## Classify fashion images on the MNIST data

# What would be an appropriate metric to evaluate your models? Why? (Hint: No code required.)

Since no specific cost is given, accuracy seems like the best metric to be used since we assume that all class are equally important. Accuracy is also intuitive to understand the performance of each of the models. Hence, in this assignment, we will choose accuracy as the mertric of choice.

## Get the data and show some example images from the data.

```
In [1]: %capture
        import warnings
        warnings.filterwarnings('ignore')
         import pandas as pd
         import numpy as np
In [2]: from keras.datasets import fashion_mnist
         # Load MNIST dataset
         (X_train, y_train), (X_test, y_test) = fashion_mnist.load_data()
In [3]: # Look at the dimensions
        print(f"X_train: {X_train.shape}")
         print(f"y_train: {y_train.shape}")
        print(f"X_test: {X_test.shape}")
print(f"y_test: {y_test.shape}")
       X_train: (60000, 28, 28)
       y_train: (60000,)
       X_test: (10000, 28, 28)
       y_test: (10000,)
In [4]: # Visualize some items in a grid
        import matplotlib.pyplot as plt
         fig, axs = plt.subplots(5, 5, figsize=(10,10))
         for i, ax in enumerate(axs.flatten()):
            ax.imshow(X_train[i], cmap="binary")
            ax.axis("off")
            ax.set_title(f"Label: {y_train[i]}")
         plt.tight_layout()
         plt.show()
```



Train a simple fully connected single hidden layer network to predict the items. Remember to normalize the data similar to what we did in class. Make sure that you use enough epochs so that the validation error begins to level off - provide a plot of the training history.

```
In [5]: from sklearn.model_selection import train_test_split
        prng = np.random.RandomState(20240329) # ensure we have the same split as in last class
        # intentionally choose a small train set to decrease computational burden
        X_train, X_val, y_train, y_val = train_test_split(X_train, y_train, test_size=0.3, random_state=prng)
        print(f"X_train: {X_train.shape}")
        print(f"y_train: {y_train.shape}")
        print(f"X_val: {X_val.shape}")
        print(f"y_val: {y_val.shape}")
        print(f"X_test: {X_test.shape}")
print(f"y_test: {y_test.shape}")
       X_train: (42000, 28, 28)
       y_train: (42000,)
       X_val: (18000, 28, 28)
       y_val: (18000,)
       X_test: (10000, 28, 28)
y_test: (10000,)
In [6]: # Benchmark #1 (silly):
        from sklearn.metrics import accuracy_score
        from statistics import mode
        most_frequent = mode(y_train)
        print(f"Most frequent element is: {most_frequent}")
        accuracy_most_frequent = accuracy_score(y_val, np.repeat(most_frequent, len(y_val)))
        print(f"Accuracy for our no-brainer model: {round(accuracy_most_frequent, 4)}")
         summary_df = pd.DataFrame({'Model': ['Benchmark'],
                                     'Train accuracy': [round(accuracy_score(np.array([most_frequent] * len(y_train))
                                    'Val accuracy': [round(accuracy_score(np.array([most_frequent] * len(y_val)), y_
                                    'Test accuracy': [round(accuracy_score(np.array([most_frequent] * len(y_test)),
                                   })
        summary_df
       Most frequent element is: 7
       Accuracy for our no-brainer model: 0.0944
Out[6]:
               Model Train accuracy Val accuracy Test accuracy
        0 Benchmark
                             0.1024
                                                           0.1
                                          0.0944
In [7]: def update_summary(summary_df, model_name, train_score, val_score, test_score):
            if model_name not in summary_df.Model.values:
                 summary_df.loc[len(summary_df.index)] = [model_name,
                                                           '{:.4f}'.format(train_score),
                                                           '{:.4f}'.format(val_score),
                                                           '{:.4f}'.format(test_score)]
            else:
                 summary_df.loc[summary_df.Model == model_name] = [model_name,
                                                           '{:.4f}'.format(train score),
                                                           '{:.4f}'.format(val_score),
                                                           '{:.4f}'.format(test_score)]
In [8]: from sklearn.pipeline import Pipeline
        # Benchmark #2 (RF):
        from sklearn.ensemble import RandomForestClassifier
        from sklearn.preprocessing import FunctionTransformer
        def flatten_data(X):
            return X.reshape(X.shape[0], -1)
        transformer = FunctionTransformer(flatten_data)
        rf_model = Pipeline(
            [("preprocess", transformer),
              ("rf", RandomForestClassifier(random_state = prng))
             ], verbose=True
        rf_model.fit(X_train, y_train)
```

```
predictions_rf = rf_model.predict(X_val)
         accuracy_score(y_val, predictions_rf)
        [Pipeline] ...... (step 1 of 2) Processing preprocess, total=
        [Pipeline] ...... (step 2 of 2) Processing rf, total= 37.4s
Out[8]: 0.881166666666667
In [9]:
         update_summary(summary_df, 'Random Forest', accuracy_score(y_train, rf_model.predict(X_train)), accuracy_sc
         summary_df
Out[9]:
                   Model Train accuracy Val accuracy Test accuracy
               Benchmark
                                0.1024
                                            0.0944
         1 Random Forest
                                1.0000
                                             0.8812
                                                          0.8699
In [10]: from keras.utils import to_categorical
         print(f"Dimension of y: {y_train.shape}")
         # Convert target variables to categorical
         num_classes = 10
         y_sets = [y_train, y_test, y_val]
         y_train, y_test, y_val = [to_categorical(y, num_classes=num_classes) for y in y_sets]
         print(f"Dimension of y: {y_train.shape}")
        Dimension of y: (42000,)
        Dimension of y: (42000, 10)
In [11]: from keras.models import Sequential
         from keras.layers import Input, Flatten, Rescaling, Dense
         from keras.utils import to_categorical
         # Build the simple fully connected single hidden layer network model
         model = Sequential([
             Input(shape=X_train.shape[1:]),
             Flatten(),
             Rescaling(1./255),
             Dense(255, activation='relu'),
             Dense(num_classes, activation='softmax')
         ])
         # Compile the model
         model.compile(loss='categorical_crossentropy', optimizer='adam', metrics=['accuracy'])
         print(model.summary())
       Model: "sequential"
```

Layer (type)	Output Shape	Param #
flatten (Flatten)	(None, 784)	0
rescaling (Rescaling)	(None, 784)	0
dense (Dense)	(None, 255)	200,175
dense_1 (Dense)	(None, 10)	2,560

Total params: 202,735 (791.93 KB)

Trainable params: 202,735 (791.93 KB)

Non-trainable params: 0 (0.00 B)

Accuracy for keras single hidden layer: 0.8867

None

```
In [12]: import keras
# Fit the model
keras.utils.set_random_seed(20240329) # for reproducibility
history = model.fit(X_train, y_train, validation_data=(X_val, y_val), epochs=100, batch_size=512, verbose=0
In [13]: # Evaluation of the model on the validation set
scores = model.evaluate(X_val, y_val)
print(f"Accuracy for keras single hidden layer: {round(scores[1], 4)}")

563/563 ________ 0s 414us/step - accuracy: 0.8847 - loss: 0.5014
```

Before building the simple fully connected single hidden layer network, we build a naive most frequent model and a Random Forest model as benchmarks.

