.6666667  350      0.6316513  0.09441835
##   0.4   9         0.6               0.6666667  400      0.6319480  0.09444327
##   0.4   9         0.6               0.6666667  450      0.6328992  0.09471359
```

```
##   0.4   9         0.6               0.6666667  500      0.6340171  0.09661392
##   0.4   9         0.6               0.7222222   50      0.6308749  0.09607557
##   0.4   9         0.6               0.7222222  100      0.6259156  0.09017210
##   0.4   9         0.6               0.7222222  150      0.6257327  0.09179004
##   0.4   9         0.6               0.7222222  200      0.6272042  0.09355982
##   0.4   9         0.6               0.7222222  250      0.6294034  0.09448758
##   0.4   9         0.6               0.7222222  300      0.6309765  0.09648000
##   0.4   9         0.6               0.7222222  350      0.6310212  0.09652441
##   0.4   9         0.6               0.7222222  400      0.6314236  0.09635506
##   0.4   9         0.6               0.7222222  450      0.6324602  0.09761853
##   0.4   9         0.6               0.7222222  500      0.6325781  0.09729931
##   0.4   9         0.6               0.7777778   50      0.6318017  0.09560963
##   0.4   9         0.6               0.7777778  100      0.6290497  0.09334690
##   0.4   9         0.6               0.7777778  150      0.6288749  0.09442533
##   0.4   9         0.6               0.7777778  200      0.6301025  0.09534507
##   0.4   9         0.6               0.7777778  250      0.6322895  0.09740980
##   0.4   9         0.6               0.7777778  300      0.6335862  0.09861572
##   0.4   9         0.6               0.7777778  350      0.6343992  0.09931596
##   0.4   9         0.6               0.7777778  400      0.6349561  0.10012187
##   0.4   9         0.6               0.7777778  450      0.6357243  0.09964319
##   0.4   9         0.6               0.7777778  500      0.6358869  0.10039721
##   0.4   9         0.6               0.8333333   50      0.6355049  0.09984145
##   0.4   9         0.6               0.8333333  100      0.6317163  0.09592842
##   0.4   9         0.6               0.8333333  150      0.6318749  0.09661399
##   0.4   9         0.6               0.8333333  200      0.6330700  0.09766740
##   0.4   9         0.6               0.8333333  250      0.6339683  0.09833676
##   0.4   9         0.6               0.8333333  300      0.6349967  0.09977744
##   0.4   9         0.6               0.8333333  350      0.6357772  0.10010816
##   0.4   9         0.6               0.8333333  400      0.6364439  0.10051037
##   0.4   9         0.6               0.8333333  450      0.6374357  0.10161400
##   0.4   9         0.6               0.8333333  500      0.6377853  0.10192310
##   0.4   9         0.6               0.8888889   50      0.6362690  0.09980893
##   0.4   9         0.6               0.8888889  100      0.6317082  0.09498027
##   0.4   9         0.6               0.8888889  150      0.6322854  0.09602948
##   0.4   9         0.6               0.8888889  200      0.6332691  0.09671590
##   0.4   9         0.6               0.8888889  250      0.6344845  0.09745128
##   0.4   9         0.6               0.8888889  300      0.6350130  0.09753195
##   0.4   9         0.6               0.8888889  350      0.6356065  0.09765778
##   0.4   9         0.6               0.8888889  400      0.6361918  0.09861812
##   0.4   9         0.6               0.8888889  450      0.6369032  0.09947394
##   0.4   9         0.6               0.8888889  500      0.6371105  0.09978700
##   0.4   9         0.6               0.9444444   50      0.6386023  0.10004636
##   0.4   9         0.6               0.9444444  100      0.6337163  0.09588300
##   0.4   9         0.6               0.9444444  150      0.6338789  0.09660644
##   0.4   9         0.6               0.9444444  200      0.6345130  0.09674830
##   0.4   9         0.6               0.9444444  250      0.6361796  0.09819641
##   0.4   9         0.6               0.9444444  300      0.6375170  0.09919740
##   0.4   9         0.6               0.9444444  350      0.6379438  0.09962652
##   0.4   9         0.6               0.9444444  400      0.6383341  0.09994519
##   0.4   9         0.6               0.9444444  450      0.6390901  0.10109978
##   0.4   9         0.6               0.9444444  500      0.6394600  0.10199792
##   0.4   9         0.6               1.0000000   50      0.6395901  0.10295294
##   0.4   9         0.6               1.0000000  100      0.6355008  0.09920839
##   0.4   9         0.6               1.0000000  150      0.6355333  0.09846501
```

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##   0.4   9         0.6               1.0000000  200      0.6364804  0.09969422
##   0.4   9         0.6               1.0000000  250      0.6370292  0.09962516
##   0.4   9         0.6               1.0000000  300      0.6376755  0.10090363
##   0.4   9         0.6               1.0000000  350      0.6386105  0.10145863
##   0.4   9         0.6               1.0000000  400      0.6399885  0.10241129
##   0.4   9         0.6               1.0000000  450      0.6408502  0.10317144
##   0.4   9         0.6               1.0000000  500      0.6407364  0.10307180
##   0.4   9         0.8               0.5000000   50      0.6187126  0.08499979
##   0.4   9         0.8               0.5000000  100      0.6135867  0.08056191
##   0.4   9         0.8               0.5000000  150      0.6136598  0.08095123
##   0.4   9         0.8               0.5000000  200      0.6161395  0.08354538
##   0.4   9         0.8               0.5000000  250      0.6176232  0.08524830
##   0.4   9         0.8               0.5000000  300      0.6185744  0.08581160
##   0.4   9         0.8               0.5000000  350      0.6205133  0.08716604
##   0.4   9         0.8               0.5000000  400      0.6215621  0.08770157
##   0.4   9         0.8               0.5000000  450      0.6222897  0.08822671
##   0.4   9         0.8               0.5000000  500      0.6227775  0.08921781
##   0.4   9         0.8               0.5555556   50      0.6235783  0.08855160
##   0.4   9         0.8               0.5555556  100      0.6168021  0.08435175
##   0.4   9         0.8               0.5555556  150      0.6185053  0.08341757
##   0.4   9         0.8               0.5555556  200      0.6209280  0.08615289
##   0.4   9         0.8               0.5555556  250      0.6219971  0.08641168
##   0.4   9         0.8               0.5555556  300      0.6240539  0.08828241
##   0.4   9         0.8               0.5555556  350      0.6250539  0.08907404
##   0.4   9         0.8               0.5555556  400      0.6270579  0.09120094
##   0.4   9         0.8               0.5555556  450      0.6269156  0.09118044
##   0.4   9         0.8               0.5555556  500      0.6273099  0.09088728
##   0.4   9         0.8               0.6111111   50      0.6243222  0.08990843
##   0.4   9         0.8               0.6111111  100      0.6220336  0.08777813
##   0.4   9         0.8               0.6111111  150      0.6220702  0.08867029
##   0.4   9         0.8               0.6111111  200      0.6241718  0.09057215
##   0.4   9         0.8               0.6111111  250      0.6261270  0.09169587
##   0.4   9         0.8               0.6111111  300      0.6268709  0.09289761
##   0.4   9         0.8               0.6111111  350      0.6279278  0.09382940
##   0.4   9         0.8               0.6111111  400      0.6280009  0.09348396
##   0.4   9         0.8               0.6111111  450      0.6288465  0.09441099
##   0.4   9         0.8               0.6111111  500      0.6295294  0.09448387
##   0.4   9         0.8               0.6666667   50      0.6303180  0.09257881
##   0.4   9         0.8               0.6666667  100      0.6253344  0.08957789
##   0.4   9         0.8               0.6666667  150      0.6256718  0.08984922
##   0.4   9         0.8               0.6666667  200      0.6283384  0.09182092
##   0.4   9         0.8               0.6666667  250      0.6295253  0.09212225
##   0.4   9         0.8               0.6666667  300      0.6303871  0.09322241
##   0.4   9         0.8               0.6666667  350      0.6315050  0.09460449
##   0.4   9         0.8               0.6666667  400      0.6320131  0.09575386
##   0.4   9         0.8               0.6666667  450      0.6327813  0.09587409
##   0.4   9         0.8               0.6666667  500      0.6334846  0.09656314
##   0.4   9         0.8               0.7222222   50      0.6313627  0.09533213
##   0.4   9         0.8               0.7222222  100      0.6261961  0.09117765
##   0.4   9         0.8               0.7222222  150      0.6263913  0.09174691
##   0.4   9         0.8               0.7222222  200      0.6282286  0.09289886
##   0.4   9         0.8               0.7222222  250      0.6295172  0.09457711
##   0.4   9         0.8               0.7222222  300      0.6316269  0.09663189
##   0.4   9         0.8               0.7222222  350      0.6330903  0.09740272
```

