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A game theory based exit selection model for evacuation

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Abstract

How do evacuees find their way to escape from a fire zone? This will be a significant question that should be considered for modeling the evacuation process. Generally, a building consists of enclosure areas such rooms, walkways and stairs. The principal problem in wayfinding is to select the way out when occupants egress from a multi-exit area. The choice of exits will depend on how groups of evacuees interact. Non-cooperative game theory deals largely with how intelligent individuals interact with one another in an effort to achieve their own goals—to leave the fire zone as fast as possible. This article presents a game theory based exit choice model for evacuation. It has been integrated in an evacuation model and demonstrates that the evacuees' interaction can affect the evacuation pattern and clearance time of a multi-exit zone.

Keywords: Evacuation; Game theory; Selection of exits

1. Introduction

Evacuation of occupants from the hazardous region(s) is per se a way to reduce the ill effects of a fire disaster in a building and predicting evacuation pattern is useful in emergency management. Indeed, it has been considered by building maintenance professionals [1] as well as building control officers [2] that the evacuation system is the most important means for fire safety protection in buildings. In the circumstances, many evacuation models, such as EXODUS [3], SIMULEX [4,5], EGRESS [6,7], SGEM [8–10] etc have been developed to assist building designers to predict the evacuation pattern. Most of the models have focused on modeling the flow of evacuees and the behavior of crowd flow has not been comprehensively studied, especially the behavioral reaction of the evacuees during their movement.

One of the critical behavioral reactions of people that may affect the escape process is the choice of exit. Choice of exit is one of the most complex aspects of people's movement. Generally, a building consists of enclosure areas such rooms, walkways and stairs. Besides the final exits of the building, the term 'exit' here also refers to the openings of an area through which people can escape from one area to another inside the building. The principal problem in wayfinding is to select the way out when occupants egress from a multi-exit area. Therefore, choice of exit is the selection of routes from one point to another. In case of emergency, an individual's choice of escape route can be regarded as a wayfinding which involves perception and cognition. His or her decision will be affected by what has been seen in the environment and what has been formulated in his or her mind—the cognitive map. When several people are finding their ways to leave a hazardous zone, an individual's decision may be affected by other people's actions. In other words, interaction of people will be a process that should be considered in modeling the evacuation pattern in a zone of multi-exits. Non-cooperative game theory [11,12] deals largely with how intelligent individuals interact with one another in an effort to achieve their own goals—to leave the fire zone as fast as possible. This article presents a game theory-based method that can be incorporated within an evacuation model and effectively models the exit selection process in an evacuation process.

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2. Previous studies

Early engineering models used to predict people's movement, such as EVACNET [13], applied no behavioral rules. They relied on the physical movement of the population, and the physical representation of the building geometry to influence and determine occupant egress. Recently, social/architectural scientists, such as Passini [14], Ozel [15], Proulx [16,17], Sime [18] and Canter [19] have pointed out that one of the dominant factors affecting evacuation patterns is the evacuees' behavioral reactions accompanying their movement. Their studies have identified the contributing factors and provide valuable information for studying wayfinding process. On the basis of these studies, some engineering models can incorporate some psychological rules to model the response pattern of evacuees. However, how the rules can be applied in a dynamic process, especially when the reaction of an individual is affected dynamically by others, has rarely been discussed.

Garbrecht [20,21] studied the difference between random walk and random path selection strategies in normal situations. The first describes a movement in a labyrinth where a person makes a random choice at each intersection. The latter refers to an initial random choice of a complete path from origin to destination. It has been shown that the two ways of route selection will lead to different results even if the random mechanism is assumed to be uniform amongst all alternatives. This indicates that an individual has pre-select a route based on his or her knowledge of the environment, the final destination may be altered if he or she has changed the choice during the movement. The transient change of route choice is critical to the escape process in that under an emergency situation, an individual may endeavor to achieve his or her own goal—to leave the hazardous zone as fast as possible. One of the interim goals to achieve the final goal will be to avoid congestion. This will be particular obvious if the evacuees can notice the movement and reactions of the others such as in a space with large population—such as a stadium, an auditorium, etc.

