Optimal Crowd Evacuation

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Abstract

This paper deals with the optimal allocation of routes, destination, and departure times to members of a crowd, for instance in case of an evacuation or another hazardous situation in which the people need to leave the area as quickly as possible. The generic approach minimizes the evacuation times, considering the demand dependent waiting times at bottlenecks within the considered infrastructure. We present the mathematical optimization problem for both the optimal instructions, and the continuum model describing the pedestrian flow dynamics.

The key contribution of the approach is that it solves the evacuation problem considering the entire solution space in a continuous manner (i.e. both the time dimension and the routing), implying that for each location and for each time instant the optimal path towards the most favorable exit is calculated, taking into consideration the traffic flow operations along the routes. The approach is generic in the sense that different network loading models can be used, and that a variety of components can be added to the optimization objective without loss of generality.

Next to presenting the framework and the mathematical model, we propose an iterative numerical solver to compute the optimal instructions. We demonstrate the abilities and opportunities of this optimization framework with two case studies.

1 Introduction

As evacuation procedures are becoming an established part of large-scale event risk assessment plans, event organizers are increasingly challenged to proof that their plans are in order and visitors can efficiently leave the premises in case of an emergency. Over the years numerous evacuation models have been proposed to assist in this endeavor. A good overview of these models is given by e.g. [1]. All proposed evacuation prediction models models are capable of computing an effective evacuation. However, these models cannot predict the most effective one achievable.

In contrast to the accumulation of literature treating pedestrian evacuation behavior and pedestrian evacuation models, there have only been limited attempts to optimize pedestrian evacuation behavior. According to [2], evacuation efficiency of an enclosure is determined by a combination of configuration (i.e. building lay-out), environment (i.e. hazard development), behavior (e.g. pedestrian decision making processes) and procedure (e.g. evacuation instructions) aspects of the process. Until now, the behavioral and environmental aspects of the evacuation process have in all optimization attempts been considered as stipulations. Some researchers did try to find more optimal or actually optimize of the configuration of the infrastructure. [3, 4, 5] iteratively searched for the optimal setting of the characteristics of the infrastructure. To the authors knowledge only [6] describes a computational effort to maximize outflow through the optimization of an architectural intervention.

Next to the optimizations of evacuation behavior, researchers tried to optimize evacuation instructions and as such the pedestrians' movement decisions. Several researchers used a game-theoretic approach to assign static (a.o. [7]) exit choices. Most researchers, however, used models that optimize dynamic network flows to divide pedestrians over the network. The approach to the optimization of the flows differs greatly between the various attempts. Where [8] optimizes the flow through the exits through the management of pedestrian velocities using a feedback control algorithm, [9] uses a dynamic programming approach to allocate exit choices based on the combination of distance, link distributions and exit width. In [10] a Multi-objective Programming (MOP) approach is mentioned to allocate pedestrians over entire routes. All of these attempts optimize pedestrian movements through models that do not account for pedestrian interactions. Furthermore, due to the use of network flow diagrams, the continuous nature of pedestrian movement is often disregarded. Both issues imply that the current solutions, even though often presented in a microscopic pedestrian model, are either not realistic or sub-optimal.

In contrast to the previously mentioned attempts, [11] does attempt simulate pedestrian behavior correctly. The optimization entails an iterative process in which the solution of a quickest transshipment problem (routing fractions on the nodes of the network) is recomputed iteratively based on the actual realization of the last solution in a microscopic model. Although the approach works well for small scenarios, it has difficulties balancing the solutions of both models for larger scenarios, resulting in a lack of convergence of the solution results. Although not mentioned by [11], it is hypothesized that the computational effort of their approach also increases severely for larger scenarios due to the use of a microscopic pedestrian model.

To the authors' knowledge none of the presented optimization frameworks provides an efficient way to optimize optimal evacuation instructions, for both small and large scenarios. We aim to introduce a mathematical optimization framework that computes optimal instructions efficiently. Furthermore, we propose an iterative numerical solver to compute the optimal instructions. Departure times, destinations and routes of individual members of a crowd are considered in the evacuation instructions. The approach minimizes the evacuation times, considering the demand dependent waiting times at bottlenecks within the considered infrastructure. This optimization framework, which is continuous in time and space, is generic with respect to the macroscopic pedestrian flow model that is also presented in this paper.

In the remainder of the paper first the mathematical preliminaries are elaborated upon, which consists of the definition of paths, costs, and aggregated pedestrian flow characteristics. Subsequently, the modeling framework is described. Accordingly, the pedestrian flow model is introduced in section 4. The optimal routing model and its implementation are discussed in sections 5 and 6. Two case studies showing the (numerical) characteristics of the approach provide feedback on the actual capabilities and opportunities of the framework. The paper ends with conclusions and future research directions.

2 Mathematical preliminaries

In this section we introduce the two-dimensional flow characteristics, including the necessary definitions. Then, we introduce the principle of admissible paths, which is the basis for the path cost function. Finally, we shortly describe the pedestrian flow characteristics, which are the basics of the network loading model.

2.1 Notation and definitions

Let us first define the infrastructure from which the people need to evacuate. The area is defined by $\Omega \subset \mathbb{R}$. Within this area, there are M obstacles that cannot be penetrated. These are defined by areas $B_m \subset \Omega$ for $m = 1,...,M$. Finally, we define J exit areas $D_j \subset \Omega$, which represent the safe havens in area Ω.

For the sake of simplicity, we assume a one-level area. Multi-level areas (e.g. buildings) will turn out to be relatively straightforward generalizations of the evacuation problem and will not be considered explicitly in this paper.

2.2 Admissible paths

As we will show in the remainder of this paper, we will look for the optimal path $\vec{x}^*_{[t,T)}$ starting at time instant t from any location $\vec{x}(t) \in \Omega$. These paths are defined as follows:

$$\vec{x}_{[t,T)} = \{\vec{\xi}(s) \in \Omega | t \leq s \leq T, \vec{\xi}(s) \notin B_m\} \tag{1}$$

Here, t denotes the departure time, and T denotes the terminal time when the evacuee either reaches one of the safe havens or when the evacuation is otherwise ended (i.e. total simulation time has elapsed). A requirement of the paths is that they need to be continuous and differentiable. That is, the derivative of the path x to t exists everywhere and is equal to $\vec{v}(t)$.