The simple fully connected single hidden layer network consists of:

- Normalize layer to like we did in class
- A single hidden layer with 255 neurons

Random Forest

model.

• An output layer with 10 neurons representing 10 classes and uses 'softmax' as activation function

```
In [14]:

def plot_history(fit_history):
    plt.plot(fit_history['accuracy'], label='Training Accuracy')
    plt.plot(fit_history['val_accuracy'], label='Validation Accuracy')
    plt.xlabel('Epoch')
    plt.ylabel('Accuracy')
    plt.title('Training and Validation Accuracy')
    plt.legend()
    plt.show()

plot_history(history.history)
```



1.0000

```
In [15]: update_summary(summary_df, 'Single Hidden Layer', model.evaluate(X_train, y_train)[1], scores[1], scores[1]
```

2	Single Hidden Layer	0.9575	0.8867	0.8764	
Fr	om the summary table, the s	imple fully co	nnected single	hidden layer network outperforms the benchmark and rando	m
fo	rest models. Although on the	train data, th	ne simple netwo	orks does not generalize as perfect as the random forest, on	
va	alidation and test data, it perf	orms much b	etter, meaning	it avoid the overfitting problem that exist in the random forest	1

0.8699

0.8812

From the history graph, it seems the validation accuracy flattens out pretty quickly. We can probably stop the training earlier and still obtain the same performance on the validation and test data.

Experiment with different network architectures and settings (number of hidden layers, number of nodes, regularization, etc.). Train at least 3 models. Explain what you have tried and how it worked.

#### Model: "sequential\_1"

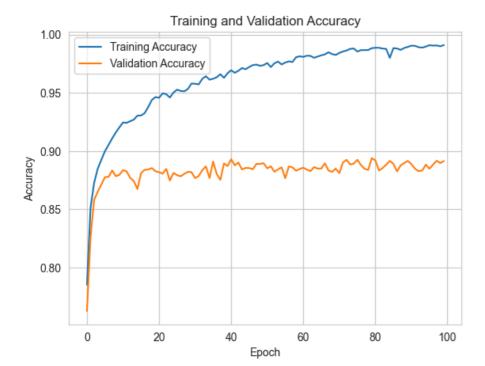
Layer (type)	Output Shape	Param #
flatten_1 (Flatten)	(None, 784)	0
rescaling_1 (Rescaling)	(None, 784)	0
dense_2 (Dense)	(None, 255)	200,175
dense_3 (Dense)	(None, 1020)	261,120
dense_4 (Dense)	(None, 510)	520,710
dense_5 (Dense)	(None, 10)	5,110

Total params: 987,115 (3.77 MB)

Trainable params: 987,115 (3.77 MB)

Non-trainable params: 0 (0.00 B)

None



In [20]: update\_summary(summary\_df, '3 Hidden Layers', hidden3\_model.evaluate(X\_train, y\_train)[1], hidden3\_scores[1
summary\_df

1313/1313 — 3s 2ms/step - accuracy: 0.9839 - loss: 0.0482
313/313 — 1s 3ms/step - accuracy: 0.8831 - loss: 0.9080

Out[20]:

	Model	Train accuracy	Val accuracy	Test accuracy
0	Benchmark	0.1024	0.0944	0.1
1	Random Forest	1.0000	0.8812	0.8699
2	Single Hidden Layer	0.9575	0.8867	0.8764
3	3 Hidden Layers	0.9834	0.8914	0.8855

```
In [21]: from keras.layers import Dropout
         # Build the regularized 3 hidden layers network model
         reg_hidden3_model = Sequential([
             Input(shape=X_train.shape[1:]),
             Flatten(),
             Rescaling(1./255),
             Dense(255, activation='relu'),
             Dropout(0.2),
             Dense(1020, activation='relu'),
             Dropout(0.2),
             Dense(510, activation='relu'),
             Dense(num_classes, activation='softmax')
         ])
         # Compile the model
         reg_hidden3_model.compile(loss='categorical_crossentropy', optimizer='adam', metrics=['accuracy'])
         print(reg_hidden3_model.summary())
```

Model: "sequential\_2"

Layer (type)	Output Shape	Param #
flatten_2 (Flatten)	(None, 784)	0
rescaling_2 (Rescaling)	(None, 784)	0
dense_6 (Dense)	(None, 255)	200,175
dropout (Dropout)	(None, 255)	0
dense_7 (Dense)	(None, 1020)	261,120
dropout_1 (Dropout)	(None, 1020)	0
dense_8 (Dense)	(None, 510)	520,710
dense_9 (Dense)	(None, 10)	5,110

Total params: 987,115 (3.77 MB)

Trainable params: 987,115 (3.77 MB)

Non-trainable params: 0 (0.00 B)

None

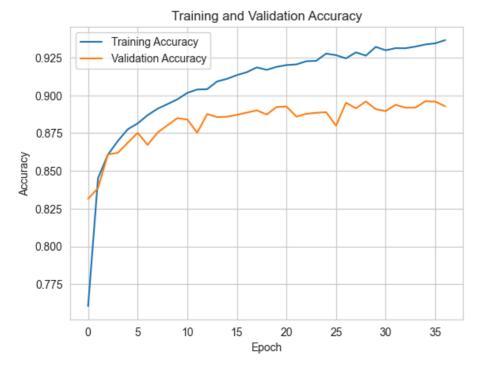
```
In [22]: from keras.callbacks import EarlyStopping
    # Fit the model with EarlyStopping
    keras.utils.set_random_seed(20240329) # for reproducibility
    reg_hidden3_history = reg_hidden3_model.fit(X_train, y_train, validation_data=(X_val, y_val), epochs=100, b
```

Epoch 37: early stopping

```
In [23]: # Evaluation of the model on the validation set
  reg_hidden3_model_scores = reg_hidden3_model.evaluate(X_val, y_val)
  print(f"Accuracy for keras regularized 3 hidden layers: {round(hidden3_scores[1], 4)}")
```

