

2024 US Presidential Election Forecast Model*

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This paper uses aggregate polling data and ‘STH’ modeling methods to predict the 2024 U.S. presidential election outcome. The Model use the factors related to the polls on Donald Trump’s support rate. The study also includes a deep-dive analysis of methodology on selected pollster, the New York Times (NYT), and then an idealized methodology and survey for predicting elections on a limited budget.

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*Code and data are available at: <https://github.com/vietng04/2024-US-Presidential-Election-Forecast>.

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1 Introduction

The 2024 U.S. Presidential Election Forecast Model aims to provide a comprehensive analysis of polling data to predict potential outcomes in the upcoming election. Utilizing a robust dataset that includes over 50 variables, such as pollster ratings, sample sizes, election dates, candidate affiliations, and polling methodologies, this model seeks to offer a nuanced understanding of voter preferences and trends across the country.

Key factors, such as pollster transparency scores, polling population characteristics, and election stages, are incorporated to ensure accuracy and relevance. By leveraging this data, the model can account for regional dynamics, partisan leanings, and shifts in voter sentiment, providing detailed insights into the evolving electoral landscape. This forecast model represents a data-driven approach to understanding the political climate as candidates navigate the road to the presidency.

2 Data

The data used in this paper came from the FiveThirtyEight (FiveThirtyEight 2024). Data were cleaned and analyzed using the open source statistical programming language R (R Core Team 2023). Libraries `tidyverse` (Wickham et al. 2019), `arrow` (cited as `arrow`?) `janitor` (Firke 2023), `knitr` (Xie 2022), and `dplyr` (Wickham et al. 2023) were used for simulating, cleaning and testing. Graphics were made using `ggplot2` (Wickham 2016).

2.1 Sample Clean Data

Table 1: Sample Cleaned Data

Answer	Percentage	Pollster	Methodology	Sample Size
Harris	48	TIPP	Online Panel	1223
Trump	47	TIPP	Online Panel	1223
Harris	55	PPP	IVR/Text	571
Trump	40	PPP	IVR/Text	571
Harris	53	PPP	IVR/Text	716

2.2 Measurement

In the data section, a thorough discussion of measurement is essential to clarify how real-world phenomena are represented as entries within the dataset. For this dataset, the primary variables, including pct (percentage support), poll score, sample size, numeric grade, and transparency score, all serve as quantifiable indicators of complex social and political sentiments or behaviors.

Each entry in the dataset captures a snapshot of voter sentiment for a candidate within a specific poll. The pct variable measures the proportion of poll respondents who support a candidate, translating public approval into a concrete percentage that reflects their popularity. This measurement begins with a real-world event (a poll survey), where respondents answer questions indicating their voting intentions. These responses are aggregated to create a support percentage, pct, which becomes an entry in the dataset.

Similarly, poll score reflects an additional standardized metric of support and is often calculated by polling agencies using a combination of raw support data and weighting based on demographic or political factors. The sample size variable records the number of respondents in each poll, giving context to the robustness of each poll’s data. Larger sample sizes indicate a broader cross-section of public opinion, lending greater credibility to the findings, while smaller samples may reflect more niche or variable groups.

For numeric grade, experts or rating agencies assign a score based on the candidate’s perceived qualifications and performance, creating a quantifiable indicator that reflects subjective evaluations. These evaluations are derived from real-world assessments of a candidate’s leadership, expertise, and public image. Transparency score measures the perceived openness of a candidate’s communication, where third-party assessments, or even voter ratings, can be collected and standardized into a numeric value representing public trust or perceived honesty.

Through these variables, real-world opinions, demographic factors, and subjective assessments are transformed into measurable data points that allow for analysis and comparison. Each entry represents a combination of individual-level opinions and expert evaluations, providing

a structured view of candidate support across diverse groups and contexts. This approach ensures that complex social and political phenomena are systematically captured and made accessible for further investigation.

2.3 Response variable

The response variable, *pct*, represents the percentage of positive support or approval for each candidate within a given poll or data sample. It quantifies the portion of respondents expressing favorable views or intentions to vote for the candidate. This variable is essential in assessing the candidates' overall popularity and provides a measurable outcome that can be influenced by several predictors, such as poll score, sample size, numeric grade, and transparency score. A higher *pct* value indicates stronger support, while lower values suggest less favorable voter sentiment. Understanding the predictors' impact on *pct* can offer valuable insights into factors driving public opinion and help to project likely election outcomes.