Note that the paths are not necessarily loop-less. Consider for instance a situation where an evacuee has to wait somewhere for a certain amount of time. It may be optimal for the evacuee to move to a location where he / she is out of danger for the duration of the wait.

Since we assume that the paths are differentiable, we could analogously describe the path in terms of the *velocity path*:

$$\vec{v}_{[t,T)} = \{\vec{v}(s) \in \Gamma(s, \vec{\xi}(s)) | t \leq s \leq T\} \tag{2}$$

where

$$\vec{\xi}(s) = \vec{\xi}(t) + \int_t^s \vec{v}(\tau) d\tau \tag{3}$$

and where $\Gamma(s, \vec{\xi})$ denotes the set of velocities that yield admissible paths at time s and location $\vec{\xi}$.

2.3 Path or velocity path costs

For each path $\vec{x}_{[t,T)}$, or equivalently, the velocity path $\vec{v}_{[t,T)}$, we define the path costs $J(t,\vec{x}(t)|\vec{v}_{[t,T)})$ as follows:

$$J(t,\vec{x}(t)|\vec{v}_{[t,T)}) = \int_t^T L(s,\vec{x}(s),\vec{v}(s))ds + \phi(T,\vec{x}(T)) \tag{4}$$

subject to

$$\frac{d}{dt}\vec{x} = \vec{v} \tag{5}$$

In this expression, the function $L(t,\vec{x},\vec{v})$ denotes the so-called running cost, which is the cost added to the total cost during the infinitesimal period $[s,s+dt)$. The running cost is a function of the time s, the location $\vec{x}(s)$ and the velocity $\vec{v}(s)$. This cost can reflect different factors, such as the travel time or cost incurred because the evacuee is too close to a hazard (fire, smoke). When we only consider travel times, we can choose $L = 1$.

The function $\phi(T,\vec{x}(T))$ denotes the so-called terminal cost. This expresses the cost incurred being at a certain location $\vec{x}(T)$ at time instant T. T denotes the terminal time, which is either the end of the evacuation period (denoted by $T = t_1$) or the time an evacuee arrives at one of the safe havens.

In the former case, we would set the cost of not being at one of the safe havens at the end of the evacuation to a high value, e.g.:

$$\phi(T,\vec{x}(T)) = \begin{cases} d_j & \vec{x}(T) \in D_j, T < t_1 \\ \infty & \vec{x}(T) \notin \bigcup_j D_j, T = t_1 \end{cases} \tag{6}$$

where d_j denotes the cost of arriving at one of the destinations j. Note that this allows us to differentiate between preferences for a specific safe haven.

2.4 Pedestrian flow characteristics

In the remainder of the paper, we present a network loading model that is continuous in time and space in detail. This model describes the dynamics of the densities $\rho(t,\vec{x})$ representing the mean number of pedestrians (per unit area) at time instant t and location x, in relation to the flows and the velocities. In this respect, note that the density $\rho(t,\vec{x})$ is a scalar, while the velocity $\vec{v}(t,\vec{x})$ is a two-dimensional vector. The speed is defined as the absolute value of the velocity, and hence is a scalar. The flow is also a two-dimensional vector, which is defined by $q(t,\vec{x}) = \rho(t,\vec{x})v(t,\vec{x})$.

When determining the speed, we assume dependence on the densities according to the well know relation:

$$v^e(t,\vec{x}) = V(\rho(t,\vec{x})) \tag{7}$$

which in fact denotes the fundamental relation for pedestrian flows. The direction is determined by the prevailing route choice.

3 Modeling framework

The aim of the framework is to evacuate as many people as possible within the allotted time period $[t_0, t_1)$, taking into consideration the limited supply available due to the infrastructure and possible queues due to these supply limitations, and the physical capabilities of the evacuees.

To this end, we determine the *optimal cost function* $W(t,\vec{x})$ with $t_0 \leq t \leq t_1$ and $\vec{x} \in \Omega$ that describes the minimal costs (e.g. time) to get to one of the safe havens D_j for $j = 1, ..., J$. That is:

$$W(t,\vec{x}) = \min_{\vec{v}_{[t,T)}} J(t,\vec{x}|\vec{v}_{[t,T)}) \tag{8}$$

subject to $\frac{d}{dt}\vec{x} = \vec{v}$ and $\vec{v}(s) \in \Gamma(s,\vec{x}(s))$. Note that $\Gamma(s,\vec{x}(s))$ can be used to describe the influence of obstacles present, as well as physical limitations of the evacuee (including limitations in the speed due to other evacuees present). That is: the evacuee cannot walk into an obstacle and the speed of an evacuee will be a function of the density $\rho(s,\vec{x}(s))$ at the location.

Note that this function describes the minimum individual evacuation cost, which does not necessarily imply that the overall costs are minimized. Later, we will make plausible under which conditions / assumptions both are actually the same.

If the minimum costs are known, determining the optimal path can be achieved easily by determining the directions in which the costs decrease the quickest [13], i.e.:

$$\vec{v}^*(t,\vec{x}) = \nabla W(t,\vec{x}) \tag{9}$$

Note that by necessity, we have $|\vec{v}^*(t,\vec{x})| \leq \min(w, \vec{v}^e(t,\vec{x}))$.

The pedestrian flows are expressed as continuum flows through the infrastructure, and are modeled by means of simple conservation equations, yielding the densities $\rho(t,\vec{x})$ (and speeds and flows) as functions of time and space. This is explained in detail in the next section.

The objective of the evacuation problem is to find the optimal cost function $\tilde{W}(t,\vec{x})$ (and thus the optimal paths) that minimizes the evacuation costs of the evacuees, given the evacuation flows (and thus densities $\tilde{\rho}(t,\vec{x})$). That means that we would need to solve a dynamic assignment problem in continuous time and space until we achieve consistency between the optimal evacuation routes (and staging, i.e. departure times) and the resulting flows.

In the remainder of this paper, we discuss the network loading model and the optimal evacuation path problem in more detail. We provide the mathematical background as well as a simple recipe to solve the problem numerically.

4 Network loading by first-order pedestrian flow modeling

In this section, we introduce the pedestrian flow model that is used to load the network. The model used is a relatively simple macroscopic first-order pedestrian flow model based on the conservation of pedestrians equation and the fundamental speed-density relation. We first show the mathematical model, after which we present the numerical solution approach.