**563/563** — **1s** 2ms/step - accuracy: 0.8910 - loss: 0.3622 Accuracy for keras regularized 3 hidden layers: 0.8914

In [24]: plot\_history(reg\_hidden3\_history.history)



```
In [25]: update_summary(summary_df, 'Regularized 3 Hidden Layers', reg_hidden3_model.evaluate(X_train, y_train)[1],
summary_df

1313/1313 ________ 2s 2ms/step - accuracy: 0.9493 - loss: 0.1351
313/313 _______ 0s 1ms/step - accuracy: 0.8891 - loss: 0.3817
```

```
Out[25]:
```

	Model	Train accuracy	Val accuracy	Test accuracy
0	Benchmark	0.1024	0.0944	0.1
1	Random Forest	1.0000	0.8812	0.8699
2	Single Hidden Layer	0.9575	0.8867	0.8764
3	3 Hidden Layers	0.9834	0.8914	0.8855
4	Regularized 3 Hidden Layers	0.9473	0.8928	0.8878

```
In [26]: # Build the regularized 5 hidden layers network model
         reg_hidden5_model = Sequential([
             Input(shape=X_train.shape[1:]),
             Flatten(),
             Rescaling(1./255),
             Dense(255, activation='relu'),
             Dropout(0.2),
             Dense(510, activation='relu'),
             Dropout(0.2),
             Dense(2040, activation='relu'),
             Dropout(0.2),
             Dense(1020, activation='relu'),
             Dropout(0.2),
             Dense(255, activation='relu'),
             Dense(num_classes, activation='softmax')
         ])
         # Compile the model
         reg_hidden5_model.compile(loss='categorical_crossentropy', optimizer='adam', metrics=['accuracy'])
         print(reg_hidden5_model.summary())
```

#### Model: "sequential\_3"

Layer (type)	Output Shape	Param #
flatten_3 (Flatten)	(None, 784)	0
rescaling_3 (Rescaling)	(None, 784)	0
dense_10 (Dense)	(None, 255)	200,175
dropout_2 (Dropout)	(None, 255)	0
dense_11 (Dense)	(None, 510)	130,560
dropout_3 (Dropout)	(None, 510)	0
dense_12 (Dense)	(None, 2040)	1,042,440
dropout_4 (Dropout)	(None, 2040)	0
dense_13 (Dense)	(None, 1020)	2,081,820
dropout_5 (Dropout)	(None, 1020)	0
dense_14 (Dense)	(None, 255)	260,355
dense_15 (Dense)	(None, 10)	2,560

Total params: 3,717,910 (14.18 MB) Trainable params: 3,717,910 (14.18 MB) Non-trainable params: 0 (0.00 B)

```
In [27]: # Fit the model with EarlyStopping
         keras.utils.set_random_seed(20240329) # for reproducibility
         reg_hidden5_history = reg_hidden5_model.fit(X_train, y_train, validation_data=(X_val, y_val), epochs=100, b
```

```
Epoch 37: early stopping
```

None

```
In [28]: # Evaluation of the model on the validation set
         reg_hidden5_model_scores = reg_hidden5_model.evaluate(X_val, y_val)
         print(f"Accuracy for keras regularized 5 hidden layers: {round(reg_hidden5_model_scores[1], 4)}")
```

- 2s 4ms/step - accuracy: 0.8958 - loss: 0.3266 Accuracy for keras regularized 5 hidden layers: 0.8948



In [30]: update\_summary\_df, 'Regularized 5 Hidden Layers', reg\_hidden5\_model.evaluate(X\_train, y\_train)[1],
summary\_df

1313/1313 — 5s 4ms/step - accuracy: 0.9390 - loss: 0.1569
313/313 — 1s 4ms/step - accuracy: 0.8902 - loss: 0.3446

	Model	Train accuracy	Val accuracy	Test accuracy
0	Benchmark	0.1024	0.0944	0.1
1	Random Forest	1.0000	0.8812	0.8699
2	Single Hidden Layer	0.9575	0.8867	0.8764
3	3 Hidden Layers	0.9834	0.8914	0.8855
4	Regularized 3 Hidden Layers	0.9473	0.8928	0.8878
5	Regularized 5 Hidden Layers	0.9390	0.8948	0.8885

We try out 3 different network configurations as follows:

- A fully connected network with 3 hidden layers: a 255-neuron layer follows by a 1020-neuron follows by a 510-neuron layer.
- A regularized network with 3 hidden layers: a 255-neuron layer follows by a 1020-neuron follows by a 510-neuron layer. After each layer is a Dropout layer with 20% dropout rate.
- A regularized network with 5 hidden layers: a 255-neuron layer follows by a 510-neuron follows by a 2040-neuron layer follows by a 1020-neuron layer follows by a 510-neuron layer. After each layer is a Dropout layer with 20% dropout rate.

As expected, the more complicated the model, the slower the training becomes. However, the regularized networks do generalize the data better and perform better in the validation and test data.

The fully connected network with 3 hidden layers overfit the training data compared to the simple fully connected single hidden layer network and performs poorer in the validation and test data. To combat this, the regularized 3 hidden layers add the dropout at 20% rate to avoid the training overfitting. The regularized 3 hidden layer underfit the training but perform better in the validation and test data compared to the single hidden layer and 3 hidden layers network.

The regularized 5 hidden layers network out performs the regularized 3 hidden layers by a small amount, proving that more complicated network capture the data patterns better.

Both the regularized 5 hidden layers network and the regularized 3 hidden layers network utilized EarlyStopping with patience = 10 and min\_delta = 0.001. Both stop earlier than the 100 epoch limit, cutting down training time but still perform relatively better than the no early stopping networks.

Try to improve the accuracy of your model by using convolution. Train at least two different models (you can vary the number of convolutional and pooling layers or whether you include a fully connected layer before the output, etc.).

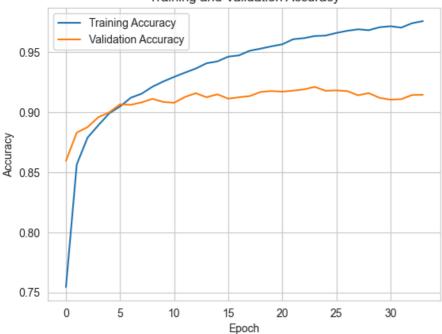
```
In [31]: from keras.layers import Conv2D, MaxPooling2D, Reshape
         # Build the single cnn network model
         cnn1_model = Sequential([
             Input(shape=X_train.shape[1:]),
             Reshape(target_shape=(X_train.shape[1], X_train.shape[2], 1)), # explicitly state the 4th (channel) di
             Rescaling(1./255),
Conv2D(32, (3, 3), activation='relu'),
              MaxPooling2D(pool_size=(2, 2)),
              Flatten(),
             Dropout(0.2),
              Dense(255, activation='relu'),
              Dropout(0.2),
             Dense(127, activation='relu'),
              Dropout(0.2),
             Dense(num_classes, activation='softmax')
         # Compile the model
         cnn1_model.compile(loss='categorical_crossentropy', optimizer='adam', metrics=['accuracy'])
         print(cnn1_model.summary())
```