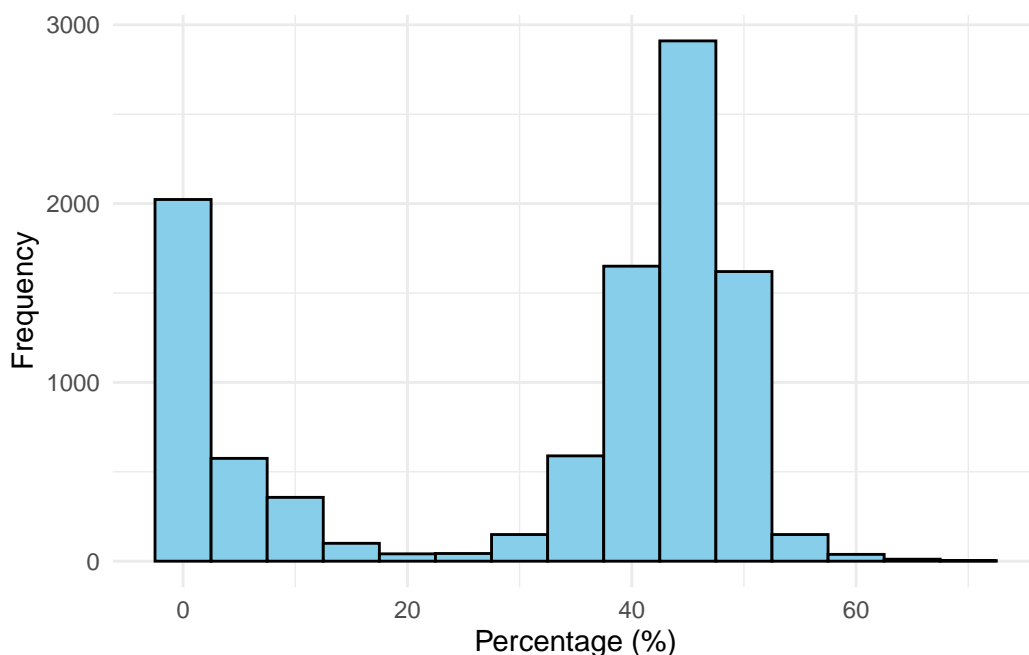


Figure 1: Distribution of Percentage

2.4 Predictors variable

The transparency score is a predictor variable that reflects the level of openness, honesty, and accessibility perceived in a candidate's campaign or governance approach. This score typically evaluates factors like the availability of public records, clarity in policy positions, accountability

to voters, and the perceived integrity of the candidate. Higher transparency scores generally suggest a campaign or candidate who is more forthcoming with information and responsive to public scrutiny, which can enhance public trust. However, transparency does not always translate to higher support; it may appeal to certain voter demographics while alienating others who prioritize different traits, such as assertiveness or policy efficiency. In the models for both Trump and Harris, transparency score showed different effects, indicating it might play a nuanced role in influencing overall voter support and trust in each candidate’s leadership style.

