P0.24:

$$E(t) = E_0 e^{-\frac{1}{2}\left(\frac{t}{T}\right)^2} \cos(\omega_0 t)$$

$$E(\omega) = \frac{1}{\sqrt{2\pi}} E_0 \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{t}{T}\right)^2} \cos(\omega_0 t)\, e^{i\omega t}\, dt$$

$$\cos(\omega_0 t) = \frac{e^{i\omega_0 t} + e^{-i\omega_0 t}}{2}$$

$$\frac{1}{2\sqrt{2\pi}} E_0 \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{t}{T}\right)^2} \left(e^{i\omega_0 t} + e^{-i\omega_0 t}\right) e^{i\omega t}\, dt$$

$$\left(e^{i\omega_0 t} + e^{-i\omega_0 t}\right) e^{i\omega t} = e^{it(\omega+\omega_0)} + e^{it(\omega-\omega_0)}$$

$$\Rightarrow e^{-\frac{1}{2}\left(\frac{t}{T}\right)^2}\left(e^{i(\omega+\omega_0)t} + e^{i(\omega-\omega_0)t}\right) = e^{-\frac{1}{2}(t/T)^2 + i(\omega+\omega_0)t} + e^{-\frac{1}{2}(t/T)^2 + i(\omega-\omega_0)t}$$

$$\Rightarrow \frac{1}{2\sqrt{2\pi}} E_0 \left[\int_{-\infty}^{\infty} e^{-\frac{1}{2}(t/T)^2 + i(\omega+\omega_0)t}\, dt + \int_{-\infty}^{\infty} e^{-\frac{1}{2}(t/T)^2 + i(\omega-\omega_0)t}\, dt\right]$$

$$\int_{-\infty}^{\infty} e^{-at^2+bt}\, dt \qquad -at^2+bt \Rightarrow -a\left(t^2 - \frac{b}{a}t\right) \quad \left(\frac{1}{2}\cdot\frac{-b}{a}\right)^2 \Rightarrow -a\left(t^2 - \frac{b}{a}t + \frac{b^2}{4a^2} - \frac{b^2}{4a^2}\right)$$

$$\hookrightarrow -a\left(\left(t-\frac{b}{2a}\right)^2 - \frac{b^2}{4a^2}\right) \Rightarrow -a\left(t-\frac{b}{2a}\right)^2 + \frac{b^2}{4a}$$

$$a = \frac{1}{2T^2}$$

$$b = i(\omega+\omega_0)$$

$$\int_{-\infty}^{\infty} e^{-\frac{1}{2T^2}\left(t - \frac{i(\omega+\omega_0)}{4T^2}\right)^2 + \frac{-(\omega+\omega_0)^2 T^2}{2}}\, dt \qquad u$$

$$\hookrightarrow e^{\frac{-(\omega+\omega_0)^2 T^2}{2}} \int_{-\infty}^{\infty} e^{-\frac{1}{2T^2}\left(t - \frac{i(\omega+\omega_0)}{4T^2}\right)^2}\, dt \qquad u = \left(t - \frac{i(\omega+\omega_0)}{4T^2}\right) \qquad du = dt$$

$$\hookrightarrow \int_{-\infty}^{\infty} e^{-\frac{1}{2T^2}u^2}\, du$$

$$\int_{-\infty}^{\infty} e^{au^2}\, du = \sqrt{\frac{\pi}{a}}$$

$$\Rightarrow \sqrt{\frac{2\pi}{T^2}} = T\sqrt{2\pi}$$

$$\hookrightarrow \frac{1}{2\cancel{\sqrt{2\pi}}} E_0 \left[e^{\frac{-(\omega+\omega_0)^2 T^2}{2}} \cdot T\cancel{\sqrt{2\pi}} + e^{\frac{-(\omega-\omega_0)^2 T^2}{2}} \cdot T\cancel{\sqrt{2\pi}}\right]$$

$$\boxed{E(\omega) = \frac{T}{2} E_0 \left(e^{\frac{-(\omega+\omega_0)^2 T^2}{2}} + e^{\frac{-(\omega-\omega_0)^2 T^2}{2}}\right)}$$

