HISTOGRAM PROCESSING
(DIGITAL IMAGE PROCESSING)

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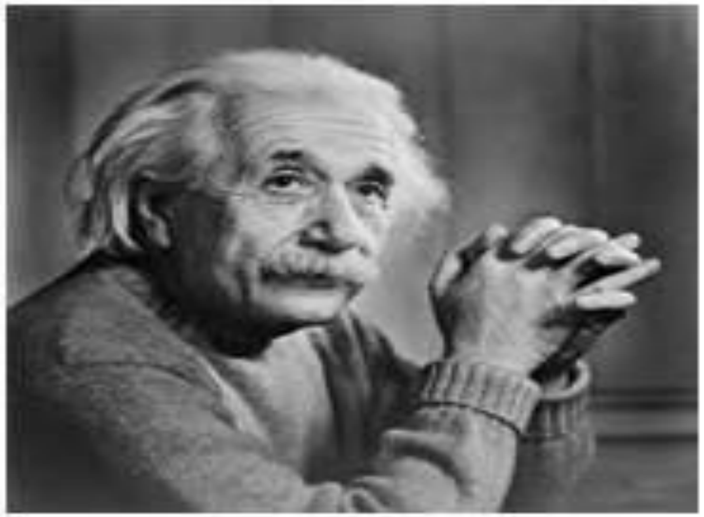
Image Enhancement

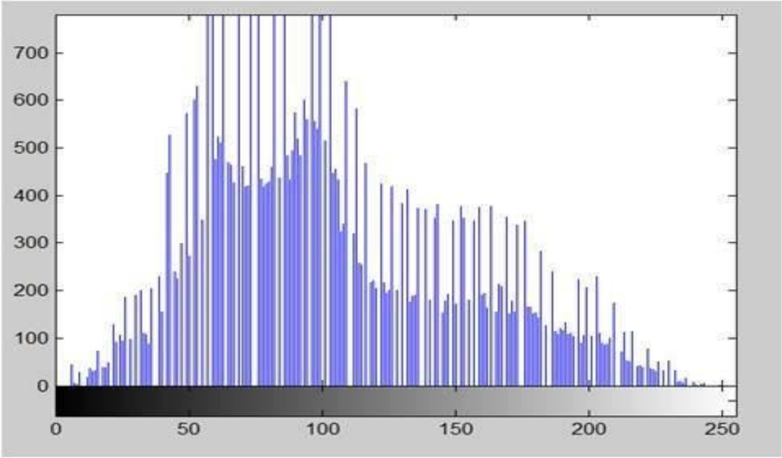
- Brightness and contrast
- Histogram Equalization
  - Image histogram
  - Histogram equalization techniques
  - Adaptive histogram equalization
  - Applications
- Image fitering
  - Convolution
  - Noise removal

Python
OpenCV

Histogram

- Histograms shows frequency.
- An image histogram, shows **frequency of pixels intensity values**.
  - In an image histogram, the x-axis shows the gray level intensities and the y-axis shows the frequency of these intensities.



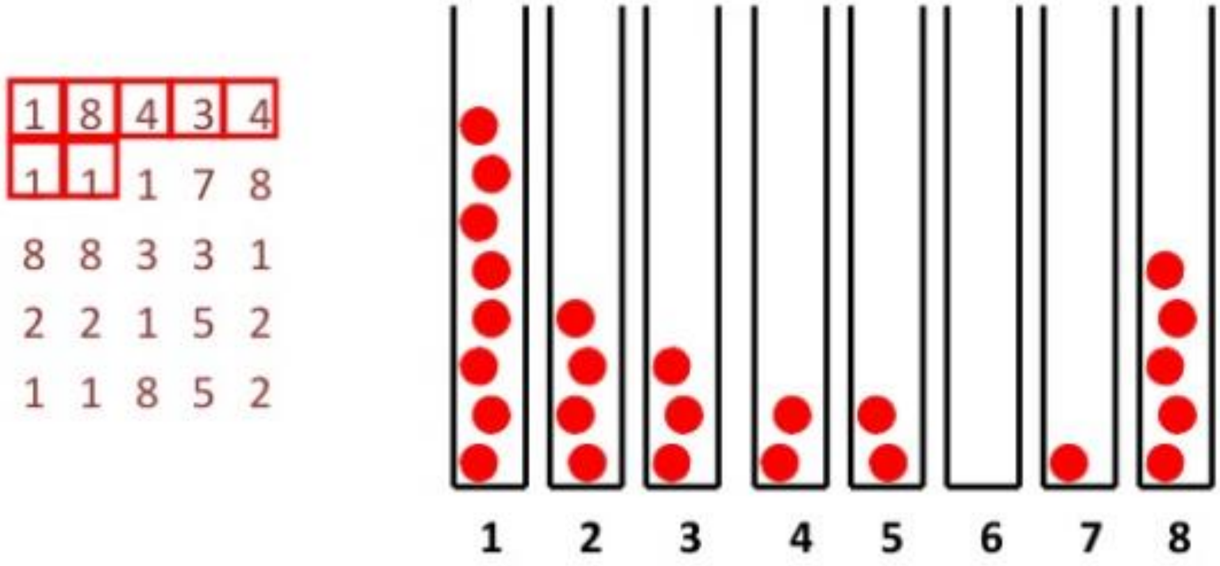


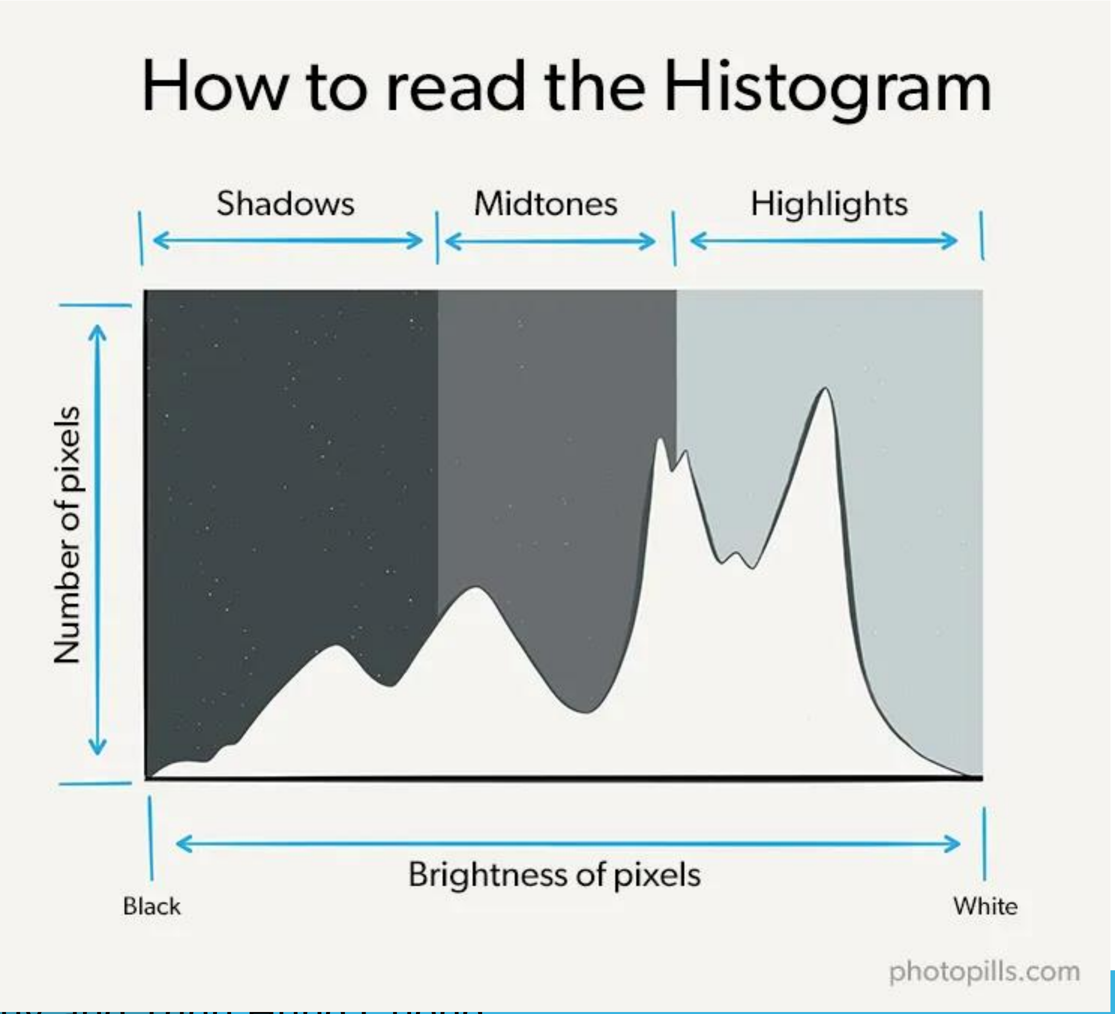
The histogram of the above picture of the Einstein would be something like this

Gray level histograms

- To measure a histogram:
  - For B-bit image, initialize 2^B counters with 0
  - Loop over all pixels x,y
  - When encountering gray level f [x,y]=i, increment counter $\#i$
- Normalized histogram can be thought of as an estimate of the probability distribution of the continuous signal amplitude
- Use fewer, larger bins to trade off amplitude resolution against sample size