4.1 First-order pedestrian flow theory

As with vehicular traffic flow, pedestrian flow can be described both at the microscopic level (individual pedestrians) and at the macroscopic level (flow representation). In this contribution, a macroscopic approach is taken where flow is described in terms of the dynamics of pedestrian densities $\rho(t,\vec{x})$ in time and space. The proposed model is similar to the well known kinematic wave model [14] for vehicular traffic, with the exception that next to the fundamental diagram, also the flow direction needs to be considered.

The dynamic network loading model used here is described by a simple conservation of pedestrians equation. That is, assuming that the velocity $v(t,x)$ is known, the flow propagation satisfies:

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho\cdot\vec{v})}{\partial x}=\frac{\partial\rho}{\partial t}+\frac{\partial\vec{q}}{\partial x}=r-s \tag{10}$$

subject to initial conditions:

$$\rho(t_0,\vec{x})=\rho_0(\vec{x}) \tag{11}$$

for all $\vec{x}\in\Omega$. Here $r=r(t,\vec{x})$ denotes a source term where evacuees flow into the area Ω; $s=s(t,\vec{x})$ denotes a sink term, where evacuees leave the area Ω (i.e. at the safe destinations D_j).

In the proposed framework, the (optimal) direction of the flow is determined at the path choice level. This yields a unitary vector $\vec{\gamma}^*(t,\vec{x})$ that points into the optimal walking direction. This vector is defined by:

$$\vec{\gamma}^*(t,\vec{x})=\frac{\nabla W(t,\vec{x})}{||\nabla W(t,\vec{x})||} \tag{12}$$

That is, the unitary vector points into the steepest descent direction of the minimum cost function $W(t,\vec{x})$.

For the (absolute) speed $||\vec{v}||$ we assume that the flow behaves according to the fundamental relation between density and speed, i.e. $||\vec{v}(t,\vec{x})||=V(\rho(t,\vec{x}))$. Alternatively, we use the flow-density relation, which then results in:

$$\vec{q}(t,\vec{x})=\vec{\gamma}^*(t,\vec{x})\cdot Q(\rho(t,\vec{x})) \tag{13}$$

Note that for the sake of simplicity, we assume that the absolute flow is only a function of the density. We will use a simple linear speed-density function, i.e.:

$$V(\rho)=v^0\cdot(1-\rho/\rho_{jam})=1.34\cdot(1-\rho/5.4) \tag{14}$$

Note that with the presence of obstacles $B_m\in\Omega$, we need to respect that no evacuees can flow into the obstacles. This means that the velocities satisfy:

$$\vec{v}(t,\vec{x})\cdot n_m(t,\vec{x})\geq 0 \tag{15}$$

where $n_m(t,\vec{x})$ denotes the outward pointing normal vector of the boundary of obstacle B_m. In other words, the inner product between the speed and the normal vector cannot be negative, since this would mean evacuees flowing into the obstacle.

As mentioned earlier, the resulting model has properties similar to the kinematic wave model for vehicular traffic. First and foremost, strong solutions do not necessarily exist while multiple weak solutions may be possible. The correct weak solution is the solution that maximizes the flow (entropy condition). Moreover, we will see rarefaction waves and the formation of shocks, similar to the classic kinematic wave model.

The mathematical properties of the flow model proposed here will however not be investigated further. However, many of the approaches used for the kinematic wave model can also be applied to 2D (e.g. method of characteristics). This has important consequences for potential (numerical) solution approaches (e.g. Godunov and variational theory).

4.2 Numerical solution using Lax-Friedrich

Now we have specified the mathematical model, we look briefly into numerical solution approaches. To keep matters simple, we have opted for a simple approach: the Lax-Friedrich scheme [16].

Let us consider an equidistant mesh where the cells are Δx by Δy. Let i and j denote the cell index in the x and y direction. Let Δt denote the time step size. The Lax-Friedrich scheme is a so-called space centered finite difference scheme that describes the changes in the density of cell (i,j) as a function of its neighboring cells. Let $f = \gamma_x^* \cdot Q(\rho)$ and $g = \gamma_y^* \cdot Q(\rho)$ denote the fluxes in the x and the y direction, and let $\rho_{i,j}^n$ denote the density in cell (i,j) at time instant $t_n = n\Delta t$. The Lax-Friedrich scheme is now given by:

$$\rho_{i,j}^{n+1} = \frac{1}{4}\left(\rho_{i+1,j}^n + \rho_{i,j+1}^n + \rho_{i-1,j}^n + \rho_{i,j-1}^n\right) - \frac{\tau_x}{2}\left(f_{i+1,j}^n - f_{i-1,j}^n\right) - \frac{\tau_y}{2}\left(g_{i,j+1}^n - g_{i,j-1}^n\right) \quad (16)$$

where $\tau_x = \Delta t/\Delta x$ and $\tau_y = \Delta t/\Delta y$.

Now, with the correct initial and boundary conditions, the model can be solved numerically. It is beyond the scope of the paper to discuss the properties of the scheme, but application examples will be given in the section on case studies.

5 Optimal dynamic routing in continuous time and space with exogenous speeds

In this section, we introduce the optimal routing model where the (maximum) speeds are assumed to be known. In the next section, we will show how to use the speeds determined by the network loading model.

5.1 Path choice modeling by dynamic programming

For the path choice modeling in continuous time and space, we will use the approach first described in [13]. The key to this approach is that instead of explicitly determining the optimal paths, we will determine the optimal direction (and speed) at each location and at each time instant. More specifically, the dynamic programming approach will determine the optimal velocity (direction times speed) towards the (nearest) destination D_j.

As we have seen in the above, the optimal velocity $\vec{v}^*(t,\vec{x})$ of an evacuee moving in a two-dimensional area Ω is a function of the minimum actual cost $W(t,\vec{x})$ towards a safe haven. Note that these optimal velocities describe not only the path choice, but also the destination choice as well as the evacuation staging (departure times).

The minimum actual cost $W(t,\vec{x})$ can be determined by solving the so-called *Hamilton-Jacobi-Bellman* (HJB) equation (see [12] and [13]):

$$-\frac{\partial}{\partial t}W(t,\vec{x}) = H(t,x,\nabla W) \tag{17}$$

with terminal conditions reflecting the penalty of not arriving at the safe haven before the end time t_1:

$$W(t_1,\vec{x}) = \infty \tag{18}$$

and boundary conditions describing the cost or preference of arriving at a specific destination D_j:

$$W(t,\vec{x}) = d_j \tag{19}$$

for $\vec{x} \in D_j$ and $t \leq t_1$.

