① $(1+5z^{-1}-3z^{-2}-5z^{-4}-2z^{-5})(1z^{-1}+2z^{-4}+5z^{-5})$

$= \cancel{z^{-1}} + \cancel{2z^{-4}} + \cancel{5z^{-5}} + \cancel{5z^{-2}} + \cancel{10z^{-5}} + \cancel{25z^{-6}} \cancel{-3z^{-3}} \cancel{-6z^{-6}} - \cancel{15z^{-7}}$
$\cancel{-5z^{-5}} - \cancel{10z^{-8}} - 25z^{-9} \cancel{-2z^{-6}} - 4z^{-9} - 10z^{-10}$

$= 1z^{-1} + 5z^{-2} - 3z^{-3} + 2z^{-4} + 10z^{-5} + 17z^{-6} - 15z^{-7} - 10z^{-8} - 29z^{-9} - 10z^{-10}$

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② $\dfrac{(1+3z^{-1}+6z^{-2}+9z^{-3}+17z^{-4}+12z^{-5})}{(1+2z^{-2}+3z^{-3})} \left(\dfrac{z^5}{z^5}\right) \Rightarrow \dfrac{(z^5+3z^4+6z^3+9z^2+17z+12)}{z^5+2z^3+3z^2}$

$$
\begin{array}{r|l}
 & 1 + 3z^{-1} + 4z^{-2} \\
z^5+2z^3+3z^2 & z^5 + 3z^4 + 6z^3 + 9z^2 + 17z + 12 + 0z^{-1} + 0z^{-2} \\
 & -z^5 - 0z^4 - 2z^3 - 3z^2 \\
 & 0 + 3z^4 + 4z^3 + 6z^2 + 17z + 12 \\
 & -3z^4 + 0z^3 - 6z^2 - 9z - 0 \\
 & 4z^3 + 0z^2 + 8z + 12 \\
 & -4z^3 \quad -8z - 12 \\
 & 0
\end{array}
$$

$\hookrightarrow \boxed{1+3z^{-1}+4z^{-2}}$

③

$$\frac{1}{2\pi}\int_{\omega_1}^{\omega_2} e^{j\omega n}\,d\omega + \frac{1}{2\pi}\int_{-\omega_2}^{-\omega_1} e^{j\omega n}\,d\omega$$

$$\hookrightarrow \frac{1}{2\pi}\left(\frac{1}{jn} e^{j\omega n}\Big|_{\omega_1}^{\omega_2} + \frac{1}{jn} e^{j\omega n}\Big|_{-\omega_2}^{-\omega_1}\right)$$

$$\Rightarrow \frac{1}{\pi n 2j}\left(e^{jn\omega_2} - e^{jn\omega_1} + e^{-jn\omega_1} - e^{-jn\omega_2}\right)$$

$$\Rightarrow \frac{1}{\pi n}\left[\left(\frac{e^{jn\omega_2} - e^{-jn\omega_2}}{2j}\right) - \left(\frac{e^{jn\omega_1} - e^{-jn\omega_1}}{2j}\right)\right]$$

$$\Rightarrow \frac{1}{n\pi}\left[\sin(n\omega_2) - \sin(n\omega_1)\right] \Rightarrow \boxed{\frac{\sin(n\omega_2) - \sin(n\omega_1)}{n\pi}}$$

④

$$\begin{bmatrix} 1 & 2 & 2 \\ 1 & 2 & -1 \\ -2 & 2 & 2 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 & 1 \\ 2 & 3 & 1 \\ -1 & -3 & 1 \end{bmatrix} = \begin{bmatrix} -1+4-2 & 2+6-6 & 1+2+2 \\ -1+4+1 & 2+6+3 & 1+2-1 \\ 2+4-2 & -4+6-6 & -2+2+2 \end{bmatrix}$$

$$= \boxed{\begin{bmatrix} 1 & 2 & 5 \\ 4 & 11 & 2 \\ 4 & -4 & 2 \end{bmatrix}}$$

$1+e^{j\pi}=0$

(5) $\left(e^{j(-7\pi/2)}+e^{j\pi}+e^{-j\pi/2}\right)^{*} \Rightarrow e^{j(7\pi/2)}+e^{-j\pi}+e^{j\pi/2}$

$\Rightarrow \sin(7\pi/2)+\sin(-\pi)+\sin(\pi/2)$

$\Rightarrow -1+0+1=0$

$\cos(\cancel{7\pi/2})^{0}+\cos(-\pi)+\cos(\cancel{\pi/2})^{0} \Rightarrow -1$

$\Rightarrow \boxed{-1+0j \quad \text{in } a+bi \text{ form}}$