Chapter 8
Relations

Objectives

- 8.1-Relations and their Properties
- 8.2-N-ary Relations and their Applications
- 8.3-Representing relations
- 8.5-Equivalence Relations

8.1 Relations and Their Applications

- Introduction
- Functions as Relations
- Relations on a Set
- Properties of Relations
- Combining Relations

Introduction

- Relations can be used to solve problems such as determining which pairs of cities are linked by airline flights in a network, finding a viable order for the different phases of a complicated project, or producing a useful way to store information in computer databases

8.1- Relations… : Introduction

DEFINITION 1

Let A and B be sets. A *binary relation from A to B* is a subset of $A \times B$.

Notations

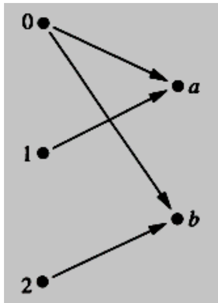
$a\,R\,b$: $(a, b) \in R$

$a \not R\, b$: $(a, b) \notin R$.

Example 1: A: set of students , B: set of courses
AxB = { (Hellen, Math1), (Jacob, C Language), ……. }

Example 2: A: set of cities , B: set of states of USA
AxB = { (Boulder, Colorado), (Bangore, Maine), ……. }

Example 3:
A = { 0, 1, 2 }
B= { a, b }
R= { (0,a) , (0,b) ,
(1,a), (2,b) }



R	a	b
0	×	×
1	×	
2		×

FIGURE 1 Displaying the Ordered Pairs in the Relation R from Example 3.

8.1- Relations… : Functions as Relations

Function: $f : A \rightarrow B$

Graph of f: $G = \{ (a,b) \mid a \in A, b \in B \}$

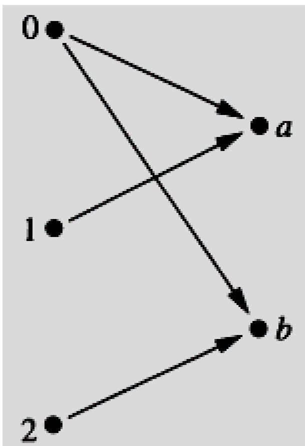
➔ $G \subseteq A \times B$

➔ G is a relation

➔ Relations are a **generalization** of functions. But a relation may not be a function.

➔ Example:

(R is not a function) ➔



R	*a*	*b*
0	×	×
1	×	
2		×

8.1- Relations… : Relations on a Set

DEFINITION 2

A *relation on the set* A is a relation from A to A.

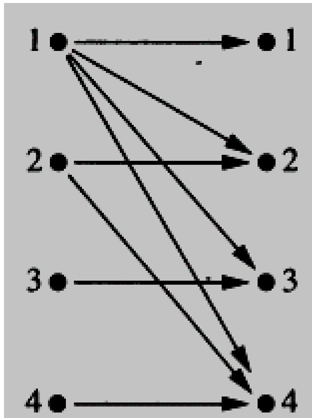
In other words, a relation on a set A is a subset of $A \times A$.

EXAMPLE 4

Let A be the set $\{1, 2, 3, 4\}$. Which ordered pairs are in the relation $R = \{(a, b) \mid a \text{ divides } b\}$?

Solution: Because (a, b) is in R if and only if a and b are positive integers not exceeding 4 such that a divides b, we see that

$$R = \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (2, 4), (3, 3), (4, 4)\}.$$



R	1	2	3	4
1	×	×	×	×
2		×		×
3			×	
4				×

In-class Examples:
Example 5, page 521
Example 6: page 522

8.1- Relations… : Properties of Relations

- Reflexive / Irreflexive
- Symmetric/ Asymmetric / Antisymmetric
- Transitive
- Inverse relations
- Complementary relations

8.1- Relations… : Properties of Relations

DEFINITION 3

A relation R on a set A is called *reflexive* if $(a, a) \in R$ for every element $a \in A$.

R is reflexive $\leftrightarrow \forall a \in A, (a,a) \in R$

Example 7: page 522

Example 9: page 523

R is irreflexive $\leftrightarrow \forall a \in A, (a,a) \notin R$

A= {0,1}.