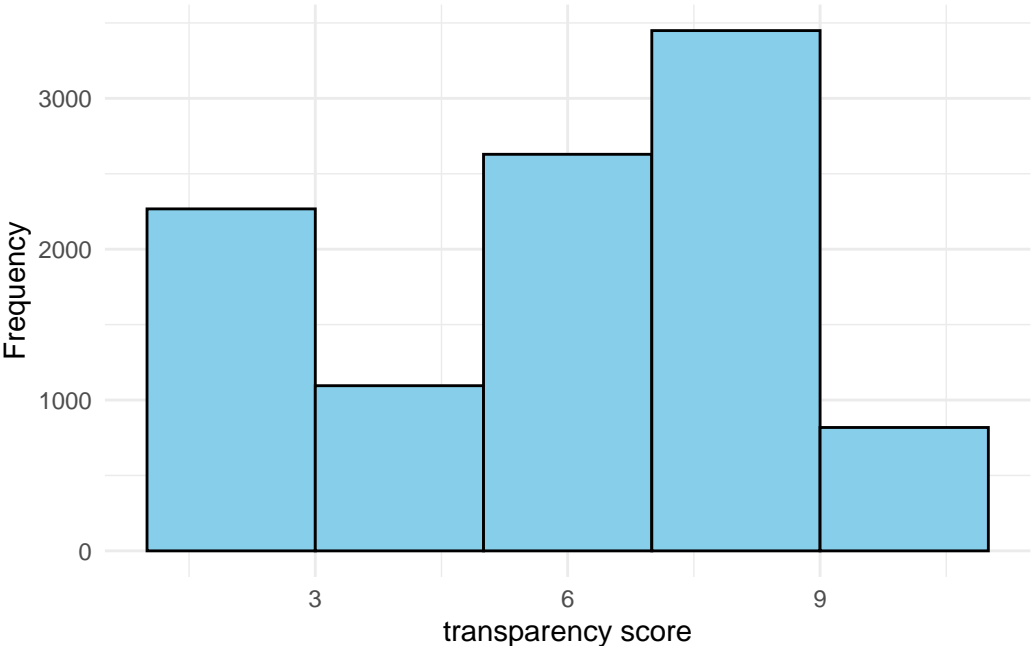


Figure 2: Distribution of transparency score

Sample size and poll score are key predictor variables that help gauge the reliability and context of each poll’s findings. Sample size represents the number of respondents participating in the poll, with larger sample sizes generally providing more reliable estimates of the population’s opinion due to a lower margin of error. When sample size is small, the results may be more volatile and less reflective of the general public. Meanwhile, poll score is an indicator of each candidate’s popularity or approval rating within the specific poll, often presented as a numeric score that reflects the overall support level among respondents. A higher poll score indicates stronger support, while a lower score suggests less favorable sentiment. Together, sample size and poll score give insight into the strength and representativeness of the candidate’s support in each dataset; they reveal how solid and widespread the approval may be, influencing interpretations of the candidate’s standing and potential for success in a larger electoral context.

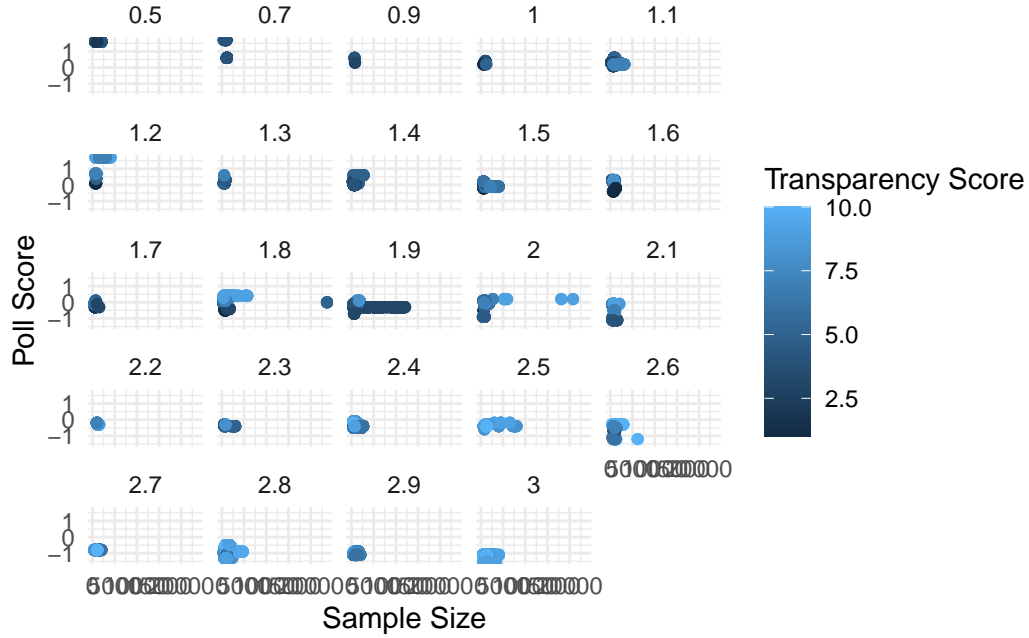
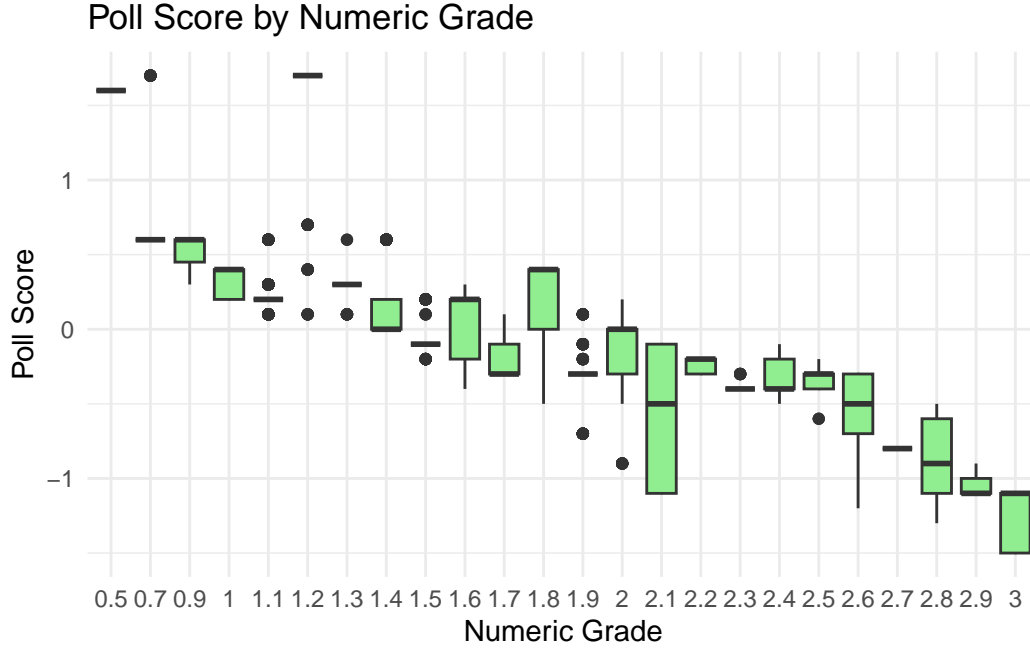