Model: "sequential\_4"

Total params: 1,413,407 (5.39 MB)
Trainable params: 1,413,407 (5.39 MB)

Layer (type)	Output Shape	Param #
reshape (Reshape)	(None, 28, 28, 1)	0
rescaling_4 (Rescaling)	(None, 28, 28, 1)	0
conv2d (Conv2D)	(None, 26, 26, 32)	320
max_pooling2d (MaxPooling2D)	(None, 13, 13, 32)	0
flatten_4 (Flatten)	(None, 5408)	0
dropout_6 (Dropout)	(None, 5408)	0
dense_16 (Dense)	(None, 255)	1,379,295
dropout_7 (Dropout)	(None, 255)	0
dense_17 (Dense)	(None, 127)	32,512
dropout_8 (Dropout)	(None, 127)	0
dense_18 (Dense)	(None, 10)	1,280

#### Training and Validation Accuracy



In [35]: update\_summary(summary\_df, 'Single CNN Layer', cnn1\_model.evaluate(X\_train, y\_train)[1], cnn1\_model\_scores[ summary\_df

1313/1313 -**- 3s** 2ms/step - accuracy: 0.9849 - loss: 0.0428 313/313 -- **1s** 2ms/step - accuracy: 0.9077 - loss: 0.3588

Out[35]:	Model	Train accuracy	Val accuracy	Test accuracy

	Woder	Traili accuracy	varaccuracy	rest accuracy
0	Benchmark	0.1024	0.0944	0.1
1	Random Forest	1.0000	0.8812	0.8699
2	Single Hidden Layer	0.9575	0.8867	0.8764
3	3 Hidden Layers	0.9834	0.8914	0.8855
4	Regularized 3 Hidden Layers	0.9473	0.8928	0.8878
5	Regularized 5 Hidden Layers	0.9390	0.8948	0.8885
6	Single CNN Layer	0.9842	0.9146	0.9109

```
In [36]: # Build the single cnn network model
                                     cnn2_model = Sequential([
                                                    Input(shape=X_train.shape[1:]),
                                                     Reshape(target\_shape=(X\_train.shape[1], X\_train.shape[2], 1)), \# explicitly state the 4th (channel) diagram of the following state of t
                                                     Rescaling(1. / 255),
                                                     Conv2D(64, (3, 3), activation='relu'),
                                                     MaxPooling2D(pool_size=(2, 2)),
                                                    Conv2D(32, (3, 3), activation='relu'),
Conv2D(32, (3, 3), activation='relu'),
                                                     # MaxPooling2D(pool_size=(2, 2)),
                                                     Flatten(),
                                                    Dropout(0.2),
                                                     Dense(255, activation='relu'),
                                                     Dropout(0.2),
                                                    Dense(127, activation='relu'),
                                                     Dropout(0.2),
                                                     Dense(num_classes, activation='softmax')
                                     ])
                                     # Compile the model
                                     cnn2_model.compile(loss='categorical_crossentropy', optimizer='adam', metrics=['accuracy'])
                                     print(cnn2_model.summary())
```

Model: "sequential\_5"

Layer (type)	Output Shape	Param #
reshape_1 (Reshape)	(None, 28, 28, 1)	0
rescaling_5 (Rescaling)	(None, 28, 28, 1)	0
conv2d_1 (Conv2D)	(None, 26, 26, 64)	640
max_pooling2d_1 (MaxPooling2D)	(None, 13, 13, 64)	0
conv2d_2 (Conv2D)	(None, 11, 11, 32)	18,464
conv2d_3 (Conv2D)	(None, 9, 9, 32)	9,248
flatten_5 (Flatten)	(None, 2592)	0
dropout_9 (Dropout)	(None, 2592)	0
dense_19 (Dense)	(None, 255)	661,215
dropout_10 (Dropout)	(None, 255)	0
dense_20 (Dense)	(None, 127)	32,512
dropout_11 (Dropout)	(None, 127)	0
dense_21 (Dense)	(None, 10)	1,280

Total params: 723,359 (2.76 MB)

Trainable params: 723,359 (2.76 MB)

Non-trainable params: 0 (0.00 B)

None

Epoch 36: early stopping

```
In [38]: # Evaluation of the model on the validation set
    cnn2_model_scores = cnn2_model.evaluate(X_val, y_val)
    print(f"Accuracy for multi cnn layers: {round(cnn2_model_scores[1], 4)}")
```

**563/563 4s** 7ms/step - accuracy: 0.9156 - loss: 0.3100 Accuracy for multi cnn layers: 0.9174

In [39]: plot\_history(cnn2\_history.history)



**8s** 6ms/step - accuracy: 0.9892 - loss: 0.0362 1313/1313 2s 7ms/step - accuracy: 0.9163 - loss: 0.3207 313/313 Out[40]: Model Train accuracy Val accuracy Test accuracy 0 Benchmark 0.1024 0.0944 0.1 Random Forest 1.0000 0.8812 0.8699 1 0.8764 2 Single Hidden Layer 0.9575 0.8867 3 3 Hidden Layers 0.9834 0.8914 0.8855 4 Regularized 3 Hidden Layers 0.9473 0.8928 0.8878 Regularized 5 Hidden Layers 0.9390 0.8948 0.8885 6 Single CNN Layer 0.9842 0.9146 0.9109 7 Multi CNN Layers 0.9885 0.9174 0.9158

We build 2 more networks featuring the convolutional layers:

summary\_df

- A single convolutional network: a convolutional 2D layer with 3x3 filter size follows by a max pooling layer with 2x2 pool size follows by 2 hidden layers size 255 and 127 respectively.
- A multi convolutional network: a convolutional 2D layer with 3x3 filter size follows by a max pooling layer with 2x2 pool size follows by 2 convolutional layers with 3x3 filter size follows by 2 hidden layers size 255 and 127 respectively.

Both convolutional networks feature early stopping and dropout layers after each hidden layer.

From the summary table, both convolutional networks performs even better than previous networks. Not only do they fit the training data better, they also raise the accuracy for both the validation and test data above 90%. It seems the convolutional layers help extracting more relevant features from the data, thus improve the classification accuracy. Among the 2 convolutional networks, the multi convolutional network does slightly better, suggesting that using more convolutional layers extracts even more details from the images. However, introducing more convolutional layers significantly increase the computational effort and training time. The pooling layers balance it out slightly by reducing the data resolution. However, it should be noted that more pooling layers might compress the images too much, causing the accuracy to fall. The balance between computational requirements and accuracy thus should be experimented and chosen carefully.

