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Claim: Suppose A is a square matrix with eigenvectors v and w, and corresponding eigenvalues λ_v and λ_w. If $\lambda_v \neq \lambda_w$ then v and w are linearly independent.

By the definition of eigenvectors,
$A\vec{v} = \lambda_v\vec{v}$
$A\vec{w} = \lambda_w\vec{w}$

Proof by contrapositive
Assume: v and w are linearly dependent
means $\exists c \in \mathbb{R} : c\vec{v} = \vec{w}$ by thm __ (in this theorem there is no mention of the coefficients having to be nonzero)
W.M.S. $\lambda_v = \lambda_w$

$A\vec{w} = \lambda_v\vec{w}$
⇒ $A(c\vec{v}) = \lambda_w(c\vec{v})$ [subsitutiton]
⇒ $c(A\vec{v}) = c\lambda_w\vec{v}$ [theorem 3.5.10.1]
⇒ $c(\lambda_v\vec{v}) = c\lambda_w\vec{v}$ [substitution]
⇒ $c\lambda_v\vec{v} - c\lambda_w\vec{v} = 0$
⇒ $c(\lambda_v\vec{v} - \lambda_w\vec{v}) = 0$ no, this is not the reason c=/=0
$c \neq 0$ because v and w are linearly dependent so $\lambda_w\vec{v} - \lambda_v\vec{v} = 0$
⇒$\lambda_w\vec{v} - \lambda_v\vec{v} = 0$
⇒$\vec{v}(\lambda_w - \lambda_v) = 0$ put scalars on the left
by the definition of eigenvectors $\vec{v} \neq 0$ good
$\implies \lambda_w - \lambda_v = 0$
$\therefore \lambda_w = \lambda_v$

because of proof by contrapositive
$\therefore$ If $\lambda_v \neq \lambda_w$ then v and w are linearly independent.