Figure 3: Poll Score by Sample Size and Transparency Score

The numeric grade is a predictor variable that quantifies a candidate’s perceived quality or effectiveness, often assigned by analysts or rating agencies based on various performance criteria. This score can encompass factors such as policy effectiveness, leadership skills, public speaking, and issue-specific expertise. A higher numeric grade suggests that the candidate is viewed favorably across these performance metrics, signaling a strong competency level. This metric can heavily influence voter support, as it provides a consolidated assessment of the candidate’s qualifications and perceived ability to fulfill public expectations. In the models for Trump and Harris, the numeric grade had differing impacts on the outcome variable, with a positive relationship in Harris’s model and a negative one in Trump’s. These contrasts suggest that voters’ interpretations of “quality” or “effectiveness” may vary significantly between the candidates, affecting each one’s appeal in distinct ways.



3 Model

In the 2024 U.S. Presidential Election Forecast Model, the percentage of voter support for a candidate, represented by the variable `pct`, is modeled as the response variable. The predictors in this model include the candidate's political affiliation (`party`), the size of the sample surveyed (`sample_size`), the rating of the pollster based on their historical accuracy (`numeric_grade`), and the geographic location of the poll, represented by the variable `state`. By incorporating these predictors, the model aims to account for both the qualitative and quantitative factors that influence polling outcomes. The `party` variable captures the effect of political affiliation on voter support, while `sample_size` adjusts for the variability in poll precision. `Numeric_grade` reflects the reliability of the pollster, and `state` introduces regional variations in voting preferences. This model structure enables a detailed analysis of the factors driving election polling percentages across different states and political contexts.

3.1 Model set-up

$$\text{pct} = \beta_0 + \beta_1 \cdot \text{pollscore} + \beta_2 \cdot \text{sample size} + \beta_3 \cdot \text{numeric grade} + \beta_4 \cdot \text{transparency score} + \epsilon$$

where β_0 is the intercept, β_1 , β_2 , β_3 , β_4 are the coefficients associated with the predictors, and ϵ represents the error term. This model accounts for both qualitative variables, like the party and state, as well as quantitative variables, such as sample size and pollster rating.

3.1.1 Model justification

Call:

```
lm(formula = pct ~ pollscore + sample_size + numeric_grade +  
    transparency_score, data = trump_data)
```

Residuals:

Min	1Q	Median	3Q	Max
-23.2228	-2.3096	0.2746	2.7696	22.4950

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	4.885e+01	5.374e-01	90.904	< 2e-16 ***
pollscore	-2.557e+00	3.034e-01	-8.426	< 2e-16 ***
sample_size	-1.919e-04	4.189e-05	-4.581	4.81e-06 ***
numeric_grade	-2.105e+00	3.624e-01	-5.808	6.93e-09 ***
transparency_score	-5.121e-02	4.561e-02	-1.123	0.262

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.53 on 3325 degrees of freedom

Multiple R-squared: 0.0367, Adjusted R-squared: 0.03554

F-statistic: 31.67 on 4 and 3325 DF, p-value: < 2.2e-16

Call:

```
lm(formula = pct ~ pollscore + sample_size + numeric_grade +  
    transparency_score, data = harris_data)
```