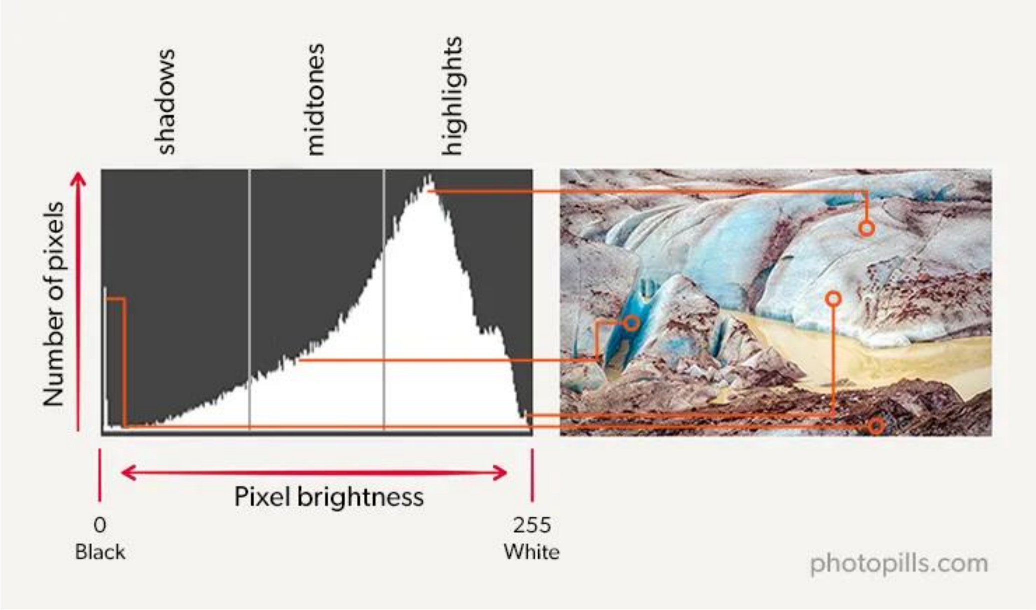
Histogram calculation



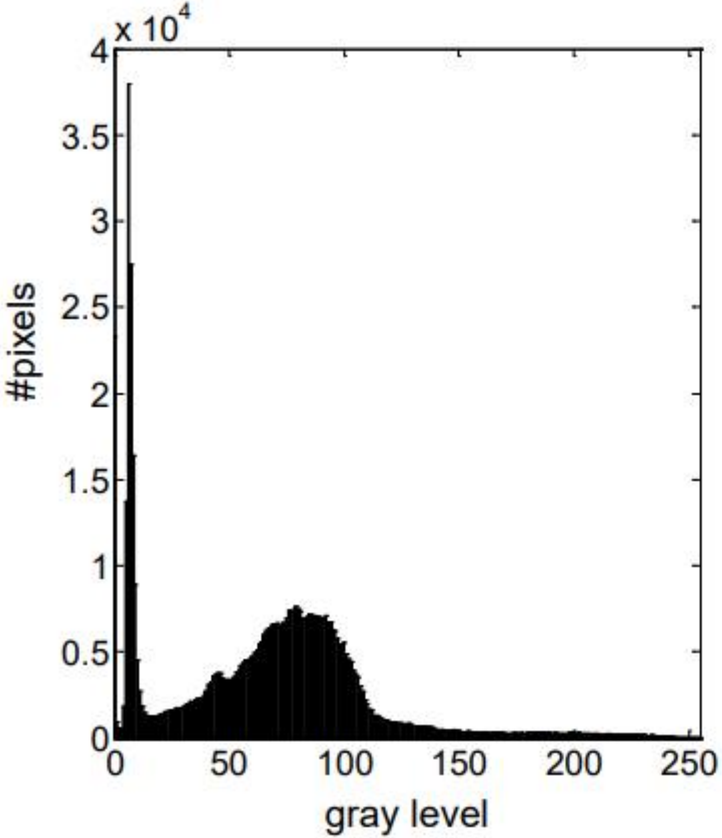

How to read the Histogram
Shadows
Midtones
Highlights
Number of pixels
Brightness of pixels
Black
White
photopills.com

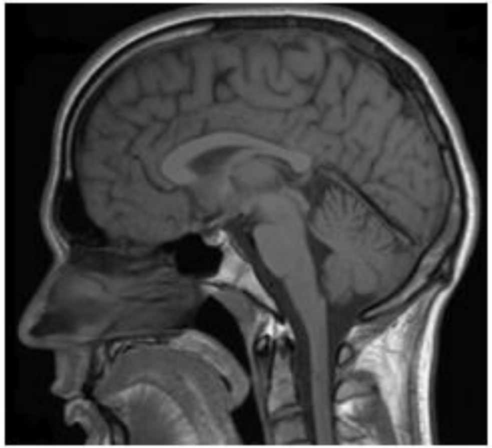
How to read the histogram

- Along the horizontal axis (x) and from left to right you have:
  - First the black tones, with pure black on the left edge.
  - Then come the shadows.
  - Then the midtones.
  - Followed by the highlights.
  - And finally, the white tones, with pure white on the right edge.



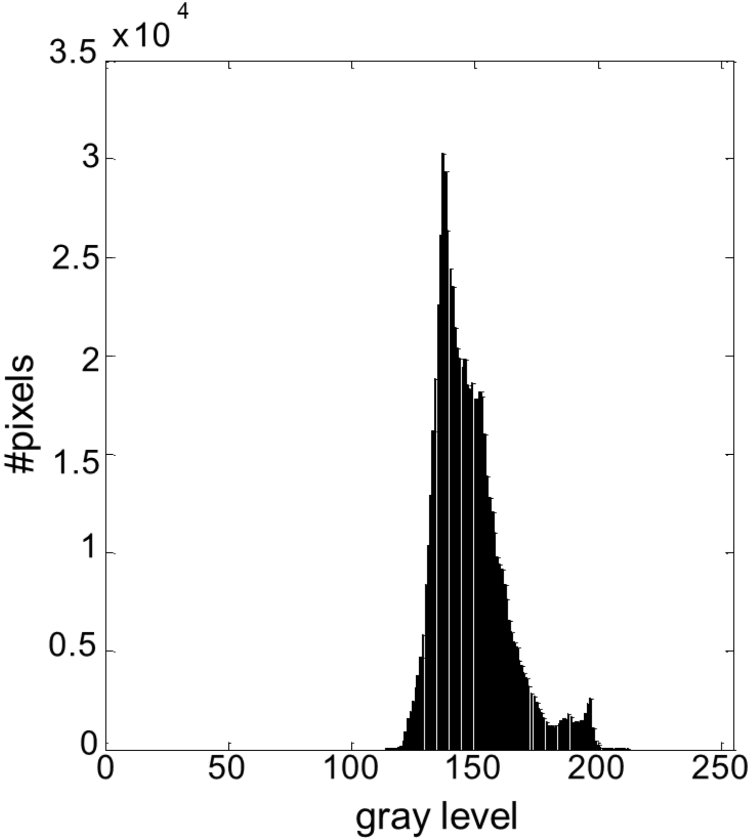
Gray level histograms





Brain image

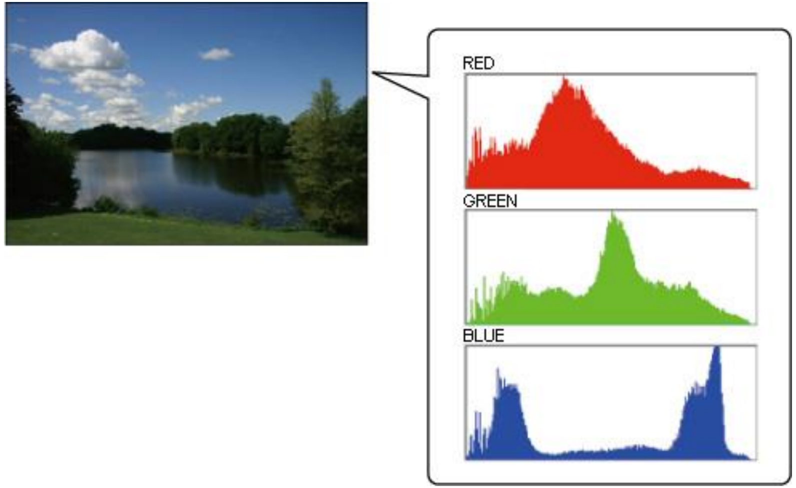
Gray level histograms

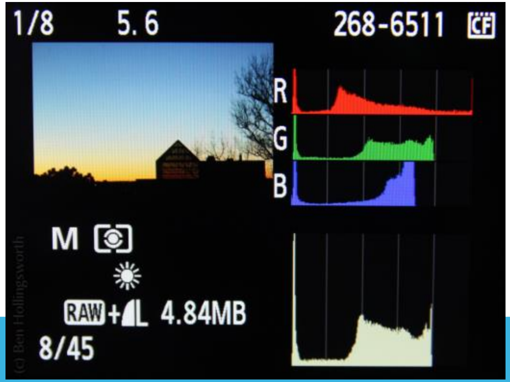


Bay image

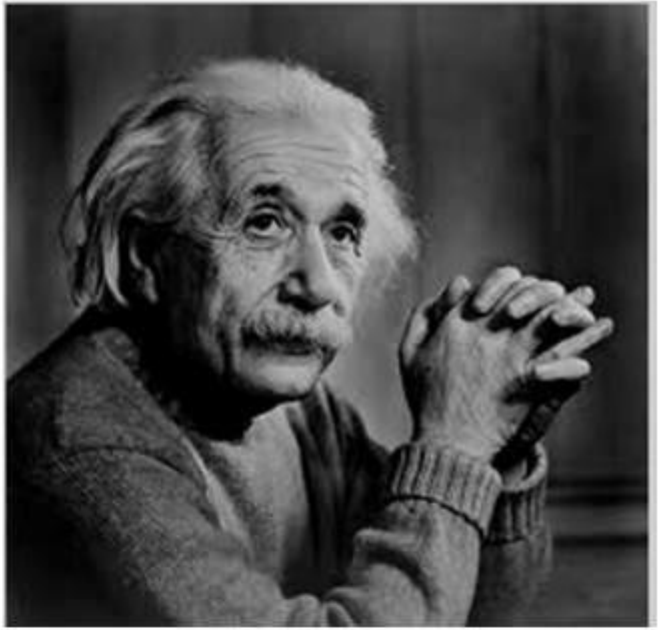
Histogram

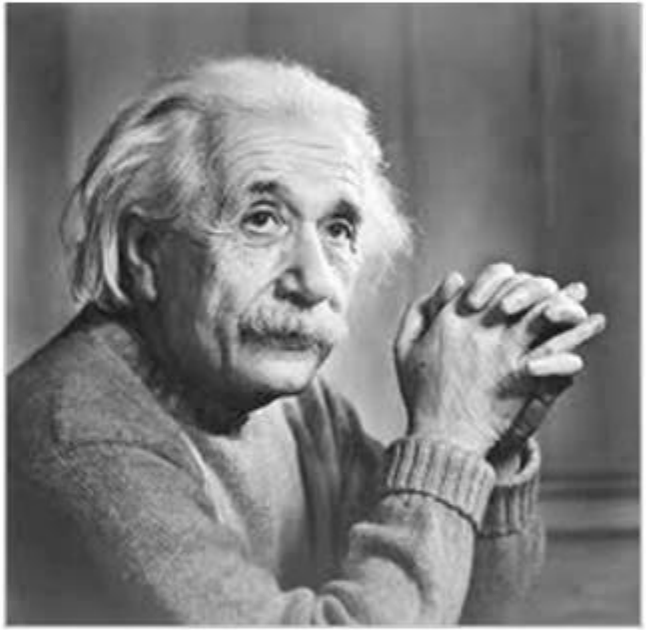
- Applications of Histograms:
  - We can predict about an image by just looking at its histogram. Its like looking an x ray of a bone of a body.
  - For brightness purposes.
  - To equalize an image used in adjusting contrast of an image.
  - Histogram has wide use in thresholding an image.





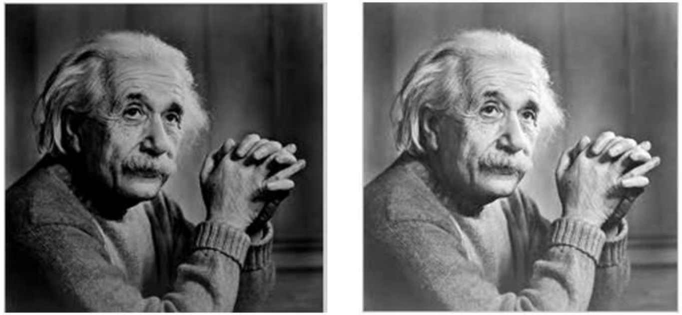
Which is brighter?



