Nate Viewegh - HW 3

A.) $h(\omega) = \frac{1}{3+j\omega}$

1.) $x(t) = 3$

$y(t) = x(t) * h(\omega) \Rightarrow \boxed{y(t) = 1}$

2.)
$x(t) = 3\sqrt{2}\cos(3t)$

$h(\omega) = \frac{1}{3+j3} \Rightarrow \frac{1}{\sqrt{18}} \Rightarrow \frac{\sqrt{2}}{6}$

$\Rightarrow \theta = \arctan(1) = \frac{\pi}{4}$

$h(\omega) = \frac{\sqrt{2}}{6} e^{-j\pi/4}$

$y(t) = \frac{\sqrt{2}}{6} e^{-j\frac{\pi}{4}} \cdot 3\sqrt{2}\, e^{j3t} \Rightarrow \boxed{y(t) = \cos(3t - \frac{\pi}{4})}$

3.) $x(t) = 5\cos(4t) \Rightarrow 5e^{j4t}$

$H(j\omega) = \frac{1}{3+j4} \Rightarrow \theta = \arctan(4/3)$
$= 0.927$

$H = \frac{1}{5} e^{-j.927}$ $\boxed{y(t) = \cos(4t - 0.927)}$

4.) $x(t) = \delta(t)$

$y = \mathcal{L}\{1 \cdot \frac{1}{3+s}\}$

$\mathcal{L}\{\delta(t)\} = 1$

$\mathcal{L}\{H\} = \frac{1}{3+s}$

$\boxed{y(t) = e^{-3t}}$

5.) $x(t) = u(t)$

$\mathcal{L}\{x(t)\} = \frac{1}{s}$

$y(s) = \frac{1}{s} \cdot \frac{1}{3+s}$

$\frac{A}{s} + \frac{B}{s+3} = \frac{1}{s(3+s)}$

$A + B = 0$ $\quad A = \frac{1}{3}$

$3A = 1$ $\quad B = -\frac{1}{3}$

$\mathcal{L}^{-1}\left\{ \frac{1}{3s} - \frac{1}{3(s+3)} \right\}$

$\boxed{y(t) = \frac{1}{3} - \frac{1}{3}e^{-3t}}$

6.) $x(t) = 1$

$H = \frac{1}{3} \Rightarrow \boxed{y(t) = \frac{1}{3}}$

$\boxed{\text{IN Radians}}$

B.) $h(t) = e^{-12t}u(t) \Rightarrow \int h(t) \cdot e^{-j\omega} \rightarrow \frac{1}{12+j\omega}$

$x(t) = 12 + 26\cos(5t) + 45\cos(9t) + 80\cos(16t)$

$x_1(t) = 12 \quad H = \frac{1}{12}$

$H(\omega) = \frac{1}{\sqrt{12^2+\omega^2}\, e^{j\arctan(\frac{\omega}{12})}}$

$$y_1(t) = H \cdot x_1 \Rightarrow y_1(t) = 1$$

$$x_2(t) = 26\cos(5t) \Rightarrow 26e^{j5t}$$

$$H = \frac{1}{13e^{j.394}} \cdot 26e^{j5t}$$

$$\Bigg\} \; y_2(t) = 2\cos(5t - .394)$$

$$x_3(t) = 45\cos(9t) \Rightarrow 45e^{j9t}$$

$$H = \frac{1}{15 \quad e^{j11.31}}$$

$$\Bigg\} \; y_3(t) = 3\cos(9t - 0.644)$$

$$x_4(t) = 80\cos(16t) \Rightarrow 80e^{j16t}$$

$$H = \frac{1}{20e^{j.927}}$$

$$\Bigg\} \; y_4(t) = 4\cos(16t - .927)$$

$$\boxed{y(t) = 1 + 2\cos(5t - .394) + 3\cos(9t - 0.644) + 4\cos(16t - .927)}$$

c.) $i(t) = \cos(500t) + \cos(900t)$

$v(t) = 13\cos(500t + \theta_1) + 15\cos(900t + \theta_2)$

$Z_L = j\omega L$

$X = R + j\omega L$

$$13e^{j(500t + \theta_1)} = e^{j500t} \cdot \sqrt{R^2 + \omega^2 L^2}\, e^{j\arctan(\frac{L}{R}) \cdot 500}$$

$$15e^{j(900t + \theta_2)} = e^{j900t} \cdot \sqrt{R^2 + \omega^2 L^2}\, e^{j\tan^{-1}(\frac{\omega L}{R}) \cdot 500}$$

$$13e^{j500t + \theta_1 j - j900t} = \sqrt{R^2 + \omega L^2}\, e^{j\tan^{-1}(\frac{\omega L}{R})500}$$

$$\sqrt{R^2 + 500^2 L^2} = 13$$

$$\sqrt{R^2 + 900^2 L^2} = 15$$

$$\Rightarrow \left.\begin{array}{l} R^2 + 500^2 L^2 = 169 \\ R^2 + 900^2 L^2 = 225 \end{array}\right\} \; 56e4\, L^2 = 56$$

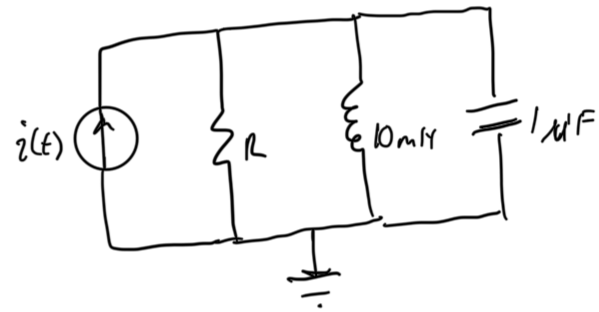
$$\boxed{\begin{array}{l} L = 0.01 \text{ H} \\ R = 12\,\Omega \end{array}}$$

$$i \;\; = \;\; i + i_1 + i$$

D.)

$L = 10\ mH$

$C = 1\ \mu F$



$i_s(t) = i_R + i_L + i_C$

$i(t) = \frac{1}{R}V(t) + C\frac{dV}{dt} + i_L(t)$

$\hookrightarrow i_s(t) = \frac{d^2 i}{dt} + \frac{1}{RC}\frac{di}{dt} + \frac{1}{LC}i(t)$

$\hookrightarrow s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \Rightarrow s^2 + Bs + C = 0$

$\frac{B \pm \sqrt{B^2 - 4(1)(C)}}{2(1)} \} = 0$

$\frac{1}{R^2C^2} - \frac{4}{LC} = 0 \Rightarrow \frac{1}{R^2C^2} = 4e8$

$R = 50\ \Omega$