Some irreflexive relations:

{} , { (0,1) } , {(1,0) } , { (0,1), (1,0) }

Some non-irreflexive relations

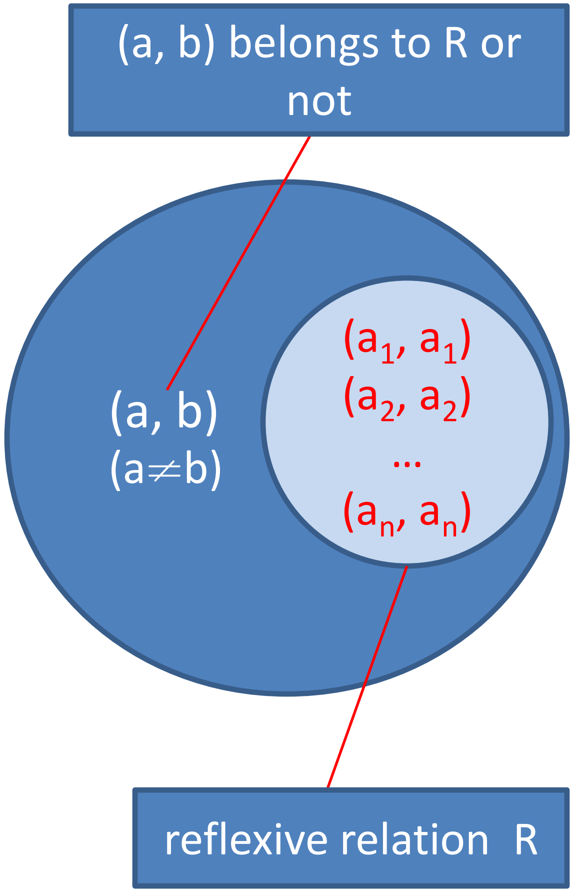
{(0,0)}, {(0,1), (1,1)}, { (0,0), (0,1), (1,0)}

Reflexive and Irreflexive properties are **boundaries.**

Example 16/p.525.

How many *reflexive* relations are there on a set with n elements?

- **Solution.**
- Let A be $\{a_1, a_2, \ldots, a_n\}$.
- A relation on A is a subset of $A \times A$, which has $n \times n$ elements.
- Every relation R on A is reflexive if and only if R is a subset of $A \times A$ and R contains all n ordered pairs (a_i, a_i) for $i = 1, 2, \ldots, n$. **R may have (a, b) or not** (where $a \neq b$).
- There are $n \times n - n$ ordered pairs (a, b) where $a \neq b$.
- So, there are $2^{n \times n - n}$ reflexive relation on A (using product rule to count).



8.1- Relations… : Properties of Relations

DEFINITION 4

A relation R on a set A is called *symmetric* if $(b, a) \in R$ whenever $(a, b) \in R$, for all $a, b \in A$. A relation R on a set A such that for all $a, b \in A$, if $(a, b) \in R$ and $(b, a) \in R$, then $a = b$ is called *antisymmetric*.

R is symmetric $\leftrightarrow \forall (a,b) \in R \rightarrow (b,a) \in R$

Example 10: page 523

Example 12: page 524

R is asymmetric $\leftrightarrow \forall (a,b) \in R \rightarrow (b,a) \notin R$

A = { 0,1}

{(0,1)} and {(1,0)} are asymmetric.

R is antisymmetric $\leftrightarrow \forall a,b \in A \rightarrow (a,b) \in R$ ^ $(b,a) \in R \rightarrow a=b$

A= {0,1}

{}, {(0,1)}, { (0,0), (1,0), (1,1)} are antisymmetric

{(0,0), (0,1), (1,0), (1,1)} is not antisymmetric ($0 \neq 1$)

8.1- Relations… : Properties of Relations

DEFINITION 5

A relation R on a set A is called *transitive* if whenever $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$, for all $a, b, c \in A$.

Example 13: page 524
Example 15: page 524

8.1- Relations… : Properties of Relations

Inverse Relations:

R is a relation from the set A to the set B

Inverse relation R^{-1} of R is a relation from B to A and

$\forall(a,b) \in R \rightarrow (b,a) \in R^{-1}$.

Complementary Relations:

R is a relation from the set A to the set B

Complementary relation ~R of R is a relation from A to B and and ~R contains ordered pairs $\{ (a,b) \mid (a,b) \notin R \}$.