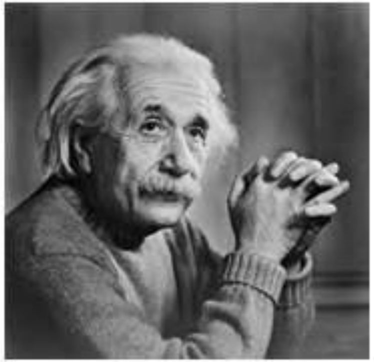


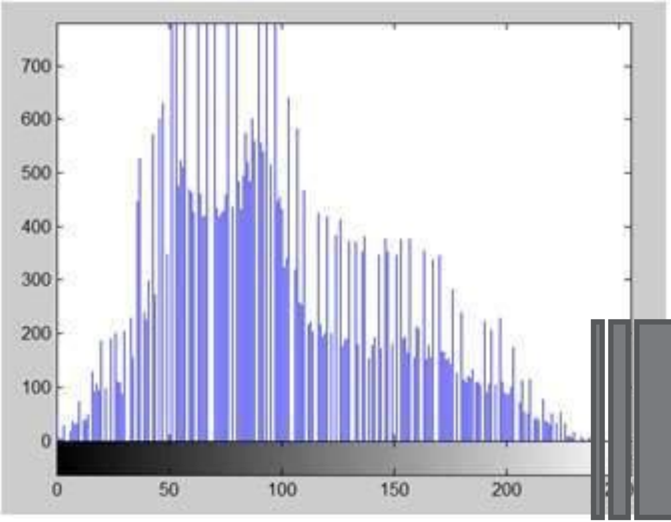
Brightness

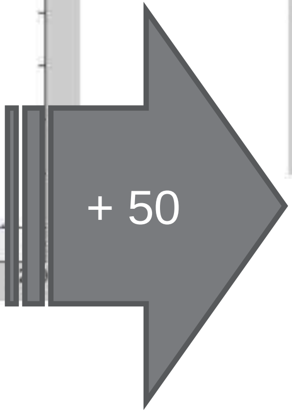
- Brightness can be defined as the amount of energy output by a source of light (Wiki)
- Brightness is the perceived intensity of light coming from a source
  - Brightness is a relative term. It depends on your visual perception.

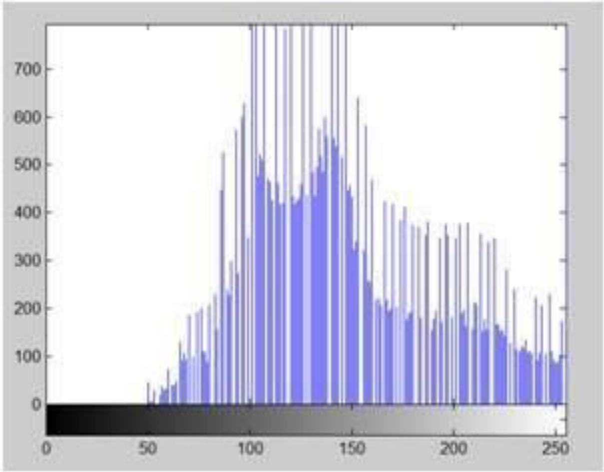


Brightness enhancement

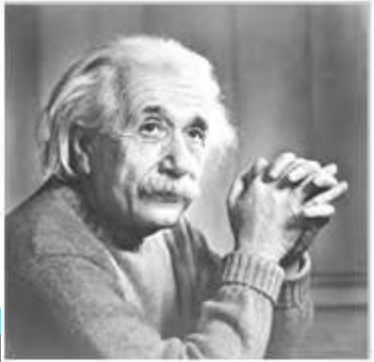




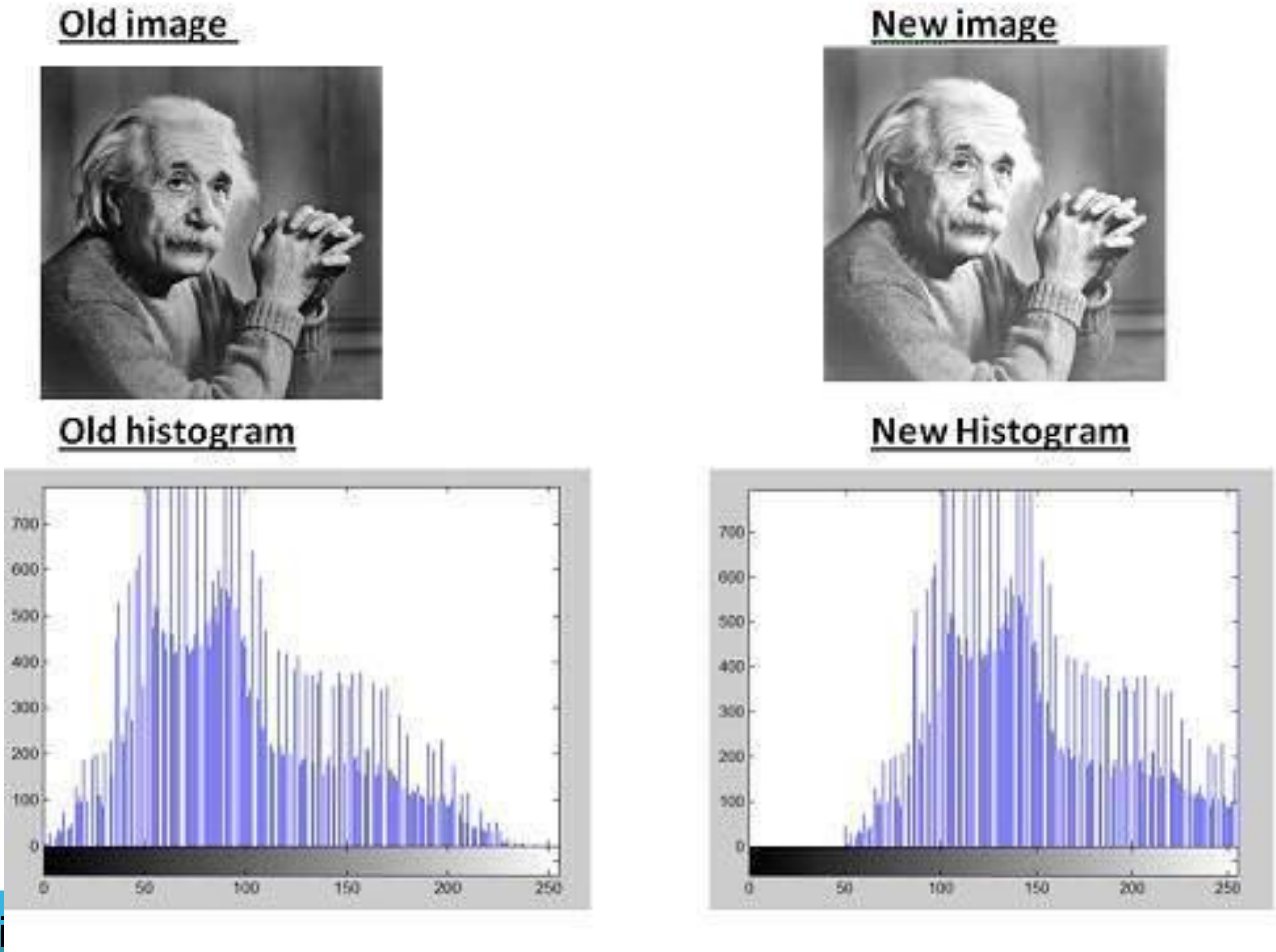




Sliding histogram



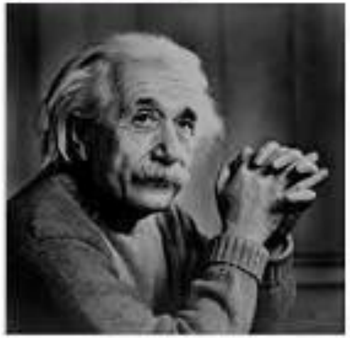
+ 50



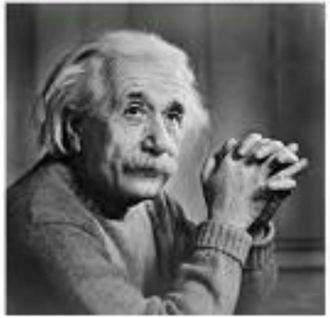
Old image
New image
Old histogram
New Histogram
700
600
500
400
300
200
100
0
0
50
100
150
200
250

- 80

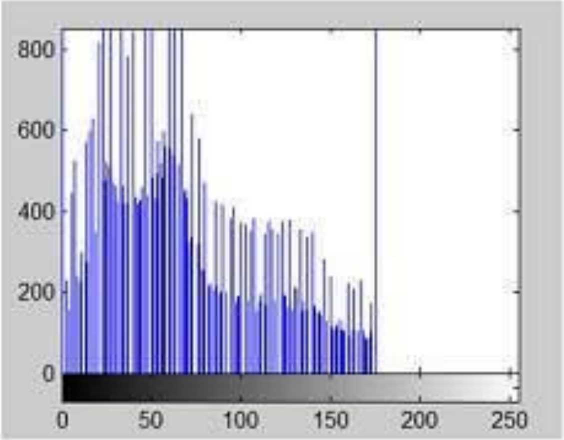
New image.



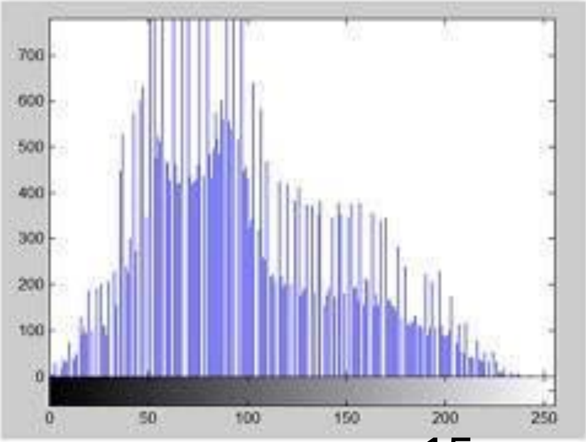
Original image.



New Histogram.



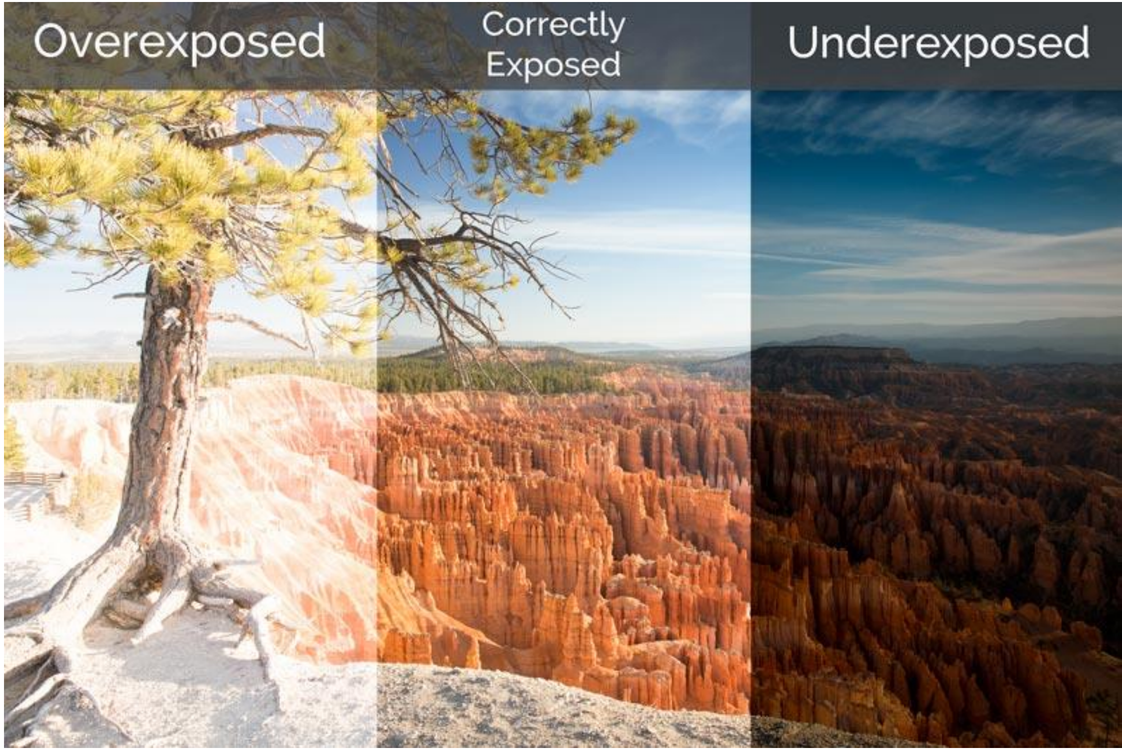
Original Histogram.



