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[5 → 1]

Let: A is an $n \times n$ matrix

Assume: A has a multiplicative inverse.

By the theorem 3.5 5. and determinant definition for square matrix.

There is nothing in 3.5 about determinents.

Hint: Apply det( ) to both sides of definition of inverse:

A A^-1 = I

$\implies det(A) \neq 0$

[1 → 2]

Let A a square matrix, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$det(A) \neq 0$

$\implies ad - bc \neq 0$

Can't assume A is 2x2

how

that means $ad \neq 0 | bc \neq 0 | ad \neq bc$

so there has to be a leading 1 in every rows of row reduced A

$\implies$ A columns span $\mathbb{R}^n$

[2 → 3]

A columns span $\mathbb{R}^n$ and A is a square matrix

$\implies$ there are n leading 1s in rows and there are n columns in RR A

$\implies$ there are n leading 1s in columns when RR A

$\implies$ A columns are linearly independent. good (could also use Thm 2.17)

[3 → 4]

$Ax = b$

$\Rightarrow x \in \mathbb{R}^n$ [def of product in 3.5]

every columns of A are linearly independent

$\implies Ax$ will produce b that every rows are linearly independent

$\implies Ax = b$ has a unique solution, $x \in \mathbb{R}^n$, for each $b \in \mathbb{R}^n$

Need also show there will always be a solution

[4 → 5]

$Ax = b$ has a unique solution, $x \in \mathbb{R}^n$, for each $b \in \mathbb{R}^n$

~~Let $x = A^{-1}$~~

x is nx1 and A^(-1) would need to be nxn

$\implies Ax = AA^{-1}$ [substitution]

$\implies b = I$ [def 3.5]

using theorem 3.5 part 5. we know that $AA^{-1} = A^{-1}A$

$\implies$ A has a multiplicative inverse.