Properties of Relations

Consider the set A={1,2,3,4} and some relations

R1	1	2	3	4
1	X	X		
2		X		X
3	X		X	
4		x		X

Reflexive
Main diagonal

R2	1	2	3	4
1		X	X	
2	X			X
3	X			X
4		X	X	

Symmetric

R3	1	2	3	4
1	X	X	X	
2	X	X	X	
3		X	X	
4				

Transitive
(1,2) (2,1) → (1,1)
(1,3) (3,2) → (1,2)
(2,1) (1,2) → (2,2)
(2,1) (1,3) → (2,3)
(2,3) (3,2) → (2,2)
(3,2) (2,3) → (3,3)

8.1- Relations… : Combining Relations

Unions, Intersections and Differences of two relations:

EXAMPLE 17

Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4\}$. The relations $R_1 = \{(1, 1), (2, 2), (3, 3)\}$ and $R_2 = \{(1, 1), (1, 2), (1, 3), (1, 4)\}$ can be combined to obtain

$$\boxed{\begin{aligned} R_1 \cup R_2 &= \{(1, 1), (1, 2), (1, 3), (1, 4), (2, 2), (3, 3)\}, \\ R_1 \cap R_2 &= \{(1, 1)\}, \\ R_1 - R_2 &= \{(2, 2), (3, 3)\}, \\ R_2 - R_1 &= \{(1, 2), (1, 3), (1, 4)\}. \end{aligned}}$$

8.1- Relations… : Combining Relations

DEFINITION 6

Let R be a relation from a set A to a set B and S a relation from B to a set C. The *composite* of R and S is the relation consisting of ordered pairs (a, c), where $a \in A$, $c \in C$, and for which there exists an element $b \in B$ such that $(a, b) \in R$ and $(b, c) \in S$. We denote the composite of R and S by $S \circ R$.

EXAMPLE 20

R is the relation from $\{1, 2, 3\}$ to $\{1, 2, 3, 4\}$

$$R = \{(1, 1), (1, 4), (2, 3), (3, 1), (3, 4)\}$$

S is the relation from $\{1, 2, 3, 4\}$ to $\{0, 1, 2\}$

$$S = \{(1, 0), (2, 0), (3, 1), (3, 2), (4, 1)\}$$

$$S \circ R = \{(1, 0), (1, 1), (2, 1), (2, 2), (3, 0), (3, 1)\}.$$

R	S	SoR
(1,1)	(1,0)	(1,0)
(1,4)	(4,1)	(1,1)
(2,3)	(3,1) (3,2)	(2,1) (2,2)
(3,1)	(1,0)	(3,0)
(3,4)	(4,1)	(3,1)

DEFINITION 7

Let R be a relation on the set A. The powers R^n, $n = 1, 2, 3, \ldots$, are defined recursively by

$$R^1 = R \quad \text{and} \quad R^{n+1} = R^n \circ R.$$

The definition shows that $R^2 = R \circ R$, $R^3 = R^2 \circ R = (R \circ R) \circ R$, and so on.

EXAMPLE 22 Let $R = \{(1, 1), (2, 1), (3, 2), (4, 3)\}$. Find the powers R^n, $n = 2, 3, 4, \ldots$.

$R^2 = R \circ R = \{(1, 1), (2, 1), (3, 1), (4, 2)\}$

$R^3 = R^2 \circ R = \{(1, 1), (2, 1), (3, 1), (4, 1)\}$

$R^4 = \{(1, 1), (2, 1), (3, 1), (4, 1)\}$

.....

8.1- Relations… : Combining Relations

THEOREM 1

The relation R on a set A is transitive if and only if $R^n \subseteq R$ for $n = 1, 2, 3, \ldots$.

Proof: Page 527

8.2- n-ary Relations and Their Applications

- Theory of Relational Databases

DEFINITION 1

Let $A_1, A_2, \ldots, A_n$ be sets. An *n-ary relation* on these sets is a subset of $A_1 \times A_2 \times \cdots \times A_n$. The sets $A_1, A_2, \ldots, A_n$ are called the *domains* of the relation, and n is called its *degree*.

Primary key column

Primary domain

Table - dbo.Phone_tbl

Column Name	Data Type	Allow Nulls
ModelNo	nvarchar(50)	☐
Manufacturer	nvarchar(50)	☐
Weight	int	☐
MP3_support	bit	☐

Degree of 4

Extension: collection of current records.

Intension: current records.

Table - dbo.Phone_tbl

ModelNo	Manufacturer	Weight	MP3_support	Price
D5000	Samsung	115	True	260
L6	Motorola	126	False	200
NULL	NULL	NULL	NULL	NULL

Record (n-tuple)

N-ary Relations ….

TABLE 1 Students.

Student_name	*ID_number*	*Major*	*GPA*
Ackermann	231455	Computer Science	3.88
Adams	888323	Physics	3.45
Chou	102147	Computer Science	3.49
Goodfriend	453876	Mathematics	3.45
Rao	678543	Mathematics	3.90
Stevens	786576	Psychology	2.99

Which domains are primary keys? Suppose that no record can be added.

