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| State | Finished |
| Completed on | Friday, 3 December 2021, 8:38 AM |
| Time taken | 1 hour 47 mins |
| Marks | 21.00/25.00 |
| Grade | 8.40 out of 10.00 (84%) |

Question 1

Incorrect

Mark 0.00 out of 1.00

Let the random variable X have a discrete **uniform distribution** on the set of integers $\{3, 6, 9, 12, 15, 18, 21\}$. Find the **mean** and **variance** of X .

- ☐ a. $E(X) = 2, V(X) = 6$
- ☐ b. $E(X) = 4, V(X) = 4$
- ☐ c. $E(X) = 12, V(X) = 36$
- ☐ d. $E(X) = 12, V(X) = 12$
- ☒ e. None of others



If $X \sim \text{Unif}()$ and values are in $\{a, a+1, \dots, b\}$, then

- The **mean** of X is $\mu = E(X) = \frac{a+b}{2}$
- The **variance** of X is $\sigma^2 = V(X) = \frac{(b-a+1)^2 - 1}{12}$

Given a discrete **uniform distribution** X whose values are integers in $\{3, 6, 9, 12, 15, 18, 21\}$.

Let $Y = X/3$, then Y has a discrete **uniform distribution** on the set of integers $\{1, 2, 3, 4, 5, 6, 7\}$.

$$E(Y) = (1+7)/2 = 4 \Rightarrow E(X) = 3E(Y) = 12.$$

$$V(Y) = [(7-1+1)^2 - 1]/12 = 4 \Rightarrow V(X) = 36.$$

The correct answer is: $E(X) = 12, V(X) = 36$

Question 2

Incorrect

Mark 0.00 out of 1.00

$$\mu = \frac{8}{0.4} = 20$$

Consider a sequence of independent Bernoulli trials with $p = 0.4$. After the seventh success occurs, what is the expected number of trials to obtain the eighth success?

- ☐ a. None of the others
- ☐ b. 4
- ☐ c. 3
- ☒ d. 20
- ☐ e. 2.5

$$\frac{1}{0.4} = \frac{10}{4} = 2.5$$

✗

Lack of Memory Property of Geometric distribution

After the sixth success, the expected number of trials to obtain the eighth success = The expected number of trials to obtain the first success = $E(X) = 1/p = 1/0.4 = 2.5$

The correct answer is: 2.5

Question **3**

Correct

Mark 1.00 out of 1.00

Let \leq be "less than or equal". Let the random variable X have a discrete uniform distribution on the intergers $1 \leq x \leq 35$. Determine the mean and variance of X .

- ☐ a. 17 and 102
- ☐ b. None of the others
- ☐ c. 17.5 and 102
- ☒ d. 18 and 102



The correct answer is: 18 and 102

Question 4

Incorrect

Mark 0.00 out of 1.00

A dangerous computer virus attacks a folder consisting of 50 files. Files are affected by the virus independently of one another. Each file is affected with the probability 0.02. What is the probability that more than 3 files are affected by this virus?

- ☐ a. 0.06
- ☐ b. None of the others
- ☐ c. 0.814
- ☐ d. 0.018
- ☒ e. 0.078



Let X be the number of files affected by the virus.

Then $X \sim \text{Binom}(n = 50, p = 0.02)$.

We wish to find $P(X > 3)$.

$$P(X > 3) = 1 - P(X \leq 3) = 0.018$$

The correct answer is: 0.018

Question **5**

Correct

Mark 1.00 out of 1.00

A store receives a shipment of 1000 phones. Suppose the probability that a phone is defective is 0.1%. Let X be the number of defective phones in the shipment.

What kind of distribution does X have?

- ☐ a. Poisson
- ☐ b. Negative Binomial
- ☐ c. Geometric
- ☒ d. Binomial
- ☐ e. Hypergeometric



The correct answer is: Binomial

Question **6**

Correct

Mark 1.00 out of 1.00

The probability that an individual is left-handed is 0.11. In a class of 40 students, what is the probability of finding five left-handers?

- ☐ a. None of the other choices is correct
- ☒ b. 0.179
- ☐ c. 0.000
- ☐ d. 0.125
- ☐ e. 0.11



The correct answer is: 0.179

Question 7

Correct

Mark 1.00 out of 1.00

An array of 30 LED bulbs is used in an automotive light. The probability that a bulb is defective is 0.001 and defective bulbs occur independently.

Determine the probability that an automotive light has **no** defective bulb.