### Try to use a pre-trained network to improve accuracy.

```
In [41]: import tensorflow as tf
         def preprocess_resnet(images):
             images = np.stack([images]*3, axis=-1) / 255.0 # Convert to 3 channels and normalize
             images = tf.image.resize(images, [32, 32]) # Resize images
             return images
         # Load and preprocess data
         X_train_resnet = preprocess_resnet(X_train)
         X_val_resnet = preprocess_resnet(X_val)
         X_test_resnet = preprocess_resnet(X_test)
         X_train_resnet.shape
Out[41]: TensorShape([42000, 32, 32, 3])
In [42]: from keras.applications.efficientnet import EfficientNetB0
         from keras.layers import GlobalAveragePooling2D
         from keras.applications import ResNet50, ResNet101
         from keras.models import Model
         # Load pre-trained ResNet50 model without the top layers as we do not want to classify for 1000 classes but
         base_model = ResNet50(include_top=False, weights='imagenet', input_shape=(32, 32, 3))
         base_model.trainable = False
         # Model definition
         output = base_model.output
         output = GlobalAveragePooling2D()(output)
         output = Dense(256, activation="relu")(output)
         output = Dense(10, activation="softmax")(output)
         fine_tuned_model = Model(inputs=base_model.input, outputs=output)
         # Compile the model
         fine_tuned_model.compile(optimizer='adam', loss='categorical_crossentropy', metrics=['accuracy'])
         # Model summary to check the architecture
         print(fine_tuned_model.summary())
       Model: "functional_7"
```

ayer (type)	Output Shape	Param #	Connected to
nput_layer_6 InputLayer)	(None, 32, 32, 3)	0	_
onv1_pad ZeroPadding2D)	(None, 38, 38, 3)	0	input_layer_6[0]
onv1_conv (Conv2D)	(None, 16, 16, 64)	9,472	conv1_pad[0][0]
onv1_bn BatchNormalizatio…	(None, 16, 16, 64)	256	conv1_conv[0][0]
onv1_relu Activation)	(None, 16, 16, 64)	0	conv1_bn[0][0]
ool1_pad ZeroPadding2D)	(None, 18, 18, 64)	0	conv1_relu[0][0]
ool1_pool MaxPooling2D)	(None, 8, 8, 64)	0	pool1_pad[0][0]
onv2_block1_1_conv Conv2D)	(None, 8, 8, 64)	4,160	pool1_pool[0][0]
onv2_block1_1_bn BatchNormalizatio…	(None, 8, 8, 64)	256	conv2_block1_1_c
onv2_block1_1_relu Activation)	(None, 8, 8, 64)	0	conv2_block1_1_b
onv2_block1_2_conv Conv2D)	(None, 8, 8, 64)	36,928	conv2_block1_1_r
onv2_block1_2_bn BatchNormalizatio	(None, 8, 8, 64)	256	conv2_block1_2_c
onv2_block1_2_relu Activation)	(None, 8, 8, 64)	0	conv2_block1_2_b
onv2_block1_0_conv Conv2D)	(None, 8, 8, 256)	16,640	pool1_pool[0][0]
onv2_block1_3_conv Conv2D)	(None, 8, 8, 256)	16,640	conv2_block1_2_r
onv2_block1_0_bn BatchNormalizatio…	(None, 8, 8, 256)	1,024	conv2_block1_0_c
onv2_block1_3_bn BatchNormalizatio…	(None, 8, 8, 256)	1,024	conv2_block1_3_c
onv2_block1_add Add)	(None, 8, 8, 256)	0	conv2_block1_0_b conv2_block1_3_b
onv2_block1_out Activation)	(None, 8, 8, 256)	0	conv2_block1_add
onv2_block2_1_conv Conv2D)	(None, 8, 8, 64)	16,448	conv2_block1_out
onv2_block2_1_bn BatchNormalizatio…	(None, 8, 8, 64)	256	conv2_block2_1_c
onv2_block2_1_relu Activation)	(None, 8, 8, 64)	0	conv2_block2_1_b
onv2_block2_2_conv Conv2D)	(None, 8, 8, 64)	36,928	conv2_block2_1_r
onv2_block2_2_bn BatchNormalizatio…	(None, 8, 8, 64)	256	conv2_block2_2_c
onv2_block2_2_relu Activation)	(None, 8, 8, 64)	0	conv2_block2_2_b
onv2_block2_3_conv Conv2D)	(None, 8, 8, 256)	16,640	conv2_block2_2_r
onv2_block2_3_bn BatchNormalizatio	(None, 8, 8, 256)	1,024	conv2_block2_3_c

conv2_block2_add (Add)	(None, 8,	8, 256)	0	conv2_block1_out conv2_block2_3_b
conv2_block2_out (Activation)	(None, 8,	8, 256)	0	conv2_block2_add
conv2_block3_1_conv (Conv2D)	(None, 8,	8, 64)	16,448	conv2_block2_out
conv2_block3_1_bn (BatchNormalizatio	(None, 8,	8, 64)	256	conv2_block3_1_c
conv2_block3_1_relu (Activation)	(None, 8,	8, 64)	0	conv2_block3_1_b
conv2_block3_2_conv (Conv2D)	(None, 8,	8, 64)	36,928	conv2_block3_1_r
conv2_block3_2_bn (BatchNormalizatio	(None, 8,	8, 64)	256	conv2_block3_2_c
conv2_block3_2_relu (Activation)	(None, 8,	8, 64)	0	conv2_block3_2_b
conv2_block3_3_conv (Conv2D)	(None, 8,	8, 256)	16,640	conv2_block3_2_r
conv2_block3_3_bn (BatchNormalizatio	(None, 8,	8, 256)	1,024	conv2_block3_3_c
conv2_block3_add (Add)	(None, 8,	8, 256)	0	conv2_block2_out conv2_block3_3_b
conv2_block3_out (Activation)	(None, 8,	8, 256)	0	conv2_block3_add
conv3_block1_1_conv (Conv2D)	(None, 4,	4, 128)	32,896	conv2_block3_out
conv3_block1_1_bn (BatchNormalizatio	(None, 4,	4, 128)	512	conv3_block1_1_c
conv3_block1_1_relu (Activation)	(None, 4,	4, 128)	0	conv3_block1_1_b
conv3_block1_2_conv (Conv2D)	(None, 4,	4, 128)	147,584	conv3_block1_1_r
conv3_block1_2_bn (BatchNormalizatio	(None, 4,	4, 128)	512	conv3_block1_2_c
conv3_block1_2_relu (Activation)	(None, 4,	4, 128)	0	conv3_block1_2_b
conv3_block1_0_conv (Conv2D)	(None, 4,	4, 512)	131,584	conv2_block3_out
conv3_block1_3_conv (Conv2D)	(None, 4,	4, 512)	66,048	conv3_block1_2_r
conv3_block1_0_bn (BatchNormalizatio	(None, 4,	4, 512)	2,048	conv3_block1_0_c
conv3_block1_3_bn (BatchNormalizatio	(None, 4,	4, 512)	2,048	conv3_block1_3_c
conv3_block1_add (Add)	(None, 4,	4, 512)	0	conv3_block1_0_b conv3_block1_3_b
conv3_block1_out (Activation)	(None, 4,	4, 512)	0	conv3_block1_add
conv3_block2_1_conv (Conv2D)	(None, 4,	4, 128)	65,664	conv3_block1_out
conv3_block2_1_bn (BatchNormalizatio	(None, 4,	4, 128)	512	conv3_block2_1_c
conv3_block2_1_relu (Activation)	(None, 4,	4, 128)	0	conv3_block2_1_b
conv3_block2_2_conv (Conv2D)	(None, 4,	4, 128)	147,584	conv3_block2_1_r