15

Brightness enhancement

- Exposure problem
  - Over-exposed photo
  - Under-exposed photo
  - Flash
  - RAW image
  - HDR technology

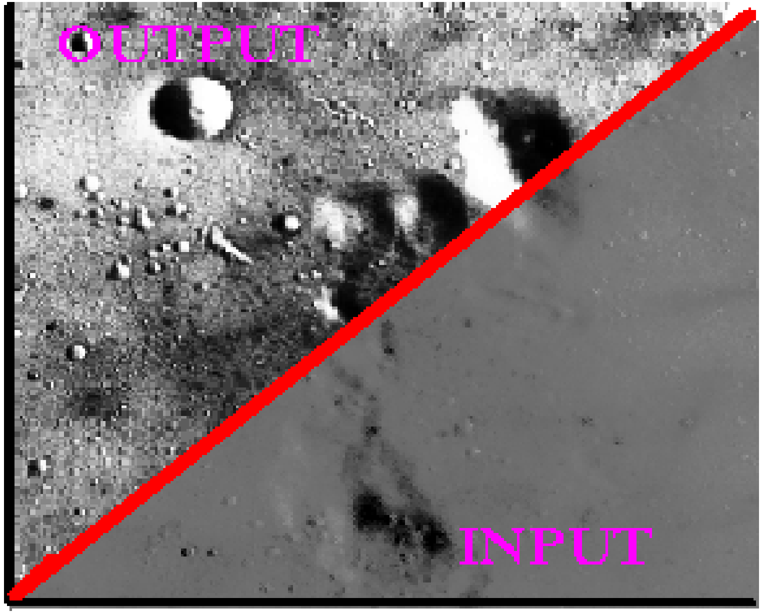


High-dynamic-range photographs are generally achieved by capturing multiple standard-exposure images, often using exposure bracketing, and then later merging them into a single HDR image, usually within a photo manipulation program.

Contrast

- the difference between maximum and minimum pixel intensity in an image.