Gwynne and Schneider [22,23] have also proposed exit selection behavior models. The models concern the response of occupants to exit selection and re-direction. The occupants' decision-making is adaptive according to their familiarity with the structure, the visibility of the exit and the length of queues at the exits. The exit selection behavior is mainly modeled as the passive response to the extent of the crowding and only final exits of the building is considered. In reality, however, the occupants will predict their evacuation efficiency based on others' walking direction and make a decision. Besides the final exits, the selection of exits will also occur whenever occupants egress from any enclosure inside a building.

In most evacuation models, the exit choice of an individual may be modeled by a pre-selection process on the basis of some wayfinding rules. Checking the shortest distance and the inter-person distance dynamically will be an approach to manipulate the transient situations. However, modeling the dynamic interaction of people with respect to the congestion state of the exits and the actions of other evacuees during the process is rare.

3. Model development

If the interactive decision process of the evacuees is rational, game theory can be adopted to describe the interactive behavior. In a game, several agents (the evacuees) strive to maximize their (expected) utility index by choosing particular courses of action (selecting particular route), and each agent's final utility payoffs will depend on the profile of courses of action chosen by all agents. The interactive situation, specified by the set of participants, the possible courses of action of each agent, and the set of all possible utility payoffs, is called a game. When evacuees and the congestion state of a route achieve a *Nash Equilibrium*[1] [24], the strategy[2] is optimal, i.e. all evacuees select exits based on the strategy in that the clearance time is the shortest.

Many modern buildings will comprise rooms, walkway and stairs forming a multi-zone complex. Such building system can be represented by a network system and the evacuation problem can be resolved as a network flow problem with nodes and links representing rooms and communication paths. O'Neill has commented that the network structure of nodes and their activity links is analogous to the topological paths between choice points within a building layout [25]. Choice points can occur at route (e.g. corridor) intersections and route turns [26]. An individual will select his or her route 'step by step' with the route choice point taken at every node of his evacuation path. Random path selection strategies [20,21] obviously do not represent the actual behavior. We can then define $P_{n,k}$ (D_i, L_i, B_i) as the probability that person n select the exit at time step k, for the movement from a node to another node. D_i is crowd density at exit i, L_i is distance person n to the exit i and B_i is the width of exit i. $P_{n,k}$ (D_i, L_i, B_i) will be computed on the basis of game theory at each time step with the assumption that every evacuee will select the 'best' exit based on the $P_{n,k}$ (D_i, L_i, B_i) at each time step.

4. Exit selection process

Game theory is a branch of mathematics devoted to the logic of decision-making in social interactions. The principal objective of game theory is to determine, through formal reasoning alone, what strategies the players ought

[1] If there is a set of strategies with the property that no player can benefit by changing his or her strategy while the other players keep their strategies unchanged, then that set of strategies and the corresponding payoffs constitute the Nash Equilibrium.

[2] A player's strategy is the action or the plan of actions this player chooses out of his set of strategies—a plan for playing the game.

to choose in order to pursue their own interests rationally and what outcomes will result if they do so. All players are advisable and do not know what strategies the other side players choose. In other words, it is not possible to increase a player's own benefit using the erroneous strategies of others and an optimal strategy will be found to all players at *Nash Equilibrium*.

The process of an evacuee's evacuation is mainly influenced by his or her interaction with the environment and other evacuees. In the worst scenario, we assume that all the evacuees are selfish during an emergency and the interaction between them is competitive. Therefore, finding an exit to leave a hazardous space can be attributed as playing a game in that each of the evacuees will pursue his or her interests—leaving the hazardous place as quickly as possible. At each decision point, they will not know what *strategies* the other side evacuees choose. An estimate of evacuation cost must be taken into account with expectation to the reaction of other evacuees at other exits. The thought process for person a may be as follows: "if the congestion at exit k was serious, selection for exit k would not be preferred, and people will try to seek an alterative exit; however, as other evacuees may think similarly and cause congestion on the alterative exit, the best decision may be to take a chance with exit k after all". This infinite chain of hypotheses is typical for a game situation.

