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# COMPREHENSIVE MOTION PLANNING AND SLOSHING SUPPRESSING CONTROL FOR LIQUID TRANSPORTATION SYSTEMS


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# 1. INTRODUCTION

**Figure 1.** Liquid sloshing system applications

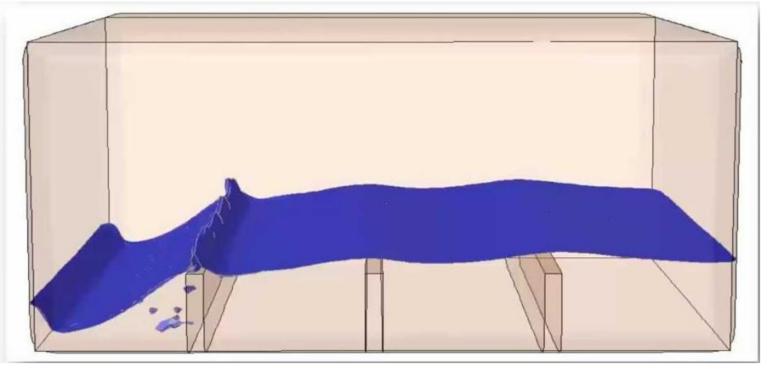
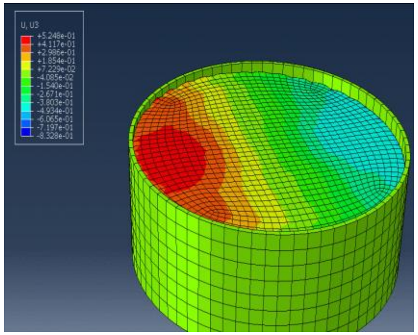
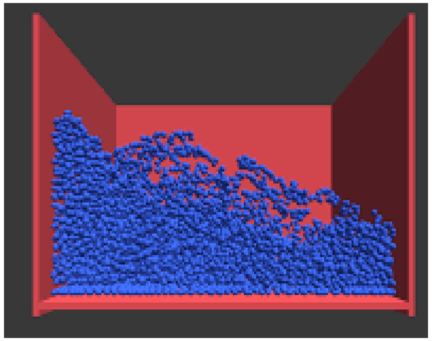


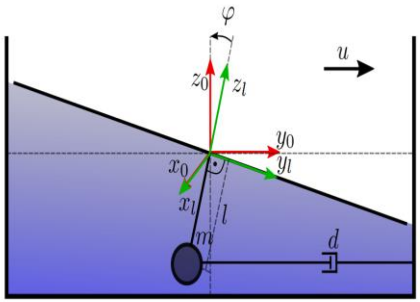
**Figure 2.** Anti wave baffles



Finite Element Method [1]



Smoothed Particle Hydrodynamics [2]




Equivalent Discrete Mechanical Model[3]

**Figure 1.** Different approaches of liquid sloshing models


[1] Kolukula, Siva Srinivas, and P. Chellapandi. "Nonlinear Finite Element Analysis of Sloshing." *Advances in Numerical Analysis* (2013).
[2] Schörgenhumer, Markus, and Andreas Eitzlmayr. "Modeling of liquid sloshing with application in robotics and automation." *IFAC-PapersOnLine* 52.15 (2019): 253-258.
[3 ] Bauer, Helmut F. "Nonlinear mechanical model for the description of propellant sloshing." *AIAA Journal* 4.9 (1966): 1662-1668.

# 1. INTRODUCTION

**problem formulation**

- container moving from A to B
- suppress the liquid sloshing height
- ensure maximum liquid sloshing height in acceptable range
- avoid static obstacle

**solution**

motion planning:
- generating path from A to B
- maximum liquid sloshing height in acceptable range

extended observer
- estimate unmeasured state variable
- estimate disturbance

controller:
- trajectory tracking
- liquid sloshing suppressing

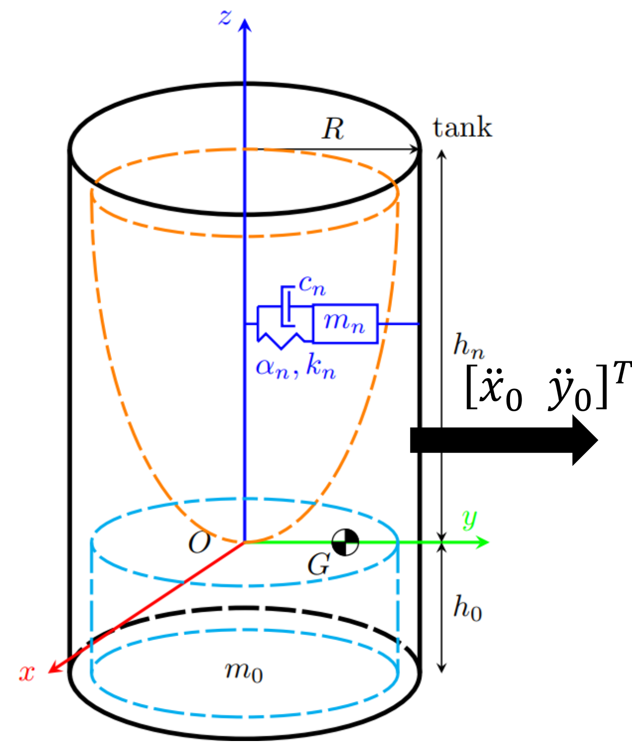
## 2.1 Mathematical Model

**Euler-Lagrange equation**

$$\begin{cases} \dfrac{d}{dt}\left(\dfrac{\partial T}{\partial \dot{x}_n}\right) - \dfrac{\partial T}{\partial x_n} + \dfrac{\partial V}{\partial x_n} = -\dfrac{\partial D}{\partial \dot{x}_n} \\ \dfrac{d}{dt}\left(\dfrac{\partial T}{\partial \dot{y}_n}\right) - \dfrac{\partial T}{\partial y_n} + \dfrac{\partial V}{\partial y_n} = -\dfrac{\partial D}{\partial \dot{y}_n} \end{cases}$$