- ☒ a. 0.970
- ☐ b. 0.030
- ☐ c. 1/30
- ☐ d. 0.029
- ☐ e. None of the others



Let X be the number of defective bulbs in an automotive light. Then $X \sim \text{Binom}(n = 30, p = 0.001)$.

We wish to find $P(X = 0) = 0.970$

The correct answer is: 0.970

Question 8

Correct

Mark 1.00 out of 1.00

Given the probability distribution of a discrete random variable X.

| | | | | |
|------|-----|-----|-----|-----|
| X | 3 | 4 | 7 | 9 |
| P(X) | 0.1 | 0.2 | 0.4 | 0.3 |

Find E(X) and V(X)

- ☐ a. 6.6, 5.9
- ☐ b. 5.9, 5.4
- ☐ c. None of these
- ☒ d. 6.6, 4.44
- ☐ e. 5.75, 4



Your answer is correct.

$$E(X) = 3 \cdot 0.1 + 4 \cdot 0.2 + 7 \cdot 0.4 + 9 \cdot 0.3 = 6.6$$

$$E(X^2) = 3^2 \cdot 0.1 + 4^2 \cdot 0.2 + 7^2 \cdot 0.4 + 9^2 \cdot 0.3 = 48$$

$$V(X) = E(X^2) - E(X)^2 = 48 - 6.6^2 = 4.44$$

The correct answer is:

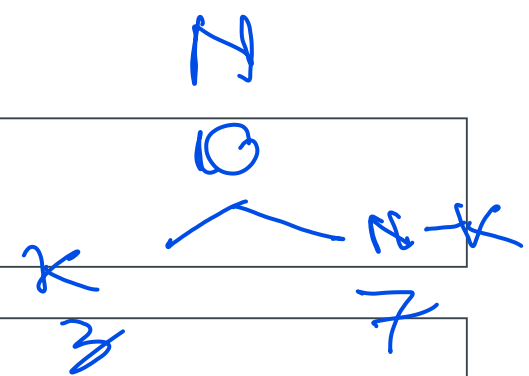
6.6, 4.44

Question 9

Correct

Mark 1.00 out of 1.00

$$\sum_{x=0}^2 \frac{C_3^x \cdot C_7^{5-x}}{C_{10}^5}$$



Suppose that X has a hypergeometric distribution with $N = 10$, $n = 5$, and $K = 3$. Determine $P(X < 2)$.

- ☐ a. None of the others
- ☒ b. 0.5
- ☐ c. 0.92
- ☐ d. 0.08
- ☐ e. 0.64

$$\begin{aligned} P(X < 2) &= P(X=0) + P(X=1) \checkmark \\ &= \frac{C_7^5}{C_{10}^5} + \frac{3 \cdot C_7^4}{C_{10}^5} \end{aligned}$$

The correct answer is: 0.5

Question 10

Correct

Mark 1.00 out of 1.00

Let the random variable X be a *Poisson distribution* with mean of 0.8. Find the probability that $X > 1$.

- ☐ a. 0.550
- ☒ b. 0.191
- ☐ c. 0.809
- ☐ d. 0.640
- ☐ e. None of the other choices is correct

$$\begin{aligned} P(X > 1) &= 1 - P(X \leq 1) \\ &= 1 - \sum_{x=0}^1 \frac{e^{-0.8} \cdot 0.8^x}{x!} \end{aligned}$$

$X \sim \text{Poisson}(\lambda = 0.8)$

$P(X > 1) = 1 - P(X \leq 1) = 1 - P(X = 0) - P(X = 1) = 0.191$

The correct answer is: 0.191

Question 11

Correct

Mark 1.00 out of 1.00

$$p = 20\%$$

An exciting computer game is released. Twenty percent of players will buy an **advanced version** of the game. Among 20 users, what is the expected number of people who will buy the **advanced version**? \longrightarrow

mean

- ☐ a. 20
- ☒ b. 4
- ☐ c. 5
- ☐ d. 10
- ☐ e. None of the others

$$\mu = np = 20 \cdot 20\% \checkmark$$

Let X be the number of players who will buy the advanced version among 20 players.