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##   0.4   9         0.8               0.7222222  400      0.6334358  0.09767747
##   0.4   9         0.8               0.7222222  450      0.6337854  0.09848994
##   0.4   9         0.8               0.7222222  500      0.6334683  0.09803753
##   0.4   9         0.8               0.7777778   50      0.6334805  0.09746787
##   0.4   9         0.8               0.7777778  100      0.6292286  0.09350318
##   0.4   9         0.8               0.7777778  150      0.6304562  0.09444448
##   0.4   9         0.8               0.7777778  200      0.6313464  0.09611833
##   0.4   9         0.8               0.7777778  250      0.6333260  0.09791809
##   0.4   9         0.8               0.7777778  300      0.6342000  0.09863429
##   0.4   9         0.8               0.7777778  350      0.6350862  0.09948237
##   0.4   9         0.8               0.7777778  400      0.6356268  0.09982867
##   0.4   9         0.8               0.7777778  450      0.6353910  0.09942358
##   0.4   9         0.8               0.7777778  500      0.6357447  0.09898748
##   0.4   9         0.8               0.8333333   50      0.6344804  0.09612077
##   0.4   9         0.8               0.8333333  100      0.6303667  0.09310922
##   0.4   9         0.8               0.8333333  150      0.6316797  0.09516308
##   0.4   9         0.8               0.8333333  200      0.6323911  0.09593058
##   0.4   9         0.8               0.8333333  250      0.6336797  0.09660932
##   0.4   9         0.8               0.8333333  300      0.6348260  0.09688691
##   0.4   9         0.8               0.8333333  350      0.6361105  0.09790045
##   0.4   9         0.8               0.8333333  400      0.6368910  0.09836971
##   0.4   9         0.8               0.8333333  450      0.6374641  0.09892991
##   0.4   9         0.8               0.8333333  500      0.6376064  0.09902474
##   0.4   9         0.8               0.8888889   50      0.6356512  0.09910185
##   0.4   9         0.8               0.8888889  100      0.6321065  0.09673522
##   0.4   9         0.8               0.8888889  150      0.6320456  0.09632814
##   0.4   9         0.8               0.8888889  200      0.6337407  0.09769020
##   0.4   9         0.8               0.8888889  250      0.6349398  0.09854617
##   0.4   9         0.8               0.8888889  300      0.6360373  0.09981553
##   0.4   9         0.8               0.8888889  350      0.6368097  0.10051289
##   0.4   9         0.8               0.8888889  400      0.6366512  0.10080359
##   0.4   9         0.8               0.8888889  450      0.6377975  0.10182609
##   0.4   9         0.8               0.8888889  500      0.6377040  0.10189010
##   0.4   9         0.8               0.9444444   50      0.6377609  0.10098302
##   0.4   9         0.8               0.9444444  100      0.6341594  0.09634995
##   0.4   9         0.8               0.9444444  150      0.6334602  0.09658576
##   0.4   9         0.8               0.9444444  200      0.6349236  0.09793839
##   0.4   9         0.8               0.9444444  250      0.6363707  0.09908918
##   0.4   9         0.8               0.9444444  300      0.6367975  0.09993746
##   0.4   9         0.8               0.9444444  350      0.6374276  0.10019181
##   0.4   9         0.8               0.9444444  400      0.6378707  0.10050251
##   0.4   9         0.8               0.9444444  450      0.6386023  0.10115819
##   0.4   9         0.8               0.9444444  500      0.6392609  0.10154893
##   0.4   9         0.8               1.0000000   50      0.6392039  0.10150291
##   0.4   9         0.8               1.0000000  100      0.6347813  0.09850468
##   0.4   9         0.8               1.0000000  150      0.6348748  0.09829393
##   0.4   9         0.8               1.0000000  200      0.6356105  0.09918773
##   0.4   9         0.8               1.0000000  250      0.6371674  0.10038950
##   0.4   9         0.8               1.0000000  300      0.6387039  0.10195457
##   0.4   9         0.8               1.0000000  350      0.6387974  0.10128321
##   0.4   9         0.8               1.0000000  400      0.6390048  0.10237893
##   0.4   9         0.8               1.0000000  450      0.6400616  0.10263642
##   0.4   9         0.8               1.0000000  500      0.6402039  0.10309164
##   0.4  10         0.6               0.5000000   50      0.6171719  0.08390880
```