equations of motion

**assumption:** consider acceleration of the container $[\ddot{x}_0 \; \ddot{y}_0]$ as control input




Equation of Motion in matrix form: $\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{G}(\boldsymbol{q}) = \boldsymbol{Q}\boldsymbol{u}$

$$\boldsymbol{q} = [x_1 \; y_1 \; x_0 \; y_0]^T$$

$$\bar{\eta} = \frac{\xi_{1n}^2 h m_n}{m_F R}\sqrt{x_1^2 + y_1^2}$$

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## 2.2 Flat output of the liquid sloshing system

**Euler-Lagrange equation of motion:** $\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}}) + \boldsymbol{G} = \boldsymbol{Q}\boldsymbol{u}$

Ignoring inertial cross-coupling terms, and small nonlinear terms. Setting

$$v_{x1} = x_1 + \frac{\beta}{6} x_1^3$$

$$v_{y1} = y_1 + \frac{\beta}{6} y_1^3$$

Inspired by [4]

$$\begin{bmatrix} \mathbf{x}_s \\ \mathbf{y}_s \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{f2s} & \mathbf{0}_{5\times6} \\ \mathbf{0}_{5\times6} & \mathbf{A}_{f2s} \end{bmatrix} \begin{bmatrix} \mathbf{z}_{s1} \\ \mathbf{z}_{s2} \end{bmatrix}$$

**flat domain:** $\dot{\boldsymbol{z}} = \boldsymbol{A}_{flat}\boldsymbol{z} + \boldsymbol{B}_{flat}\boldsymbol{v},$

$$\boldsymbol{z} = \begin{bmatrix} z_1 & z_2 & \dot{z}_1 & \dot{z}_2 & \ddot{z}_1 & \ddot{z}_1 & z_1^{(3)} & z_2^{(3)} \end{bmatrix}^{\boldsymbol{T}}$$



[4] Lévine, Jean, and Dinh Viet Nguyen. "Flat output characterization for linear systems using polynomial matrices." *Systems & control letters* 48.1 (2003): 69-75.

## 2.3 Flatness-based trajectory

**flat output**

$$z_1 = f(x_1, \dot{x}_1, x_0, \dot{x}_0)$$

$$z_1(t) = z_1(t_i) + \left(\frac{t - t_i}{t_f - t_i}\right)^{r+2} \left(\sum_{k=0}^{r+1} \alpha_k \left(\frac{t - t_i}{t_f - t_i}\right)^k\right)$$

$$z_2 = z_{2i} + (z_{2f} - z_{2i})\left(\frac{z_1 - z_{1i}}{z_{1f} - z_{1i}}\right)\left(c_2\left(\frac{z_1 - z_{1i}}{z_{1f} - z_{1i}}\right)^2 + c_1\left(\frac{z_1 - z_{1i}}{z_{1f} - z_{1i}}\right) + c_0\right)$$

**state variables (similar for y-axis)**

$$x_1 = f_{x_1}(\ddot{z})$$
$$x_0 = f_{x_0}(z_1, \dot{z}_{,1}\ \ddot{z}_1)$$
$$u_x = f_{u_x}\left(z_1, \dot{z}_1, \ddot{z}_1, z_1^{(3)}, z_1^{(4)}\right)$$

$$[\boldsymbol{x}_r \quad \dot{\boldsymbol{x}}_r \quad \ddot{\boldsymbol{x}}_r]^T$$

## 2.3 Flatness-based trajectory

$$M(q)\ddot{q} + C(q,\dot{q}) + G(q) = Qu$$

$\xrightarrow{\text{flat transformation}}$

$$\dot{z} = A_y.\mathbf{z} + \mathbf{B_y}v$$

**flat domain optimization problem**

$$\min\ J = \sum_{=1}^{N} \Delta t[k]$$

$$s.t.\ \ \mathbf{z}[k+1] = \mathbf{z}[k] + \Delta t[k].\left(A_{flat}z[k] + B_{flat}v\right)$$

$$|\bar{\eta}_n| < \bar{\eta}_{max}$$

$$\|\mathbf{u}\| < u_{max}$$

$$Rect = \sum_{j=1}^{2}\|x - s_j\| + \|y - s_{j+2}\| > \|s_1 - s_2\| + \|s_3 - s_4\|$$

$$\mathbf{z}[N] = \mathbf{z}_f\ ; \mathbf{z}[1] = \mathbf{z}_i$$

**inverse-flat transformation**

$$z_1 = f(x_1, \dot{x}_1, x_0, \dot{x}_0)$$

$$x_1 = f_{x_1}(\ddot{z})$$

$$x_0 = f_{x_0}(z_1, \dot{z}_1\ \ddot{z}_1)$$

$$u_x = f_{u_x}\left(z_1, \dot{z}_1, \ddot{z}_1, z_1^{(3)}, z_1^{(4)}\right)$$

**reference**

$$[\boldsymbol{x}_r\quad \dot{\boldsymbol{x}}_r\quad \ddot{\boldsymbol{x}}_r]^T$$

