Question 1: Euclid’s algorithm and Bezout’s identity

gcd(2024, 1000 + m) and lcm(2024, 1000 + m), with m = 120

* gcd(2024, 1120) and lcm(2024, 1120)

Consider gcd(2024, 1120):

2024 = 1120 x 1 + 904 🡨 gcd(1120, 904)

1120 = 904 x 1 + 216 🡨 gcd(904, 216)

904 = 216 x 4 + 40 🡨 gcd(216, 40)

216 = 40 x 5 + 16 🡨 gcd(40, 16)

40 = 16 x 2 + 8 🡨 gcd(16, 8)

16 = 8 x 2 + 0 🡨 gcd(8, 0)

The result is: gcd(2024, 1120) = 8

* lcm(2024, 1120) = (2024 x 1120) / 8 = 283360

Question 2: Recurrence relation

With and

Step 1: Find the characteristic equation:

For the recurrence relation , the characteristic equation is obtained by assuming a solution of the form . Substituting this into the recurrence relation gives:

Dividing through by (assuming r 0) gives:

⬄

Step 2: Solve the characteristic equation:

To solve the quadratic equation , I use the quadratic formula , where *a* = 1, 𝑏 = −8 and 𝑐 = 15*:* r = =

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Step 3: Form the general solution:

The general solution to the recurrence relation is:

Step 4: Determine the constants using initial conditions:

I use the initial conditions and to find the constants A and B.

1. For 𝑛 = 0:
2. For 𝑛 = 1:

So, I have the system of linear equations:

⬄

Step 5: Write the particular solution:

The constants A and B are both ​. Therefore, the particular solution to the recurrence relation is: ⬄

So, the solution to the recurrence relation ​ with and is:

Question 3: Set

a. Create a set Γ of characters from your case-insensitive non diacritical full name. For example, the set corresponding with “Tôn Đức Thắng” is Δ = {A, C, D, G, H, N, O, T, U}.

b. Find the union, intersect, non-symmetric difference, and symmetric difference of Γ and Δ, where Γ and Δ are from question 3a.

a. My full name is: “Nguyễn Đình Việt Hoàng”

Γ = {N, G, U, Y, E, D, I, H, V, T, O, A}

Δ = {A, C, D, G, H, N, O, T, U}

Union of Γ and Δ: Γ ∪ Δ = {N, G, U, Y, E, D, I, H, V, T, O, A, C}

Intersect of Γ and Δ: Γ ∩ Δ = {N, G, U, D, H, T, O, A}

Non-symmetric difference of Γ and Δ: Γ ∖ Δ = {Y, E, I, V}

Symmetric difference of Γ and Δ: Γ Δ Δ = (Γ ∖ Δ) ∪ (Δ ∖ Γ) = {Y, E, I, V} ∪ {C} = {Y, E, I, V, C}

Question 4:  Logical

A businessman has a golden chain with 149 links. He rented a villa from an old man for 149 weeks with the condition: Every week, he must pay the old man 1 link from his golden chain. What is the minimum number of cuts that he must use to pay the rent in 149 weeks?

Note:

- He can get the gold links back from the old man. For example: in week 1 he paid 1 link of chain, in week 2 he paid 2 links of chain and received "in return" 1 link of chain from the old man.

- Cut chain links can still be used to pay rent.

- He is very smart and good at logic.

To solve this problem most effectively, we need to find a way to minimize the number of cuts the businessman must make to pay 1 golden link per week for 149 weeks. We will use a smart divide-and-conquer approach to achieve this goal.

1. Analysis:

The businessman can return the chain link in segments of increasing length, then take the chain back from the old man to reuse for the following weeks. We will need to find a cut that allows the businessman to pay exactly 1 link per week without having to cut more each week.

1. Method:

The best way to do this is to split the chain into segments of length 1, 2, 4, 8, etc., in powers of 2. This gives us flexibility in paying the chain each week.

1. The method for cutting the golden chain:

To simplify, we will use the binary system:

Week 1: Pay 1 link.

Week 2: Return 2 links and get 1 link back.

Week 3: Pay 1 link.

Week 4: Return 4 links and get 3 links back.

Week 5: Pay 1 link.

Week 6: Return 2 links and get 1 link back.

Week 7: Pay 1 link.

Week 8: Return 8 links and get 7 links back.

And so on...

With a total of 149 weeks, the 149-link chain can be represented in binary form: 149 = 128 + 16 + 4 + 1

1. Perform cutting steps:

Cut 1 link: This is the first cut to create a segment with 1 link.

Cut 3 links: Cut to create a segment with 3 links (used to create a segment with 2 links and 1 remaining link).

Cut 7 links: Cut to create a segment with 7 links (used to create segments with 4 links, 2 links, and 1 link).

Cut 15 links: Cut to create a segment with 15 links (use to create segments with 8 links, 4 links, 2 links, and 1 link).

Cut 31 links: Cut to create a segment with 31 links (use to create segments with 16 links, 8 links, 4 links, 2 links, and 1 link).

Cut 63 links: Cut to create a segment with 63 links (use to create segments with 32 links, 16 links, 8 links, 4 links, 2 links, and 1 link).

Thus, we have segments cut according to powers of 2, plus the necessary segments for odd numbers. The total number of cuts needed will be a power of 2 to achieve 149 links:

1. Conclusion

* The minimum number of cuts an entrepreneur must make to be able to pay rent each week for 149 weeks is: 7 cuts.