➔ Student_name or ID_number because these columns contain different values.

➔ Student_name x ID_number may be used as primary domain (composite key).

8.2- n-ary Relations…: Operations

- Selection
- Projection
- Join

8.2…. Selection Operation

DEFINITION 2

Let R be an n-ary relation and C a condition that elements in R may satisfy. Then the *selection operator* s_C maps the n-ary relation R to the n-ary relation of all n-tuples from R that satisfy the condition C.

TABLE 1 Students.

Student_name	*ID_number*	*Major*	*GPA*
Ackermann	231455	Computer Science	3.88
Adams	888323	Physics	3.45
Chou	102147	Computer Science	3.49
Goodfriend	453876	Mathematics	3.45
Rao	678543	Mathematics	3.90
Stevens	786576	Psychology	2.99

$S_{\text{Major='Computer Science'}}$

$P_{\text{Student\_name,GPA}}$

Student_name	*ID_number*	*Major*	*GPA*
Ackermann	231455	Computer Science	3.88
Chou	102147	Computer Science	3.49

8.2…: Projection Operation

DEFINITION 3

The *projection* $P_{i_1 i_2, \ldots, i_m}$ where $i_1 < i_2 < \cdots < i_m$, maps the n-tuple $(a_1, a_2, \ldots, a_n)$ to the m-tuple $(a_{i_1}, a_{i_2}, \ldots, a_{i_m})$, where $m \leq n$.

TABLE 1 Students.

Student_name	*ID_number*	*Major*	*GPA*
Ackermann	231455	Computer Science	3.88
Adams	888323	Physics	3.45
Chou	102147	Computer Science	3.49
Goodfriend	453876	Mathematics	3.45
Rao	678543	Mathematics	3.90
Stevens	786576	Psychology	2.99

$P_{\text{Student\_name,GPA}}$ $P_{1,4}$

Student_name	*GPA*
Ackermann	3.88
Adams	3.45
Chou	3.49
Goodfriend	3.45
Rao	3.90
Stevens	2.99

8.2… : Projection

TABLE 3 Enrollments.

Student	*Major*	*Course*
Glauser	Biology	BI 290
Glauser	Biology	MS 475
Glauser	Biology	PY 410
Marcus	Mathematics	MS 511
Marcus	Mathematics	MS 603
Marcus	Mathematics	CS 322
Miller	Computer Science	MS 575
Miller	Computer Science	CS 455

$P_{1,2}$

TABLE 4 Majors.

Student	*Major*
Glauser	Biology
Marcus	Mathematics
Miller	Computer Science

8.2… : Join Operation

DEFINITION 4

Let R be a relation of degree m and S a relation of degree n. The *join* $J_p(R,S)$, where $p \le m$ and $p \le n$, is a relation of degree $m+n-p$ that consists of all $(m+n-p)$-tuples $(a_1, a_2, \ldots, a_{m-p}, c_1, c_2, \ldots, c_p, b_1, b_2, \ldots, b_{n-p})$, where the m-tuple $(a_1, a_2, \ldots, a_{m-p}, c_1, c_2, \ldots, c_p)$ belongs to R and the n-tuple $(c_1, c_2, \ldots, c_p, b_1, b_2, \ldots, b_{n-p})$ belongs to S.

TABLE 5 Teaching_assignments.

Professor	*Department*	*Course_number*
Cruz	Zoology	335
Cruz	Zoology	412
Farber	Psychology	501
Farber	Psychology	617
Grammer	Physics	544
Grammer	Physics	551
Rosen	Computer Science	518
Rosen	Mathematics	575

TABLE 6 Class_schedule.

Department	*Course_number*	*Room*	*Time*
Computer Science	518	N521	2:00 P.M.
Mathematics	575	N502	3:00 P.M.
Mathematics	611	N521	4:00 P.M.
Physics	544	B505	4:00 P.M.
Psychology	501	A100	3:00 P.M.
Psychology	617	A110	11:00 A.M.
Zoology	335	A100	9:00 A.M.
Zoology	412	A100	8:00 A.M.

TABLE 7 Teaching_schedule.

Professor	*Department*	*Course_number*	*Room*	*Time*
Cruz	Zoology	335	A100	9:00 A.M.
Cruz	Zoology	412	A100	8:00 A.M.
Farber	Psychology	501	A100	3:00 P.M.
Farber	Psychology	617	A110	11:00 A.M.
Grammer	Physics	544	B505	4:00 P.M.
Rosen	Computer Science	518	N521	2:00 P.M.
Rosen	Mathematics	575	N502	3:00 P.M.