## 2.4 Comparing different motion planning approaches

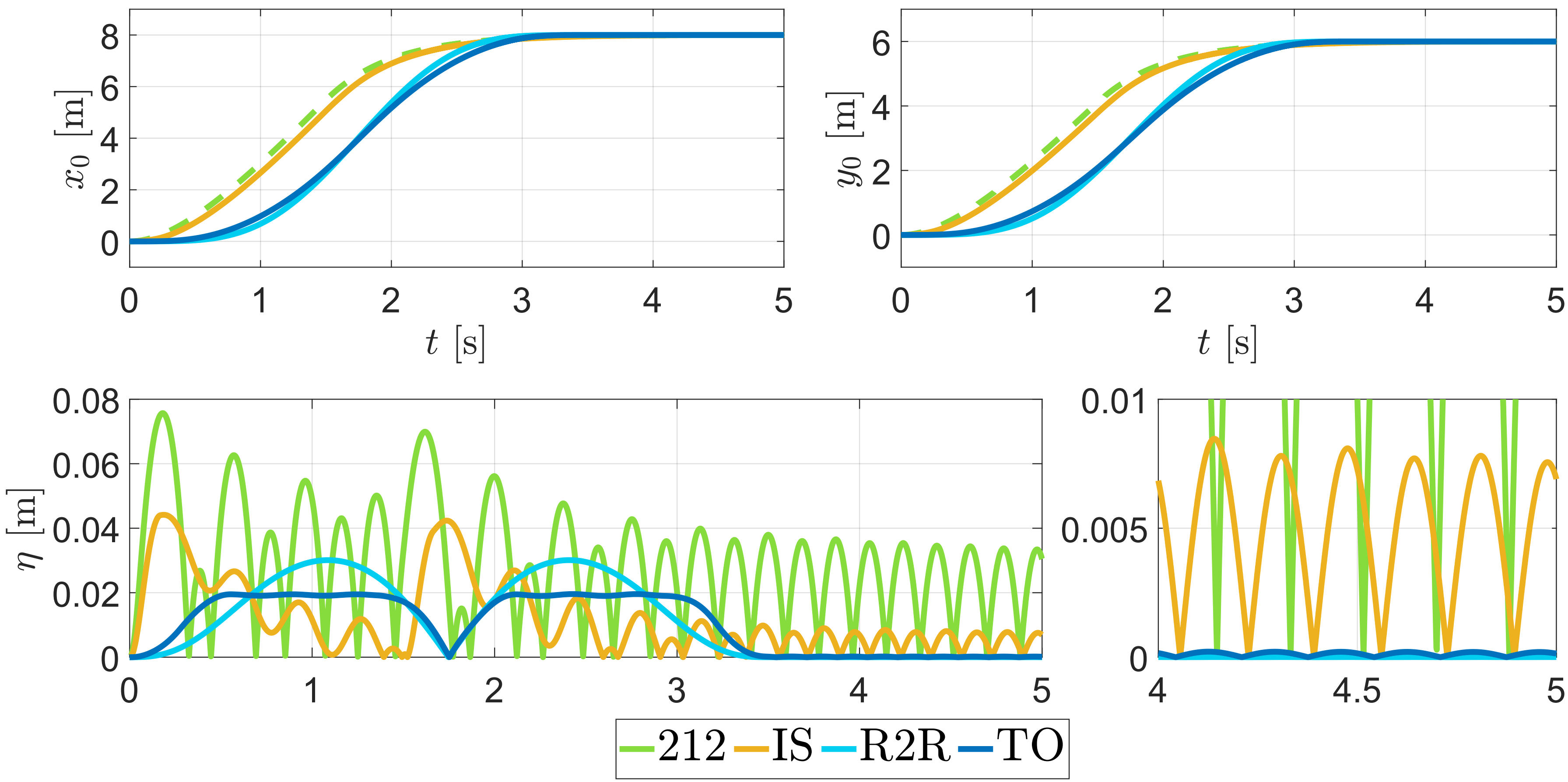