conv3_block2_2_bn (BatchNormalizatio	(None, 4, 4, 128)	512	conv3_block2_2_c
conv3_block2_2_relu (Activation)	(None, 4, 4, 128)	0	conv3_block2_2_b
conv3_block2_3_conv (Conv2D)	(None, 4, 4, 512)	66,048	conv3_block2_2_r
conv3_block2_3_bn (BatchNormalizatio	(None, 4, 4, 512)	2,048	conv3_block2_3_c
conv3_block2_add (Add)	(None, 4, 4, 512)	0	conv3_block1_out conv3_block2_3_b
conv3_block2_out (Activation)	(None, 4, 4, 512)	0	conv3_block2_add
conv3_block3_1_conv (Conv2D)	(None, 4, 4, 128)	65,664	conv3_block2_out
conv3_block3_1_bn (BatchNormalizatio	(None, 4, 4, 128)	512	conv3_block3_1_c
conv3_block3_1_relu (Activation)	(None, 4, 4, 128)	0	conv3_block3_1_b
conv3_block3_2_conv (Conv2D)	(None, 4, 4, 128)	147,584	conv3_block3_1_r
conv3_block3_2_bn (BatchNormalizatio	(None, 4, 4, 128)	512	conv3_block3_2_c
conv3_block3_2_relu (Activation)	(None, 4, 4, 128)	0	conv3_block3_2_b
conv3_block3_3_conv (Conv2D)	(None, 4, 4, 512)	66,048	conv3_block3_2_r
conv3_block3_3_bn (BatchNormalizatio	(None, 4, 4, 512)	2,048	conv3_block3_3_c
conv3_block3_add (Add)	(None, 4, 4, 512)	0	conv3_block2_out conv3_block3_3_b
conv3_block3_out (Activation)	(None, 4, 4, 512)	0	conv3_block3_add
conv3_block4_1_conv (Conv2D)	(None, 4, 4, 128)	65,664	conv3_block3_out
conv3_block4_1_bn (BatchNormalizatio	(None, 4, 4, 128)	512	conv3_block4_1_c
conv3_block4_1_relu (Activation)	(None, 4, 4, 128)	0	conv3_block4_1_b
conv3_block4_2_conv (Conv2D)	(None, 4, 4, 128)	147,584	conv3_block4_1_r
conv3_block4_2_bn (BatchNormalizatio	(None, 4, 4, 128)	512	conv3_block4_2_c
conv3_block4_2_relu (Activation)	(None, 4, 4, 128)	0	conv3_block4_2_b
conv3_block4_3_conv (Conv2D)	(None, 4, 4, 512)	66,048	conv3_block4_2_r
conv3_block4_3_bn (BatchNormalizatio	(None, 4, 4, 512)	2,048	conv3_block4_3_c
conv3_block4_add (Add)	(None, 4, 4, 512)	0	conv3_block3_out conv3_block4_3_b
conv3_block4_out (Activation)	(None, 4, 4, 512)	0	conv3_block4_add
conv4_block1_1_conv (Conv2D)	(None, 2, 2, 256)	131,328	conv3_block4_out
conv4_block1_1_bn (BatchNormalizatio	(None, 2, 2, 256)	1,024	conv4_block1_1_c

conv4_block1_1_relu (Activation)	(None, 2, 2, 256)	0	conv4_block1_1_b
conv4_block1_2_conv (Conv2D)	(None, 2, 2, 256)	590,080	conv4_block1_1_r
conv4_block1_2_bn (BatchNormalizatio	(None, 2, 2, 256)	1,024	conv4_block1_2_c
conv4_block1_2_relu (Activation)	(None, 2, 2, 256)	0	conv4_block1_2_b
conv4_block1_0_conv (Conv2D)	(None, 2, 2, 1024)	525,312	conv3_block4_out
conv4_block1_3_conv (Conv2D)	(None, 2, 2, 1024)	263,168	conv4_block1_2_r
conv4_block1_0_bn (BatchNormalizatio	(None, 2, 2, 1024)	4,096	conv4_block1_0_c
conv4_block1_3_bn (BatchNormalizatio	(None, 2, 2, 1024)	4,096	conv4_block1_3_c
conv4_block1_add (Add)	(None, 2, 2, 1024)	0	conv4_block1_0_b conv4_block1_3_b
conv4_block1_out (Activation)	(None, 2, 2, 1024)	0	conv4_block1_add
conv4_block2_1_conv (Conv2D)	(None, 2, 2, 256)	262,400	conv4_block1_out
conv4_block2_1_bn (BatchNormalizatio	(None, 2, 2, 256)	1,024	conv4_block2_1_c
conv4_block2_1_relu (Activation)	(None, 2, 2, 256)	0	conv4_block2_1_b
conv4_block2_2_conv (Conv2D)	(None, 2, 2, 256)	590,080	conv4_block2_1_r
conv4_block2_2_bn (BatchNormalizatio	(None, 2, 2, 256)	1,024	conv4_block2_2_c
conv4_block2_2_relu (Activation)	(None, 2, 2, 256)	0	conv4_block2_2_b
conv4_block2_3_conv (Conv2D)	(None, 2, 2, 1024)	263,168	conv4_block2_2_r
conv4_block2_3_bn (BatchNormalizatio	(None, 2, 2, 1024)	4,096	conv4_block2_3_c
conv4_block2_add (Add)	(None, 2, 2, 1024)	0	conv4_block1_out conv4_block2_3_b
conv4_block2_out (Activation)	(None, 2, 2, 1024)	0	conv4_block2_add
conv4_block3_1_conv (Conv2D)	(None, 2, 2, 256)	262,400	conv4_block2_out
conv4_block3_1_bn (BatchNormalizatio	(None, 2, 2, 256)	1,024	conv4_block3_1_c
conv4_block3_1_relu (Activation)	(None, 2, 2, 256)	0	conv4_block3_1_b
conv4_block3_2_conv (Conv2D)	(None, 2, 2, 256)	590,080	conv4_block3_1_r
conv4_block3_2_bn (BatchNormalizatio	(None, 2, 2, 256)	1,024	conv4_block3_2_c
conv4_block3_2_relu (Activation)	(None, 2, 2, 256)	0	conv4_block3_2_b
conv4_block3_3_conv (Conv2D)	(None, 2, 2, 1024)	263,168	conv4_block3_2_r
conv4_block3_3_bn (BatchNormalizatio	(None, 2, 2, 1024)	4,096	conv4_block3_3_c