J_2 , 2 common columns

➔ Result table haves 3+4-2=5 columns

8.3- Representing Relations

- Using matrices
- Using Directed Graphs (Digraphs)

8.3…. : Representing Relations Using Matrices

A relation from $A = \{a_1, a_2, \ldots, a_m\}$ to $B = \{b_1, b_2, \ldots, b_n\}$.

$$\mathbf{M}_R = [m_{ij}],\ m_{ij} = \begin{cases} 1 \text{ if } (a_i, b_j) \in R, \\ 0 \text{ if } (a_i, b_j) \notin R. \end{cases}$$

EXAMPLE 1

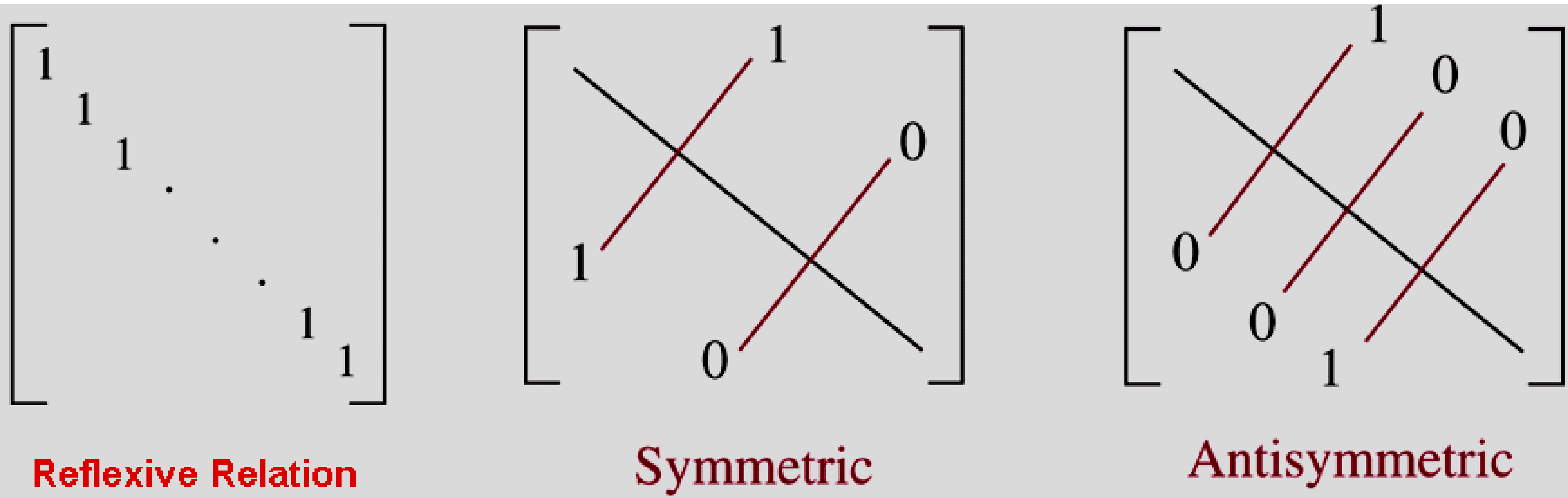
Suppose that $A = \{1, 2, 3\}$ and $B = \{1, 2\}$. Let R be the relation from A to B containing (a, b) if $a \in A$, $b \in B$, and $\boxed{a > b}$. What is the matrix representing R if $a_1 = 1$, $a_2 = 2$, and $a_3 = 3$, and $b_1 = 1$ and $b_2 = 2$?

Solution: Because $R = \{(2, 1), (3, 1), (3, 2)\}$, the matrix for R is

$$\mathbf{M}_R = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 1 \end{bmatrix}.$$

The 1s in $\mathbf{M}_R$ show that the pairs (2, 1), (3, 1), and (3, 2) belong to R. The 0s show that no other pairs belong to R.

8.3… : Matrices of Special Relations



8.3…. Matrices for Unions, Intersections

$$\mathbf{M}_{R_1 \cup R_2} = \mathbf{M}_{R_1} \vee \mathbf{M}_{R_2} \quad \text{and} \quad \mathbf{M}_{R_1 \cap R_2} = \mathbf{M}_{R_1} \wedge \mathbf{M}_{R_2}.$$

EXAMPLE 4

Suppose that the relations R_1 and R_2 on a set A are represented by the matrices

$$\mathbf{M}_{R_1} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}.$$

What are the matrices representing $R_1 \cup R_2$ and $R_1 \cap R_2$?