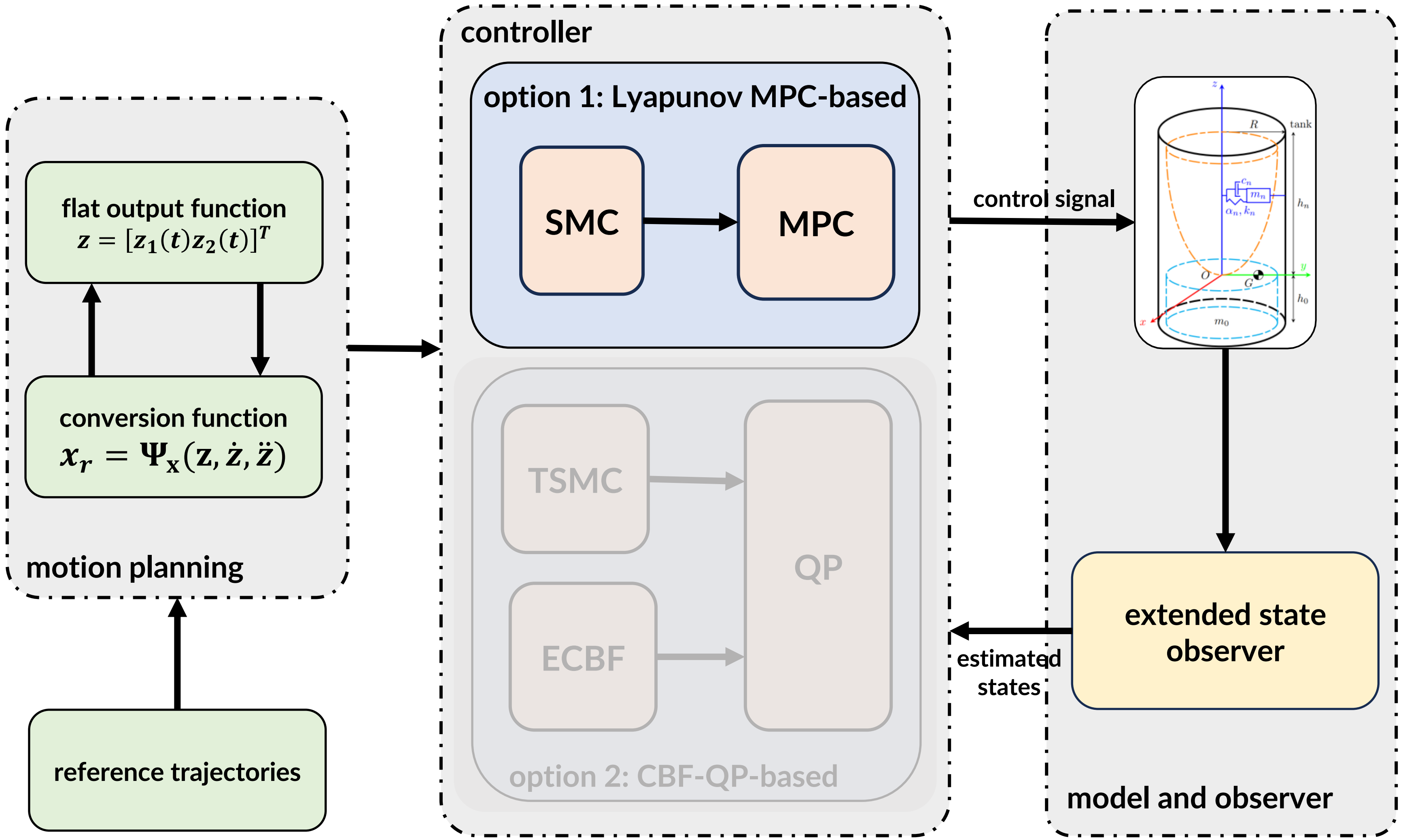
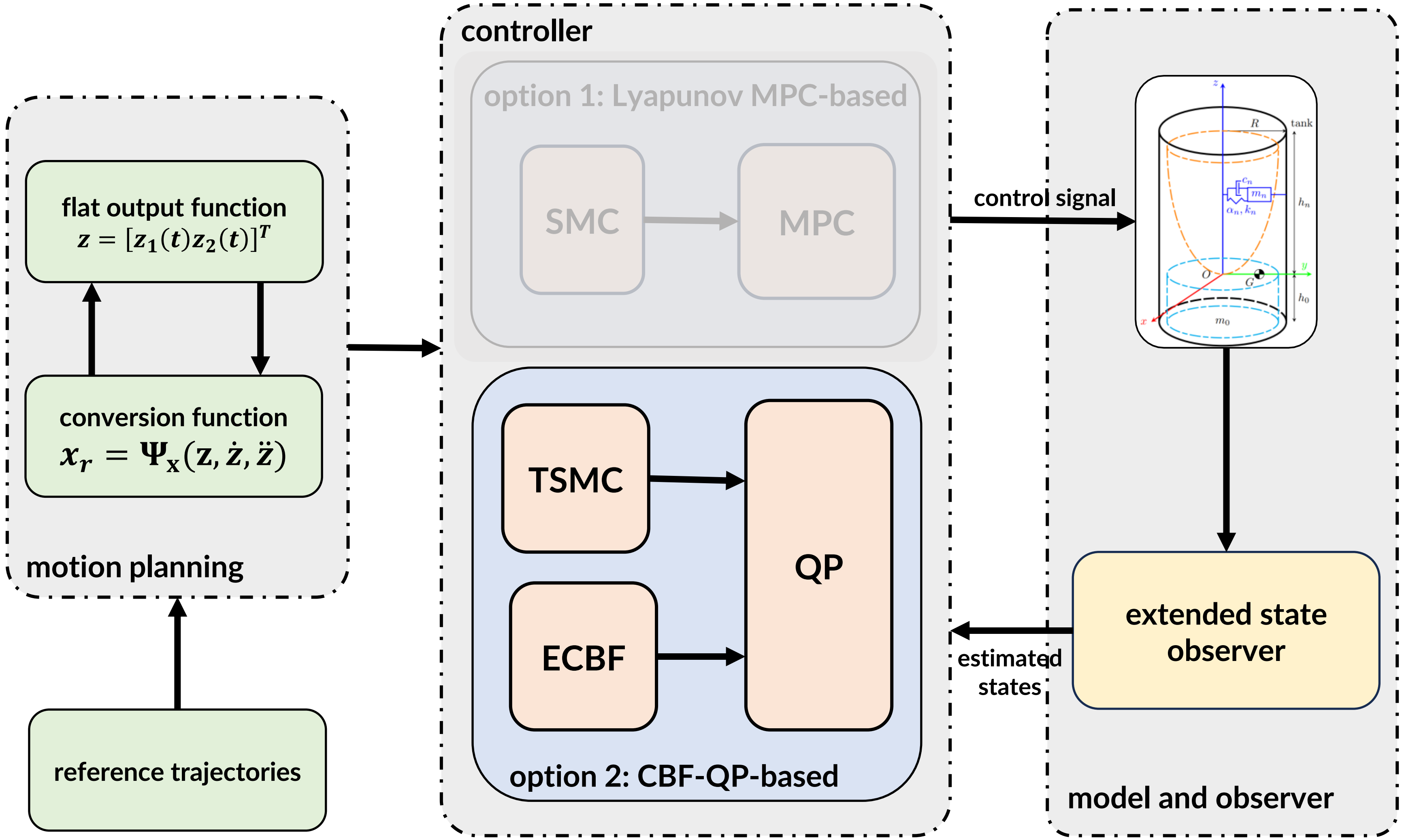



