Discrete Mathematics Final Exam

Time: 60m

1. Which the following propositions is FALSE

A. If 1+1=3 or 1+1=2, then 2+2=4 and 2+2=1

B. If 1<0, then 1 = 0

C. 1+1=3 if and only if 1+2=0

D. If 2+1=3, then 4=5-1

2. Let P be the statement "Lucy has an invention" and Q be the statement "Lucy gets a prize". Which of the following English statements can be used for "P → Q"?

    (i) She will get a prize if she has an invention

    (ii) A prize is necessary for having an invention

A. Both (i) and (ii)   B. (ii) only

C. (i) only   D. None of the others

3. Study the following computer code segment:

    x:= 5

    If (1+1=0) OR (2+2=4) then x:=x+1

    If (1+1=2) AND (1+2=3) then x:=2x+1

What are values of x after the codes execute?

A. 13   B. 5   C. 11   D. 6

4. Premise: Everyone, who is over 30, can read newspaper.

Premise: Mr. Bean cannot read newspaper.

Premise: Mrs. Bean is 26 years old.

Conclusion 1: he is not over 30.

Conclusion 2: she cannot read newspaper.

A. The conclusion 1 is logical and the conclusion 2 is not.

B. Both conclusions are illogical.

C. The conclusion 2 is logical and the conclusion 1 is not.

D. Both conclusions are logical.

5. Study the following arguments:

(i) All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.

(ii) If Mai is smart, then she knows French. But she doesn't know French. So, she is not smart.

(i) is ... and (ii) is ...

A. illogical, logical   B. illogical, illogical

C. logical, illogical   D. logical, logical

6. How many rows appear in a truth table for the proposition

$(q \rightarrow \neg p) \vee (\neg p \rightarrow \neg q)$?

A. 2   B. 4   C. 8   D. 16

7. If $f: Z \rightarrow Z$; $f(x) = 2x+3$. Which of the following statements is true?

(i) f is one-to-one

(ii) f is onto

A. (i) only   B. (ii) only

C. Both (i) and (ii)   D. None of the others

8. Let f be floor function and g be ceiling function. Which of the following is true?

A. $g(-4.5) = -4$ B. $g(5) = 6$

C. $f(-6.1) = -6$ D. $f(9.9) = 10$

9. Let U = {0,1,2,3,4,5,6,7,8,9}. Given the subsets A = {1,2,3,4,8}, B = {1,5,6,7,8}. The bit string representing the subset A - B is

A. 00 1110 0000   B. 00 1110 0010

C. 01 1110 0110   D. None of the others

10. Given A={0,Ø,1}. Find the cardinality of P(AxA).

A. 512   B. 9   C. 81   D. 8

11. Let $a_n = a_{n-1} + a_{n-2}$ for all n>2. If $a_1 = -1$ and $a_4 = 11$, find a_2

A. 6   B. 5   C. 11   D. 16

12. The function $f(x) = 5x\log x + x^3 + 2^x + 2\log^8 x + 30$ is

A. $\Theta(2^x)$ B. $\Theta(x^3)$ C. $\Theta(x\log x)$ D. $\Theta(\log^6 x)$

13. Encrypt the message HE by translating the letters into numbers (the character A is translated to 0), applying the encryption function f (p) = (p + 3) mod 26, and then translating the numbers back into letters. Encrypted form:

A. KH   B. QH   C. TK   D. HQ   E. None of the others

14. Consider the algorithm:

Procedure f(a_1, a_2, ...,a_n: integer)

    for i:=1 to n:

        if $a_i > 0$ then print(a_i)

Determine the time complexity of the algorithm.

A. $\Theta(2^n)$ B. $\Theta(n)$ C. $\Theta(n\log n)$ D. $\Theta(\log n)$

15. Which of the following function is big-oh of n^3?

A. 2^n B. $n!$ C. $n^3 \log n$ D. $\log^4 n$

16. Let a = -17 div 5 and b = -17 mod 5. Find a + b

A. -1   B. 0   C. 1   D. 7

17. Which of the following statements is false?

A. $0 \in \emptyset$ B. $\emptyset \subset \{0\}$ C. $\{x\} \subseteq \{x\}$ D. $|\{ \emptyset \}|=1$

18. Suppose a mod 3 = 2 and b mod 6 = 4, find ab mod 3

A. 2   B. 8   C. 0   D. -1   E. None of the others

19. Which memory locations are assigned by the hashing function h(k) = k mod 101 to the records of insurance company customers with the Social Security Number 104578690

A. 58   B. 81   C. 90   D. 10

20. Let S be the set defined recursively by: 5 is in S, and if x is an element of S then x+5 is an element of S. What is S?