```
##   0.4  10         0.6               0.5000000  100      0.6135704  0.08072961
##   0.4  10         0.6               0.5000000  150      0.6158468  0.08279240
##   0.4  10         0.6               0.5000000  200      0.6186313  0.08540774
##   0.4  10         0.6               0.5000000  250      0.6199483  0.08682124
##   0.4  10         0.6               0.5000000  300      0.6220946  0.08767713
##   0.4  10         0.6               0.5000000  350      0.6234401  0.08926005
##   0.4  10         0.6               0.5000000  400      0.6246352  0.09005420
##   0.4  10         0.6               0.5000000  450      0.6242653  0.08986827
##   0.4  10         0.6               0.5000000  500      0.6252205  0.09060068
##   0.4  10         0.6               0.5555556   50      0.6204524  0.08602780
##   0.4  10         0.6               0.5555556  100      0.6187776  0.08576929
##   0.4  10         0.6               0.5555556  150      0.6201475  0.08669194
##   0.4  10         0.6               0.5555556  200      0.6231393  0.08948184
##   0.4  10         0.6               0.5555556  250      0.6248303  0.08979896
##   0.4  10         0.6               0.5555556  300      0.6262896  0.09159850
##   0.4  10         0.6               0.5555556  350      0.6273465  0.09236181
##   0.4  10         0.6               0.5555556  400      0.6279359  0.09279700
##   0.4  10         0.6               0.5555556  450      0.6287652  0.09395684
##   0.4  10         0.6               0.5555556  500      0.6292123  0.09435343
##   0.4  10         0.6               0.6111111   50      0.6244279  0.08986536
##   0.4  10         0.6               0.6111111  100      0.6233059  0.08809321
##   0.4  10         0.6               0.6111111  150      0.6255132  0.09028181
##   0.4  10         0.6               0.6111111  200      0.6274766  0.09114322
##   0.4  10         0.6               0.6111111  250      0.6287855  0.09278068
##   0.4  10         0.6               0.6111111  300      0.6298993  0.09380466
##   0.4  10         0.6               0.6111111  350      0.6310252  0.09502535
##   0.4  10         0.6               0.6111111  400      0.6315862  0.09588377
##   0.4  10         0.6               0.6111111  450      0.6323016  0.09570666
##   0.4  10         0.6               0.6111111  500      0.6324276  0.09616571
##   0.4  10         0.6               0.6666667   50      0.6246514  0.08918052
##   0.4  10         0.6               0.6666667  100      0.6237043  0.08847348
##   0.4  10         0.6               0.6666667  150      0.6253872  0.09058153
##   0.4  10         0.6               0.6666667  200      0.6274156  0.09159208
##   0.4  10         0.6               0.6666667  250      0.6292896  0.09354617
##   0.4  10         0.6               0.6666667  300      0.6303220  0.09534743
##   0.4  10         0.6               0.6666667  350      0.6316960  0.09574448
##   0.4  10         0.6               0.6666667  400      0.6325131  0.09623296
##   0.4  10         0.6               0.6666667  450      0.6324033  0.09630028
##   0.4  10         0.6               0.6666667  500      0.6333911  0.09727499
##   0.4  10         0.6               0.7222222   50      0.6260213  0.09180386
##   0.4  10         0.6               0.7222222  100      0.6242937  0.09019279
##   0.4  10         0.6               0.7222222  150      0.6275091  0.09211080
##   0.4  10         0.6               0.7222222  200      0.6291066  0.09302758
##   0.4  10         0.6               0.7222222  250      0.6306147  0.09467237
##   0.4  10         0.6               0.7222222  300      0.6310944  0.09416074
##   0.4  10         0.6               0.7222222  350      0.6322244  0.09545675
##   0.4  10         0.6               0.7222222  400      0.6328504  0.09646180
##   0.4  10         0.6               0.7222222  450      0.6333098  0.09704118
##   0.4  10         0.6               0.7222222  500      0.6337325  0.09772085
##   0.4  10         0.6               0.7777778   50      0.6293790  0.09369827
##   0.4  10         0.6               0.7777778  100      0.6283424  0.09330230
##   0.4  10         0.6               0.7777778  150      0.6305090  0.09539015
##   0.4  10         0.6               0.7777778  200      0.6325415  0.09677211
##   0.4  10         0.6               0.7777778  250      0.6343707  0.09846974
```

```
##   0.4  10         0.6               0.7777778  300      0.6349439  0.09893480
##   0.4  10         0.6               0.7777778  350      0.6359438  0.09988816
##   0.4  10         0.6               0.7777778  400      0.6362650  0.10065375
##   0.4  10         0.6               0.7777778  450      0.6371755  0.10133423
##   0.4  10         0.6               0.7777778  500      0.6375251  0.10149923
##   0.4  10         0.6               0.8333333   50      0.6289197  0.09218096
##   0.4  10         0.6               0.8333333  100      0.6295578  0.09222582
##   0.4  10         0.6               0.8333333  150      0.6321350  0.09489968
##   0.4  10         0.6               0.8333333  200      0.6341065  0.09638897
##   0.4  10         0.6               0.8333333  250      0.6354479  0.09785657
##   0.4  10         0.6               0.8333333  300      0.6363666  0.09877356
##   0.4  10         0.6               0.8333333  350      0.6369560  0.09856868
##   0.4  10         0.6               0.8333333  400      0.6379275  0.09950944
##   0.4  10         0.6               0.8333333  450      0.6382446  0.09997075
##   0.4  10         0.6               0.8333333  500      0.6378747  0.09932622
##   0.4  10         0.6               0.8888889   50      0.6314358  0.09557284
##   0.4  10         0.6               0.8888889  100      0.6309765  0.09538044
##   0.4  10         0.6               0.8888889  150      0.6338667  0.09730009
##   0.4  10         0.6               0.8888889  200      0.6349480  0.09784448
##   0.4  10         0.6               0.8888889  250      0.6369723  0.10003059
##   0.4  10         0.6               0.8888889  300      0.6372650  0.10054024
##   0.4  10         0.6               0.8888889  350      0.6375942  0.10079411
##   0.4  10         0.6               0.8888889  400      0.6388706  0.10221157
##   0.4  10         0.6               0.8888889  450      0.6395373  0.10264012
##   0.4  10         0.6               0.8888889  500      0.6397771  0.10265429
##   0.4  10         0.6               0.9444444   50      0.6347813  0.09735917
##   0.4  10         0.6               0.9444444  100      0.6333748  0.09764609
##   0.4  10         0.6               0.9444444  150      0.6356674  0.09854360
##   0.4  10         0.6               0.9444444  200      0.6367121  0.09967598
##   0.4  10         0.6               0.9444444  250      0.6375983  0.10092146
##   0.4  10         0.6               0.9444444  300      0.6389885  0.10183546
##   0.4  10         0.6               0.9444444  350      0.6393747  0.10219913
##   0.4  10         0.6               0.9444444  400      0.6401104  0.10282455
##   0.4  10         0.6               0.9444444  450      0.6406592  0.10382935
##   0.4  10         0.6               0.9444444  500      0.6409153  0.10358597
##   0.4  10         0.6               1.0000000   50      0.6359723  0.09829400
##   0.4  10         0.6               1.0000000  100      0.6355211  0.09805372
##   0.4  10         0.6               1.0000000  150      0.6366511  0.09846949
##   0.4  10         0.6               1.0000000  200      0.6378056  0.09970454
##   0.4  10         0.6               1.0000000  250      0.6392974  0.10105832
##   0.4  10         0.6               1.0000000  300      0.6397283  0.10097951
##   0.4  10         0.6               1.0000000  350      0.6405779  0.10158021
##   0.4  10         0.6               1.0000000  400      0.6411307  0.10239427
##   0.4  10         0.6               1.0000000  450      0.6412689  0.10286306
##   0.4  10         0.6               1.0000000  500      0.6415616  0.10377966
##   0.4  10         0.8               0.5000000   50      0.6172898  0.08441972
##   0.4  10         0.8               0.5000000  100      0.6146476  0.08188958
##   0.4  10         0.8               0.5000000  150      0.6182857  0.08499542
##   0.4  10         0.8               0.5000000  200      0.6207653  0.08661224
##   0.4  10         0.8               0.5000000  250      0.6222450  0.08774622
##   0.4  10         0.8               0.5000000  300      0.6233384  0.08853258
##   0.4  10         0.8               0.5000000  350      0.6246433  0.08997641
##   0.4  10         0.8               0.5000000  400      0.6250376  0.09044480
##   0.4  10         0.8               0.5000000  450      0.6260294  0.09149008
```