PO.15:

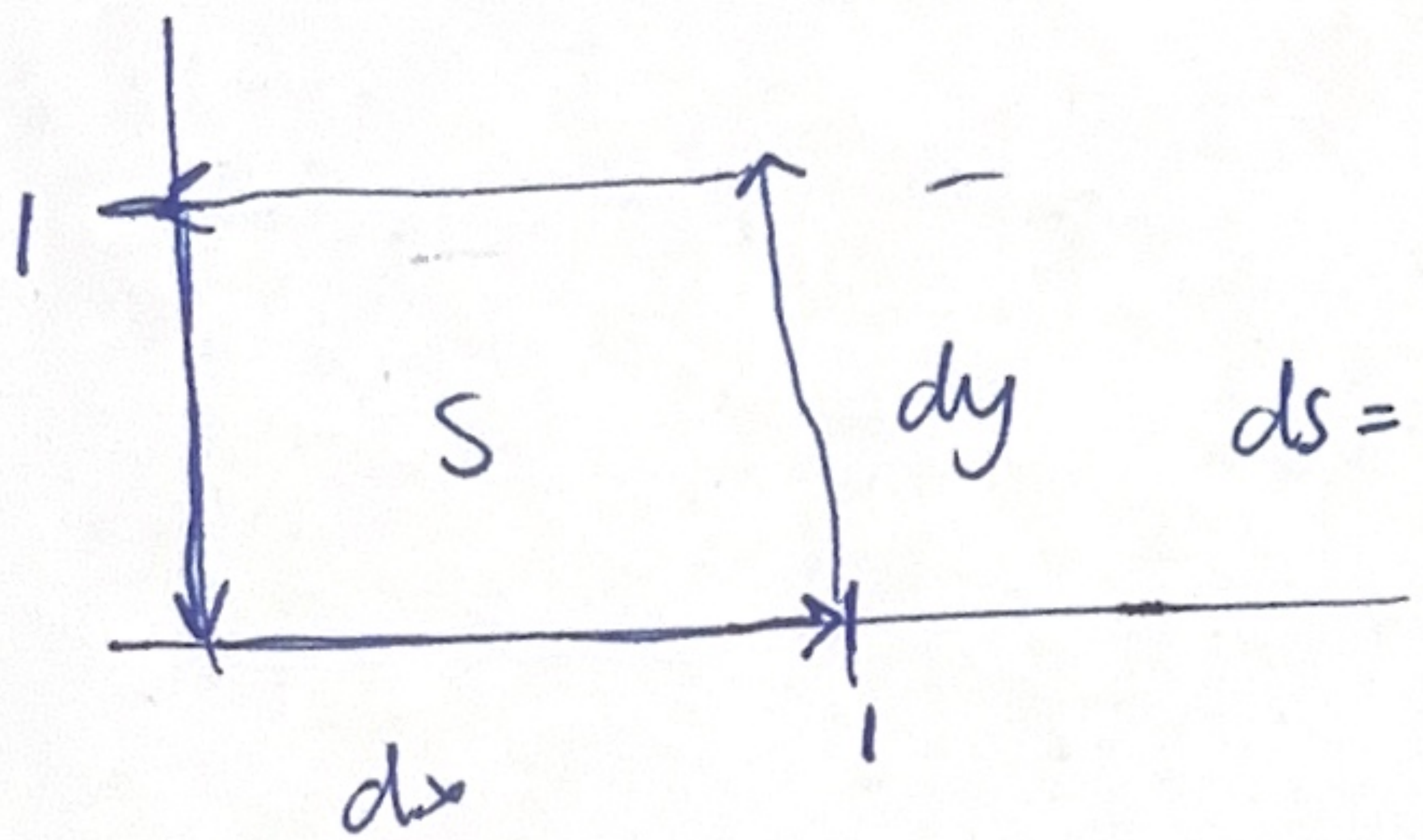
$$\frac{a-ib = \sqrt{a^2+b^2}\, e^{-i\tan^{-1}\left(\frac{b}{a}\right)}}{a+ib = \sqrt{a^2+b^2}\, e^{i\tan^{-1}\left(\frac{b}{a}\right)}} = e^{-2i\tan^{-1}\left(\frac{b}{a}\right)}$$

$$\hookrightarrow \quad \frac{a-ib}{a+ib} = e^{-2i\tan^{-1}\left(\frac{b}{a}\right)}$$