Contrast enhancement

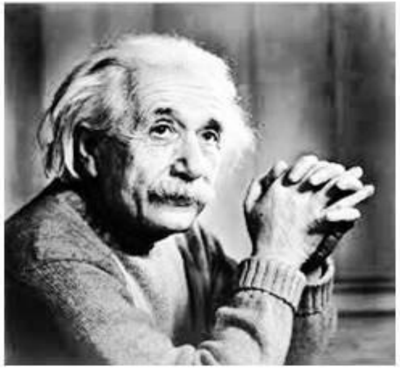


Histogram equalization example

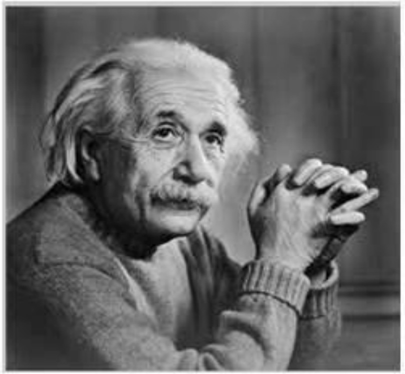
Original image *Bay*

... after histogram equalization

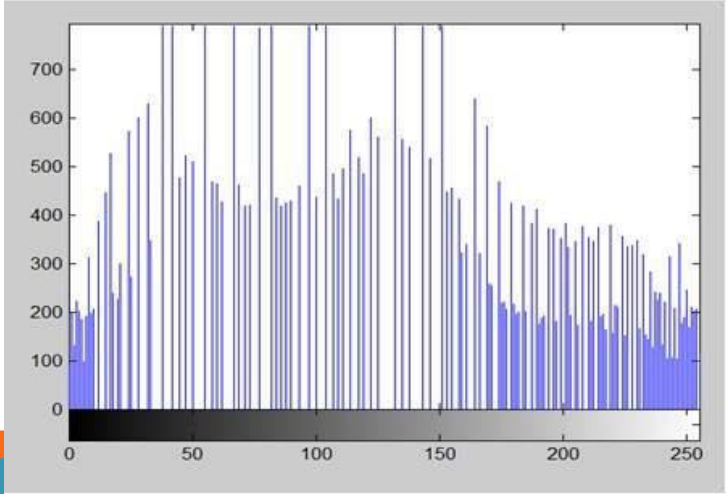
New Image



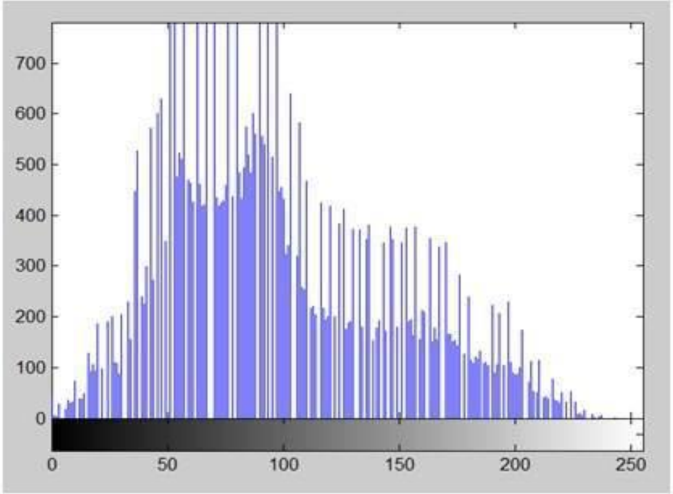
Old image

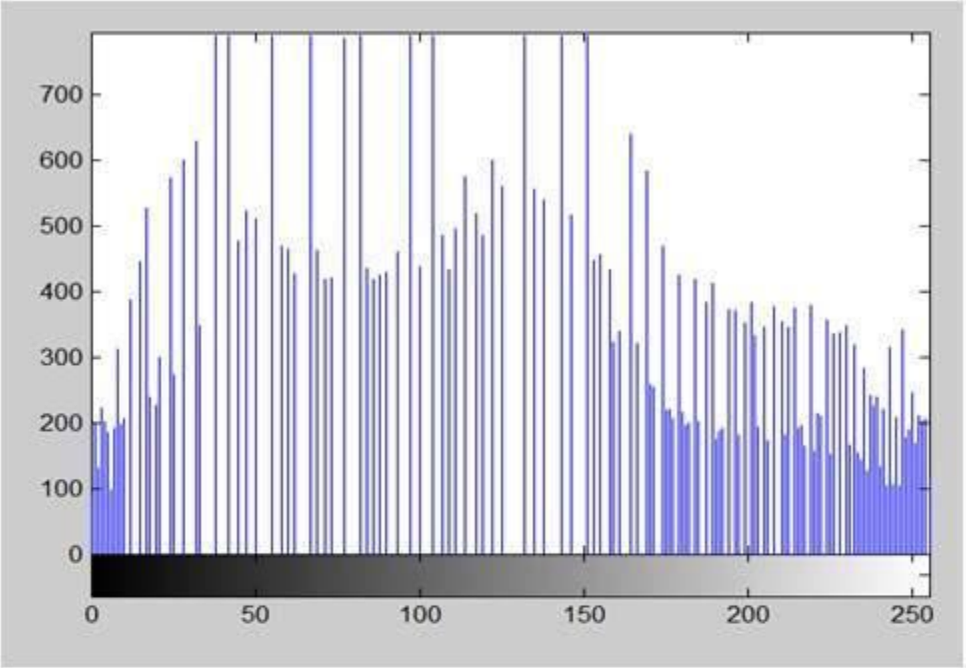


New Histogram



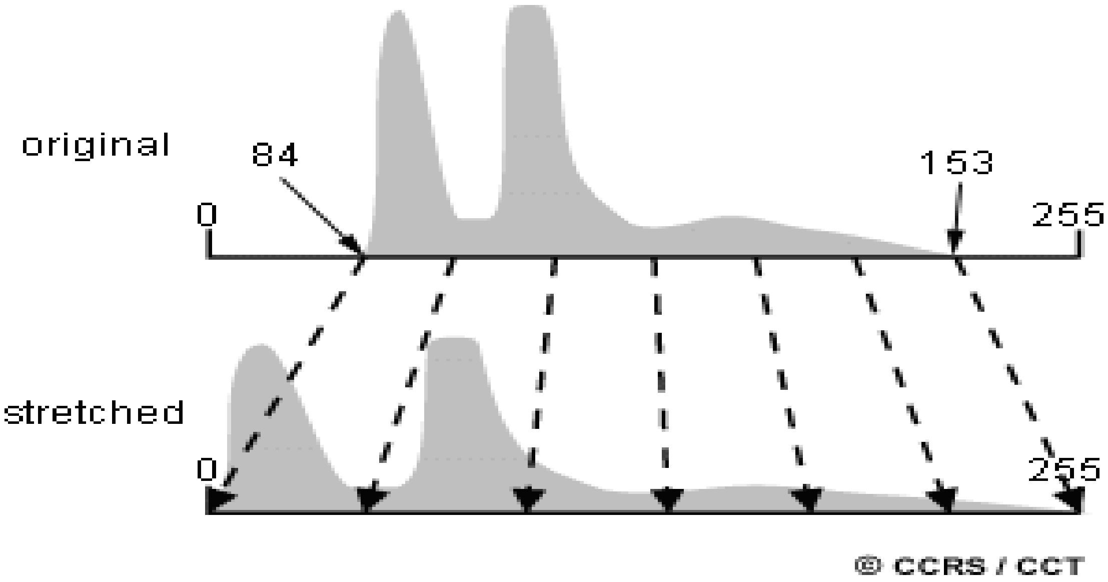
Old Histogram




700
600
500
400
300
200
100
0
0
50
100
150
200
250

Contrast enhancement

- Histogram Stretching
  - Increasing the contrast of an image



Contrast enhancement

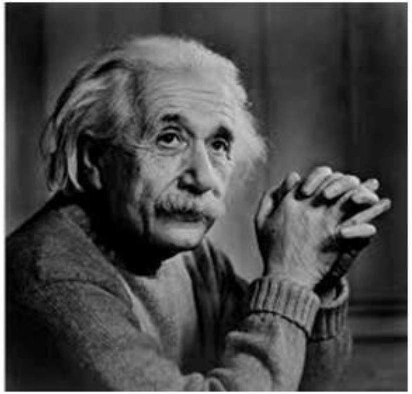
- Histogram Stretching

$$g(x,y) = \frac{f(x,y) - fmin}{fmax - fmin} \times (2^{bpp} - 1)$$

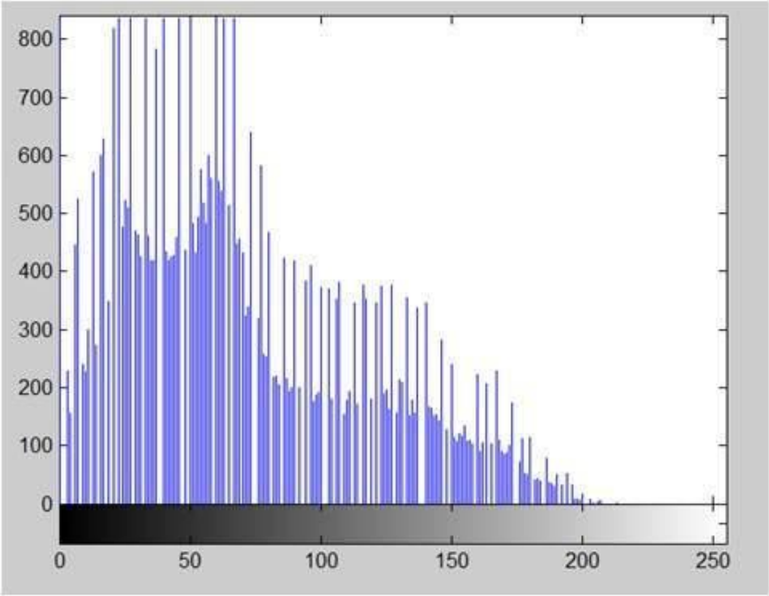
  - f(x,y) denotes the intensity value of each pixel
  - fmin and fmax are the minimum and maximum pixel intensity, respec.
  - bpp is the number of bits per pixel. In our case, the image is 8bpp, $bpp = 8$.

Contrast enhancement

- Histogram Stretching - example



Contrast = 225

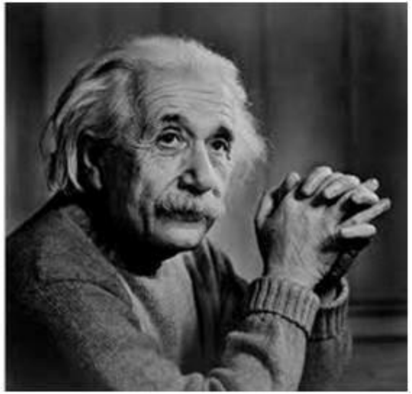


- The minimum value is 0 and the maximum value is 225. So the formula in our case is

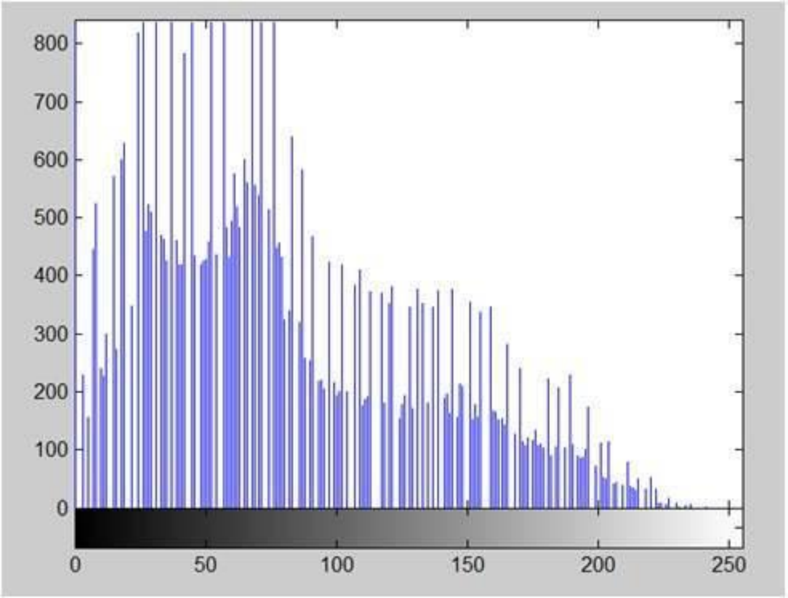
$$g(x,y) = \frac{f(x,y) - 0}{225 - 0} * 255$$

Contrast enhancement

- Histogram Stretching - example

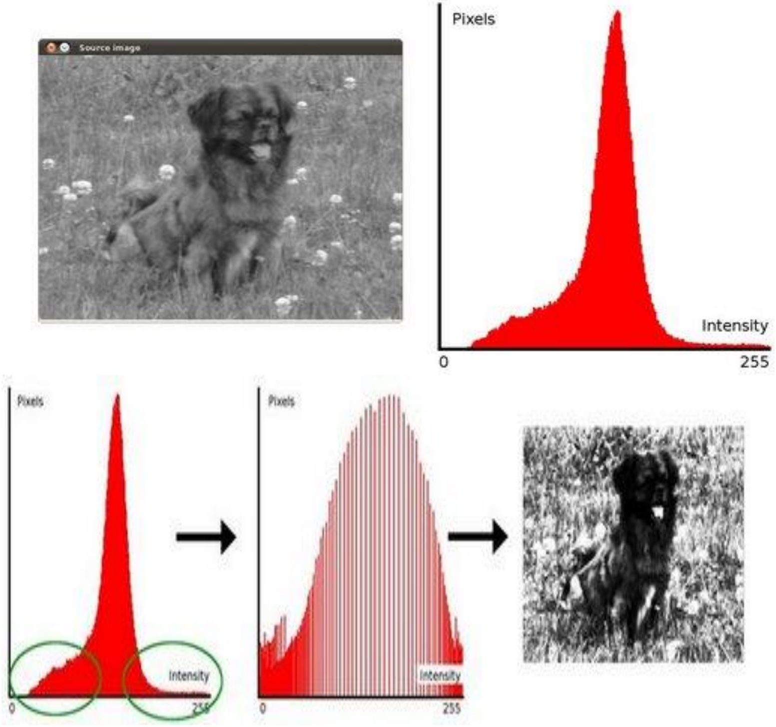


Contrast = 255



Contrast enhancement

- Histogram Equalization
  - a technique to adjust contrast levels and expand the intensity range in a digital image.
  - Thus, it enhances the image which makes information extraction and further image processing easier.