```
##   0.4  10         0.8               0.5000000  500      0.6262408  0.09166139
##   0.4  10         0.8               0.5555556   50      0.6211759  0.08762947
##   0.4  10         0.8               0.5555556  100      0.6194646  0.08578281
##   0.4  10         0.8               0.5555556  150      0.6218995  0.08779069
##   0.4  10         0.8               0.5555556  200      0.6238547  0.08908511
##   0.4  10         0.8               0.5555556  250      0.6255417  0.09054554
##   0.4  10         0.8               0.5555556  300      0.6273506  0.09144008
##   0.4  10         0.8               0.5555556  350      0.6278993  0.09193818
##   0.4  10         0.8               0.5555556  400      0.6286432  0.09322765
##   0.4  10         0.8               0.5555556  450      0.6294318  0.09366778
##   0.4  10         0.8               0.5555556  500      0.6293261  0.09342629
##   0.4  10         0.8               0.6111111   50      0.6220621  0.08649868
##   0.4  10         0.8               0.6111111  100      0.6213141  0.08555169
##   0.4  10         0.8               0.6111111  150      0.6245336  0.08860920
##   0.4  10         0.8               0.6111111  200      0.6264522  0.08986709
##   0.4  10         0.8               0.6111111  250      0.6282733  0.09288477
##   0.4  10         0.8               0.6111111  300      0.6300416  0.09482806
##   0.4  10         0.8               0.6111111  350      0.6302245  0.09416409
##   0.4  10         0.8               0.6111111  400      0.6307285  0.09495139
##   0.4  10         0.8               0.6111111  450      0.6308911  0.09448603
##   0.4  10         0.8               0.6111111  500      0.6311106  0.09513572
##   0.4  10         0.8               0.6666667   50      0.6257449  0.08961184
##   0.4  10         0.8               0.6666667  100      0.6256799  0.08836368
##   0.4  10         0.8               0.6666667  150      0.6281270  0.09061990
##   0.4  10         0.8               0.6666667  200      0.6296188  0.09285547
##   0.4  10         0.8               0.6666667  250      0.6318627  0.09411584
##   0.4  10         0.8               0.6666667  300      0.6329724  0.09460958
##   0.4  10         0.8               0.6666667  350      0.6336594  0.09615566
##   0.4  10         0.8               0.6666667  400      0.6338341  0.09662878
##   0.4  10         0.8               0.6666667  450      0.6344805  0.09678460
##   0.4  10         0.8               0.6666667  500      0.6347366  0.09670717
##   0.4  10         0.8               0.7222222   50      0.6278546  0.09320252
##   0.4  10         0.8               0.7222222  100      0.6271961  0.09244641
##   0.4  10         0.8               0.7222222  150      0.6288668  0.09397027
##   0.4  10         0.8               0.7222222  200      0.6303668  0.09470576
##   0.4  10         0.8               0.7222222  250      0.6315700  0.09518548
##   0.4  10         0.8               0.7222222  300      0.6324318  0.09644628
##   0.4  10         0.8               0.7222222  350      0.6339805  0.09760416
##   0.4  10         0.8               0.7222222  400      0.6345211  0.09867156
##   0.4  10         0.8               0.7222222  450      0.6348870  0.09875408
##   0.4  10         0.8               0.7222222  500      0.6347976  0.09836966
##   0.4  10         0.8               0.7777778   50      0.6283139  0.09364745
##   0.4  10         0.8               0.7777778  100      0.6281392  0.09346151
##   0.4  10         0.8               0.7777778  150      0.6304968  0.09455110
##   0.4  10         0.8               0.7777778  200      0.6327244  0.09603710
##   0.4  10         0.8               0.7777778  250      0.6346349  0.09811018
##   0.4  10         0.8               0.7777778  300      0.6348707  0.09831015
##   0.4  10         0.8               0.7777778  350      0.6359113  0.09829109
##   0.4  10         0.8               0.7777778  400      0.6363341  0.09882956
##   0.4  10         0.8               0.7777778  450      0.6364194  0.09979356
##   0.4  10         0.8               0.7777778  500      0.6370495  0.10038132
##   0.4  10         0.8               0.8333333   50      0.6313952  0.09565101
##   0.4  10         0.8               0.8333333  100      0.6304969  0.09595030
##   0.4  10         0.8               0.8333333  150      0.6339277  0.09873709
```