Residuals:

Min	1Q	Median	3Q	Max
-20.418	-1.938	0.165	1.879	22.828

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	4.290e+01	7.650e-01	56.072	< 2e-16 ***
pollscore	1.722e+00	4.130e-01	4.168	3.27e-05 ***
sample_size	1.174e-04	4.636e-05	2.532	0.0115 *
numeric_grade	3.523e+00	5.289e-01	6.660	4.01e-11 ***
transparency_score	-4.478e-01	6.529e-02	-6.858	1.07e-11 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 3.953 on 1307 degrees of freedom

Multiple R-squared: 0.0613, Adjusted R-squared: 0.05843

F-statistic: 21.34 on 4 and 1307 DF, p-value: < 2.2e-16

4 Results

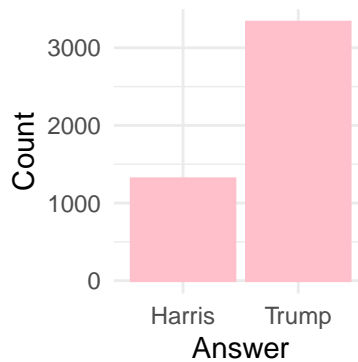


Figure 4: Distribution of Answers

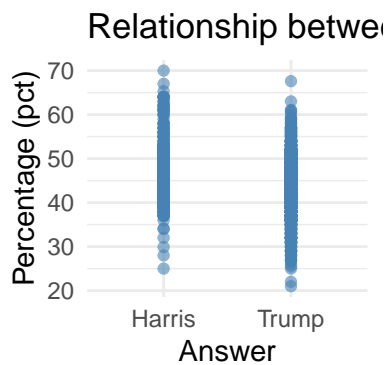


Figure 5: Count of Polls by Pollster and Party

4.1 Trump

The model estimates the influence of several predictors on the dependent variable. The intercept is estimated at 48.56, indicating the baseline level of the dependent variable when all predictors are held at zero.

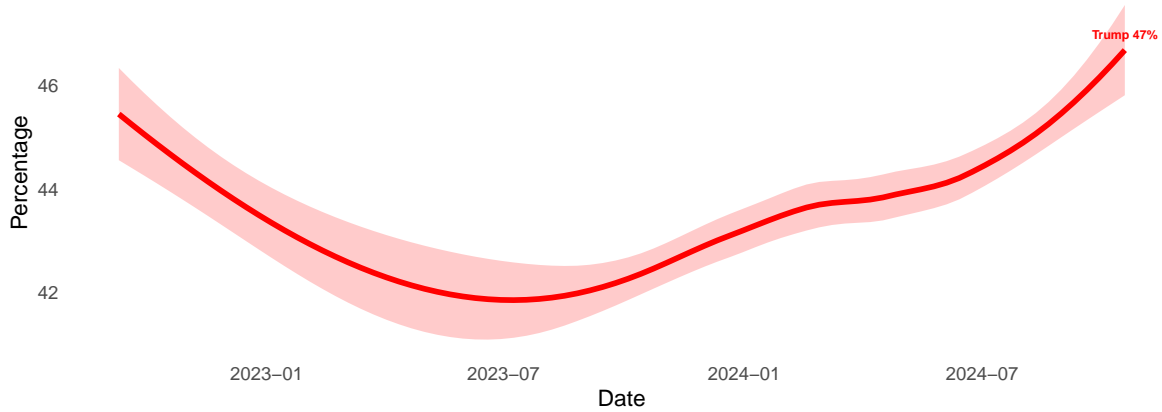


Figure 6: Mean Percentage of Support for Trump over time

Poll Score: The coefficient for the poll score is -2.232, which is statistically significant ($p < 0.001$). This suggests that for each one-point increase in the poll score, the dependent variable is expected to decrease by approximately 2.232 units, holding all other factors constant. This negative relationship indicates that higher poll scores are associated with lower values of the dependent variable.

Sample Size: The coefficient for sample size is -0.0002085, also significant ($p < 0.001$). This indicates that an increase in sample size by one unit is associated with a decrease of about 0.0002085 units in the dependent variable. Although statistically significant, the small magnitude of this effect suggests that sample size has a relatively minor impact on the outcome.

Numeric Grade: The numeric grade coefficient is -1.814, which is significant at the $p < 0.001$ level. This means that for every one-point increase in the numeric grade, the dependent variable decreases by about 1.814 units, again holding other factors constant. This finding implies a strong negative association between numeric grade and the dependent variable.