The so-called Hamilton function H is defined by:

$$H(t,\vec{x},\nabla W) = \min_{v\in\Gamma(t,\vec{x})} \left[L(t,\vec{x},\vec{v}) + \vec{v}\cdot\nabla W\right] \tag{20}$$

Here, $\Gamma(t,\vec{x})$ denotes the set of admissible velocities. This includes the admissible directions as well as the possible walking speeds, which are influenced by both infrastructure and flow conditions (i.e. density); L denotes the so-called running costs, which describes the cost incurred over a short time interval $[t,t+dt)$, given the time t, the location $\vec{x}$ and the velocity $\vec{v}$.

In [13], we discuss existence and uniqueness of solutions to the HJB equation. It turns out that to determine the value function, we need to look for a special kind of (weak) solution to the HJB equation, namely the viscosity solution. The latter can be uniquely determined.

It is important to note that in this contribution, we consider a dynamic problem, in the sense that the densities, speeds, and thus the optimal route choice will change over time (and space) during the simulation period. As such, static approximations of the optimal route choice problem (see [13]) are not applicable.

5.2 Problem specification for the evacuation problem

Let us briefly look at the problem specification for the evacuation problem. First of all, we define the cost. For now, we will assume that evacuees will aim to minimize their evacuation times. This implies $L = 1$ (since each period $[t,t+dt)$ adds 1 times the time period length to the overall cost J).

In addition, we could penalize walking too close to any of the obstacles B_m. To describe this, let $||B_m - \vec{x}||$ denote the (minimum) distance between $\vec{x}$ and the obstacle B_m, then an appropriate running cost component would be $\beta\cdot\exp(-||B_m-\vec{x}||/S)$ where β and S are parameters to be specified.

In the remainder, we thus use the following running cost specification:

$$L = 1 + \beta\cdot\exp(-||B_m-\vec{x}||/S) \tag{21}$$

This leaves us with the specification of the admissible velocities. We assume that these velocities are determined by two factors. First of all, the obstacles restrict the walking directions[1]. Eq. (15) describes how the admissible set $\Gamma(t,\vec{x})$ is shaped by the obstacles. Second of all, the traffic conditions determine the possible speeds. Given the local density $\rho(t,\vec{x})$, the choice of velocities is limited by the fundamental diagram as follows:

$$||\vec{v}(t,\vec{x})|| \leq V(\rho(t,\vec{x})) \tag{22}$$

5.3 Numerical solution scheme

For the sake of simplicity, we will use a finite difference approach to numerically solve the HJB equation. We will use the same mesh as used in the pedestrian flow model described in the previous section. In [13], the details of the approach are given.

6 Optimal evacuation by dynamic assignment in time and space

The approach proposed in this section is based on the approach first proposed in [15]. However, a critical difference is that the pedestrian flow model and - as a consequence - the path choice model directly captures the reduction in the walking speeds (and the incurred waiting times, etc.) due to (high) densities. On the contrary, the model proposed in [15] tries to find consistency between flow propagation and path choice by recomputing the speeds at the end of the flow computation based on the predicted densities.

Let us briefly describe the different steps in the approach proposed in this paper.

1. *Initialization.* We first set the iteration index at zero, i.e. $k = 0$. We determine the initial density profile $\rho(t_0,\vec{x}) = \rho_0(\vec{x})$ for all $\vec{x} \in \Omega$. Having not yet computed the optimal paths, we set $\rho^{(0)}(t,\vec{x}) = \rho_0(\vec{x})$ for all $\vec{x} \in \Omega$ and $t \in [t_0,t_1]$. Based on these densities, and the geometry of the obstacles, we determine the admissible velocities $\Gamma^{(1)}(t,\vec{x})$.

2. *Optimal path calculation*. Given the set of admissible velocities, we solve Eq. (17) - (19). Based on the optimal cost $W^{(k)}(t,\vec{x})$ that is determined, we compute the optimal velocities (direction and speeds) for iteration k. The optimal directions $\gamma_*^k(t,\vec{x})$ are used in the next step.

3. *Flow propagation*. The optimal direction $\gamma_*^k(t,\vec{x})$ is fixed while the densities $\rho^{(k)}(t,\vec{x})$ are determined by solving Eq. (10) - (11) using the numerical scheme presented in this paper.

4. *Update average densities*. The densities computed in the previous step are used to re-determine the optimal paths (step 2). However, using the densities directly will cause oscillating behavior hampering convergence of the scheme. This is why we introduce an exponentially smoothed density $\bar{\rho}^{(k)}(t,\vec{x}) = (1-\alpha)\bar{\rho}^{(k-1)}(t,\vec{x}) + \alpha\rho^{(k)}(t,\vec{x})$ for some value of $0 \leq \alpha \leq 1$. The smoothed densities are used to determine the set of admissible velocities $\Gamma^{(k)}(t,\vec{x})$ that are used in step 2.

[1]Note that the high running costs of being close to obstacles will automatically steer the evacuees away from the obstacles which may make adding restrictions on the admissible velocities unnecessary

5. *Check for convergence and continue.* In this final step, we check if the scheme has converged. To this end, we will check if the optimal cost functions between two subsequent iterations have not changed significantly, i.e. $\max_{t,\vec{x}} |W^{(k)}(t,\vec{x}) - W^{(k-1)}(t,\vec{x})| \leq \varepsilon$. If this is not yet the case, we go to step 2 for the next iteration step.

7 Case studies

In this section we show the (numerical) characteristics of the approach and the solutions that result from its application by means of two case studies. The first case is a simple maze, showing the optimal route choice through the maze. In the second case the area has two exits, where the size (and thus capacity) of one of the exits is increased. This case shows the optimal destination choice of the evacuees. We emphasize that we do not intend to put forward realistic case studies, but aim to show the characteristics and face-validity of the approach.

7.1 Case 1: optimal route choice through a simple maze

The first case study that we will consider aims to illustrate the workings of the evacuation route computations for a simple maze. Fig. 1 illustrates this case. The evacuees are initially on the left-hand-side (black area) with a density of $2.5P/m^2$. The safe area is located on the right of the 50 m by 50 m area. The figure shows the walls and the doors the evacuees need to use in order to reach the safe area. There are six doors in total. The doors have different widths (door 1 and 2 are wide (2.5 m); door 3 to 6 are narrow doors (1.25 m)). Clearly, there are multiple routes available.