A. S={5,10,15,20,…}   B. S={0,5,10,15,20,…}

C. S={0,1,2,3,4,…}   D. S={…,-10,-5,0,5,10 ,…}

21. Suppose a_n is defined recursively by: $a_1 = 4$ and $a_{n+1} = 3a_n$ if n>0. What is a_3?

A. 36   B. 48   C. 108   D. 12   E. None of the others

22. Consider the algorithm:

Procedure T(n: positive integer)

   If n=1 then T(n) := 3

   else T(n) := 3 + T(n-1)

What is the output if input is: n = 4?

A. 12   B. 9   C. 15   D. None of the others

23. How many **functions** are there from the set {10, 11, 111} to the set {11, 01, 100, 111}?

A. 64   B. 81   C. 24   D. 0   E. None of the others

24. How many bit strings of length 10 begin with 100 or 01?

A. 384   B. 352   C. 1024   D. 512   E. None of the others

25. Let B be the set {a, b, c}. How many one-to-one functions are there from B to BxB?

A. 504   B. 6   C. 729   D. None of the others

26. Study a simple graph having the degree sequence {6,5,5,4,4,4,3,3,2,2,2, 1,1}. If the graph has n edges, then n =

A. 21   B. 42   C. 19   D. 20   E. None of the others

27. If every Euler circuit in $K_{2,6}$ is a path of length n, find n

A. 12   B. 8   C. 24   D. The graph does not have no Euler circuit

28. Suppose that f(n) = 2f(n/2) + 3 when n is an even positive integer, and f(1) = 5. Find f(16)

A. 125   B. 61   C. 29   D. 253   E. None of the others

29. Determine whether each of these sequences is graphic
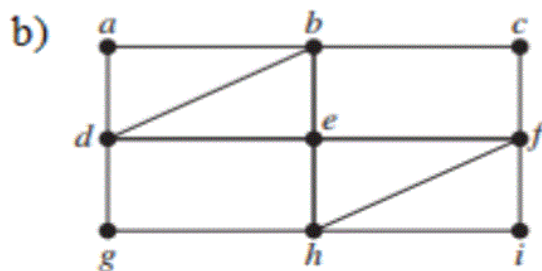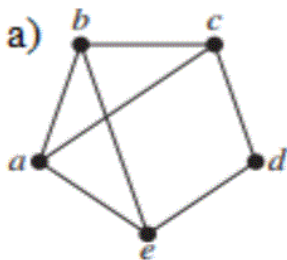 (i)   5, 4, 3, 2, 2, 0
  (ii)   6, 5, 4, 3, 2, 1

A. (i) only   B. (ii) only
C. Both (i) and (ii)   D. None of the others
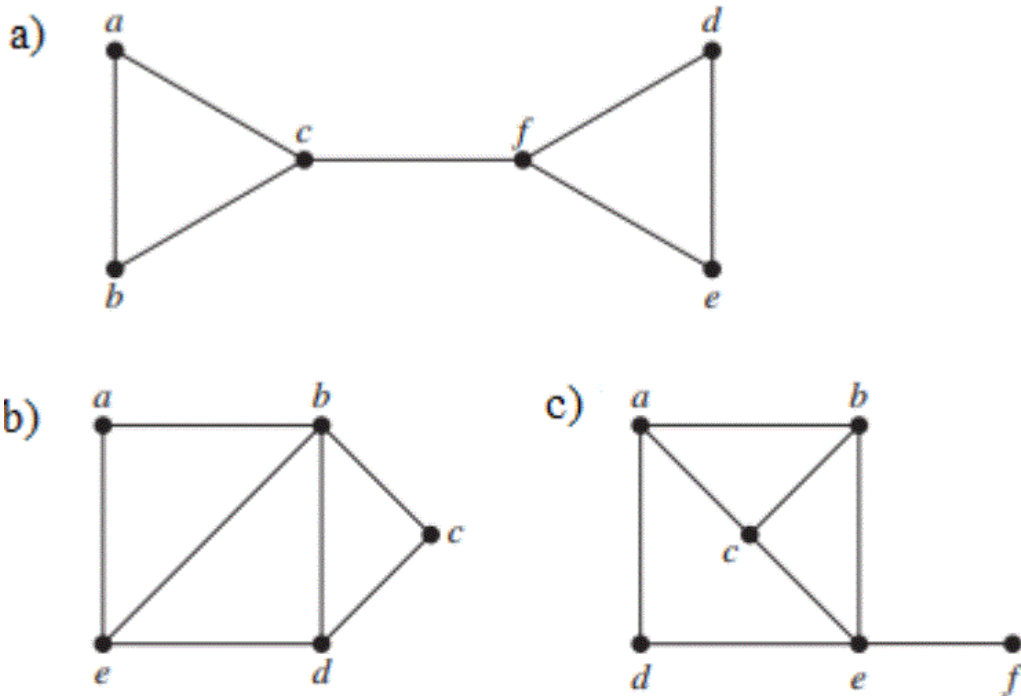
30.

Determine whether the given graph has an Euler circuit.