**Figure 2. Container position and sloshing height – feed-forward control**

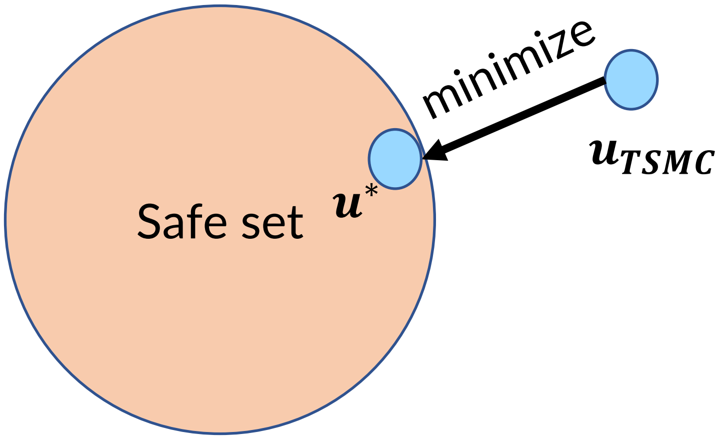
# 3. CONTROL METHODOLOGY

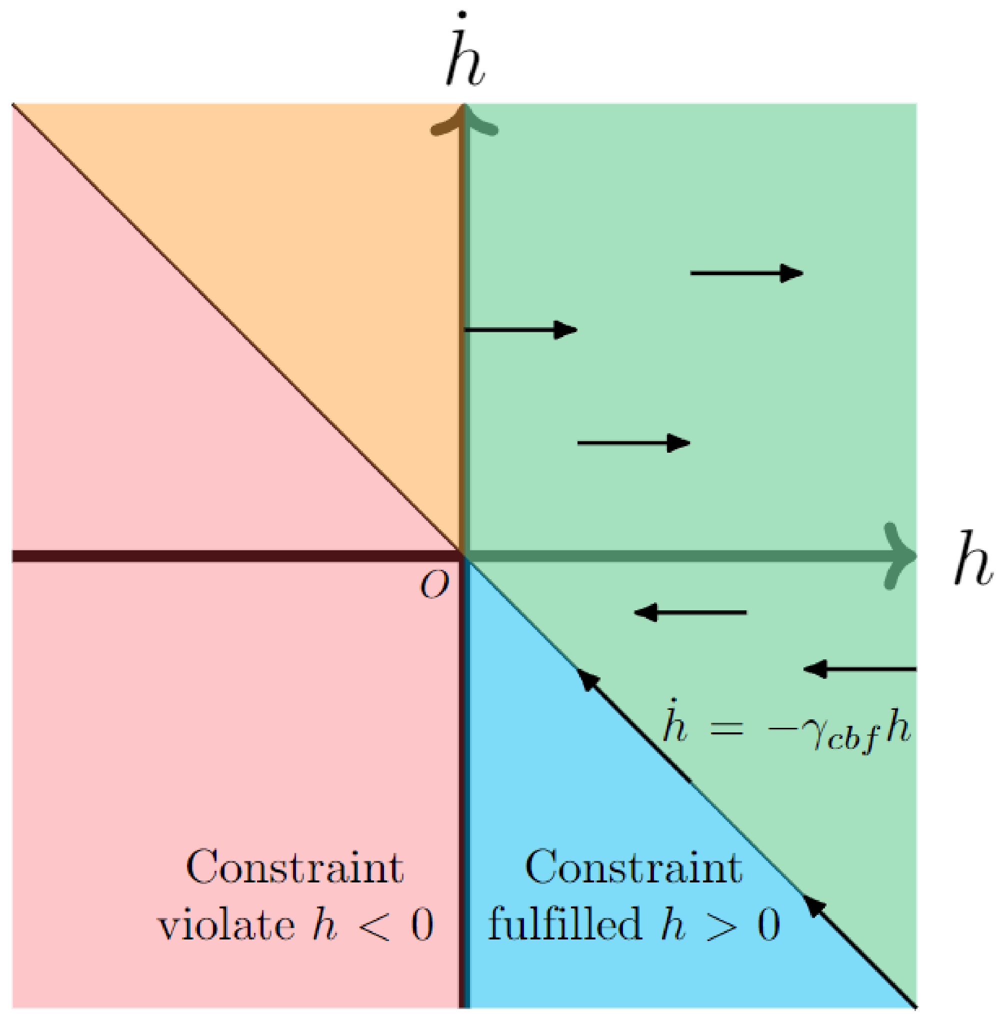


# 3. CONTROL METHODOLOGY



## TSMC-CBF




Quadratic Programming (QP):