The figure shows the results of the initial evacuation route computation (i.e. iteration 1, assuming zero density, that is free speeds anywhere, anytime) for $t = 0$. For this situation, the area was discretized using cells of 0.25 m by 0.25 m, while using a time step op 0.1s (both for the route choice computation and flow propagation). We simulated a period of 250 s. The colors indicate the minimum cost (or in this case, time) to reach the safe area on the right. The optimal route that the evacuees take from any location is determined by the steepest descent path (i.e. perpendicular to the iso-cost curves). For the free flow conditions situation, this will for instance mean that when reaching door 2, evacuees are best off choosing door 4. Note that since zero density is assumed for the entire simulation period, the optimal routes are equal for all time instances.

7.1.1 Example results for maze case study

When applying the flow model using these free flow optimal routes, queueing occurs at different locations in the network. Since the evacuees will not react to the reduced speeds caused by the high densities, these queues will persist during a large part of the simulation (actually, some evacuees will still be left at the end of the simulation). Fig. 2 shows the results for two time steps (after 25 s and after 125 s). Maximum (jam) densities ($5.4P/m^2$) are observed at various locations, yielding low speeds and hence a slow evacuation. Furthermore, we clearly see that in this case, the capacities of the - especially wide - doors are not fully used (evacuees have the

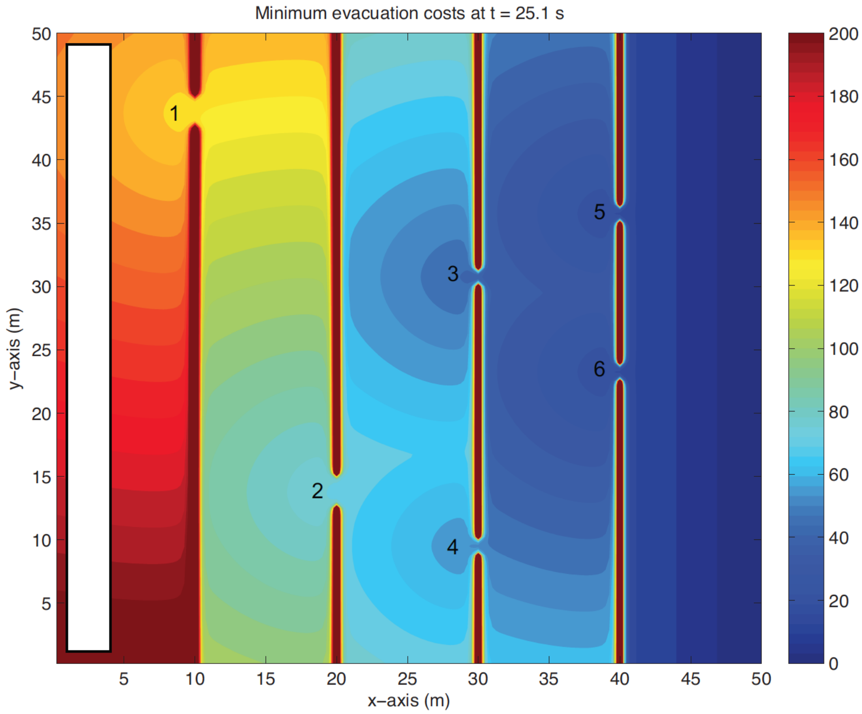

Figure 1: Minimum cost of getting to the exit on the right in case of unhindered walking.

tendency to curl around the doors). Also, available capacity from other doors is not used either: doors 3 and 5 are not used, while queuing occurs at door 4 and (to a lesser extent) door 6. In this end, this results in an incomplete evacuation of the 300 (or so) evacuees, about 140 of which are not able to get our of the area within the 250 s simulation period. It is important to note that this is obviously an intermediate result (result from the initial iteration) that does not reflect any realistic (or optimal) situation. It does illustrate, however, that taking into consideration the congestion in finding the optimal evacuation routes is very important.

Let us see how applying the procedure proposed in this paper would yield a better evacuation strategy. To this end, we have applied the proposed iterative optimization scheme (with $\alpha = 0.05$) using a maximum number of iterations of 200. Fig. 3 shows the results of iteration 30 at two different time stamps (50s and 125s). The two graphs clearly show the dynamic nature of the evacuation scheme. First of all, for $t = 25s$, we see that the wide doors are utilized more effectively (flow spans the entire width of the doors). Second of all, we see that after some time, doors 3 and 5 are used as well (see snapshot at $t = 125s$. In fact, it turns out that in the end, the available capacity of all doors is used fully.

These favorable properties of the evacuation scheme allows clearing the area well within the 250s period. In fact, for this iteration the total evacuation time was 235 seconds, showing that the approach leads to large improvements compared to the free flow scheme.

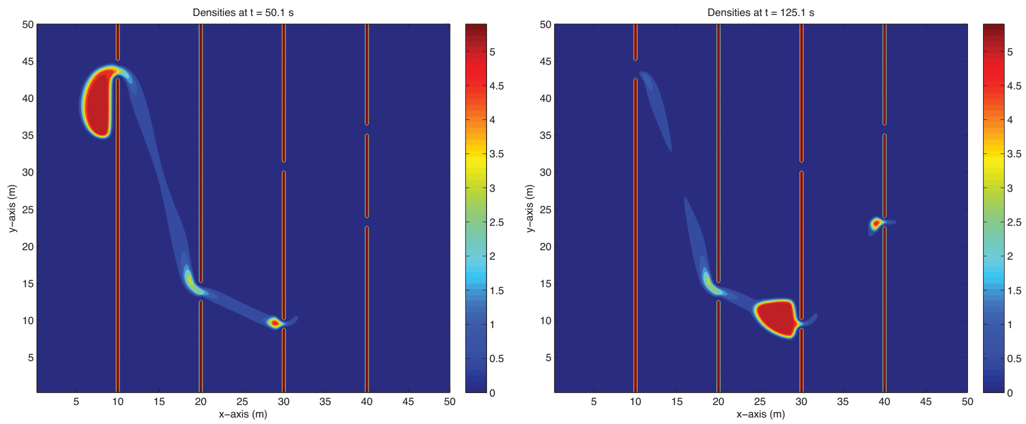

Figure 2: Evacuee assignment using optimal free flow routing.