Solution: The matrices of these relations are

$$\mathbf{M}_{R_1 \cup R_2} = \mathbf{M}_{R_1} \vee \mathbf{M}_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\mathbf{M}_{R_1 \cap R_2} = \mathbf{M}_{R_1} \wedge \mathbf{M}_{R_2} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

8.3- ……Matrix for SoR

Boolean product :

$$\mathbf{M}_{S \circ R} = \mathbf{M}_R \odot \mathbf{M}_S$$

EXAMPLE 5

Find the matrix representing the relations $S \circ R$, where the matrices representing R and S are

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_S = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}.$$

Solution: The matrix for $S \circ R$ is $\mathbf{M}_{S \circ R} = \mathbf{M}_R \odot \mathbf{M}_S = \begin{bmatrix} \boxed{1} & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}.$

(1.0)+(0.0)+(1.1)=1
(1.1)+(0.0)+(1.0)=1
(1^0)+(0.1)+(1.1)=1

In particular, $\mathbf{M}_{R^n} = \mathbf{M}_R^{[n]}$

8.3…. : Representing Relations Using Digraphs

DEFINITION 1

A *directed graph*, or *digraph*, consists of a set V of *vertices* (or *nodes*) together with a set E of ordered pairs of elements of V called *edges* (or *arcs*). The vertex a is called the *initial vertex* of the edge (a, b), and the vertex b is called the *terminal vertex* of this edge.

An edge of the form (a, a) is represented using an arc from the vertex a back to itself. Such an edge is called a **loop.**

EXAMPLE 7

The directed graph with vertices a, b, c, and d, and edges (a, b), (a, d), (b, b), (b, d), (c, a), (c, b), and (d, b) is displayed in Figure 3.

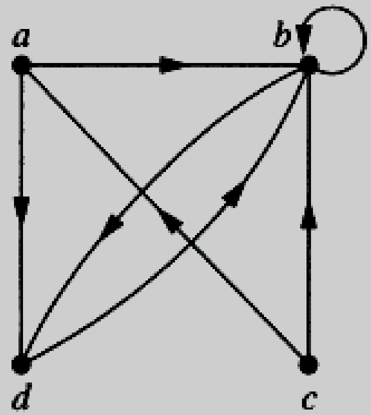
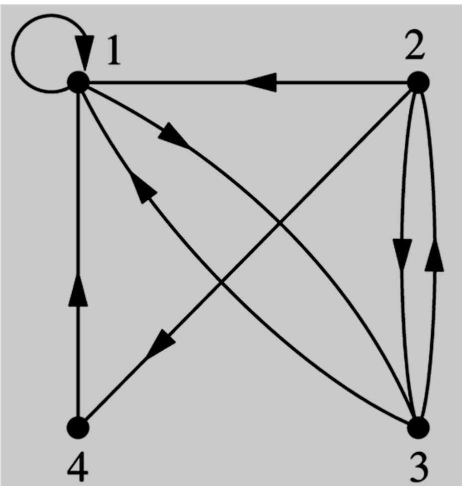
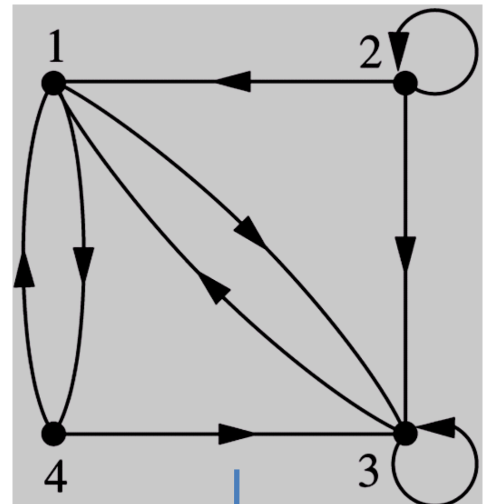