$$\boldsymbol{u}^* = \arg\min_{\boldsymbol{u}\in\mathbb{C}} \|\boldsymbol{u} - \boldsymbol{u}_{TSMC}\|^2$$

subject to

$$H = L_{\bar{\mathcal{A}}}H + L_{\bar{\mathcal{B}}}H\boldsymbol{u} \geq -\alpha_{cbf}.H$$

## 4.1 First scenario

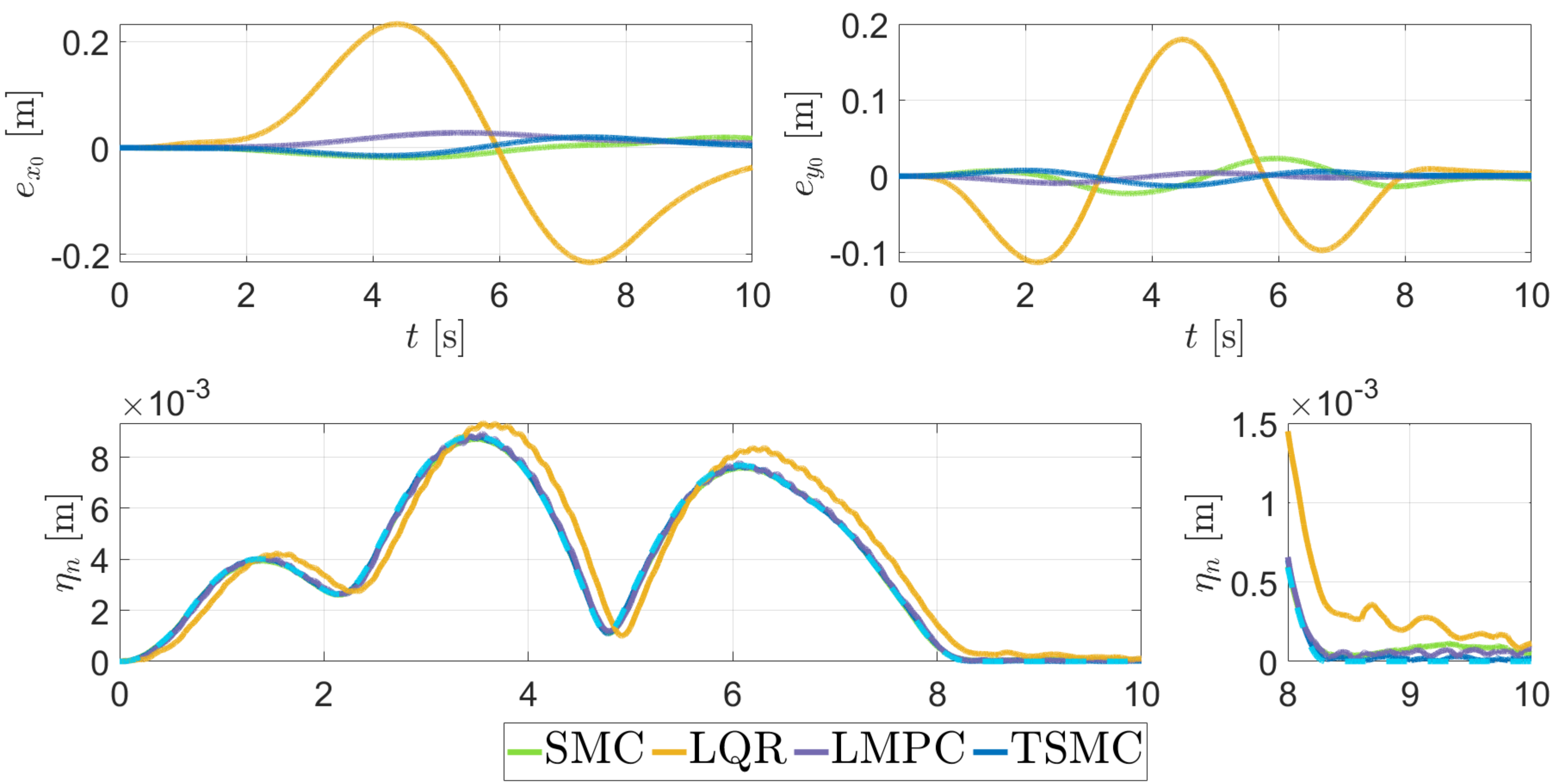



**Figure 3. Container position and sloshing height – time optimal trajectory**

# 4. NUMERICAL SIMULATION

## 4.2 Second scenario

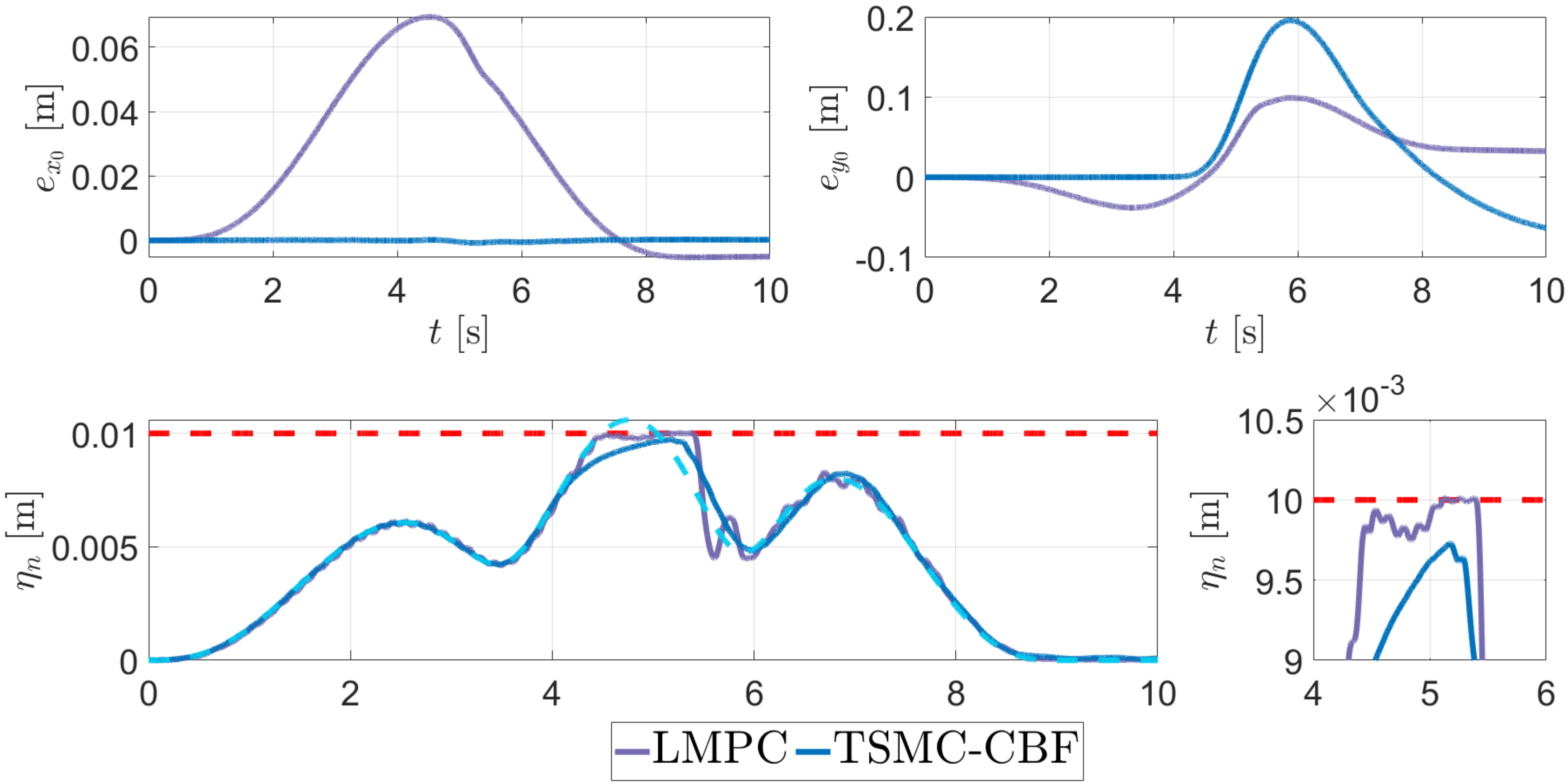



**Figure 4. Container position and sloshing height – output constraint**

## 4.3 Sloshing height constraint for rest-to-rest trajectory

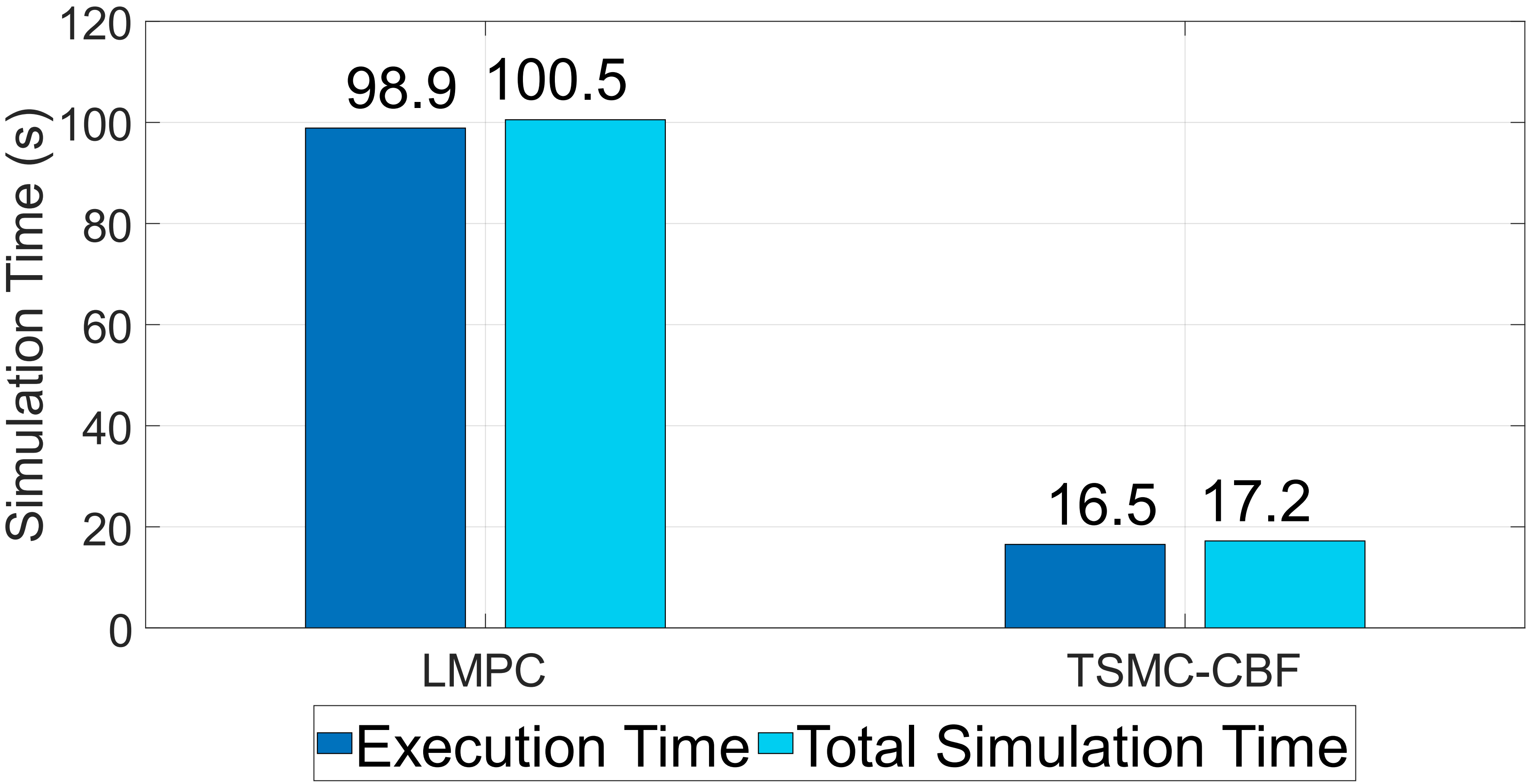



**Figure 5. Controller's simulation time comparison**

# 6. Conclusion

Main contribution:

- Novel flatness modeling
- Flatness-based Rest-to-rest and Time-optimal trajectory planning
- Flexible and stable control via LMPC:
- Robust control strategy with TSMC-CBF:

Disadvantage:

- Offline trajectory generation
- Model dependency
- Computationally expensive
- The control methodology is not validated through an experiment

# Related publication and awards

## Awards

- 3rd Prize – HUST Student Research Award (2023–2024)