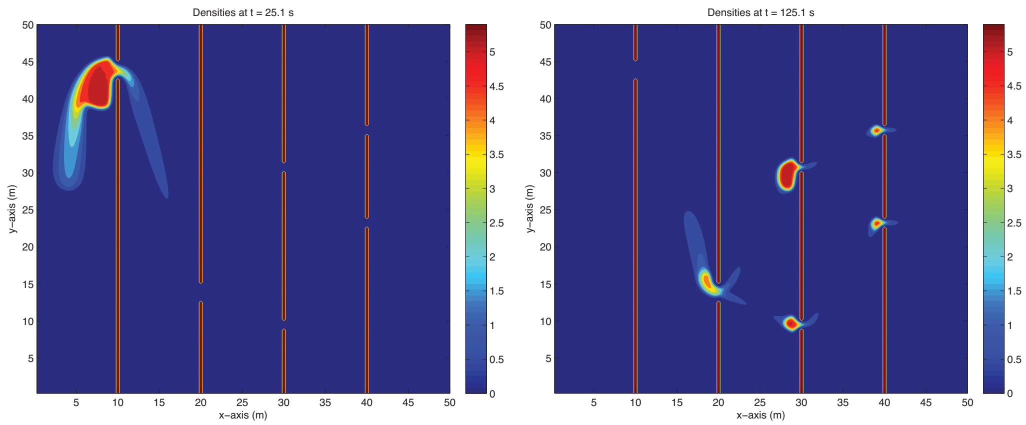

Figure 3: Evacuee assignment (iteration 30) for two time stamps.

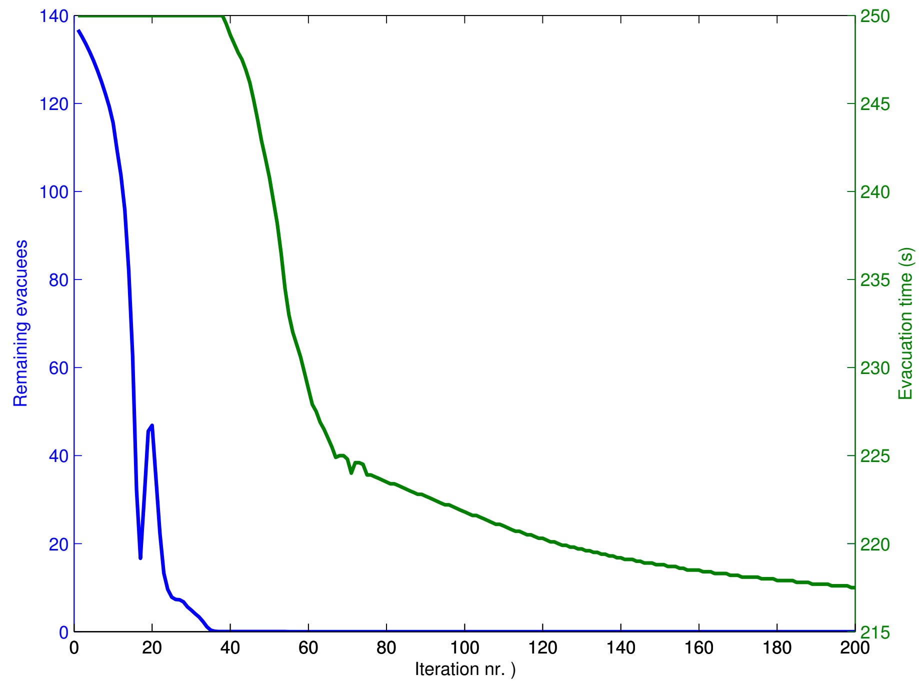

Figure 4: Minimum cost of getting to the exit on the right in case of unhindered walking.

7.1.2 Convergence properties

Although it is beyond the scope of this paper to go into the details of the approach, let us briefly discuss some of the properties of the approach. Fig. 4 shows the changes in the scheme's effectiveness over 200 iterations. The figure shows that initially, not all evacuees can leave the area in time. However, from iteration 35 onward, all evacuees are able to leave the area in time. In fact, the evacuation time reduces (almost) monotonically to about 217 s (for iteration 200). Note that the speed of convergence changes over the iterations. This is partially due to the fact that the scheme (seems) to converge to a stable minimum evacuation duration, but also because at two time instances, the update factor α is reduced (at iteration 20 and at iteration 63) by a factor of two to ensure convergence. This shows that the properties of the numerical schemes needs further investigation, and improvement, although the approach appears to be working given the results presented here.

7.2 Case 2: optimal exit choice

The second case study aims to illustrate the workings of the evacuation framework for a choice between two exits (safe areas). Fig. 5 illustrates this case. The evacuees are initially uniformly distributed over the full area with a density of $2P/m^2$. The pedestrians can leave the area through two exits: one on the left hand side of the 50 m by 50 m area, the second one opposite to the first

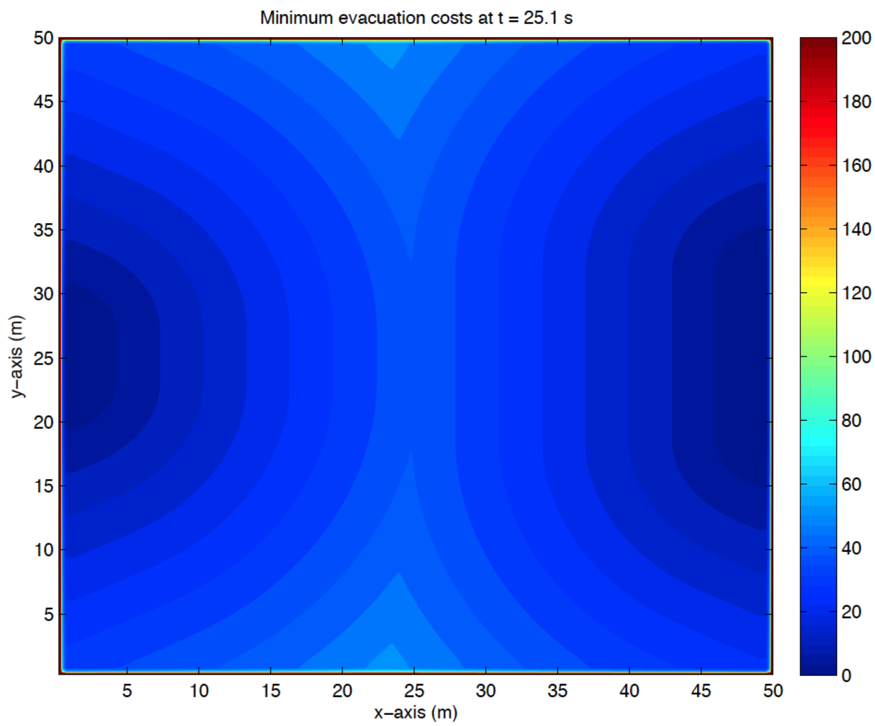

Figure 5: Minimum cost of getting to the exits.