As people in a fire will normally not be in a panic condition [14], it is reasonable to assume that the evacuees will be rational, that is, their behavior will be based on the observed situation and logical reasoning. Each individual will always choose his or her maximum utility (or minimum cost) route with respect to the states of the exits so as to minimize the escape time.

We know that the maximum flow rate through an exit (the number of persons walking through an exit in unit time) is limited and in proportion to the width of the exit. Therefore, when the number of evacuees who decide to egress through an exit increase to a certain amount, they must queue at the exit. Thus, the crowding is formed at the vicinity of the exit and congestion may occur due to possible conflicts among evacuees. It is obvious that the expected travel time of an evacuee depends on both the queue length and the distance to the exit L. The queue length is related to the crowd density D and the width of the exit B.

In this paper, we propose a game theory approach to model the behavior of selecting exits. The approach has been used in traffic assignment problems [27]. There are two steps incorporated in the model. In the first step, we treat all the evacuees as a "whole entity" and assign them to the exits. A game is envisaged between the crowd of evacuees seeking an exit to minimize the expected travel time and a "virtual entity" imposing the blockage influence on the evacuees to maximize the expected travel time. This is assumed to be a two-player, non-cooperative, zero sum game. In this game, the evacuees will guess which exit will be congested and the "virtual entity" will guess which exit will be chosen. According to the spirit of game theory, the mixed strategy *Nash equilibrium* for this game gives a reasonable probability-based result of exit selection. Actually, the result of this step indicates the attraction of all the exits to the crowd. In the second step, we need to determine the decision of each evacuee. The factor of distance to the exits is considered. This information is used to adjust the probability values obtained in step one. Fig. 1 briefly illustrates this approach.

As a general case, we consider an example in that N persons are anxious to evacuate from an enclosure (e.g. a room). The enclosure has M exits leading to other enclosures or the outside of the building. The initial positions of the evacuees are distributed randomly at the start of evacuation. Pure strategies $S_1 = \{\alpha_i\}$ (for, $1 \leqslant i \leqslant m$) of the crowd will be formed when all evacuees choose the exit i. A pure strategy is a completely deterministic strategy and occurs when a player assigns probability 1 to a specific action. The pure strategies of the "virtual entity" are $S_2 = \{\beta_j\}$ (for, $1 \leqslant j \leqslant m$) representing that exit j is imposed capacity restriction. The payoff matrix $\mathbf{A}$ of the evacuees can be expressed as follows:

$$\mathbf{A} = \begin{array}{c} \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{array} \begin{array}{c} \begin{array}{cccc} \alpha_1 & \alpha_2 & \cdots & \alpha_m \end{array} \\ \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{bmatrix} \end{array}, \qquad (1)$$

where $\{\alpha_1, \alpha_2, \ldots, \alpha_m\}$, $\{\beta_1, \beta_2, \ldots, \beta_m\}$ are the strategies of the two players, respectively. The expected payoff a_{ij} is the

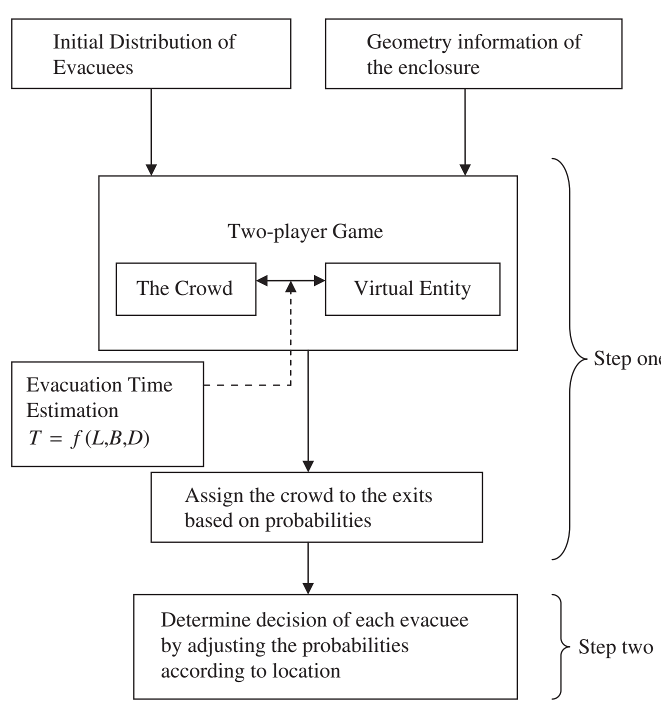

Fig. 1. Approach of modeling.