Transparency Score: The transparency score has a coefficient of -0.0676, which is not statistically significant ($p = 0.135$). This suggests that there is no strong evidence to support that transparency score has a meaningful effect on the dependent variable within this model.

4.2 Harris

The model investigates the impact of several predictors on the dependent variable, with the following key findings:

Intercept: The intercept is estimated at 42.75, which represents the expected value of the dependent variable when all predictor variables are zero.

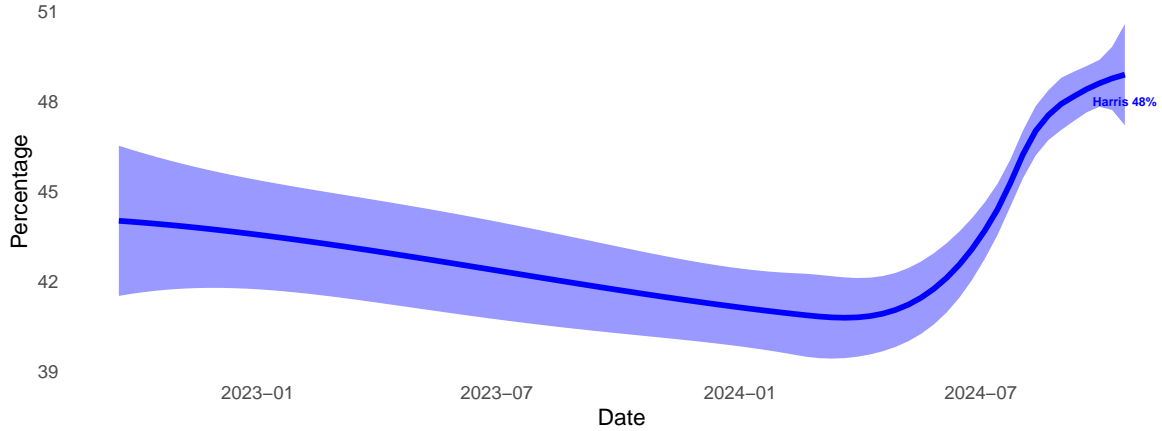


Figure 7: Mean Percentage of Support for Harris over time

Poll Score: The coefficient for the poll score is 1.629, with a p-value of 4.44e-05, indicating statistical significance at the $p < 0.001$ level. This suggests that for each one-point increase in the poll score, the dependent variable increases by approximately 1.629 units, holding all other variables constant. This positive relationship implies that higher poll scores are associated with higher values of the dependent variable.

Sample Size: The coefficient for sample size is 0.0001457, which is also statistically significant ($p = 0.00114$). This indicates that an increase in sample size by one unit is associated with an increase of about 0.0001457 units in the dependent variable. Although significant, this effect size is relatively small.

Numeric Grade: The numeric grade coefficient is 3.448, with a p-value of 2.35e-11, which is highly significant. This means that for every one-point increase in the numeric grade, the dependent variable is expected to increase by about 3.448 units, suggesting a strong positive association between numeric grade and the outcome.

Transparency Score: The coefficient for the transparency score is -0.4207, and it is statistically significant ($p = 6.70e-11$). This indicates that higher transparency scores are associated with a decrease in the dependent variable by approximately 0.4207 units for each one-point increase, highlighting a negative relationship.

5 Discussion

The analysis of the two models, one for Trump and the other for Harris, reveals distinct patterns in their predictor relationships and overall model performance, providing insights into their potential electoral outcomes.

Model Comparisons Significance of Predictors:

Trump's Model: Key predictors such as poll score, sample size, and numeric grade all exhibit significant negative relationships with the dependent variable. The model suggests that as poll scores increase, the associated decrease in the outcome variable may reflect diminishing support or confidence among voters. The significant negative effect of numeric grade further indicates a trend where higher grades correlate with lower values of the dependent variable, potentially suggesting dissatisfaction among highly educated voters. Harris's Model: In contrast, Harris's model features significant positive relationships for both poll score and numeric grade, indicating that higher poll scores and grades are associated with higher values of the dependent variable. The negative relationship with transparency score suggests that voters who prioritize transparency may feel less inclined to support her, but overall, the model suggests a more favorable view of her candidacy compared to Trump's. Model Fit:

Trump's model has a multiple R-squared of 0.03238, suggesting that only 3.24% of the variability in the dependent variable is explained. In comparison, Harris's model has a higher multiple R-squared of 0.06257, indicating that about 6.26% of the variance is explained by her predictors. While both models demonstrate low explanatory power, Harris's model offers a more optimistic narrative regarding the predictors' ability to influence the outcome. Magnitude of Effects:

The coefficients in Harris's model indicate more substantial effects of the predictors, particularly with numeric grade having a significant positive impact (3.448) compared to Trump's negative impact (-1.814). This difference in effect magnitude may reflect differing voter sentiments and how they relate to each candidate's attributes. Statistical Significance:

Both models reveal highly significant p-values for several predictors, indicating that the relationships identified are statistically robust. However, the overall model significance is more pronounced in Harris's model, particularly in light of the lower R-squared values for both models. Likely Outcome Considering the predictors and their implications for voter sentiment, Harris appears to have the upper hand in this analysis. The positive associations with poll scores and numeric grades in her model suggest a stronger alignment with voter preferences. Additionally, the higher R-squared value and overall model fit imply that her candidacy resonates more favorably with the electorate.

Moreover, the contrast in the effect of the transparency score could indicate differing voter priorities. Harris's potential appeal to voters valuing transparency may be a double-edged sword, yet it underscores a critical difference in how the two candidates are perceived regarding their public image.

Appendix

A Discussion of the survey’s methodology of Siena/NYT

Table 2: Sample Siena/NYT Data

Pollster	Methodology	Population	Sample Size	Grade Rating	Pollscore
Siena/NYT	Live Phone	rv	857	3	-1.5
Siena/NYT	Live Phone	rv	857	3	-1.5
Siena/NYT	Live Phone	rv	857	3	-1.5
Siena/NYT	Live Phone	rv	857	3	-1.5
Siena/NYT	Live Phone	rv	857	3	-1.5

A.1 Background and Intro

The New York Times/Siena College Poll is a collaborative effort between The New York Times and the Siena College Research Institute, renowned for its methodological rigor and accuracy. The Table 2 above shows sample data of Siena/NYT from clean data, which contains some useful information. According to FiveThirtyEight’s pollster ratings, this partnership holds a 3.0-star rating as shown Table 2, indicating high reliability. Based on the section Methodological standards from FAQs of FiveThirtyEight(Radcliffe and Morris 2023), “Pollsters must also be able to answer basic questions about their methodology, including but not limited to the polling mode used (e.g., telephone calls, text messages, online panels), the source of their sample, their weighting criteria and the source of the poll’s funding. In most cases, a detailed written methodology statement is sufficient to satisfy this criterion, but we may contact pollsters directly to clarify methodological details and follow up occasionally to ensure the pollster is still meeting our standards.” Their data will ensure that pollsters maintain methodological standards. More specific details about NYT/SEINA’s methodology are discussed below.

A.2 Population, Frame, and Sample

- Population: The target population comprises likely voters in the United States, focusing on those eligible and expected to participate in the upcoming election.
- Frame: The sampling frame is constructed from voter registration lists, which are comprehensive databases of registered voters across various states.
- Sample: A stratified random sample is drawn from the voter registration lists, ensuring representation across key demographics such as age, gender, race, education, and geographic location. A topline sample size of at least 100 (Radcliffe and Morris 2023).

A.3 Sample Recruitment Method

Participants are recruited through a combination of landline and mobile phone calls (like the Methodology in Table 2). This dual approach addresses the decline in landline usage and aims to reach a broader demographic spectrum. The use of voter registration lists enhances the likelihood of contacting registered voters, thereby improving the sample's representativeness.

A.4 Sampling Approach's trade-offs

A.4.1 Advantages

Enhances representativeness by ensuring all key subgroups are included. Improves precision in estimates for specific subpopulations.

A.4.2 Disadvantages

More complex and time-consuming compared to simple random sampling. Requires accurate and up-to-date information on the stratification variables. Handling Non-Response:

A.5 How is non-response handled

Non-response is addressed through weighting adjustments. Post-stratification weights are applied to align the sample with known population parameters, such as age, gender, race, and education. This process mitigates biases arising from differential response rates among various demographic groups.

A.6 What is good and bad about the questionnaire

A.6.1 Advantages

- Questions are crafted to be clear and unbiased, minimizing misinterpretation. The questionnaire covers a broad range of topics, providing comprehensive insights into voter opinions.

A.6.2 Disadvantages

- The length of the questionnaire may lead to respondent fatigue, potentially affecting the quality of responses
- The mode of administration (telephone interviews) might introduce social desirability bias, where respondents provide answers they perceive as socially acceptable rather than their true beliefs.