```
##   0.4  10         0.8              0.8333333  200      0.6358666  0.10093461
##   0.4  10         0.8              0.8333333  250      0.6368463  0.10112212
##   0.4  10         0.8              0.8333333  300      0.6382324  0.10253173
##   0.4  10         0.8              0.8333333  350      0.6387527  0.10276261
##   0.4  10         0.8              0.8333333  400      0.6384926  0.10292023
##   0.4  10         0.8              0.8333333  450      0.6390251  0.10299588
##   0.4  10         0.8              0.8333333  500      0.6395657  0.10404007
##   0.4  10         0.8              0.8888889   50      0.6325537  0.09497401
##   0.4  10         0.8              0.8888889  100      0.6318789  0.09603459
##   0.4  10         0.8              0.8888889  150      0.6333830  0.09643838
##   0.4  10         0.8              0.8888889  200      0.6358626  0.09850555
##   0.4  10         0.8              0.8888889  250      0.6368788  0.09902169
##   0.4  10         0.8              0.8888889  300      0.6366959  0.09871100
##   0.4  10         0.8              0.8888889  350      0.6373422  0.09911549
##   0.4  10         0.8              0.8888889  400      0.6389275  0.10073266
##   0.4  10         0.8              0.8888889  450      0.6382934  0.10029720
##   0.4  10         0.8              0.8888889  500      0.6386145  0.10034410
##   0.4  10         0.8              0.9444444   50      0.6338911  0.09637563
##   0.4  10         0.8              0.9444444  100      0.6340496  0.09680654
##   0.4  10         0.8              0.9444444  150      0.6347853  0.09747162
##   0.4  10         0.8              0.9444444  200      0.6368259  0.10000274
##   0.4  10         0.8              0.9444444  250      0.6374438  0.10023950
##   0.4  10         0.8              0.9444444  300      0.6382771  0.10121364
##   0.4  10         0.8              0.9444444  350      0.6386633  0.10143193
##   0.4  10         0.8              0.9444444  400      0.6392812  0.10221886
##   0.4  10         0.8              0.9444444  450      0.6393299  0.10188044
##   0.4  10         0.8              0.9444444  500      0.6393259  0.10152519
##   0.4  10         0.8              1.0000000   50      0.6363910  0.10062242
##   0.4  10         0.8              1.0000000  100      0.6362731  0.09857047
##   0.4  10         0.8              1.0000000  150      0.6376349  0.10035597
##   0.4  10         0.8              1.0000000  200      0.6392893  0.10179116
##   0.4  10         0.8              1.0000000  250      0.6407161  0.10304049
##   0.4  10         0.8              1.0000000  300      0.6410982  0.10365961
##   0.4  10         0.8              1.0000000  350      0.6412690  0.10334349
##   0.4  10         0.8              1.0000000  400      0.6418258  0.10394236
##   0.4  10         0.8              1.0000000  450      0.6419559  0.10425644
##   0.4  10         0.8              1.0000000  500      0.6422323  0.10463925
##
## Tuning parameter 'gamma' was held constant at a value of 0
## Tuning
##  parameter 'min_child_weight' was held constant at a value of 1
## Accuracy was used to select the optimal model using the largest value.
## The final values used for the model were nrounds = 300, max_depth = 1, eta
##  = 0.4, gamma = 0, colsample_bytree = 0.8, min_child_weight = 1 and subsample
##  = 0.7777778.
```

The model underwent pre-processing, including centering and scaling of all features, followed by 10-fold cross-validation and down-sampling to handle class imbalance. The final model was selected using a accuracy metric, with the best configuration being eta = 0.4, max_depth = 1, gamma = 0, colsample_bytree = 0.8, min_child_weight = 1, and subsample = 0.7. While more extensive fine-tuning could potentially improve performance further, the decision to limit the search was based on balancing model accuracy and computational efficiency. The selected parameters provided a good trade-off, ensuring the model was trained within a reasonable timeframe while still achieving a accuracy score of 0.6743901.

```r
# Make predictions on the test set
xgb_predictions <- predict(xgb_model_pca, newdata = x_test)

# Assuming the true labels are stored in test_target
xgb_conf_matrix <- confusionMatrix(xgb_predictions, y_test)

xgb_conf_matrix
```

```
## Confusion Matrix and Statistics
##
##           Reference
## Prediction    X0    X1
##         X0 25629  1214
##         X1 12704  2458
##
##                Accuracy : 0.6687
##                  95% CI : (0.6641, 0.6732)
##     No Information Rate : 0.9126
##     P-Value [Acc > NIR] : 1
##
##                   Kappa : 0.14
##
##  Mcnemar's Test P-Value : <2e-16
##
##             Sensitivity : 0.6686
##             Specificity : 0.6694
##          Pos Pred Value : 0.9548
##          Neg Pred Value : 0.1621
##              Prevalence : 0.9126
##          Detection Rate : 0.6101
##    Detection Prevalence : 0.6390
##       Balanced Accuracy : 0.6690
##
##        'Positive' Class : X0
##
```

```r
xgb_accuracy <- as.numeric(xgb_conf_matrix$overall["Accuracy"])
```

Main Findings

Comparing the Model

```r
# Create a data frame with the model names and their corresponding precision
accuracy_table <- data.frame(
  Model = c("Random Forest (RF)", "Extreme Gradient Boosting (XGB)", "Logistic Regression (LR)"),
  Accuracy = c(rf_accuracy, xgb_accuracy, lr_accuracy)
)

# Order the data frame by decreasing precision
accuracy_table <- accuracy_table[order(-accuracy_table$Accuracy), ]

# Print the table using knitr::kable
kable(accuracy_table, caption = "Model Accuracy")
```

Table 5: Model Accuracy

	Model	Accuracy
3	Logistic Regression (LR)	0.6983216
2	Extreme Gradient Boosting (XGB)	0.6686585
1	Random Forest (RF)	0.6658493

The table presents the Accuracy of three models, listed in decreasing order:

Logistic Regression (LR) achieved the highest Accuracy at 0.7053684. This means it was the most reliable in correctly predicting repayment cases (true positives), minimizing false positives the most effectively among the models tested.

eXtreme Gradient Boosting (XGB) followed with a Accuracy of 0.6686585, slightly lower than Logistic Regression. While it was still good at correctly identifying true positives, it did so at a slightly higher rate of false positives compared to LR.

Random Forest (RF) had the lowest Accuracy at 0.6652303, indicating it was the least effective at minimizing false positives when predicting repayment.

Interestingly, the results contradict our initial expectations. Given the reputation of XGBoost (XGB) as a powerful model for handling complex relationships, we anticipated it would perform better in terms of Precision. However, Logistic Regression (LR) outperformed XGBoost (XGB) in this case, highlighting that even simpler models can sometimes deliver better results, especially when the dataset is relatively straightforward and PCA have make the data much easier to predict.

Principle Component

We will first make a table of principle components in our model, since there are 25 PCs, we will only show the first 12 PCs here.