FIGURE 3 **A Directed Graph.**

8.3…. Digraphs

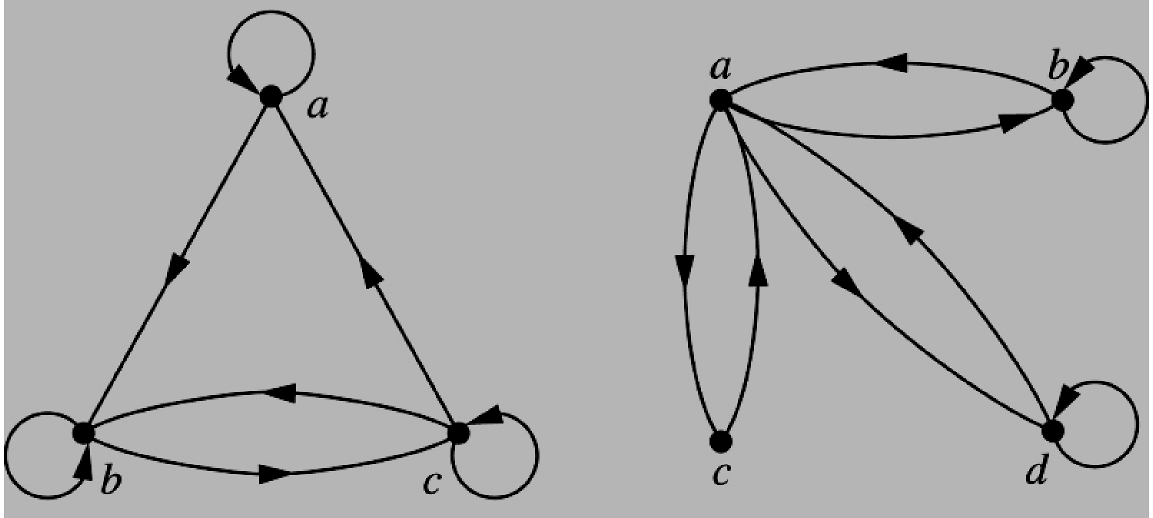
$R = \{(1, 1), (1, 3), (2, 1), (2, 3), (2, 4), (3, 1), (3, 2), (4, 1)\}$





$R = \{(1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (3, 1), (3, 3), (4, 1), (4, 3)\}.$

8.3….. Digraphs



(a) Directed graph of *R* (b) Directed graph of *S*

- It is reflexive (loops)
- It is not symmetric
  (a,b) $\in$ R but (b,a) $\notin$ R
- It is not antisymmetric:
  (b,c), (c,b) $\in$ R but b $\neq$ c
- It is not transitive:
  (a,b), (b,c) $\in$ R but (a,c) $\notin$ R

- It is not reflexive (no loop)
- It is symmetric
- It is not antisymmetric:
  (a,c), (c,a) $\in$ R but a $\neq$ c
- It is not transitive
  (c,a), (a,b) $\in$ R but (c,b) $\notin$ R

8.5- Equivalence Relations

In this section we will study relations with a particular combination of properties that allows them to be used to relate objects that are similar in some way.

DEFINITION 1

A relation on a set A is called an *equivalence relation* if it is reflexive, symmetric, and transitive.

DEFINITION 2

Two elements a and b that are related by an equivalence relation are called *equivalent*. The notation $a \sim b$ is often used to denote that a and b are equivalent elements with respect to a particular equivalence relation.

Steps to show that a relation is a equivalence:
Step 1: Show that it is reflexive : aRa
Step 2: Show that it is symmetric : $aRb \rightarrow bRa$
Step 3: Show that it is transitive: $aRb, bRc \rightarrow aRc$

Equivalence Relations….

Page 556, 557

EXAMPLE 1

Let R be the relation on the set of integers such that aRb if and only if $a = b$ or $a = -b$. In Section 8.1 we showed that R is reflexive, symmetric, and transitive. It follows that R is an equivalence relation.

EXAMPLE 2

Let R be the relation on the set of real numbers such that aRb if and only if $a - b$ is an integer. Is R an equivalence relation?

EXAMPLE 3

Congruence Modulo m Let m be a positive integer with $m > 1$. Show that the relation

$$R = \{(a, b) \mid a \equiv b \pmod{m}\}$$

is an equivalence relation on the set of integers.

EXAMPLE 4

Suppose that R is the relation on the set of strings of English letters such that aRb if and only if $l(a) = l(b)$, where $l(x)$ is the length of the string x. Is R an equivalence relation?

Equivalence Classes

DEFINITION 3

Let R be an equivalence relation on a set A. The set of all elements that are related to an element a of A is called the *equivalence class* of a. The equivalence class of a with respect to R is denoted by $[a]_R$. When only one relation is under consideration, we can delete the subscript R and write $[a]$ for this equivalence class.

In other words, if R is an equivalence relation on a set A, the equivalence class of the element a is $[a]_R = \{s \mid (a, s) \in R\}$.

If $b \in [a]_R$, then b is called a **representative** of this equivalence class. Any element of a class can be used as a representative of this class. That is, there is nothing special about the particular element chosen as the representative of the class.

EXAMPLE 8

Let R be the relation on the set of integers such that aRb if and only if $a = b$ or $a = -b$. In Section 8.1 we showed that R is reflexive, symmetric, and transitive. It follows that R is an equivalence relation. What is the equivalence class?