Then X has a binomial distribution with parameter $n = 20$, $p = 0.2$

$$\implies E(X) = np = 20 \cdot 0.2 = 4$$

The correct answer is: 4

Question **12**

Correct

Mark 1.00 out of 1.00

Given the cumulative distribution function

$$F(x) = \begin{cases} 0 & x < 1 \\ 0.7 & 1 \leq x < 4 \\ 0.9 & 4 \leq x < 7 \\ 1 & 7 \leq x \end{cases}$$

Find $P(X \leq 2)$ and $P(X > 4)$

- ☐ a. 0.3, 0.1
- ☐ b. 0.7, 0.9
- ☐ c. None of the other choices is correct
- ☐ d. 0.3, 0.9
- ☒ e. 0.7, 0.1



$$P(X \leq 2) = F(2) = 0.7$$

$$P(X > 4) = 1 - P(X \leq 4) = 1 - F(4) = 1 - 0.9 = 0.1$$

The correct answer is: 0.7, 0.1

Question **13**

Correct

Mark 1.00 out of 1.00

Give $f(x) = 0,75 \cdot 0,25^x$, $x = 0, 1, 2, \dots$ is the probability mass function. Which the following statement is NOT TRUE?

- ☐ a. $P(X = 2) = 3/64$
- ☐ b. All of the others
- ☒ c. $P(X \geq 1) = 48/64$
- ☐ d. $P(X \leq 2) = 63/64$



The correct answer is: $P(X \geq 1) = 48/64$

$$P(X \geq 1) = \sum_{x=1}^{20} C_{20}^x \cdot 0.1^x \cdot (1-0.1)^{20-x}$$

Question 14

Correct

Mark 1.00 out of 1.00

$$p = 0.1$$

Suppose the probability that item produced by a certain machine will be defective is 0.1. Find the probability that 20 items will contain **at least one** defective item. Assume that the quality of successive items is independent.

- at least one
- ☐ a. 0.270
 - ☐ b. 0.122
 - ☐ c. None of these
 - ☒ d. 0.878
 - ☐ e. 0.730

x

$$n = 20$$

$$P(X \geq 1) = 1 - P(X < 1) = 1 - P(X = 0) = 1 - C_{20}^0 \cdot 0.1^0 \cdot 0.9^{20}$$

Let X be the number of defective items then X has binomial distribution with parameter $n = 20$, $p = 0.1$.

The desired probability is $P(X > 0) = 1 - P(X = 0) = 0.878$.

The correct answer is: 0.878

Question **15**

Correct

Mark 1.00 out of 1.00

A multiple choice test contains 25 questions, each with four answers. Assume a student just guesses on each question. What is the probability that the student answers more than 22 questions correctly?

- ☒ a. $2.46558 \cdot 10^{-12}$
- ☐ b. 1.15463×10^{-14}
- ☐ c. None of these
- ☐ d. 0.096770



The correct answer is: $2.46558 \cdot 10^{-12}$

Question **16**

Correct

Mark 1.00 out of 1.00

The number of 113-calls in HCM city, has a Poisson distribution with a mean of 7 calls per day. The probability that there are 4 calls tomorrow is ____

- ☐ a. None of the other choices is correct
- ☐ b. 0.195
- ☒ c. 0.091
- ☐ d. 0.060
- ☐ e. 0.149



Let X be the number of 113-calls in a day.

$X \sim \text{Poisson}(\lambda)$ and $E(X) = \lambda = 7$ (= mean of 7 calls/day)

$$f(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$P(X = 4) = e^{-7} (7^4) / 4! = 0.091$$

The correct answer is: 0.091

Question 17

Correct

Mark 1.00 out of 1.00

A computer user tries to recall her password. She knows it can be one of 6 possible passwords. She tries her passwords until she finds the right one.

Find the expected number of passwords she uses.

- ☐ a. 5
- ☐ b. 3
- ☐ c. 4
- ☒ d. 6
- ☐ e. None of the others

mean

$$\mu = \frac{1}{p} = \frac{1}{\frac{1}{6}} = 6$$



Let X be the number of passwords she uses until she finds the right one.

Then $X \sim \text{Geometric}(p = 1/6)$

$E(X) = 1/p = 6$.

The correct answer is: 6

Question **18**

Correct

Mark 1.00 out of 1.00

Consider the time to recharge the flash in a camera. The probability that a camera passes the test is 0.9, and the cameras perform independently. What is the probability that the second failure is obtained at the fifth test?

- ☐ a. 0.003
- ☐ b. None of the others
- ☐ c. 0.047
- ☐ d. 0.035
- ☒ e. 0.029



Let X be the number of tests until the second failure.

Then X has a negative binomial distribution with parameter $p = 0.1$ and $r = 2$.