```
# Get the loadings (rotation matrix)
loadings_matrix <- pca_result$rotation

# Select the first 12 principal components
selected_loadings <- loadings_matrix[, 1:12]

# Create a cleaner table display with kable
kable(selected_loadings,
      caption = "Loadings for the First 12 Principal Components (PCs)",
      digits = 3,  # Format numeric values to 3 decimal places
      col.names = paste("PC", 1:12))  # Provide descriptive column names
```

Table 6: Loadings for the First 12 Principal Components (PCs)

	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9	PC 10	PC 11	PC 12
NAME_CONTRACT_TYPE	0.024	0.196	-0.156	0.038	-0.138	0.133	-0.530	0.113	-0.071	0.194	0.040	-0.137
CODE_GENDERM	0.109	0.006	0.021	-0.004	-0.082	-0.135	-0.042	-0.064	0.111	0.412	0.022	0.345
FLAG_OWN_CARY	0.112	0.066	-0.021	0.002	0.015	-0.171	0.027	-0.066	0.103	0.391	0.021	0.365
FLAG_OWN_REALTY	-0.053	-0.025	-0.002	-0.019	0.072	-0.106	0.205	-0.216	-0.072	0.146	0.137	-0.251

	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9	PC 10	PC 11	PC 12
CNT_CHILDREN	0.174	-0.025	-0.137	0.022	0.146	-0.468	-0.123	0.171	-0.018	-0.135	-0.004	-0.114
AMT_INCOME_TOTAL	0.146	0.093	0.054	-0.017	0.007	-0.021	0.104	0.074	0.014	0.260	0.009	0.151
AMT_CREDIT	0.059	0.466	-0.145	0.046	-0.055	0.049	0.184	-0.004	0.054	-0.050	-0.008	0.000
AMT_ANNUITY	0.082	0.413	-0.095	0.030	-0.047	0.024	0.149	0.036	0.051	0.073	0.004	0.017
AMT_GOODS_PRICE	0.060	0.463	-0.137	0.044	-0.050	0.045	0.202	-0.009	0.052	-0.062	-0.012	0.026
NAME_TYPE_SUITE	0.010	-0.021	0.047	0.025	-0.008	0.101	0.001	0.098	0.005	0.029	0.024	0.178
NAME_EDUCATION_TYPE	0.080	0.073	0.066	0.014	0.080	0.019	0.177	0.190	-0.022	0.106	-0.036	0.173
NAME_FAMILY_STATUS	0.170	0.098	-0.137	0.021	0.076	-0.369	-0.036	-0.065	-0.012	-0.057	0.025	0.066
NAME_HOUSING_TYPE	-0.060	0.044	-0.042	0.006	0.065	-0.096	-0.053	-0.265	-0.158	0.098	0.167	-0.105
REGION_POPULATION_RELATIVE	0.104	0.164	0.297	-0.168	0.156	-0.038	-0.082	0.000	0.122	0.001	0.003	-0.065
DAYS_EMPLOYED	-0.407	0.039	0.025	-0.006	-0.108	-0.160	-0.005	0.069	0.063	-0.016	-0.029	0.038
DAYS_REGISTRATION	-0.144	0.045	0.082	-0.025	-0.008	0.084	-0.053	-0.215	-0.062	-0.123	0.032	0.044
DAYS_ID_PUBLISH	-0.138	0.048	-0.016	0.003	0.006	-0.164	-0.125	-0.092	-0.072	-0.019	0.004	0.190
FLAG_EMP_PHONE	0.406	-0.039	-0.026	0.006	0.109	0.160	0.005	-0.071	-0.064	0.016	0.029	-0.038
FLAG_WORK_PHONE	0.125	-0.020	0.000	0.014	-0.027	0.078	-0.207	-0.056	-0.155	-0.315	0.004	0.334
FLAG_CONT_MOBILE	-0.007	0.017	-0.025	0.005	-0.021	0.007	-0.027	-0.030	-0.018	-0.061	0.013	0.015
FLAG_PHONE	-0.004	0.056	0.069	0.004	0.054	0.040	-0.186	-0.066	-0.177	-0.290	0.011	0.392
FLAG_EMAIL	0.042	0.020	0.038	-0.020	0.028	-0.003	0.032	0.132	-0.009	0.151	-0.009	0.027
OCCUPATION_TYPE	-0.288	0.020	0.031	-0.011	-0.113	-0.154	-0.026	-0.075	-0.101	-0.004	-0.036	0.046
CNT_FAM_MEMBERS	0.876	0.019	-0.173	0.028	0.152	-0.542	-0.122	0.112	-0.022	-0.141	0.008	-0.073
REGION_RATING_CLIENT	-0.037	-0.197	-0.413	0.207	-0.160	0.048	0.130	-0.005	-0.131	0.030	0.017	0.140
REGION_RATING_CLIENT_W_CITY	-0.036	-0.200	-0.407	0.207	-0.168	0.043	0.122	-0.014	-0.125	0.021	0.018	0.142
HOUR_APPR_PROCESS_START	0.088	0.183	0.192	-0.098	0.076	0.027	-0.046	0.081	0.067	-0.105	-0.091	0.033
REG_REGION_NOT_LIVE_REGION	0.057	0.012	0.019	-0.028	-0.273	-0.032	-0.115	0.302	-0.137	-0.154	-0.140	0.082
REG_REGION_NOT_WORK_REGION	0.117	0.052	0.262	-0.081	-0.411	-0.142	-0.093	0.065	-0.427	0.012	0.071	-0.060
LIVE_REGION_NOT_WORK_REGION	0.103	0.059	0.214	-0.081	-0.342	-0.152	-0.053	0.064	-0.421	0.085	0.144	-0.106

	PC 1	PC 2	PC 3	PC 4	PC 5	PC 6	PC 7	PC 8	PC 9	PC 10	PC 11	PC 12
REG_CITY_NOT_LIVE_CITY	-0.089	-0.056	0.035	-0.002	-0.271	0.005	0.055	0.252	0.233	-0.170	-0.204	0.074
REG_CITY_NOT_WORK_CITY	0.081	-0.063	0.046	-0.020	-0.397	-0.068	-0.079	-0.253	0.396	-0.109	-0.041	-0.004
LIVE_CITY_NOT_WORK_CITY	0.068	-0.034	0.042	-0.024	-0.323	-0.096	-0.119	-0.418	0.322	-0.037	-0.060	-0.039
ORGANIZATION_TYPE	-0.187	0.030	0.047	-0.013	-0.088	-0.116	-0.008	-0.125	0.127	-0.077	-0.042	0.080
EXT_SOURCE_2	0.008	0.158	0.174	-0.074	-0.161	-0.023	-0.007	-0.084	-0.059	-0.034	-0.036	0.157
EXT_SOURCE_3	-0.073	0.055	-0.009	-0.024	-0.043	-0.036	-0.013	-0.259	-0.082	-0.059	-0.003	0.031
OBS_30_CNT_SOCIAL_CIRCLE	-0.009	-0.018	-0.217	-0.477	-0.011	-0.004	-0.008	-0.114	-0.110	-0.069	-0.432	-0.017
DEF_30_CNT_SOCIAL_CIRCLE	-0.015	-0.032	-0.168	-0.453	-0.031	-0.048	-0.033	-0.130	-0.089	-0.078	-0.452	-0.078
OBS_60_CNT_SOCIAL_CIRCLE	-0.009	-0.017	-0.217	-0.478	-0.011	-0.004	-0.008	-0.113	-0.110	-0.069	-0.430	-0.017
DEF_60_CNT_SOCIAL_CIRCLE	-0.014	-0.033	-0.156	-0.421	-0.032	-0.050	-0.035	-0.144	-0.102	-0.085	-0.507	-0.083
DAYS_LAST_PHONE_CHANGE	-0.002	-0.179	-0.006	-0.015	-0.096	-0.002	-0.098	-0.176	-0.133	-0.079	-0.008	-0.295
AMT_REQ_CREDIT_BUREAU_QRT	-0.012	-0.017	-0.015	-0.004	-0.001	-0.003	-0.003	-0.025	-0.002	-0.061	-0.004	-0.036
DOCUMENT_COUNT	-0.007	0.186	-0.135	-0.035	-0.120	-0.137	-0.544	-0.143	-0.075	-0.214	-0.030	-0.127
DAY_EMPLOYED_PERCENT	-0.401	-0.041	0.023	-0.004	-0.107	-0.164	-0.001	-0.079	-0.068	-0.016	-0.031	0.037
AGES	-0.319	0.102	0.027	0.003	-0.039	-0.012	-0.007	-0.211	-0.074	-0.015	-0.068	0.033
CREDIT_INCOME_PERCENT	-0.028	-0.032	-0.227	-0.074	-0.067	-0.054	-0.077	-0.058	-0.038	-0.273	-0.009	-0.110

The principal component loadings reveal how original features contribute to the key dimensions of variance in the dataset. The first principal component (PC1) appears heavily influenced by variables related to employment and age, such as DAYS_EMPLOYED, DAY_EMPLOYED_PERCENT, and AGES, suggesting it captures a general demographic and work-history trend. PC2 is dominated by financial attributes like AMT_CREDIT, AMT_ANNUITY, AMT_GOODS_PRICE, and DOCUMENT_COUNT, indicating it represents a customer's financial application profile. Meanwhile, PC3 reflects regional credit behavior and affordability, with strong negative loadings from REGION_RATING_CLIENT, REGION_RATING_CLIENT_W_CITY, and CREDIT_INCOME_PERCENT. Together, these components help reduce the dataset's complexity while preserving critical variance tied to customer background, financial standing, and regional credit risk.