Solution: Because an integer is equivalent to itself and its negative in this equivalence relation, it follows that $[a] = \{-a, a\}$. This set contains two distinct integers unless $a = 0$. For instance, $[7] = \{-7, 7\}$, $[-5] = \{-5, 5\}$, and $[0] = \{0\}$.

Equivalence Classes and Partitions

THEOREM 1 Proof: page 560

Let R be an equivalence relation on a set A. These statements for elements a and b of A are equivalent: (*i*) aRb (*ii*) $[a] = [b]$ (*iii*) $[a] \cap [b] \neq \emptyset$

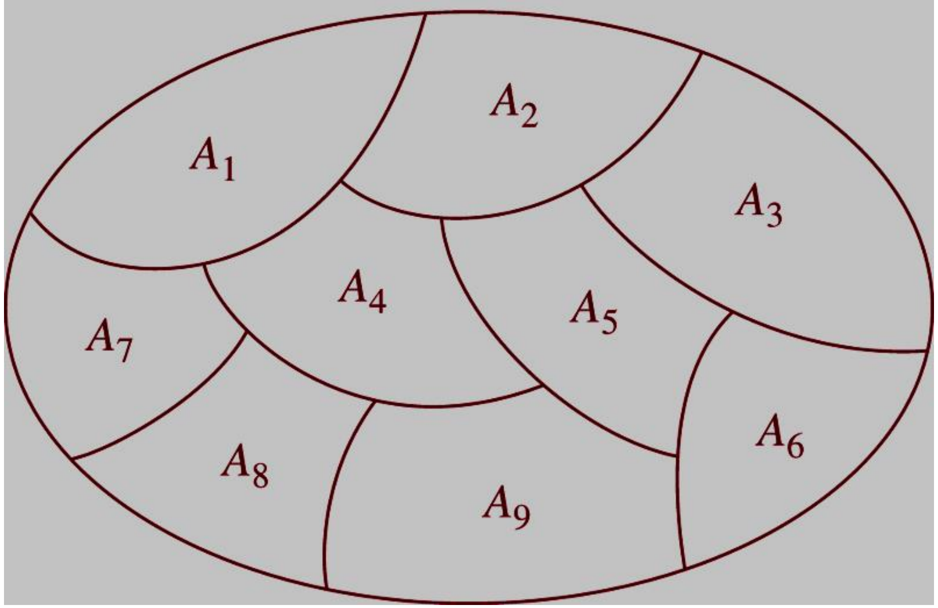


Figure 1: A Partition of a Set

Consider the relation on the set N:
R = { (a,b) | a = b (mod 4) }

Equivalence classes:
a mod (4)=0
{ 4,8,12,16,20,24,28,32,...}
{ 1,5,9,13,17, ...}
{ 2,6,10,14,18,...}
{ 3,7,11,15,19,.... }
→ N is partitioned into 4 different parts

Equivalence Classes and Partitions

THEOREM 2

Let R be an equivalence relation on a set S. Then the equivalence classes of R form a partition of S. Conversely, given a partition $\{A_i \mid i \in I\}$ of the set S, there is an equivalence relation R that has the sets A_i, $i \in I$, as its equivalence classes.

Example 13 shows how to construct an equivalence relation from a partition.

EXAMPLE 13

List the ordered pairs in the equivalence relation R produced by the partition $A_1 = \{1, 2, 3\}$, $A_2 = \{4, 5\}$, and $A_3 = \{6\}$ of $S = \{1, 2, 3, 4, 5, 6\}$, given in Example 12.

Solution: The subsets in the partition are the equivalence classes of R. The pair $(a, b) \in R$ if and only if a and b are in the same subset of the partition. The pairs $(1, 1)$, $(1, 2)$, $(1, 3)$, $(2, 1)$, $(2, 2)$, $(2, 3)$, $(3, 1)$, $(3, 2)$, and $(3, 3)$ belong to R because $A_1 = \{1, 2, 3\}$ is an equivalence class; the pairs $(4, 4)$, $(4, 5)$, $(5, 4)$, and $(5, 5)$ belong to R because $A_2 = \{4, 5\}$ is an equivalence class; and finally the pair $(6, 6)$ belongs to R because $\{6\}$ is an equivalence class. No pair other than those listed belongs to R.

Summary

- 8.1-Relations and their Properties
- 8.2-N-ary Relations and their Applications
- 8.3-Representing relations
- 8.5-Equivalence Relations

Thanks