

# MIDTERM ESSAY

## Discrete Structures

### Regulations

You should solve and submit this essay to your theory Google classroom within 14 days, from Apr. 1<sup>st</sup>, 2024, to the end of Apr. 14<sup>th</sup>, 2024. Late submission is not accepted.

This is an individual midterm essay. Any case of plagiarism will get 0.

Students need to solve all the problems and submit a doc/docx file named by your StudentID, using our faculty's format. English is required for high-quality classes. Format violations will cost from 10% to 50% of your total scores.

### Problems

#### Part 1

Students need to solve all the problems in part 1.

#### Problem 1: Password (1 score)

A hacker is trying to hack a password. He knows that this password has 3 characters, each of which is a distinct number from 1 to 9. He also learns from his trials that:

- a. 472: one number is correct but in an incorrect position.
- b. 581: one number is correct but in an incorrect position.
- c. 483: one number is correct and in the correct position.
- d. 317: two numbers are correct but in incorrect positions.
- e. 956: all numbers are incorrect.

Please help him to find the password with good reasoning.

#### Problem 2: Conditional statements (1 score)

State the converse, inverse, contrapositive, and non-conditional-form negation of these conditional statements in natural language:

- a. "If a man, holding a belief which he was taught in childhood or persuaded of afterwards, keeps down and pushes away any doubts which arise about it in his mind, purposely avoids the reading of books and the company of men that call in question or discuss it, and regards as impious those questions which cannot easily be asked without disturbing it - the life of that man is one long sin against mankind."

*The Ethics of Belief* (1877) by William K. Clifford.

- b. "If existing agricultural knowledge were everywhere applied, the planet could feed twice its present population."

*The Lessons of History* (1968) by Will and Ariel Durant.

- c. "But even if the initial colonists had consisted of only 100 people and their numbers had increased at a rate of only 1.1 percent per year, the colonists' descendants would have reached that population ceiling of 10 million people within a thousand years."

*Guns, Germs, and Steel* (1997) by Jared Diamond.

- d. "If anyone looked out of their window now, even beady-eyed Mrs. Dursley, they wouldn't be able to see anything that was happening down on the pavement."

*Harry Potter and the Philosopher's Stone* (1997) by J. K. Rowling

### Problem 3: Fallacies (1 scores)


















Give a real-life example for each type of fallacy in chapter 1. Reference materials are needed. Paraphrase the materials, using your own words.

### Part 2

Let  $\overline{abcd}$  be the 4-digit number combined by the last 4 digits in your StudentID. For example, StudentID 520H1234 has  $\overline{abcd} = 1234$ .

### Problem 4: Tarski's world (2 scores)

Giving the following Tarski's world.

|   | 1   | 2   | 3   | 4   | 5   | 6   | 7  | 8   | 9   |
|---|---|---|---|---|---|---|--|---|---|
| A |   |  |   |   |   |   |  |  |   |
| B |   |   |   |   |   |   |  |   |   |
| C |   |  |   |  |   |   |  |   |  |
| D |   |  |  |   |   |   |  |  |   |
| E |   |   |   |  |  |  |  |   |   |
| F |  |   |   |   |   |   |  |   |   |
| G |   |   |   |   |   |   |  |   |   |
| H |   |  |   |   |  |   |  |   |   |
| I |  |   |   |   |   |   |  |   |   |

Items are named by their positions. These notations are defined:

- Triangle(x), meaning “x is a triangle,”
- Circle(x), meaning “x is a circle,”
- Square(x), meaning “x is a square,”
- Red(x), meaning “x is red,”
- Green(x), meaning “x is green,”
- Orange(x), meaning “x is orange,”
- RightOf(x, y), meaning “x is to the right of y (but possibly in a different row),”
- LeftOf(x, y), meaning “x is to the left of y (but possibly in a different row),”
- AboveOf(x, y), meaning “x is to the above of y (but possibly in a different column),”
- BelowOf(x, y), meaning “x is to the below of y (but possibly in a different column).”

The domain for all variables is the set of objects in Tarski’s world shown in the picture.

a. Modify the above Tarski's world as follows:

- If  $\overline{abcd} \% 7 = 0$  then add a red square in A5.
- If  $\overline{abcd} \% 7 = 1$  then add a green triangle in F3.
- If  $\overline{abcd} \% 7 = 2$  then delete the item at G7.
- If  $\overline{abcd} \% 7 = 3$  then delete the item at H5.
- If  $\overline{abcd} \% 7 = 4$  then change the item at E4 into an orange square.
- If  $\overline{abcd} \% 7 = 5$  then change the item at E5 into a green circle.
- If  $\overline{abcd} \% 7 = 6$  then change the item at E6 into a red triangle.

(Example: For StudentID 522H1234, we have  $1234 \% 7 = 2$ , so we delete item at G7.)

Re-draw your new Tarski's world.

b. Determine the truth or falsity of all the following statements, based on the modified Tarski's world. Give the reasons for your justification.