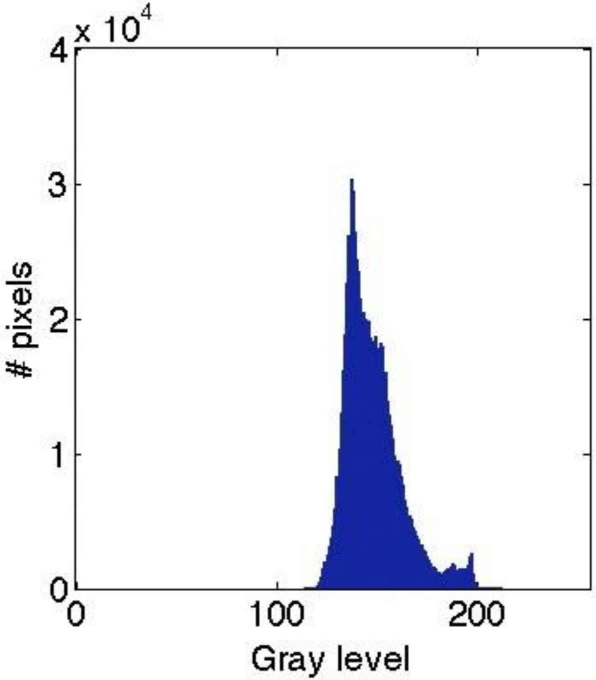


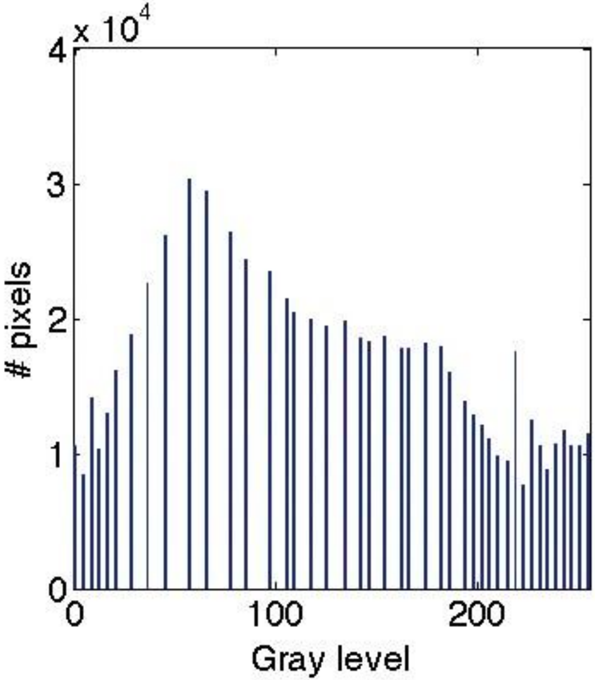
Histogram equalization example

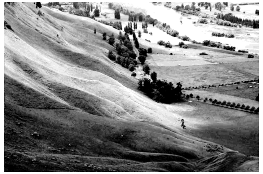
Original
image *Bay*

... after histogram
equalization

Histogram equalization example

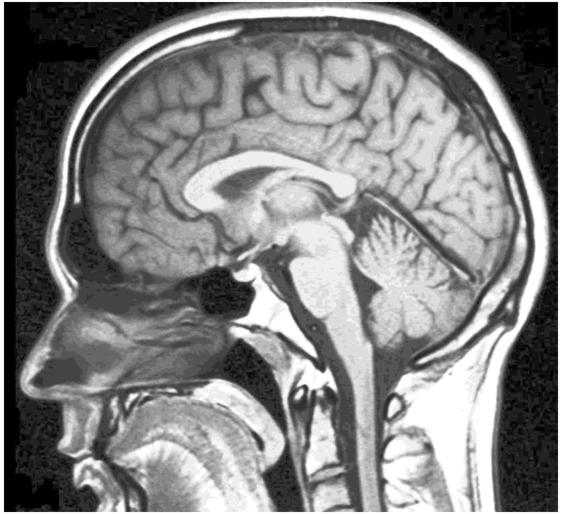






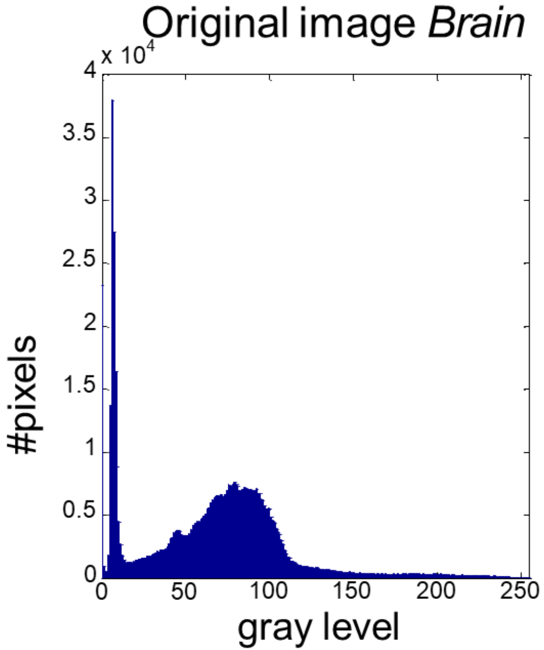
Histogram equalization example

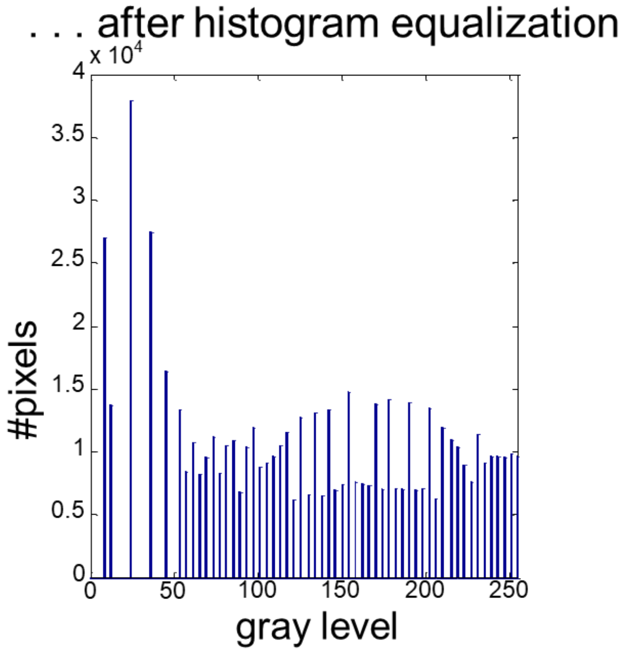
Original image
Brain



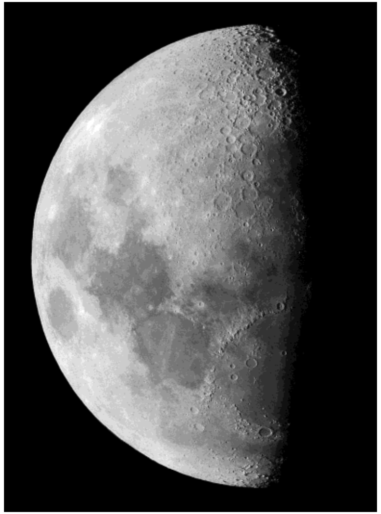
... after histogram equalization

Histogram equalization example

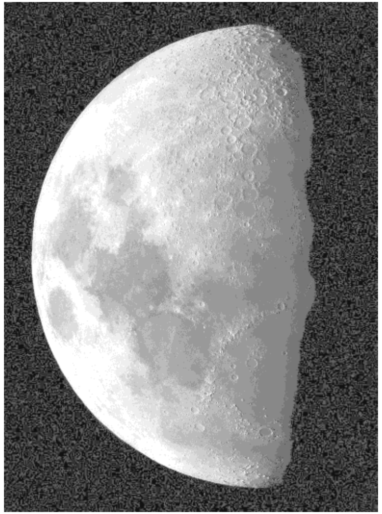




Histogram equalization example

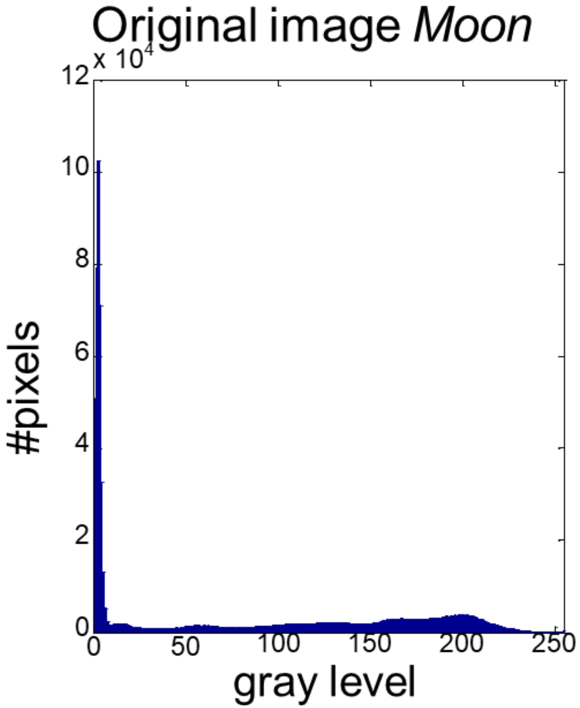


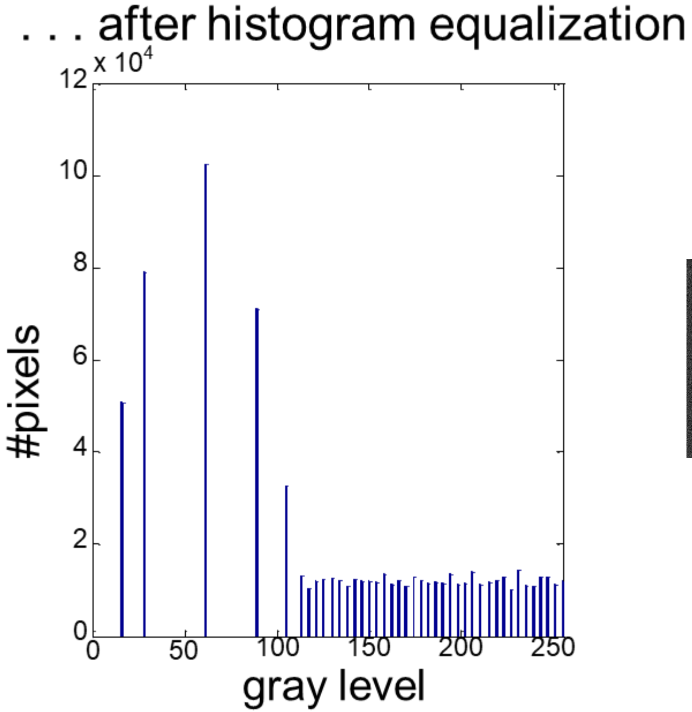
Original image
Moon

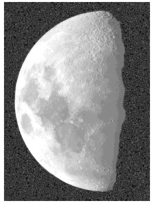


... after histogram
equalization

Histogram equalization example







Histogram Equalization - An algorithm

1. Convert the input image into a grayscale image
2. Find frequency of occurrence for each pixel value i.e. histogram of an image (values lie in the range [0, 255] for any grayscale image)
3. Calculate Cumulative frequency of all pixel values
4. Divide the cumulative frequencies by total number of pixels and multiply them by maximum graycount (pixel value) in the image

For example, consider an image having total 25 pixels having 8 distinct pixel values. All the steps have been applied to the histogram of the original image.

Gray level	0	1	2	3	4	5	6	7
Number of pixels	0	0	0	6	14	5	0	0
Cumulative frequency	0	0	0	$\frac{6}{25}$	$\frac{20}{25}$	$\frac{25}{25}$	$\frac{25}{25}$	$\frac{25}{25}$
Result of multiplication	0	0	0	2	6	7	7	7

PMF – Probability mass function
CDF = Cumulative density function

1	2	7	5	6
7	2	3	4	5
0	1	5	7	3
1	2	5	6	7
6	1	0	3	4

Image pixel values

PMF

Pixel	Count	PMF
0	2	2/25
1	4	4/25
2	3	3/25
3	3	3/25
4	2	2/25
5	4	4/25
6	3	3/25
7	4	4/25

CDF

Gray Level Value	CDF	CDF * (Levels-1) = CDF*7
0	0.11	0
1	0.22	1
2	0.55	3
3	0.66	4
4	0.77	5
5	0.88	6
6	0.99	6
7	1	7

EXAMPLE

0	1	3	4
1	2	2	3
1	3	4	4
3	2	5	2

(a)

i	$\hat{h}_i$	$\hat{C}_i$	$7\hat{C}_i$
0	1/16	1/16	0
1	3/16	4/16	2
2	4/16	8/16	4
3	4/16	12/16	5
4	3/16	15/16	7
5	1/16	16/16	7
6	0/16	16/16	7
7	0/16	16/16	7

(b)

0	2	5	7
2	4	4	5
2	5	7	7
5	4	7	4

(c)

i	$\hat{h}_i$
0	1/16
1	0/16
2	3/16
3	0/16
4	4/16
5	4/16
6	0/16
7	4/16

(d)

Figure 5.9. Numerical example of histogram equalization: (a) a 3-bit image, (b) normalized histogram and CDF, (c) the equalized image, and (d) histogram of the result.

Example

N Bit?

1	8	4	3	4
1	1	1	7	8
8	8	3	3	1
2	2	1	5	2
1	1	8	5	2

8 Bit?

$$\begin{bmatrix} 52 & 55 & 61 & 66 & 70 & 61 & 66 & 70 \\ 62 & 60 & 54 & 90 & 108 & 85 & 67 & 71 \\ 63 & 65 & 66 & 110 & 140 & 104 & 63 & 72 \\ 64 & 70 & 70 & 120 & 152 & 106 & 71 & 69 \\ 67 & 75 & 68 & 106 & 124 & 88 & 68 & 68 \\ 68 & 80 & 60 & 72 & 77 & 66 & 58 & 75 \\ 69 & 85 & 64 & 58 & 55 & 61 & 65 & 83 \\ 70 & 90 & 69 & 68 & 65 & 72 & 78 & 90 \end{bmatrix}$$

Histogram Equalization

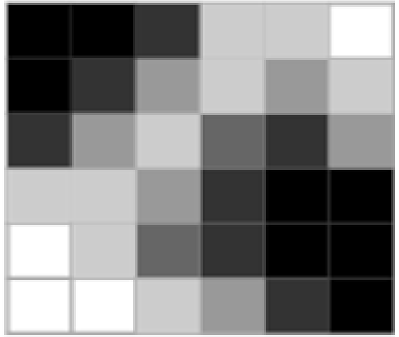
Ex. 1

- Write a pseudo code to equalize histogram of a grayscale image.

