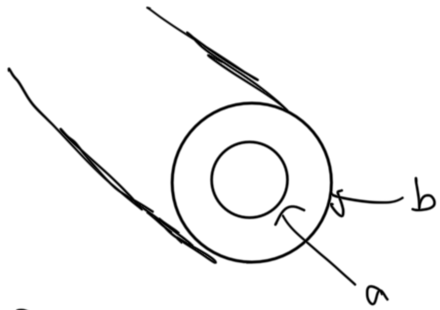
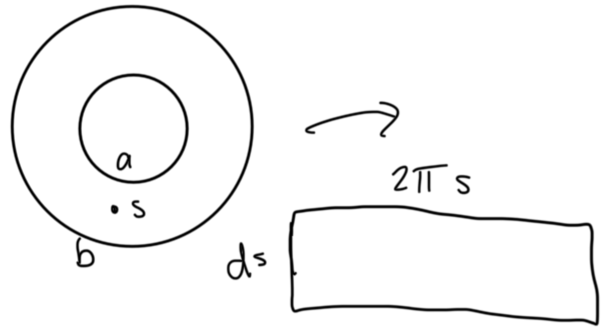
HW 1

P 1.1.)





$$\nabla \cdot E = \frac{\rho}{\varepsilon_0} \Rightarrow \int_{vol.} \bar{E} \cdot \hat{n}\, dA = \frac{Q_{enc}}{\varepsilon_0} \qquad \rho = \frac{k}{s^2} \qquad dA = 2\pi s\, ds$$

$s < a \Rightarrow E = 0 \Rightarrow Q_{enc} = 0$

$$Q_{enc} = \int_a^s \rho \cdot dA$$

$$\hookrightarrow 2\pi k \int_a^s \frac{1}{s}\, ds$$

$$= 2\pi k \left(\ln|s| \Big|_a^s \right)$$

$$= 2\pi k \ln\left(\frac{s}{a}\right)$$

$a < s < b \Rightarrow \displaystyle\int_a^b \bar{E} \cdot \hat{n}\, dA = \frac{Q_{enc}}{\varepsilon_0}$

$E \displaystyle\int dA = \frac{Q_{enc}}{\varepsilon_0} \qquad dA = 2\pi s\, ds$

$$E = \frac{Q_{enc}}{\varepsilon_0\, dA} = \frac{k \ln(s/a)}{s \varepsilon_0}$$

$s > b \qquad Q_{enc} = \displaystyle\int_a^b \rho\, dA \quad \ldots = 2\pi k \ln(b/a)$

$$E = \frac{Q_{enc}}{\varepsilon_0\, dA} = \frac{k \ln(b/a)}{s \varepsilon_0}$$

1.2.) $E(r,t) = E_0 \cos(k \cdot r - \omega t + \phi)$ show that

$k \perp E_0$ and $\dfrac{d\phi}{dt} = 0 \qquad B(r,t) = \dfrac{k \times E_0}{\omega} \cos(k \cdot r - \omega t + \phi)$

$$\nabla \times E = -\frac{\partial B}{\partial t} \Rightarrow \oint E \cdot dl = -\frac{\partial}{\partial t}\int B \cdot dA$$

$$\nabla \times E \qquad \nabla \times E_0 \cos(u), \quad u = k \cdot r - \omega t + \phi$$

$$\hookrightarrow = E_0 \sin(u)\, u', \quad u' = k \Rightarrow \nabla \times E = E_0 \times k \sin(k \cdot r - \omega t + \phi)$$

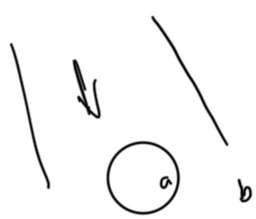
$$\nabla \times E) = -\frac{\partial B}{\partial t} = -\int E_0 \times k \sin(u)\, dt$$

$$\frac{du}{dt} = -\omega$$

$$\hookrightarrow \frac{E_0 \times k}{\omega} \cos u \Rightarrow \underline{B = \frac{E_0 \times k}{\omega} \cos(k \cdot r - \omega t + \phi)}$$

P1.3

$$J = k/s\, \hat{z} \qquad \nabla \times \frac{B}{\mu_0} = \epsilon_0 \frac{\partial E}{\partial t} + J$$



$$\oint B \cdot dl = \mu_0 I_{enc} \qquad s < a \Rightarrow B = 0$$

$a < s < b$: $\oint B \cdot dl = \mu I_{enc}$ $\qquad \int dl = 2\pi s$

$$I_{en} = \int_a^s J \cdot dA \qquad dA = ds \overset{2\pi s}{\boxed{}}$$

$$\hookrightarrow \int_a^s 2\pi k\, ds = 2\pi k(s-a) \rightarrow B(2\pi s) = 2\pi k(s-a)\mu_0$$

$$\underline{B = \frac{k\mu_0(s-a)}{s}}$$

$s > b$:

$$I_{enc} = \int_a^b 2\pi k\, ds = 2\pi k(b-a) \qquad B(2\pi s) = 2\pi k(s-a)\mu_0$$

