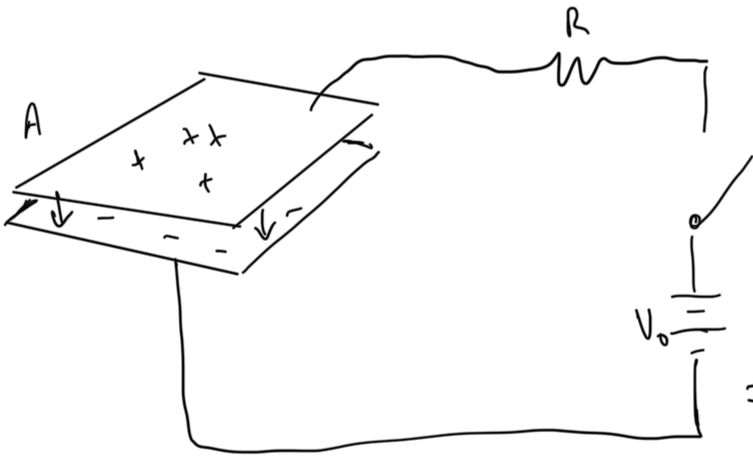
Faraday's Law

$$\oint E \cdot dA = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{A\varepsilon_0} = \frac{V}{d}$$

$$\hookrightarrow Q = \underbrace{\left(\frac{\varepsilon_0 A}{d}\right)}_{C} V$$



$$Q = CV$$

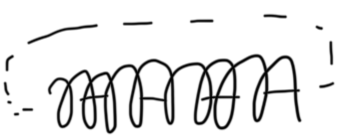
$$dW = dQ \cdot V$$

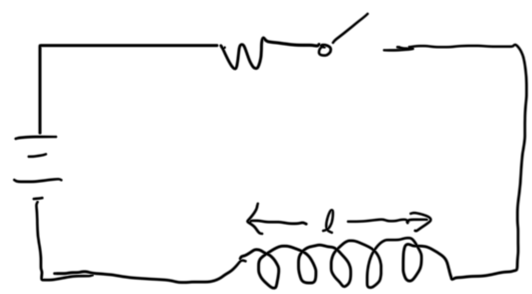
$$\hookrightarrow \int_0^{Q_{max}} \frac{dQ\, Q}{C}$$

$$\Rightarrow E = \frac{1}{2} C V^2$$

$$= \frac{1}{2}\left[\frac{\varepsilon_0 A}{d}\right]\left[E d\right]^2 =$$

$$\text{Energy} = \underbrace{\left[\frac{1}{2}\varepsilon_0 E^2\right]}_{\text{energy density}} \underbrace{(Ad)}_{\substack{\text{volume}\\ \text{of space}\\ \text{between plates}}}$$





$$\oint \vec{B} \cdot d\vec{l} = \mu I_{enc} \Rightarrow B \cdot L = \mu I_{enc}$$

$$\Rightarrow B = \frac{\mu N I}{L} = \mu n I$$

$$n = \frac{N}{L} \quad \frac{\#\text{turns}}{\text{length}}$$

$$I = \frac{B l}{\mu N}$$

$$dW = V dQ = \frac{d\Phi}{dt} \cdot dQ$$

$$\int \frac{d\Phi}{dt} \cdot dQ = \int L I \, dI$$

\# of loops

$$\Phi = N \int \vec{B} \cdot d\vec{A} = NBA = Nn\mu IA$$

$$\Phi = LI$$

$$\hookrightarrow \underbrace{[Nn\mu A]}_{L} I$$

or

$$[N^2 \ell \mu A] I$$

$$\mathcal{E} = \frac{d\Phi}{dt} = L\frac{dI}{dt}$$

$$E = L\frac{I^2}{2} \Rightarrow \frac{N^2 \mu A}{l} \cdot \frac{Bl}{\mu N} \cdot \frac{1}{2}$$

$$E = \underbrace{\frac{B^2}{2\mu}}_{\substack{\text{magnetic}\\ \text{energy}\\ \text{density}}} \underbrace{[A\ell]}_{\substack{\text{Volume enclosed}\\ \text{by inductor}}}$$