- Manually do histogram equalization for the following image:

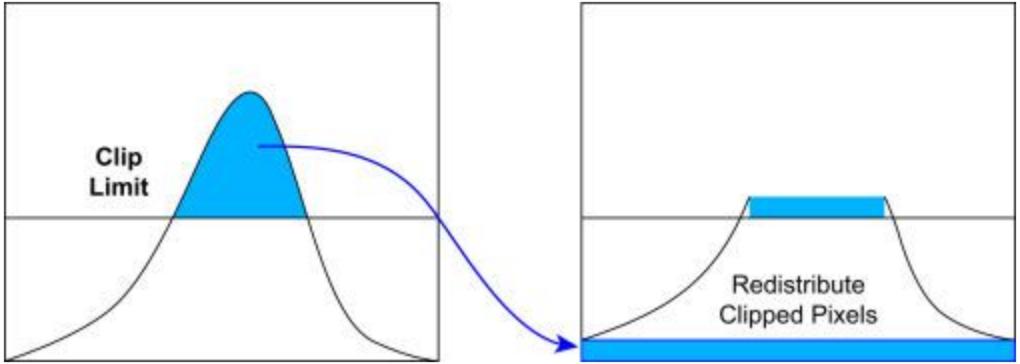
3-bit image

0	0	1	4	4	5
0	1	3	4	3	4
1	2	4	2	1	3
4	4	3	1	0	0
5	4	2	1	0	0
5	5	4	3	1	0



Contrast-limited histogram equalization

- If any histogram bin is above the specified contrast limit, those pixels are clipped and distributed uniformly to other bins before computing the cumulative distribution function
  - limit the overamplification of noise



https://www.sciencedirect.com/topics/computer-science/histogram-equalization
https://docs.opencv.org/4.x/d5/daf/tutorial_py_histogram_equalization.html

Contrast-limited histogram equalization

https://www.semanticscholar.org/paper/Real-Time-Processing-of-Contrast-Limited-Adaptive-Kokufuta-Maruyama/787ce60813fde958e6a3f21ad5072afc587c4241

```
Excess = 0;
// step 1
for (i = 0; i < N; i++) {
  if (h[i] > β) {
    Excess += h[i] − β;   h[i] = β;
} }
// step 2
m = Excess / N;
for (i = 0; i < N; i++) {
  if (h[i] < β − m) {
    h[i] += m;  Excess −= m;
  } else if (h[i] < β) {
    Excess += h[i] − β;  h[i] = β;
} }
// step 3
while (Excess > 0) {
  for (i = 0; i < N; i++) {
    if (Excess > 0) {
      if (h[i] < β) {
        h[i] += 1;  Excess −= 1;
} } } }
```

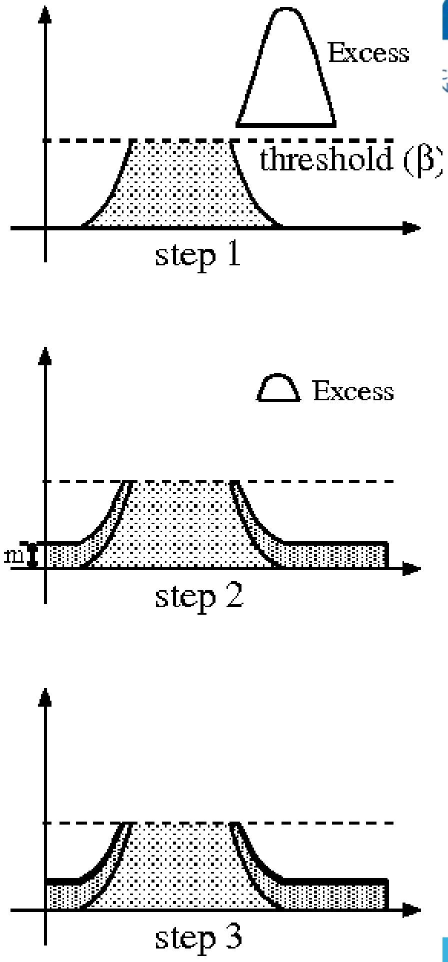
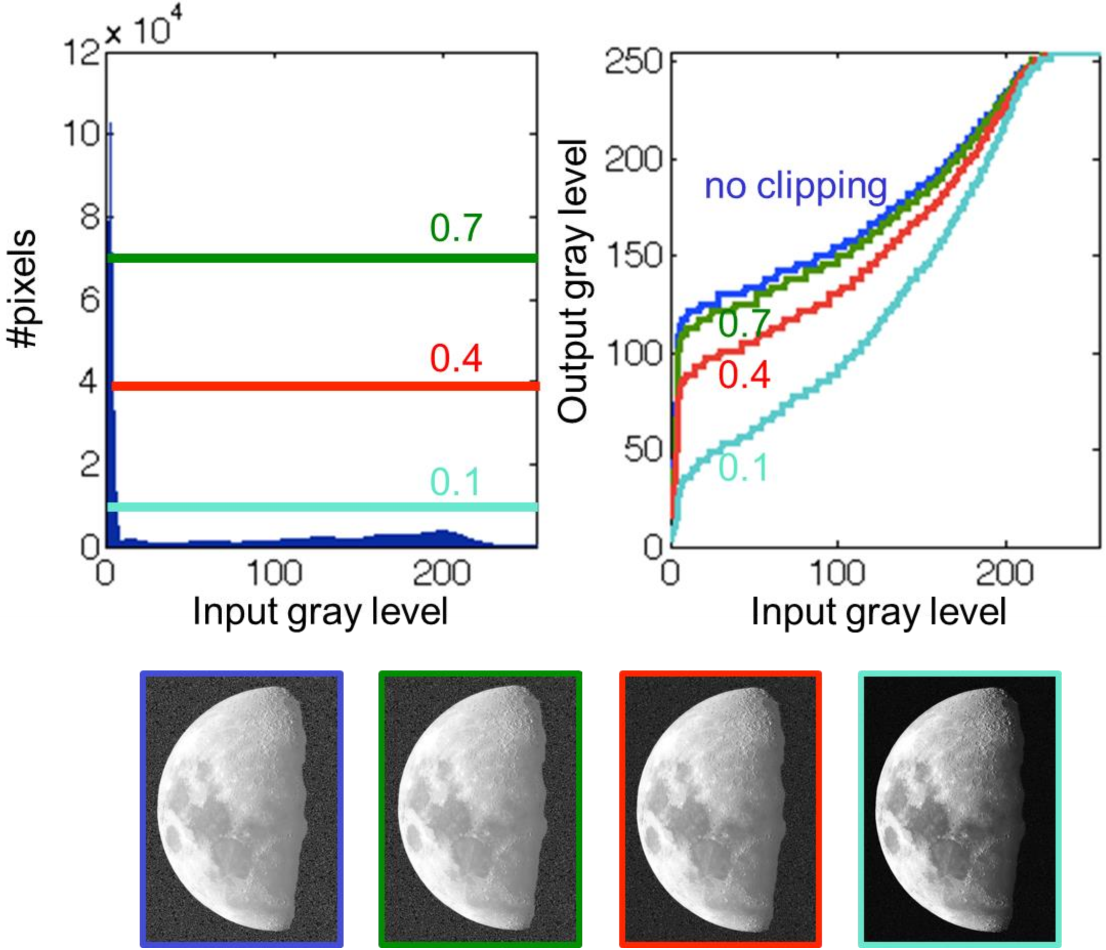


Figure 1. Distribution of histogram for CLAHE

Contrast-limited histogram equalization



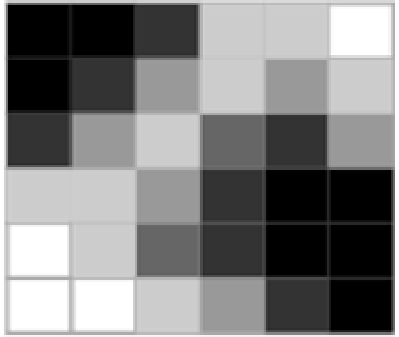
Contrast-limited Histogram Equalization

Ex. 2

Given the specified contrast limit is 6.
Manually do contrast-limited histogram equalization for the following image:

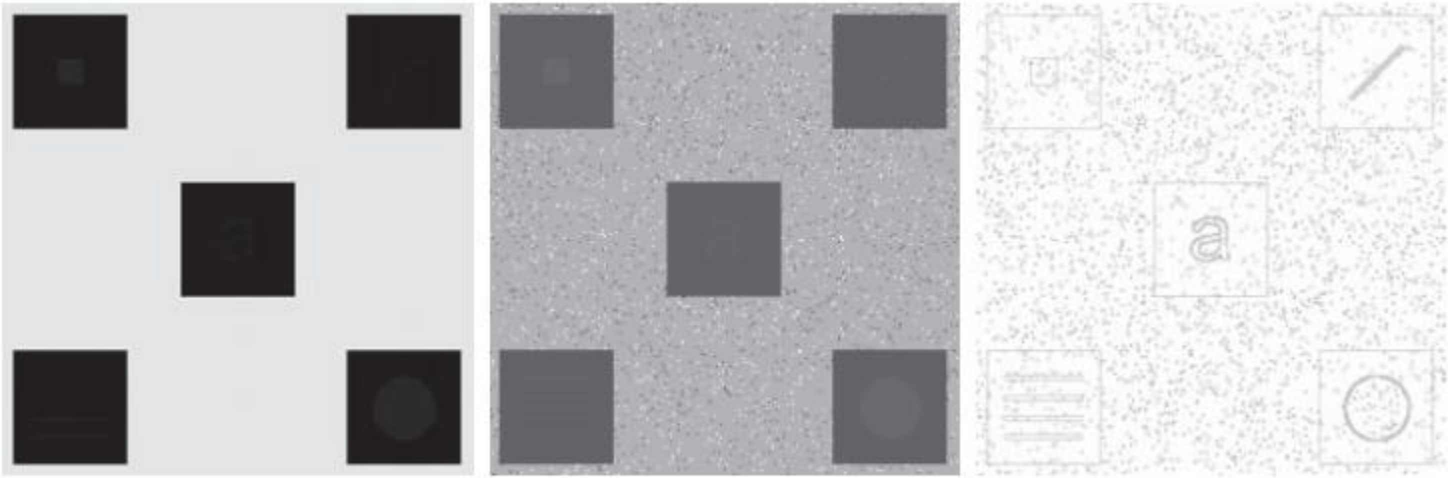
3-bit image

0	0	1	4	4	5
0	1	3	4	3	4
1	2	4	2	1	3
4	4	3	1	0	0
5	4	2	1	0	0
5	5	4	3	1	0



Adaptive (local) histogram equalization

- The global approach is suitable for overall enhancement, but generally fails when the objective is to enhance details over small areas in an image.



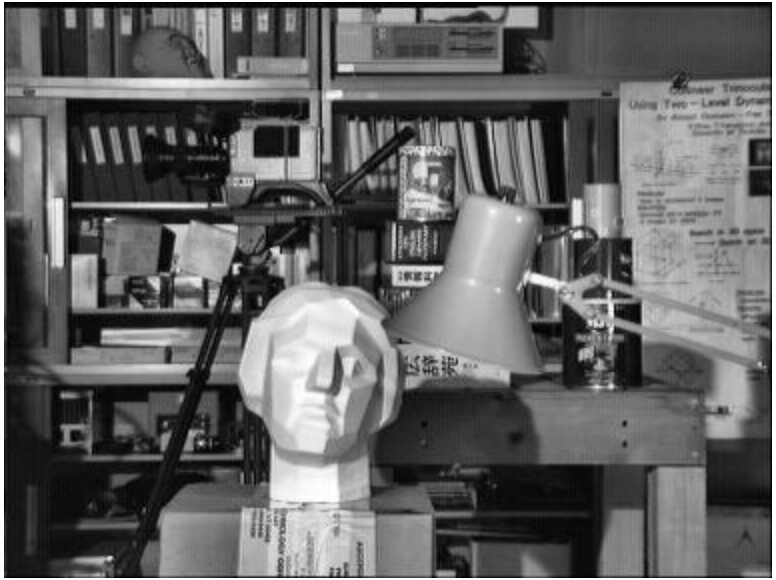
a b c

FIGURE 3.26 (a) Original image. (b) Result of global histogram equalization. (c) Result of local histogram equalization.

