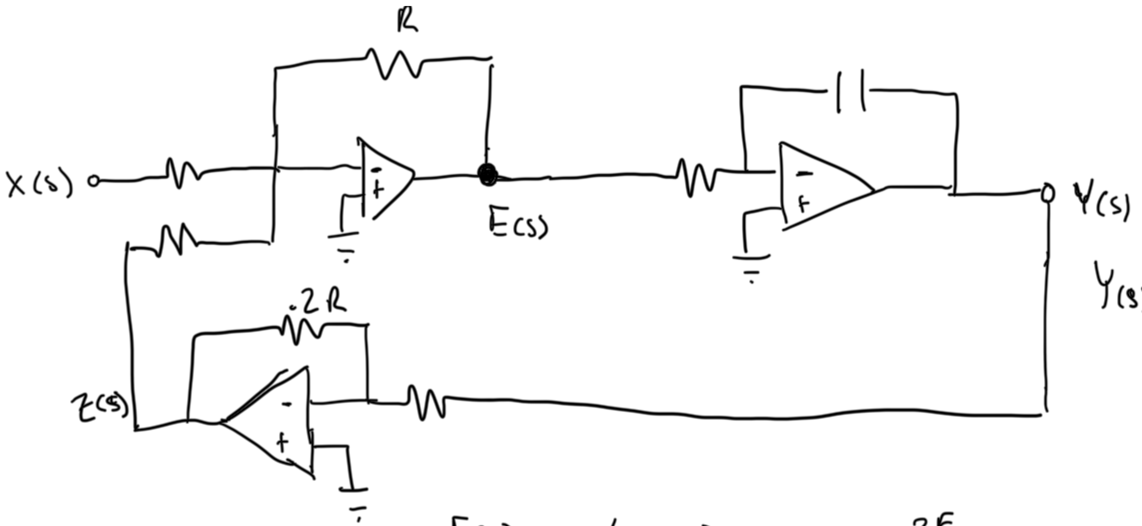


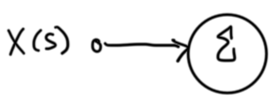
$$Y(s) = \frac{-E}{RCs}$$

$$E(s) = -(X+Z) = -\left(X + \frac{.2E}{RCs}\right) \rightarrow E = \frac{X}{1 + \frac{.2}{RCs}}$$

$$Z = -.2Y = \frac{.2E}{RCs}$$

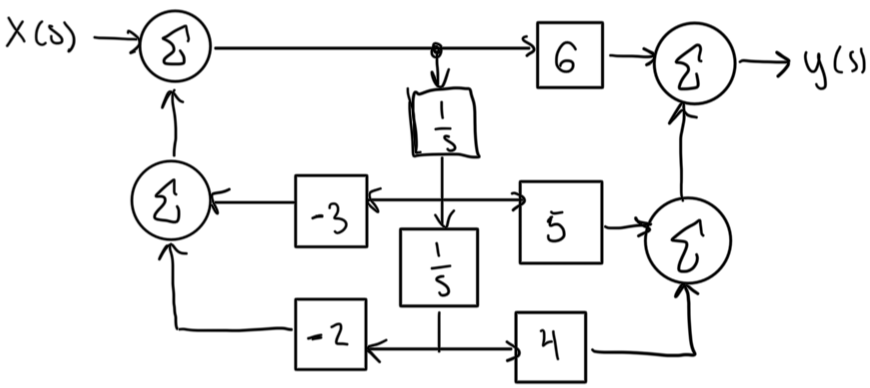
$$Y = \frac{-E}{RCs} \qquad Y = \frac{X}{RCs\left(1 + .2/RCs\right)}$$

or



HW 7:

A.) $$H(\omega) = \frac{(4-6\omega^2) + j5\omega}{(2-\omega^2) + j3\omega} = \frac{Y(s)}{X(s)}$$



B.) $h(t) = e^{t}u(t) - e^{-5t}u(t)$ $G(s) = K$

1.) closed loop poles @ $\{-1, -3\}$

$H(s) = \frac{1}{s-1} - \frac{1}{s+5}$ with proportional feedback

$Q(s) = \frac{H(s)}{1+KH(s)}$

$Q(s) = \frac{1}{6K + s^2 + 4s - 5}$ $(s+1)(s+3)$

$\hookrightarrow 6K + \cancel{s^2} + \cancel{4s} - 5 = \cancel{s^2} + \cancel{4s} + 3$

$\boxed{K = \frac{4}{3}}$

2.) Real repeated roots

$s^2 + 4s - 5 + 6K = 0 \Rightarrow b^2 - 4ac = 0$

$\hookrightarrow 16 - 4(1)(-5+6K) = 0$

$16 + 20 - 24K = 0$

$K = \frac{36}{24} \Rightarrow \boxed{K = \frac{3}{2}}$

C.) An unstable LTI system

$h(t) = (e^{3t} - e^{2t})u(t)$

1.) Show proportional feedback alone cannot stabilize the system

$Q(s) = \frac{1}{K + s^2 - 5s + 6}$ if $b^2 - 4ac < 0$

if $b^2 - 4ac > 0$

$25 - 4(1)(K+6) > 0$

$K > \frac{1}{4} \Rightarrow$ complex $\Rightarrow$ unstable

$25 - 4(1)(K+6)$

$25 - 4K + 24 \Rightarrow 1 - 4K$

therefore $K < \frac{1}{4}$

so $\frac{5 \pm \text{something positive}}{2}$ $\Rightarrow$ unstable

2.) $G(s) = Ks$ show stability when $p \geq -\sqrt{6}$

$$Q(s) = \frac{1}{s^2 + (K-5)s + 6}$$

$$-K+5 \pm \sqrt{(K-5)^2 - 24}$$

$$(K-5)^2 - 24 = 0$$

$$K - 5 = 2\sqrt{6}$$

$$K = 2\sqrt{6} + 5$$

if $K > 2\sqrt{6} + 5$ then $p \geq -\sqrt{6}$

3.) show $G(s) = K_1 + K_2 s$ both poles @ -10

$$(s+10)^2 = s^2 + 20s + 100$$

$$Q(s) = \frac{1}{s^2 + (K_2 - 5)s + (6 + K_1)} = s^2 + 20s + 100$$

$$\boxed{\begin{aligned} K_1 &= 94 \\ K_2 &= 25 \end{aligned}}$$

D.) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y(t) = x(t)$ $\quad G(s) = K_1 + K_2 s$

$s^2 - 2s + 5$

1.) make stable and critically damped

$$Q(s) = \frac{1}{s^2 + (K_2 - 2)s + K_1 + 5} \qquad \lim_{s \to 0} Q(s) = \frac{1}{K_1 + 5} = 0.01 \qquad \underline{K_1 = 95}$$

$b^2 - 4ac = 0$ $\quad (K_2 - 2)^2 - 4(1)(100) = 0 \Rightarrow K_2 - 2 = 20 \Rightarrow \underline{K_2 = 22}$