Viet M. Bui
vietbui20@augustana.edu

Claim: $\mathbb{S}$ spans $\mathbb{L}$, $Q \in \mathbb{L} \land P \in \mathbb{S} : Q$ is written as a linear combination of point of $\mathbb{S}$, the coefficient of P is not zero. If $\mathbb{S}'$ is the set obtained from $\mathbb{S}$ by replacing P with Q, then $\mathbb{S}'$ also spans $\mathbb{L}$

Define: $S = \{T_1, T_2, \ldots, T_n, P\}$,$S' = \{T_1, T_2, \ldots, T_n, Q\}$
Assume:

- $\mathbb{S}$ spans $\mathbb{L}$
- $Q \in \mathbb{L}$
- $P \in \mathbb{S}$
- $Q = c_1T_1 + \ldots + c_nP_n + c_pP, c_p \neq 0$
  > W.M.S. $\mathbb{S}'$ spans $\mathbb{L}$

Let: $A \in \mathbb{L}, A \notin \mathbb{S}$
$A = d_1T_1 + \ldots + d_nT_n + d_pP$ ($A \in spanS$, definition of span)
Perform arithmetic of $Q = c_1T_1 + \ldots + c_nP_n + c_pP, c_p$
$c_pP = Q - (c_1T_1 + \ldots + c_nT_n)$
$P = \frac{1}{c_p}(Q - (c_1T_1 + \ldots + c_nT_n))$ (can do since $c_p \neq 0$)
$A = d_1T_1 + \ldots + d_nT_n + d_p(\frac{1}{c_p}(Q - (c_1T_1 + \ldots + c_nT_n)))$ (substitution)
$A = (d_1 - \frac{d_pc_1}{c_p})T_1 + \ldots + (d_n - \frac{d_pc_n}{c_p})T_n + \frac{d_p}{c_p}Q$
A can be written as linear combination of other point in S'
⇒ $A \in spanS'$
Therefore S' spans $\mathbb{L}$