Rafael C. Gonzalez, Richard E. Woods, [2018], Digital image processing, 4th edition, Pearson.

Adaptive (local) histogram equaliza

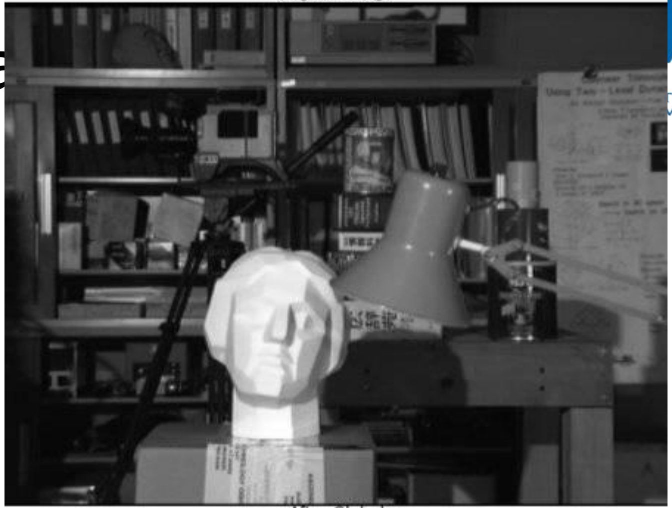
- Histogram is not confined to a particular region



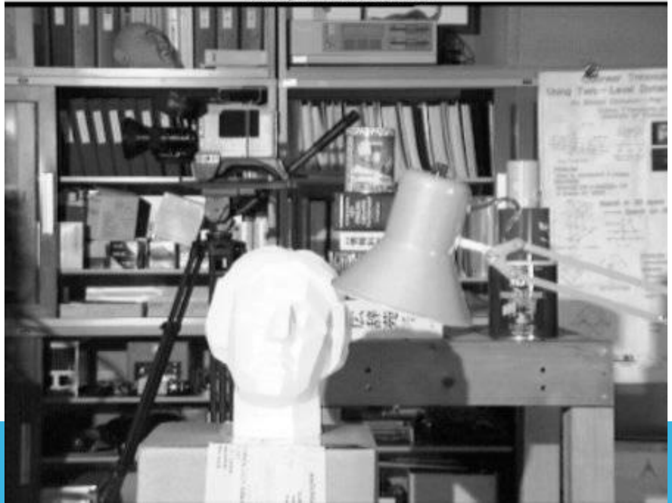
After Local Histogram Equalization

https://docs.opencv.org/4.x/d5/daf/tutorial_py_histogram_equalization.html

Original Image

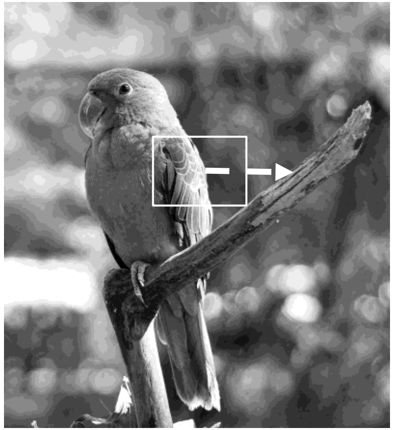


After Global
Histogram Equalization

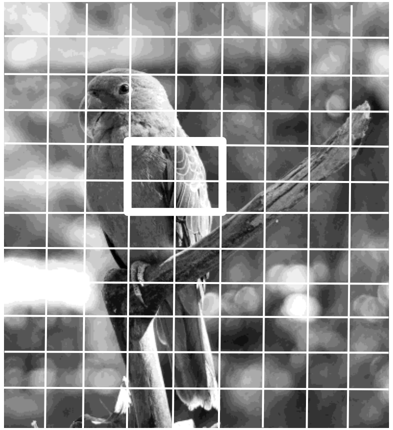


Adaptive histogram equalization

- Histogram equalization based on a histogram obtained from a portion of the image



Sliding window approach: different histogram (and mapping) for every pixel



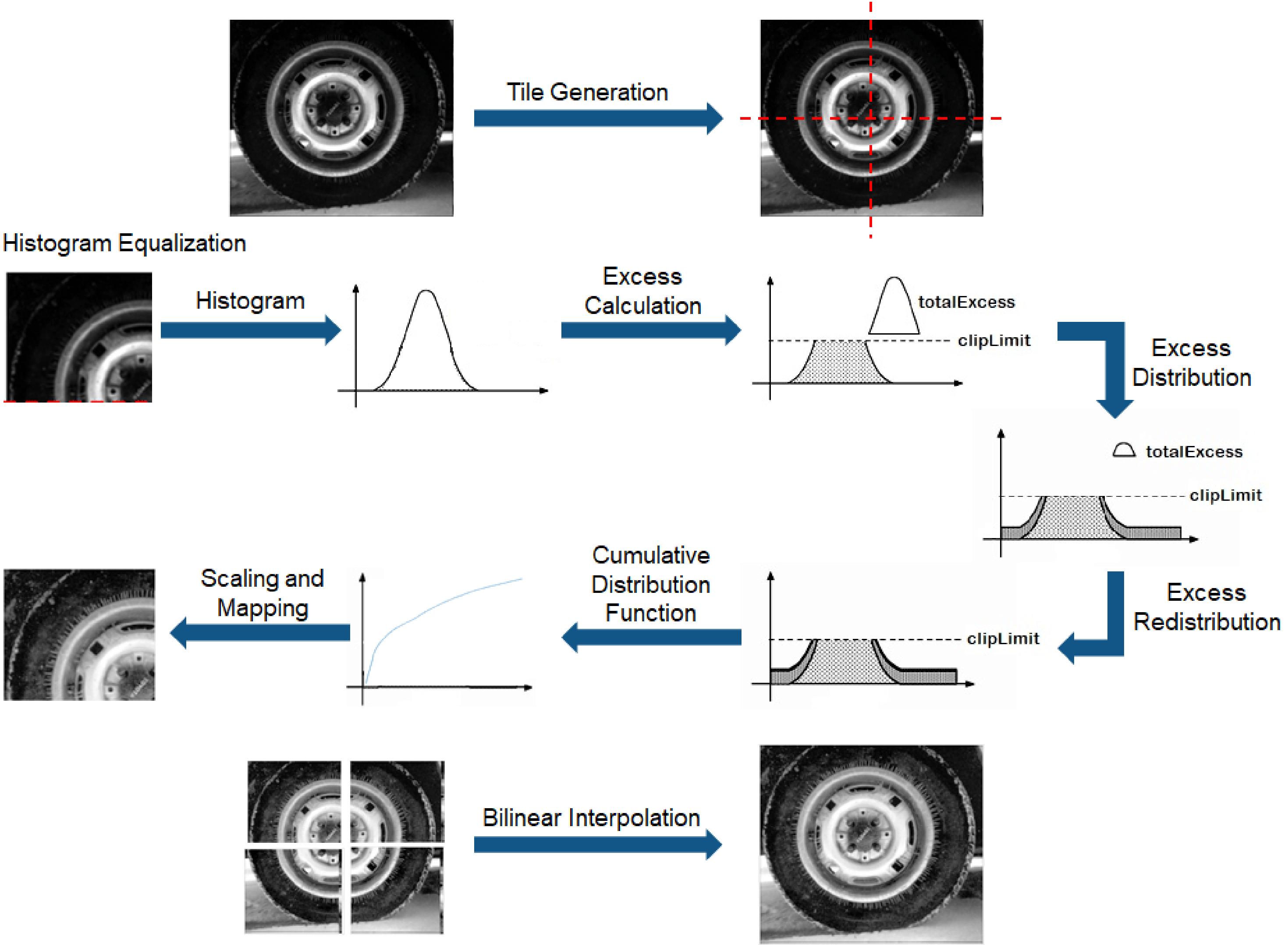
Tiling approach: image is divided into small blocks called "tiles" (size is 8x8 in OpenCV). Then each of these blocks are histogram equalized as usual.

- Limit contrast expansion in flat regions of the image, e.g., by clipping histogram values (“Contrast-limited adaptive histogram equalization”) [Pizer, Amburn et al. 1987]

CLAHE (Contrast-Limited Adaptive Histogram Equalization)

- Image is divided into small blocks called "tiles" (size is 8x8 in OpenCV)
  - Then each of these blocks are histogram equalized as usual.
- In a tile, histogram would confine to a small region (unless there is noise). If noise is there, it will be amplified.
  - To avoid this, contrast limiting is applied. If any histogram bin is above the specified contrast limit (by default 40 in OpenCV), those pixels are clipped and distributed uniformly to other bins before applying histogram equalization.
- After equalization, to remove artifacts in tile borders, bilinear interpolation is applied.

CLAHE



https://es.mathworks.com/

Adaptive histogram equalization

Original image *Parrot*

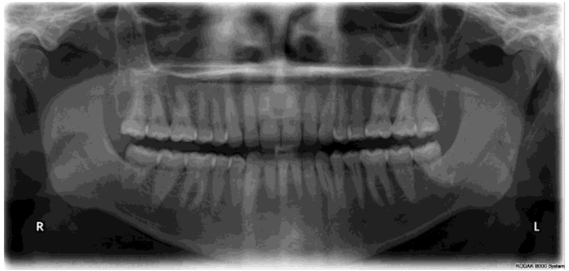
Global histogram equalization

Adaptive histogram equalization, 8x8 tiles

Adaptive histogram equalization, 16x16 tiles

Adaptive histogram equalization

Original image *Dental Xray*



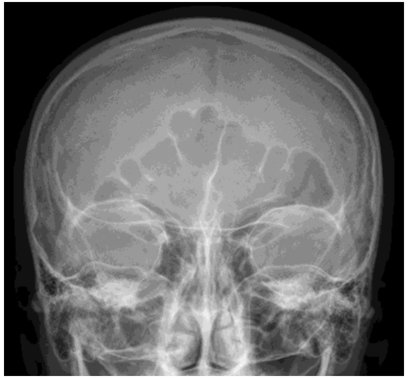
Global histogram equalization

Adaptive histogram equalization, 8x8 tiles

Adaptive histogram equalization, 16x16 tiles

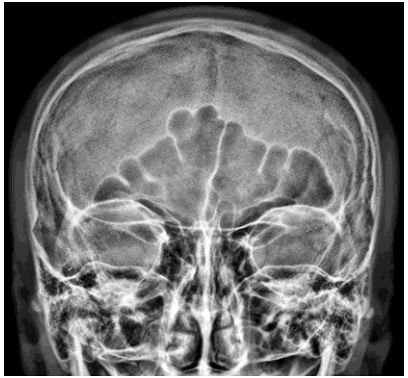
Adaptive histogram equalization

Original image *Skull Xray*



Global histogram equalization

Adaptive histogram equalization, 8x8 tiles



Adaptive histogram equalization, 16x16 tiles

References

- https://www.photopills.com/articles/exposure-photography-guide-2
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- https://slideplayer.com/slide/9293085/

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- https://web.stanford.edu/class/cs101/image-6-grayscale.html