A.7 Summary

Overall, the New York Times/Siena College Poll employs robust methodologies to ensure accurate and reliable insights into voter preferences and behaviors.

B The idealized Methodology and Survey of forecasting the US presidential election

Idealized Methodology and Survey for Forecasting the US Presidential Election

1. Budget Allocation Overview

With a total budget of \$100K, the funds would be allocated as follows:

Survey Design and Platform Costs: \$5,000

Respondent Recruitment (Incentives and Advertising): \$60,000

Sampling Management and Data Validation: \$15,000

Analysis and Poll Aggregation Infrastructure: \$10,000

Contingency and Miscellaneous Costs: \$10,000

2. Sampling Approach

Target Population: The target population would be registered voters across the United States, ensuring representation from different demographics including age, gender, ethnicity, and political affiliation.

Sampling Method:

Stratified Random Sampling: To ensure proportional representation of various subgroups (e.g., regions, demographics), we will divide the population into strata and randomly select respondents from each stratum.

Sample Size: The goal is to survey 10,000 respondents to maintain a balance between precision and budget constraints.

Recruitment Strategy:

Online Panels: Use reputable online survey panels to reach diverse respondents.

Targeted Advertising: Run social media and digital ads to recruit additional respondents, particularly in underrepresented areas.

Incentives: Offer monetary incentives (e.g., \$5 per respondent) to increase participation.

3. Data Validation

To maintain data integrity, the following checks will be implemented:

Response Quality Control: Implement screening questions and attention checks to ensure respondents are engaged.

IP Address Monitoring: Ensure unique responses by blocking multiple submissions from the same IP address.

Completion Time Analysis: Flag responses that fall outside reasonable time frames (e.g., completed too quickly).

Ethical Considerations: All data will be collected anonymously, and respondents will be informed of their rights, including the right to withdraw from the survey at any time. Personal data will not be shared or used beyond the scope of this survey.

4. Poll Aggregation and Methodology

Weighting: Apply post-stratification weighting based on census data to ensure the sample aligns with the US voter demographics.

Error Margin and Confidence Intervals: Report results with a calculated margin of error (typically around $\pm 3\%$) and a 95% confidence interval.

Aggregation of Poll Results: Collect and combine data from multiple waves of polling to smooth out any anomalies and better predict trends.

Adjustments for Non-Response Bias: Implement statistical adjustments to account for known biases in survey responses.

5. Implementation Platform

The survey will be hosted on Google Forms to streamline data collection and integration. A direct link to the live survey is included below:

Access the Survey Here: [US Presidential Election 2024: Voter Opinion and Key Issues Survey](#)

6. Survey Copy

Below is a more comprehensive sample of the survey questions:

Survey Title: US Presidential Election 2024: Voter Opinion and Key Issues Survey

Introduction: Thank you for participating in our comprehensive survey on the upcoming US presidential election. Your responses are vital for understanding public sentiment and will remain confidential. Participation is voluntary, and you may skip any question or withdraw at any time.

Q1. Are you a registered voter?

Yes

No

Q2. Which state do you reside in?

Q3. What is your age group?

18-24

25-34

35-44

45-54

55-64

65+

Q4. Which racial or ethnic group(s) do you identify with? (Select all that apply)

White/Caucasian

Black/African American

Hispanic/Latino

Asian/Pacific Islander

Native American

Other (please specify)

Prefer not to say

Q5. What is your highest level of education?

Less than high school

High school diploma or GED

Some college, no degree

Associate degree (e.g., AA, AS)

Bachelor's degree (e.g., BA, BS)
Master's degree (e.g., MA, MS, MEd)
Professional degree (e.g., JD, MD)
Doctoral degree (e.g., PhD, EdD)
Prefer not to say

Q6. What is your gender?

Male
Female
Non-binary/Third gender
Prefer not to say

Q7. Which candidate do you currently intend to vote for in the upcoming presidential election?

Kamala Harris
Donald Trump
Other
Undecided

Q8. What is your political affiliation?

Democrat
Republican
Independent
Other
Prefer not to say

Q9. How confident are you in your choice for the presidential election?

Very confident
Somewhat confident
Not very confident

Not confident at all

Q10. What is the most important issue influencing your vote? (Choose the top 3)

Healthcare

Economy

Immigration

Climate Change

Education

National Security

Social Justice

Other (please specify)

Q11. How often do you follow news and updates about the election?

Daily

Weekly

Occasionally

Rarely

Never

End of Survey: Thank you for your time and valuable input. Your responses will contribute significantly to our understanding of public opinion regarding the upcoming presidential election.

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