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Claim: If an $n \times n$ matrix has n L.I. eigenvectors then there exists an invertible P and diagonal matrix D such that $A = PDP^{-1}$

Proof:

Assume: $AP = PD$ You can't start by assuming this!

We ~~know~~ Define P and then show it makes the above equation true.

$P = [v_1 \quad v_2 \quad \dots \quad v_n]$ what are the v s ?

$AP = A\,[v_1 \quad v_2 \quad \dots \quad v_n]$ [substitution]

$= [Av_1 \quad Av_2 \quad \dots \quad Av_n]$ [~~theorem 3.5~~] Definition matrix mult.

$= [\lambda_1 v_1 \quad \lambda_2 v_2 \quad \dots \quad \lambda_n v_n]$ [definition of eigenvectors and eigenvalues] good ( but you never told your reader that the vs were eigenvectors with eigenvalues __)

$$= [v_1 \quad v_2 \quad \dots \quad v_n] \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ & & \dots & \\ 0 & 0 & \dots & \lambda_n \end{bmatrix}$$

$= PD$ [D is diagonal ]

Since P has n L.I. eigenvectors, columns of P matrix are L.I. by theorem ___

also mean the product of $PD$ has unique solution [Big Theorem 3→4] equations have solutions, what do you mean here?

$\Rightarrow P$ is invertible [Big Theorem 4→5]

$\Rightarrow APP^{-1} = PDP^{-1}$ [times both side by P^-1]

$\Rightarrow A = PDP^{-1}$ [definition of multiplicative inverse] yes

$\therefore$ there exists an invertible P and diagonal matrix D such that $A = PDP^{-1}$