- 2nd Prize – HUST Student Research Award (2024–2025)

## Related publication

- **Khanh Nguyen Viet**, Hue Luu Thi, Thanh Cao Duc, Huy Nguyen Danh, Minh Nhat Vu, Tung Lam Nguyen.Time-optimal motion planning and anti-sloshing control for a container under disturbances. IEEE Access (**SCIE, Q1, Scopus**), 2025.
- **Khanh Nguyen Viet**, Minh Do Duc, Thanh Cao Duc, Lam Nguyen Tung.Anti-sloshing control: Flatness-based trajectory planning and tracking control with an integrated extended state observer. IET Cyber-Systems and Robotics (**Q3, Scopus**), 2024.
- **Viet Khanh Nguyen**, Hue Luu Thi, Duc Thanh Cao, Dang Huu Bang, Tung Lam Nguyen.The Non-Flatness Property of the Liquid Sloshing System and an Approximate Approach. 2024 International Conference on Advanced Technologies for Communications (ATC), IEEE.

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# Related publication and awards

- **Khanh Nguyen Viet**, Hue Luu Thi, Minh Do Duc, Thanh Cao Duc, Huy Nguyen Danh, Nguyen Tung Lam.Input Shaping Integrated with Lyapunov-Based Model Predictive Control for Anti-Sloshing Problem. 3rd International Conference on Advances in Information and Communication Technology, Springer (**Q4, Scopus**), 2024.