```
summary(logistic_model_pca)
```

```
##
## Call:
## NULL
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
```

```
## (Intercept)  0.007717   0.010958   0.704 0.481267
## PC1          0.211586   0.011078  19.099  < 2e-16 ***
## PC2         -0.258736   0.011192 -23.118  < 2e-16 ***
## PC3         -0.244532   0.011159 -21.913  < 2e-16 ***
## PC4          0.009888   0.011057   0.894 0.371190
## PC5         -0.286219   0.011216 -25.518  < 2e-16 ***
## PC6          0.111815   0.010995  10.169  < 2e-16 ***
## PC7         -0.086030   0.011216  -7.670 1.71e-14 ***
## PC8          0.246439   0.011196  22.012  < 2e-16 ***
## PC9          0.135254   0.011036  12.256  < 2e-16 ***
## PC10         0.109602   0.011799   9.289  < 2e-16 ***
## PC11         0.060179   0.011070   5.436 5.44e-08 ***
## PC12        -0.168292   0.011325 -14.860  < 2e-16 ***
## PC13        -0.386541   0.011348 -34.061  < 2e-16 ***
## PC14        -0.014886   0.011298  -1.318 0.187634
## PC15        -0.105334   0.011382  -9.254  < 2e-16 ***
## PC16        -0.145654   0.011159 -13.052  < 2e-16 ***
## PC17        -0.100346   0.011067  -9.067  < 2e-16 ***
## PC18         0.145245   0.012261  11.846  < 2e-16 ***
## PC19        -0.095278   0.012397  -7.685 1.52e-14 ***
## PC20        -0.157884   0.015058 -10.485  < 2e-16 ***
## PC21         0.198506   0.011404  17.406  < 2e-16 ***
## PC22         0.037401   0.010963   3.412 0.000646 ***
## PC23         0.271150   0.011304  23.986  < 2e-16 ***
## PC24        -0.083087   0.011766  -7.062 1.65e-12 ***
## PC25        -0.017245   0.011558  -1.492 0.135708
## PC26        -0.023470   0.011014  -2.131 0.033093 *
## PC27         0.104467   0.010961   9.531  < 2e-16 ***
## PC28         0.079682   0.010977   7.259 3.91e-13 ***
## PC29         0.006429   0.011069   0.581 0.561373
## PC30        -0.114044   0.011071 -10.301  < 2e-16 ***
## PC31         0.078812   0.011226   7.020 2.21e-12 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 55138  on 39773  degrees of freedom
## Residual deviance: 48306  on 39742  degrees of freedom
## AIC: 48370
##
## Number of Fisher Scoring iterations: 4
```

The logistic regression output shows how each principal component (PC) contributes to predicting the likelihood of a loan being repaid on time (X0). Several components—including PC1, PC2, PC3, PC5, PC8, and PC13—exhibit large, highly significant coefficients, indicating their strong influence on the model's predictions. For instance, PC1 and PC8 have large positive coefficients, suggesting that higher values along these dimensions increase the likelihood of a positive outcome (on-time repayment). In contrast, components like PC2, PC3, PC5, and PC13 have strong negative associations, implying higher values along these PCs are linked to delayed or missed repayments. Most predictors are statistically significant at the 0.001 level, reflecting a robust model fit. The model also shows a substantial reduction in deviance from 55,138 to 48,373, indicating good explanatory power, while an AIC of 48,437 supports its relative efficiency among competing models.

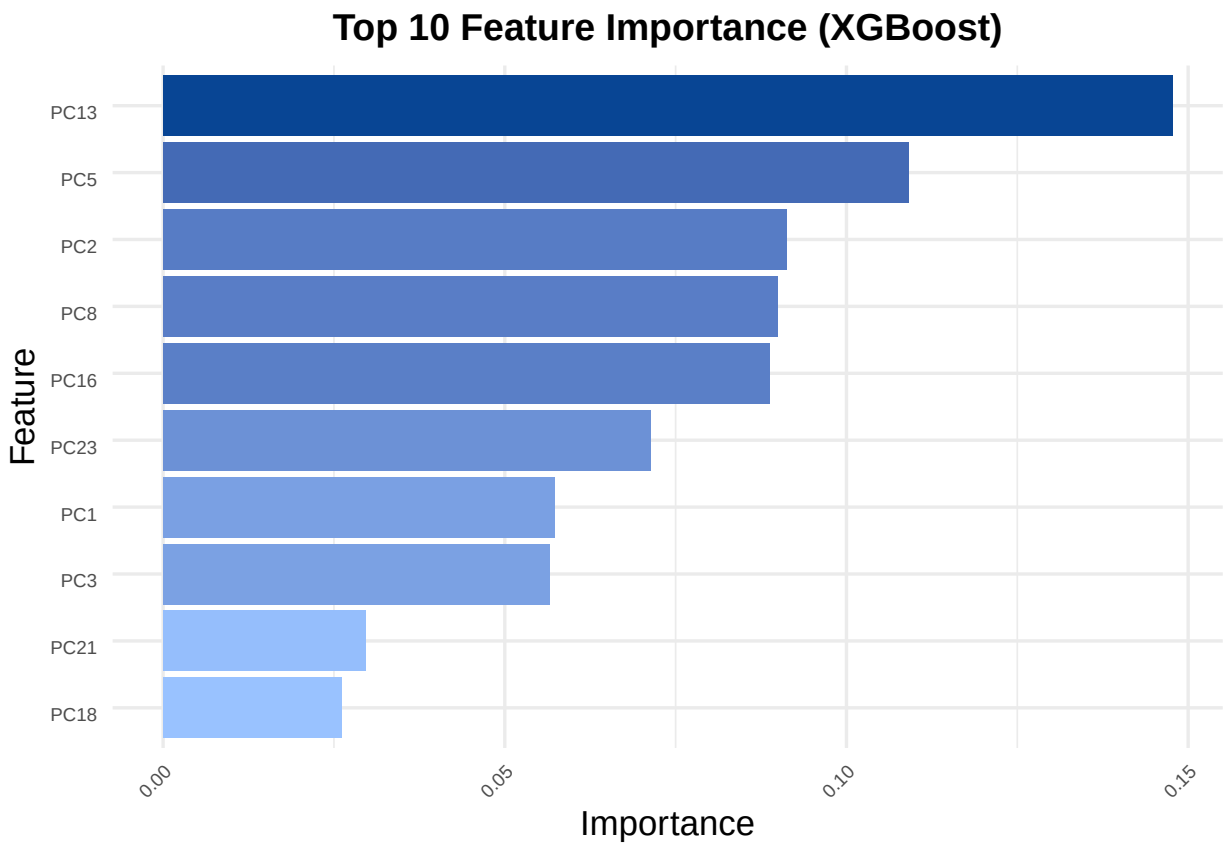
Important Factors (XGBoost)

We will make a chart of the top 10 most important factors from the XGBoost model since it is the second best model that we have.