cost representing the total evacuation time of evacuees under scenario (α_i, β_j) with all evacuees moving toward the exit i. a_{ij} may be computed on the basis of the crowd density at the exit i, the width of exit i, the distance to exit i and the strategy β_j.

According to game theory, if the matrix (1) satisfies the following condition:

$$\max_i \min_j a_{ij} = \min_j \max_i a_{ij} = a_{kl} \tag{2}$$

an optimal pure strategy situation (α, β) will exist. It should be noted that "optimal" here means "equilibrium" not "best" when players are extremely pessimistic and choose their own best strategy from the worst ones. The optimal pure strategies may be more than one or do not exist.

When formula (2) cannot be satisfied, there will be no solution for pure strategies but a mixed strategy solution will then exist. In game theory, a mixed strategy is used to describe a strategy compromising of possible moves and a probability distribution which corresponds to how frequently each move is chosen [24]. A player adopts a mixed strategy by choosing his actions randomly, using fixed probabilities. In other words, players may instead randomly select from among these *pure* strategies with certain probabilities. Randomizing one's own choice in this way is called a *mixed* strategy. Nash showed in 1951 [24] that any finite strategic-form game has equilibrium if mixed strategies are allowed.

For the above exit selection process, a mixed strategy is considered as the probability of exit choice. To seek the equilibrium (optimal) solution, we consider the following couple of mathematical linear programming problems:

$$\max\ z = \sum_i x_i,$$

$$\begin{cases} \sum_i a_{ij} x_i \leqslant 1, & j = 1, \ldots, m, \\ x_i \geqslant 0, & i = 1, \ldots, m, \end{cases} \tag{3}$$

$$\min\ w = \sum_j y_i$$

$$\begin{cases} \sum_i a_{ij} y_j \geqslant 1, & i = 1, \ldots, m, \\ y_i \geqslant 0, & j = 1, \ldots, m, \end{cases} \tag{4}$$

where z, w are the objective functions, $z = w = 1/v$; v is the value of the games, i.e. the expected evacuation time of the evacuees (or the expected earned value of the virtual entity); a_{ij} are elements of the payoff (lose) matrix. If $\bar{\mathbf{x}}$, $\bar{\mathbf{y}}$ are the optimal solutions of the problems (3) and (4), respectively, then the value of the game is

$$v = \frac{1}{\sum_{i=1}^{m} \bar{x}_i}. \tag{5}$$

The optimal solution of the game is $(\mathbf{x}^*, \mathbf{y}^*)$, where

$$\mathbf{x}^* = v\bar{\mathbf{x}}, \quad \mathbf{y}^* = v\bar{\mathbf{y}}, \tag{6}$$

x_i^* is the probability that the evacuees choose exit i; y_j^* is the probability that the virtual entity choose exit j to impose blockage.

For the above couple of problems (3) and (4) only one of them needs to be solved because the solution of one problem can be derived from another. In this paper, we aim to find the probability of exit choice, so problem (3) is solved. As the classical method, we adopt simplex method to solve the linear programming problem and obtain the probability distribution $\mathbf{x}^*$ assigning the crowd to the exits.

In the step one, the above game-based algorithm only considers the relation between the congestion situation near the exits and the egress time. Common sense tells us that an evacuee will not choose the farther exit unless the closer exit is congested. Because the distances to the exits also influence the exit choice process, we should modify $\mathbf{x}^*$ by the location information in the step two. The final probabilities of exit choice should be

$$p_i^* = \frac{x_i^*}{l_i \sum_{j=1}^{m} x_j^* / l_j}, \tag{7}$$

where $l_i (1 \leqslant i \leqslant m)$ is the distance to the exit i.