- **Viet Khanh Nguyen**, Hue Luu Thi, Duc Thanh Cao, Tung Lam Nguyen, Duc Minh Do, Thanh Ha Vo.Control Strategy for Liquid Transfer Using a Four-Wheel Mecanum Mobile Robot Platform. 9th International Conference on Applying New Technology in Green Buildings, IEEE, 2024.
- Minh Do Duc**, Khanh Nguyen Viet**, Thanh Cao Duc, Ho Thanh Hieu, Duc Duong Minh, Lam Nguyen Tung.Flatness-Based Nonlinear Control for Path Planning and Tracking of Sloshing Liquid Container. Journal of Science and Technology, June 2023.
- Cao Duc Thanh, **Nguyen Viet Khanh**, Tran Thi Thanh Thao, Nguyen Van Minh, Nguyen Danh Huy, Nguyen Tung Lam.Control of Liquid Oscillations in Horizontal Motion Using Flatness-Based Trajectory Planning. Journal of Military Science and Technology, 2024.
- Thanh Cao Duc, **Khanh Nguyen Viet**, Minh Do Duc, Lam Nguyen Tung.Flatness-Based Nonlinear Approach to Liquid Sloshing in a 2D Moving Container. Vietnam International Conference and Exhibition on Control and Automation (VCCA), 2024 (Accepted).

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