- $\forall x, \text{Circle}(x) \rightarrow \text{Green}(x)$
- $\forall x, \text{Triangle}(x) \rightarrow \sim \text{Orange}(x)$
- $\exists x$  such that  $\text{Red}(x) \wedge \text{Triangle}(x)$
- $\exists x$  such that  $\sim \text{Green}(x) \wedge \text{BelowOf}(x, E4)$
- $\forall x, \text{Square}(x) \rightarrow \text{RightOf}(E5, x)$ .
- $\exists x$  such that  $\text{AboveOf}(E5, x) \wedge \text{LeftOf}(x, E5)$ .
- There is a triangle x such that for all squares y, x is above y.
- For all circles x, there is a square y such that y is to the right of x.
- There is a circle x and there is a square y such that y is below x.
- For all circles x and for all triangles y, x and y have the same color.

### Problem 5: Symbolic form (1 score)

Let p = “it is windy”; q = “it is thundering”; r = “it is raining”; s = “it is lightning”.

There are some statements:

- It is windy but it isn't raining.
- It is windy, thundering but it isn't raining.
- It is raining without thundering and lightning.
- Windiness is a necessary condition for rain.
- Windiness is a sufficient condition for rain.
- Whenever it is lightning, it will be thundering.
- The necessary and sufficient condition for thundering is lightning.

Using  $p, q, r, s$  and logical connectives to write the symbolic form of:

- If  $\overline{abcd} \% 2 = 0$ 
  - Write statements  $a, d, f, g$ .
- If  $\overline{abcd} \% 2 = 1$ 
  - Write statements  $b, c, e, g$ .

(Example: For StudentID 522H1234, we have  $1234 \% 2 = 0$ , so he/she need to solve  $a, d, f, g$ .)

#### Problem 6: Equivalence (2 scores)

Let  $p, q, r$  be statement variables. Prove that the following pair of statements are logically equivalent by 2 methods: (a) using truth table; and (b) using logical equivalence laws.

- If  $\overline{abcd} \% 3 = 0$ 
  - $\sim [(\sim p \wedge \sim \sim q) \vee \sim (p \vee r)] \equiv (r \vee p) \wedge (\sim q \vee p)$
- If  $\overline{abcd} \% 3 = 1$ 
  - $\sim [(\sim p \vee q) \vee \sim (p \wedge \sim (p \vee q))] \equiv p \wedge \sim (p \vee q)$
- If  $\overline{abcd} \% 3 = 2$ 
  - $\sim (p \vee \sim (q \wedge r)) \wedge \sim (\sim q \vee (p \vee q)) \equiv (r \wedge q) \wedge \sim (q \vee p)$

(Example: For StudentID 522H1234, we have  $1234 \% 3 = 1$ , so he/she needs to prove for the second pair.)

#### Part 3

In part 3, students need to install and learn the Prolog language to perform some simple queries. Students need to solve all the problems in part 3.

#### Problem 7: Prolog (2 scores)

Go to SWI-Prolog's website: <http://www.swi-prolog.org/>, download and install SWI-Prolog.

- Find and run the file **{SWI-Prolog}\demo\likes.pl** on your computer. Execute the following queries and capture the results. Use debug mode to explain how to calculate these results.

**mild(dahl).**

**indian (X) .**

**likes (sam, X) .**

- b. Write file **hello1.pl** to print "Hello World". Capture the result.
- c. Write a file **hello2.pl** that allows printing "Hello nam", with "nam" entered by the user. Capture the result.

## Rubric

| Criteria     | Scale     | 1                               | 2  | 3   |
|--------------|-----------|---------------------------------|--|---|
|              | Score /10 | 0 score                         | 1/2 score  | Full score  |
| Problem 1    | 1         | Do nothing or wrongly.          | Right password but the reasoning is not good enough. | Right password and good reasoning.  |
| Problem 2    | 1         | Do nothing or wrongly.          | Correct half of the requirements                     | All correct.  |
| Problem 3    | 1         | Do nothing or wrongly.          | >80% of examples are correct; but bad references.    | All examples are correct; and good references from newspapers, books, websites... |
| Problem 4    | 2         | Do nothing or wrongly. Wrong a) | Right result but wrong explanation.                  | Right result and right explanation.   |
| Problem 5    | 1         | Do nothing or wrongly.          | Correct <50%.  | All are correct.  |
| Problem 6a   | 1         | Do nothing or wrongly.          | Correct <50% of the table.                           | The table is correct.   |
| Problem 6b   | 1         | Do nothing or wrongly.          | Correct <50%. Not state the laws' name.              | All are correct. Used laws' names are given.                                      |
| Problem 7a   | 1         | Do nothing or wrongly.          | Correct results but incorrect explanation.           | Correct results and correct explanation.  |
| Problem 7b   | 0.5       | Do nothing or wrongly.          | /  | Correct result.   |
| Problem 7c   | 0.5       | Do nothing or wrongly.          | /  | Correct result.   |
| <b>Total</b> | 10        |                                 |  |   |

The end.