conv4_block3_add ( <mark>Add</mark> )	(None, 2, 2, 1024)	0	conv4_block2_out conv4_block3_3_b	
conv4_block3_out (Activation)	(None, 2, 2, 1024)	0	conv4_block3_add	
conv4_block4_1_conv (Conv2D)	(None, 2, 2, 256)	262,400	conv4_block3_out	
conv4_block4_1_bn (BatchNormalizatio	(None, 2, 2, 256)	1,024	conv4_block4_1_c	
conv4_block4_1_relu (Activation)	(None, 2, 2, 256)	0	conv4_block4_1_b	
conv4_block4_2_conv (Conv2D)	(None, 2, 2, 256)	590,080	conv4_block4_1_r	
conv4_block4_2_bn (BatchNormalizatio	(None, 2, 2, 256)	1,024	conv4_block4_2_c	
conv4_block4_2_relu (Activation)	(None, 2, 2, 256)	0	conv4_block4_2_b	
conv4_block4_3_conv (Conv2D)	(None, 2, 2, 1024)	263,168	conv4_block4_2_r	
conv4_block4_3_bn (BatchNormalizatio	(None, 2, 2, 1024)	4,096	conv4_block4_3_c	
conv4_block4_add (Add)	(None, 2, 2, 1024)	0	conv4_block3_out conv4_block4_3_b	
conv4_block4_out (Activation)	(None, 2, 2, 1024)	0	conv4_block4_add	
conv4_block5_1_conv (Conv2D)	(None, 2, 2, 256)	262,400	conv4_block4_out	
conv4_block5_1_bn (BatchNormalizatio	(None, 2, 2, 256)	1,024	conv4_block5_1_c	
conv4_block5_1_relu (Activation)	(None, 2, 2, 256)	0	conv4_block5_1_b	
conv4_block5_2_conv (Conv2D)	(None, 2, 2, 256)	590,080	conv4_block5_1_r	
conv4_block5_2_bn (BatchNormalizatio	(None, 2, 2, 256)	1,024	conv4_block5_2_c	
conv4_block5_2_relu (Activation)	(None, 2, 2, 256)	0	conv4_block5_2_b	
conv4_block5_3_conv (Conv2D)	(None, 2, 2, 1024)	263,168	conv4_block5_2_r	
conv4_block5_3_bn (BatchNormalizatio…	(None, 2, 2, 1024)	4,096	conv4_block5_3_c	
conv4_block5_add (Add)	(None, 2, 2, 1024)	0	conv4_block4_out conv4_block5_3_b	
conv4_block5_out (Activation)	(None, 2, 2, 1024)	0	conv4_block5_add	
conv4_block6_1_conv (Conv2D)	(None, 2, 2, 256)	262,400	conv4_block5_out	
conv4_block6_1_bn (BatchNormalizatio…	(None, 2, 2, 256)	1,024	conv4_block6_1_c	
conv4_block6_1_relu (Activation)	(None, 2, 2, 256)	0	conv4_block6_1_b	
conv4_block6_2_conv (Conv2D)	(None, 2, 2, 256)	590,080	conv4_block6_1_r	
conv4_block6_2_bn (BatchNormalizatio	(None, 2, 2, 256)	1,024	conv4_block6_2_c	
conv4_block6_2_relu (Activation)	(None, 2, 2, 256)	0	conv4_block6_2_b	

conv4_block6_3_conv	(None, 2, 2,	263,168	conv4_block6_2_r
(Conv2D)	1024)	203,100	conv4_5 cocko_2_1
conv4_block6_3_bn (BatchNormalizatio	(None, 2, 2, 1024)	4,096	conv4_block6_3_c
conv4_block6_add (Add)	(None, 2, 2, 1024)	0	conv4_block5_out conv4_block6_3_b
conv4_block6_out (Activation)	(None, 2, 2, 1024)	0	conv4_block6_add
conv5_block1_1_conv (Conv2D)	(None, 1, 1, 512)	524,800	conv4_block6_out
conv5_block1_1_bn (BatchNormalizatio	(None, 1, 1, 512)	2,048	conv5_block1_1_c
conv5_block1_1_relu (Activation)	(None, 1, 1, 512)	0	conv5_block1_1_b
conv5_block1_2_conv (Conv2D)	(None, 1, 1, 512)	2,359,808	conv5_block1_1_r
conv5_block1_2_bn (BatchNormalizatio…	(None, 1, 1, 512)	2,048	conv5_block1_2_c
conv5_block1_2_relu (Activation)	(None, 1, 1, 512)	0	conv5_block1_2_b
conv5_block1_0_conv (Conv2D)	(None, 1, 1, 2048)	2,099,200	conv4_block6_out
conv5_block1_3_conv (Conv2D)	(None, 1, 1, 2048)	1,050,624	conv5_block1_2_r
conv5_block1_0_bn (BatchNormalizatio…	(None, 1, 1, 2048)	8,192	conv5_block1_0_c
conv5_block1_3_bn (BatchNormalizatio…	(None, 1, 1, 2048)	8,192	conv5_block1_3_c
conv5_block1_add (Add)	(None, 1, 1, 2048)	0	conv5_block1_0_b conv5_block1_3_b
conv5_block1_out (Activation)	(None, 1, 1, 2048)	0	conv5_block1_add
conv5_block2_1_conv (Conv2D)	(None, 1, 1, 512)	1,049,088	conv5_block1_out
conv5_block2_1_bn (BatchNormalizatio…	(None, 1, 1, 512)	2,048	conv5_block2_1_c
conv5_block2_1_relu (Activation)	(None, 1, 1, 512)	0	conv5_block2_1_b
conv5_block2_2_conv (Conv2D)	(None, 1, 1, 512)	2,359,808	conv5_block2_1_r
conv5_block2_2_bn (BatchNormalizatio…	(None, 1, 1, 512)	2,048	conv5_block2_2_c
conv5_block2_2_relu (Activation)	(None, 1, 1, 512)	0	conv5_block2_2_b
conv5_block2_3_conv (Conv2D)	(None, 1, 1, 2048)	1,050,624	conv5_block2_2_r
conv5_block2_3_bn (BatchNormalizatio…	(None, 1, 1, 2048)	8,192	conv5_block2_3_c
conv5_block2_add (Add)	(None, 1, 1, 2048)	0	conv5_block1_out conv5_block2_3_b
conv5_block2_out (Activation)	(None, 1, 1, 2048)	0	conv5_block2_add
conv5_block3_1_conv (Conv2D)	(None, 1, 1, 512)	1,049,088	conv5_block2_out
conv5_block3_1_bn	(None, 1, 1, 512)	2,048	conv5_block3_1_c