one, on the right hand side. In order to show the dynamics of the choice behavior, we have made a series of scenarios, in which the width of the exit on the right hand side gradually increases. Fig. 5 shows the results of the initial evacuation route computation, assuming an empty area. The colors indicate the minimum time to get to a safe destination. Pedestrians choose the exit for which their costs are minimal. The optimal route can, again, be determined by the steepest descent path. This implies that pedestrians initially located on the left-hand side of the area will move to the left exit, while pedestrians initially located on the right-hand side of the area will move to the right exit. Similar to the previous case, when the flow model is applied using the free flow optimal routes, queuing occurs upstream of both exits. As pedestrians will not change their decision in reaction to the reduced speeds, the queues occur until the end of the simulation. After a number of iteration steps, where local densities (low travel times) have been included in the route and exit choice, the density pattern over the area has changed, see fig. 6. We can see that more pedestrians use the exit on the right (the area with high densities is larger). This can also be seen from some statistics from the simulation, shown in fig. 7. The total evacuation time reduces when the right exit width increases (thus its capacity increases). This reduction in evacuation time can be contributed to the fact that pedestrians redistribute over the exits, such that more pedestrians use the exit on the right hand side. When the exit on the right hand side is about three times the size of exit on the left hand side, about 40% of the evacuees uses the exit on the left and 60% of the evacuees uses the exit on the right.

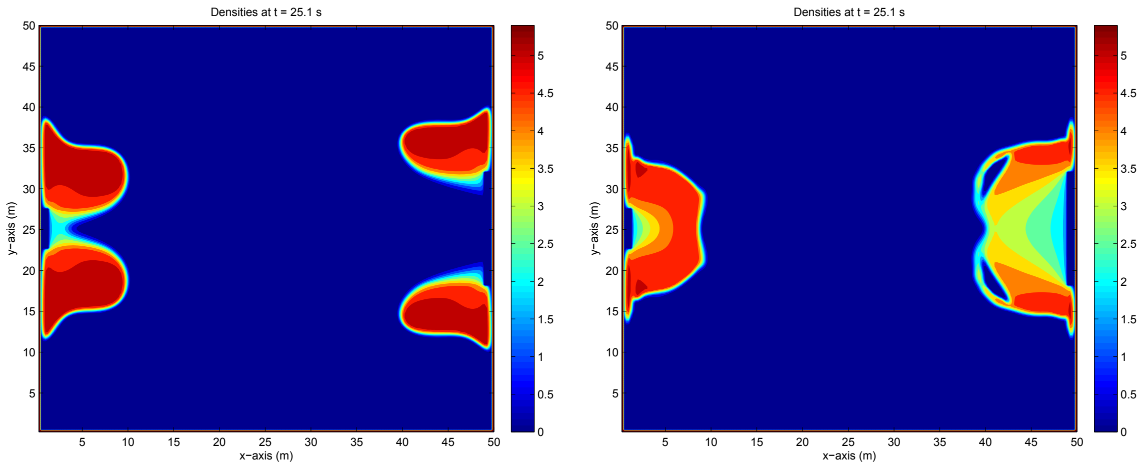

Figure 6: Evacuee assignment for the two exits. The figure on the left shows the densities for free routing, the figure on the right shows the optimized densities.

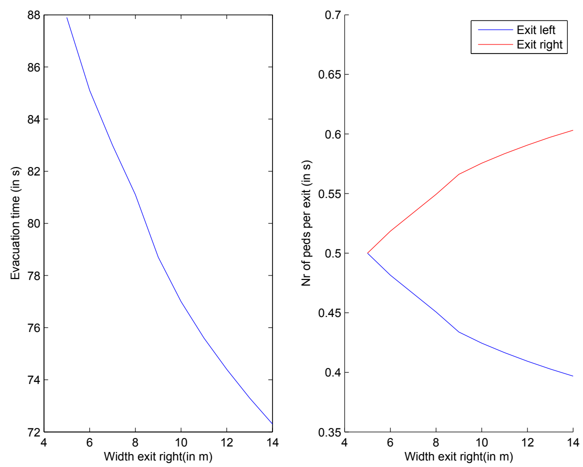

Figure 7: Evacuation time (on the left) and distribution over the two exits (on the right) for the different exit widths.

8 Conclusion and future research directions

This contribution describes a novel, generic approach to determine optimal evacuation instructions. The paper focuses on the theory, showing that optimization of the pedestrian decision behavior consisting of exit, departure, and route choice can indeed yield substantial improvements in evacuation times compared to for instance shortest distance paths. The workings of this optimization framework have been shown for two case studies: one in which the optimal route choice has been illustrated, while the second case study demonstrates the optimal destination choice. Unlike previous attempts to optimize evacuations, this approach covers optimal allocation of routes, destination, and departure times, including pedestrian behavior in the form of the well-known speed-density relation, and a first-order continuum pedestrian flow model, comparable to the LWR model for vehicular traffic. As the pedestrians dynamics are described by a macroscopic pedestrian flow model, the approach is applicable to larger infrastructures, or other large-scale applications (many pedestrians present).

However, both for the model to describe pedestrian dynamics and the optimization model a number of assumptions has been made in order to simplify the models. Now we have shown the proper workings and seen the promising results, future research can be pursued on different practical and theoretical issues.

From a theoretical perspective, the key issues that need to be considered deal with proving existence and uniqueness of solutions, but also proving optimality. More specifically, issues that will be considered in the ensuring of our research are:

- *Proving optimality for a system perspective.* The approach that we have proposed is in fact a dynamic user optimal assignment. Although the case studies show improvements achieved by application of the approach, system optimality has not been proven. In future work, we will investigate under which circumstances system optimality can be guaranteed.

- *Analyzing and improving the mathematical flow model (spontaneous phase transitions, turbulent flows, etc).* The model proposed in this paper is simple, not reflecting some of the key properties in pedestrian flows. Examples of these properties are the faster-is-slower effect, turbulence, etc. These characteristics are not yet included in the model, but will have severe impact on the flow propagation, and may have to be considered in order to allow computation of robust and efficient evacuation schemes. Furthermore, the mathematical properties of continuum pedestrian flow models have not been studies in detail, which is required for e.g. the improvement of numerical schemes, etc.