5. Integration with evacuation model

The aforesaid conditions can be implemented in a fine grid evacuation model such as the SGEM [8]. Fig. 2 shows the outline algorithm of the computation at each time step. The computation process will be iterated at each time step and is basically a dynamic simulation process.

In order to illustrate the output of the results, a two-room setting, which is shown in Fig. 3, is taken for the

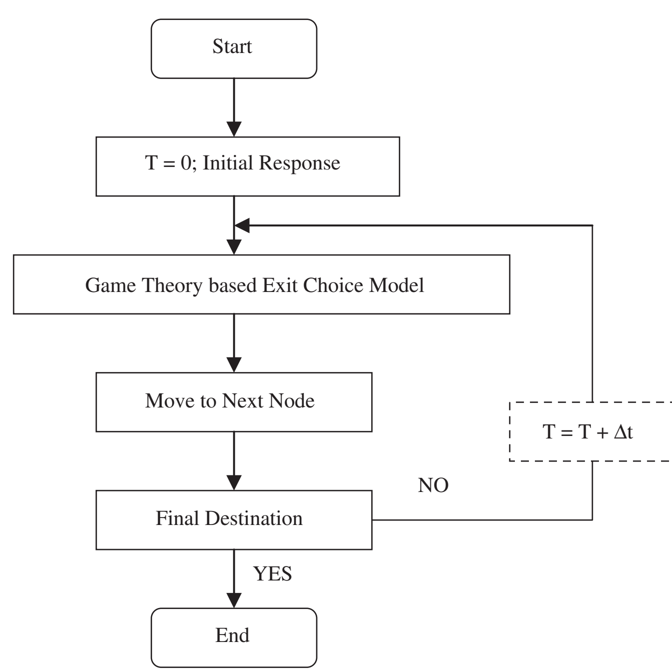

Fig. 2. The evacuation process for each person.

analysis. It is supposed that Room 2 will accommodate 100 persons. All occupants will escape from room 2 to room 1 and then pass exit 1 and exit 2 to reach the place of safety.

Snapshots of the simulation output from the SGEM model illustrating the transient evacuation pattern are shown in Fig. 4. An output from the same model without adding the game theory route choice model is used for comparison. Fig. 5 shows the cumulative outflow of evacuees given by the two simulations.

Table 1 shows a reasonable distribution of evacuees amongst the two exits. The simulation output provides a more rational result than merely considering the travel distance from the exit. Fig. 3(a) and (b) show a change in strategy of an evacuee, X, when both congestion and travel distance are taken into consideration. Fig. 4(a) and (b) show the transient flow through the two exits. It indicates a more rational flow through exit 1 in Fig. 4(a).

6. Concluding remarks

A novel approach to model the dynamic exit selection process of evacuees has been presented in this article. Evacuees perceive the actions of other evacuees and the environmental situations, and respond to their cognition to decide their escape route. In a space with high crowd density where occupants can notice the actions of other occupants, an individual will formulate his or her strategy of leaving on the basis of the crowd's actions, the travel distance to the exits, the effect of the environmental stimuli and the cognitive map (familiarity of the exits). At present, a non-cooperative game theory model has been established to examine how the rational interacting behavior of the evacuees will affect the evacuation pattern. The mixed-strategy *Nash Equilibrium* for the game describes the equilibrium for the evacuees and the congestion states of exits. The influence of other factors, such as the familiarity, has not been considered in this study. Nevertheless, the approach presented can rationalize the interaction of the evacuees with the environment. Further works will be required to examine the effect of familiarity and environmental stimuli as well as other 'grouping' effect on exit selection process.

