Sinusodial Response Example

$$H(j\omega) = \frac{y(t)}{x(t)}$$ $\Rightarrow$ only true when $x(t)$ is sinusodial ie depends on 1 ω

$$\overbrace{\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 7y}^{} = 5\frac{dx}{dt}$$

$$x = \cos(\omega t) = e^{j\omega t}$$

$$y = ? \Rightarrow y = H(j\omega)\,x(t)$$

$$= \underbrace{H(j\omega)}_{H} e^{j\omega t}$$

$$-\omega^2 H\cancel{e^{j\omega t}} + 2j\omega H\cancel{e^{j\omega t}} + 7H\cancel{e^{j\omega t}} = 5\cancel{e^{j\omega t}}$$

$$\hookrightarrow H(-\omega^2 + 2j\omega + 7) = 5j\omega$$

$$H(j\omega) = \frac{5j\omega}{-\omega^2 + 2j\omega + 7} = Ae^{j\phi} \Rightarrow y = e^{j\phi}e^{j\omega}$$

$$\Rightarrow y = Ae^{j(\omega t + \phi)}$$

$$H(j\omega) = \frac{5\omega e^{j\pi/2}}{\sqrt{(7-\omega^2) + (2\omega)^2}\, e^{j\tan^{-1}\left(\frac{2\omega}{7-\omega^2}\right)}}$$

$A \rightarrow$

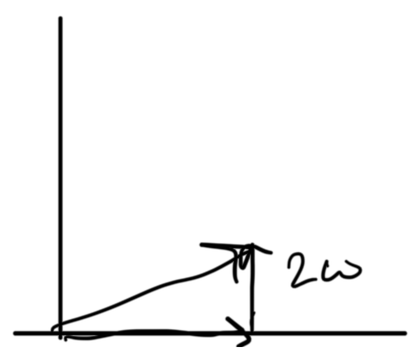
ϕ

$$\frac{\pi}{2} - \tan^{-1}\left(\frac{2\omega}{7-\omega^2}\right) =$$

$$\Rightarrow y(t) = \mathrm{Re}\{Ae^{j(\omega t + \phi)}\}$$

$$= A\cos(\omega t + \phi)$$

$$y(t) = A\cos(\omega t + \phi)$$



at what point does $\phi = 0$

$7-\omega^2$

$$\frac{\pi}{2} - \tan^{-1}(\ ) = 0$$

$$\tan^{-1}(\ ) = \frac{\pi}{2} \Rightarrow \tan(\ ) = \frac{\pi}{2} \quad \frac{2\omega}{7-\omega^2} = \frac{1}{0}$$

$$\hookrightarrow 7-\omega^2 = 0$$

$$\omega = \pm\sqrt{7}$$

Ex:

$$\frac{d^2y(t)}{dt^2} + \beta \frac{dy(t)}{dt} + 25\,y(t) = 21\sqrt{3}\,x(t)$$

$$x(t) = \cos(\omega t) = e^{j\omega t} \qquad \text{find } H(j\omega) = \frac{y(t)}{x(t)}$$

$$y = e^{j\omega t} H(j\omega)$$

$$= e^{j\omega t} H$$

$$-\omega^2 \cancel{e^{j\omega t}} H + \beta j\omega \cancel{e^{j\omega t}} H + 25 \cancel{e^{j\omega t}} H = 21\sqrt{3}\,\cancel{e^{j\omega t}}$$

$$\Rightarrow H(-\omega^2 + \beta j\omega + 25) = 21\sqrt{3}\,e^{j\omega t}$$

$$H(j\omega) = \frac{21\sqrt{3}}{-\omega^2 + \beta j\omega + 25}$$

$$y \propto A e^{\lambda_1 t} + B e^{\lambda_2 t}$$

underdamped

$\lambda_{1 \text{ or } 2} \Rightarrow +$ unstable

$\lambda \Rightarrow -$ stable

lor 2

$\lambda_1 = \lambda_2 \Rightarrow$ Real

critically damped

$$\frac{dy}{dt} + a\,y(t) = b\,\frac{dx}{dt} + c\,x(t)$$

$$x(t) = 9 + 15\cos(12t) \Rightarrow 9 + 15e^{j12t}$$
$$y(t) = 5 + 13\cos(12t + 0.2487) \Rightarrow 5 + 13e^{j(12t+0.2487)}$$

$H(j\omega)$

$a5 = c9$ b/c $x(t) = 9$ $y(t) = 5$

$\hookrightarrow 0 + a5 + b0 + c9$

$$H(0j) = \frac{5}{9}$$

$$x(t) = 15\cos(12t) \Rightarrow 15e^{j12}$$
$$y(t) = 13\cos(12t + .2487) \Rightarrow 13e^{j(12t+.2487)}$$

$$H(j12) = \frac{13}{15}e^{-j.2487}$$

$$x = e^{j\omega t}$$
$$y = H(j\omega)e^{j\omega t} \quad \ldots \Rightarrow \quad H(j\omega) = \frac{bj\omega + c}{j\omega + a}$$

$$H(j12) = \frac{13}{13}e^{-j.2487} = \frac{12bj + c}{12j + a}$$

solvable b/c imag = imag, real = real

$$H(j0) = \frac{5}{9} = \frac{c}{a}$$

then 3 eqn.