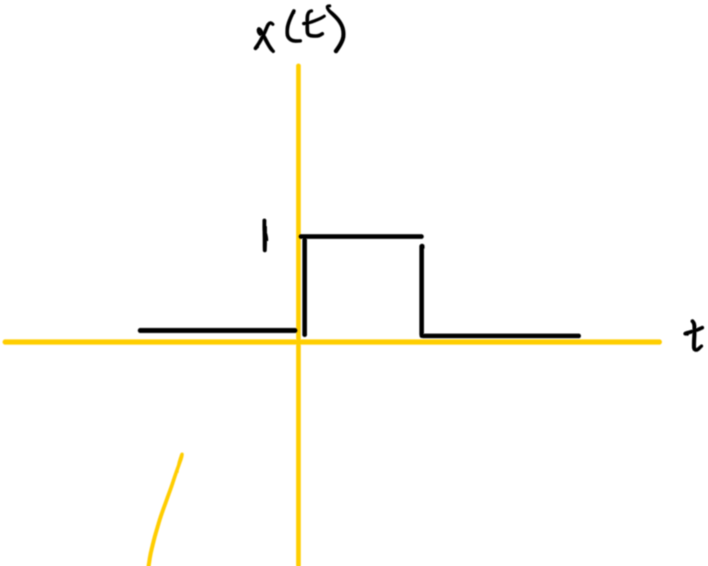
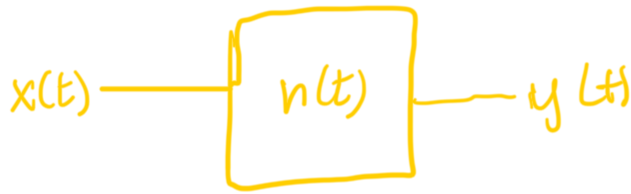
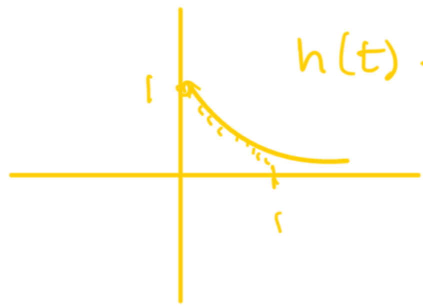
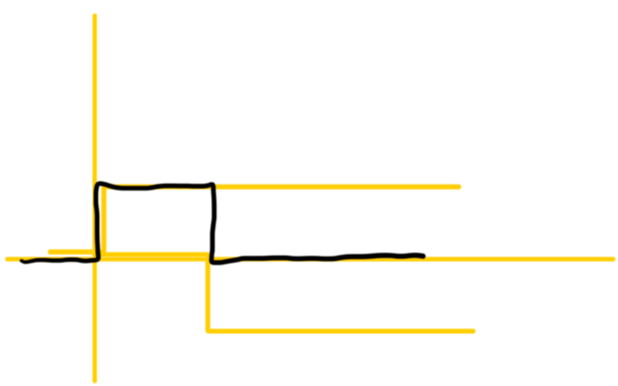
8/29 Convolution:

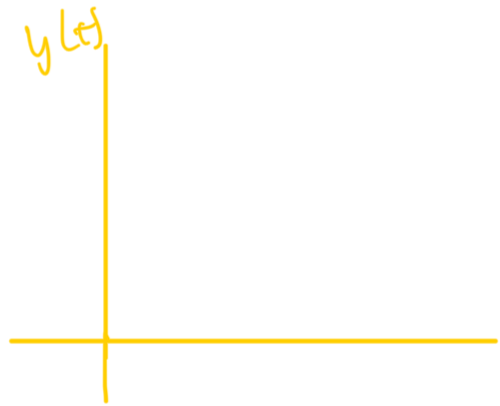




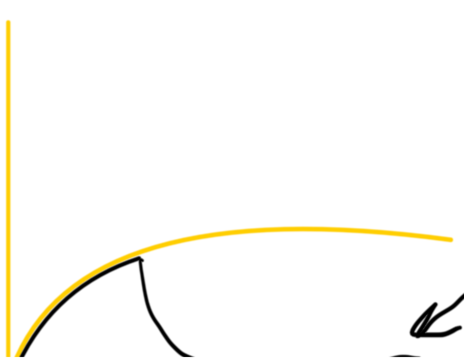


$h(t) = e^{-t}$

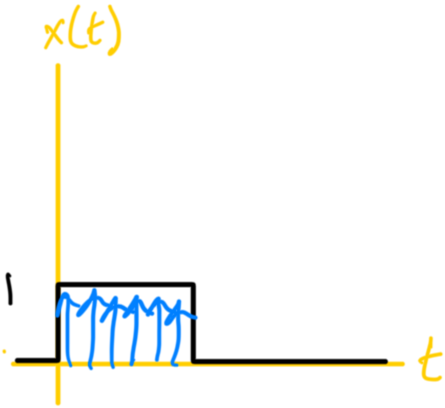




$$s(t) = \int_0^t h(t)\,dt = (1-e^{-t})$$



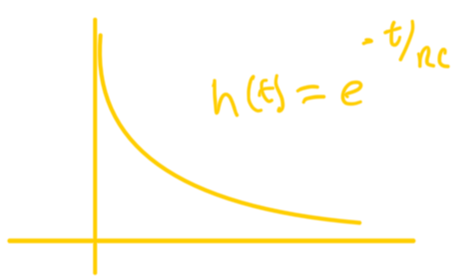
convolution : break up into an infinite # of delta functions



$$\int h(\tau)\, x(t-\tau)\, d\tau$$

Ex: $h(t)$

Impulse response

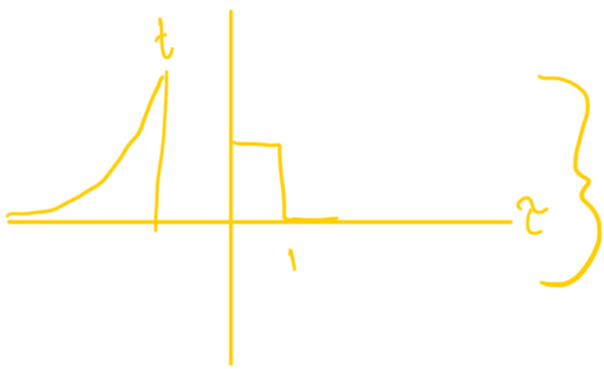


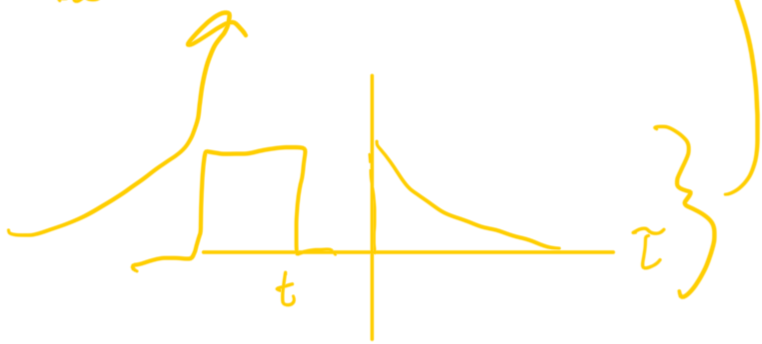
$h(t) = e^{-t/RC}\, u(t) \quad x(t) = u(t) - u(t-1)$

convolution

$$y(t) = h(t) * x(t) = \int_{-\infty}^{t} h(\tau)\, x(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{t} x(\tau)\, h(t-\tau)\, d\tau$$





$t<0 \Rightarrow 0$

$0<t<1 \Rightarrow RC\left[1-e^{-t/RC}\right]$

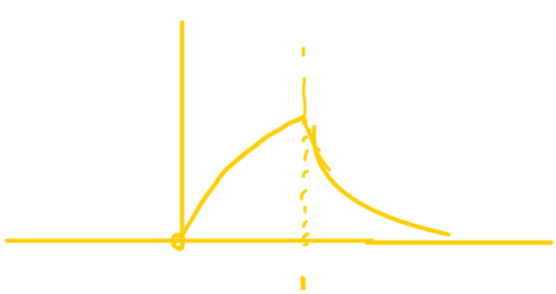
$t>1 \Rightarrow RC\, e^{-t/RC}\left[e^{1/RC}-1\right]$

@ $t=1 \Rightarrow RC\left[1-e^{-1/RC}\right]$

$$\int_0^t (1)\, e^{-\tau/RC}\, d\tau$$

$$= -RC\, e^{-\tau/RC}\Big|_0^t$$

$$RC\left[1-e^{-t/RC}\right]$$



$$\int_{t-1}^{t} (1)\, e^{-\tau/RC}\, d\tau$$

$$\hookrightarrow RC\left[e^{\frac{-(t-1)}{RC}} - e^{-t/RC}\right]$$

Ex: $h(t) \Rightarrow$

$x(t) \Rightarrow$

$y(t) = \max$