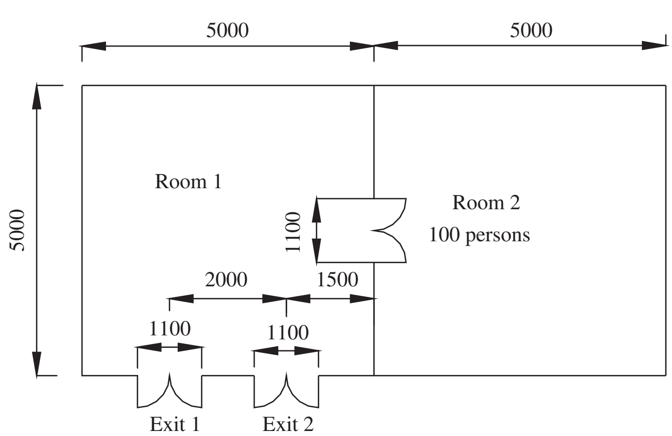

Fig. 3. Layout plan.

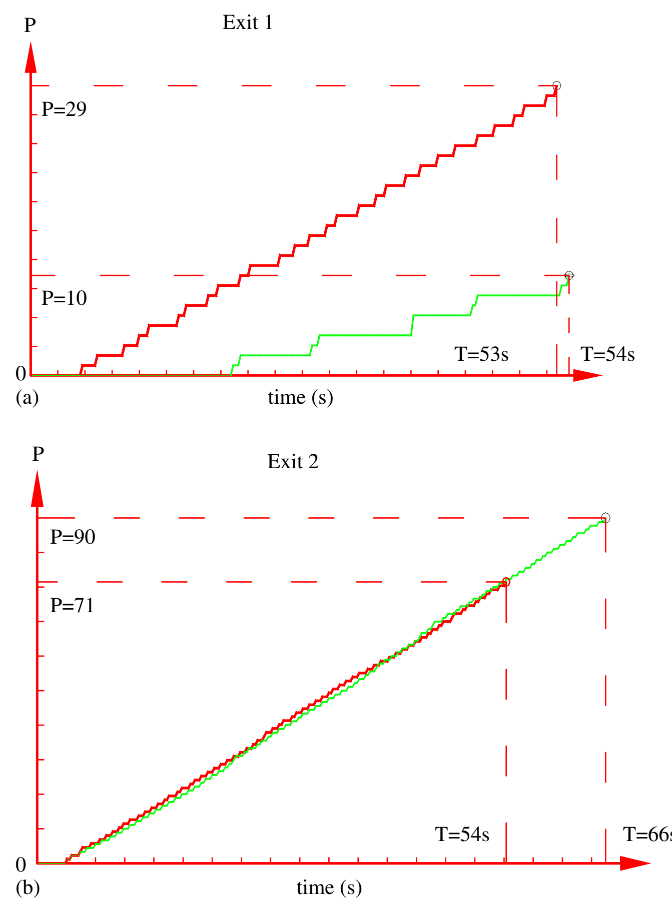

Fig. 5. Cumulative evacuees passing through (a) exit 1 and (b) exit 2.

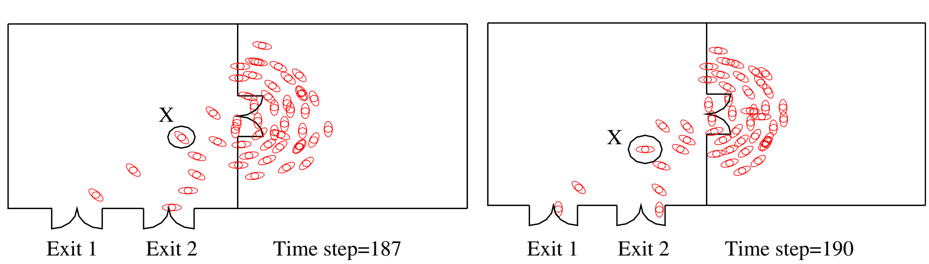

Fig. 4. (a) Snapshots of the simulation output at time step 187. (b) Snapshots of the simulation output at time step 190.

Table 1
Exiting pattern for a 2-room/2-exit situation

		Simulation with game theory route choice model	Simulation without game theory route choice model
Exit 1	P	29	10
	T	53	54
Exit 2	P	71	90
	T	54	66

P = cumulative number of persons passing through the exit.
T = clearance time.

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