PO.11

$$\oint_C F\cdot dl = \int_S (\nabla\times F)\cdot ds \qquad F(x,y,z) = y^2\hat{x} + xy\,\hat{y} + x^2 z\,\hat{z}$$

$$\nabla\times F = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & xy & x^2 z \end{vmatrix} = -2xz\,\hat{y} - y\,\hat{z}$$



$ds = dx\,dy\,\hat{z}$

$$\int_S (\bar{\nabla}\times\bar{F})\cdot d\bar{s} = \int_S (-2xz\,\hat{y} - y\,\hat{z})\cdot dx\,dy\,\hat{z} = \int_0^1\int_0^1 -y\,dx\,dy = -\frac{y^2}{2}\Big|_0^1 = \boxed{-\frac{1}{2}}$$

$$\oint_C F\cdot dl \qquad \hat{z}=0 \Rightarrow F = y^2\hat{x} + xy\,\hat{y}$$

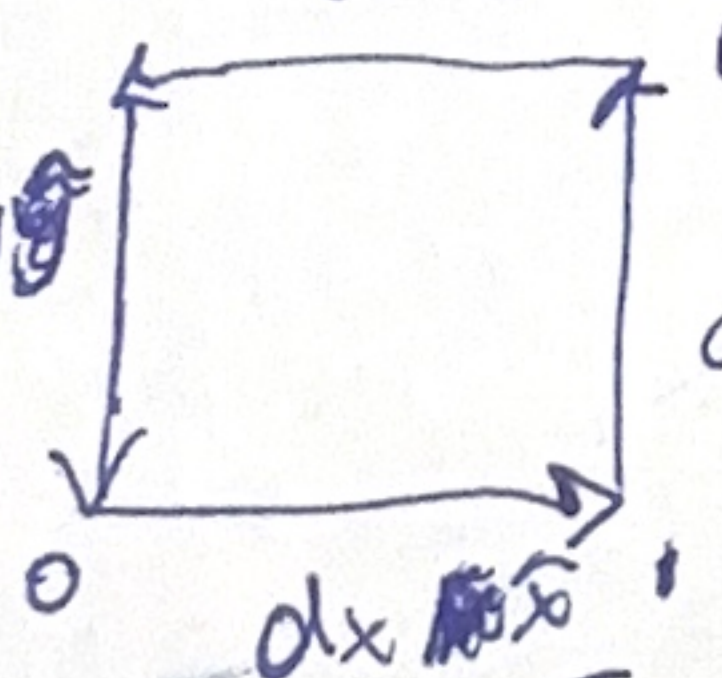
$\oint F\cdot dl = \int_S (\nabla\times F)\cdot dS$ ✓

$dx\,\hat{x} \rightarrow \oint ((1^2)\hat{x} + x(1)\hat{y})\cdot dx\,\hat{x} = \int_1^0 1\,dx = -1$

$\oint (y^2\hat{x} + (0)y\,\hat{y})\cdot dy\,\hat{y}$

$dy\,\hat{y}$

$= \int_1^0 0\,dy = 0$



$dy\,\hat{y}$

$\oint_0^1 (y^2\hat{x} + (1)y\,\hat{y})\cdot dy\,\hat{y} = \int_0^1 y\,dy = \frac{1}{2}$

$\int 0^2\hat{x} + 0\,\hat{y}\,dx = 0$

$$\oint F\cdot dl = 0 + \frac{1}{2} - 1 + 0 = \boxed{-\frac{1}{2}}$$