Regarding the numerical approach, we deem research is necessary on - among other things - the following issues:

- *Improving numerical approach for flow propagation model.* For the flow propagation model, we have used a very simple numerical discretization scheme. This Lax-Friedrich scheme is well know for its strong numerical diffusion, not respecting the strong hyperbolic nature of the first order model. Future research will focus on improving the numerical solution methods.

- *Analysis and improvements of DTA algorithm.* There are many improvements conceivable regarding the algorithms presented in this paper. Examples are incremental assignment rather than subsequent all-or-nothing assignments, alternative approaches to update the densities and optimal costs, etc. Furthermore, convergence, uniqueness and optimality of the numerical solutions need to be investigated.

Finally, to make the approach applicable to real cases, a number of issues need to be tackled:

- *Translating theoretical instructions to practical instructions.* The evacuation instructions computed via the approach presented here can be rather complex, since they will vary over time and space during the course of the evacuation. In the next phases of the research, we need to consider the practical translation of these instructions, either by simplifying them or by including restrictions during the optimization that reflect the applicability of the instructions in the instructions. Although the model can provide "practical" guidelines to help improve evacuation of crowd in real emergency situations, further demonstration of this issue is needed.
- *Consideration of evacuee compliance in modeling and optimization.* Another issue which is important in the modeling part of the presented work is the user-compliance and behavior. It is well known that compliance can be limited, which obviously will have severe impact on the validity of the predictions and the effectiveness of the instructions.
- *Calibration and validation.* The model results, especially with respect to evacuation instructions, need to be calibrated and validated. This can be done using either real-life data (when available), or experiments.
- *Explore alternative applications (e.g. planning).* The approach put forward has many other applications than pedestrian flow. Furthermore, the macroscopic nature of the approach allows for applications at different scales.

Despite the issues that need to be tackled, we believe that the approach put forward here has clear potential in terms of providing a strong theoretical basis for evacuation plan design. The cases presented in this paper clearly reveal this potential.

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References

[1] Castle, C.J.E. and P.A. Longley. Building Evacuation in Emergencies: A review and Interpretation of Software for Simulating Pedestrian Egress, in *Geospatial Technologies and Homeland Security*, D. Sui, Editor. Vol. 94, 2008, pp. 209-228, Springer Netherlands.

[2] Gwynne, S., E.R. Galea, M. Owen, P.J. Lawrence, and L. Filippidis. A Review of the Methodologies used in the Computer Simulation of Evacuation from the Built Environment. *Building and Environment*, Vol. 34, 1999, pp. 741-749.

[3] Shiwakoti, N., M. Sarvi, G. Rose, and M. Burd, Enhancing the Safety of Pedestrians during Emergency Egress - Can we learn from biological Entities? In *Transportation Research Record: Journal of the Transportation Research Board, No. 2137*, Transportation Reserach Board of the National Academies, Washington, D.C., 2009, pp. 31-37.

[4] Shiwakoti, N., and M. Sarvi. Enhancing the Panic Escape of Crowd through Architectural Design. *Transportation Research Part C: Emerging Technologies*, 2013, in press.

[5] Twarogowska, M., P. Goatin., R. Duvigneau, and N. Aissiouene. On the Hughes' Models of Crowd Dynamics. In *ESF Research Conference on Applied Partial Differential Equations in Physics, Biology and Social Sciences: Classical and Modern Perspectives*, 2012, Bellaterra (Spain).

[6] Johansson, A., and D. Helbing. Pedestrian Flow Optimization with a Genetic Algorithm based on Boolean grids. In *Pedestrian and Evacuation Dynamics 2005*, N. Waldau, P. Gattermann, H. Knoflacher, and M. Schreckenberg, Editors. 2005, Springer-Verlag Berlin Heidelberg, pp. 267-272.

[7] Ehtamo, H., S. Heliovaara, S. Hostikka, and T. Korhonen. Modeling Evacuees' Exit Selection with Best Response Dynamics. In *Pedestrian and Evacuation Dynamics 2008*, W.W.F. Klingsch, C. Rogsch, A. Schadschneider, and M. Schreckenberg, Editors. 2008, Springer-Verlag: Heidelberg, pp. 309-319.

[8] Shende, A., M. Singh, and P. Kachroo. Optimization-based Feedback Control for Pedestrian Evacuation from an Exit Corridor. *IEEE transactions on intelligent transportation systems*, Vol. 12, No. 4, 2011, pp. 1167-1176.

[9] Xu, X., M. Zhong, and C. Shi. Optimal Exit Choice Algorithm for Pedestrian Evacuation Dynamics. In *Pedestrian and Evacuation Dynamics*. 2012. Zurich.

[10] Zong, X., S. Xiong, Z. Fang, and Q. Li. Multi-objective Optimization for Massive Pedestrian Evacuation using Ant Colony Algorithm. In *Proceedings of the First international conference on Advances in Swarm Intelligence ICSI'10*, 2010.

[11] Kneidl, A.T., M. Thiemann, D. Hartmann, and A. Borrmann. Combining Pedestrian Simulation with a Network Flow Optimization to support Security Staff in handling an Evacuation of a Soccer Stadium. In *Proceedings of the European Conference Forum 2011*, 2011. Cork.

[12] Fleming, W.H., and H.M. Soner, *Controlled Markov Processes and Viscosity Solutions. Applications of Mathematics*, Vol. 25. Springer-Verlag. 1993.

[13] Hoogendoorn, S.P. and P.H.L. Bovy. Pedestrian Route-choice and Activity Scheduling Theory and Models. *Transportation Research Part B: Methodological*, Vol. 38, No. 2, 2004, pp. 169-190, .

[14] Lighthill, M.J., and G.B. Whitham. On Kinematic waves. II: A Theory of Traffic Flow on Long Crowded Roads. *Proceedings of the Royal Society*, Vol. 229A, No. 4, 1955, pp. 317345.

[15] Hoogendoorn, S.P. and P.H.L. Bovy. Dynamic user-optimal Assignment in Continuous Time and Space. *Transportation Research Part B: Methodological*. Vol. 38, No. 7, 2004, pp. 571-592.

[16] Hirsch, C., 1995. *Numerical Computation of Internal and External Flows*. 1995. John Wiley & Sons