We wish to find $P(X = 5)$.

$$P(X = 5) = \binom{4}{1} 0.1^2 (0.9)^3 = 0.029$$

The correct answer is: 0.029

Question **19**

Incorrect

Mark 0.00 out of 1.00

Network breakdowns are unexpected rare events that occur every 3 weeks, on the average. Compute the probability of more than one breakdown during a 12-week period.

- ☐ a. None of the others
- ☐ b. 0.908
- ☒ c. 0.982
- ☐ d. 0.092
- ☐ e. 0.927



Let X be the number of network breakdowns during a 12-week period. Then X has a Poisson distribution with mean of 4 (breakdowns per 12 weeks).

We wish to find $P(X > 1)$

$$= 1 - P(X=0) - P(X=1) = 0.908$$

The correct answer is: 0.908

Question **20**

Correct

Mark 1.00 out of 1.00

Let X be a binomial random variable with $p=0.1$ and $n=10$. Calculate the following probability: $P(X>2)$ and $P(X\leq 8)$. . .

- ☐ a. 0.0702 and 0
- ☐ b. 0.702 and 0.999
- ☐ c. 0.702 and 1
- ☒ d. None of others.
- ☐ e. 0.9892 and 1

$$P(X > 2) = P(X \geq 3)$$
$$=$$



The correct answer is: None of others.

Question **21**

Correct

Mark 1.00 out of 1.00

Assume that each of your calls to a popular radio station has a probability of 0.04 of connecting, that is, of not obtaining a busy signal. Assume that your calls are independent.

What is the probability that your first call that connects is your 8th call?

- ☐ a. 0.020
- ☐ b. 0.040
- ☐ c. 0.025
- ☒ d. 0.030
- ☐ e. None of the others



Let X be the number of calls until the first connect. Then $X \sim \text{Geometric}(p=0.04)$. We wish to compute $P(X=8)$.

$$P(X=8) = (0.96)^7 \cdot (0.04) = 0.030$$

The correct answer is: 0.030

Question **22**

Correct

Mark 1.00 out of 1.00

You roll a **fair die** until it shows a six. What is the expected number of times of rolling?

- ☐ a. 3
- ☐ b. 4
- ☐ c. None of these
- ☒ d. 6
- ☐ e. 12



Your answer is correct.

Let X be the number of times of rolling until a six comes up. Then $X \sim \text{Geometric}(p=1/6)$.

$E(X) = 1/p = 6$.

The correct answer is:

6

Question **23**

Correct

Mark 1.00 out of 1.00

Suppose that the random variable X has a geometric distribution with parameter $p = 0.3$. Find $P(X > 2)$.

- ☒ a. 0.49
- ☐ b. 0.21
- ☐ c. None of the others
- ☐ d. 0.79
- ☐ e. 0.7



$$P(X > 2) = 1 - P(X = 1) - P(X = 2)$$

$$= 1 - 0.3 - 0.7 \cdot 0.3 = 0.49$$

The correct answer is: 0.49

Question **24**

Correct

Mark 1.00 out of 1.00

Messages arrive at a switchboard in a Poisson manner at an average rate of five per hour. Find the probability for each of the following event: "No message arrives within one hour"

- ☐ a. 0.4046
- ☐ b. None of the other choices is correct
- ☒ c. 0.0067
- ☐ d. 0.4406
- ☐ e. 0.4460

$$\begin{aligned} \lambda &= 5 \rightarrow 1 \text{ hour} \\ P(X=0) &= \frac{e^{-\lambda} \lambda^x}{x!} = \frac{e^{-5} \cdot 5^0}{0!} \end{aligned}$$

The correct answer is: 0.0067

Question **25**

Correct

Mark 1.00 out of 1.00

Given the probability distribution of X.

| | | | | | |
|------|---|-----|----|-----|-----------|
| X | 0 | 1 | 2 | 3 | Otherwise |
| P(X) | a | 0.1 | 2a | 0.3 | 0 |

Find E(X).

- ☒ a. 1.8
- ☐ b. None of the others
- ☐ c. 2
- ☐ d. 1.5
- ☐ e. 2.2



First, determine a such that $a + 0.1 + 2a + 0.3 = 1$.

$\implies a = 0.2$

$E(X) = 0 \cdot 0.2 + 1 \cdot 0.1 + 2 \cdot 0.4 + 3 \cdot 0.3 = 1.8$

The correct answer is: 1.8