$k\mu(b-a)$

$$B = \frac{\mu_0 \ldots}{s}$$

1.7 $\nabla^2 B - \frac{1}{c^2}\frac{\partial^2 B}{\partial t^2} = 0$ $\quad B = \frac{k \times E_0}{\omega}\cos(k\cdot r - \omega t + \phi)$

$$\nabla^2 B = \nabla^2 \cos(u) = -\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial}\sin(u)\right) = \cancel{\frac{\partial^2 u}{\partial r^2}\sin u} + \frac{\partial u}{\partial r}\frac{\partial u}{\partial r}\cos(u)$$

$\frac{\partial u}{\partial r} = k$

$\frac{\partial^2 u}{\partial r} = 0$

$$= -\frac{k \times E_0}{\omega} k^2 \cos(u) \qquad \frac{\partial^2 B}{\partial t^2} = -\frac{k \times E_0}{\omega}\omega^2 \cos(u)$$

$$-\frac{k \times E_0}{\omega} k^2 \cos(u) + \frac{(k \times E_0)\omega}{c^2}\cos(u) \qquad c = \frac{\omega}{k}$$

$$\hookrightarrow -k^2\, k \times E_0 \cos(u) + \frac{k^2\, k \times E_0}{\omega}\cos(u) = 0 \quad \checkmark$$

1.9 $E(r,t) = E_0 \cos(k(\hat{u}\cdot r - ct) + \Phi)$ is a sol'n to $\nabla^2 E - \mu_0 \varepsilon_0 \frac{\partial^2 E}{\partial t^2} = 0$

a.)

$$\nabla^2 E = -E_0 k^2 \hat{u}^2 \cos(u) \qquad \frac{\partial^2 E}{\partial t^2} = -k^2 c^2 E_0 \cos(u) \qquad \frac{1}{c^2} = \mu_0 \varepsilon_0$$

$\hat{u}^2 = 1$

$$-E_0 k^2 \cos u + E_0 \underbrace{\mu_0 \varepsilon_0 k^2 c^2}_{\cancel{\frac{1}{c^2}} k^2 \cancel{c^2}} \cos u = -E_0 k^2 \cos u + E_0 k^2 \cos u = 0 \quad \checkmark$$

b.)

$$k(\hat{u}\cdot r - ct) + \Phi = \lambda \leftarrow \text{const.}$$

$$\hat{u}\cdot r - ct = \frac{\lambda - \Phi}{k} \Rightarrow r = \frac{\lambda - \Phi}{k} + ct$$

c.) $(\hat{u}\cdot r - ct) + \phi = \lambda \qquad v = \Delta r / \Delta t$

$\hookrightarrow \quad (\hat{u}\cdot(r+\Delta r)-c(t+\Delta t))+\phi = \lambda$

$$\cancel{k}\ \hat{u}\cdot(r+\Delta r)-c(t+\Delta t))\cancel{+\phi} = \cancel{k}(\hat{u}\cdot r - ct)\cancel{+\phi}$$

$$\cancel{r}+\Delta r \cancel{-tc} - \Delta tc = \cancel{r}\cancel{-ct}$$

$$\frac{\Delta r - \Delta t c =}{\Delta t} \frac{0}{} = \frac{\Delta r}{\Delta t} = c \Rightarrow \underline{v = c}$$

d.) $\psi(r,t) = k(r - ct_0) + \phi$

$\psi_2 - \psi_1 = 2\pi \Rightarrow kr_2 - ct_0k + \phi - kr_1 + ct_0k - \phi = 2\pi$

$= k(r_2 - r_1) = 2\pi \qquad \lambda = r_2 - r_1$

$= k\lambda = 2\pi$

$$\lambda = \frac{2\pi}{k}$$

e.) $\nabla\cdot E = -\frac{\rho}{\varepsilon_0}$ show $E_0 \times \hat{u} = 1$ or $E_0\cdot\hat{u} = 0$

in a vaccum $\rho = 0 \Rightarrow \nabla\cdot E = 0$

$\left.\text{if } E_0\times\hat{u} = 1 \text{ ie } E_0 \perp \hat{u} \text{ then } \nabla\cdot E = 0\right\\} \ 0=0 \checkmark$