A. b) only   B. a) only
C. Both a) and b)   D. None of the others

31.

Determine whether the given graph has a Hamilton circuit.

a)

b)

c)

A. b) only   B. a) only

C. b) and c)   D. None of the others

32.

Study the statements:

(i) $K_{2,3}$ has no an Euler circuit, but has an Euler path.

(ii) $K_{2,3}$ has no a Hamilton circuit, but has an Hamilton path.

Which statement is true?

A. Both (i) and (ii)   B. (i) only

C. (ii) only   D. None of the others

33.

Given the adjacency matrix of an undirected graph with vertices {m, n, p}

	m	*n*	*p*
m	2	1	3
n	1	1	1
p	3	1	0

How many paths of length 2 are there from the vertex m to the vertex p in this graph?

A. 7   B. 6   C. 14   D. 10   E. None of the others

34. Use Huffman coding algorithm to encode the word "football". What is the average number of bits required to encode a character?

A. 2.5   B. 2.25   C. 2.45   D. 2

35. Which codes are prefix codes?

    (i) a: 101,   b: 010,   c: 1101,   d: 110

    (ii) a: 110,   b: 101,   c: 1110,   d: 1001

A. (ii) only   B. (i) only

C. Both (i) and (ii)   D. None of the others

    36. Study the following prefix expression: + - * 2 3 5 / * 2 3 3. It will be evaluated to ....

A. 3   B. 2   C. 4   D. 6   E. None of the others

37. Given the coding scheme: a: 00, b: 01, c: 10, d: 110, e: 111. Find the word represented by: 1100001

A. dab   B. abc   C. eab   D. cab

38. Which of the following is false in the case of a spanning tree of a graph G?

A. It can be cyclic   B. It is a subgraph of the G

C. It includes every vertex of the G   D. It is tree that spans G

39. How many edges must be removed from a connected graph with 8 vertices and 12 edges to produce a spanning tree?

A. 5   B. 6   C. 7   D. 4

40. A full 5-ary tree with 45 leaves has _____ internal vertices

A. 11   B. 12   C. 13   D. 14

41. Construct a binary search tree for the numbers: 5, 3, 6, 2, 4, 7. How many comparisons are used to locate the number "2"?

A. 3   B. 2   C. 4   D. 5

42. Find the value of the expression 5 4 + 6 3 / 7 - *
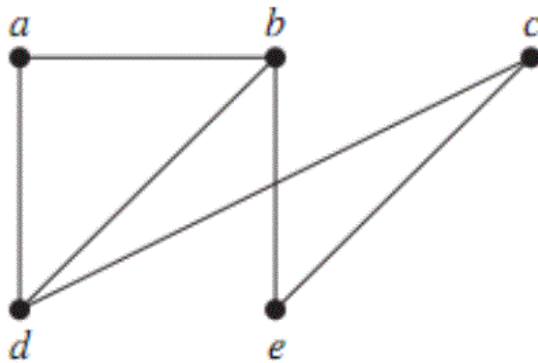
A. 9   B. -9   C. -7.5   D. -45   E. None of the others

43. A full binary tree with 12 internal vertices has _____ edges.

A. 24   B. 13   C. 12   D. 25

44.

How many *cut vertices* does the graph below have?

A. 0   B. 1   C. 2   D. 3

45. Every Hamilton circuit for W_7 has length ____

A. 7   B. 8   C. 16   D. The graph has no Hamilton circuit

46. How many 1-entries in the adjacency matrix of the graph $K_{2,3}$?

A. 5   B. 6   C. 12   D. 13

47. Which of the following integers are congruent to -15 modulo 9?

A. 21   B. 4   D. -21   D. -3   E. None of the others

48. Find the base 7 expansion of 186

A. 354   B. 331   C. 413   D. 345

49. If a, b are positive integers such that gcd(a, b) = 10 and ab = 700, find lcm(a, b)

A. 70   B. 600   C. 800   D. None of the others

50. What is the negation of the proposition "If it is not raining then the soccer game will be held"

A. It is not raining and the soccer game is held

B. If it is not raining then the soccer game will not be held

C. If it is raining then the soccer game will not be held

D. None of the others