```
xgb_model <- xgb_model_pca$finalModel
importance_matrix <- xgb.importance(model = xgb_model)

# Top 10 important features
top_10 <- head(importance_matrix, 10)

# Professional plot with color and styling
ggplot(top_10, aes(x = reorder(Feature, Gain), y = Gain, fill = Gain)) +
  geom_bar(stat = "identity") +
  coord_flip() +
  scale_fill_gradient(low = "#99c2ff", high = "#084594") +  # Color gradient
  theme_minimal(base_size = 13) +
  theme(plot.title = element_text(face = "bold", size = 14, hjust = 0.5),
        axis.text.x = element_text(angle = 45, hjust = 1, size = 7, vjust = 1),
        axis.text.y = element_text(hjust = 1, size = 7, vjust = 1),
        legend.position = "None") +
  labs(
    title = "Top 10 Feature Importance (XGBoost)",
    x = "Feature",
    y = "Importance"
  )
```



The chart displays the top 10 most important features based on their importance scores. The bar lengths indicate each feature's contribution to the model, with longer bars representing higher importance. PC13

stands out as the most important feature, followed by PC5, PC2, and others.

Conclusions

In this project, we implemented Principal Component Analysis (PCA) to reduce dimensionality and trained three machine learning models—Logistic Regression, Random Forest, and XGBoost—on the transformed dataset to classify loan repayment status. PCA enabled us to eliminate multicollinearity, reduce noise, and retain the most informative structure of the data.

Among the models, Logistic Regression with PCA achieved the highest test set accuracy at 70.5%, outperforming both XGBoost (66.9%) and Random Forest (66.5%). This was a surprising result, as simpler models like Logistic Regression are not always expected to outperform more complex ensemble methods. However, the dimensionality reduction provided by PCA made the data more linearly separable, favoring logistic regression's performance.

Analysis of the principal component loadings and model coefficients revealed several key factors:

PC1, which was among the strongest positive predictors, had high contributions from variables like DAYS_EMPLOYED, DAY_EMPLOYED_PERCENT, and AGES, suggesting that employment stability and age are positively associated with repayment likelihood.

PC2 and PC5, which were strong negative predictors, were heavily influenced by financial application variables such as AMT_CREDIT, AMT_ANNUITY, and DOCUMENT_COUNT, implying that higher credit amounts and document requirements may correlate with increased default risk.

PC13, the most important feature in the XGBoost model, also had strong contributions from CNT_CHILDREN, NAME_FAMILY_STATUS, and CNT_FAM_MEMBERS, indicating the significance of household composition and family obligations in assessing repayment risk.

The combination of dimensionality reduction and rigorous cross-validation allowed us to uncover these patterns while maintaining high predictive performance. This modeling pipeline highlights that even in high-dimensional settings, simpler models can be powerful when paired with the right pre-processing techniques and well-interpreted feature engineering.

Limitations

This project leveraged PCA and Logistic Regression to reduce dimensionality and improve computational efficiency. While this approach effectively enhanced model interpretability and generalization, there are several areas where the analysis could be further strengthened.

First, computational constraints limited the breadth of hyperparameter tuning and restricted the training of more complex models such as deep neural networks or support vector machines. With more powerful computational resources, we could explore a wider hyperparameter space and more advanced algorithms that may capture non-linear patterns more effectively.

Second, the dataset was confined to a subset of six countries in the CIS and Southeast Asia regions. This narrow geographic scope may limit the model's generalizability. Incorporating more diverse regions and up-to-date financial behavior data would improve both the robustness and global applicability of the predictive models.

Third, although PCA helps reduce redundancy and multicollinearity, it transforms features into latent components, which can make interpretation less intuitive. While we analyzed component loadings, some loss of transparency in individual variable effects may impact practical implementation and stakeholder communication.

Finally, several important predictors were demographic in nature (e.g., gender, age, employment history), which, while predictive, raise concerns about fairness and potential bias. Despite efforts to minimize these

risks, future work should explore fairness-aware modeling techniques to ensure equitable outcomes across different population groups.

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