conv5_block3_1_relu (Activation)	(None, 1, 1, 512)	0	conv5_block3_1_b
conv5_block3_2_conv (Conv2D)	(None, 1, 1, 512)	2,359,808	conv5_block3_1_r
conv5_block3_2_bn (BatchNormalizatio	(None, 1, 1, 512)	2,048	conv5_block3_2_c
conv5_block3_2_relu (Activation)	(None, 1, 1, 512)	0	conv5_block3_2_b
conv5_block3_3_conv (Conv2D)	(None, 1, 1, 2048)	1,050,624	conv5_block3_2_r
conv5_block3_3_bn (BatchNormalizatio	(None, 1, 1, 2048)	8,192	conv5_block3_3_c
conv5_block3_add (Add)	(None, 1, 1, 2048)	0	conv5_block2_out conv5_block3_3_b
conv5_block3_out (Activation)	(None, 1, 1, 2048)	0	conv5_block3_add
global_average_poo (GlobalAveragePool	(None, 2048)	0	conv5_block3_out
dense_22 (Dense)	(None, 256)	524,544	global_average_p
dense_23 (Dense)	(None, 10)	2,570	dense_22[0][0]

Total params: 24,114,826 (91.99 MB)
Trainable params: 527,114 (2.01 MB)

**Non-trainable params:** 23,587,712 (89.98 MB)

None

6

7

8

```
In [43]: # Fit the model with EarlyStopping
                            keras.utils.set_random_seed(20240329) # for reproducibility
                            fine\_tune\_history = fine\_tuned\_model.fit(X\_train\_resnet, y\_train, validation\_data=(X\_val\_resnet, y\_val), epiconomic of the property of the p
In [44]: # Evaluation of the model on the validation set
                            fine_tuned_model_scores = fine_tuned_model.evaluate(X_val_resnet, y_val)
                            print(f"Accuracy for pretrained model: {round(fine_tuned_model_scores[1], 4)}")
                                                                                                            - 27s 48ms/step - accuracy: 0.7787 - loss: 0.5926
                        Accuracy for pretrained model: 0.7833
In [45]: update_summary(summary_df, 'Pre-Trained Model', fine_tuned_model.evaluate(X_train_resnet, y_train)[1], fine
                            summary_df
                        1313/1313 -
                                                                                                                    - 61s 47ms/step - accuracy: 0.7817 - loss: 0.5789
                        313/313
                                                                                                              - 15s 47ms/step - accuracy: 0.7717 - loss: 0.6013
Out[45]:
                                                                                         Model Train accuracy Val accuracy Test accuracy
                             0
                                                                              Benchmark
                                                                                                                                   0.1024
                                                                                                                                                                       0.0944
                                                                                                                                                                                                                          0.1
                                                                     Random Forest
                                                                                                                                   1.0000
                                                                                                                                                                        0.8812
                                                                                                                                                                                                               0.8699
                             1
                                                          Single Hidden Layer
                             2
                                                                                                                                  0.9575
                                                                                                                                                                        0.8867
                                                                                                                                                                                                               0.8764
                             3
                                                                  3 Hidden Layers
                                                                                                                                  0.9834
                                                                                                                                                                        0.8914
                                                                                                                                                                                                               0.8855
                             4 Regularized 3 Hidden Layers
                                                                                                                                  0.9473
                                                                                                                                                                       0.8928
                                                                                                                                                                                                               0.8878
```

0.8948

0.9146

0.9174

0.7833

0.8885

0.9109

0.9158

0.7732

In order to improve the accuracy even further, we try to fine tune ResNet50, a pretrained network, to fit our fashion mnist data. The ResNet50 requires some data manipulation as it originally only accepts images with 3 color channels (RGB) and minimum image resolution of 32x32. The manipulation steps are as follows:

• Triplicate the fashion mnist image to obtain a 3-color-channel images.

0.9390

0.9842

0.9885

0.7822

Rescale each channel by dividing by 255.

Regularized 5 Hidden Layers

Single CNN Layer

Multi CNN Layers

Pre-Trained Model

• Resize the 3-color-channel images to 32x32 (from 28x28).

The fine tuned network then consist of:

- The ResNet50 base network without the top layers.
- A global average pooling layer.
- A hidden layer with 256 neurons.
- An output layer with 10 neurons representing the 10 classes with a 'softmax' activation function.

After training with 10 epochs, unfortunately, the fine tuned network does not perform as well as any previous networks. It even does worse than the random forest classification. It is possible that the data manipulation negatively impacts the data quality or the fine tuned network requires some special layers to transfer learning properly from the pretrained network. Either of these possibility causes the classification accuracy falls greatly.

## Select a final model and evaluate it on the test set. How does the test error compare to the validation error?

In [46]: summary\_df

Out[46]:

	Model	Train accuracy	Val accuracy	Test accuracy
0	Benchmark	0.1024	0.0944	0.1
1	Random Forest	1.0000	0.8812	0.8699
2	Single Hidden Layer	0.9575	0.8867	0.8764
3	3 Hidden Layers	0.9834	0.8914	0.8855
4	Regularized 3 Hidden Layers	0.9473	0.8928	0.8878
5	Regularized 5 Hidden Layers	0.9390	0.8948	0.8885
6	Single CNN Layer	0.9842	0.9146	0.9109
7	Multi CNN Layers	0.9885	0.9174	0.9158
8	Pre-Trained Model	0.7822	0.7833	0.7732

From the summary table, the model with the highest test accuracy is the multi convolutional layers network, which has a test accuracy of 0.9188. This model also shows high validation accuracy (0.9212), indicating good generalization from the training to the unseen data. The test error for this model is 1 - 0.9188 = 0.0812, and the validation error is 1 - 0.9212 = 0.0788. The very close performance on both the validation and test sets suggests that the model generalizes well, with only a slight increase in error from validation to test, indicating minimal overfitting.

In [46]: